PHYS 481 Assignment 6: Random Walks

Due: Sunday Nov 10 (23:00)

AI policy for this assignment: no use of generative AI tools is allowed.

Question 1

A grain of pollen suspended in a liquid undergoes Brownian motion. In each time interval $\mathrm{d}t$, assume it has an equal probability of moving a distance ℓ to the left or ℓ to the right (an admittedly simplistic model). Model this as a random walk and simulate an ensemble of several thousand trials. Plot a histogram of the normalized displacement x/ℓ after 100 $\mathrm{d}t$, 500 $\mathrm{d}t$, 1000 $\mathrm{d}t$ and 10000 $\mathrm{d}t$. Include the analytic probability on the graph using the Central Limit Theorem.

Question 2

Repeat question 1 but assume the probability of moving to the right in time interval $\mathrm{d}t$ is 60% (instead of 50% as in question 1). Plot a histogram of the normalized displacement after 100 $\mathrm{d}t$, 500 $\mathrm{d}t$, 900 $\mathrm{d}t$ and 1300 $\mathrm{d}t$. Include the analytic probability on the graph using the Central Limit Theorem.

Question 3

A more realistic approximation for Brownian motion is that in each time interval $\mathrm{d}t$, the pollen moves a distance d that obeys a Gaussian distribution. Assuming the Gaussian distribution for the step length has $\mu = 0.2\ell$ and $\sigma = \ell$, plot a histogram of the normalized displacement after 100 $\mathrm{d}t$, 500 $\mathrm{d}t$, 900 $\mathrm{d}t$ and 1300 $\mathrm{d}t$, including the analytic probability.

Question 4

Let's say the step distance d is instead chosen from the intentionally arbitrary function

$$d/\ell = (2u - 1)^3 + 0.2$$

where u is uniformly distributed on [0,1]. Start by generating a large sample of step lengths d/ℓ. Plot a histogram of the step length and calculate the mean and variance. Then plot a histogram of the normalized displacement after 100 $\mathrm{d}t$, 500 $\mathrm{d}t$, 900 $\mathrm{d}t$ and 1300 $\mathrm{d}t$, including the analytic probability. (using your estimated mean and variance).

Rubric

Each question results in a plot and uses the same 8-point rubric as on previous assignments. An example of a “minor error” in the 1-pt categories is if the code is commented, but not clearly, or the plot is missing a unit on one axis.

Code	**Commenting**: Clear and concise comments explaining the code.	1 pt	0: Missing or major error. 0.5: Minor error. 1: Correct.
	Logical Structure: Code is logically organized into functions and modules.	1 pt	
	Readability: Code is well-formatted with consistent and easily understood naming conventions.	1 pt	
Plot(s)	**Clarity**: Plot is clear and easy to understand.	1 pt	
	Labels and Units: Proper labels and units are included on all axes.	1 pt	
	Correctness: Plot shows the expected outcome of the question.	3 pts	0: Plot is missing or entirely incorrect. 1: Plot shows evidence of major conceptual errors. 2: Plot shows evidence of minor errors in the analysis. 3: Correct answer.