PHYS 481 Assignment 3: ODEs

Due: Thursday Oct 3 (23:00)

This assignment will introduce some ODE techniques. This material is discussed well in Chapters 22 and 23 of https://pythonnumericalmethods.berkeley.edu/notebooks/Index.html . Please use only numpy and matplotlib (not scipy).

Question 1

a. **[1 pt]** For the equation $y'' + \sin(t) + 1 = 0$ write down the set of first-order ODEs in terms of the state vector $S = [y, y']^T$ (see notes, or chapter 22). This is only about 1 or 2 lines. (Don't overthink it!)
b. **[8 pts]** Write a general function to integrate second-order ODEs using Euler's method. The function should accept a function $F = dS/dt$ (which depends on S and t), an interval, an initial value vector S and a number of steps to integrate. It should return the step points x_n and the solution to the ODE y_n at all the step points. Use it to integrate the ODE from part a on the interval $[0, \frac{\pi}{2}]$ with the initial values $y(0) = 0$ and $y'(0) = 1$. Compare the results using different numbers of steps and compare to the analytic result $y = c_1 + c_2 x - \frac{x^2}{2} + \sin(x)$.
c. **[8 pts]** Repeat part b but use RK4 integration instead of Euler's method.
d. **[1 pt]** Using Euler's method in part b, each step yields a new value (at t_n) that is systematically too large. Why? [Hint: What order of accuracy is Euler's method? Where does the error come from? Does that error have a consistent sign?]

Question 2

a. **[8 pts]** For $y'' + \sin(t) + 1 = 0$ (same ODE as in question 1) and the boundary values $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 1$, use a shooting method to solve the equation on the interval $[0, \frac{\pi}{2}]$. Compare to the analytic result.
b. **[8 pts]** Repeat question a using a finite difference method with a varying number of grid points (5,10,20,50).

Question 3 [8 pts]

The deflection $y(x)$ of a 1-D beam is governed by the following ODE:

$$EI\frac{d^2y}{dx^2} = \frac{1}{2}\omega_0(Lx - x^2)\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$$

where EI is a parameter known as the "flexural rigidity" that depends on the material and its cross-section, L is the length of the beam and ω_0 is the load per unit length applied to the beam. If $L = 5$ m, $EI = 1.8 \times 10^7$ Nm^2, $\omega_0 = 1.5 \times 10^4$ N/m and $y(0) = y(L) = 0$, find the beam deflection $y(x)$ on the interval $[0, L]$. [https://pythonnumericalmethods.berkeley.edu/notebooks/chapter23.06-Summary-and-Problems.html question 10]

Rubric

Question 1a is graded as correct/incorrect (1/0) with no part marks. Question 1d is graded as correct/partially correct/incorrect (1/0.5/0).

All other questions are worth 8 points each, assessed according to the following rubric. An example of a “minor error” in the 1-pt categories is if the code is commented, but not clearly, or the plot is missing a unit on one axis.

<table>
<tr><td rowspan="3">Code</td><td>Commenting: Clear and concise comments explaining the code.</td><td>1 pt</td><td rowspan="5">0: Missing or major error.
0.5: Minor error.
1: Correct.</td></tr>
<tr><td>Logical Structure: Code is logically organized into functions and modules.</td><td>1 pt</td></tr>
<tr><td>Readability: Code is well-formatted with consistent and easily understood naming conventions.</td><td>1 pt</td></tr>
<tr><td rowspan="3">Plot(s)</td><td>Clarity: Plot is clear and easy to understand.</td><td>1 pt</td></tr>
<tr><td>Labels and Units: Proper labels and units are included on all axes.</td><td>1 pt</td></tr>
<tr><td>Correctness: Plot shows the expected outcome of the question.</td><td>3 pts</td><td>0: Plot is missing or entirely incorrect.
1: Plot shows evidence of major conceptual errors.
2: Plot shows evidence of minor errors in the analysis.
3: Correct answer.</td></tr>
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