Assignment 3 Daniel Crawford

7. a) Factors: $P(X_1)$, $P(X_2 | X_1)$, $P(X_4 | X_1)$, $P(X_5 | X_2, X_4)$, $P(X_3 | X_2)$, $P(X_6 | X_3, X_5)$

Elimination: X_1, X_2, X_3, X_4, X_5

$$f_1(X_2, X_4) = \sum_{X_1} P(X_1) P(X_2 | X_1) P(X_4 | X_1)$$

$$f_2(X_3, X_4, X_5) = \sum_{X_2} f_1(X_2, X_4) P(X_5 | X_2, X_4) P(X_3 | X_2)$$

$$f_3(X_4, X_5) = \sum_{X_3} f_2(X_3, X_4, X_5) P(X_6 | X_3, X_5)$$

$$f_4(X_5) = \sum_{X_4} f_3(X_4, X_5)$$

$$f_5(X_6) = \sum_{X_5} f_4(X_5)$$

Time: $O(5e^4)$

space: $O(5e^3)$

b) Best elimination order: $X_1, X_2, \ldots, X_{2n-1}$

- As we saw above, that solution is optimal simply because each variable after X_n eliminates one function per step.

Time: $O(nd^3)$

space: $O(nd^3)$

8. a)

A	B	C	w
0	1	0	w_1
0	1	1	w_2
0	1	1	1
0	0	1	w_3
1	0	1	w_4
1	1	0	w_5
1	1	1	w_6
1	0	0	w_7
1	0	1	w_8
0	0	0	1
1	1	1	1

0 $w_1 = (1-0.9) \cdot 0.6 \cdot (1-0.3)$

1 $w_2 = (1-0.9) \cdot 0.6 \cdot 0.3$

0 $w_3 = (1-0.9) \cdot (1-0.6) \cdot 0.3$

1 $w_4 = 0.9 \cdot (1-0.1) \cdot (0.7)$

0 $w_5 = 0.9 \cdot 0.1 \cdot (1-0.7)$

1 $w_6 = 0.9 \cdot 0.1 \cdot 0.7 \leftarrow$

0 $w_7 = 0.9 \cdot (1-0.1) \cdot (1-0.7)$

1 $w_8 = 0.9 \cdot (1-0.1) \cdot (0.7 \leftarrow$

b) $P(A=1) = \frac{W_4 + W_5 + W_6 + W_7 + W_8 + 1}{7}$

$P(B=1 \mid A=1) = \frac{W_5 + W_6}{W_4 + W_5 + W_6 + W_7 + W_8 + 1}$

$P(B=1 \mid A=0) = \frac{W_1 + W_2 + 1}{W_1 + W_2 + W_3 + 2}$

$P(C=1 \mid A=1) = \frac{W_4 + W_6 + W_8 + 1}{W_4 + W_5 + W_6 + W_7 + W_8 + 1}$

$P(C=1 \mid A=0) = \frac{W_2 + W_3 + 2}{W_1 + W_2 + W_3 + 2}$

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3) a) Time $= O(3K^3)$

space $= O(3k^2)$

Because each evidence variable has 3 variables predicting them, these variables have k different states. Therefore to compute all states we will need to compute k^3 different combinations 3 times. space requires only $3k^2$ to hold all results.

b) Time $= O(3K)$

space $= O(3)$

simple condition only 3 results and 3k possible combinations to compute

c) Case 1: $O(d)$ time to compute $X_1, \ldots, X_3$

Time: $O(d + 3k^3)$

space: $O(d + 3k^2)$

Case 2: Time $O(d + 3k)$

space $O(d + 3)$

4. a) $L = \prod_{D} \prod_{i}^{m} \theta_m^{g_i(x)} (1-\theta_i)^{1-g_i(x)}$

$LL = \sum_{D} \sum_{i=0}^{m} \log(\theta_i^{g_i(x)} (1-\theta_i)^{1-g_i(x)})$

$= \sum_{D} \sum_{i=0}^{m} g_i(x) \log \theta_i + (1-g_i(x)) \log(1-\theta_i)$

b) $\frac{dLL}{d\theta_i} = \sum_{D} \sum_{i=0}^{m} \frac{g_i(x)}{\theta} - \frac{1}{1-\theta_i} + \frac{g_i(x)}{1-\theta_i} = \sum\sum \frac{g_i(x) - \theta_i g_i(x) - \theta + \theta_i g_i(x)}{\theta(1-\theta_i)}$

$= \sum_{D} \sum_{i=0}^{m} g_i(x) - \theta_i = 0$

$\theta_i = 1D1 \sum_{i=0}^{m} g_i(x)$

6. a) Functions: $\phi(X_1), \phi(X_1, X_2), \phi(X_1, X_3), \phi(X_2, X_3, X_4), \phi(X_4, X_5), \phi(X_4, X_6)$
$\phi(X_5, X_7), \phi(X_6, X_7)$

Assume K states in all $X_1, \ldots, X_6$

Time complexity: $O(6K^4)$
Space complexity: $O(6K^3)$

EXP: There will be 6 functions to get network, and there are K^4 combinations to be computed. Space complexity requires one less variable, so it only needs $6K^3$

b) use: $X_1, X_2, X_3, X_4, X_5, X_6$

To prove, consider example $X_6, X_5, X_4, X_2, X_3, X_1$

For each example we will be using multiple functions for each node.

8. a)

A	B	C	
0	0	1	w_1
0	1	1	w_2
0	1	1	1
0	0	1	w_3
1	0	1	w_4
0	1	0	w_5
1	1	0	w_6
1	0	0	w_7
1	0	1	w_8

$w_1 = (1-0.3)(1-0.8)\cdot 0.5$ $\quad w_2 = (1-0.3)\cdot 0.8 \cdot 0.5$

$w_3 = (1-0.3)\cdot(1-0.8)\cdot(1-0.5)$ $\quad w_4 = 0.3\cdot(1-0.5)\cdot 0.1$

$w_5 = (1-0.3)\cdot(0.8)\cdot 0.5$ $\quad w_6 = 0.3\cdot 0.5\cdot(1-0.1)$

$w_7 = 0.3\cdot(1-0.5)\cdot 0.1$ $\quad w_8 = 0.3\cdot(1-0.5)\cdot(1-0.1)$

b)

$$P(A=1) = \frac{w_4 + w_6 + w_7 + w_8}{4}$$

$$P(B=1 \mid A=1) = \frac{w_6}{w_4 + w_6 + w_7 + w_8}$$

$$P(B=1 \mid A=0) = \frac{w_2 + 1 + w_5}{w_1 + w_2 + 1 + w_3 + w_5}$$

$$P(C=1 \mid A=1) = \frac{w_4 + w_8}{w_4 + w_6 + w_7 + w_8}$$

$$P(C=1 \mid A=0) = \frac{w_1 + w_2 + 1 + w_3}{w_1 + w_2 + 1 + w_3 + w_5}$$