Subjective Parental Beliefs: Their Measurement and Role

Orazio P. Attanasio (Yale/IFS/NHH/NBER)
Flávio Cunha (Rice)
Pamela Jervis (Universidad de Chile/IFS)

Household decision making and human capital in life cycle models

DSE - Bonn University

August 19th and 20th, 2021

Outline

Introduction

- Substantial body of research on the development of human capital:
  - Almond and Currie (2011).
  - Cunha et al. (2006), Cunha and Heckman (2008), Cunha et al. (2010).
  - The Lancet Series (2007, 2010).
  - Developing countries: Attanasio et al. (2013), Grantham-McGregor et al. (2012), Helmers and Patnam (2011)...

Introduction

- Substantial body of research on the development of human capital:
  - Almond and Currie (2011).
  - Cunha et al. (2006), Cunha and Heckman (2008), Cunha et al. (2010).
  - The Lancet Series (2007, 2010).
  - Developing countries: Attanasio et al. (2013), Grantham-McGregor et al. (2012), Helmers and Patnam (2011)...
- Human capital formation is a complex process:
  - Human capital is multi-dimensional (cognitive, non-cognitive, health...);
  - Skill formation is a dynamic process;
  - Dimensions of human capital interact both within and across periods;
  - Both genes and the environment are important inputs.

Introduction

- Key research questions:
  - $\Rightarrow$ How does human capital develop?
  - $\Rightarrow$ How persistent is human capital?
  - $\Rightarrow$ What role different factors play?
  - $\Rightarrow$ What role, if any, can policy play to remedy early deficiencies among children?
  - $\Rightarrow$,,, and how do these effective policies work?
  - $\Rightarrow$ What is the role of parental investment and how does it interact with other inputs?

Introduction

- Key research questions:
  - ⇒ How does human capital develop?
  - ⇒ How persistent is human capital?
  - ⇒ What role different factors play?
  - ⇒ What role, if any, can policy play to remedy early deficiencies among children?
  - ⇒ ,,, and how do these effective policies work?
  - ⇒ What is the role of parental investment and how does it interact with other inputs?
  - ⇒ What determines parental investment?

Introduction

- The goal of this paper is understanding parental behaviour.
- Parental investment will depend:
  - Resources;
  - Preferences;
  - Beliefs (perception of expected returns of investment).

Introduction

- The goal of this paper is understanding parental behaviour.
- Parental investment will depend:
  - Resources;
  - Preferences;
  - Beliefs (perception of expected returns of investment).
- We will focus in particular on the role of beliefs.

Introduction

- The goal of this paper is understanding parental behaviour.
- Parental investment will depend:
  - Resources;
  - Preferences;
  - Beliefs (perception of expected returns of investment).
- We will focus in particular on the role of beliefs.
- Do parents know the production function?

Background

- A few years ago, a large clustered RCT of a parenting stimulation program was ran in Colombia.

Background

- A few years ago, a large clustered RCT of a parenting stimulation program was ran in Colombia.
- That parenting stimulation program had some impacts on a variety of outcomes.
- These results were published in a paper in the BMJ, 2013.

Background

- A few years ago, a large clustered RCT of a parenting stimulation program was ran in Colombia.
- That parenting stimulation program had some impacts on a variety of outcomes.
- These results were published in a paper in the BMJ, 2013.
- The next step in the research agenda was to understand how those outcomes were achieved.
- We estimated a production function for child development.

Background

- A few years ago, a large clustered RCT of a parenting stimulation program was ran in Colombia.
- That parenting stimulation program had some impacts on a variety of outcomes.
- These results were published in a paper in the BMJ, 2013.
- The next step in the research agenda was to understand how those outcomes were achieved.
- We estimated a production function for child development.
- We took into account the endogeneity of parental investment but:
  - Did not use the RCT as an instrument (allowing the intervention a direct effect on child developemt);

Background

- A few years ago, a large clustered RCT of a parenting stimulation program was ran in Colombia.
- That parenting stimulation program had some impacts on a variety of outcomes.
- These results were published in a paper in the BMJ, 2013.
- The next step in the research agenda was to understand how those outcomes were achieved.
- We estimated a production function for child development.
- We took into account the endogeneity of parental investment but:
  - Did not use the RCT as an instrument (allowing the intervention a direct effect on child developemt);
  - Did not require parents to know the exact nature of the production function of child development.
- The paper is now published in the AER, 2019.

Summary of the results on the Colombian intervention

1. The technology of skill formation is well approximated by a Cobb Douglas.
2. Its parameters are not affected by the intervention.

Summary of the results on the Colombian intervention

1. The technology of skill formation is well approximated by a Cobb Douglas.
2. Its parameters are not affected by the intervention.
3. Human capital at the beginning of the period matter.

Summary of the results on the Colombian intervention

1. The technology of skill formation is well approximated by a Cobb Douglas.
2. Its parameters are not affected by the intervention.
3. Human capital at the beginning of the period matter.
4. Parental investment also matters.
5. Allowing for endogenous parental investment may be important but it is difficult.
    - Evidence of compensating parental investment.
    - Parental background has mainly an effect through parental investment.

Summary of the results on the Colombian intervention

1. The technology of skill formation is well approximated by a Cobb Douglas.
2. Its parameters are not affected by the intervention.
3. Human capital at the beginning of the period matter.
4. Parental investment also matters.
5. Allowing for endogenous parental investment may be important but it is difficult.
   - Evidence of compensating parental investment.
   - Parental background has mainly an effect through parental investment.
6. The program shifts significantly the distribution of parental investment .
7. The program does not seem to have a direct effect on child development.

Interpreting the results of the Colombian intervention

- The results indicated that the impact was explained almost exclusively by an increase in parental investment.

Interpreting the results of the Colombian intervention

- The results indicated that the impact was explained almost exclusively by an increase in parental investment.
- Why did parents increase investment?

Interpreting the results of the Colombian intervention

- The results indicated that the impact was explained almost exclusively by an increase in parental investment.
- Why did parents increase investment?
- Could it be because parents change their perception of the technology of skill formation?

Interpreting the results of the Colombian intervention

- The results indicated that the impact was explained almost exclusively by an increase in parental investment.
- Why did parents increase investment?
- Could it be because parents change their perception of the technology of skill formation?
- In this paper we use data that elicit information about beliefs directly.

Contribution

- Our contributions has two original elements:
  1. We propose a methodology to elicit subjective beliefs using a measrement system.
  2. We se the beliefs data to estimate a subjective production function of HK that can be compared to tha objective production function.

Collecting and using data on subjective beliefs

We designed a module that elicited parental subjective beliefs about the process of child development.

Collecting and using data on subjective beliefs

We designed a module that elicited parental subjective beliefs about the process of child development.

We will:

- Describe the beliefs elicitation survey instrument;

Collecting and using data on subjective beliefs

We designed a module that elicited parental subjective beliefs about the process of child development.

We will:

- Describe the beliefs elicitation survey instrument;
- Estimate the subjective expected returns to parental investment;
- Analyse the cross sectional variation of subjective expected returns;

Collecting and using data on subjective beliefs

We designed a module that elicited parental subjective beliefs about the process of child development.

We will:

- Describe the beliefs elicitation survey instrument;
- Estimate the subjective expected returns to parental investment;
- Analyse the cross sectional variation of subjective expected returns;
- Estimate the *perceived* technology of skill formation;
- Compare the ‘true’ and perceived technology of skill formation;

Collecting and using data on subjective beliefs

We designed a module that elicited parental subjective beliefs about the process of child development.

We will:

- Describe the beliefs elicitation survey instrument;
- Estimate the subjective expected returns to parental investment;
- Analyse the cross sectional variation of subjective expected returns;
- Estimate the *perceived* technology of skill formation;
- Compare the ‘true’ and perceived technology of skill formation;
- Relate perceived returns and perceived technology parameters to actual investment choices.

Collecting and using data on subjective beliefs

We designed a module that elicited parental subjective beliefs about the process of child development.

We will:

- Describe the beliefs elicitation survey instrument;
- Estimate the subjective expected returns to parental investment;
- Analyse the cross sectional variation of subjective expected returns;
- Estimate the *perceived* technology of skill formation;
- Compare the 'true' and perceived technology of skill formation;
- Relate perceived returns and perceived technology parameters to actual investment choices.
- Without data on beliefs we cannot distinguish different hypotheses to explain changes in investment behaviour:

Outline

What drives parental investment?

- The evidence from the parenting stimulation program indicates that parents invest ‘too little’;
- This is consistent with some anecdotal evidence;

What drives parental investment?

- The evidence from the parenting stimulation program indicates that parents invest 'too little';
- This is consistent with some anecdotal evidence;
- It is also consistent with anthropological and sociological studies in the US:
    - Annette Lareau (2003) *Unequal Childhoods: Class, Race, and Family Life*.
    - Robert Putnam (2015) *Our Kids*.
- Concerted cultivation vs pursue of natural growth.

What drives parental investment?

- In a standard model, determinants of parental investment are:
  - Financial and material resources
  - Preferences for human capital
  - Beliefs about the technology of skill formation

What drives parental investment?

- In a standard model, determinants of parental investment are:
    - Financial and material resources
    - Preferences for human capital
    - Beliefs about the technology of skill formation
- Investment behaviour contains information on preferences and beliefs.

What drives parental investment?

- In a standard model, determinants of parental investment are:
  - Financial and material resources
  - Preferences for human capital
  - Beliefs about the technology of skill formation
- Investment behaviour contains information on preferences and beliefs.
- In general it is not possible to disentangle them, without further assumptions or information.

What drives parental investment?

- In a standard model, determinants of parental investment are:
  - Financial and material resources
  - Preferences for human capital
  - Beliefs about the technology of skill formation
- Investment behaviour contains information on preferences and beliefs.
- In general it is not possible to disentangle them, without further assumptions or information.
- Why do parents choose low levels of parental investment?

What drives parental investment?

- In a standard model, determinants of parental investment are:
  - Financial and material resources
  - Preferences for human capital
  - Beliefs about the technology of skill formation
- Investment behaviour contains information on preferences and beliefs.
- In general it is not possible to disentangle them, without further assumptions or information.
- Why do parents choose low levels of parental investment?
  - Are they financially constraint?
  - Don't love their children?

What drives parental investment?

- In a standard model, determinants of parental investment are:
  - Financial and material resources
  - Preferences for human capital
  - Beliefs about the technology of skill formation
- Investment behaviour contains information on preferences and beliefs.
- In general it is not possible to disentangle them, without further assumptions or information.
- Why do parents choose low levels of parental investment?
  - Are they financially constraint?
  - Don't love their children?
  - Do they *perceive* the returns to that investment as being low?

What drives parental investment?

- In a standard model, determinants of parental investment are:
  - Financial and material resources
  - Preferences for human capital
  - Beliefs about the technology of skill formation
- Investment behaviour contains information on preferences and beliefs.
- In general it is not possible to disentangle them, without further assumptions or information.
- Why do parents choose low levels of parental investment?
  - Are they financially constraint?
  - Don't love their children?
  - Do they *perceive* the returns to that investment as being low?
- In this paper we use data that elicit information about beliefs directly.

A Conceptual Framework

We assume parents maximise an objective function, which depends on:

- household consumption (C_i),
- the child's human capital at the end of the period ($H_{i,1}$), and,
- possibly, parental investments (X_i).

A Conceptual Framework

We assume parents maximise an objective function, which depends on:

- household consumption (C_i),
- the child's human capital at the end of the period ($H_{i,1}$), and,
- possibly, parental investments (X_i).

We assume that parents' objective function is Cobb-Douglas in the three arguments:

$$U\left(C_i, H_{i,1}, X_i\right) = \ln C_i + \lambda_i \ln H_{i,1} + \kappa_i \ln X_i$$

A Conceptual Framework

We assume parents maximise an objective function, which depends on:

- household consumption (C_i),
- the child's human capital at the end of the period $(H_{i,1})$, and,
- possibly, parental investments (X_i).

We assume that parents' objective function is Cobb-Douglas in the three arguments:

$$U\left(C_i, H_{i,1}, X_i\right) = \ln C_i + \lambda_i \ln H_{i,1} + \kappa_i \ln X_i$$

- subject to a budget constraint (one-period model and no saving or borrowing):

$$C_i + P_i X_i = Y_i. \tag{1}$$

A Conceptual Framework

We assume parents maximise an objective function, which depends on:

- household consumption (C_i),
- the child's human capital at the end of the period $(H_{i,1})$, and,
- possibly, parental investments (X_i).

We assume that parents' objective function is Cobb-Douglas in the three arguments:

$$U\left(C_i, H_{i,1}, X_i\right) = \ln C_i + \lambda_i \ln H_{i,1} + \kappa_i \ln X_i$$

- subject to a budget constraint (one-period model and no saving or borrowing):

$$C_i + P_i X_i = Y_i. \tag{1}$$

- subject to the *true* technology of skill formation:

$$\ln H_{i,1} = \delta_0 + \delta_1 \ln H_{i,0} + \delta_2 \ln X_i + \delta_3 [\ln H_{i,0} \ln X_i] + \epsilon_i + \nu_i, \tag{2}$$

A Conceptual Framework

- In standard models, parents are assumed to know such a process and make choices subject to the budget constraint and such a technology of skill formation.

A Conceptual Framework

- In standard models, parents are assumed to know such a process and make choices subject to the budget constraint and such a technology of skill formation.
- We relax this assumption.

A Conceptual Framework

- In standard models, parents are assumed to know such a process and make choices subject to the budget constraint and such a technology of skill formation.
- We relax this assumption.
- Let Ω_i denote the parent's information set or parental beliefs about the process of child development.

A Conceptual Framework

- In standard models, parents are assumed to know such a process and make choices subject to the budget constraint and such a technology of skill formation.
- We relax this assumption.
- Let Ω_i denote the parent's information set or parental beliefs about the process of child development.
- We assume parents know the *right functional form* for the technology of skill formation,
- but we not impose the assumption that they know the correct values for the parameters of the technology of skill formation

A Conceptual Framework

- In standard models, parents are assumed to know such a process and make choices subject to the budget constraint and such a technology of skill formation.
- We relax this assumption.
- Let Ω_i denote the parent's information set or parental beliefs about the process of child development.
- We assume parents know the *right functional form* for the technology of skill formation,
- but we not impose the assumption that they know the correct values for the parameters of the technology of skill formation
- Hence, the *subjective* technology of skill formation is:

$$E\left(\ln H_{i,1}\middle|\,\Omega_i\right) = \mu_{i,0} + \mu_{i,1}\ln H_{i,0} + \mu_{i,2}\ln X_i + \mu_{i,3}[\ln H_{i,0}\ln X_i] + \epsilon_i, \quad (3)$$

A Conceptual Framework

- The *true* technology of skill formation determines the evolution of the child's human capital, while the *subjective* technology of skill formation determines parental investment choices.

A Conceptual Framework

- The *true* technology of skill formation determines the evolution of the child's human capital, while the *subjective* technology of skill formation determines parental investment choices.
- The problem of parents is to maximise expected utility:

$$\max_{X_i} E\left[\ln C_i + \lambda_i \ln H_{i,1} + \kappa_i \ln X_i \middle| \Omega_i, X_i\right]$$

subject to (1), (2) and (3).

Parental Investment Equation

- From the F.O.C., it is possible to derive the following investment equation:

$$X_i = \left(\frac{\lambda_i \left(\mu_{i,2} + \mu_{i,3} \ln H_{i,0}\right) + \kappa_i}{1 + \lambda_i \left(\mu_{i,2} + \mu_{i,3} \ln H_{i,0}\right) + \kappa_i} \right) \frac{Y_i}{P_i}$$

Parental Investment Equation

- From the F.O.C., it is possible to derive the following investment equation:

$$X_i = \left(\frac{\lambda_i \left(\mu_{i,2} + \mu_{i,3} \ln H_{i,0}\right) + \kappa_i}{1 + \lambda_i \left(\mu_{i,2} + \mu_{i,3} \ln H_{i,0}\right) + \kappa_i} \right) \frac{Y_i}{P_i}$$

- As the investment equation makes clear, a given investment behaviour is driven by a combination of parental preferences and subjective beliefs about the parameters of the technology of skill formation.

Is it possible to identify the Model?

- The investment equation and the technology of skill formation can be estimated ...
    - ... if there are variables in the investment equation that do not enter directly in the technology of skill formation.
- Identification of the technology of skill formation does not require the assumption that parents know the 'true' technology of skill formation;

Is it possible to identify the Model?

- The investment equation and the technology of skill formation can be estimated …
  - … if there are variables in the investment equation that do not enter directly in the technology of skill formation.
- Identification of the technology of skill formation does not require the assumption that parents know the 'true' technology of skill formation;
- However, without further restrictions or information it is not possible to identify separately the beliefs parameters (μ_i's) from the preference parameters (λ_i and κ_i).

Outline

Parenting Stimulation Program: Aims and Timeline

- The RCT was designed to evaluate the effect of a parenting stimulation program delivered through weekly home visits.

Parenting Stimulation Program: Aims and Timeline

- The RCT was designed to evaluate the effect of a parenting stimulation program delivered through weekly home visits.
- The Jamaica curriculum (*Reach-Up and Learn*) was adapted to Colombia.

Parenting Stimulation Program: Aims and Timeline

- The RCT was designed to evaluate the effect of a parenting stimulation program delivered through weekly home visits.
- The Jamaica curriculum (*Reach-Up and Learn*) was adapted to Colombia.
- The curriculum aims to promote child development in an integrated manner (language, cognitive, motor and socio-emotional skills).

Parenting Stimulation Program: Aims and Timeline

- The RCT was designed to evaluate the effect of a parenting stimulation program delivered through weekly home visits.
- The Jamaica curriculum (*Reach-Up and Learn*) was adapted to Colombia.
- The curriculum aims to promote child development in an integrated manner (language, cognitive, motor and socio-emotional skills).
- The evaluation sample included 1,429 children aged 12-24 months at baseline living in 96 semi-urban towns.

Parenting Stimulation Program: Aims and Timeline

- The RCT was designed to evaluate the effect of a parenting stimulation program delivered through weekly home visits.
- The Jamaica curriculum (*Reach-Up and Learn*) was adapted to Colombia.
- The curriculum aims to promote child development in an integrated manner (language, cognitive, motor and socio-emotional skills).
- The evaluation sample included 1,429 children aged 12-24 months at baseline living in 96 semi-urban towns.
- Data was collected through:
  - Household module: SES, labour supply, health, time use, child care arrangements;
  - Child assessment and maternal report on child development;
  - Data on home visitors and data on visits (frequency & quality);
  - Intake of micronutrients (maternal reports).

Measures of Child Development and Parental Investments

- Baseline: collected before the intervention started in 2009-2010
  - Initial conditions: measure of $\ln H_{i,0}$:
    BSID III (cognitive, receptive language, expressive language) and MacArthur-Bates Language Scale (MLI-I and MLI-II).
  - Intervention assignment: $d_i \in \{0, 1\}$.

Measures of Child Development and Parental Investments

- Baseline: collected before the intervention started in 2009-2010
  - Initial conditions: measure of $\ln H_{i,0}$:
    BSID III (cognitive, receptive language, expressive language) and MacArthur-Bates Language Scale (MLI-I and MLI-II).
  - Intervention assignment: $d_i \in \{0, 1\}$.
- Follow up one: collected 18 months after the baseline, at the end of the program, in the last few months of 2011
  - Child development measure, $\ln H_{i,1}$: BSID III (cognitive, receptive language, expressive language) and MacArthur-Bates Language Scale (MLI-III).
  - Measure parental investment X_i: UNICEF Family Care Indicators (materials, activities) and time diary for child.

Measures of Child Development and Parental Investments

- Baseline: collected before the intervention started in 2009-2010
  - Initial conditions: measure of $\ln H_{i,0}$:
    BSID III (cognitive, receptive language, expressive language) and MacArthur-Bates Language Scale (MLI-I and MLI-II).
  - Intervention assignment: $d_i \in \{0, 1\}$.
- Follow up one: collected 18 months after the baseline, at the end of the program, in the last few months of 2011
  - Child development measure, $\ln H_{i,1}$: BSID III (cognitive, receptive language, expressive language) and MacArthur-Bates Language Scale (MLI-III).
  - Measure parental investment X_i: UNICEF Family Care Indicators (materials, activities) and time diary for child.
- Follow up two: about two years after the end of the program (fall 2013)
  - Measure parental beliefs: Beliefs Elicitation Survey Instrument (primary caregivers - mostly mothers-)

Measurement system

- In order to relate observed variables to the abstract constructs that enter equations such as the *true* technology of skill formation, we use relatively standard latent variable models, which in some cases we extend to accommodate the nature of the data available and use all the information we have efficiently.

Measurement system

- In order to relate observed variables to the abstract constructs that enter equations such as the *true* technology of skill formation, we use relatively standard latent variable models, which in some cases we extend to accommodate the nature of the data available and use all the information we have efficiently.
- Let the latent factor $\theta_{i,j}$, where $\theta_{i,j} \in \{\ln H_{i,0}, \ln X_i, \ln H_{i,1}\}$ and j represents:
    - time-0 natural log of child's human capital ($\ln H_{i,0}$),
    - natural log of parental investment ($\ln X_i$), or
    - time-1 natural log of child's human capital ($\ln H_{i,1}$).

Measurement system

- In order to relate observed variables to the abstract constructs that enter equations such as the *true* technology of skill formation, we use relatively standard latent variable models, which in some cases we extend to accommodate the nature of the data available and use all the information we have efficiently.
- Let the latent factor $\theta_{i,j}$, where $\theta_{i,j} \in \{\ln H_{i,0}, \ln X_i, \ln H_{i,1}\}$ and j represents:
    - time-0 natural log of child's human capital ($\ln H_{i,0}$),
    - natural log of parental investment ($\ln X_i$), or
    - time-1 natural log of child's human capital ($\ln H_{i,1}$).
- We use the measurement system to construct hypothetical scenarios to elicit subjective beliefs.

Measurement system

- In order to relate observed variables to the abstract constructs that enter equations such as the *true* technology of skill formation, we use relatively standard latent variable models, which in some cases we extend to accommodate the nature of the data available and use all the information we have efficiently.
- Let the latent factor $\theta_{i,j}$, where $\theta_{i,j} \in \{\ln H_{i,0}, \ln X_i, \ln H_{i,1}\}$ and j represents:
  - time-0 natural log of child's human capital ($\ln H_{i,0}$),
  - natural log of parental investment ($\ln X_i$), or
  - time-1 natural log of child's human capital ($\ln H_{i,1}$).
- We use the measurement system to construct hypothetical scenarios to elicit subjective beliefs.
- We use estimates to use informative and salient variables for the scenarios.

Measurement system

- In order to relate observed variables to the abstract constructs that enter equations such as the *true* technology of skill formation, we use relatively standard latent variable models, which in some cases we extend to accommodate the nature of the data available and use all the information we have efficiently.
- Let the latent factor $\theta_{i,j}$, where $\theta_{i,j} \in \{\ln H_{i,0}, \ln X_i, \ln H_{i,1}\}$ and j represents:
    - time-0 natural log of child's human capital ($\ln H_{i,0}$),
    - natural log of parental investment ($\ln X_i$), or
    - time-1 natural log of child's human capital ($\ln H_{i,1}$).
- We use the measurement system to construct hypothetical scenarios to elicit subjective beliefs.
- We use estimates to use informative and salient variables for the scenarios.
- We implicitly assume that mothers share the same measurement system.

Measurement system

- We express the relationship between measure k, $m_{i,j,k}$, and latent factors, $\theta_{i,j}$, by the following equation:

$$m^*_{i,j,k} = \alpha_{j,k,0} + \alpha'_{j,k,1} z_i + \beta_{j,k}\theta_{i,j} + \epsilon_{i,j,k}; \qquad \theta_{i,j} = \ln H_{i,0}, \ln H_{i,1}, \ln X_i. \quad (4)$$

  where $\epsilon_{i,j,k}$ represents measurement error and z_i are observable variables that can potentially enter the relationship between human capital and its markers represented by equation (4).

- We assume that the joint distribution of the unobservable factors θ_i is a mixture of log normals, while the measurement error $\epsilon_{i,j,k}$ is a normal random variable, independent across different measures k.
- We estimate all of its parameters (subject to location and scaling parameters) and recover the distribution of the latent factors.

Measurement system

We use slightly different types of single index models for three different types of measures of the latent factors:

- Dichotomous variables $m_{i,j,k} \in \{0,1\}$: $Prob\{m_{i,j,k} = 1\} = Pr\{m^*_{i,j,k} \geq 0\}$;
- Polytomous variables $m_{i,j,k} \in \{1, 2, ..., L\}$:
  $Prob\{m_{i,j,k} = l\} = Pr\{c_{l-1} \leq m_{i,j,k} \leq c_l\}$, where $c_0 = -\infty$;
- Continuous variables: $m_{i,j,k} = m^*_{i,j,k}$;

Beliefs Elicitation Survey Instrument

- We use the output of the measurement system to construct hypothetical scenarios of human capital at the beginning of period and parental investments.
- We also the measurement system for outcomes to construct possible outcomes.

Beliefs Elicitation Survey Instrument

- We use the output of the measurement system to construct hypothetical scenarios of human capital at the beginning of period and parental investments.
- We also the measurement system for outcomes to construct possible outcomes.
- Effectively we ask mothers to map different hypothetical scenarios in terms of initial conditions and investment into a set of outcomes.
- Mothers report their expectations about the impact of initial human capital and parental investment on end-of-period human capital.

Beliefs Elicitation Survey Instrument

- We use the output of the measurement system to construct hypothetical scenarios of human capital at the beginning of period and parental investments.
- We also the measurement system for outcomes to construct possible outcomes.
- Effectively we ask mothers to map different hypothetical scenarios in terms of initial conditions and investment into a set of outcomes.
- Mothers report their expectations about the impact of initial human capital and parental investment on end-of-period human capital.
- For each scenario, we ask several questions, so to obtain several measures of the mapping.

Constructing Hypothetical Scenarios

- We ask the mother to consider a hypothetical 9 months old child.
- We present the mother with four possible scenarios of inputs:
  - Hypothetical Scenarios for Human Capital at the Beginning of the Period: low and high:
  - We use the estimated parameters $\alpha_{MLII,w,0}$ and $\beta_{MLII,w,0}$ from the IRT model to present scenarios using only a small set of carefully selected words.

Constructing Hypothetical Scenarios

- We ask the mother to consider a hypothetical 9 months old child.
- We present the mother with four possible scenarios of inputs:
  - Hypothetical Scenarios for Human Capital at the Beginning of the Period: low and high:
  - We use the estimated parameters $\alpha_{MLII,w,0}$ and $\beta_{MLII,w,0}$ from the IRT model to present scenarios using only a small set of carefully selected words.
  - By selecting salient words (high $\beta_{MLII,w,0}$) and varying degrees of difficulty (different values of $\alpha_{MLII,w,0}$), we can use a small set of MLI I items to construct informative groups of "easy" and "difficult" words.

Constructing Hypothetical Scenarios

- We ask the mother to consider a hypothetical 9 months old child.
- We present the mother with four possible scenarios of inputs:
  - Hypothetical Scenarios for Human Capital at the Beginning of the Period: low and high:
  - We use the estimated parameters $\alpha_{MLII,w,0}$ and $\beta_{MLII,w,0}$ from the IRT model to present scenarios using only a small set of carefully selected words.
  - By selecting salient words (high $\beta_{MLII,w,0}$) and varying degrees of difficulty (different values of $\alpha_{MLII,w,0}$), we can use a small set of MLI I items to construct informative groups of "easy" and "difficult" words.
  - Then, we define the first scenario - verbal description of H_0^L - as one in which the child can only say the easy words.
  - Finally, we define the second scenario - H_0^H - as one in which the child can understand and say both the easy and the difficult words.

Constructing Hypothetical Scenarios

- We ask the mother to consider a hypothetical 9 months old child.
- We present the mother with four possible scenarios of inputs:
  - Hypothetical Scenarios for Human Capital at the Beginning of the Period: low and high:
    - We use the estimated parameters $\alpha_{MLII,w,0}$ and $\beta_{MLII,w,0}$ from the IRT model to present scenarios using only a small set of carefully selected words.
    - By selecting salient words (high $\beta_{MLII,w,0}$) and varying degrees of difficulty (different values of $\alpha_{MLII,w,0}$), we can use a small set of MLI I items to construct informative groups of "easy" and "difficult" words.
    - Then, we define the first scenario - verbal description of H_0^L - as one in which the child can only say the easy words.
    - Finally, we define the second scenario - H_0^H - as one in which the child can understand and say both the easy and the difficult words.

Constructing Hypothetical Scenarios

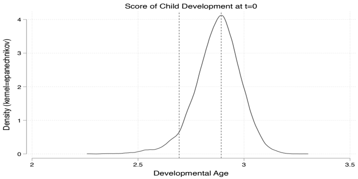
- We ask the mother to consider a hypothetical 9 months old child.
- We present the mother with four possible scenarios of inputs:
    - Hypothetical Scenarios for Parental Investments: low and high:

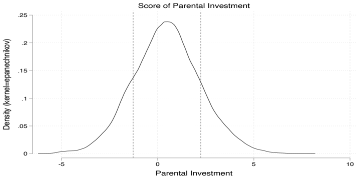
Constructing Hypothetical Scenarios

- We ask the mother to consider a hypothetical 9 months old child.
- We present the mother with four possible scenarios of inputs:
  - Hypothetical Scenarios for Parental Investments: low and high:
    - We identify salient items of such a factor and choose these measures to define X^L and X^H scenarios for parental investment.

Constructing Hypothetical Scenarios

- The estimates of the IRT model and the definitions of the scenarios can be used to establish what part of the relevant domains are spanned by the scenarios.



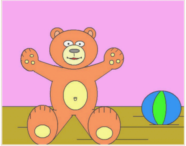


Hypothetical Scenarios

- We use visual cues to describe the scenarios.

Hypothetical Scenarios

- We use visual cues to describe the scenarios.
- 'High' and 'Low' levels of Parental Investment.



Beliefs Elicitation Survey Instrument

- Finally, for each of the four scenarios we construct, we asked the mother at what age a hypothetical child would start saying 3 sets of words in that particular scenario.

Beliefs Elicitation Survey Instrument

- Finally, for each of the four scenarios we construct, we asked the mother at what age a hypothetical child would start saying 3 sets of words in that particular scenario.
- These words are chosen using the same approach used to choose the words that define the scenarios.

Beliefs Elicitation Survey Instrument

- Finally, for each of the four scenarios we construct, we asked the mother at what age a hypothetical child would start saying 3 sets of words in that particular scenario.
- These words are chosen using the same approach used to choose the words that define the scenarios.
- The three set of words were chosen to be:
  - Easy (first set)
  - Medium (second set) and
  - Hard (third set)

Beliefs Elicitation Survey Instrument

- Finally, for each of the four scenarios we construct, we asked the mother at what age a hypothetical child would start saying 3 sets of words in that particular scenario.
- These words are chosen using the same approach used to choose the words that define the scenarios.
- The three set of words were chosen to be:
  - Easy (first set)
  - Medium (second set) and
  - Hard (third set)
- Each set of words defines a measure of the subjective expectations corresponding with a specific scenario.

Beliefs Elicitation Survey Instrument

- These words were selected on the basis of the measurement system parameters. We used words from the MLI-II on baseline data for children between 19-24 months.

Beliefs Elicitation Survey Instrument

- These words were selected on the basis of the measurement system parameters. We used words from the MLI-II on baseline data for children between 19-24 months.
- All the chosen words, w, had high loading factors' $\beta_{MLIII,w,1}$ and different values of $\alpha_{MLIII,w,1}$'s.

Beliefs Elicitation Survey Instrument

- These words were selected on the basis of the measurement system parameters. We used words from the MLI-II on baseline data for children between 19-24 months.
- All the chosen words, w, had high loading factors' $\beta_{MLIII,w,1}$ and different values of $\alpha_{MLIII,w,1}$'s.
- The use of different groups of words with different levels of difficulty allows us to investigate and address measurement error in maternal responses about expected levels of child development.

Beliefs Elicitation Survey Instrument

Beliefs Elicitation Survey Instrument

Beliefs Elicitation Survey Instrument

Beliefs Elicitation Survey Instrument - Descriptive Statistics

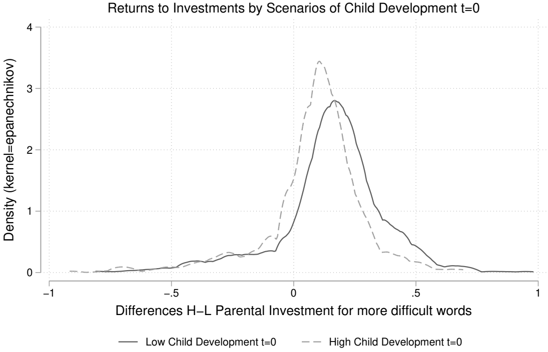
Table: Expected Child Development at the end of the period: Maternal Answers

VARIABLES			Mean	St. Dv.	Min	Max
Low Child Development at $t = 0$	Low Investment	easy	18.4	6.3	9	48
		medium	23.4	7.3	10	48
		hard	29.3	8.8	11	48
	High Investment	easy	15.6	5.4	9	48
		medium	20.0	6.4	9	48
		hard	24.8	7.9	10	48
High Child Development at $t = 0$	Low Investment	easy	14.3	4.5	9	45
		medium	17.9	5.4	9	47
		hard	22.3	7.0	10	48
	High Investment	easy	13.4	4.9	9	48
		medium	16.5	5.5	9	48
		hard	20.2	6.9	9	48

Observations: 1,017.

Beliefs Elicitation Survey Instrument - Descriptive Statistics

- Using the data collected we can get some idea about returns to investment:



Outline

A Methodological approach to use beliefs data

- Stage 1: Location, Scaling and Measurement System:
    - Dimensionality reduction. We use factor models to reduce the dimension of the data.

A Methodological approach to use beliefs data

- Stage 1: Location, Scaling and Measurement System:
  - Dimensionality reduction. We use factor models to reduce the dimension of the data.
  - We need to establish a metric for child development so wethat can meaningfully estimate subjective and objective parameters of the technology of skill formation.

A Methodological approach to use beliefs data

- Stage 1: Location, Scaling and Measurement System:
  - Dimensionality reduction. We use factor models to reduce the dimension of the data.
  - We need to establish a metric for child development so wethat can meaningfully estimate subjective and objective parameters of the technology of skill formation.
- Stage 2: Estimation of expected returns and subjective technology of skill formation: We describe how to use maternal answers from the beliefs elicitation survey instrument to estimate returns and parameters of the subjective technology of skill formation.

A Methodological approach to use beliefs data

- Stage 1: Location, Scaling and Measurement System:
    - Dimensionality reduction. We use factor models to reduce the dimension of the data.
    - We need to establish a metric for child development so wethat can meaningfully estimate subjective and objective parameters of the technology of skill formation.
- Stage 2: Estimation of expected returns and subjective technology of skill formation: We describe how to use maternal answers from the beliefs elicitation survey instrument to estimate returns and parameters of the subjective technology of skill formation.
- Stage 3: Estimation of objective technology of skill formation: We describe how to use objective data on child's human capital at baseline and follow-up one and exogenous variation in parental investment (in follow-up one) induced by the randomization to control or treatment arm.

Location and Scale of Child Development

- As our conceptual framework deals with unobserved latent factors representing child's human capital (and parental investment) for which we have a number of measures or markers, we need to establish the metric that is going to be used for such factors.

Location and Scale of Child Development

- As our conceptual framework deals with unobserved latent factors representing child's human capital (and parental investment) for which we have a number of measures or markers, we need to establish the metric that is going to be used for such factors.
- Each BSID-III subscale is measured in terms of the raw score based in the number of items for which the child receives credit (i.e., 1 point).
- As the raw scores are positively correlated with the child's age, we exploit this property to construct both the location and the scale of the child's latent stock of human capital at both periods of our model: $\ln H_{i,0}$ and $\ln H_{i,1}$.

Location and Scale of Child Development

- As our conceptual framework deals with unobserved latent factors representing child's human capital (and parental investment) for which we have a number of measures or markers, we need to establish the metric that is going to be used for such factors.
- Each BSID-III subscale is measured in terms of the raw score based in the number of items for which the child receives credit (i.e., 1 point).
- As the raw scores are positively correlated with the child's age, we exploit this property to construct both the location and the scale of the child's latent stock of human capital at both periods of our model: $\ln H_{i,0}$ and $\ln H_{i,1}$.
- In the education literature, these location and scale are known as "age-equivalent score."

Converting to "age-equivalent score": establishing a metric

- We first convert the BSID-III subscales of child development (the ability to perform certain tasks) into "age-equivalent score".

$$devage_{i,j} = \pi_0^j + \pi_1^j \times \ln(BSID_{i,j}^{raw}),$$

- In our analysis, the age-equivalent expressive language BSID-III score is used as an anchor for the other measures of child's human capital.

Converting to "age-equivalent score": establishing a metric

- We first convert the BSID-III subscales of child development (the ability to perform certain tasks) into "age-equivalent score".

$$devage_{i,j} = \pi_0^j + \pi_1^j \times \ln(BSID_{i,j}^{raw}),$$

- In our analysis, the age-equivalent expressive language BSID-III score is used as an anchor for the other measures of child's human capital.
- As a result, our factors of child's human capital $\ln H_{i,0}$ and $\ln H_{i,1}$ have both the location and scale of age-equivalent expressive language score.

Converting to "age-equivalent score": establishing a metric

- We first convert the BSID-III subscales of child development (the ability to perform certain tasks) into "age-equivalent score".

$$devage_{i,j} = \pi_0^j + \pi_1^j \times \ln(BSID_{i,j}^{raw}),$$

- In our analysis, the age-equivalent expressive language BSID-III score is used as an anchor for the other measures of child's human capital.
- As a result, our factors of child's human capital $\ln H_{i,0}$ and $\ln H_{i,1}$ have both the location and scale of age-equivalent expressive language score.
- In what follows, we assume that the measures of human capital are all located and scaled according to age-equivalent scores of expressive language scale in the BSID-III.

Methodological Approach: What we recover?

- Consider two hypothetical levels of human capital at the beginning of the period: H_0^H and H_0^L, and
- Two hypothetical levels of parental investment: X^H and X^L.

Methodological Approach: What we recover?

- Consider two hypothetical levels of human capital at the beginning of the period: H_0^H and H_0^L, and
- Two hypothetical levels of parental investment: X^H and X^L.
- The cartesian product of these two hypothetical levels of H_0 and X produces four hypothetical scenarios that we present to each mother in our study.

Methodological Approach: What we recover?

- Consider two hypothetical levels of human capital at the beginning of the period: H_0^H and H_0^L, and
- Two hypothetical levels of parental investment: X^H and X^L.
- The cartesian product of these two hypothetical levels of H_0 and X produces four hypothetical scenarios that we present to each mother in our study.
- Given a scenario $s = (s_h, s_x)$, for $s \in S = \{(H_0^L, X^L), (H_0^L, X^H), (H_0^H, X^L), (H_0^H, X^H)\}$ and $s_h, s_x = L, H$, the mother then provides answers from our beliefs elicitation survey instrument that we can connect to $E\left(\ln H_{i,1} \middle| \Omega_i, s\right)$.

Methodological Approach: What we recover?

- Consider two hypothetical levels of human capital at the beginning of the period: H_0^H and H_0^L, and
- Two hypothetical levels of parental investment: X^H and X^L.
- The cartesian product of these two hypothetical levels of H_0 and X produces four hypothetical scenarios that we present to each mother in our study.
- Given a scenario $s = (s_h, s_x)$, for $s \in S = \{(H_0^L, X^L), (H_0^L, X^H), (H_0^H, X^L), (H_0^H, X^H)\}$ and $s_h, s_x = L, H$, the mother then provides answers from our beliefs elicitation survey instrument that we can connect to $E\left(\ln H_{i,1} | \, \Omega_i, s\right)$.
- We can then use these expectations to estimate return to investment:

$$r_i\left(H_0^H\right) = E\left(\ln H_{i,1} | \, \Omega_i, (H_0^H, X^H)\right) - E\left(\ln H_{i,1} | \, \Omega_i, (H_0^H, X^L)\right)$$

Estimation of Subjective Technology of Skill Formation

- We need to translate maternal answers to measures that correspond to the our model.
- In the MLI-II survey questionnaire, mothers can choose one out of two mutually exclusive alternatives.

Estimation of Subjective Technology of Skill Formation

- We need to translate maternal answers to measures that correspond to the our model.
- In the MLI-II survey questionnaire, mothers can choose one out of two mutually exclusive alternatives.
- A probit model where child development θ_i is the latent factor in the index: $m^*_{i,w} = \alpha_w + \beta_w \theta_i + \epsilon_{i,w}$ is used, where in the case of MLI-II, $\theta_i = \ln H_{i,1}$.
- We need to adapt the IRT model for the MLI-II (centered around 24 months) so that our analysis is consistent with the way we asked mothers about beliefs.

Estimation of Subjective Technology of Skill Formation

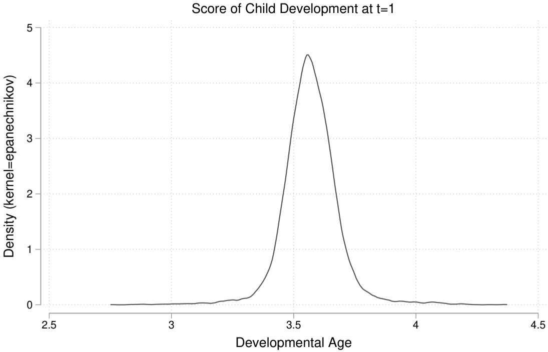
- We need to translate maternal answers to measures that correspond to the our model.
- In the MLI-II survey questionnaire, mothers can choose one out of two mutually exclusive alternatives.
- A probit model where child development θ_i is the latent factor in the index: $m^*_{i,w} = \alpha_w + \beta_w \theta_i + \epsilon_{i,w}$ is used, where in the case of MLI-II, $\theta_i = \ln H_{i,1}$.
- We need to adapt the IRT model for the MLI-II (centered around 24 months) so that our analysis is consistent with the way we asked mothers about beliefs.

  Let $M_{i,q} = 1$ if the child can say all the three words considered in category q. The probability that $M_{i,q} = 1$ for child i with development $\ln H_{i,1}$ is:

$$\Pr\left(M_{i,q} = 1 | \ln H_{i,1}\right) = \left[\prod_{w_q=1}^{3} \left[1 - \Phi\left(-\alpha_{w_q} - \beta_{w_q} \ln H_{i,1}\right)\right]\right]$$

- We need to adapt mothers answers so that the resulting factors have the same metric as the factors used in estimating the objective production function.

Estimating the factor scores



Estimation of Subjective Technology of Skill Formation

- The estimated factors can be seen as different error-ridden measures of our target subjective moment, $E\left(\ln H_{i,1} \middle| \Omega_i, s\right)$.
  Specifically, we assume:

$$\ln H_{i,1,s,q} = E\left(\ln H_{i,1} \middle| \Omega_i, s\right) + \eta_{i,1,s,q}$$

Estimation of Subjective Technology of Skill Formation

- The estimated factors can be seen as different error-ridden measures of our target subjective moment, $E\left(\ln H_{i,1} \middle| \Omega_i, s\right)$.
  Specifically, we assume:

$$\ln H_{i,1,s,q} = E\left(\ln H_{i,1} \middle| \Omega_i, s\right) + \eta_{i,1,s,q}$$

- Given the IRT models estimated, we can compute a score for $\ln H_0^{s_h}$ and $\ln X^{s_x}$ corresponding to each scenario.

Estimation of Subjective Technology of Skill Formation

- The estimated factors can be seen as different error-ridden measures of our target subjective moment, $E\left(\ln H_{i,1}\middle|\,\Omega_i, s\right)$.
  Specifically, we assume:

$$\ln H_{i,1,s,q} = E\left(\ln H_{i,1}\middle|\,\Omega_i, s\right) + \eta_{i,1,s,q}$$

- Given the IRT models estimated, we can compute a score for $\ln H_0^{s_h}$ and $\ln X^{s_x}$ corresponding to each scenario.
- If we combine the *subjective* technology of skill formation and equation above for a specific scenario $s = (s_h, s_x)$, we obtain:

$$\ln H_{i,1,s,q} = \mu_{0,i} + \mu_{1,i} \ln H_0^{s_h} + \mu_{2,i} \ln X^{s_x} + \mu_{3,i} \left[\ln H_0^{s_h} \ln X^{s_x}\right] + \eta_{i,1,s,q}.$$

Estimation of Subjective Technology of Skill Formation

- The estimated factors can be seen as different error-ridden measures of our target subjective moment, $E\left(\ln H_{i,1} \middle| \Omega_i, s\right)$.
  Specifically, we assume:

$$\ln H_{i,1,s,q} = E\left(\ln H_{i,1} \middle| \Omega_i, s\right) + \eta_{i,1,s,q}$$

- Given the IRT models estimated, we can compute a score for $\ln H_0^{s_h}$ and $\ln X^{s_x}$ corresponding to each scenario.
- If we combine the *subjective* technology of skill formation and equation above for a specific scenario $s = (s_h, s_x)$, we obtain:

$$\ln H_{i,1,s,q} = \mu_{0,i} + \mu_{1,i} \ln H_0^{s_h} + \mu_{2,i} \ln X^{s_x} + \mu_{3,i} \left[\ln H_0^{s_h} \ln X^{s_x}\right] + \eta_{i,1,s,q}.$$

- The equality requires that we assume that $E\left(\epsilon_i \middle| \ln H_0^{s_h}, \ln X^{s_x}\right) = 0$.
- The equation can be seen as a factor model where the μ_i's are the factors and where the factor loadings are known.

Estimation of Subjective Technology of Skill Formation

- We can then estimate, for each individual, the vector $\{\widehat{\mu}_{i,0}, \widehat{\mu}_{i,1}, \widehat{\mu}_{i,2}, \widehat{\mu}_{i.3}\}$ by running an OLS regression separately for each mother in our sample.

Estimation of Subjective Technology of Skill Formation

- We can then estimate, for each individual, the vector $\{\widehat{\mu}_{i,0}, \widehat{\mu}_{i,1}, \widehat{\mu}_{i,2}, \widehat{\mu}_{i.3}\}$ by running an OLS regression separately for each mother in our sample.
- Alternatively, we can improve the precision of the individual-level estimates by using the Swamy (1967) estimator, to obtain, for each household, an efficient estimator for the vector $\{\widehat{\mu}_{i,0}, \widehat{\mu}_{i,1}, \widehat{\mu}_{i,2}, \widehat{\mu}_{i,3}\}$ and its variance-covariance matrix.

Estimation of Subjective Technology of Skill Formation

- We can then estimate, for each individual, the vector $\{\widehat{\mu}_{i,0}, \widehat{\mu}_{i,1}, \widehat{\mu}_{i,2}, \widehat{\mu}_{i.3}\}$ by running an OLS regression separately for each mother in our sample.
- Alternatively, we can improve the precision of the individual-level estimates by using the Swamy (1967) estimator, to obtain, for each household, an efficient estimator for the vector $\{\widehat{\mu}_{i,0}, \widehat{\mu}_{i,1}, \widehat{\mu}_{i,2}, \widehat{\mu}_{i,3}\}$ and its variance-covariance matrix.
- This effectively is equivalent to a GLS estimator.

Swamy estimator

Step 1: For each i

$$\begin{aligned} \hat{m}_{\psi,i} &= (Z_i'Z_i)^{-1}(Z_i'lnh_i) \\ \hat{\sigma}_i^2 &= \frac{\hat{\epsilon}_i'\hat{\epsilon}_i}{T-L} \\ \hat{V}_i &= \hat{\sigma_i^2}(Z_i'Z_i)^{-1} \end{aligned}$$

Swamy estimator

Step 1: For each i

$$\begin{aligned}\hat{m}_{\psi,i} &= (Z_i'Z_i)^{-1}(Z_i'lnh_i)\\ \hat{\sigma}_i^2 &= \frac{\hat{\epsilon}_i'\hat{\epsilon}_i}{T-L}\\ \hat{V}_i &= \hat{\sigma_i^2}(Z_i'Z_i)^{-1}\end{aligned}$$

Step 2 : compute $\bar{m} = \frac{1}{N}\sum_{i=1}^{N}\hat{m}_{\psi,i}$

$$\begin{aligned}\hat{\Gamma} &= \frac{1}{N-1}[\sum_{i=1}^{N}\hat{m}_{\psi,i}\hat{m}_{\psi,i}' - N\bar{m}_{\psi}\bar{m}_{\psi}'] - \frac{1}{N}\sum_{i=1}^{N}\hat{V}_i\\ \hat{\Pi}_i &= \hat{\sigma}_i I + Z_i\hat{\Gamma}Z_i'\end{aligned}$$

Swamy estimator

Step 1: For each i

$$\begin{aligned}\hat{m}_{\psi,i} &= (Z_i'Z_i)^{-1}(Z_i'lnh_i)\\ \hat{\sigma}_i^2 &= \frac{\hat{\epsilon}_i'\hat{\epsilon}_i}{T-L}\\ \hat{V}_i &= \hat{\sigma_i^2}(Z_i'Z_i)^{-1}\end{aligned}$$

Step 2 : compute $\bar{m} = \frac{1}{N}\sum_{i=1}^{N}\hat{m}_{\psi,i}$

$$\begin{aligned}\hat{\Gamma} &= \frac{1}{N-1}[\sum_{i=1}^{N}\hat{m}_{\psi,i}\hat{m}_{\psi,i}' - N\bar{m}_{\psi}\bar{m}_{\psi}'] - \frac{1}{N}\sum_{i=1}^{N}\hat{V}_i\\ \hat{\Pi}_i &= \hat{\sigma}_i I + Z_i\hat{\Gamma}Z_i'\end{aligned}$$

Step 3: efficiently estimate mean MSE and its variance

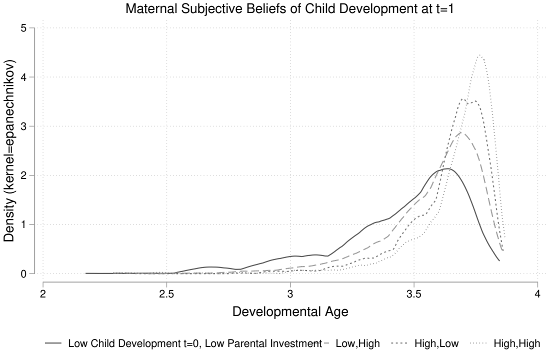
$$\begin{aligned}\hat{\mu}_{\psi} &= (\sum_{i=1}^{N}Z_i'\hat{\Pi}_iZ_i)^{-1}(\sum_{i=1}^{N}Z_i'\hat{\Pi}_ilnh_i)\\ Var(\hat{\mu}_{\psi}) &= \frac{1}{N}\sum_{i=1}^{N}(\hat{\Gamma}+\hat{V}_i)^{-1}\end{aligned}$$

Outline

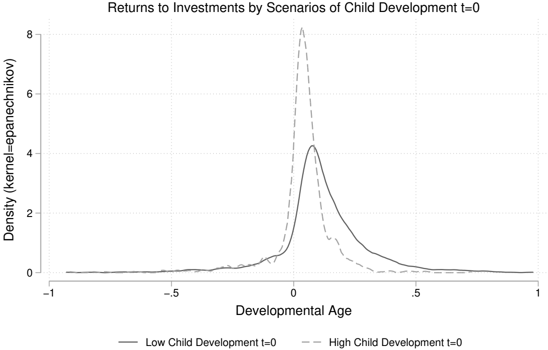
Results

- We will show results on:
  - Subjective Expected Returns
  - Perceived and True Production Functions
  - Beliefs in the cross section
  - Investment and beliefs

Subjective Beliefs under different Scenarios



Subjective Expected Returns by initial condition

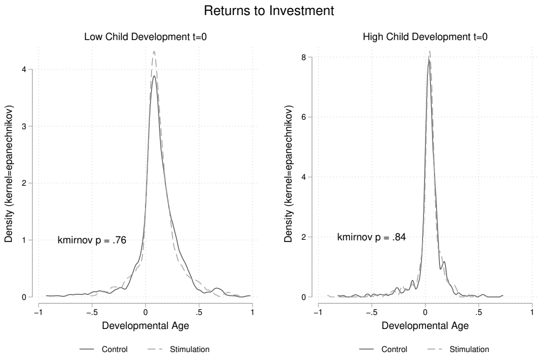


Subjective Expected Child Development Returns on Parental Investment

VARIABLES	Return to Low Dev. Child Development at $t = 0$		Return to High Dev. Child Development at $t = 0$	
Mother's age	0.002*	0.002*	0.000	0.000
	(0.001)	(0.001)	(0.001)	(0.001)
Mother's education (primary)	0.075	0.075	0.008	0.007
	(0.068)	(0.068)	(0.087)	(0.086)
Mother's education (secondary and more)	0.072	0.072	0.021	0.020
	(0.070)	(0.070)	(0.086)	(0.086)
Mother's depression (CES-D)	-0.005	-0.005	0.002	0.002
	(0.012)	(0.012)	(0.008)	(0.008)
Mother's IQ (standardized Raven's Score)	0.011	0.011	0.018***	0.018***
	(0.007)	(0.007)	(0.005)	(0.005)
Dummy for Male (child)	-0.012	-0.012	-0.005	-0.005
	(0.013)	(0.013)	(0.008)	(0.008)
Dummy for Treatment		-0.001		0.006
		(0.013)		(0.010)
R^2	0.009	0.009	0.028	0.029
F	1.506	1.338	2.944	2.545
Observations	1017	1017	1017	1017

Standard errors (in parentheses) are clustered at municipality level, $^{*}\ p < 0.10$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$

Subjective Expected Returns by initial conditions and Intervention



Estimation of Objective Technology of Skill Formation

- We explore the dimensionality reduction provided by the measurement system and only use the factor scores for each measure of child development (both in baseline and follow-up one) and investments (in follow-up one).

Estimation of Objective Technology of Skill Formation

- We explore the dimensionality reduction provided by the measurement system and only use the factor scores for each measure of child development (both in baseline and follow-up one) and investments (in follow-up one).
- We have $K = 4$ continuous measures of human capital at the beginning of the period. Furthermore, all of these measures have location and scale in age-equivalent scores (e.g for $\ln \mathbf{H_0}$):

$$\mathbf{m_{i,\ln H_0}} = \alpha_{\ln \mathbf{H_0},\mathbf{0}} + \beta_{\ln \mathbf{H_0}} \ln H_{i,0} + \epsilon_{\mathbf{i},\ln \mathbf{H_0}}$$

Estimation of Objective Technology of Skill Formation

- The objective estimation of the technology of skill formation has three steps.
  1. We factor analyze equations for each factor separately.
  2. We use estimated intercepts, factor loadings, and variances of the factors, to generate the factor scores:
  $$\hat{\theta}_i = \left(\beta_\theta{}'\mathbf{\Sigma}_\theta^{-1}\beta_\theta\right)^{-1}\left(\beta_\theta{}'\mathbf{\Sigma}_\theta^{-1}\mathbf{m}_{\mathbf{i},\theta}\right)$$
  where $\hat{\theta}_i = \ln \hat{H}_{i,0}, \ln \hat{H}_{i,1}, \ln \hat{X}_i$
  3. Let $\delta = (\delta_0, \delta_1, \delta_2, \delta_3)'$ and $\Psi_i = \left(1, \ln \hat{H}_{i,0}, \ln \hat{X}_i, \ln \hat{H}_{i,0} \ln \hat{X}_i\right)$, so that we can write the *true* technology of skill formation in the following form:
  $$\ln \hat{H}_{i,1} = \Psi_i \delta + \epsilon_i + \nu_i$$

Estimation of Objective Technology of Skill Formation

- The objective estimation of the technology of skill formation has three steps.
  1. We factor analyze equations for each factor separately.
  2. We use estimated intercepts, factor loadings, and variances of the factors, to generate the factor scores:
  $$\hat{\theta}_i = \left(\beta_\theta{}'\mathbf{\Sigma}_\theta^{-1}\beta_\theta\right)^{-1}\left(\beta_\theta{}'\mathbf{\Sigma}_\theta^{-1}\mathbf{m}_{\mathbf{i},\theta}\right)$$
  where $\hat{\theta}_i = \ln\hat{H}_{i,0}, \ln\hat{H}_{i,1}, \ln\hat{X}_i$
  3. Let $\delta = (\delta_0, \delta_1, \delta_2, \delta_3)'$ and $\Psi_i = \left(1, \ln\hat{H}_{i,0}, \ln\hat{X}_i, \ln\hat{H}_{i,0}\ln\hat{X}_i\right)$, so that we can write the *true* technology of skill formation in the following form:
  $$\ln\hat{H}_{i,1} = \Psi_i\delta + \epsilon_i + \nu_i$$

- We estimate the 'true' technology of skill formation taking into account the endogeneity of investment.
- We use the random assignment as an instrument for parental investments in children.

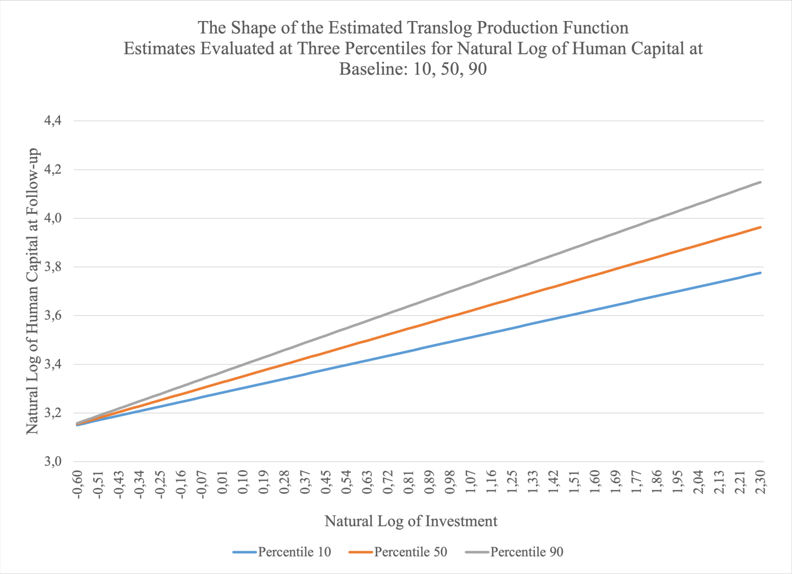
Estimation of Objective Technology of Skill Formation

$$\ln H_{i,1} = \delta_0 + \delta_1 \ln H_{i,0} + \delta_2 \ln X_i + \delta_3 [\ln H_{i,0} \ln X_i] + \epsilon_i + \nu_i,$$

Table: Objective Estimation of the Production Function

Dependent Variable: Log of Human Capital at Follow Up		
	Cobb Douglas	Translog
Intercept	2.265 (0.067)	2.886 (0.178)
Log of Human Capital at Baseline	0.377 (0.045)	0.151 (0.075)
Log of Investments at Follow Up	0.248 (0.096)	-0.386 (0.194)
Log of Investment x Log of Human Capital at Baseline		0.230 (0.061)
Control function	-0.173 (0.098)	-0.203 (0.098)

The Shape of the Estimated Translog Production Function



Perceived technology of skill formation

It will be remembered that mother's perceived technology of skill formation is:

$$E\left(\ln H_{i,1} \middle| \Omega_i\right) = \mu_{i,0} + \mu_{i,1} \ln H_{i,0} + \mu_{i,2} \ln X_i + \mu_{i,3}[\ln H_{i,0} \ln X_i] + \epsilon_i,$$

Our procedure yields estimates of the coefficients for each mother:

Table: Estimation of Perceived Production Function

Dependent Variable: Expected Human Capital at Follow Up				
	Cobb Douglas	Fraction $\lvert t \rvert > 2$	Translog	Fraction $\lvert t \rvert > 2$
Intercept	2.498 (0.044)	93.11%	1.554 (0.109)	48.87%
Human Capital at Baseline	0.337 (0.013)	73.84%	0.663 (0.036)	44.64%
Investment	0.128 (0.008)	46.21%	1.088 (0.096)	23.89%
Investment x Human Capital at Baseline			-0.332 (0.032)	19.96%

Perceived and true technology of skill formation

- Perceived technology of skill formation:

$$ElnH_1 = \mu_{0,i} + \mu_{1,i} lnH_0 + \mu_{2,i} lnX + \mu_{3,i}[ln(h_0)\ln(X)]$$

- 'True ' technology of skill formation:

$$ElnH_{i,1} = \delta_0 + \delta_1 \ln H_{i,0} + \delta_2 \ln X_i + \delta_3[\ln h_{i,0} \ln X_i] + \epsilon_i$$

Perceived and true technology of skill formation

- Perceived technology of skill formation:

$$ElnH_1 = \mu_{0,i} + \mu_{1,i} lnH_0 + \mu_{2,i} lnX + \mu_{3,i}[ln(h_0)\ln(X)]$$

- 'True ' technology of skill formation:

$$ElnH_{i,1} = \delta_0 + \delta_1 \ln H_{i,0} + \delta_2 \ln X_i + \delta_3[\ln h_{i,0} \ln X_i] + \epsilon_i$$

- Cobb Douglas production function (δ_3 and $\mu_{3,i} = 0$):
  - On average, $\mu_{1,i}$ is smaller than δ_1: (0.337 vs 0.377) .
  - On average, $\mu_{2,i}$ is much smaller than δ_2: (0.128 vs 0.248).

Perceived and true technology of skill formation

- Perceived technology of skill formation:

$$ElnH_1 = \mu_{0,i} + \mu_{1,i} lnH_0 + \mu_{2,i} lnX + \mu_{3,i}[ln(h_0)\ln(X)]$$

- 'True ' technology of skill formation:

$$ElnH_{i,1} = \delta_0 + \delta_1 \ln H_{i,0} + \delta_2 \ln X_i + \delta_3[\ln h_{i,0} \ln X_i] + \epsilon_i$$

- Cobb Douglas production function (δ_3 and $\mu_{3,i} = 0$):
  - On average, $\mu_{1,i}$ is smaller than δ_1: (0.337 vs 0.377) .
  - On average, $\mu_{2,i}$ is much smaller than δ_2: (0.128 vs 0.248).
- Translog production function (δ_3 and $\mu_{3,i} \neq 0$)
  - The patterns of coefficients is very different .
  - $\mu_{1,i}, \mu_{2,i}$, and $\mu_{3,i}$ equal to $0.663, 1.088, -0.332$ on average;
  - δ_1, δ_2, and δ_3 equal to $0.151, -0.386, 0.230$

Subjective Estimates and SE characteristics: Cobb Douglas

VARIABLES	μ_1	μ_1	μ_2	μ_2
Dummy for Treatment	-0.093	-0.082	-0.025	-0.022
	(0.066)	(0.058)	(0.074)	(0.054)
Dummy for Male	-0.056	-0.061	-0.101	-0.108
(child)	(0.068)	(0.067)	(0.067)	(0.068)
Standardized Human Capital at Baseline	0.022	0.019	0.028	0.018
	(0.034)	(0.033)	(0.036)	(0.033)
Standardized Household Wealth	-0.018	-0.016	0.017	0.021
	(0.034)	(0.034)	(0.036)	(0.034)
Mother's age	0.010*	0.010*	0.008	0.009
	(0.005)	(0.005)	(0.005)	(0.005)
Mother's education	0.205	0.199	0.075	0.116
(primary)	(0.436)	(0.429)	(0.383)	(0.380)
Mother's education	0.212	0.206	0.116	0.157
(secondary and more)	(0.423)	(0.417)	(0.379)	(0.377)
Mother's depression	-0.012	-0.012	-0.045	-0.051
(CES-D)	(0.056)	(0.056)	(0.063)	(0.063)
Mother's IQ	-0.005	-0.004	0.102***	0.101***
(standardized Raven's Score)	(0.035)	(0.035)	(0.037)	(0.035)
Standardized values of mean μ_1		0.045		
		(0.043)		
Standardized values of mean μ_2				0.099**
				(0.047)
R^2	0.008	0.010	0.019	0.029
Observations	1017	1017	1017	1017

Standard errors (in parentheses) are clustered at municipality level.
sym* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Subjective Estimates and SE characteristics: Translog

VARIABLES	μ_1	μ_1	μ_2	μ_2	μ_3	μ_3
Dummy for Treatment	-0.102	-0.096	-0.077	-0.076	0.079	0.080
	(0.064)	(0.063)	(0.063)	(0.064)	(0.062)	(0.064)
Dummy for Male	-0.081	-0.084	-0.082	-0.082	0.077	0.077
(child)	(0.072)	(0.071)	(0.067)	(0.068)	(0.067)	(0.068)
Standardized Human Capital at Baseline	0.009	0.006	0.002	0.002	0.002	0.002
	(0.033)	(0.031)	(0.032)	(0.032)	(0.032)	(0.031)
Standardized Household Wealth	-0.046	-0.045	-0.045	-0.045	0.051	0.051
	(0.036)	(0.036)	(0.035)	(0.035)	(0.035)	(0.035)
Mother's age	0.010*	0.010*	0.009	0.009	-0.008	-0.008
	(0.006)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
Mother's education	0.255	0.252	0.219	0.219	-0.227	-0.227
(primary)	(0.285)	(0.282)	(0.183)	(0.183)	(0.172)	(0.172)
Mother's education	0.225	0.223	0.173	0.173	-0.172	-0.172
(secondary and more)	(0.279)	(0.276)	(0.186)	(0.186)	(0.177)	(0.177)
Mother's depression	-0.042	-0.040	-0.051	-0.051	0.051	0.051
(CES-D)	(0.060)	(0.061)	(0.065)	(0.065)	(0.064)	(0.064)
Mother's IQ	-0.021	-0.020	-0.012	-0.012	0.024	0.024
(standardized Raven's Score)	(0.034)	(0.034)	(0.034)	(0.034)	(0.034)	(0.034)
Standardized values of mean μ_1		0.022				
		(0.046)				
Standardized values of mean μ_2				0.001		
				(0.040)		
Standardized values of mean μ_3						-0.001
						(0.039)
R^2	0.013	0.014	0.011	0.011	0.013	0.013
Observations	1017	1017	1017	1017	1017	1017

Beliefs and Investments FU-I: Translog

VARIABLES	Log Investment		Time		Activities		Materials	
Standardized values of mean μ_0	1.002**	0.579	2.578**	1.995	1.323***	1.009**	-0.218	-0.560*
	(0.395)	(0.406)	(1.205)	(1.262)	(0.434)	(0.426)	(0.337)	(0.328)
Standardized values of mean μ_1	0.878**	0.493	2.458*	1.918	1.280***	1.002**	-0.351	-0.664*
	(0.398)	(0.403)	(1.240)	(1.287)	(0.448)	(0.437)	(0.344)	(0.334)
Standardized values of mean μ_2	1.338***	0.595	3.278**	2.324*	0.900*	0.315	0.654	0.063
	(0.450)	(0.495)	(1.276)	(1.375)	(0.473)	(0.475)	(0.500)	(0.520)
Standardized values of mean μ_3	1.286***	0.566	3.277***	2.346*	0.923**	0.358	0.557	-0.016
	(0.419)	(0.463)	(1.195)	(1.298)	(0.448)	(0.447)	(0.470)	(0.490)
Dummy for Treatment	0.162*	0.174*	0.068	0.076	0.265***	0.278***	0.001	0.008
	(0.094)	(0.091)	(0.224)	(0.221)	(0.098)	(0.097)	(0.086)	(0.083)
Child's age at FU-I	-0.000	-0.000	-0.050**	-0.050**	-0.016*	-0.015*	0.001	0.001
	(0.008)	(0.008)	(0.022)	(0.022)	(0.008)	(0.008)	(0.008)	(0.008)
Dummy for Male	-0.006	-0.003	-0.009	0.003	0.041	0.045	-0.020	-0.023
	(0.062)	(0.062)	(0.172)	(0.173)	(0.064)	(0.064)	(0.058)	(0.057)
Standardized Human Capital at Baseline	0.166***	0.138***	0.214***	0.174**	0.120***	0.099**	0.136***	0.113***
	(0.035)	(0.035)	(0.081)	(0.080)	(0.041)	(0.040)	(0.034)	(0.035)
Standardized Household Wealth	0.149***	0.103***	0.023	-0.043	0.104***	0.072**	0.172***	0.133***
	(0.035)	(0.034)	(0.087)	(0.089)	(0.032)	(0.032)	(0.034)	(0.035)
Mother's IQ		0.130***		0.132		0.097***		0.120***
(standardized Raven's Score)		(0.038)		(0.106)		(0.037)		(0.036)
R^2	0.080	0.126	0.022	0.036	0.063	0.096	0.062	0.089
Observations	1017	1017	1017	1017	1017	1017	1017	1017

Standard errors (in parentheses) are clustered at municipality level.
$^{*}\ p < 0.10$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$

Beliefs and Investments FU-II: Translog

VARIABLES	Log Investment		Time		Activities		Materials	
Standardized values of mean μ_0	0.327	-0.067	-0.312	-0.507	0.986*	0.802	0.068	-0.342
	(0.394)	(0.400)	(0.711)	(0.715)	(0.547)	(0.569)	(0.376)	(0.374)
Standardized values of mean μ_1	0.315	-0.030	-0.340	-0.507	1.040*	0.881	0.009	-0.355
	(0.428)	(0.433)	(0.726)	(0.729)	(0.568)	(0.590)	(0.405)	(0.406)
Standardized values of mean μ_2	1.024**	0.267	-0.268	-0.681	0.832*	0.453	1.067**	0.306
	(0.463)	(0.457)	(0.755)	(0.787)	(0.495)	(0.510)	(0.463)	(0.450)
Standardized values of mean μ_3	1.089**	0.362	-0.233	-0.634	0.944*	0.580	1.083**	0.350
	(0.458)	(0.446)	(0.705)	(0.736)	(0.484)	(0.501)	(0.426)	(0.412)
Dummy for Treatment	-0.073	-0.057	0.029	0.052	-0.141	-0.128	0.006	0.020
	(0.083)	(0.079)	(0.115)	(0.116)	(0.087)	(0.085)	(0.089)	(0.085)
Child's age at FU-II	0.166	0.104	0.098	0.073	-0.481	-0.503	0.281	0.217
	(0.483)	(0.477)	(0.663)	(0.673)	(0.467)	(0.466)	(0.448)	(0.433)
Dummy for Male	-0.074	-0.077	0.025	0.016	-0.080	-0.087	-0.013	-0.015
	(0.066)	(0.066)	(0.086)	(0.085)	(0.062)	(0.062)	(0.059)	(0.059)
Standardized Human Capital at Baseline	0.114***	0.085***	0.104*	0.090*	0.043	0.029	0.151***	0.123***
	(0.032)	(0.030)	(0.055)	(0.053)	(0.030)	(0.029)	(0.036)	(0.033)
Standardized Household Wealth	0.181***	0.140***	0.104**	0.084	0.125***	0.105**	0.199***	0.156***
	(0.035)	(0.033)	(0.051)	(0.052)	(0.042)	(0.042)	(0.032)	(0.030)
Mother's IQ		0.149***		0.057		0.079*		0.162***
(standardized Raven's Score)		(0.039)		(0.050)		(0.041)		(0.039)
R^2	0.070	0.116	0.016	0.030	0.035	0.048	0.089	0.133
Observations	1017	1017	1017	1017	1017	1017	1017	1017

Standard errors (in parentheses) are clustered at municipality level.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Outline

Conclusions and further work

- We have elicited parental beliefs about the production function.
- We have shown how to relate answers about developmental age under different scenarios to beliefs about returns to investment and parameters of the production function.
- We find that parents think that the productivity of investment is much higher for low initial conditions than higher initial conditions.