PHYS 481 Assignment 1: Numerical differentiation

Due: Thursday Sept 12 (by 23:00)

AI policy for this assignment: no use of generative AI tools is allowed.

Code should be clean, well-commented and readable. Use functions where appropriate to break your code into small, logical pieces. Avoid having too much code outside of functions; a few lines is OK, but long blocks are not. The template file for this assignment will help you match the expected standards. Please also see the rubric at the end of this document. The green "tips" boxes will help make sure your code is similar to the code in the expected solution, which will make the quizzes easier.

1. This assignment will use publicly available data from NASA for THEMIS-D (aka THEMIS-P3), one of 5 research satellites launched in 2007 to study Earth's magnetosphere. THEMIS-D has a highly elliptical, near-equatorial orbit with a period of about 1 day [https://nssdc.gsfc.nasa.gov/nmc/spacecraft/display.action?id=2007-004D]. Download the position of THEMIS-D during the day 2023/07/31 from the site https://sscweb.gsfc.nasa.gov/cgi-bin/Locator.cgi and plot the satellite's orbit.

Select the satellite and enter the stop/start times on the start page. Under "Output Options", use "GEI/TOD XYZ" coordinates (Geocentric Equatorial Inertial: see https://sscweb.gsfc.nasa.gov/users_guide/Appendix_C.shtml for a definition of the coordinate system). When you hit "Submit query and wait for output", the site will generate a (large) data table that you can copy and paste into a text file.

Plot the orbits, first with the 3 components of position as a function of time and then as position projected to the XY and XZ planes. Use units of Earth radii (R_E) for position and hours for time to make the plot more readable. Include a circle for the Earth on the XY and XZ plots.

Tips: Use only the numpy and matplotlib packages for the entire assignment. Use the open() function and the readline() method to read the file. Use text methods like split() to parse the file. Convert the data to numpy array(s) before plotting. Use matplotlib.pyplot to plot the data.

2. Check conservation of energy (gravitational potential+kinetic energy per unit mass) using the data, assuming a simple 1/r gravitational potential for the Earth and using a forward finite difference scheme to compute the kinetic energy.

Be careful of the sign on the potential and use at least 5 significant digits for any physical constant. You may drop the edge points after differencing.

a. Plot the kinetic energy per unit mass, the potential energy per unit mass, and their sum (the total energy per unit mass) on the same plot as a function of time.
b. Plot just the total energy per unit mass (KE+PE) on its own plot.

Tip: Use vectorized numpy operations like:

```
a=np.sin(b[1:,:])*4
```

3. Repeat question 2 using 3-point, 5-point and 7-point central finite difference estimators to compute the kinetic energy. Plot just the total energy per unit mass (KE+PE), not the individual parts (KE and PE).
4. The Earth is not exactly spherical (mostly because it rotates), and the gravitational potential is therefore not strictly the same as for a point mass (a monopole). A more accurate representation for the Earth's gravitational potential is

$$\phi = -\frac{\mu}{r} + \frac{J_2}{2r^5}(3z^2 - r^2)$$

with μ =3.98600440× 10^{14} m^3/s^2, J_2 =1.75553× 10^{25} m^5/s^2, r the radial distance and z the distance above the equator (z in the GEI system). Plot the total energy per unit mass (KE+PE) using the 5-point stencil.

In the "A1" folder on D2L:

- This file
- A Gradescope link for submitting the answer
- a1_template.ipynb: A Jupyter template to help you get started. Includes the questions, tips, etc. You can use this as a starting point for your assignment.

What to turn in

Please submit a single Jupyter (.ipynb) or PDF file to Gradescope containing:

Part I [32 pts]: Code and resulting plots for questions 1-4

Part II [12 pts]: Written responses to the following (½ page or less for each question):

1. [4 pts] Convince me that your plots in question 1 show a plausible orbit for THEMIS-D (give 3 good reasons).
2. [4 pts] Is the dominant systematic error when using forward finite differencing (in question 2) due to numerical error or model error? Why?
3. [4 pts] Does the data support the conclusion that the Earth's mass distribution is slightly non-spherical?

Rubric

Part I:

Each question is worth 8 points, assessed according to the following rubric. An example of a "minor error" in the 1-pt categories is if the code is commented, but not clearly, or the plot is missing a unit on one axis.

Code	**Commenting**: Clear and concise comments explaining the code.	1 pt	0: Missing or major error. 0.5: Minor error. 1: Correct.
	Logical Structure: Code is logically organized into functions and modules.	1 pt	
	Readability: Code is well-formatted with consistent and easily understood naming conventions.	1 pt	
Plot(s)	**Clarity**: Plot is clear and easy to understand.	1 pt	
	Labels and Units: Proper labels and units are included on all axes.	1 pt	
	Correctness: Plot shows the expected outcome of the question.	3 pts	0: Plot is missing or entirely incorrect. 1: Plot shows evidence of major conceptual errors. 2: Plot shows evidence of minor errors in the analysis. 3: Correct answer.

Part II:

Q1. 4 points:

- Overall text is clear and concise [0/0.5/1 pt]
- 3 clear reasons are given [0/1 pt for each reason]

Q2 and Q3. 4 points each:

- Overall text is clear and concise [0/1/2 pt]
- Answer is correct [0/1/2 pt]