PHYS 481 Assignment 8: Poisson's Equation

Due: Friday Dec 6 at 23:00

AI policy for this assignment: no use of generative AI tools.

Question 1 [8 pts]

[Griffiths example 3.4] Two infinitely long grounded metal plates lie parallel to the xz plane, one at $y = 0$, the other at $y = a = 5$ cm. They are connected at $x = \pm b = \pm 4$ cm by narrower metal strips maintained at a constant potential $V_0 = 4$ Volts to form a rectangular channel. A thin layer of insulation at each corner allows the sides to be maintained at different potentials.

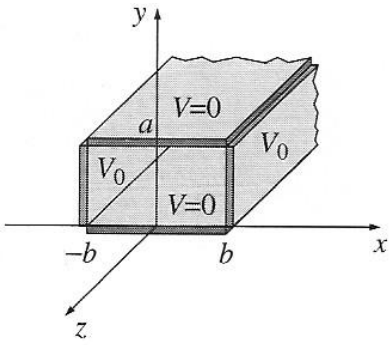

Figure 3.20

An analytic solution to this boundary value problem can be obtained through separation of variables as:

$$V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5,\ldots} \frac{1}{n} \frac{\cosh(n\pi x/a)}{\cosh(n\pi b/a)} \sin(n\pi y/a)$$

Plot a contour plot of the analytic result, using contours from 0 to 4V in steps of 0.1V. Label the contour lines legibly.

WARNING: you will need a very large number of terms to get an accurate answer (order 10^3 or so), and the higher-order terms involve the ratio of two extremely large numbers. You will need to rearrange the analytic result into a more tractable form to prevent overflow.

Question 2 [8 pts]

Repeat Q1 numerically using Jacobi iteration. Plot a contour plot of the potential and compare to the analytic result (i.e. contour plots of both the numerical solution and the absolute difference between the numerical and analytic solutions).

Question 3 [8 pts]

Consider question 2 again, except with a uniform charge density filling the space between the plates. Use 3 different charge densities: $\rho = 1\ \mathrm{nC/m^3}$, $100\ \mathrm{nC/m^3}$, and $1000\ \mathrm{nC/m^3}$. There is no need to find an analytic solution for comparison.

Question 4 [8 pts]

Repeat question 3 with $\rho = 100\ \mathrm{nC/m^3}$ using a direct matrix solution rather than Jacobi iteration. Compare your result with the result from question 2.

Rubric

Each question results in a plot or animation and uses the same 8-point rubric as on previous assignments. An example of a “minor error” in the 1-pt categories is if the code is commented, but not clearly, or the plot is missing a unit on one axis.

Code	**Commenting**: Clear and concise comments explaining the code.	1 pt	0: Missing or major error. 0.5: Minor error. 1: Correct.
	Logical Structure: Code is logically organized into functions and modules.	1 pt	
	Readability: Code is well-formatted with consistent and easily understood naming conventions.	1 pt	
Plot(s)	**Clarity**: Plot is clear and easy to understand.	1 pt	
	Labels and Units: Proper labels and units are included on all axes.	1 pt	
	Correctness: Plot shows the expected outcome of the question.	3 pts	0: Plot is missing or entirely incorrect. 1: Plot shows evidence of major conceptual errors. 2: Plot shows evidence of minor errors in the analysis. 3: Correct answer.