Population Growth with Limited Resources

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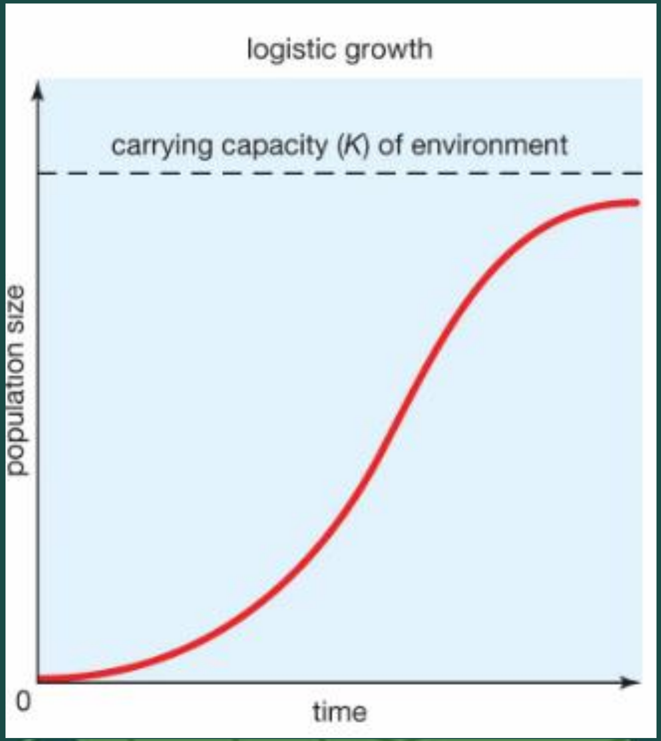
Population Dynamics

- Study of change in population over time.
- Interaction of populations with their environment.
- By limiting the resources available, we can model realistic population growth
- Sources of variation include seasons, natural disasters, disease.



[1] https://www.worldatlas.com/r/w960-q80/upload/9f/4d/df/shutterstock-231214222.jpg [2] https://www.britannica.com/science/population-ecology/Logistic-population-growth.jpg

Models for Population Growth

- <u>Exponential Growth Model</u>
- Unlimited resources
- Population grows unchecked
- Unrealistic

- <u>Logistic Growth Model</u>
- Limited resources
- Growth levels off at carrying capacity, K.
- Realistic

The Logistic Equation

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

r is the growth rate.

P is the population.

K is the carrying capacity.

[3] https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/08%3A_Introduction_to_Differential_Equations/8.04%3A_The_Logistic_Equation

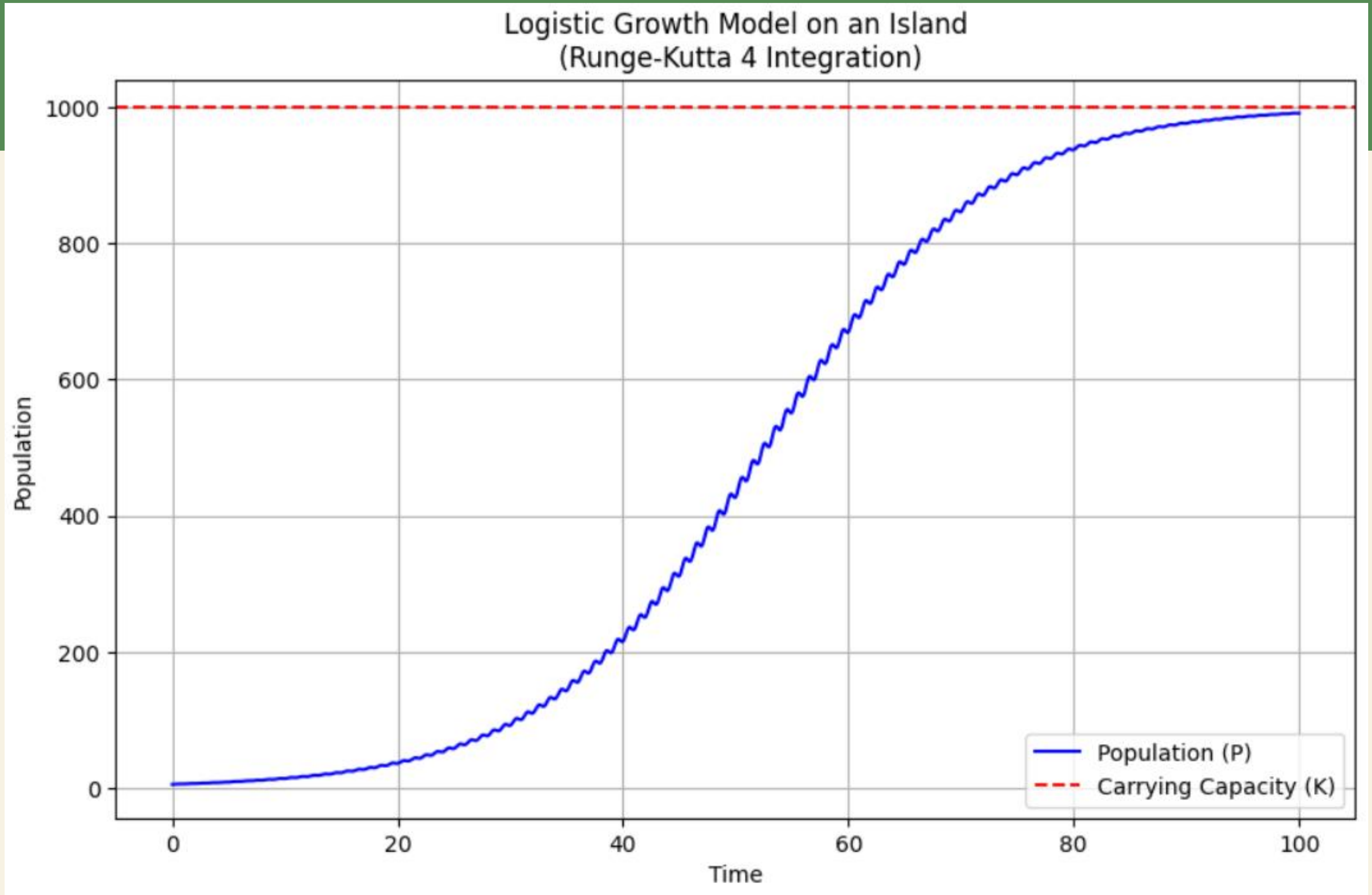
Logistic Growth Equation with Runge-Kutta-4 Algorithm

```python
# Function representing the derivative dP/dt
def dP_dt(P,r, K):
    return r * P * (1 - P / K)

def logistic_DE(r0, P0, K, T):

    dt = 0.01
    time_steps = int(T / dt)
    t = np.linspace(0, T, time_steps)

    r = r0*(1+(np.sin(2*np.pi*t)))

    # Initialize population array
    P = np.zeros(time_steps)
    P[0] = P0

    # Runge-Kutta 4th Order Method
    for i in range(1, time_steps):
        k1 = dP_dt(P[i-1], r[i-1], K)
        k2 = dP_dt(P[i-1] + 0.5 * k1 * dt, r[i-1], K)
        k3 = dP_dt(P[i-1] + 0.5 * k2 * dt, r[i-1], K)
        k4 = dP_dt(P[i-1] + k3 * dt, r[i-1], K)
        P[i] = P[i-1] + (k1 + 2 * k2 + 2 * k3 + k4) * dt / 6

    return t, P, K

t, P, K = logistic_DE(0.2, 5, 1000, 100)
```

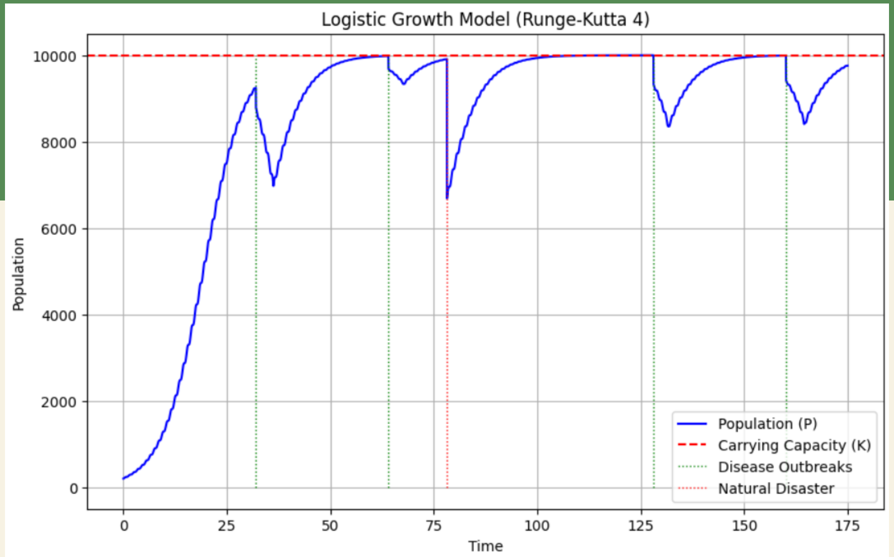
Methods

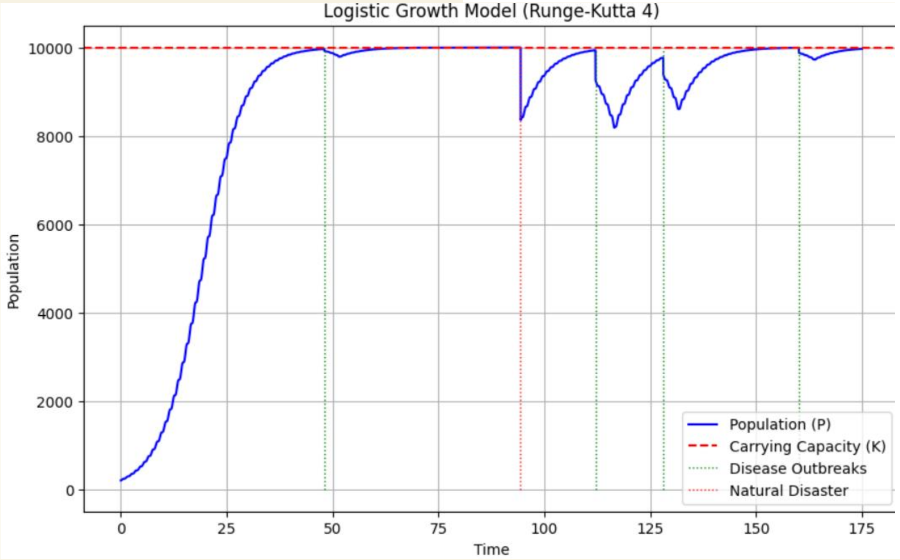
- We use Runge-Kutta (4^{th} order) to solve this DE, and added in our variations that affect r and/or P.
- Seasons: $r = r_0(1 + \sin(\omega t)) = r_0 + r_0 \sin(\omega t)$
- Diseases: Correlated with population, variable amount of decreases
- Disasters: Happen at random intervals, variable amount of decrease

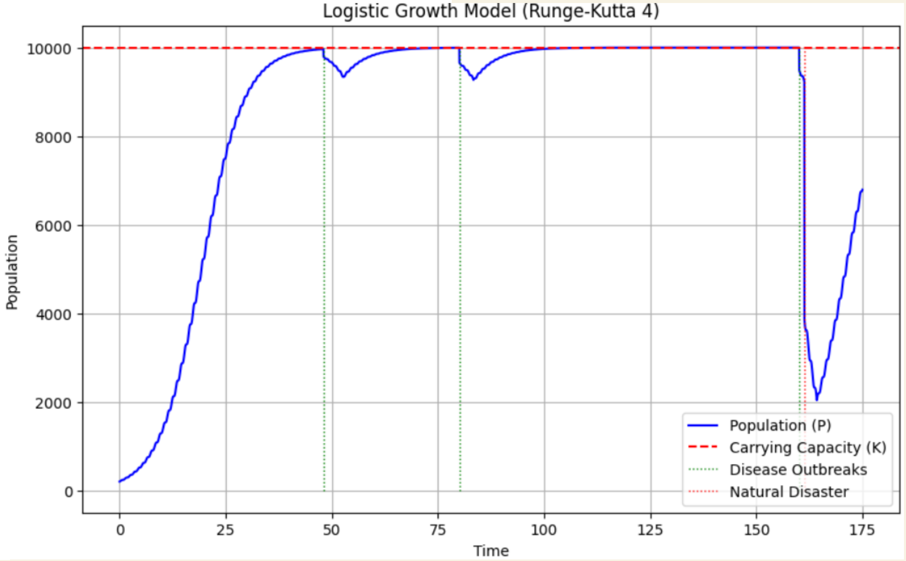
Limited “Island” Experiment



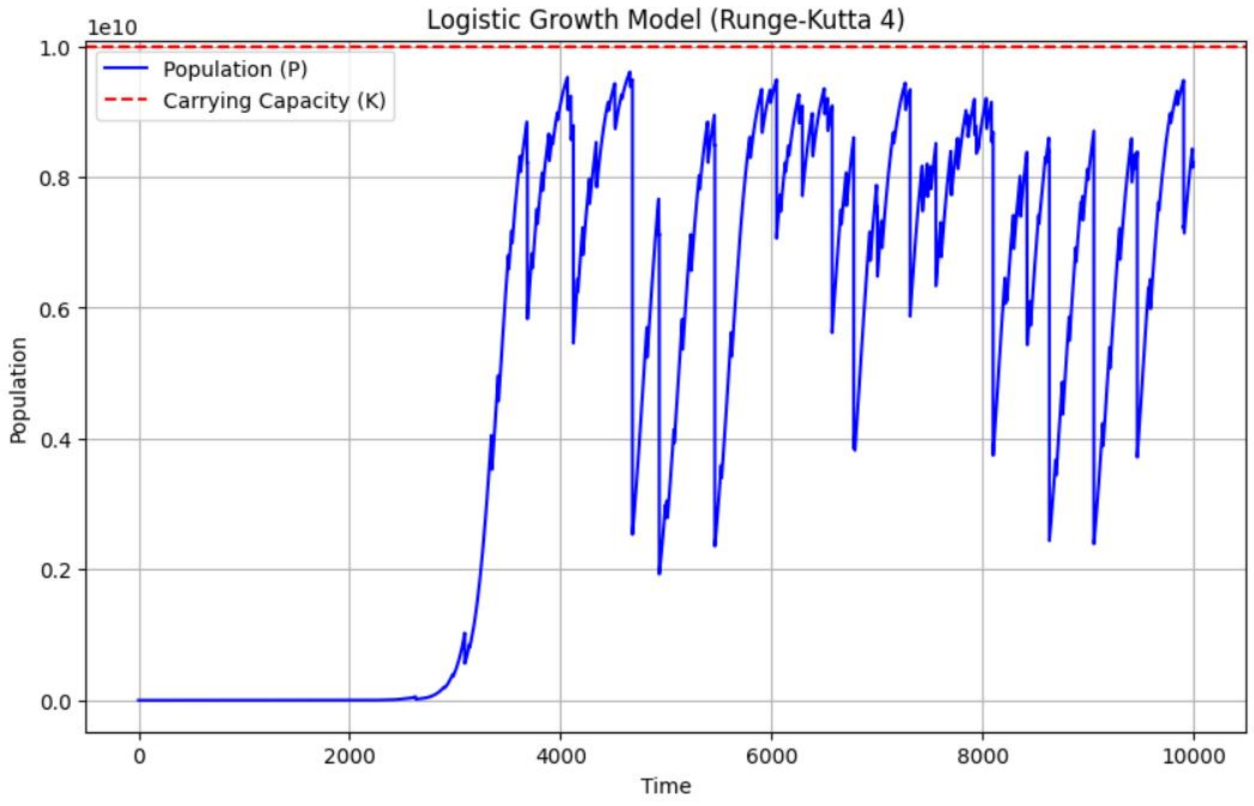
Adding in Disease and Disaster Dynamics

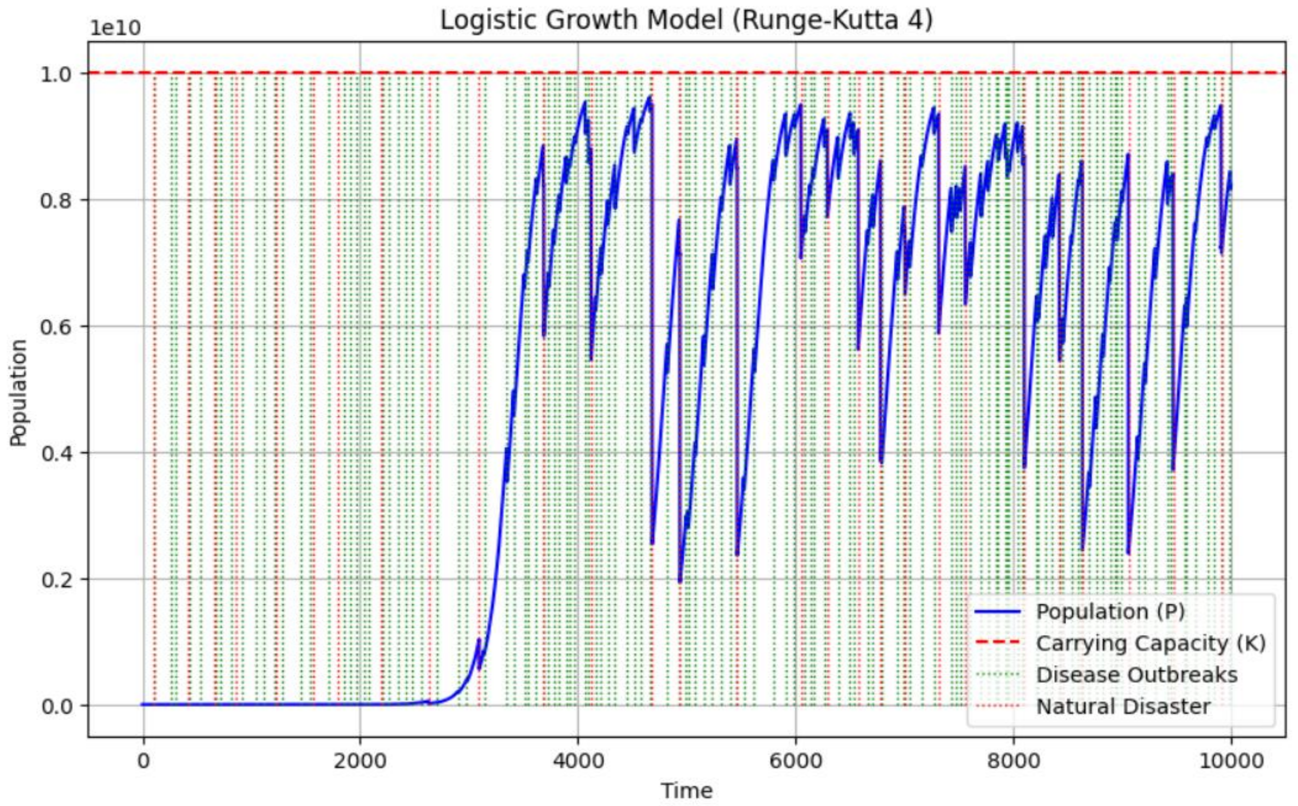






Increasing Carrying Capacity to 10 billion





References

[1] World Atlas. (Unknown). https://www.worldatlas.com/r/w960-q80/upload/9f/4d/df/shutterstock-231214222.jpg

[2] Population Ecology. (2024, September 13). https://www.britannica.com/science/population-ecology/Calculating-population-growth

[3] The Logistic Equation. (2024, August 17). https://math.libretexts.org/@go/page/2559

[4] One Planet, How Many People? A Review of Earth's Carrying Capacity. (2012, June). https://na.unep.net/geas/archive/pdfs/GEAS_Jun_12_Carrying_Capacity.pdf