Econometric Methods for Endogenously Sampled Time Series: The Case of Commodity Price Speculation in the Steel Market

George Hall, Brandeis University
John Rust, Georgetown University

Conference in Honor of Dan McFadden's 80th birthday

June 6, 2018

The Steel Service Center (SSC) Industry

- The US metals distribution industry includes about 7,500 companies with combined annual revenue of about $215 billion.
- The industry includes metals service centers and companies that distribute metals other than steel, but not companies that distribute precious metals or metal ores.
- Companies in the industry distribute steel and other primary metals and metal products and may also provide sawing, shearing, bending, leveling, cleaning, or edging services.
- Steel products are made from carbon or alloy, stainless, or specialty steels, and come in the form of sheets, plates, bars, rods, tubes, and structural items like rails and I-beams.

Photo of a typical steel service center

A semi-robotic steel service center

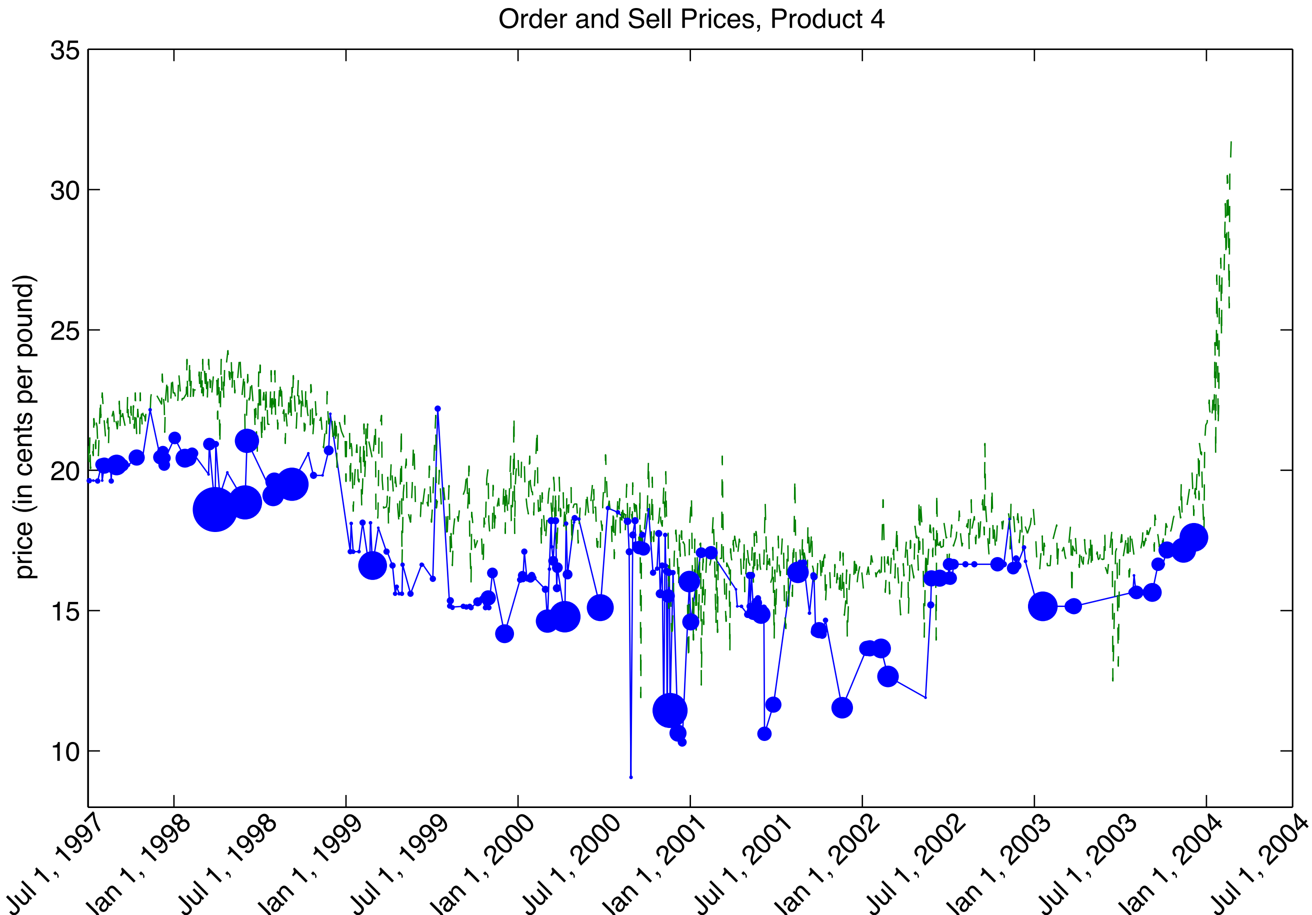
The Steel Service Center (SSC) Industry

- Major companies include Reliance Steel & Aluminum and Ryerson (US), IMS International Metal Service (France), Klöckner (Germany), as well as the distribution arms of integrated manufacturers such as ArcelorMittal (Luxembourg) and ThyssenKrupp (Germany).
- The US SSC industry is fairly concentrated: the 50 largest companies generate about 25-50% of revenue.
- Reliance Steel had $10 billion revenues in 2014 from 300 locations in 39 states and 12 countries besides US, with 20,000 transactions/day and average order size of $1780.
- "Despite all the consolidation, IBISWorld estimates the industry still has about 10,600 locations operated by more than 7,500 companies. Thus, the metals service center industry remains highly fragmented and intensely competitive within localized areas."

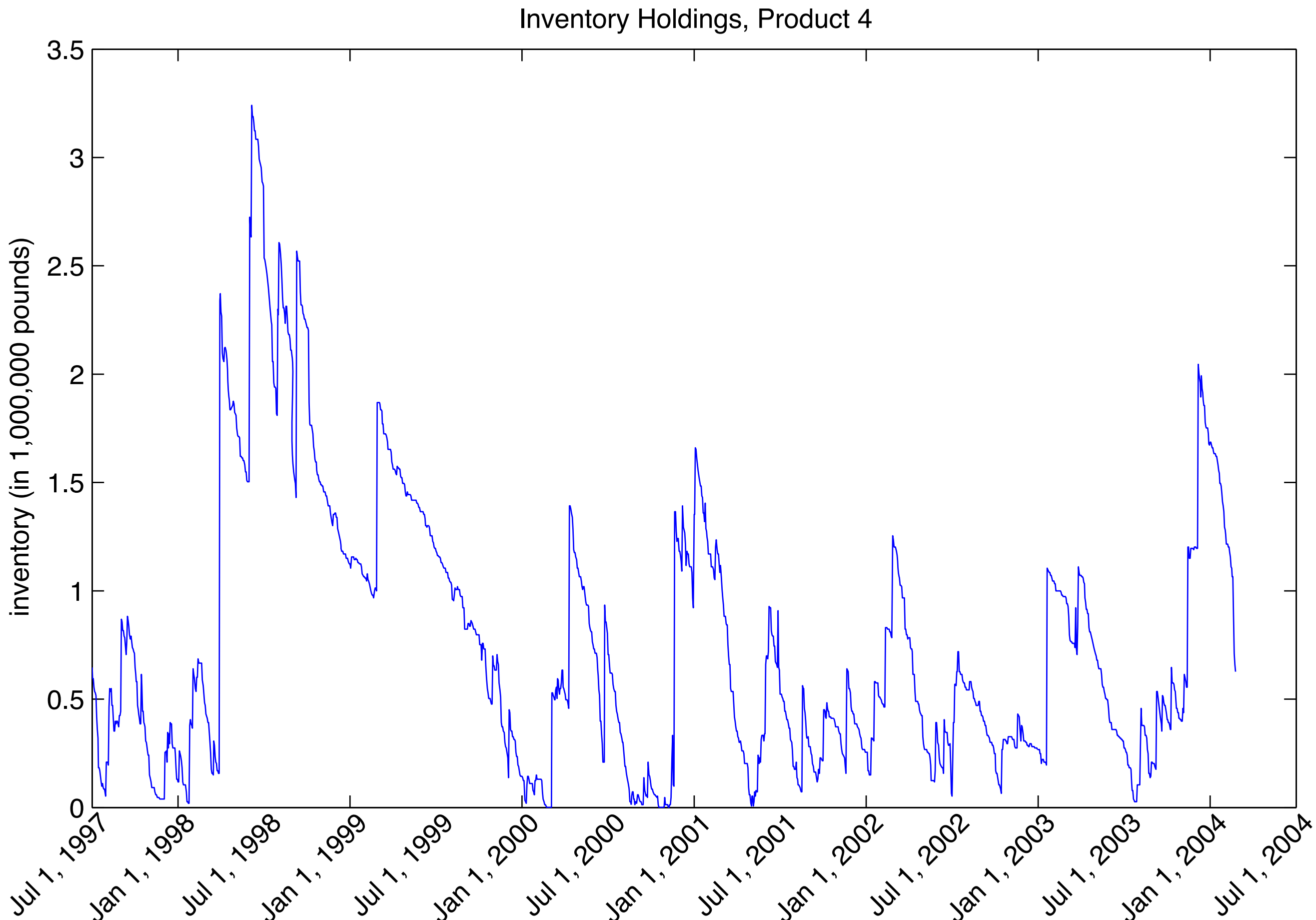
Steel Dataset

- We have data on a *steel service center* (SSC) which is basically a middleman in the market for steel products
- The company buys large quantities of different steel products at wholesale prices and sells these products in smaller batches to retail customers at a markup. *There is minimal additional processing or manufacturing done to the steel it sells.*
- We have *daily data* on *every transaction* that company made on over 8900 products it trades in between July 1, 1997 and November 29, 2006.
- Gross revenues: $650M, net revenues, $ 400M, cost of wholesale steel: $425M from 143098 sales transactions from 853 customers and 52577 purchases.
- Sales of the top 100 products (with sales of $1M or more) account for 68% of gross revenues.

Steel Prices, 1 inch plate, 96x240



Steel Inventories, 1 inch plate, 96x240



Some Basic Facts

- There is second degree price discrimination: different customers pay different prices for the same types of steel on the same day.
- There are quantity discounts and distance matters: markups are higher for customers who are closer to the firm.
- There is considerable day-to-day and within-day variation in retail prices: wholesale price variation explains about 3/4 of the variation in retail prices
- Retail prices seem only weakly related to current inventory holdings.
- Purchases are made infrequently and vary considerably in size and are more volatile than sales. There is no stable inventory/sales relationship and stockouts and near stockouts occur regularly.

The Endogenous Sampling Problem

- Obviously the company makes profits in part from markups but also in part from *commodity price speculation* — "buy low and sell high"
- For 1 inch 96x240 plate, we observe sales on 1372 days but purchases only on 331 days.
- So at the very least, we have *irregularly sampled time series*
- More problematically, we have *endogenously sampled times series — we only observe wholesale prices on the days the company buys steel, not on days that it does not buy steel.*
- What about data from a "steel exchange"? Sorry, no luck. *Steel exchanges don't exist.* The steel market can be described as a "telephone market" populated by thousands of middlemen, i.e. the SSC's like the firm we are studying.

A Model of a Profit-Maximizing SSC

- Consider a steel middleman. At the start of each day t, the middleman observes his current inventory q_t and the current wholesale price p_t^b (which he cannot affect).
- The middleman pays a fixed cost K every time he purchases steel.
- He can buy unlimited quantities of steel, $q_t^b > 0$ at p_t^b which we assume ca be delivered immediately. After the arrival of this order, inventory equals $q_t + q_t^b$.
- The wholesale price evolves according to a Markov process $\{p_t^b, x_t\}$ with transition density $g(p_t^b, x_t | p_{t-1}^b, x_{t-1})$ where x_t denotes a vector of other state variables that are useful for predicting the level of demand and future prices of steel.

The Model, continued

- Each day, with probability $\eta(p_t^b, x_t)$ a customer arrives (i.e calls the middleman trying to negotiate a purchase). Conditional on an arrival, the quantity the customer wishes to buy q_t^d is a draw from a density $m(q_t^d | p_t^b, x_t)$.
- The customer reveals his location and the quantity q_t^d he wishes to purchase. The middleman also observes a vector z_t of other characteristics of the customer, but does not know the customer's reservation value p_t^r for steel.
- The middleman has a belief about the customer's possible reservation values given by a conditional density $f(p_t^r | z_t, x_t, q_t^d, p_t^b)$.

The Model, continued

- We assume there are no large repeat purchasers. and each customer adopts an optimal search strategy in the market for steel, given his exogenously determined demand q_t^d for steel. The customer's problem is to obtain q_t^d at the lowest possible price.
- In standard search models, a reservation price strategy is optimal, so the customer will be willing to buy q_t^d from the middleman provided the middleman's quoted retail price p_t^s is less than the customer's reservation price p_t^r.
- We assume the middleman quotes a take-it-over leave it price p_t^s.

Optimal price discrimination

- One feature of the pricing rule that will emerge from certain specifications for $f(p^r|z, x, p, q^d)$ is quantity discounts, i.e. p^r will be a decreasing function of quantity demanded, q^d. This is an implication of the assumption that increases in q^d shift the distribution of reservation values to the left (i.e. customers who demand larger quantities also expect to get lower prices).
- Given the Markovian, infinite horizon nature of this problem, we deduce that the middleman's optimal policy take the form of two decision rules

$$\begin{aligned} q_t^b &= q^b(p_t^b, x_t, q_t) \\ p_t^s &= p^s(p_t^b, x_t, z_t, q_t, q_t^d) \end{aligned}$$

Optimal price discrimination

- We can show that the firm's optimal retail pricing problem can be written as follows

$$p^s(q^d, q, p, x, z) =$$
$$\underset{p^s}{argmax} \left[\min[q, q^d][1 - F(p^s|z, x, p^b, q^d)][p^s - t_c - c_t^s] \right]$$

where t_c is the per-pound transport cost and c_t^s is an appropriately defined *shadow price of steel.*

- The optimal retail price is then:

$$p_t^s = t_c + c_t^s + \frac{1 - F(p_t^s|z_t, x_t, p_t^b, q_t^d)}{f(p_t^s|z_t, x_t, p_t^b, q_t^d)}$$

- This is identical to Myerson's (1981) optimal reserve price for an auction with one buyer and one seller.

Optimal purchasing (speculation) strategy

- Hall and Rust (2007) generalized the work of Scarf (1960) to show that the optimal strategy for ordering steel takes the form of a *generalized* (S, s) *strategy*
- Instead of two fixed constants, S and s, the optimal strategy is given by (S, s) *bands* i.e. functions $S(p, x)$ and $s(p, x)$ satisfying $S(p, x) \geq s(p, x) \geq 0$, where we have

$$q_t^b(p_t^b, x_t, q_t) = \begin{cases} S(p_t^b, x_t) - q & \text{if } q < s(p_t^b, x_t) \\ 0 & \text{otherwise} \end{cases}$$

where $S(p, x)$ is the *target inventory level* and $s(p, x)$ is the *order threshold.*

The shadow price of steel

- The marginal value of steel inventory is the partial derivative

$$\frac{\partial}{\partial q} V(p_t^b, q_t, x_t)$$

and at $q = S(p^b, x)$ we have

$$p^b = \frac{\partial}{\partial q} V(p_t^b, q_t, x_t) \quad (1)$$

- However when $q_t \geq s(p_t^b, x_t)$ (i.e. on days that the middleman does not place new orders for steel), the marginal of steel is generally not equal to the wholesale price p_t^b.

Excess inventory $\Longrightarrow$ low shadow price of steel

- When $q > S(p^b, x)$ we have

$$\frac{\partial}{\partial q} V(p_t^b, q_t, x_t) < p_t^b$$

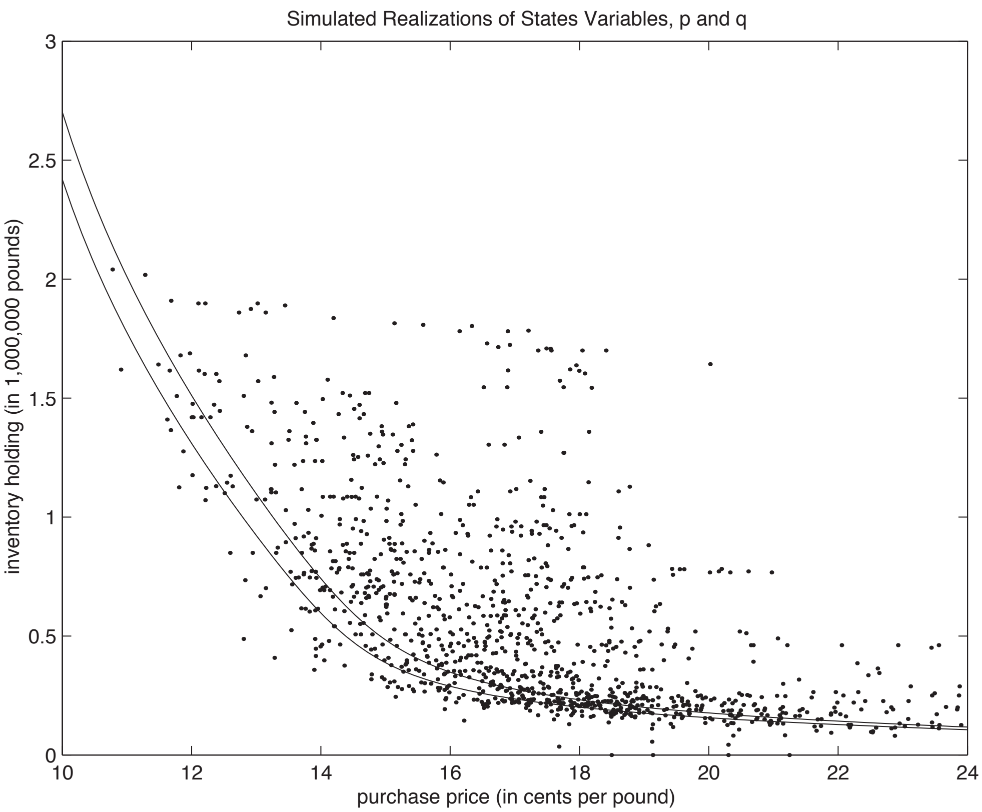
- In other words, when the middleman has "excess inventory" (i.e more than the target level $S(p_t^b, x_t)$), the marginal value of inventory is less than the current wholesale price p_t^b.
- If there is a sufficient excess of current inventory, the shadow price c_t^s can be sufficiently below the current wholesale price p_t^b to imply that $p_t^s < p_t^b$, i.e. *negative markups* over wholesale price when the firm is sufficiently overstocked.

The Endogenous Sampling Problem, Restated

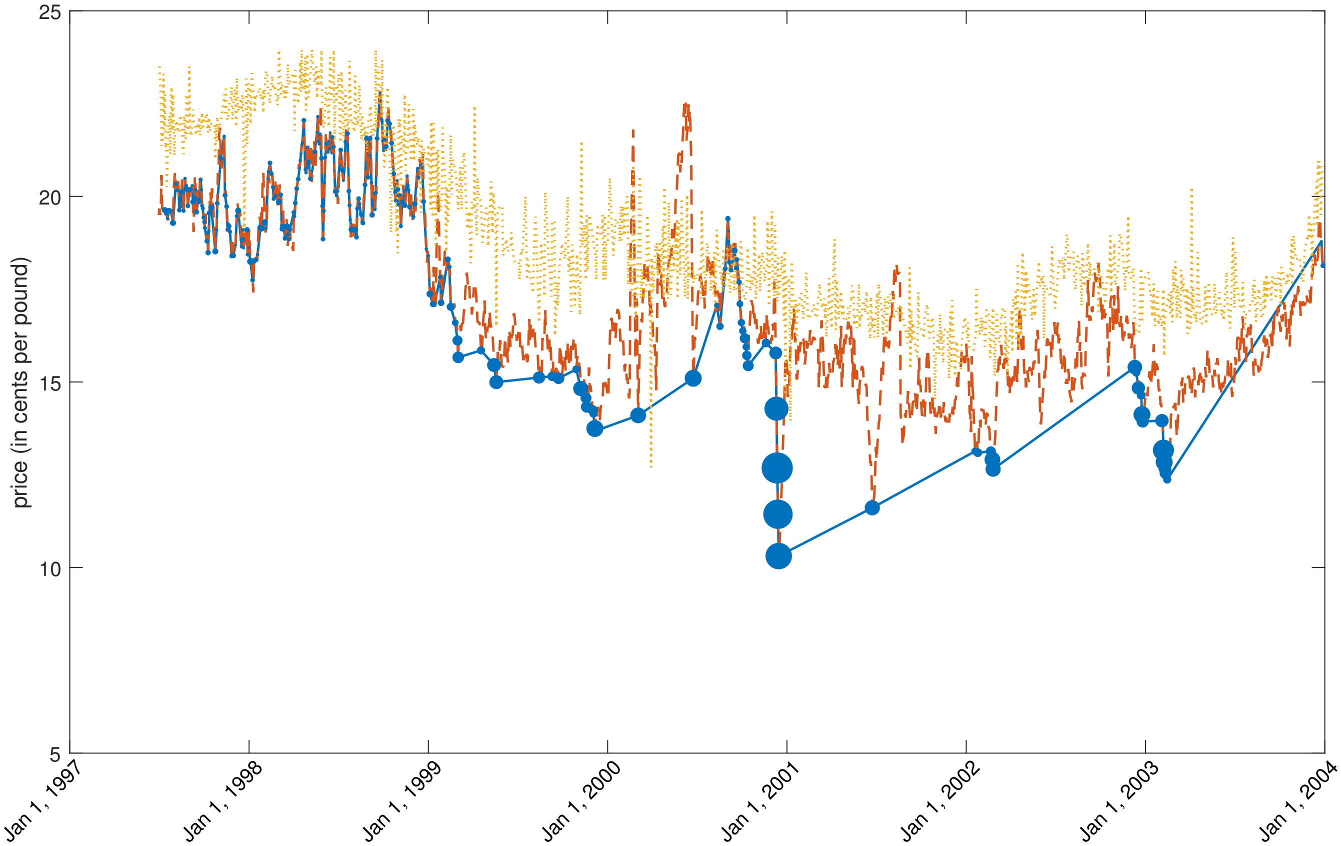
- **Endogenous Sampling Rule:**

$$p_t^b \text{ is observed } \iff q_t^b > 0 \iff q_t < s(p_t, x_t).$$

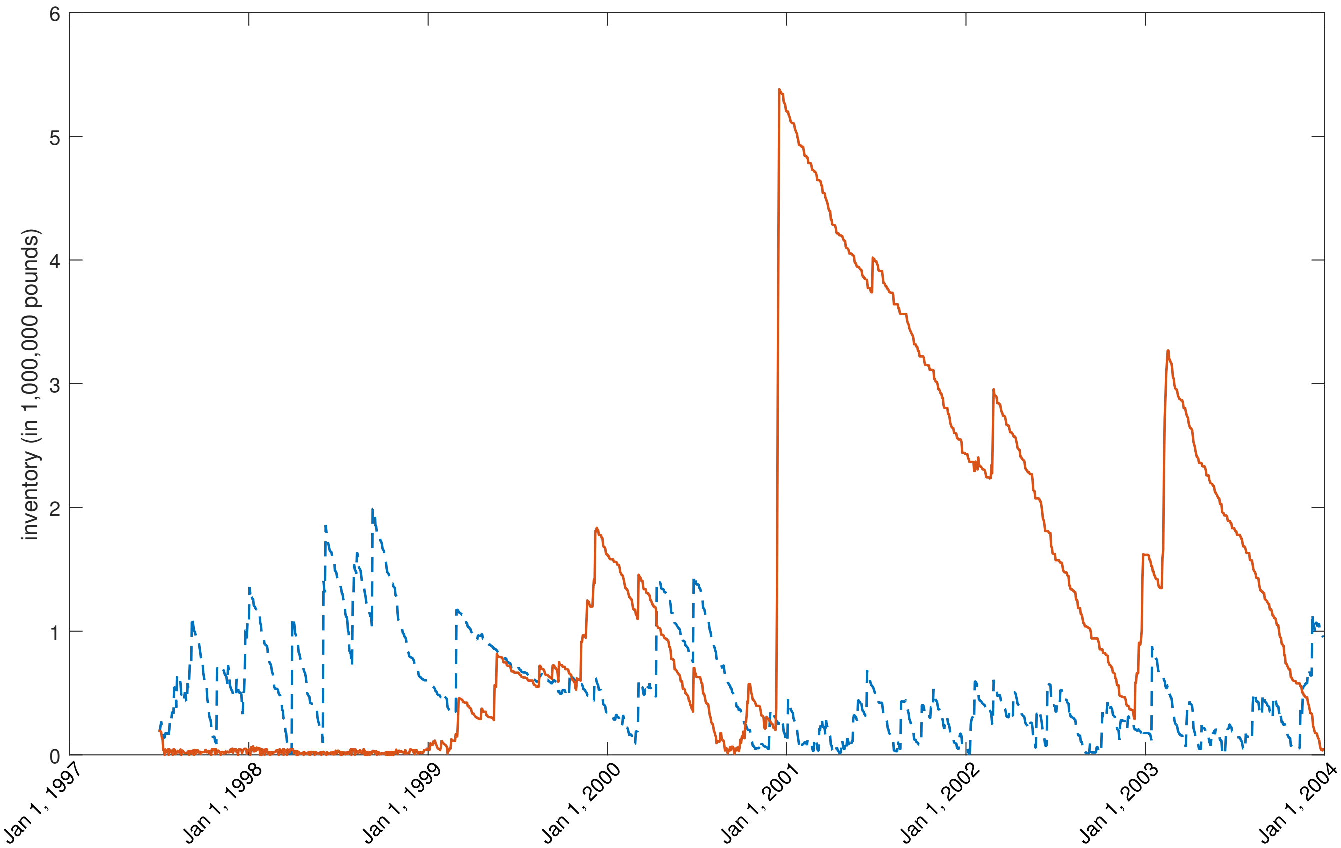
(S, s) bands and simulated (p, q) pairs



Simulated Censored Wholesale Prices



Simulated Inventories



How to estimate the model?

- Let θ denote the "parameters of interest" to be estimated. These include the parameters of the Markov transition density $g(p^b_{t+1}, x_{t+1}|p^b_t, x_t)$ for wholesale prices (and any associated "predictors" x_t that help predict future prices and sales of steel), as well as parameters characterizing retail demand (arrival rates of customers, their distributions of reservation prices, etc) and parameters of the firm's cost/profit function (the order cost K, storage costs, and the firm's cost of capital).
- How to estimate θ given endogenously sampled data?

Maximum likelihood

- We write down a transition probability for all variables, including ones we do not always observe (e.g. wholesale prices on days the firm does not buy steel).
- This is a *controlled transition probability* that depends on endogenously determined behavior of the firm: 1) the firm's retail pricing rule p^s, and 2) the firm's wholesale purchasing rule q^b as given by the (S, s) bands
- Then we *integrate out* the variables we do not observe, i.e. the wholesale prices of steel on the days the firm does not buy steel.
- But this is a *humongously high dimensional integration problem!* Recall we observe the firm for over 3000 days but observe purchases only on 300 days. So to do maximum likelihood we need to be prepared to compute *2700 dimensional integrals!*

A more practical way forward: McFadden's MSM

- "A classical method of moments estimator θ_{mm} of an unknown parameter vector θ^* minimizes the (generalized) distance from zero of empirical moments ..."
- "For some problems, the expected response function may be difficult to express analytically or to compute, but relatively easy to simulate."
- "When this function is replaced by an unbiased simulator such that the simulation errors are independent across observations and sufficiently regular in θ, the variance introduced by simulation will be controlled by the law of large numbers operating across observations, making it unnecessary to consistently estimate each expected response."
- "This is the basis for the estimation method developed in this paper, the method of simulated moments (MSM)."

Can MSM be used to estimate the steel problem?

- McFadden's 1989 *Econometrica* article focused on estimation of static discrete choice models using cross sectional (*IID*) data.
- Can it be extended to consistently estimate a dynamic model of steel pricing/speculation, where we have time series data and endogenous sampling (censoring)?
- YES! Key insight: a discrete choice model is also a censored model —- we observe individuals' *choices* but not the *utilities* of the alternatives
- But MSM simulates utilities and censors them to get simulated choices, and uses the LLN to average out the simulation noise along with random sampling noise.
- We can also do this in the steel content, but rely on *ergodic theorems* in place of LLN to average out the noise.
- "*If you can simulate it, you can estimate it.*" J.F. Richard

MSM using model data

parameter	truth	estimate	std error
K	100.0	116.6	7.4
α_0	0.90	0.89	0.17
α_1	1.16	1.14	0.02
λ_p	0.980	0.976	0.002
$\bar{\mu}_p$	18.0	18.15	0.26
$\bar{\sigma}_p$	2.5	2.48	0.07
$\bar{\mu}_q$	300.0	291.3	7.3
$\bar{\sigma}_q$	300.0	344.9	15.6
ξ	0.70	0.77	0.06
γ	500.0	256.7	149.9
η	0.600	0.587	0.002
r	0.060	0.065	0.026
ϕ	-0.075	-0.059	0.007
J-test statistic	$\chi^2(15) = 381$		

Selected actual vs fitted moments: 3/4 inch plate

moment	simulation	data
mean(buy price)	17.61	16.78
var(buy price)	3.41	4.37
mean(sell price)	18.83	19.09
var(sell price)	2.73	5.37
mean(purchase size)	674	1138
var(purchases)	50	370
mean(sale size)	156	260
var(sales)	5	5
mean(inventory)	4383	5247
mean(markup)|small sale	1.27	2.14
mean(markup)|medium sale	1.27	1.71
mean(markup)|large sale	1.26	1.63
probability of purchase	0.142	0.14
probability of sale	0.582	0.605

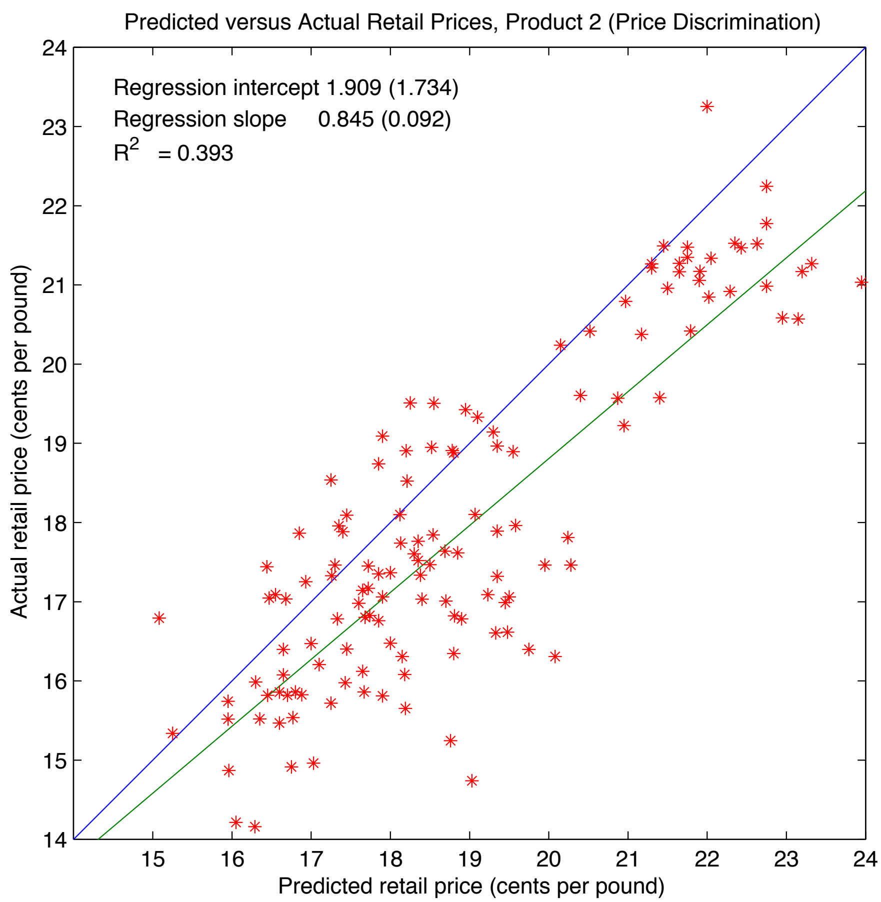
Selected actual vs fitted moments: 1 inch plate

moment	simulation	data
mean(buy price)	17.55	16.95
var(buy price)	3.88	5.58
mean(sell price)	18.36	18.96
var(sell price)	3.01	5.58
mean(purchase size)	1066	1393
var(purchases)	155	629
mean(sale size)	244	313
var(sales)	21	12
mean(inventory)	7947	7561
mean(markup)|small sale	0.80	2.02
mean(markup)|medium sale	0.80	1.79
mean(markup)|large sale	0.80	1.59
probability of purchase	0.14	0.14
probability of sale	0.572	0.594

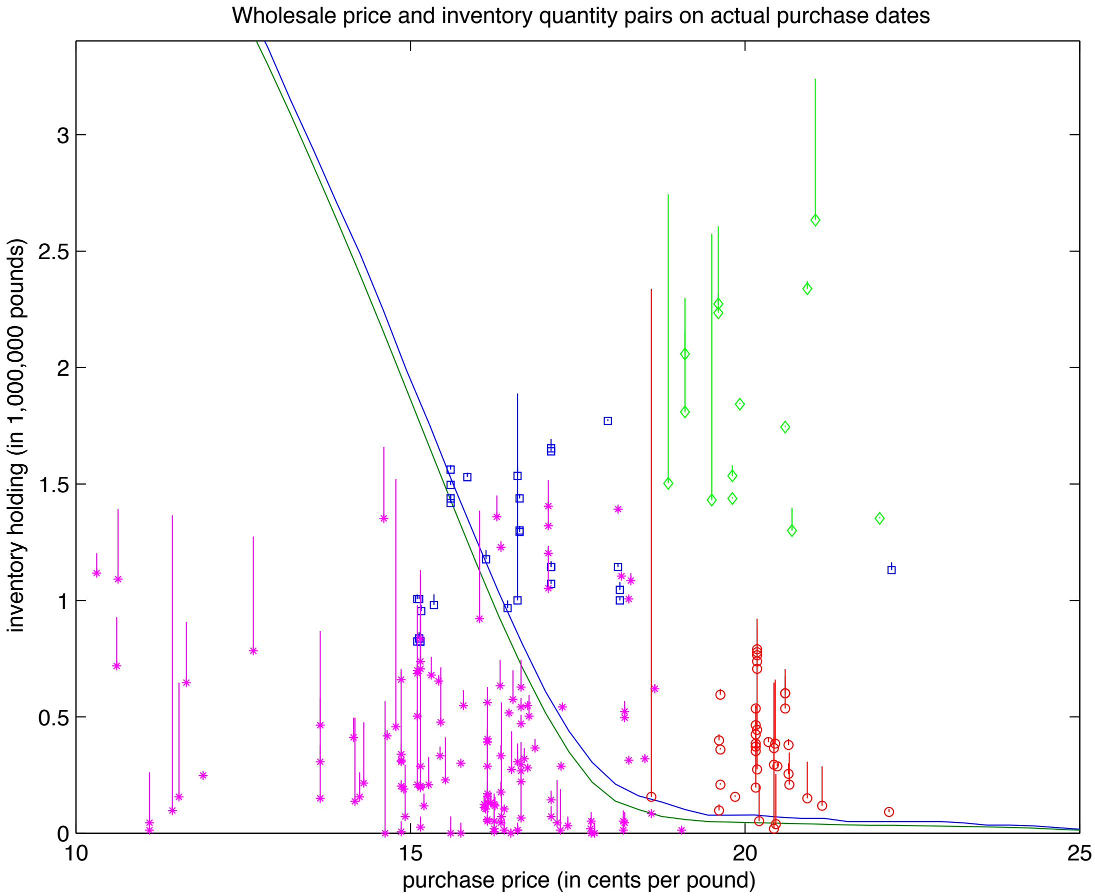
MSM estimates: 3/4 and 1 inch plate

parameter	estimate	std error	estimate	std error
r	0.0794	0.007	0.0746	0.012
K	3.05	0.33	3.36	0.94
α_0	0.553	0.07	0.55	0.045
α_1	1.041	0.005	1.014	0.002
λ_p	0.978	4.4×10^{-5}	0.979	4.5×10^{-5}
μ_p	0.0613	5.7×10^{-5}	0.0595	5.7×10^{-5}
σ_p	0.0192	3.0×10^{-4}	0.0204	3.6×10^{-4}
μ_q	6.011	0.024	6.225	0.031
σ_q	1.152	0.011	1.322	0.021
ς	0.56	0.01	0.53	0.011
ϕ_1	6.6×10^{-3}	1.7×10^{4}	6.0×10^{-3}	7.1×10^{-4}
$\chi^2(28)$	3/4 inch: 1623		1 inch: 1076	

Actual versus predicted retail prices



“The cool but ugly graph”



A profit decomposition exercise

- Finally, we use simulations of the estimated model to deduce the relative importance of capital gains versus markups for the overall profitability of the firm. The firm's profits can be written as

$$\sum_{t=1}^{T} \rho^t \pi(p_t, p_t^r, q_t^r, q_t + q_t^o) = \sum_{t=1}^{T} \rho^t (p_t^r - p_t) q_t^s + q_1 p_1 +$$
$$\sum_{t=2}^{T} \rho^t (p_t - (1+r) p_{t-1}) q_t - \sum_{t=1}^{T} \rho^t I(q_t^o) K -$$
$$\sum_{t=1}^{T} \rho^t c^h(q_t + q_t^o, p_t)$$

A profit decomposition exercise

- The first term on the right hand side of the profit decomposition can be interpreted as the discounted present value of the markup paid by the firm's retail customers over the current wholesale price.
- The third term can be interpreted as the discounted present value of the capital gains or loss from holding the steel from period $t-1$ into period t.
- The fourth, and fifth terms are the discounted present values of the order costs and the holding costs incurred by the firm over the sample period.

A profit decomposition exercise

- Since this decomposition depends on the wholesale price path between purchases, we simulate between purchase dates via importance sampling, thanks to an idea suggested by Michael Keane.
- For each interval between successive purchase dates, we simulate wholesale price paths that are consistent with the estimated law of motion and the observed purchase prices at the beginning and end of the interval.
- We truncate the simulated price process by rejecting any paths such that $q_t < s(p_t)$ for any draw within the simulated paths.

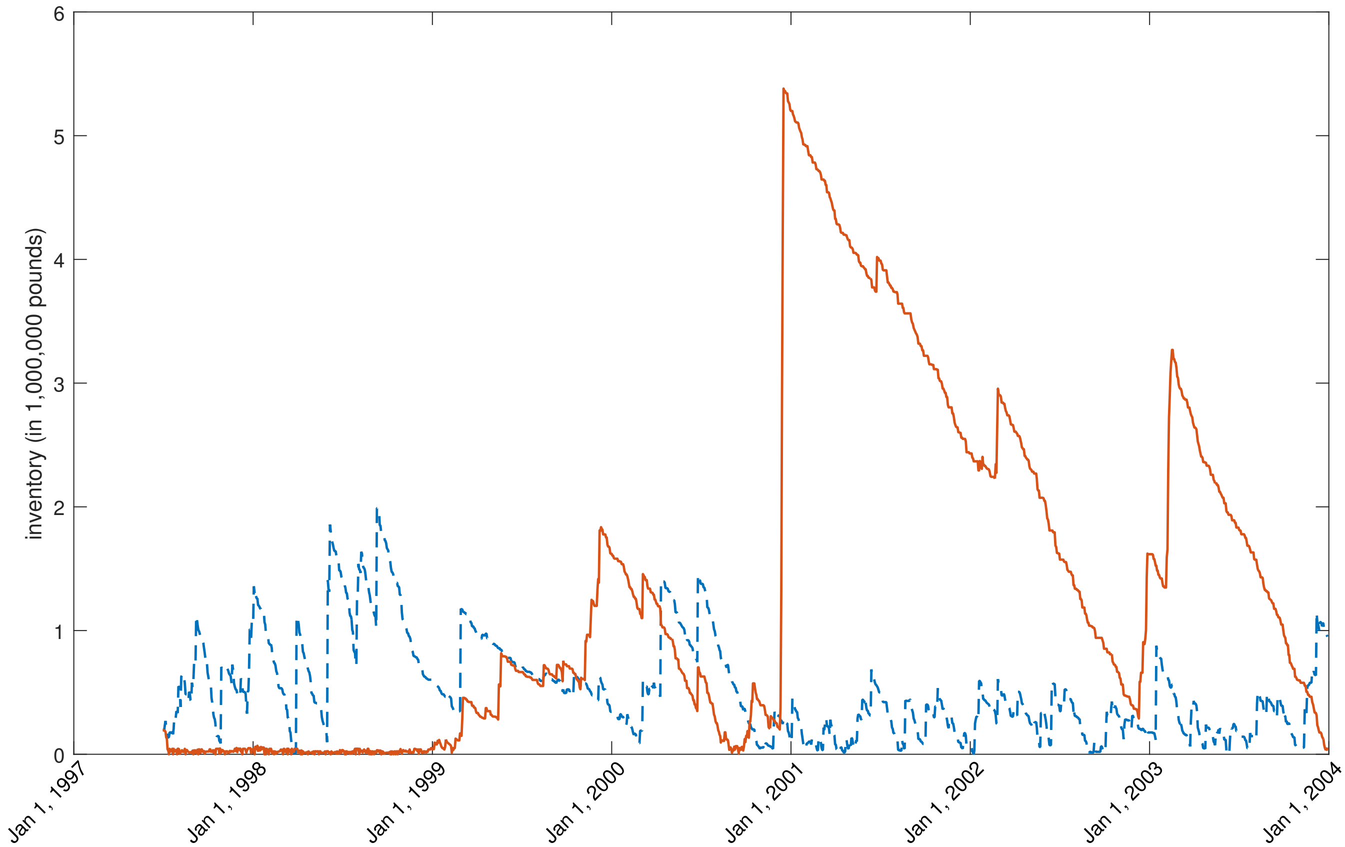
Actual vs. model profits no price discrimination

	3/4 inch plate			
	G.M.'s actual Performance		Model's Policy Prescription	
markup	$401,240	(15,034)	$330,244	(15,134)
capital gain	46,682	(16,160)	223,415	(9,462)
holding cost	-50,214	(0)	-122,592	(3,359)
order costs	-606	(0)	-743	(27)
total profits	396,102	(1,739)	430,324	(15,861)

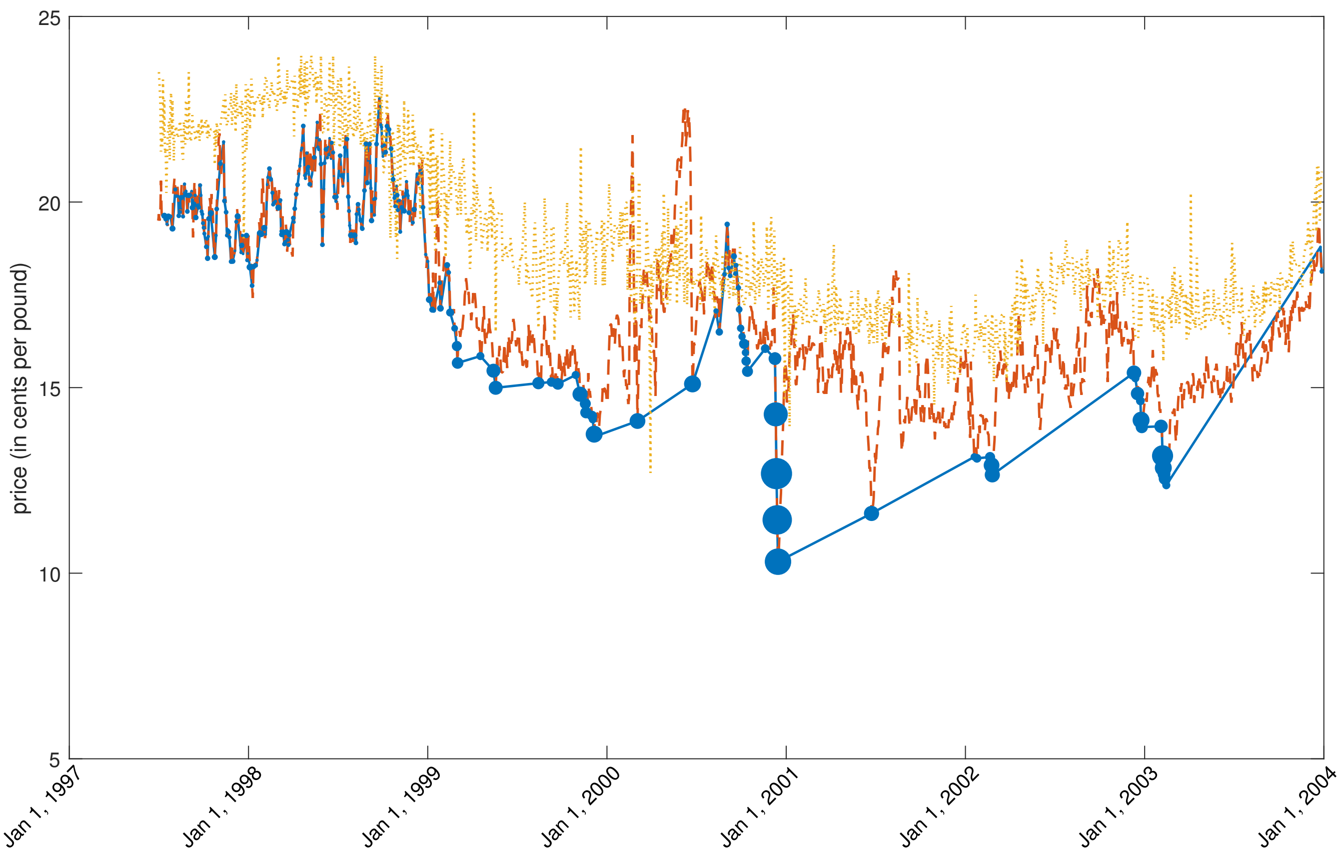
Actual vs. model profits no price discrimination

	1 inch plate			
	G.M.'s actual Performance		Model's Policy Prescription	
markup	$450,540	(15,684)	$376,997	(13,769)
capital gain	76,162	(16,739)	365,363	(14,252)
holding cost	-66,776	(0)	-197,358	(3,976)
order costs	-665	(0)	-749	(25)
total profits	459,261	(1,863)	544,252	(12,690)

Actual (dashed) vs model inventories for 3/4 inch plate



Actual vs uncensored prices (dotted), 3/4 inch plate



Actual vs. model profits, 1/2 plate, price discrimination

	Product 2			
	G.M.'s actual Performance		Model's Policy Prescription	
markup	\$364,166	(17,573)	\$510,539	(26,156)
capital gain	56,100	(18,910)	228,684	(43,920)
holding cost	-106,656	(0)	-112,085	(6,097)
order costs	-2,654	(0)	-2,711	(524)
lost sales	0	(0)	-28	(13)
total profits	310,954		624,398	(53,806)