1) $L_1 \subseteq L_2$

a) $L_1 = \{a^n b^n \mid n \geq 1\}$ $L_2 = a^* b^*$

We can see for this language everything in L_1 can be represented by L_2. However, L_1 is not irregular. So therefore $L_1 \subseteq L_2$ and L_1 is not regular does <u>NOT</u> imply L_2 is irregular

b) $L_1 \subseteq L_2$ if L_2 is not regular, then L_1 is not regular

Let $L_2 = \{a^n b^n\}$ where $n \geq 1\}$ and $L_1 = ab$

$L_1 \subseteq L_2$ in this case, but L_1 is regular and L_2 is not. Therefore $L_1 \subseteq L_2$ when L_2 is non-regular does <u>NOT</u> imply L_1 is non-regular

c) If L_1 and L_2 are not regular, then $L_1 \cup L_2$ is non-regular.

Let $L_1 = \{a^n b^n \mid n \leq m\}$ & $L_2 = \{a^n b^n \mid n > m\}$

Then $L_1 \cup L_2 = a^* b^*$ which is regular.

d) If L_1 and L_2 are not regular, then $L_1 \cap L_2$ is non-regular

$L_1 = \{a^n b^n \mid n \leq m\}$ $L_2 = \{a^n b^n \mid n \geq m\}$

Then $L_1 \cap L_2 = \{a^m b^m\}$ which is regular.

Thus, that statement is FALSE

e) If L_1 is regular and L_2 is not regular, then $L_1 \cup L_2$ is not regular

Let $L_1 = a^* b^*$ and $L_2 = \{a^n b^n \mid n \geq 1\}$ ~~then $L_1 \cup L_2 =$~~

then $L_1 \cup L_2 = a^* b^*$ which is regular

Thus, this statement is FALSE

2) a) $L = \{0^i 1^j 0^k \mid k > i + j\}$ Assume L is regular

1) Select w. $w = 0^p 1^p 0^{2p+1}$. $|w| > N$ ✓ $w \in L$ N = PUMPING LENGTH

$w = x y^i z$ $|y| \geq 1$ $|xy| \leq N$

$w = 0110000$

$i = 1$

$\underbrace{0}_{x}\underbrace{11}_{y}\underbrace{0000}_{z}$

$i = 2$

$\underbrace{0}_{x}\underbrace{111}_{y}\underbrace{0000}_{z}$ but $i = 1, j = 3$ and $k = 4$. $i + j = 4$. so $k \not> i + j$.

so L is not regular

b) $\{0^i 1^j \mid j=i \text{ or } j=2i\} = L$ Assume L is regular

Let $w = 0^N 1^{2N}$ so $|w| \geq N$ and $w \in L$

$w = xy^iz$

$i=1$

$w' = \underbrace{0}_{x}\underbrace{1}_{y}\underbrace{1}_{z}$

$i=2$

$w'' = \underbrace{0}_{x}\underbrace{11}_{y}\underbrace{1}_{z}$

But now $j = 3i$, which is not accepted by L.

Therefore, L is not regular

c) $L = \{x \in \{0,1\}^* \mid x \text{ is a palindrome}\}$ Assume L is regular

Let $w = 0^N 11\, 0^N$ so $|w| \geq N$ and $w \in L$

$w = xy^iz$ $|xy| \leq N$ $|y| \geq 1$

$i=1$

$w = xyz$ because $|xy| \leq N$, must all be 0s

$\underbrace{0}_{x}\underbrace{0}_{y}\underbrace{1100}_{z}$

$i=2$

$w = xyyz$

$\underbrace{0}_{x}\underbrace{00}_{y}\underbrace{1100}_{z}$ but w is not a palindrome here.

Thus, L is not a regular language

d) $L = \{0^{2^n} \mid n \geq 0\}$ Assume L is Regular

$w = 0^{2^N}$ $w \in L$ $|w| = 2^N$ $|w| > N$

$w = xy^iz$ $|xy| \leq N$ $|y| \geq 1$

$w' = xyyz$

$|w'| = |w| + |y| = 2^N + |y|$

$2^N < |w'| \leq 2^N + N$

$|w|$ is supposed to be of size 2^N, yet we see by pumping we can get $2^N + N$.

Therefore, L is not regular.