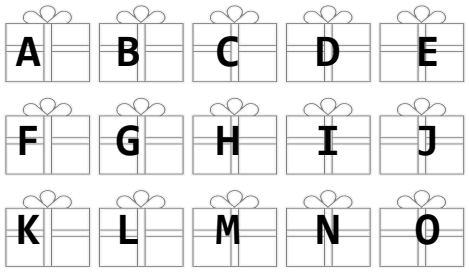
PHYS 481 Assignment 5: Monte Carlo Methods

Due: Sunday Oct 27 (23:00)

AI policy for this assignment: no use of generative AI tools is allowed.

Question 1 [8 pts]

Andrew and Barbara are playing a game, in which 15 boxes are arranged in a grid as shown below.



Prizes are put into **two (2)** randomly chosen boxes. Andrew will search the boxes row by row, so his search order is ABCDEFGHIJKLMNO. Barbara will search column by column, so her order is AFKBGLCHMDINEJO. Andrew and Barbara open their boxes together on each turn; that is, on the first turn they both open A, on the second Andrew opens B while Barbara opens F, and so on.

Who is more likely to find a prize first? Or are they both equally likely? [Question: Timothy Chow, reprinted in The Guardian, 2024]

a. Take a guess. (No points, just for fun. It's not very intuitive.)
b. **[4 pts]** Write a Python code to simulate running the experiment at least 100 thousand times. Find the 3 probabilities (i.e. Andrew wins, Barbara wins, or tie) with uncertainty estimates. Report your results as a table.
c. **[4 pts]** There is a finite number of possible games (i.e. possible ways the prizes can be distributed). Simulate them all to find the exact odds of the 3 outcomes. Report your results as a table.

Question 2 [4 pts]

Use Monte Carlo sampling to estimate the value of the following definite integral:

$$\int_0^{0.5} \sin\left(e^{3x^2+1}\right) \mathrm{d}x$$

Solve to at least 3 significant figures and give the uncertainty in your answer. Report your results clearly (the TA must be able to clearly tell what your final answer is; don't just bury it somewhere in your code).

[NOTE: Monte Carlo methods are not efficient for simple 1d integrals like this. The point of this question is to set up techniques for more complicated problems.]

Question 3 [8 pts]

Use Monte Carlo sampling to estimate the volume of a unit hypersphere of dimension n up to n =15. Plot the volume versus n including 2σ error bars and include the analytic result on the plot. The analytic result in recursive form is:

$$V_n = \begin{cases} 1 & if\ n = 0 \\ 2 & if\ n = 1 \\ \frac{2\pi}{n} V_{n-2} & otherwise \end{cases}$$

Choose the number of Monte Carlo iterations for each simulation so that the error bars are small but at least some are visible on the graph.

Question 4 [9 pts]

Define the radius r in n dimensions as $r = \sqrt{\sum_{i=1}^{n} x_i^2}$.

a) **[4 pts]** Calculate the integral

$$\int r(x_i) d^n x$$

over the unit hypercube (extending from 0 to 1 in each dimension) in 3, 4, 10 and 100 dimensions using a simple Monte Carlo integration. Report the standard uncertainty for each estimate, and make sure it's less than 0.1% of the value. Display your results as a table.

b) **[4 pts]** Repeat part a over the cube extending from 0 to 2 in each dimension.

c) **[1 pt]** If you tried to do part a using a more conventional numerical integration scheme (e.g. rectangle rule or midpoint rule) sampled on a regular grid, what problem would you run into?

Rubric

Most questions on this assignment are graded out of 4 points and have an answer that is expressed as a short table of results. The rubric for those questions is:

Code	**Commenting and readability**: Clear and concise comments explaining the code. Code is well-formatted with consistent and easily understood naming conventions.	1 pt	0: Missing or major error. 0.5: Minor error. 1: Correct.
	Logical Structure: Code is logically organized into functions and modules.	1 pt	
Output	**Correctness**: Table shows the expected outcome of the question.	2 pts	0: Answer missing or entirely incorrect. 1: Incorrect, but basically on the right track. 2: Correct .

Question 2 results in a plot and uses the same 8-point rubric as on previous assignments. An example of a "minor error" in the 1-pt categories is if the code is commented, but not clearly, or the plot is missing a unit on one axis.

Code	**Commenting**: Clear and concise comments explaining the code.	1 pt	0: Missing or major error. 0.5: Minor error. 1: Correct.
	Logical Structure: Code is logically organized into functions and modules.	1 pt	
	Readability: Code is well-formatted with consistent and easily understood naming conventions.	1 pt	
Plot(s)	**Clarity**: Plot is clear and easy to understand.	1 pt	
	Labels and Units: Proper labels and units are included on all axes.	1 pt	
	Correctness: Plot shows the expected outcome of the question.	3 pts	0: Plot is missing or entirely incorrect. 1: Plot shows evidence of major conceptual errors. 2: Plot shows evidence of minor errors in the analysis. 3: Correct answer.

Question 4c is graded out of one point as correct/incorrect.