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LARGE MARKET ASYMPTOTICS FOR DIFFERENTIATED PRODUCT DEMAND ESTIMATORS WITH ECONOMIC MODELS OF SUPPLY

BY TIMOTHY B. ARMSTRONG[1]

IO economists often estimate demand for differentiated products using data sets with a small number of large markets. This paper addresses the question of consistency and asymptotic distributions of instrumental variables estimates as the number of products increases in some commonly used models of demand under conditions on economic primitives. I show that, in a Bertrand–Nash equilibrium, product characteristics lose their identifying power as price instruments in the limit in certain cases, leading to inconsistent estimates. The reason is that product characteristic instruments achieve identification through correlation with markups, and, depending on the model of demand, the supply side can constrain markups to converge to a constant quickly relative to sampling error. I find that product characteristic instruments can yield consistent estimates in many of the cases I consider, but care must be taken in modeling demand and choosing instruments. A Monte Carlo study confirms that the asymptotic results are relevant in market sizes of practical importance.

KEYWORDS: Demand estimation, BLP instruments, weak instruments.

1. INTRODUCTION

THE SIMULTANEOUS DETERMINATION of quantities and prices is a classic problem in demand estimation. A common solution in markets with differentiated products is to use characteristics of competing products as a source of exogenous variation in prices. The idea is that a firm facing stiffer competition will set lower markups, so, as long as they are independent of demand shocks, characteristics of competing products will be valid instruments for a product's price. The use of characteristics of competing products as price instruments is common in empirical studies of differentiated product markets, and goes back at least to Bresnahan (1987) and Berry, Levinsohn, and Pakes (1995) (hereafter, BLP). I refer to instruments of this form as product characteristic instruments or, in reference to the latter paper, "BLP instruments." See footnote 2 below for more recent references that use BLP instruments.

Many empirical studies of markets with differentiated products, including those that use the BLP instruments, use data on a relatively small number of markets, each with many products, to estimate demand elasticities. For example, in their application to automobile demand, BLP use data on 20 mar-

[1]Thanks to Han Hong and Liran Einav for guidance and many useful discussions, and to Ariel Pakes, John Lazarev, Steve Berry, Phil Haile, and participants at seminars at Boston College, Stanford, and Yale and the 2010 Econometric Society World Congress for useful comments and criticism. I also thank Nail Kashaev and Huihui Li as well as the co-editors and several anonymous referees for numerous helpful comments throughout the editorial process. All remaining errors are my own.

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kets, each with about 100 products.[2] For demand models where the number of parameters grows with the number of product characteristics rather than the number of products, one might expect a small number of markets with a large number of products to give good estimates of demand.

This paper uses asymptotic approximations where asymptotics are taken in the number of products per market to examine the behavior of instrumental variables (IV) estimators of demand in large market settings, with a focus on product characteristic-based instruments. Since the BLP instruments are correlated with prices only through equilibrium markups, their validity in this setting depends crucially on the nature of competition in markets with many products. If the dependence of markups on characteristics of other products disappears as the number of products increases and does so quickly enough, the BLP instruments will lose power in large markets and estimates based on them will be inconsistent when asymptotics are taken with respect to the number of products per market. The results in this paper use the asymptotic behavior of equilibrium markups to determine when this is the case.

I find that, in certain cases, the dependence of prices on product characteristic instruments through markups disappears at a fast enough rate that the BLP instruments lead to inconsistent estimates when asymptotics are taken in the number of products per market. In particular, this is the case with the logit and random coefficients logit models when the number of products increases with the number of markets and products per firm fixed.[3]

Despite this negative result, I show that, in other settings, the dependence of markups on product characteristics remains or decreases slowly enough that the BLP instruments lead to consistent estimates under large market asymptotics. Under certain conditions, the BLP instruments lead to consistent estimates under logit demand when the number of products increases with the number of firms fixed, as long as firms are asymmetric. The asymptotic dependence of the markup on product characteristics can also be obtained when the number of products increases with the number of products per firm fixed if the dimension of the space of observed product characteristics increases with the number of products (but with enough restrictions that the dimension of the parameter space does not). I illustrate this point with a nested logit model with many nests. Finally, considering the case with many large markets, the BLP instruments will lead to consistent estimates in certain cases if the number of

[2]In addition to BLP and other papers on the automobile industry (e.g., Petrin (2002)), examples of industries with data that fit this description include personal computers (e.g., Eizenberg (2011), Bresnahan, Stern, and Trajtenberg (1997)), LCD televisions (e.g., Conlon (2012)), and other consumer goods sold at a national level. Of these papers, Bresnahan, Stern, and Trajtenberg (1997) use BLP instruments exclusively, while the others use a combination of BLP and cost instruments.

[3]The random coefficients model considered in this paper does not allow a random coefficient on price. Allowing a random coefficient on price appears to require nontrivial extensions of the methods used in this paper, and is an important topic for future research.

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markets increases quickly enough relative to the number of products in each market.

This paper gives a negative result for any setting where (i) the number of products increases with the number of markets fixed, (ii) the correlation between markups and characteristics of other products decreases quickly enough as the number of products increases (see Theorem 1 for a formal statement of "quickly enough"), and (iii) cost instruments or other sources of identification are unavailable or insufficient to identify the model alone (instruments that shift marginal cost directly do not need variation in the markup to shift prices, and therefore do not suffer from the issues brought up here). If all of these conditions hold, IV estimates of the parameters of interest will be inconsistent.

The results in this paper show that, not only are BLP instrument-based estimates inconsistent in these settings, they are asymptotically equivalent to estimates that would be obtained if instruments had zero correlation with prices. This has two important implications. First, estimates and confidence intervals (CIs) will be asymptotically no more accurate than a random guess formed without using the data. In particular, any $1-\alpha$ CI with correct coverage will cover the entire parameter space with probability approaching $1-\alpha$, thereby making it no better than a CI that reports the entire parameter space with probability $1-\alpha$ and an arbitrary point with probability α independently of the data. Second, the sampling distribution of the estimator will be poorly approximated by the normal distribution, so that standard CIs will undercover.

The second issue can be remedied by using CIs developed in the weak instrument literature (see Stock, Wright, and Yogo (2002) for a survey of this literature).[4] In particular, one can form a CI by inverting weak instrument robust tests such as those proposed by Kleibergen (2005). The resulting CI will have correct coverage, but will still suffer from the first issue described above: in cases where the asymptotics in this paper are a good description of the data generating process, the CI will be uninformative in the sense that a CI with the same properties can be generated by flipping a coin. Thus, in cases where one can determine a priori that the inconsistency results in this paper are relevant, one should use different instruments or a different specification, rather than reporting a CI that is a priori uninformative. See Section 4 for more on these issues.

[4]The weak instrument literature uses sequences of data generating processes where the correlation between instruments and endogenous variables decreases with the sample size. While this is typically done as an artificial way to obtain better finite sample approximations, this paper derives such sequences as an equilibrium outcome of a sequence of pricing games. See Pinkse and Slade (2010) for a discussion of related phenomena in the context of spatial models.

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1.1. *Related Literature*

To my knowledge, the only other paper that considers asymptotics in the number of products per market in this setting is Berry, Linton, and Pakes (2004). So as to focus on other questions involving error from simulation-based estimators and sampling error in product shares, those authors abstract from identification issues and from the supply side by placing high level conditions that assume that the instruments strongly identify the model under large market asymptotics. In contrast, the present paper asks which models of supply and demand allow product characteristic instruments to have power in a large market setting (and abstracts from the questions of simulation error and sampling error in market shares). In the cases where the present paper gives a negative answer to this question, the high level conditions for strong identification in Berry, Linton, and Pakes (2004) do not hold, and the present paper takes the further step of showing that this leads to inconsistent estimates.

In addition, there has been a recent literature proposing computational improvements and bias corrections in random coefficients demand models. Dubé, Fox, and Su (2012) and Conlon (2013) propose improved methods for computing these estimators, with the latter considering a generalized empirical likelihood estimator with improved higher order bias properties. Freyberger (2015) derives corrections for simulation error in the case where the number of markets is large relative to products per market.

More broadly, others have proposed different approaches to modeling and estimating demand in large markets, including Bajari and Benkard (2005) and Pinkse and Slade (2004). While the present paper focuses on the demand models and estimators proposed in BLP, one could make a similar point about taking into account the implications of an equilibrium model for the behavior of estimators in other settings where one deals with a small number of distinct venues in which many agents interact. Performing this type of analysis in other settings is expected to be a useful topic for future research.

This paper is also related to the literature on weak instruments (see Stock, Wright, and Yogo (2002) for a survey of this literature). That literature uses sequences of underlying distributions in which the correlation of instruments with endogenous variables shrinks with the sample size to get asymptotic approximations that better approximate finite sample distributions. This paper shows that such sequences arise endogenously from equilibrium prices when asymptotics are taken in the number of products per market in a certain class of models. Other settings in which such sequences arise naturally have been observed in the literature on spatial econometrics (see Pinkse and Slade (2010)).

While the present paper focuses on product characteristic instruments, other papers have dealt with the optimal use of other instruments when they are available, such as variables that shift marginal cost directly. In contemporaneous work, Reynaert and Verboven (2012) focus on settings where cost instruments are available and consider approximations to optimal functions of cost

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shifters and product characteristics based on the assumption of perfect competition (a setting where BLP instruments have no power). Their focus also differs from the present paper in that they focus on estimation of the distribution of the random coefficients (σ in the notation below), while the present paper focuses on the price parameter (α in the notation below). Romeo (2010) proposes other instruments for models similar to those considered here.

In addition to the literatures on weak instruments and on estimation of discrete choice models of demand, this paper relates to theoretical results on oligopoly pricing in markets where demand is characterized by a discrete choice model. Gabaix, Laibson, Li, Li, Resnick, and de Vries (2013) consider the limiting behavior of prices in large markets in a similar setting to the present paper, but focus on different questions, leading to a different formulation and different results (for example, Gabaix et al. (2013) achieve more generality in other directions by restricting attention to symmetric firms, while the present paper deals with instruments that attempt to exploit observed asymmetry between firms). Existence of equilibrium in some of the pricing games I consider follows from arguments in Caplin and Nalebuff (1991), Vives (2001), and Konovalov and Sandor (2010) or similar methods. There is also a literature examining how restrictions on demand elasticities in discrete choice models place restrictions on the possible outcomes of empirical applications (see, among others, Bajari and Benkard (2003), Ackerberg and Rysman (2005)). While some of the findings of this paper add to this body of literature, the main focus is on implications for the identifying strength of BLP instruments.

1.2. *Plan for Paper*

The paper is organized as follows. Section 2 describes the class of models being studied. Section 3 derives the asymptotic behavior of equilibrium prices and IV estimates in some commonly used models of supply and demand. Section 4 presents recommendations for diagnosing the issues brought up in this paper. Section 5 provides a Monte Carlo study. Section 6 concludes. Proofs and additional results are provided in the Supplemental Material (Armstrong (2016)).

2. THE MODEL

This section describes the class of models and estimators considered in this paper and defines some notation that will be used later. The models and much of the notation follow BLP and Berry (1994).

The researcher observes data from a single market with J products labeled 1 through J and an outside good labeled 0, and M consumers. Each product j has a price p_j, a vector of other characteristics observed by the researcher, $x_j \in \mathbb{R}^K$, and an unobserved variable ξ_j, which can be interpreted as a combination of unobserved product characteristics and aggregate preference shocks

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(p_j, x_j, and ξ_j are set to 0 for the outside good $j=0$). In addition, each individual consumer i has consumer specific unobserved components of demand ε_{ij} and ζ_i, which are independent and identically distributed (i.i.d.) across consumers. In what follows, x_j is assumed to contain a constant.

Consumer i's utility for the jth product is given by $u_{ij} = x_j'\beta - \alpha p_j + \xi_j + g(\varepsilon_{ij}, \zeta_i, x_j, p_j)$ for a known function g and parameters (α, β, σ), where $\{\varepsilon_{i,j}\}_{j=0}^J$ are independent of ζ_i, and the distribution of ζ_i is indexed by a parameter σ. The linear part is denoted by $\delta_j \equiv x_j'\beta - \alpha p_j + \xi_j$. Each consumer buys the product for which utility is the highest, and no consumer buys more than one product. Rather than individual purchasing decisions, we observe aggregate market shares, including the proportion of consumers who make no purchase (the share of the outside good). These come from aggregating purchasing decisions over the ε's and ζ's of all consumers. It is assumed that the number of consumers is large enough that sampling variation in market shares from realizations of the ε's and ζ's can be ignored, so that the market share of good j, $s_j(x, \xi, p)$, is equal to the population probability of choosing good j conditional on x, ξ, and p: $s_j(x, \xi, p) = E_{\varepsilon,\zeta} I(u_{ij} > u_{ik} \text{ all } k \neq j)$, where $E_{\varepsilon,\zeta}$ denotes expectation with respect to the distribution of $\{\varepsilon_{ij}\}_{j=0}^J$ and ζ_i. Since shares depend on ξ, α, and β only through δ, we can write them as $s_j(\delta, x, p, \sigma)$.

Most of this paper focuses on the static Bertrand supply model. There are F firms labeled 1 through F. Firm f produces the set of goods $\mathcal{F}_f \subseteq \{1, \dots, J\}$. We use the convention that firm 0 produces the outside good. Profits of firm f are given by $\sum_{k\in\mathcal{F}_f} p_k \cdot M s_k(x, p, \xi) - C_f(\{M \cdot s_k(x, p, \xi)\}_{k\in\mathcal{F}_f})$, where M is the number of consumers and C_f is firm f's cost function. Firms play a Nash–Bertrand equilibrium in prices, and rearranging the first order conditions for an interior best response gives

$$\sum_{k\in\mathcal{F}_f} (p_k - MC_k) \frac{\partial}{\partial p_j} s_k(x, p, \xi) + s_j(x, p, \xi) = 0 \tag{1}$$

for each product j owned by firm f.[5] For single product firms, this simplifies to

$$p_j = MC_j - \frac{s_j(x, p, \xi)}{\frac{d}{dp_j} s_j(x, p, \xi)}. \tag{2}$$

When new products are added to the demand system, the equilibrium price will change, so that the equilibrium price and share of good j depend on

[5]While the models of Sections 3.2, 3.3, and 3.4 have a unique equilibrium, results showing whether the pricing game in the random coefficients logit model has an equilibrium (or a unique one) in the general setting of Section 3.1 are, to my knowledge, not available in the literature. The results in that section hold for any sequence of equilibria as long as such a sequence exists, and do not impose uniqueness.

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the size of the market J. That is, even though x, MC, and ξ are sequences, prices, markups and market shares will be triangular arrays, so that a more precise notation for the equilibrium price of good j would be $p_{j,J}$ (in some cases, the instruments z_j defined below will change with J as well). To avoid extra subscripts, I use p_j to denote the price of good j when the context is clear.[6]

Finally, in all of the models below, I assume that the vector of unobserved demand shocks ξ is mean independent of observed product characteristics: $E(\xi|x) = 0$. This assumption is the exclusion restriction that provides the basis for the BLP instruments.

2.1. *Estimation*

In the models considered here, $s(\delta, x, p, \sigma)$ is invertible in its first argument (see Berry (1994), BLP). Letting $\delta(s, x, p, \sigma)$ denote the inverse with respect to the first argument, this leads to the equation

$$\delta_j(s, x, p, \sigma) = x_j'\beta - \alpha p_j + \xi_j, \tag{3}$$

which can potentially be used to estimate the demand parameters σ, β, and α. However, the parameter σ enters into a function with shares, which are endogenous. In addition, prices may be correlated with the unobserved ξ through at least two channels. First, ξ_j enters the markup $-s(\delta, x, p, \sigma)/\frac{d}{dp_j}s(\delta, x, p, \sigma)$. Second, ξ_j may be correlated with unobserved components of marginal costs if goods that are more desirable in unobserved ways are also more expensive to make in unobserved ways.

To overcome this endogeneity problem, one needs instruments that are uncorrelated with ξ and shift prices enough to identify α and σ. This paper focuses on instruments based on characteristics of other products. Since we are assuming that ξ_j is independent of the observed characteristics of all products, functions of characteristics of other products will satisfy the instrumental variables exclusion restriction. Since characteristics of other products enter price through the markup $-s(\delta, x, p, \sigma)/\frac{d}{dp_j}s(\delta, x, p, \sigma)$, they have the potential to shift prices enough to consistently estimate the model. Suppose that we

[6]While the entry decision is not explicitly modeled here, one could think of the market size J as being endogenously determined by firms' decisions of whether to pay a fixed cost of entering the market. If the number of consumers M is large relative to the entry cost, more firms will enter, and one can think of asymptotics in J as arising from asymptotics in M in a two stage model with endogenous entry (note that this interpretation requires that the information structure of the entry game is such that the exogeneity assumptions on x and ξ described below hold conditional on entry, which can be ensured by assuming that entry decisions are made before firms observe x, ξ, and marginal costs).

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use some vector-valued function $h_j(x_{-j})$ as excluded instruments. The parameter estimates minimize the generalized method of moments (GMM) criterion function

$$\left\| \frac{1}{J} \sum_{j=1}^{J} \left(\delta_j(s, x, p, \sigma) - x_j'\beta + \alpha p_j\right) z_j \right\|_{W_J}, \tag{4}$$

where $z_j = (x_j, h_j(x_{-j}))'$ and W_J is a positive definite weighting matrix with $W_J \xrightarrow{p} W$ for a strictly positive definite matrix W. Following common terminology, this paper refers to instruments of this form as product characteristic instruments or BLP instruments.

3. LARGE MARKET ASYMPTOTICS

I now turn to the question of the asymptotic behavior of demand estimates, particularly those based on product characteristic instruments, under large market asymptotics. I first state a general result relating the behavior of BLP instrument-based estimates to the asymptotic behavior of the markup. The remainder of this section is organized into subsections that show consistency or inconsistency of BLP instrument-based estimates for various settings using primitive conditions.

To give some motivation for the result, let us first consider a special case. Consider the simple logit model with many small firms in a single market. Consumer i's utility for product j takes the form $u_{i,j} = x_j'\beta - \alpha p_j + \varepsilon_{i,j} + \xi_j$, where $\varepsilon_{i,j}$ is distributed extreme value independently across products and consumers. This leads to shares taking the form $s_j(x, p, \xi) = \frac{\exp(x_j'\beta - \alpha p_j + \xi_j)}{\sum_k \exp(x_k'\beta - \alpha p_k + \xi_k)}$, which can be inverted to get (normalizing the mean utility of the outside good 0 to zero) $\log s_j - \log s_0 = x_j'\beta - \alpha p_j + \xi_j$. The derivative of firm j's share with respect to j's price is $\frac{d}{dp_j} s_j(x, p, \xi) = -\alpha s_j(x, p, \xi)(1 - s_j(x, p, \xi))$, which gives the Bertrand pricing formula, equation (2), as $p_j = MC_j + \frac{1}{\alpha(1-s_j)}$.

As long as shares converge to zero, markups of all products will converge to $1/\alpha$. If the markup were exactly equal to $1/\alpha$, product characteristic instruments would yield inconsistent estimates, since they must be correlated with markups to have identifying power. If the convergence of the markup to $1/\alpha$ is fast enough, one would expect this to be true for the actual sequence of markups. More generally, whenever the dependence of equilibrium markups on characteristics of other products decreases quickly enough with J, product characteristic instruments will lead to inconsistent estimates. The following theorem formalizes these ideas.[7]

[7]This theorem, which is used to derive inconsistency results in the random coefficients logit model in Section 3.1, applies to (α, β) with the parameter σ determining the random coeffi-

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THEOREM 1: *Let (x_j, ξ_j, MC_j) be i.i.d. with finite second moment, and let $(\hat{\alpha}, \hat{\beta}')$ be the IV estimates with σ fixed at its true value and instrument vector $z_j = (x_j, h_j(x_{-j}))$. Suppose the following:*

(i) *It holds that $\sqrt{J}\max_{1\leq j\leq J}|p_j - MC_j - b^*| \xrightarrow{p} 0$ for some constant b^*.*

(ii) *It holds that $\frac{1}{\sqrt{J}}\sum_{j=1}^{J}[z_j(x_j', MC_j, \xi_j) - Ez_j(x_j', MC_j, \xi_j)]$ converges to a nondegenerate normal distribution and $\frac{1}{J}\sum_{j=1}^{J}Eh_j(x_{-j})$ converges to some finite constant as $J \to \infty$.*

Let $(\hat{\alpha}^*, \hat{\beta}^{*\prime})$ be the same estimates obtained from data with p_j replaced by $p_j^* = MC_j + b^*$. Then $(\hat{\alpha}, \hat{\beta}')$ is inconsistent and $\|(\hat{\alpha}, \hat{\beta}) - (\hat{\alpha}^*, \hat{\beta}^{*\prime})\| \xrightarrow{p} 0$.

Theorem 1 states that, as long as markups converge to a constant at a faster than $1/\sqrt{J}$ rate, BLP instruments will lead to inconsistent estimates, even if σ is known and used in estimation. The conditions on markups are given as high level conditions, so that Theorem 1 does not require the specific structure of any of the demand specifications, supply models, or equilibrium assumptions used below. As long as the dependence of equilibrium markups on product characteristics decreases at a faster than $1/\sqrt{J}$ rate, BLP instruments will give inconsistent estimates.[8] In the simple logit model with single product firms, markups are given by $\frac{1}{\alpha(1-s_j)}$, which can be seen to converge to $1/\alpha$ at a $1/J$ rate as long as all of the market shares are roughly proportional to each other. The $1/J$ rate is fast enough to lead to inconsistent estimates by Theorem 1.

Section 3.1 gives a formal statement of these results in the random coefficients logit model, which generalizes the simple logit model used in the discussion above. Sections 3.2, 3.3, and 3.4 consider cases where the dependence of markups on product characteristics does not decrease or decreases slowly enough that BLP instruments have identifying power asymptotically. Section B in the Supplemental Material considers some other cases. In all of these cases, the unifying feature that determines whether the BLP instruments can lead to consistent estimates is whether the dependence of markups on product characteristics decreases at a slower rate than the square root of the total number of products.

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cients treated as known. Since the BLP instruments lead to inconsistent estimates in that setting, the (negative) message is essentially the same: even with σ known, product characteristic instruments do not give consistent estimates. However, extending consistency results such as those in Sections 3.2, 3.3, and 3.4 to more general specifications of random coefficients is left for future research.

[8]In the case with N markets $i = 1, \ldots, N$ with J_i products in market i and $N \to \infty$, condition (i) can be modified by replacing J with $\sum_{i=1}^{N} J_i$, and with b^* constant across both i and j, which leads to inconsistency when markups approach a constant more quickly than $1/\sqrt{\sum_{i=1}^{N} J_i}$ (see Section 3.4).

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3.1. *Random Coefficients Logit With Single Product Firms*

The random coefficients logit model, used by BLP, generalizes the simple logit, allowing for more general forms of consumer heterogeneity through random coefficients on the observed product characteristics x. Consumer i's utility for product j takes the form

$$u_{ij} = x_j'\beta - \alpha p_j + \xi_j + \sum_k x_{jk}\zeta_{ik} + \varepsilon_{ij} \equiv \delta_j + \sum_k x_{jk}\zeta_{ik} + \varepsilon_{ij},$$

where ζ_{ik} is a random coefficient on product characteristic k, and $\varepsilon_{i,j}$ is distributed extreme value independently across products and consumers. This specification assumes that there is no random coefficient on price. In practice, including a random coefficient on price can be important because of differences in price sensitivity among consumers. While it seems plausible that similar results will hold with a random coefficient on price, determining whether this is the case appears to require a nontrivial extension of the results below that does not yield to the same method of proof.

This section considers asymptotics in which the number of products increases with a single product per firm. The single product firm assumption is made for simplicity, and the results will be similar as long as products are added by increasing the number of firms rather than the number of products per firm (see Section B.3 of the Supplemental Material). It is also assumed that the dimension of the random coefficients ζ is fixed, and that new products differ only in drawing new characteristics (x_j and ξ_j) and $\varepsilon_{i,j}$ terms. As shown in Sections 3.2 and 3.3, increasing the dimension of the random coefficients or the number of products per firm can lead to dramatically different results.

Shares can be obtained by integrating the logit shares for fixed ζ, which gives, letting P_ζ be the probability measure of the random coefficients, $s_j = \int \tilde{s}_j(\delta, \zeta)\, dP_\zeta(\zeta)$, where $\tilde{s}_j(\delta, \zeta) = \frac{\exp(\delta_j + \sum_k x_{jk}\zeta_k)}{\sum_\ell \exp(\delta_\ell + \sum_k x_{\ell k}\zeta_k)}$. Differentiating under the integral and using the formulas for logit elasticities for fixed ζ gives $\frac{ds_j}{dp_j} = -\alpha \int \tilde{s}_j(\delta, \zeta)(1 - \tilde{s}_j(\delta, \zeta))\, dP_\zeta(\zeta)$ so that the Bertrand markup is $p_j - MC_j = \frac{\int \tilde{s}_j(\delta,\zeta)\, dP_\zeta(\zeta)}{\alpha \int \tilde{s}_j(\delta,\zeta)(1-\tilde{s}_j(\delta,\zeta))\, dP_\zeta(\zeta)}$. If the tails of ζ are thin enough, this can be shown to approach $1/\alpha$ quickly by truncating the integral and using bounds on the logit shares for ζ fixed, then arguing as in the simple logit model.

THEOREM 2: *In the random coefficients model with single product firms and no random coefficient on price, suppose that (x_j, ξ_j, MC_j) is bounded and i.i.d. over j, and that there exist μ and Σ such that $P(\|\zeta\| > t)$ is bounded by the corresponding tail probability for a $N(\mu, \Sigma)$ random variable. Then, for p_j arising from any sequence of Bertrand equilibria, condition* (i) *in Theorem* 1 *holds with*

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$b^* = 1/\alpha$. *Thus, under condition* (ii) *in Theorem* 1, *the BLP instrument-based estimator with* σ *known will lead to inconsistent estimates*.

Theorem 2 shows that product characteristics lead to inconsistent estimates of α and β even if the nonlinear parameter σ is known and used in estimation. The boundedness condition on (x_j, ξ_j, MC_j) is imposed for simplicity, and can be replaced by an exponential tail condition (see Section A.2 in the Supplemental Material).

3.2. *Nested Logit With Many Nests*

The results of Section 3.1 show that markups converge to a constant as $J \to \infty$ when a new idiosyncratic term $\varepsilon_{i,j}$ is added for each product, with the distribution of the random coefficients ζ staying the same. One way to avoid this negative result is to increase the dimension of ζ as the number of products increases. Increasing the dimension of ζ in a completely unrestricted way, one would end up with an increasing number of parameters, which leads to its own problems. Section B.1 of the Supplemental Material considers the nested logit model, a special case of the random coefficients logit model in which products are placed in groups, and the random coefficients are associated with group indicator variables. The nested logit model places enough structure on the random coefficients that the number of groups can be increased without increasing the number of parameters that need to be estimated. The results in Section B.1 show that under asymptotics where the dimension of ζ is increased by adding more groups, BLP instruments can have power asymptotically even with single product firms.

It should be emphasized that the nested logit model is used for tractability and is intended to illustrate the point that the negative results of Section 3.1 can be reversed if the dimension of the random coefficients increases with the number of products while the dimension of the parameter space stays fixed. One could likely achieve a similar goal through other specifications of random coefficients (for example, starting with the nested logit model with many nests and adding some random coefficients to continuous variables, as in Grigolon and Verboven (2013)), although such an approach will still require constraints on the joint distribution of random coefficients to keep the number of parameters from increasing.

3.3. *Logit With Many-Product Firms*

Now consider the logit model with multi-product firms, with the number of firms F fixed and asymptotics taken in the number of products per firm. The own and cross price elasticities can be shown to take the form $\frac{d}{dp_j}s_j(x, p, \xi) = -\alpha s_j(x, p, \xi)(1 - s_j(x, p, \xi))$ and $\frac{d}{dp_k}s_j(x, p, \xi) = \alpha s_j(x, p, \xi)s_k(x, p, \xi)$, respectively. Plugging these into the equilibrium pricing equations (1) and rearranging gives the markup of product j produced by firm f as $p_j - MC_j =$

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$\frac{1}{\alpha} + \sum_{k\in\mathcal{F}_f}(p_k - MC_k)s_k$, so that markups are constant within a firm (see Konovalov and Sandor (2010)). Letting b_f be the common markup for firm f, this gives a system of equations that define b_f for $f = 1, \dots, F$. Rearranging and plugging in the formula for shares yields

$$(5)\qquad b_f = \frac{1}{\alpha}\frac{\sum_{h=0}^{F}\sum_{k\in\mathcal{F}_h}\exp(x_k'\beta - \alpha MC_k + \xi_k - \alpha b_h)}{\sum_{h\neq f}\sum_{k\in\mathcal{F}_h}\exp(x_k'\beta - \alpha MC_k + \xi_k - \alpha b_h)}$$

$$= \frac{1}{\alpha}\frac{\sum_{h=0}^{F}\exp(-\alpha b_h)\hat{\pi}_h\bar{r}_h}{\sum_{h\neq f}\exp(-\alpha b_h)\hat{\pi}_h\bar{r}_h},$$

where $\hat{\pi}_f \equiv |\mathcal{F}_f|/J$ is the proportion of products produced by firm f and $\bar{r}_f$ is an average of the characteristics of firm f's products given by $\bar{r}_f \equiv \frac{1}{|\mathcal{F}_f|}\sum_{k\in\mathcal{F}_f}\exp(x_k'\beta - \alpha MC_k + \xi_k)$ (following the convention that firm 0 produces the outside good 0, we set $b_0 = 0$, $\hat{\pi}_0 = 1/J$, and $\bar{r}_0 = 1$).

Under a law of large numbers, $\bar{r}_f$ will converge to some $\mu_{r,f}$ for each firm f. Assuming that $\hat{\pi}_f$ also converges to some π_f for each firm f, this suggests that equilibrium prices will be determined asymptotically by the solution to equation (5) with $\bar{r}_f$ and $\hat{\pi}_f$ replaced by $\mu_{r,f}$ and π_f. This is formalized in the following theorem.

THEOREM 3: *In the simple logit model with asymptotics in the number of products per firm, suppose that (x_j, ξ_j, MC_j) is independent across all j and identically distributed within each firm with finite variance. Let $z_j = (x_j, \frac{1}{J}\sum_{k\in\mathcal{F}_f}\tilde{h}(x_k))$ for some function $\tilde{h}$ for product j owned by firm f. Let $\mu_{r,f} = E\exp(x_j'\beta - \alpha MC_j + \xi_j)$, $\mu_{h,f} = E\tilde{h}(x_j)$ and $V_f = E\binom{x_j}{\pi_f\mu_{h,f}}(x_j', \pi_f\mu_{h,f}')\xi_j^2$ for $j \in \mathcal{F}_f$ (where these quantities are assumed to be finite), and suppose $\hat{\pi}_f \to \pi_f > 0$ for some π_f for each $f = 1, \dots, F$. Let $(b_1^*, \dots, b_F^*)$ be the unique*[9] *solution to* (5) *with $\hat{\pi}_f\bar{r}_f$ replaced by $\pi_f\mu_{r,f}$ for each firm f, and let $p_j^* = MC_j + b_f^*$ for product j produced by firm f, and let $(\hat{\alpha}, \hat{\beta}')$ be the estimators defined by* (4) *with these instruments, and let $(\hat{\alpha}^*, \hat{\beta}^{*\prime})$ be defined in the same way, but with p_j^* replacing p_j. Then, if*

[9]Uniqueness follows from arguments in Konovalov and Sandor (2010); see the proof in Section A.3 in the Supplemental Material.

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$\frac{1}{J}\sum_{j=1}^{J} E z_j(x_j', p_j^*) \to M_{zx}$ *for a positive definite matrix* M_{zx},

$$\sqrt{J}\big[(\hat{\beta}', -\hat{\alpha})' - (\beta', -\alpha)'\big] \xrightarrow{d} N\left(0, (M_{zx}' W M_{zx})^{-1} M_{zx}' W \left(\sum_{f=1}^{F} \pi_f V_f\right) W M_{zx} (M_{zx}' W M_{zx})^{-1}\right),$$

and the same holds for $(\hat{\beta}^{*\prime}, -\hat{\alpha}^*)$.

Theorem 3 shows that product characteristic instruments can have identifying power in this setting if they exploit variation in $\pi_f \mu_{r,f}$ across firms. Thus, BLP instruments can have power through variation across firms, but not within firms.

3.4. *Many Large Markets*

According to the results of Section 3.1, the BLP instruments lose power in a single market (or any fixed number of markets) with many firms fast enough that estimates based on them are inconsistent. In contrast, BLP instruments will typically provide enough variation to consistently estimate these models if the market size is bounded and asymptotics are taken in the number of markets. This section considers the intermediate case where both the number of products and markets are allowed to go to infinity. To simplify the analysis, attention is restricted to the simple logit model with no random coefficients.

Some additional notation is needed to describe the results with many markets. We consider N markets, with J_i products in market i. Notation is otherwise the same as described in Section 2, except that prices, product characteristics, and so forth are now indexed by the market i as well as the product j, so that $p_{i,j}$ denotes the price of product j in market i (as before, the dependence of $p_{i,j}$ on the total market size J_i is suppressed in the notation). Let $\bar{J} = \frac{1}{N}\sum_{i=1}^{N} J_i$ be the average number of products per market. Under the asymptotics in this section, each J_i (and therefore $\bar{J}$) increases with the number of markets N, but the dependence of $\bar{J}$ and the J_i's on N is suppressed in the notation.

Let $v_N = \frac{1}{N}\sum_{i=1}^{N}(J_i - \bar{J})^2/\bar{J}^2$ and $m_3 = \frac{1}{N}\sum_{i=1}^{N}(J_i/\bar{J})^3$. Here, the J_i's are nonrandom, but v_N can be thought of as the normalized sample variance of the J_i's. The following theorem derives the asymptotic behavior of estimates based on the BLP instruments in the case where v_N is bounded away from zero.

THEOREM 4: *In the simple logit model with many large markets, suppose that* $(x_{i,j}, \xi_{i,j}, MC_{i,j})$ *is bounded and i.i.d. across i and j. Let* $(\hat{\alpha}, \hat{\beta})$ *be IV estimates with instrument vector* $z_{i,j} = (x_{i,j}, \frac{1}{J}\sum_{k\neq j}\tilde{h}(x_{i,k}))$ *for some finite variance function* $\tilde{h}$. *Suppose that* $\min_{1\leq i\leq N} J_i \to \infty$ *and that* $v_N \to v$ *for some* $v > 0$ *and* m_3

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converges to a finite constant. Then, if $N/\bar{J} \to \infty$, $(\hat{\alpha}, \hat{\beta})$ will be consistent and asymptotically normal, with a $1/\sqrt{N/\bar{J}}$ rate of convergence and asymptotic variance given in the Supplemental Material. If $N/\bar{J} \to c$ for some c, then $(\hat{\alpha}, \hat{\beta})$ will be inconsistent, and will follow a weak instrument asymptotic distribution given in the Supplemental Material. The weak instrument asymptotic distribution coincides with what would be obtained with markups equal to $1/\alpha$ in the case where $c = 0$.

Theorem 4 states that as long as there is sufficient variation in the number of products per market, the BLP instruments will use this variation to obtain consistent estimates at a rate $1/\sqrt{N/\bar{J}}$ as long as $N/\bar{J} \to \infty$. If $N/\bar{J} \to c$ for some finite c, the results give "weak instrument" asymptotics, in which the estimates do not converge and follow a nonstandard asymptotic distribution. The dependence on whether $N/\bar{J} \to \infty$ comes from an extension of condition (i) in Theorem 1 to the many market case. In general, BLP instruments require that the markup not converge to a constant more quickly than $1/\sqrt{N\bar{J}}$. Since the logit markups in market i converge to $1/\alpha$ at a $1/J_i$ rate, this means that the estimates will be inconsistent if $\sqrt{N\bar{J}}/J_i$ converges to zero uniformly over i, which (assuming $\bar{J}/J_i$ is bounded) gives the $N/\bar{J} \to 0$ condition for inconsistency and asymptotic equivalence with constant markups. Note that Theorem 4 shows that the identifying power of BLP instruments in this setting relies on variation in market size (v_N must not converge to zero). If one is not comfortable using this variation to identify demand, (for example, because of issues raised by Ackerberg and Rysman (2005)),[10] one will want to look for other instruments or a different specification.

The assumption that the data generating process for marginal costs is the same across markets includes an important assumption about how the cost function varies with market size. If marginal cost varies systematically with J_i (depending on how J_i varies with the total number of consumers in a market, this could arise from returns to scale), one can use J_i or $\sum_{k\neq j} \tilde{h}(x_{i,k})$ as cost instruments, whether or not they are correlated with markups. It should also be noted that, while Theorem 4 gives the rate at which the J_i's must increase when $\frac{1}{J} \sum_{k\neq j} \tilde{h}(x_{i,k})$ is used as the excluded instrument, other forms of BLP instruments (i.e., other functions of $\{x_{i,k}\}_{k=1}^{J_i}$) may lead to consistent estimates under weaker conditions.

[10]Ackerberg and Rysman (2005) argue that demand models such as logit and random coefficients logit place strong restrictions on how demand varies with the number of products in a market. Using product congestion as a motivation, they propose a model that leads to the number of products in a market entering mean utility directly, thereby weakening these restrictions and giving a model where variation in the number of products in a market does not have identifying power.

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4. DIAGNOSIS AND RECOMMENDATIONS FOR EMPIRICAL PRACTICE

Theorem 1 and the results in Section 3.1 state that certain specifications of supply and demand lead to BLP instruments performing poorly under certain asymptotics. On the other hand, the results in Sections 3.2, 3.3, and 3.4 show that more positive results are possible in essentially the same model, depending on the relative rate at which the number of products, firms, and markets increases. This raises the question of how one can determine which asymptotics are relevant for a given specification and data set and, in particular, whether one should worry that the asymptotic results of Theorem 1 are relevant.

As discussed in the Introduction, Theorem 1 has two consequences: (i) any valid $1-\alpha$ CI will contain the entire parameter space with asymptotic probability $1-\alpha$, making it no better than one based on tossing a coin, and (ii) standard CIs (e.g., a CI formed by adding and subtracting a normal quantile times the standard error) will undercover. To phrase the issue in terms of tests rather than CIs, (i) is an issue of statistical power (type II error), while (ii) is an issue of size (type I error).

In cases that are intermediate between the asymptotics of Theorem 1 and many market asymptotics, issue (i) may show up in a less extreme form. For example, a 95% confidence interval may cover the entire parameter space with probability 93% (which would still be a very negative result), or it may cover the entire parameter space with probability, say, 70%, while being reasonably informative with probability 30% (which, depending on the researcher's preferences, may be good enough to proceed). To examine this issue in finite samples for a particular data set, one can perform a Monte Carlo analysis to see whether it is possible to obtain informative CIs when prices are generated from some plausible supply side model and data generating process for model primitives. If the estimates and tests based on BLP instruments perform well enough in the Monte Carlos, the researcher can conclude that the negative results of Theorem 1 and Section 3.1 are not relevant. Note that this recommendation is an application of standard statistical practice: when possible, statistical tests and CIs should be accompanied with an analysis of type II error (see Cohen (1988), Ioannidis (2005)).

To deal with the second issue (undercoverage), one can report a weak instrument robust CI. When the conditions of Theorem 1 hold, the CI will have correct coverage, but will be asymptotically equivalent to a CI generated from a coin toss. A weak instrument robust CI can be formed by inverting weak instrument robust tests designed for GMM, such as those proposed by Kleibergen (2005).

Alternatively, one can use a two-step procedure, in which one first tests for identification and then reports a standard CI (formed by adding and subtracting a normal quantile times the standard error from the estimate) if evidence

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of identification is found.[11] See Wright (2003) and Andrews (2014) for identification tests for GMM models. As these tests are somewhat complicated, one can form a simpler test by treating σ as known, which leads to a linear IV model with a single endogenous variable: price. In this model, testing for identification corresponds to a standard F test: run an ordinary least squares (OLS) regression of p_j on $(x_j, h_j(x_{-j}))$, and perform an F test of the null hypothesis that the coefficients on $h_j(x_{-j})$ are zero. If one rejects with this test, this can be taken as evidence that the conclusion of Theorem 1 does not describe the data generating process.[12]

5. MONTE CARLO

This section presents the results of a Monte Carlo study of the random coefficients logit model of Section 3.1. The performance of the BLP instruments and of cost instruments is examined over a range of specifications for the number of markets, the number of products per market, and variation in the number of products per market. The data generating process for the Monte Carlo data sets is as follows. Prices are generated from a Bertrand equilibrium. For the case where the number of products per market varies, approximately 1/3 of markets have 20 products, another 1/3 of markets have 60 products, and the remaining markets have 100 products. For the case where the number of products per firm varies, approximately 1/3 of markets have 2 products per firm, another 1/3 of markets have 5 products per firm, and the remaining markets have 10 products per firm. The variable $x_{i,j}$ contains a constant and a uniform $(0, 1)$ random variable. I generate the cost shifter, $z_{i,j}$, as another uniform random variable independent of $x_{i,j}$. Marginal cost is given by $MC_{i,j} = (x'_{i,j}, z'_{i,j})\gamma + \eta_{i,j}$ for $\eta_{i,j}$ defined as follows. To generate η and ξ, I generate three independent uniform $(0, 1)$ random variables $u_{1,i,j}$, $u_{2,i,j}$, and $u_{3,i,j}$, and set $\xi_{i,j} = u_{1,i,j} + u_{3,i,j} - 1$ and $\eta_{i,j} = u_{1,i,j} + u_{2,i,j} - 1$. The variables $x_{i,j}$, $\xi_{i,j}$, and $\eta_{i,j}$ are independent across products j. Utility is given by the random coefficients model of Section 3.1, with the random coefficient on the covariate given by a $N(0, \sigma^2)$ random variable, where σ^2 is set to 9 and is estimated in

[11]If the first stage test does not find evidence of identification, one must report a weak IV robust CI.

[12]Note, however, that the results of this theorem will still be relevant if the correlation between markups and instruments would be small for data generated by the supply and demand model used by the researcher with the given N and J. In this case, rejecting with this test must be taken as evidence that the model is misspecified, since it does not match the correlation implied by the model. Thus, if one is worried about low correlation between markups and prices, and one has a particular supply side model in mind, it is good practice to supplement identification tests with specification tests that include supply side moments, so as to ensure that evidence of identification (correlation between BLP instruments and prices) is consistent with the correlation between BLP instruments and markups that the model would generate.

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TABLE I

MONTE CARLO RESULTS FOR BLP INSTRUMENTS (10 PRODUCTS PER FIRM, $\beta = (3, 6)$)

Markets	Products per Market	Median Bias of $\hat{\alpha}$	Median Absolute Deviation From α_0	Rejection Prob. at True α	Power of Test of $\alpha = 0$
1	60	−0.3698	0.6691	0.0783	0.1004
1	100	−0.3648	0.7177	0.1211	0.1381
3	20	−0.1053	0.3358	0.0390	0.3980
3	Varied	−0.0494	0.2966	0.1523	0.4649
3	60	−0.2186	0.5827	0.0200	0.1040
3	100	−0.2525	0.6383	0.1351	0.1762
20	20	0.0039	0.1140	0.0390	0.9880
20	Varied	−0.0014	0.1001	0.0400	0.9960
20	60	0.0130	0.2345	0.0230	0.7710
20	100	−0.0379	0.3154	0.0200	0.4560

the Monte Carlos. The parameters are given by $\alpha = 1$ and $\gamma = (2, 1, 1)'$ (where the last element of γ is the coefficient of the excluded cost instrument), with β taking different values depending on the design. See the Supplemental Material for additional details and results for additional specifications.[13]

Tables I, II, and III show the results for BLP instruments and cost instruments for several Monte Carlo designs. Results are reported for estimates of the price coefficient α and for a nominal level 0.05 two-sided test for α. The Monte Carlo results appear consistent with the overall result that BLP instruments perform poorly for this demand specification when the number of products is large enough relative to the number of markets, and that cost instruments do not suffer from these issues. In Table I, estimates that use BLP instruments have substantial bias and variability when the number of products per market is large and the number of markets is small (as measured by median

TABLE II

MONTE CARLO RESULTS FOR BLP INSTRUMENTS (20 MARKETS, 100 PRODUCTS PER MARKET)

β	Products per Firm	Median Bias	Median Absolute Deviation From α_0	Rejection Prob. at True α	Power of Test of $\alpha = 0$
(3, 1)	10	−0.2928	0.8090	0.1012	0.1273
(3, 1)	2	−0.3573	0.7653	0.0991	0.1241
(3, 1)	Varied	−0.0054	0.1895	0.0380	0.8440
(3, 6)	2	−0.2191	0.6897	0.1061	0.1632
(3, 6)	Varied	−0.0190	0.1659	0.0410	0.9450

[13] See also the contemporaneous work of Skrainka (2012) and Conlon (2013) for additional Monte Carlo results for BLP and cost instruments with prices generated from equilibrium play.

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TABLE III

MONTE CARLO RESULTS FOR COST INSTRUMENTS (10 PRODUCTS PER FIRM, $\beta = (3, 6)$)

Markets	Products per Market	Median Bias of $\hat{\alpha}$	Median Absolute Deviation From α_0	Rejection Prob. at True α	Power of Test of $\alpha = 0$
1	60	−0.0247	0.1749	0.1130	0.6710
1	100	−0.0196	0.1358	0.0762	0.7623
3	20	−0.0262	0.1837	0.1002	0.6092
3	Varied	−0.0122	0.1000	0.0852	0.7916
3	60	−0.0102	0.1007	0.0661	0.7768
3	100	−0.0054	0.0767	0.0662	0.8175
20	20	0.0065	0.0663	0.0220	0.7840
20	Varied	−0.0008	0.0385	0.0522	0.8554
20	60	−0.0023	0.0365	0.0641	0.8707
20	100	−0.0027	0.0298	0.0481	0.8826

bias and median absolute deviation from the true value), and perform better when the number of products per market is small. In contrast, cost instruments lead to relatively precise estimates in large market settings. Table II fixes the number of products and markets and explores how variation in other aspects of the design affects the performance of the BLP instruments. While the BLP instruments work well in some cases, the results can be very bad depending on the ownership structure and the coefficient of the product characteristic in the demand specification.

6. CONCLUSION

This paper derives asymptotic approximations for differentiated products demand estimators when the number of products is large. The question of whether product characteristic instruments have nontrivial identifying power is addressed through asymptotic correlation with equilibrium markups derived from a full model of supply and demand. The results show that certain supply and demand models constrain these instruments to have trivial power under large market asymptotics, and should therefore be avoided in cases where these asymptotic results are relevant. Other asymptotic settings (demand models and ways to add products) are shown to lead to consistent estimates and standard asymptotic distributions under large market asymptotics. The results can be used as a guide to forming a model of demand in a large market setting that is consistent with finding identification through product characteristic instruments and variation in the markups. A Monte Carlo study shows that the asymptotic results are a good description of finite sample settings of practical importance.

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REFERENCES

ACKERBERG, D. A., AND M. RYSMAN (2005): "Unobserved Product Differentiation in Discrete-Choice Models: Estimating Price Elasticities and Welfare Effects," *The RAND Journal of Economics*, 36 (4), 771–788. [1965,1974]

ANDREWS, I. (2014): "Robust Two-Step Confidence Sets, and the Trouble With the First Stage F-Statistic," Unpublished Manuscript. [1976]

ARMSTRONG, T. B. (2016): "Supplement to 'Large Market Asymptotics for Differentiated Product Demand Estimators With Economic Models of Supply'," *Econometrica Supplemental Material*, 84, http://dx.doi.org/10.3982/ECTA10600. [1965]

BAJARI, P., AND C. L. BENKARD (2003): "Discrete Choice Models as Structural Models of Demand: Some, Economic Implications of Common Approaches," Unpublished Manuscript. [1965]

——— (2005): "Demand Estimation With Heterogeneous Consumers and Unobserved Product Characteristics: A Hedonic Approach," *Journal of Political Economy*, 113 (6), 1239–1276. [1964]

BERRY, S., J. LEVINSOHN, AND A. PAKES (1995): "Automobile Prices in Market Equilibrium," *Econometrica*, 63 (4), 841–890. [1961]

BERRY, S., O. B. LINTON, AND A. PAKES (2004): "Limit Theorems for Estimating the Parameters of Differentiated Product Demand Systems," *The Review of Economic Studies*, 71 (3), 613–654. [1964]

BERRY, S. T. (1994): "Estimating Discrete-Choice Models of Product Differentiation," *The RAND Journal of Economics*, 25 (2), 242–262. [1965,1967]

BRESNAHAN, T. F. (1987): "Competition and Collusion in the American Automobile Industry: The 1955 Price War," *The Journal of Industrial Economics*, 35 (4), 457–482. [1961]

BRESNAHAN, T. F., S. STERN, AND M. TRAJTENBERG (1997): "Market Segmentation and the Sources of Rents From Innovation: Personal Computers in the Late 1980s," *The RAND Journal of Economics*, 28, S17–S44. [1962]

CAPLIN, A., AND B. NALEBUFF (1991): "Aggregation and Imperfect Competition: On the Existence of Equilibrium," *Econometrica*, 59 (1), 25–59. [1965]

COHEN, J. (1988): *Statistical Power Analysis for the Behavioral Sciences*. Hillsdale, NJ: Lawrence Erlbaum Associates. [1975]

CONLON, C. T. (2012): "A Dynamic Model of Prices and Margins in the LCD TV Industry," Manuscript, Columbia University. [1962]

——— (2013): "The Empirical Likelihood MPEC Approach to Demand Estimation," SSRN Scholarly Paper ID 2331548, Rochester, NY. [1964,1977]

DUBÉ, J.-P., J. T. FOX, AND C.-L. SU (2012): "Improving the Numerical Performance, Static and Dynamic Aggregate Discrete Choice Random Coefficients Demand Estimation," *Econometrica*, 80 (5), 2231–2267. [1964]

EIZENBERG, A. (2011): "Upstream Innovation and Product Variety in the U.S. Home PC Market," SSRN Scholarly Paper ID 1760828, Rochester, NY. [1962]

FREYBERGER, J. (2015): "Asymptotic Theory for Differentiated Products Demand Models With Many Markets," *Journal of Econometrics*, 185 (1), 162–181. [1964]

GABAIX, X., D. LAIBSON, D. LI, H. LI, S. RESNICK, AND C. G. DE VRIES (2013): "The Impact of Competition on Prices With Numerous Firms," Discussion Paper. [1965]

GRIGOLON, L., AND F. VERBOVEN (2013): "Nested Logit or Random Coefficients Logit? A Comparison of Alternative Discrete Choice Models of Product Differentiation," *Review of Economics and Statistics*, 96 (5), 916–935. [1971]

IOANNIDIS, J. P. A. (2005): "Why Most Published Research Findings Are False," *PLOS Medicine*, 2 (8), e124. [1975]

KLEIBERGEN, F. (2005): "Testing Parameters in GMM Without Assuming That They Are Identified," *Econometrica*, 73 (4), 1103–1123. [1963,1975]

KONOVALOV, A., AND Z. SANDOR (2010): "On Price Equilibrium With Multi-Product Firms," *Economic Theory*, 44 (2), 271–292. [1965,1972]

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PETRIN, A. (2002): "Quantifying the Benefits of New Products: The Case of the Minivan," *Journal of Political Economy*, 110 (4), 705–729. [1962]

PINKSE, J., AND M. E. SLADE (2004): "Mergers, Brand Competition, and the Price of a Pint," *European Economic Review*, 48 (3), 617–643. [1964]

——— (2010): "The Future of Spatial Econometrics," *Journal of Regional Science*, 50 (1), 103–117. [1963,1964]

REYNAERT, M., AND F. VERBOVEN (2012): "Improving the Performance of Random Coefficients Demand Models: The Role of Optimal Instruments," Discussion Paper. [1964]

ROMEO, C. (2010): "Filling out the Instrument Set in Mixed Logit Demand Systems for Aggregate Data," Discussion Paper. [1965]

SKRAINKA, B. S. (2012): "A Large Scale Study of the Small Sample Performance of Random Coefficient Models of Demand," SSRN Scholarly Paper ID 1942627, Rochester, NY. [1977]

STOCK, J. H., J. H. WRIGHT, AND M. YOGO (2002): "A Survey of Weak Instruments and Weak Identification in Generalized Method of Moments," *Journal of Business and Economic Statistics*, 20 (4), 518–529. [1963,1964]

VIVES, X. (2001): *Oligopoly Pricing*. MIT Press, Cambridge. [1965]

WRIGHT, J. H. (2003): "Detecting Lack of Identification in Gmm," *Econometric Theory*, 19 (2), 322–330. [1976]

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