Homework 2 Automata Theory

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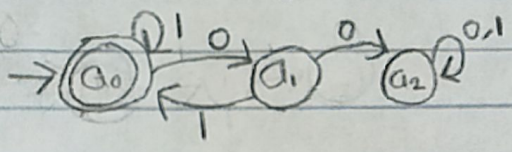
1) a) $L_1 = \{w \in \{0,1\}^* \mid 00 \text{ is not a substring of } w\}$

$L_2 = \{w \in \{0,1\}^* \mid w \text{ ends with } 01\}$

$L_1 = (Q_1, \Sigma, \delta_1, q_{s_1}, F_1)$



δ_1	1	0
a_0	a_0	a_1
a_1	a_0	a_2
a_2	a_2	a_2

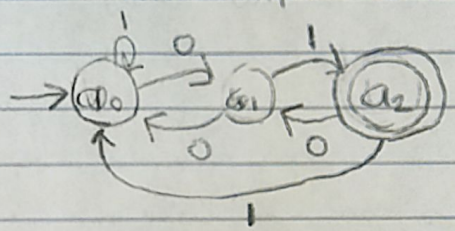
$Q_1 = \{a_0, a_1, a_2\}$

$\Sigma = \{0,1\}$

$q_{s_1} = a_0$

$F_1 = \{a_0\}$

L_2: $L_2 = \{Q_2, \Sigma, \delta_2, q_{s_2}, F_2\}$



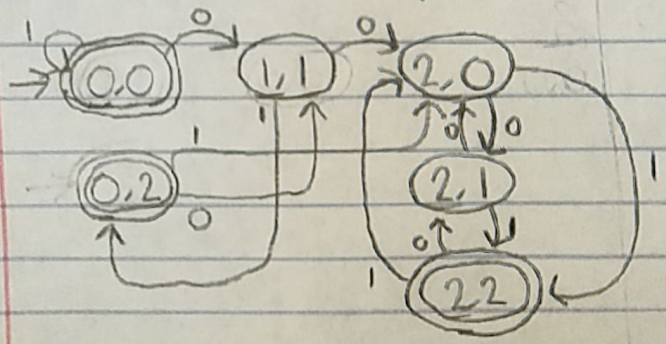
$Q_2 = \{a_0, a_1, a_2\}$

$\Sigma = \{0,1\}$

$q_{s_1} = a_0$

$F_2 = \{a_2\}$

b) $L_3 = (Q_3, \Sigma, \delta_3, q_{s_3}, F_3)$



δ_3	0	1
0,0	1,1	0,0
1,1	2,0	0,2
2,0	2,1	2,0
0,2	1,1	0,0
2,1	2,0	2,2
2,2	2,1	2,0

$q_{s_3} = (0,0)$

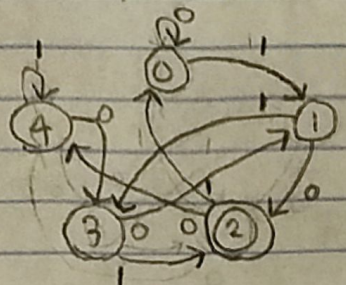
$F_3 = \{(0,0), (0,2), (2,2)\}$

c) $F_{L_1 \cap L_2} = \{(0,2)\}$

d) $F_{L_1 - L_2} = \{(0,0)\}$

2) $L = \{w \in \{0,1\}^* \mid w \equiv_5 2, \text{ when } w \text{ is treated as a binary number}\}$

$L = (Q, \Sigma, \delta, q_0, F)$



δ	0	1
0	0	1
1	2	3
2	4	0
3	1	2
4	3	4

$\Sigma = \{0,1\}$

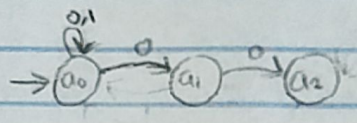
$q_0 = 0$

$F = \{2\}$

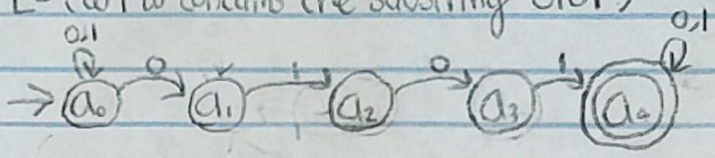
$Q = \{0,1,2,3,4\}$

3.) $\Sigma = \{0,1\}$

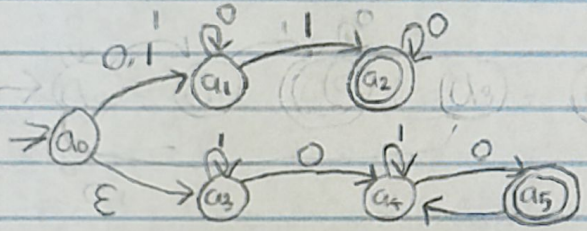
a) $L = \{w \mid w \text{ ends with } 100\}$



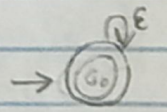
b) $L = \{w \mid w \text{ contains the substring } 0101\}$



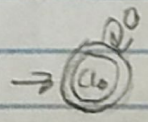
c) $L = \{w \mid w \text{ contains an even number of 0s, or exactly two 1s}\}$



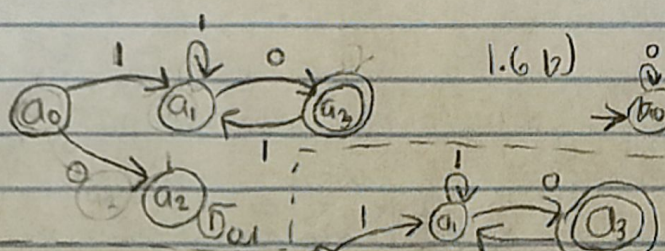
g) $L = \{\varepsilon\}$



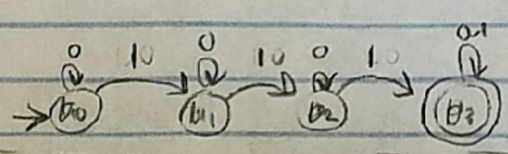
h) $L = \{0^*\}$



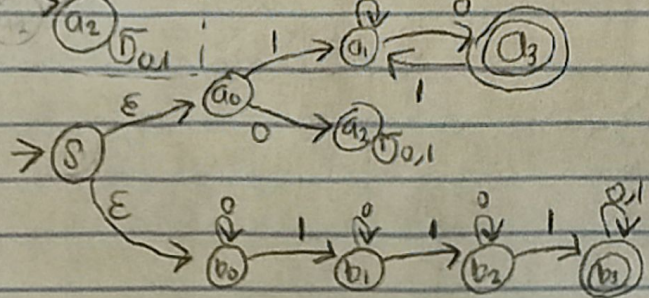
4) 1.6 a)



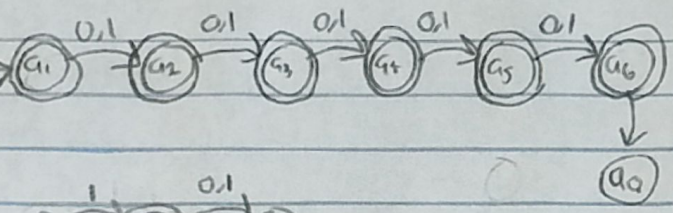
1.6 b)



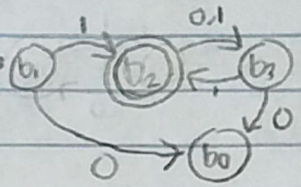
NFA:



b) 1.6 g)

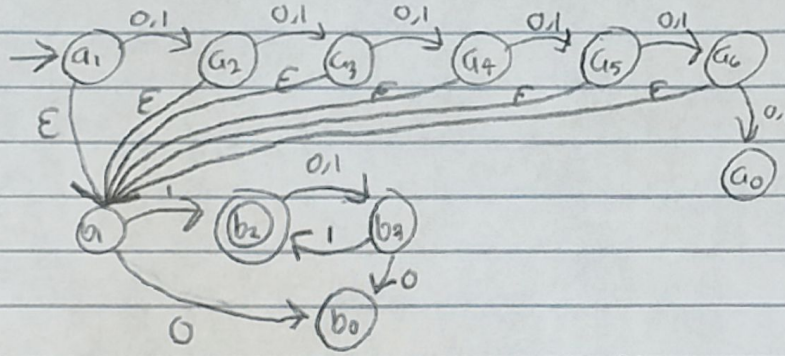


1.6 i)

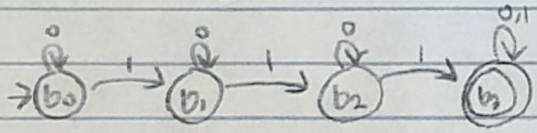


concat:

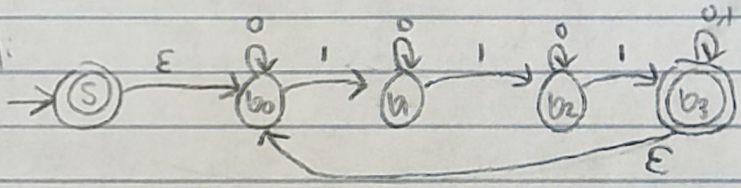
NFA:



c) 1.6 b)



NFA:

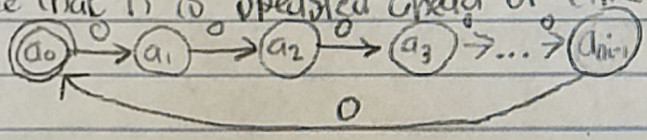


5) Let $B_n = \{a^k \mid$ where k is a multiple of $n\}$ Show that B_n is regular for any $n \geq 1$

we know that for any non-negative integer k, n and i, $a^k = a^{ni}$

Assume that n is specified ahead of time

$L_1 =$



Thus, since this can be represented as a DFA, we know that B_n is regular.