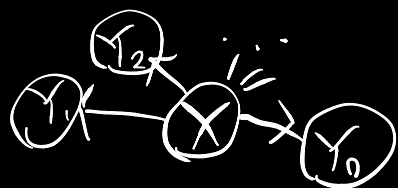
Problem 2: Exercise 6.5 from A.D.

Consider a naive bayes structure...



(a) What is the width of variable order $Y_1, \dots, Y_n, X$?

In this ordering our width is 1

This is because we will process $(X, Y_1), (X, Y_2), \dots, (X, Y_n)$ into individual buckets until all combined in X.

(b) What is the width of variable order $X, Y_1, Y_2, \dots, Y_n$

In this ordering our width is n.

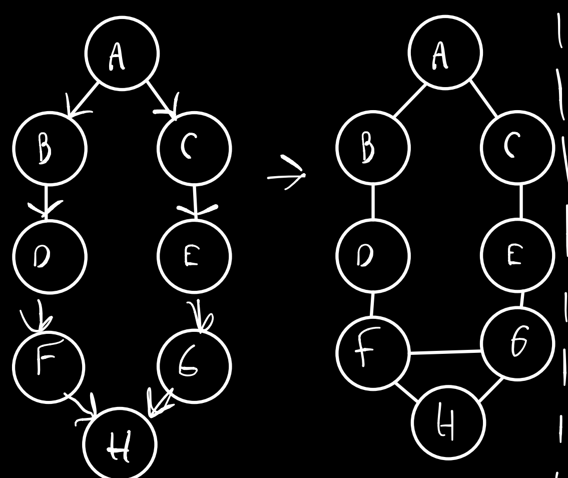
This is because our factors are $(X, Y_1), (X, Y_2), \dots, (X, Y_n)$ so then we would have a bucket of size n

Problem 3: Exercise 6.9 from A.D.

Compute elimination order for the variables in Figure 6.11 using min-degree method. In case of a tie choose variables that come first alphabetically

G:

Interaction Graph:

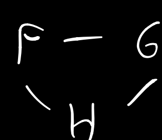


Min-degree order:

remove A, now network is

remove B, C, D, E in same way

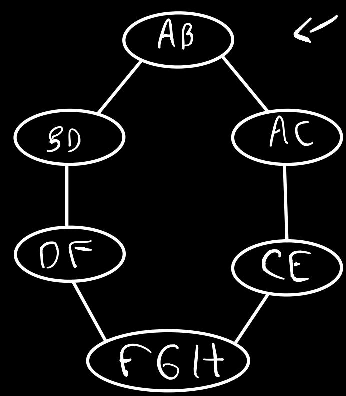
remove F, G, H now, all same degrees



Thus, our min-degree ordering is A, B, C, D, E, F, G, H

Problem 4: 6.10 in A.D.

What is the treewidth in the following network?



← treewidth = 2

Same markov network as problem 2.

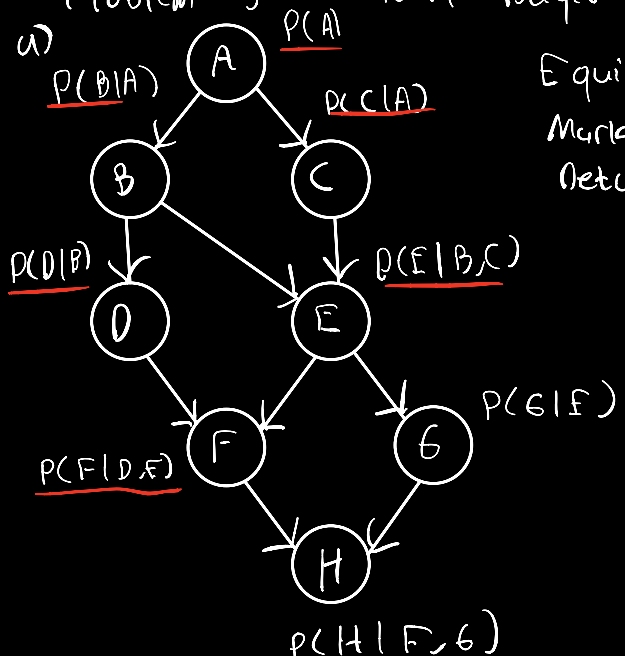
Attempt to show treewidth < 2,

unable to because FGH is a clique, must contain all 3 variables.

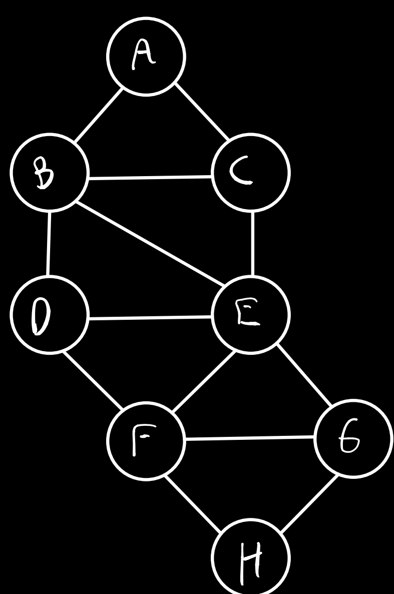
Thus, treewidth is 2 on this graph

Problem 5: Convert Bayes net to markov network...

a)

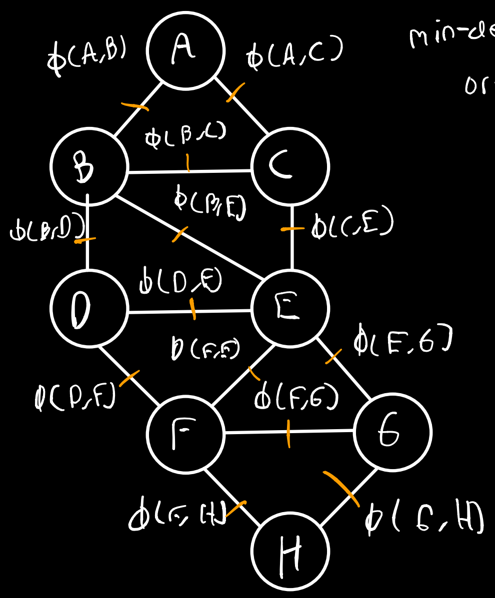


Equivalent Markov Network:

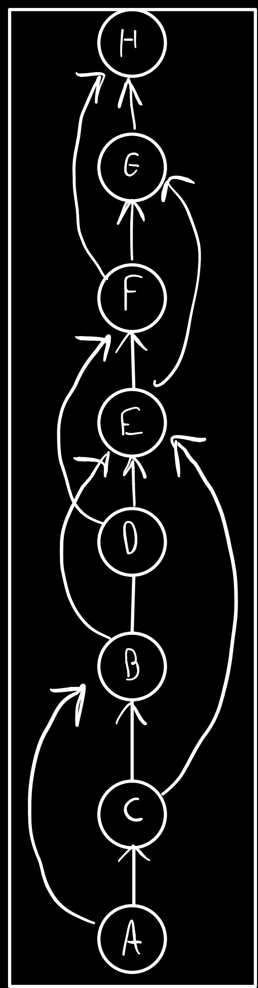


Equivalent Bayesian Network:

Markov network is already Chordal, a perfect elimination ordering will yield a pmap.



min-degree ordering : A, C, B, D, E, F, G, H ← max width=3



b)

Let H be evidence variables. Trace the operations of bucket elimination for computing P(H=h)

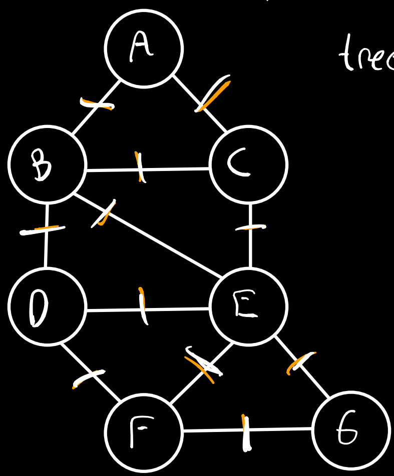
A	$P(A)P(B\mid A)P(C\mid A) \rightarrow f(B,C)$
E	$P(E\mid B,C)\ P(G\mid E)\ P(F\mid D,E) \rightarrow f(B,C,D,F,G)$
B	$f(B,C)\ f(B,C,D,F,G)\ P(D\mid B) \rightarrow f(C,D,F,G)$
C	$f(C,D,F,G) \rightarrow f(D,F,G)$
D	$f(D,F,G) \rightarrow f(F,G)$
F	$f(F,G) \rightarrow f(G)$
G	$f(G)$

$$P(H) = \frac{1}{Z} \sum_G \sum_F \sum_D \sum_C \sum_B P(D \mid B) \sum_E P(E \mid B, C) P(G \mid E) P(F \mid D, E) \sum_A P(A) P(B \mid A) P(C \mid A)$$

c)

Is the ordering (A, E, B, C, D, F, G) optimal? What is the treewidth of this network with H as evidence?

Let us first try to determine treewidth...



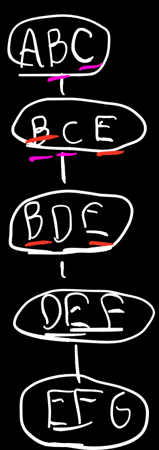
treewidth

In this decomposition, we can see that our treewidth is 2

This tells us that our algorithm is not optimal, because it had width 5.

d)

Show how junction tree algorithm will work:



① select ABC as root

② Pass to DEF $\sum_G \phi(G, E) \rightarrow \psi(E)$

③ Pass to BDE $\sum_F \phi(D, E, F) \rightarrow \psi(D, E)$

④ Pass to BCE $\sum_D \psi(D, E) \phi(B, D) \rightarrow \psi(B, E)$

⑤ Pass to ABC $\sum_E \psi(B, E) \phi(B, C, E) \psi(E) \rightarrow \psi(B, C)$

⑥ Pass to BCE $\sum_A \phi(A) \phi(A, B) \phi(A, C) \rightarrow \psi_2(B, C)$

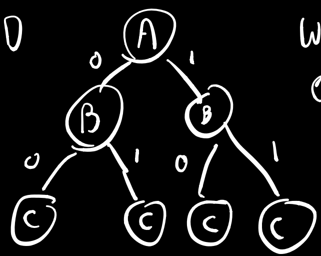
⑦ Pass to BDE $\sum_C \psi(B, C) \psi(B, C) \rightarrow \psi(B)$

⑧ Pass to DEF $\sum_B \psi(B)$

(9) Done!

Problem 6)

a) Say we are given a factor $\phi(A,B,C)$. In order to represent this, we can make a tree-CPD



which will represent our entire factor

Our leaves here represent the value of factor given the context of the path. EX: $(A=0, B=0, C=0)$

b) Instantiate_Tree(Context U, Tree Factor $\phi(Y)$):

For each context u in U:

V ← Traverse down $\phi(Y)$ to the left, return node if context node = u_{node}

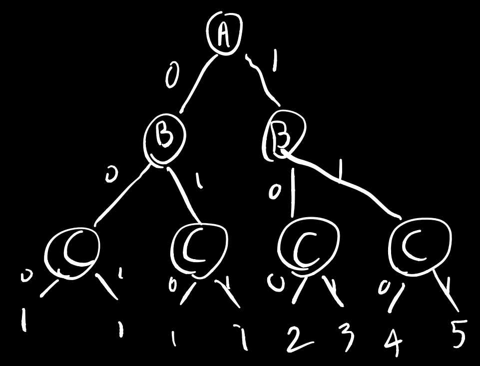
replace node V with subtree after we transition V.transition(u_{val})

repeat V ← node to the horizontal right of V until NULL

replace node V with subtree after we transition V.transition(u_{val})

return $\phi(Y)$

c) Say we had a tree-CPD



in this case, we can remove the entire left side of the tree, giving us: