Explaining Early Bidding in Informationally-Restricted Ascending-Bid Auctions

Sungjin Cho, Seoul National University
Harry J. Paarsch, University of Central Florida
John Rust, Georgetown University

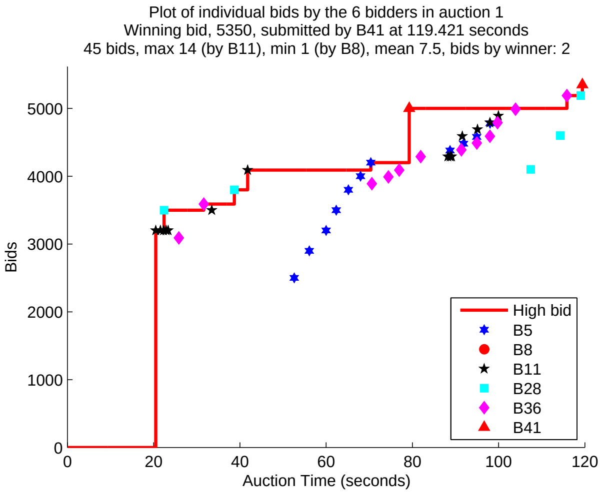
DSE 2022 Conference, ANU

December 17, 2022

What we do in this paper

- We analyze a large number of continuous time ascending bid auctions of rental cars conducted online over 2 minute intervals.
- Due to concerns about bidder collusion, the rental company designed a unique dynamic auction: the *Korean auction.*
- Bidders only know the amount of their own bids and an indicator whether their bid is the highest so far.
- *Bidders cannot observe the identities or bids placed by competing bidders, and thus do not even know the number of other bidders bidding in any given auction.*
- We are aware of only one other paper that analyzed auctions with informational restrictions similar to the Korean auctions: auctions of certificates of deposit (CDs) by the state of Texas.
- Barkley, Groeger and Miller *Journal of Econometrics* (2021) provide an empirical of analysis bidding in these auctions.
- "This market features frequent jump bidding and winning bids well above the highest losing bid, suggesting standard empirical approaches for ascending auctions may not be suitable."

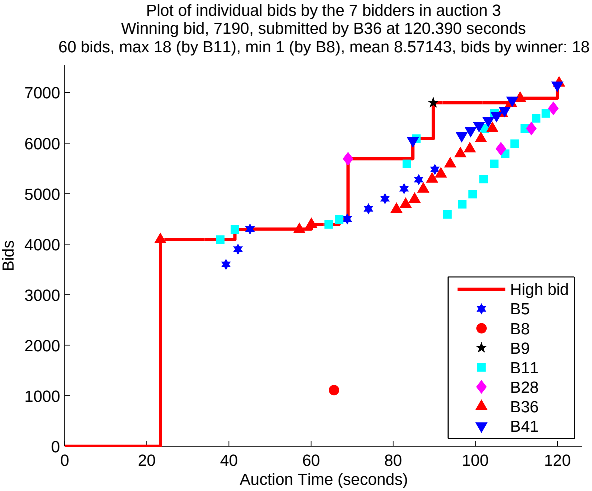
Auction 1, January 26, 2005



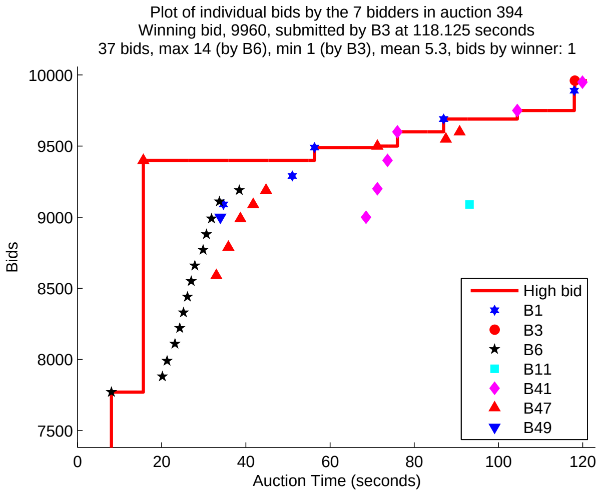
Learning via "bid creeping"

- Notice that bidder B5 makes frequent bids, each slightly higher than the previous one.
- It seems evident that B5 was trying to "probe" or "test" the market to learn what the current high bid was.
- However B5 never succeeded in placing a highest bid, and only learned that the high bid was higher than each of its sucessive bids.
- B5's last bid was $4500 placed less than 30 seconds remaining in the auction, after which B5 gave up and declined to submit any further bids.

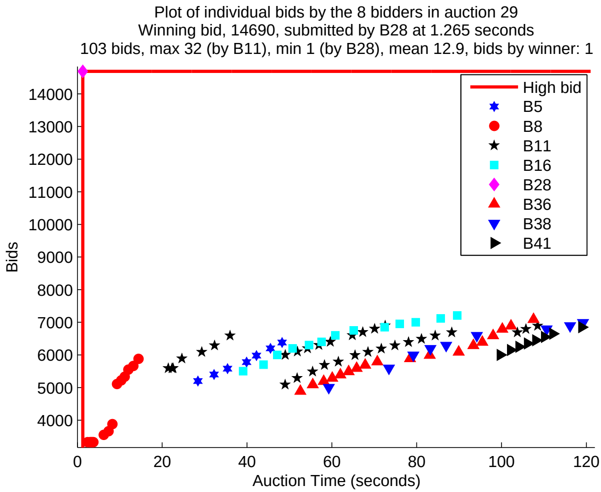
Auction 3, January 26, 2005



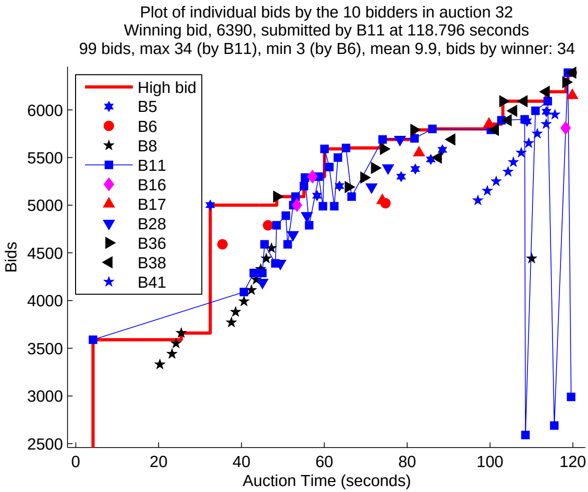
Auction 394 — bid sniping



Auction 29 — early high bidder



Auction 32 — a “crazy” bidder B11



Static vs Dynamic Auction Mechanisms

- Dynamic auction mechanisms
  - Japanese auctions: there is a continuously rising clock
  - English or *open outcry* auctions
- Static auction mechanisms
  - First price sealed bid auctions
  - Second price sealed bid auctions
- When is it better to use a static or dynamic mechanism?
- Japanese auctions are *strategically equivalent* to static second price auctions
- Under symmetry, the *Revenue Equivalence Theorem* predicts that the revenue from first and second price auctions are equal.
- But modeling bidding in an *open outcry auction* is a much more complex animal, a problem that remains unsolved.

Empirical vs theoretical auction design

- There are many different formats for auctions (i.e. rules for running auctions) such as English (open ascending bid) auctions, first price sealed bid auctions, etc.
- One could also use non-auction mechanisms to sell an item: e.g. selling lottery tickets
- Which is the best format to use? Landmark paper: "Optimal Auction Design" (1982) by Roger Myerson *Mathematics of Operations Research*
- Characterized the optimal auction mechanism for selling a single object to a finite (known) set of bidders who have valuations for the object with known distributions that are independently distributed (the *Independent Private Values* (IPV) assumption)
- Optimal market design (maximizes expected revenue to the seller): A second-price auction (Vickrey auction) with a reservation price.

Overbidding in second-price auctions

Cooper and Fang 2008, *IER*

In laboratory experiments, however, subjects are found to exhibit a consistent pattern of overbidding. Kagel et al. (1987) found that the actual bids are on average 11% above the dominant strategy bids. Kagel and Levin (1993) found that about 62% of all bids in their five-bidder SPA sessions exceed the bidder's value, while only 8% of all bids were below it. Both Kagel and Levin (1993) and Harstad (2000) further reported that experience has only a small effect in reducing overbidding in SPA.

But not in English/clock (Japanese) auctions

Kagel, Levin and Harstad (1987) *Econometrica*

The structure of English clock auctions makes it particularly clear to bidders that they don't want to bid above their private values. Once the clock price exceeds a bidder's value, it is clear that competing further to win necessarily involves losing money. The enhanced capacity of the English clock auction to produce observational learning distinguishes it most clearly, on a behavioral level, from the second-price institution.

Accounting for collusion in auction design

- "Economists are proud of their role in pushing for auctions; for example, Coase (1959) was among the first to advocate auctioning the radio spectrum. But many auctions — including some designed with the help of leading academic economists — have worked very badly." Paul Klemperer (2002) "What Really Matters in Auction Design" *Journal of Economic Perspectives*
- Can collusion-proof auction mechanisms be designed?
- Generally, no. However auction experts such as Cramton and Schwartz (2002) and Marshall and Marx (2009) have suggested that *informational restrictions* especially in ascending bid auctions can help thwart collusion.
- Main informational restrictions that have been suggested:
    1. coarsening bids to prevent signalling during auctions
    2. suppressing bidder identities
- Can also set either public or secret reserve price.

Natural experiments in auction design

- **Regime 0: (pre 2003)** *open outcry auctions* held at each rental car location, but the owners suspected bidding collusion that lowered their bids.
- **Regime 1: (2003 to 2007)** Its own unique online bidding system (via the Internet) that suppresses bidder identities and bids to try to defeat potential collusion.
- **Regime 2: (2007 to present)** The company abandoned its online auction system and sold cars in a wholesale auction house in Seoul which used open outcry auctions

Did the informational restrictions thwart collusion?

Model	Regime 0 10/1/2002 to 12/31/2022	Regime 1 1/1/2003 to 3/31/2003	Regime 1 4/1/2003 6/30/2003	P-values for two sample t-tests for equal means
EF Sonata 1.8	5279 (1048),n=81	5148 (746),n=72	4919 (700),n=89	.815,.995 .975,.976
EF Sonata 2.0	5867 (1594),n=137	6043 (1359),n=46	7161 (1432),n=17	.235,.001 .019,.005
Dynasty 3.0	11633 (2496),n=25	13043 (2458),n=23	12934 (1757),n=13	.027,.035 .016,.560
Grandeur XG 2.0	11295 (1399),n=18	11081 (1055),n=14	11123 (978),n=15	.687,.659 .692,.456
Grandeur XG 2.5	12626 (2150),n=67	11504 (1974),n=50	11827 (1356),n=78	.998,.995 .999,.157
Galloper 7	7109 (1480),n=45	7477 (1473),n=61	7776 (1263),n=53	.103,.010 .025,.123
Magnus 2.0	7614 (1170),n=11	6665 (1576),n=16	6503 (506),n=6	.957,.992 .980,.640

Is the linkage principle valid?

- *Linkage principle* if bidders' valuations are affiliated, auctions that release more information over the course of the auction will result in higher average prices compared to auctions that reveal less information.
- Our 2014 *JINDEC* paper, "Is the Linkage Principle Valid? Evidence from the Field" compared the alternative auction formats the rental company used with respect to mean revenue, and found evidence consistent with the linkage principle — the prices from the auction house were 10% higher than the company's online auction system.
- Paradox: the dynamic rental auction releases more information than a static first price sealed bid auction, yet we find that the dynamic auction results in *lower* expected revenue when bidders are rational.
- However we show the dynamic auction raises *more* revenue if bidders are boundedly rational.

Preview of our conclusions

- We characterize bidding behavior in the Korean auction and show there is *frequent early bidding.*
- We conjecture that *early bidding will not occur in a PBE.* That is, we conjecture that the only equilibrium is an *uninformative equilibrium* where all bidders wait to the last instant to submit bids.
- The uninformative equilibrium always exists and and is strategically equivalent to the equilibrium of a static first price sealed bid auction.
- We illustrate a two bidder, two period example where the *only* PBE is the uninformative equilibrium.
- We introduce a model of *rationally inattentive bidding* with *bidding frictions* in *anonymous equilibrium* that can explain early bidding.
- The model predicts that bidders have an incentive to bid early and learn via early bidding to try to win the auction without overpaying.
- Our theory implies that the learning that occurs during the Korean auction enables bidders to pay less, so expected revenue is *lower* than expected revenue in a first price sealed bid auction.

Preview of our conclusions

- Our model provides a *qualitative explanation* for the "informative early bidding" in these auctions, but . . .
- The professional bidders tend to *bid too high, too fast* compared to what our model predicts, a phenomenon we call *early overbidding*.
- We show that our estimated bidding strategies *outperform* human bidders in terms of expected profits.
- We blame the rejection of our model on the bounded rationality of the human bidders in the face of the difficult dynamic learning/DP problem in these continuous time auctions.
- The Korean auction takes advantage of irrational early overbidding, resulting in *higher* expected revenues than a static first price sealed bid auction.
- However we predict that a static second price auction would generate even higher expected revenues than the Korean auction, even without a reservation price.
- The under bidding we observe in the Korean auctions is inconsistent with the hypothesis of collusion.

Is there "straightforward bidding" in the Korean auction?

Definition of straightforward bidding

The optimal bidding strategy in a Japanese auction the involves simply *remaining in the auction until the current bid exceeds your valuation and then exit*

- In the Korean auction there is no public "price clock" broadcast to all bidders telling them what the current high bid is. Thus, not clear straightforward bidding is feasible.
- But by frequent "bid creeping" bidders can learn the high bid even if it is not broadcast to them.
- Are equilibrium bidding strategies in the Korean auction straightforward?
- **Answer:** NO. It is generally optimal to stop bidding before reaching your valuation.

Can rational game-theoretic models explain early bidding?

- There is a substantial amount of early bidding in these auctions, even though a game-theoretic analysis suggests that the informational restrictions should create strong incentives for *bid sniping* — i.e. waiting to submit a bid only in the last instant of the auction.

Definition of PBE

A *Perfect Bayesian Equilibrium* (PBE) of a dynamic game of incomplete information is a subgame perfect equilibrium, where players' beliefs are updated using Bayes rule wherever possible.

Definition of an Informative PBE

An *informative PBE* is any PBE where there is positive probability of bidding before the final instant along the equilibrium path.

- In a two bidder, two period example, we show there is no informative PBE and thus no early bidding.
- This creates a challenge: can the early bidding we observe in these auctions be explained as a PBE outcome?

The uninformative PBE

Definition of uninformative PBE

An *uninformative PBE* is a PBE where with probability 1 on the equilibrium path players do not bid at any times except for the last possible instant T in the auction. That is, all bidders *snipe* and submit bids equal to those that they would submit in a single shot first price sealed bid auction.

Theorem

If the Korean auction has a hard close, the uninformative PBE is always a PBE of the dynamic auction game.

- In an uniformative PBE, the players do not bother trying to test/probe in the early stages of the auction, so the value of learning is zero since and there is *no learning in this equilibrium.*

Do informative PBE exist?

- The bidding data allow us to easily reject the hypothesis that all bidders are playing uninformative PBE bidding strategies.
- The significant frequency of *bid sniping* could indicate that *some* bidders are trying to play the uninformative equilbrium.
- However if some players are deviating from the uninformative equilibrium, playing the uninformative equilibrium (i.e. bid sniping) may no longer be a best response.
- Can the bidding behavior we observe be rationalized as *some* PBE of this game?

Non-existence of an informative PBE, a 2x2 example

- Consider a symmetric equilibrium where in period $t = 1$ both bidders submit bids according to a single bid function $b_1(v)$, where $f(v)$ is the density of the bidders' valuations.
- Even if the bid function in period $t = 1$ is symmetric, so bids are given by $b_i = b_1(v_i), = 1, 2$ where v_i is the valuation of bidder i (a realization of the random variable $\tilde{v}$ with density $f(v)$), the relevation of information about which bid is the highest in period $t = 1$ results in *endogenous asymmetry* in the bid functions at time $t = 2$.
- if b_1 is a strictly monotonic bid function (a necessary condition for an informative equilibrium), then $b_1(v_1) > b_2(v_2)$ implies that $v_1 > v_2$.

Non-existence of an informative PBE, a 2x2 example

- The information from period 1 bids about which of the two bidders has the highest valuation is the source of informational asymmetry in period 2.
- Suppose bidder 1 learns that he has the higher valuation. Then bidder 1's posterior belief of bidder 2's valuation in period 2 is $F(v)/F(v_1)$ where $F(v)$ is the prior belief of the CDF of valuations in period 1.
- For bidder 2, their posterior belief of bidder 1's valuation in period 2 is given by $[F(v) - F(v_2)]/[1 - F(v_2)]$.
- Let $b_{2,h}(v)$ be the period two bid function for a bidder who learns they had the high bid in period 1, and let $b_{2,l}(v)$ be the bid function of a bidder who learns their bid was the low bid in period 1.

Period 2 equilibrium bid functions

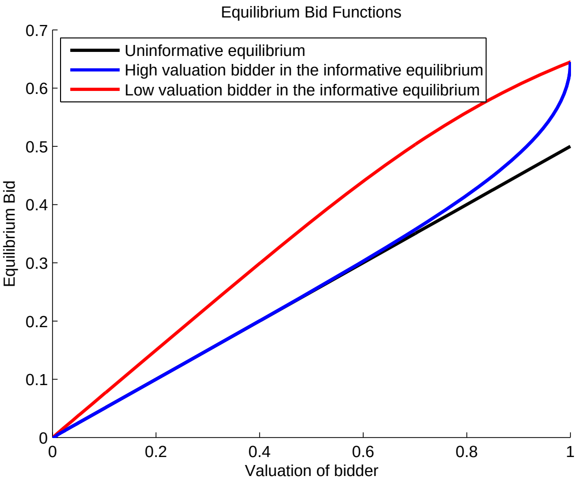
Solve the game by backward induction, In period two the equilibrium bid functions solve

$$
\begin{aligned}
b_{2,h}(v,b) &= \operatorname*{argmax}_{b' \geq b}(v-b') \times \\
&\quad \int_0^v I\{b_{2,l}(v', b_1(v')) \leq b'\} f(v') dv' / F(v) \\
b_{2,l}(v,b) &= \operatorname*{argmax}_{b' \geq b}(v-b') \times \\
&\quad \int_v^\infty I\{b_{2,h}(v', b_1(v')) \leq b'\} f(v') dv' / [1-F(v)]
\end{aligned}
$$

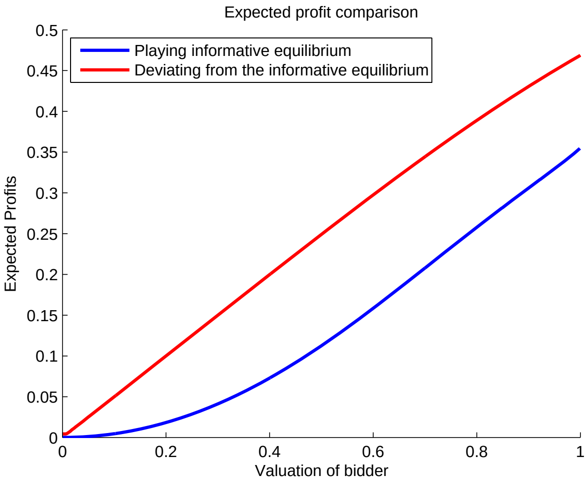
Period 1 equilibrium bid function

$$\begin{aligned}
b_1(v) &= \underset{b}{\operatorname{argmax}}(v - b_{2,h}(v,b)) \times \\
&\quad \left[\int_0^v I\{b_{2,l}(v', b_1(v')) \leq b_{2,h}(v,b)\} f(v') dv'\right] + \\
&\quad (v - b_{2,l}(v,b)) \times \\
&\quad \left[\int_v^\infty I\{b_{2,h}(v', b_1(v')) \leq b_{2,l}(v,b)\} f(v') dv'\right].
\end{aligned}$$

Equilibrium bid functions in period 2



Gain from deviating from the informative equilibrium



Difficulties of computing nontrivial PBEs

- The main difficulty is that all bidders must be endowed with *priors* over a) the number of bidders in the auction, and b) their valuations. These beliefs must be updated at each instant based on the history of bids made so far.
- Even if the history is very limited due to the informational restrictions of this auction, the history for each player includes at least, a) the current time t, b) the player's own history of bids, and c) whether the player's bid is the highest or not.
- It is extremely challenging to compute a posterior distribution over these quantities, and the dimensionality of the posterior is effectively infinite-dimensional (unless the posterior could be shown to be a member of a conjugate prior class, which seems unlikely).

Costs and benefits of informative bidding

- The main gain to placing "serious" bids early in the auction is to gather significant information on what the high bid is, and to use this to try to win without overpaying.
- However there are at least two costs of placing a serious bid: a) the bidder could mistakenly overbid and the auction rules commit the bidder to pay the *highest* bid submitted during the two minute auction.
- and b) by bidding, the bidder provides information to other bidders that could affect their subsequent bidding behavior to the detriment of the bidder in question.

A rationally inattentive model of bidding in Korean auctions

- Due to the difficulty of computing PBE and because it is not clear that there is an informative PBE that would be consistent with the bidding behavior we observe in these auctions, we adopt an alternative modeling approach.
- We develop a *behavioral DP bidding model* that assumes *bidders have rational beliefs* about the stochastic process for the high bid price in the auction.
- Using these beliefs, we solve a dynamic program to determine the optimal bidding strategy implied by these beliefs.
- We define a concept of ϵ-*anonymous equilibrium* to define approximately self-confirming beliefs of bidders in these auctions.
- It is similar to a *rational expectations equilibrium* which is also a self-confirming system of beliefs. In an ϵ-anonymous equilibrium the actual stochastic process for the highest bid during the auction is approximately equal to bidders' beliefs about this stochastic process.

Accounting for learning

- Our approach involves *learning* but employs a simpler model of *experiential learning* rather than full Bayesian updating.
- Our model is appropriate for *experienced bidders* who have participated in many auctions, and thus have well-defined and fixed beliefs about the stochastic process for the high bid in the auction.
- If a bidder has the highest bid at t, then they know it. But if the bidder does not have the highest bid at t, they must predict it based on a rational belief of the probability distribution of the high bid.
- We show that there is a significant reduction in uncertainty by learning that one has the high bid prior to the end of the auction.
- Thus the motivation for early bidding is to gather information about the current high bid, helping bidders to win the auction without paying more than necessary.

A 4 parameter DP model of rationally inattentive bidding

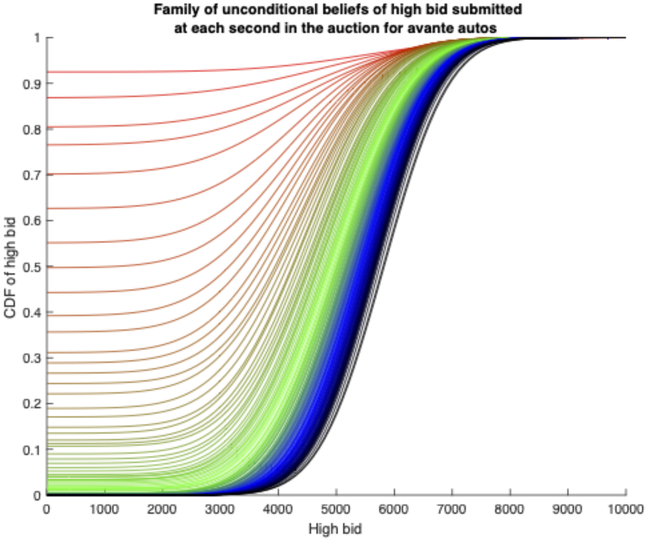
- We discretize the two minute auction into $T = 120$ one second time steps.
- Let $\tau = (v, c, p, \sigma)$ denote the *type* of the bidder, where v is the bidder's valuation of the car being auctioned, and c is the bidder's psychic *cost of submitting a bid*, p is a probability the bidder is *distracted* and cannot submit a bid, and σ is an extreme value scale parameter.
- We assume that bidders are experienced and have *fixed, rational beliefs* about the stochastic process for the high bid in auctions for homogeneous types of cars.
- Bidder beliefs are captured by a family of conditional probability distributions $\{\lambda_t(b|b_t, h_t)\}$ where $\lambda_t(b|b_t, h_t)$ is a CDF for the high bid at during the interval $(t, t+1]$ of the auction, conditioned on b_t, the bidder's highest bid up to time t (or 0 if the bidder has not bid yet), and h_t is an indicator of whether the bidder holds the high at t.

Justification for fixed beliefs

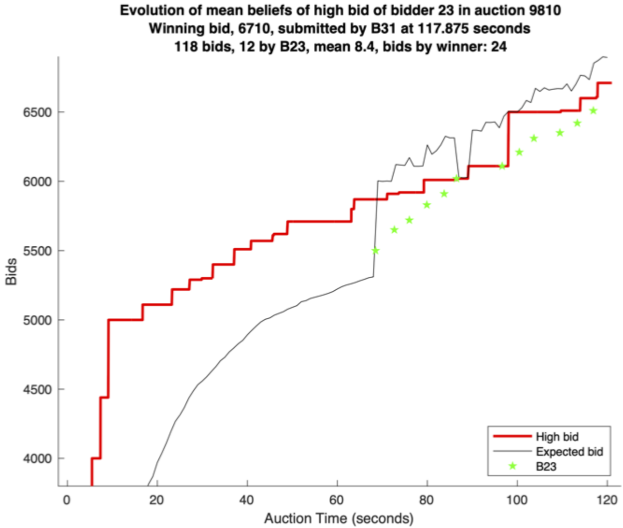
- We focus on auctions of a homogeneous class of rental cars (Hyundai Avante Elanta XD with 1.6L engines) which are unlikely to have unique characteristics that make individual auctions to be "unique" (as opposed to an auction for a Picasso or Rembrandt).
- As a result it is plausible that experienced bidders will assume there is a common stochastic process describing the evolution of the high bid in these auction, and we assume all bidders know this stochastic process.
- Thus, participating in additional auctions is unlikely to change the bidder's beliefs about this stochastic process — learning has "converged" to a rational expectation of this stochastic process.
- What a bidder does learn during an individual auction is whether he/she holds the high bid based their history of their own bids during the auction.
- Thus early bidding can be regarded as means of learning what the high bid is in order to avoid overpaying to win the auction.

2005 Hyundai Avante Elantra XD 1.6L

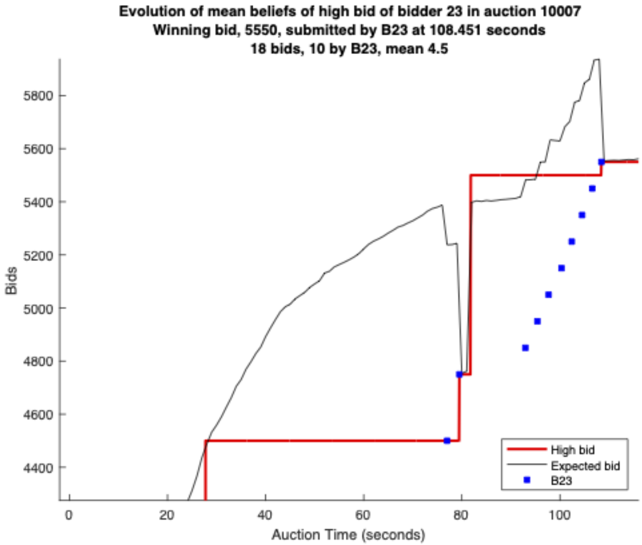
Unconditional beliefs about the high bid in the auction



Evolution of beliefs of B23 in auction 9810



Evolution of beliefs of B23 in auction 10007



Two Step Estimation Strategy

- **Step 1** Using data on 533 auctions of Avante cars, we estimate the family of beliefs $\{\lambda_t\}$ about the stochastic process for the high bids in the auction.
- **Step 2** Using $\{\hat{\lambda}_t\}$ we solve a discrete dynamic programming problem determining the optimal bidding of the bidder at each second of the auction, resulting in a family of bid functions $\{\beta_t\}$ where $\beta_t(b_t, h_t)$ is the optimal bid at the start of second t in the auction when the bidder's high bid so far is b_t and h_t is an indicator of whether the bidder has the high bid or not.
- Note the only unknown parameters in step 2 are $\tau = (v, c, p, \sigma)$. Thus, we form a likelihood $L(\tau)$ for *each bidder in each auction* resulting in auction-by-auction bidder-specific estimates of valuations, v and other parameters τ characterizing the bidder's type.
- Our goal is to see if such a model is capable of explaining the early bidding we observe in these auctions.

Bidder's DP problem

- Let $h_t = 1$ if the bidder has the highest bid up to second t in the auction, $h_t = 0$ otherwise. Let b_t be the highest bid submitted by the bidder up to second t in the auction.
- The timing is as follows. At the start of each "bidding instant" t ($t = 0, 1, \ldots, 120$), the bidder observes (b_t, h_t) and decides whether to submit a bid $b > b_t$ or not bid, which is equivalent to a non-improved bid of $b = b_t$. At $t = 0$, the auction is initialized with $b_0 = 0$ and $h_0 = 0$ for all bidders.
- The transition rule for bids by a given bidder is as follows: $b_{t+1} = b$, where b is the bid decision at time t. Thus, if the decision is not to bid, then $b_{t+1} = b_t$, otherwise if the bidder submits a bid of $b > b_t$, then $b_{t+1} = b$. Also $h_{t+1} = 1$ if b_{t+1} is the highest bid outstanding at start of second $t + 1$, otherwise $h_{t+1} = 0$.
- We assume that there is a "distraction probability" p that prevents a bidder from focusing on the auction and deciding whether to update their bid at each second t of the auction. Thus, with probability at least p, no bid is submitted and $b_{t+1} = b_t$.

Bidder's DP problem, continued

- The terminal payoff of the bidder at the conclusion of the auction at $T+1=121$ (after the final bids have been submitted so the high bid can be determined) is

$$W_{T+1}(b_{T+1},h_{T+1}) = (v - b_{T+1})I\{h_{T+1}=1\},$$

  where b_{T+1} is the bid the bidder submitted at the last possible bidding instant $T=120$ and $h_{T+1}=1$ if this was the highest bid in the auction, or 0 otherwise.

- Define $\lambda_T(b|b_T,h_T) = E\{I\{h_{T+1}=1\}|b,b_T,h_T\}$, i.e. this is the probability that the bidder will win the auction by placing a bid of b at the last possible instant $T=120$, conditioning on their information (b_T,h_T) at this instant.

Bidder's DP problem, continued

- Define the *bid-specific value function* $w_T(b, b_T, h_T)$ by

$$w_T(b, b_T, h_T) = E\{W_{T+1}(b_{T+1}, h_{T+1})|b, b_T, h_T\} = (v - b)\lambda_T(b|b_T, h_T).$$

  Thus, $w_T(b, b_T, h_T)$ is the expected payoff to the bidder from placing a final bid of b at the last possible bidding instant T in the auction, assuming the bidder is not distracted and thus able to bid.

- Define the value function $W_T(b_T, h_T, \epsilon_T)$ by

$$W_T(b_T, h_T, \epsilon_T) = \max\left[w_T(b_T, b_T, h_T) + \epsilon_T(0), \max_{b \geq b_T}[-c + \epsilon_T(1) + w_T(b, b_T, h_T)]\right]$$

  where $\epsilon_T = (\epsilon_T(0), \epsilon_T(1))$ is a bivariate Type-1 extreme value distribution that reflects idiosyncratic "noise" affecting the bidder's calculation of an optimal bid. Parameter c is the cost of "mental effort" to calculate an improved bid. We assume that passing on bidding involves zero additional mental effort.

Bidder's DP problem, continued

- If the bidder is not distracted from bidding at T their expected value is $EW_T(b_T, h_T)$, given by

$$EW_T(b_T, h_T)$$
$$= \int_{\epsilon_T} W_T(b_T, h_T, \epsilon_T) q(\epsilon_T)$$
$$= \sigma \log \left(\exp\{w_T(b_T, b_T, h_T)/\sigma\} + \exp\{\max_{b \geq b_T}[w_T(b, b_T, h_T) - c]/\sigma\} \right)$$

- However if the bidder is distracted at T and does not bid, their value is $w_T(b_T, b_T, h_T)$.
- Then at time $T-1$, the bid-specific value function is $w_{T-1}(b, b_{T-1}, h_{T-1})$ is

$$w_{T-1}(b, b_{T-1}, h_{T-1}) =$$
$$[pw_T(b, b, 1) + (1-p)EW_T(b, 1)]\, \lambda_{T-1}(b|b_{T-1}, h_{T-1}) +$$
$$[pw_T(b, b, 0) + (1-p)EW_T(b, 0)]\, [1 - \lambda_{T-1}(b|b_{T-1}, h_{T-1})].$$

Bidder's DP problem, continued

- Continuing the backward induction from $t = T, T-1, \ldots, 0$ we have solved for the optimal dynamic bidding strategy in the auction.
- The formulas for the expected value of bidding for bidders who are not distracted the same as given above, so we recursively calculate $EW_t(b_t, h_t)$, and the value of being distracted is $w_t(b_b, b_t, h_t)$, recursively for $t = T-1, T-2, \ldots, 1, 0$.
- Then at each time t the bid-specific value function is $w_t(b, b_t, h_t)$ given by

$$
\begin{aligned}
& w_t(b, b_t, h_t) = \\
& [pw_{t+1}(b, b, 1) + (1-p)EW_{t+1}(b, 1)]\, \lambda_t(b|b_t, h_t) + \\
& [pw_{t+1}(b, b, 0) + (1-p)EW_{t+1}(b, 0)]\, [1 - \lambda_t(b|b_t, h_t)].
\end{aligned}
$$

What happens if we remove the informational restriction?

- We drop the informational restriction so all bidders can see the high bid in the auction at any moment *regardless of whether they hold the high bid or not.*
- However bidder identities are still suppressed, so it continues to be an *anonymous game.*
- The resulting auction is an anonymized, electronic version of an *open outcry auction.*
- In this case beliefs about the high bid reduce to $\{\lambda_{t+1}(b_{t+1}|b_t)\}$ where b_t is the freely and publicly observed high bid at time t.

Definition: Stochastic monotonicity

$$b'_t \geq b_t \Longrightarrow \lambda_{t+1}(b_{t+1}|b'_t) \leq \lambda_{t+1}(b_{t+1}|b_t) \quad \forall b_{t+1} \tag{1}$$

Assumption: Stochastic monotonicity of beliefs

Beliefs about the high bid in the auction satisfy the stochastic monotonicity condition, (1).

What happens if we remove the informational restriction?

Theorem: No early bidding in an anonymized open outcry auction

Assume that there are no bidding frictions, $c = \sigma = 0$, but bidders may still be rationally inattentive, $p \in [0, 1]$. If beliefs satisfy the stochastic monotonicity condition, then there is no early bidding in the anonymized open outcry auction. This implies that this auction is strategically equivalent to a anonymous equilibrium version of a static first price sealed bid auction. That is, all bidders snipe and submit bids at the last possible instant T.

- Proof is by induction from the last period. There is informative bidding at T but no bidding prior to that.
- This theorem encodes the incentives for *informational free-riding* in an ascending bid auction where the high bid is publicly broadcast to all bidders "for free."

Maximum Likelihood Estimation

- We are able to estimate the parameters $\tau = (v, c, p, \sigma)$ for each bidder in each auction they participate in by maximum likelihood.
- For a given auction, we observe $\{(b_t, h_t), t = 0, \ldots, 120\}$ where b_0 is the first bid made at bidding instant $t = 0$ and b_{120} is the final bid made at $T = 120$. Let the initial conditions be $b_{-1} = h_{-1} = 0$.
- Let $L(\tau)$ be the likelihood of bids by a given bidder in a given auction

$$L(\tau) = \prod_{t=0}^{120} P_t(b_t | b_{t-1}, h_{t-1}, \tau),$$

where the probability $P_t(b' | b, h, \tau)$ is given by the MNL formula

$$P_t(b'|b,h,\tau) = \frac{\exp\{-cI\{b' \geq b\} + w_t(b', b, h)/\sigma\}}{\exp\{w_t(b, b, h)/\sigma\} + \sum_{b' \geq b} \exp\{-c + w_t(b', b, h)/\sigma\}}.$$

Maximum Likelihood Estimation

- The maximum likelihood estimator presumes that for each t and integer bid $b' > b$ there is a corresponding extreme value distributed idiosyncratic shock $\epsilon_t(b')$ associated with choosing b'. The bid-specific value function for this version of the model is

$$W_t(b_t, h_t, \epsilon_t) =$$

$$\max\left[w_t(b_t, b_t, h_t) + \epsilon_t(0), \max_{b' \geq b_t}[-c + \epsilon_t(b') + w_t(b', b_t, h_t)]\right]$$

and the expected value is

$$EW_t(b_t, h_t) = \int_{\epsilon_t} W_t(b_t, h_t, \epsilon_t) q(\epsilon_t)$$

$$= \sigma \log\left(\exp\{w_t(b_t, b_t, h_t)/\sigma\} + \sum_{b' \geq b} \exp\{[w_t(b', b_t, h_t) - c]/\sigma\}\right).$$

- This model predicts a positive probability for any integer bid $b' \geq b$ given by the logit probability above.

Quasi Maximum Likelihood Estimation

- However evaluation of the sum of exponentiated bid-specific value functions for all integer bids $b' \geq b_t$ is computationally expensive. So we propose an alternative *quasi-maximum likelihood estimator* based on an incomplete model of bidding that does not a formal theory (i.e. positive probability of) any potential bid $b' \geq b_t$.
- Under this alternative model, there are only two idiosyncratic shocks $(\epsilon_t(0), \epsilon_t(1))$ per bidding instant and the value function is

$$W_t(b_t, h_t, \epsilon_t) =$$

$$\max \left[w_t(b_t, b_t, h_t) + \epsilon_t(0), \max_{b' \geq b_t} [w_t(b', b_t, h_t) - c] + \epsilon_t(1) \right]$$

and expected value is

$$EW_t(b_t, h_t)$$

$$= \int_{\epsilon_t} W_t(b_t, h_t, \epsilon_t) q(\epsilon_t)$$

$$= \sigma \log \left(\exp\{w_t(b_t, b_t, h_t)/\sigma\} + \exp\{\max_{b' \geq b_t} [w_t(b', b_t, h_t) - c]/\sigma\} \right).$$

Quasi Maximum Likelihood Estimation

- Suppose we observe a bid of b_{t+1} at bidding instant t in bidding state (b_t, h_t).
- If $b_{t+1} > b_t$ (i.e. the bidder improved their bid), the model with only two idiosyncratic shocks per bidding instant cannot formally "explain" this bid, i.e. there is zero probability of observing "suboptimal bids" $b_{t+1} \neq \beta_t(b_t, h_t)$.
- But the QMLE assigns the following probability to a bid $b_{t+1} > b_t$

$$\Pi_t(b_{t+1}|b_t, h_t) = \frac{\exp\{w_t(b_{t+1}, b_t, h_t)/\gamma\}}{\exp\{w_t(b_{t+1}, b_t, h_t)/\gamma\} + \exp\{w_t(\beta_t(b_t, h_t), b_t, h_t)/\gamma\}},$$

where $\beta_t(b_t, h_t)$ is the *optimal bid function* at instant t given by

$$\beta_t(b_t, h_t) = \underset{b' \geq b_t}{\operatorname{argmax}}\, w_t(b', b_t, h_t).$$

and $\gamma \geq 0$ is a *smoothing parameter* or *penalty parameter* for observations $b_{t+1} \neq \beta_t(b_t, h_t)$.

Aside on the Optimal Bid Function

- In our "partial" model of bidding in the Korean auction, the optimal bid function is actually also a function of the unobserved shocks $\epsilon_t = (\epsilon_t(0), \epsilon_t(1))$. We denote this bid function by $\beta_t(b_t, h_t, \epsilon_t)$ and it is given by

$$\beta_t(b_t, h_t, \epsilon_t) =$$

$$\operatorname{argmax}\left[w_t(b_t, b_t, h_t) + \epsilon_t(0), \underset{b' \geq b_t}{\operatorname{argmax}}[w_t(b', b_t, h_t) - c] + \epsilon_t(1)\right].$$

- The relationship between $\beta_t(b, h, \epsilon)$ and $\beta_t(b, h)$ is as follows

$$\beta_t(b, h, \epsilon) = \begin{cases} \beta_t(b, h) & \text{if } w_t(b, b, h) + \epsilon(0) \leq w_t(\beta_t(b, h), b, h) + \epsilon(1) \\ b & \text{if } w_t(b, b, h) + \epsilon(0) > w_t(\beta_t(b, h), b, h) + \epsilon(1) \end{cases}$$

- Thus the optimal bid is $\beta_t(b, h)$ for any combination of private bidding shocks that makes it optimal for the bidder to improve their existing bid b, otherwise it is optimal not to improve the current bid (i.e. not bid).

Quasi Maximum Likelihood Estimation

- Thus, under the QMLE, $\Pi_t(b_{t+1}|b_t, h_t)$ is maximized at the value $\Pi_t(b_{t+1}|b_t, h_t) = 1/2$ when $b_{t+1} = \beta_t(b_t, h_t)$.
- To maximize the QMLE, parameters τ are found that make the optimal bidding function $\beta_t(b_t, h_t, \tau)$ to be as close as possible to the observed bid b_{t+1} since this maximizes $\Pi_t(b_{t+1}|b_t, h_t)$.
- The QMLE then is defined by

$$\hat{\tau} = \operatorname*{argmax}_{\tau} QL(\tau) \equiv \operatorname*{argmax}_{\tau} \prod_{t=0}^{120} P_t(b_t|b_{t-1}, h_{t-1}, \tau),$$

where $P_t(b_t|b_{t-1}, h_{t-1}, \tau)$ is given by

$$P_t(b'|b, h) = \begin{cases} 1 - \pi_t(b|b, h, \tau) & \text{if } b' = b \\ \pi_t(b'|b, h, \tau)\Pi_t(b'|b, h, \tau) & \text{if } b' \geq b \end{cases} \tag{2}$$

Quasi Maximum Likelihood Estimation

- Where $\pi_t(b'|b,h,\tau)$ is the probability of bidding given by

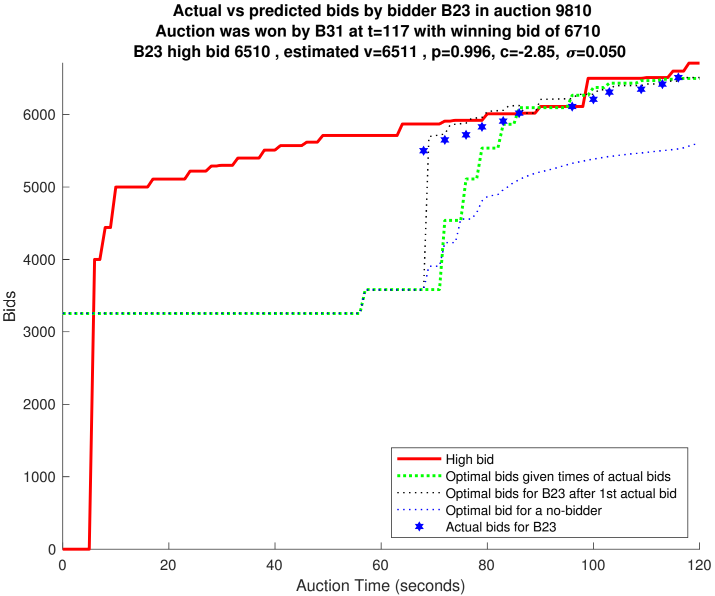
$$\pi_t(b'|b,h,\tau) = \frac{\exp\{[w_t(\beta_t(b,h),b,h,\tau) - c]/\sigma\}}{\exp\{w_t(b,b,h,\tau)/\sigma\} + \exp\{[w_t(\beta_t(b,h),b,h,\tau) - c]/\sigma\}},$$

and $\Pi_t(b'|b,h,\tau)$ is the probability of observing a potentially suboptimal bid b'

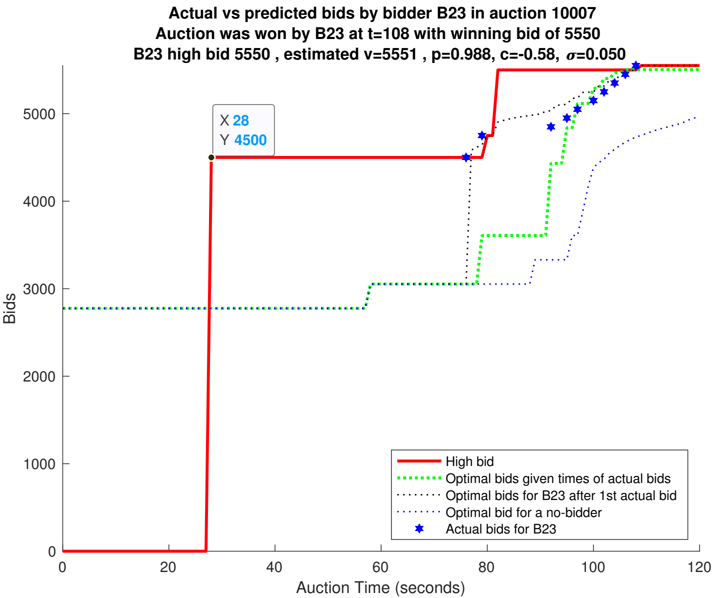
$$\Pi_t(b'|b,h,\tau) = \frac{\exp\{w_t(b',b,h,\tau)/\gamma\}}{\exp\{w_t(b',b,h,\tau)/\gamma\} + \exp\{w_t(\beta_t(b,h),b,h,\tau)/\gamma\}}.$$

- Thus, the QMLE $\hat{\tau}$ is a value of τ that maximizes the probability of the observed sequence of bids in the auction by a given bidder, even when we have an *incomplete model of bidding* — i.e. our behavioral model does not assign a positive probability to every possible bid b'.

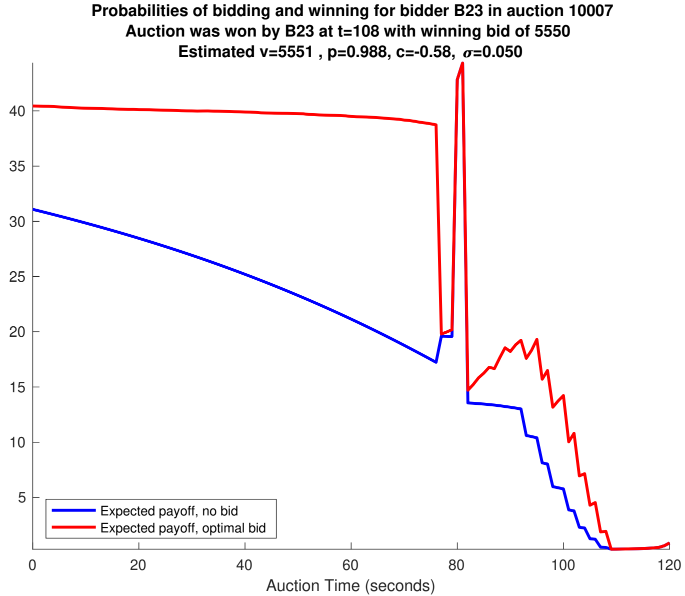
Predicted vs actual bids for B23 in auction 9810



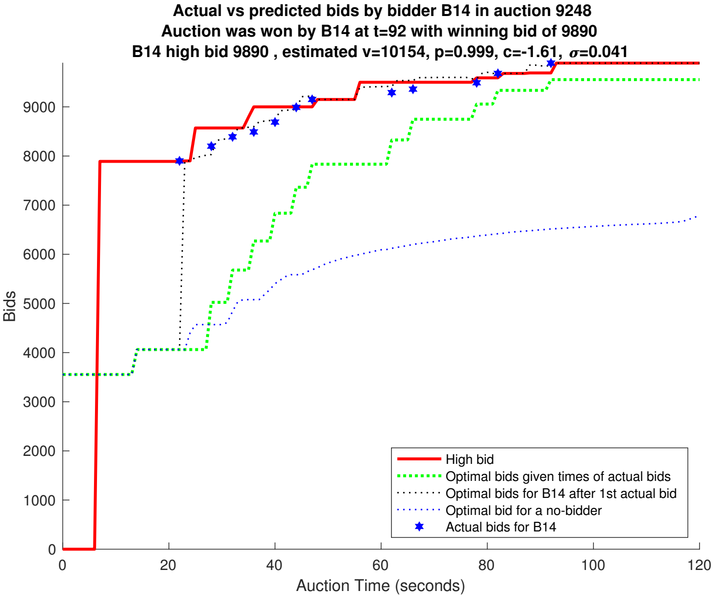
Predicted vs actual bids for B23 in auction 10007



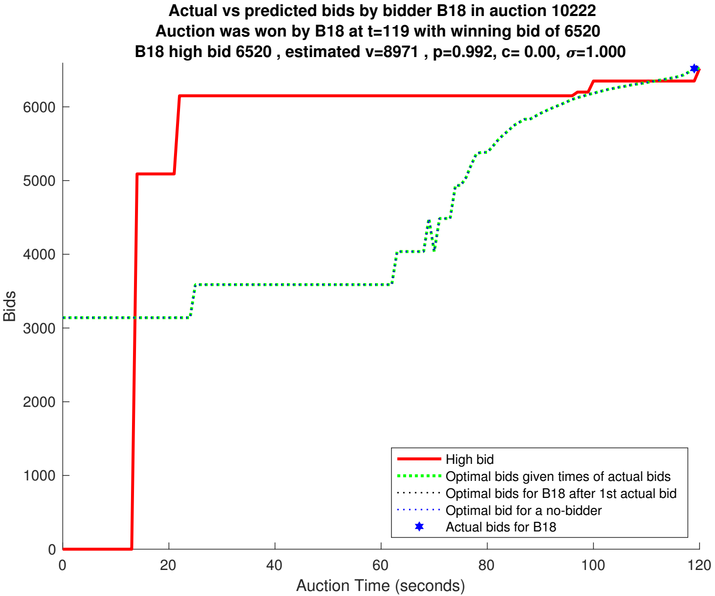
Payoffs of bidding vs not bidding: B23 in auction 10007



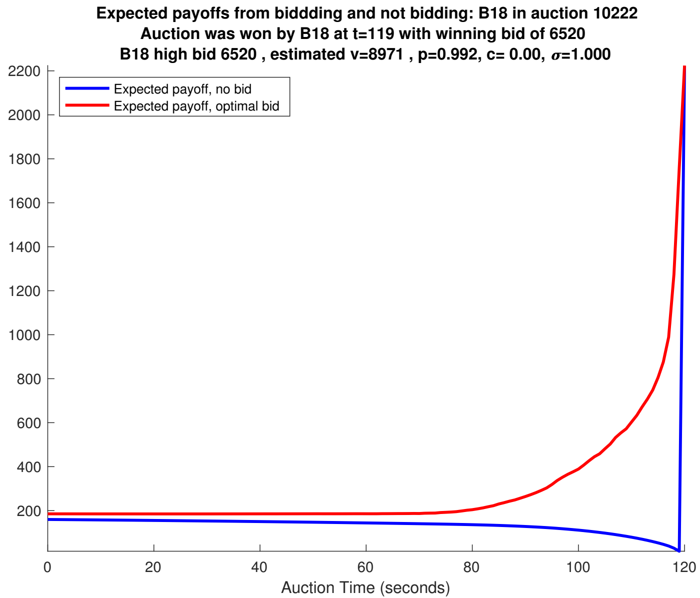
Predicted vs actual bids for B14 in auction 9248



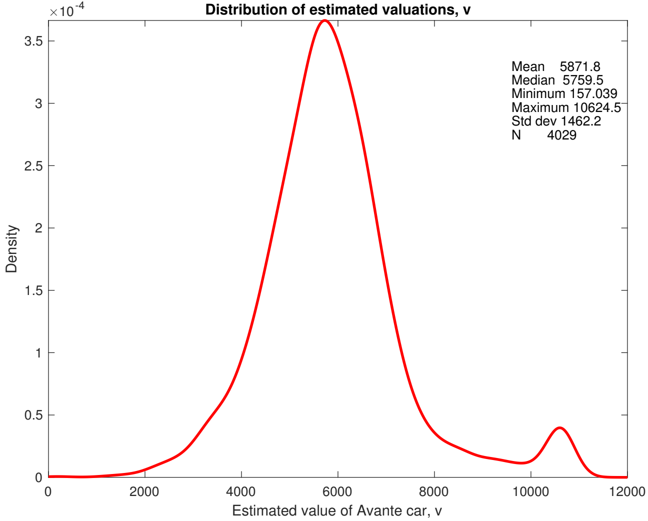
Predicted vs actual bids for B18 in auction 10222



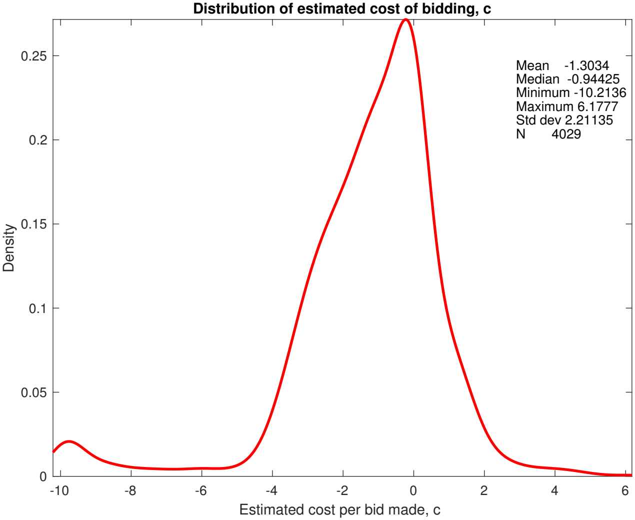
Payoffs of bidding vs not bidding: B18 in auction 10222



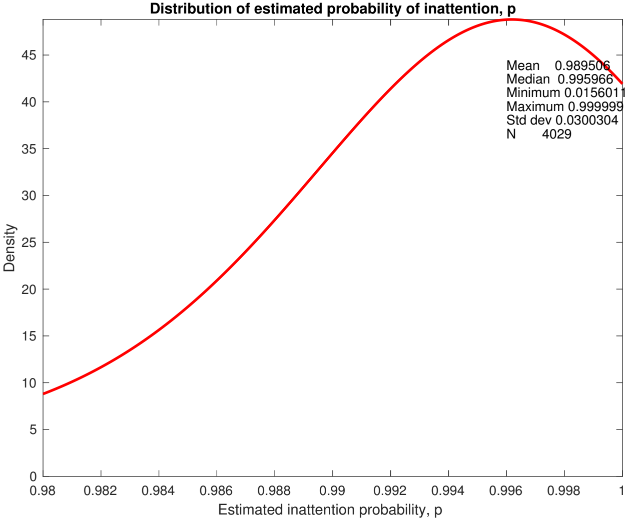
Estimation results: bidder valuations v



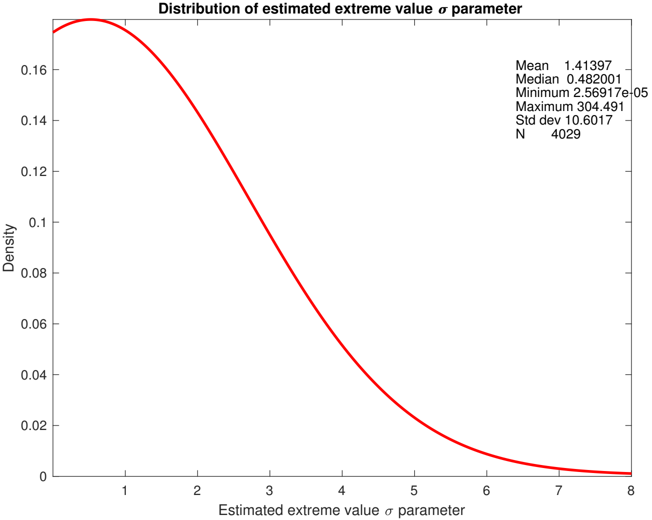
Estimation results: cost of bidding c



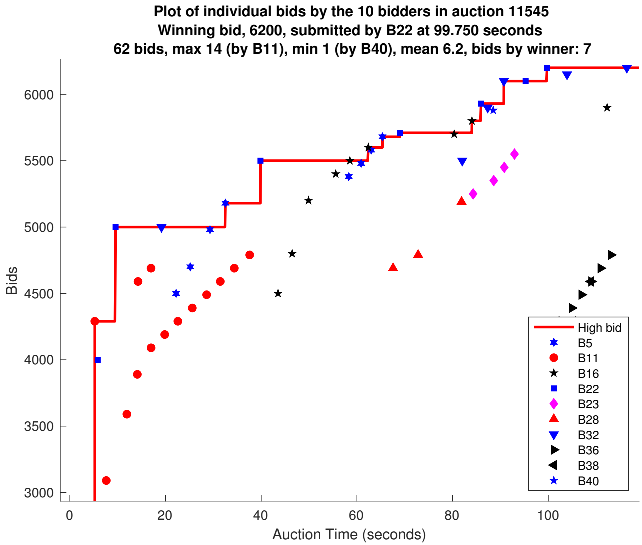
Estimation results: inattention probability *p*



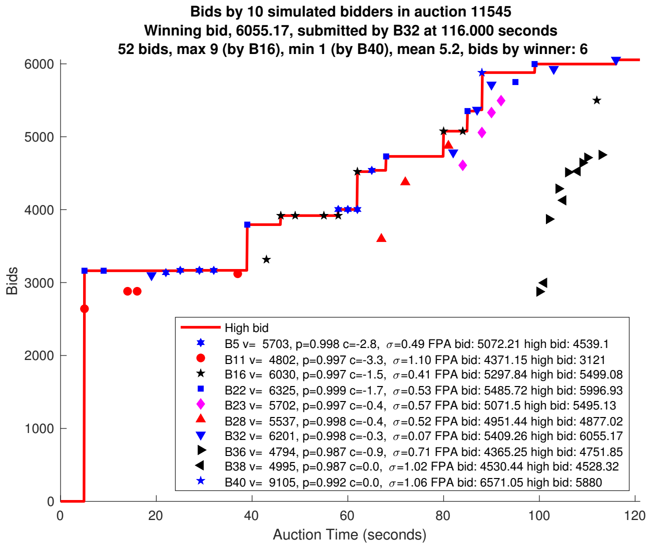
Estimation results: extreme value scale parameter σ



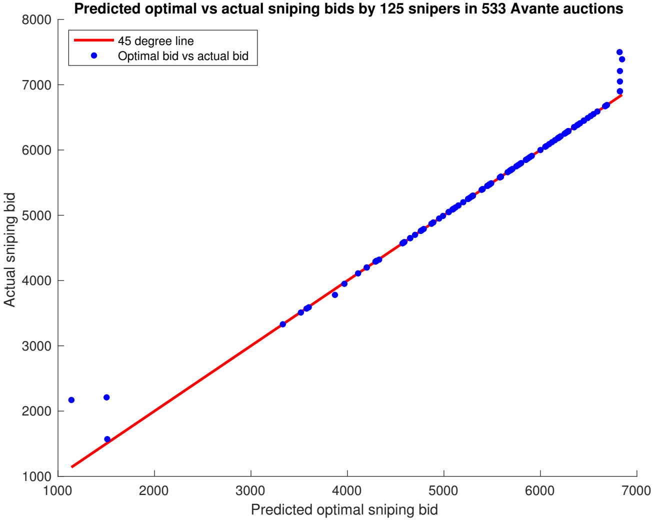
Actual outcome for auction 11545



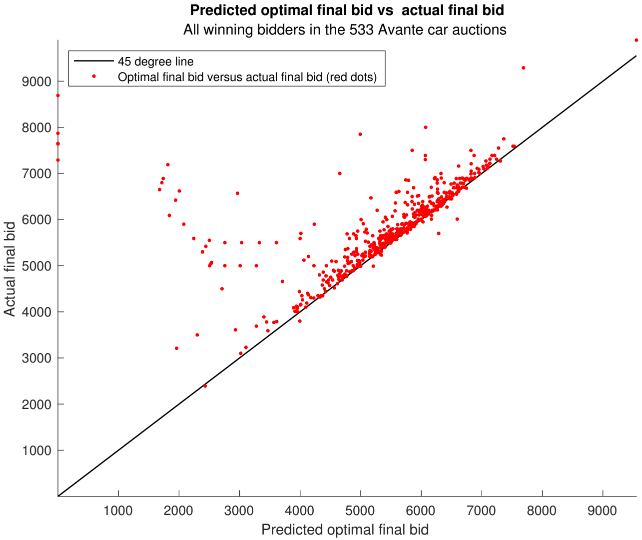
Simulated outcome for auction 11545



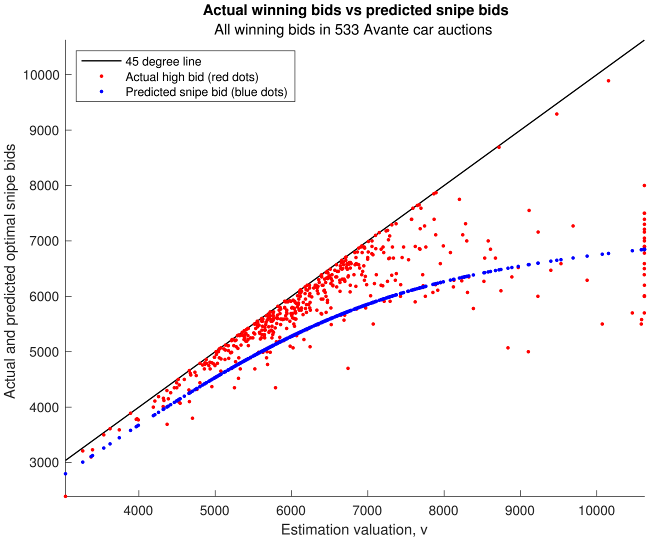
Actual vs model bids for 125 snipers



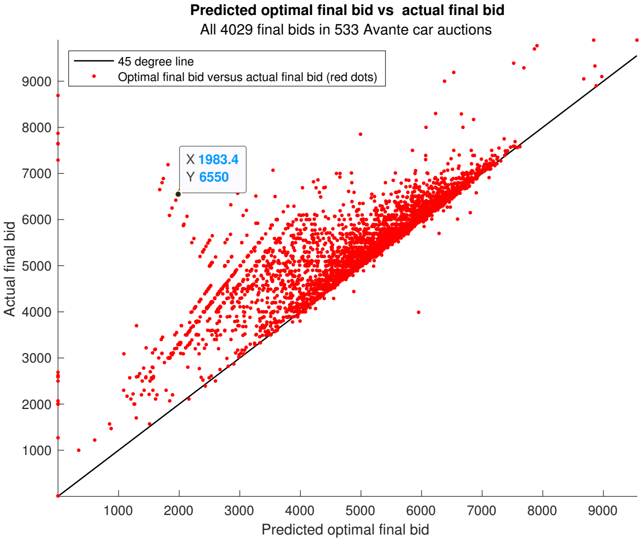
Actual vs model final bids for 533 winners



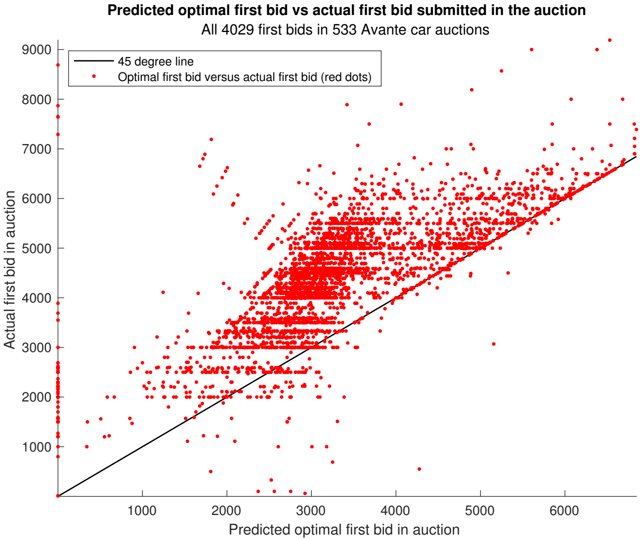
Actual high bid vs predicted snipe bids for 533 winners



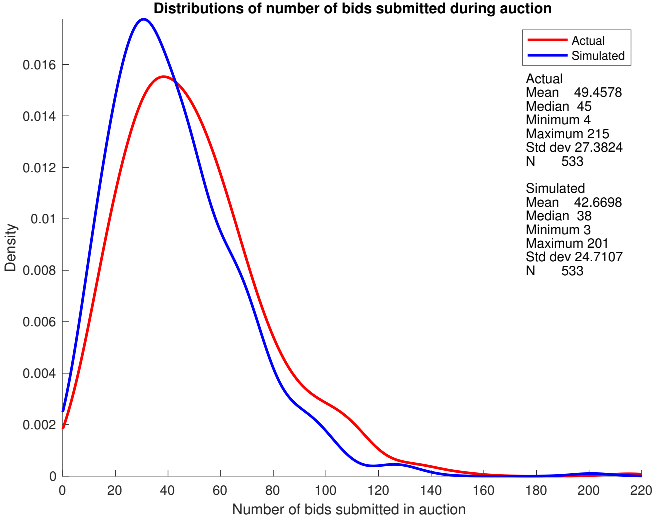
Actual vs model final bids for 4029 bidders



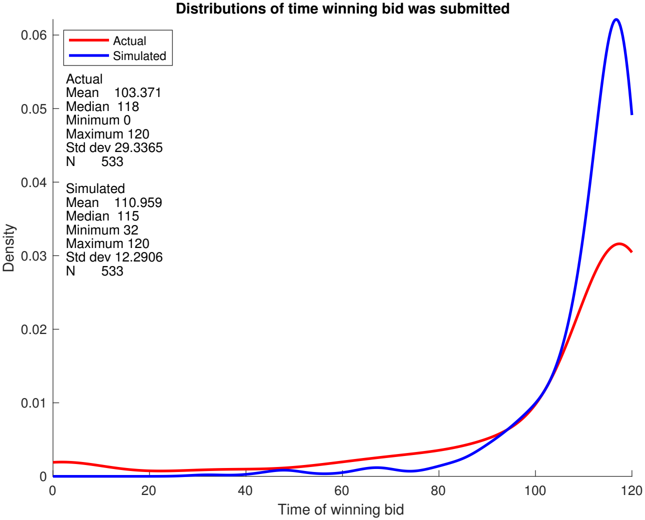
Actual vs model first bids for all 4029 bidder/auction pairs



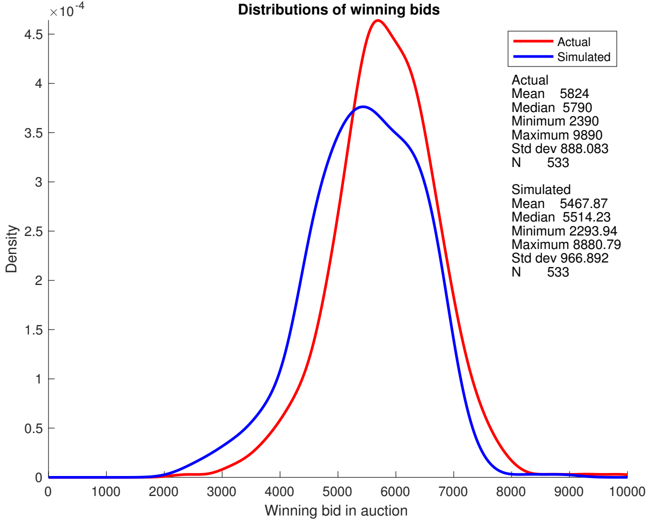
Actual vs predicted distribution of number of bids



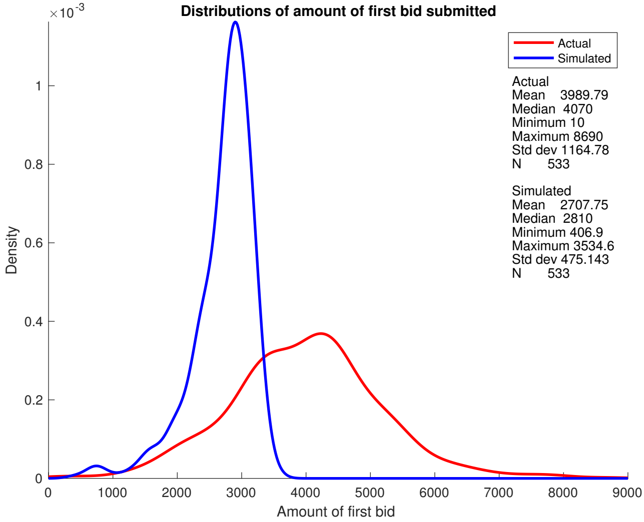
Actual vs predicted distribution time of winning bid



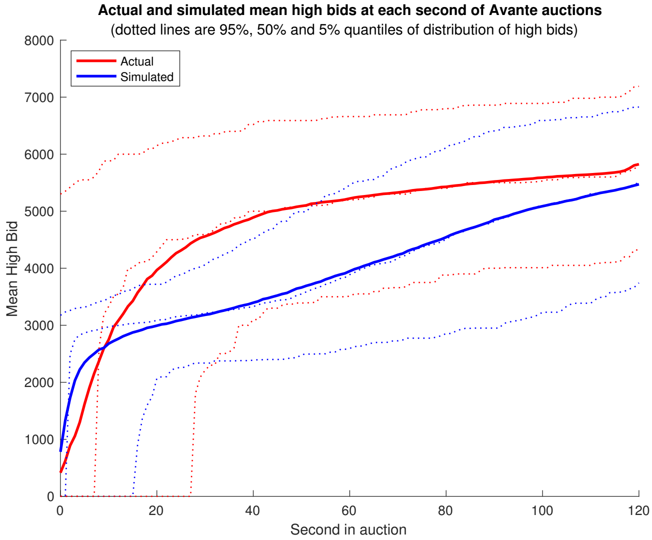
Actual vs predicted distribution of winning bids



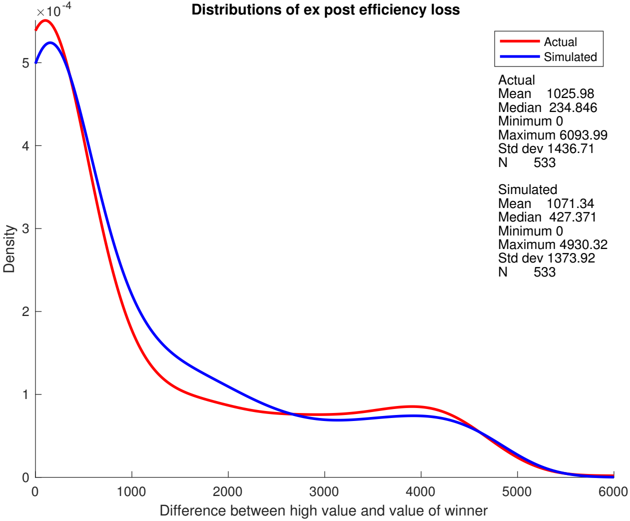
Actual vs predicted distributions of first bids



Actual vs predicted high bid trajectories



Actual vs predicted *ex post* efficiency losses



A counterfactual exercise to assess bidder rationality

- How can we convince a skeptic that human bidders are "overbidding"? After all, there is a trade-off: if a person bids less, while they do earn more conditional on winning, they also will win less often.
- To assess this we ran a "human-robot" counterfactual. For each of the 4029 bidder/auction instances we ask what would the human bidder have earned in the auction if instead the bidder followed the advice of their "robot" (i.e. their estimated DP alter-ego) and submitted the bids recommended by the robot bidder at the times they actually bid in the auction?
- That is, we assume all other human bidders' bids in the auction are unaffected by this "deviation" with the following caveat: if the robot bid ends up being the high bid in the auction, we credit the higher profits as a gain in profits to following the robot's advice.
- If the robot's high bid ends up losing the auction, then we set the bidder's profits in that auction to zero.

A counterfactual exercise to assess bidder rationality

- Let's call the "counterfactual bidder" in an auction to be one of the bidders who agrees to change their bids at the times they actually bid a *robot bidder* or *deviation bidder*.
- For all of the other human bidders in the auction, we treat their actual bids as unaffected by the change in bids by the robot bidder.
- Since the robot bidder typically bids less than what human bidders actually bid, it seems plausible that there would be no change in the bids by the other human bidders. In particular, whenever the human bidder is not the high bidder, a recommended reduction in the bid by the robot bidder would not affect the high bid and thus not even be noticed by the other bidders.
- If the human bidder is a high bidder at any point and the robot bidder recommends a lower bid, this can affect the high bid track and thus who actually wins the auction. We recompute the high bid track and the implied winning price in the auction but taking the bids of the other human bidders in the auction as being unchanged by the deviation.

A counterfactual exercise to assess bidder rationality

- There are a total of 4029 bidders in the 533 Avante car auctions. We do 4029 counterfactual simulations where in each of the counterfactuals only one of the bidders is the deviation bidder whose bids are given by what the estimated DP model for that bidder would predict (at the times the human bidder actually submitted their bids in that auction).
- All the remaining bids by the other bidders in each auction are kept fixed at the sequence of bids that were actually submitted in the auction.
- In this way, we obtain a counterfactual data set of 4029 auctions, in each one the counterfactual auction simulation predicts the impact of a deviation in the bidding strategy by only one of the bidders in the auction in the way our estimated DP model would predict.
- Intuitively, these 4029 counterfactual simulations enable us to assess the impact of a "unilateral deviation" on the profits for each bidder, for precisely the subset of auctions they participated in.

A counterfactual exercise to assess bidder rationality

- Each of the 4029 bidder/auction-specific simulations have four possible outcomes:
  1. The human bidder won the actual auction and the robot “deviation bidder” won the counterfactual auction.
     297 counterfactual simulated auctions, or 7.4% of the 4029 counterfactual simulations, had this type of outcome.
  2. The human bidder won the actual auction but the robot “deviation bidder” lost the counterfactual auction.
     236 counterfactual simulated auctions, or 5.9%, had this type of outcome.
  3. The human bidder lost the actual auction, but the robot “deviation bidder” won the the counterfactual auction.
     16 counterfactual simulated auctions, or 0.4%, had this type of outcome.
  4. The human bidder lost the actual auction and the robot “deviation bidder” lost the counterfactual auction.
     3480 counterfactual simulated auctions, or 86.4%, had this type of outcome.

Counterfactual versus actual auction outcomes

Bidder, Auctions	Win rates		Mean profits				High bids			
			All auctions		Auctions won		All auctions		Auctions won	
	Act	CF	Act	CF	Act	CF	Act	CF	Act	CF
1 163	20.2	11.6	207 (4)	126 (3)	1023 (34)	1086 (53)	5671 (6)	5311 (7)	5821 (29)	5575 (56)
3 35	37.1	11.4	282 (15)	205 (19)	760 (49)	1797 (297)	6182 (21)	5542 (30)	6330 (50)	5390 (507)
5 146	5.5	4.8	16.0 (0.5)	17.4 (0.6)	294 (23)	363 (31)	5146 (7)	4695 (9)	5955 (64)	6027 (82)
6 163	13.5	6.1	33 (0.7)	24 (0.7)	246 (10)	390 (21)	5708 (5)	5288 (6)	5925 (30)	6083 (54)
8 315	10.8	3.8	23 (0.4)	43 (0.9)	212 (10)	1126 (86)	4935 (3)	4123 (4)	5168 (32)	4476 (116)
9 323	9.9	5.0	144 (2)	153 (2)	1449 (55)	3086 (78)	5448 (3)	4799 (3)	5528 (25)	4968 (80)
10 227	12.3	5.7	68 (2)	66 (2)	550 (33)	1161 (112)	4879 (5)	4282 (6)	5566 (30)	5096 (110)
11 361	9.7	5.5	29 (0.3)	58 (1.1)	301 (8)	1049 (69)	5344 (3)	4695 (4)	5707 (20)	5124 (64)
14 167	15.6	7.2	46 (1)	33 (1)	292 (10)	454 (21)	4690 (7)	4199 (8)	5270 (46)	5412 (52)

Counterfactual versus actual auction outcomes

Bidder, Auctions	Win rates		Mean profits				High bids			
			All auctions		Auctions won		All auctions		Auctions won	
	Act	CF	Act	CF	Act	CF	Act	CF	Act	CF
15 44	6.8	6.8	4.9 (.5)	216 (27)	72 (14)	3165 (1380)	4664 (21)	3986 (28)	6370 (517)	3277 (950)
16 158	15.2	10.1	95 (2)	161 (5)	624 (29)	1589 (120)	5428 (6)	5040 (7)	5368 (39)	4610 (97)
17 181	14.9	7.7	59 (1)	66 (2)	398 (13)	857 (68)	5528 (5)	5162 (6)	5781 (27)	5597 (53)
23 148	21.6	14.9	103 (3)	132 (4)	479 (30)	891 (63)	5383 (5)	5106 (7)	5504 (24)	5290 (41)
28 225	15.6	10.7	122 (2)	117 (2)	783 (23)	1094 (49)	5641 (4)	5230 (5)	6158 (22)	5593 (51)
32 86	33.7	19.8	48 (2)	230 (12)	141 (9)	1166 (128)	5828 (9)	5499 (14)	5982 (26)	5269 (115)
36 132	6.0	3.8	15 (0.6)	76 (6)	242 (29)	2011 (752)	5675 (6)	5087 (9)	6506 (139)	4964 (572)
47 203	5.4	5.9	9.2 (0.4)	12.8 (0.3)	170 (25)	216 (15)	5661 (4)	5363 (5)	5955 (95)	5863 (79)
58 45	28.9	26.7	567 (27)	720 (32)	1961 (118)	2700 (135)	5854 (26)	5475 (33)	6500 (80)	5580 (164)

Summary of the counterfactual exercise

- Over all 4029 *actual bidder/auction outcomes* in the 533 Avante auctions, estimated total profit for the bidders was $320,144, or $79 per bidder per auction, and $601 per auction won.
- Over all 4029 *simulated counterfactual bidder/auction outcomes* in the 533 Avante auctions, estimated total profit for the bidders was $390,415, or $97 per bidder per auction, or $1247 per auction won.
- The actual "win rate" was 533/4029, or 13.2% win rate on average.
- The counterfactual "win rate" was 313/4029, or 7.8% win rate on average.
- The DP bidders bid lower than their human counterparts, win fewer auctions, but make more profits on average for the auctions they win.
- In general the human bidders bid too high too soon in the auction. The robot bidders are more patient, avoiding bidding too high early in the auction and end up bidding less on average.

Counterfactual exercise 2: all bidders are robots

- Now consider a second counterfactual simulation similar to the first one, except in this simulation *all bidders bids are those that are "recommended" by the bidding robots, i.e. the estimated DP bidding strategies.*
- However we continue to condition on the set of bidders who actually bid in each auction, and the times each bidder submitted their bids during the two minute auction.
- That is, we assume that each bidder was *paying attention* at the times they actually submitted their bids in the auction, but their robot "alter-ego" may not choose to submit a bid at those times.
- Thus, even though the simulations condition on the set of bidders and times that bids were actually submitted, the counterfactual simulations may result in fewer than the actual number of bids being submitted in the auction.
- In a few extreme cases, there may even be fewer bidders than the actual number of bidders in an auction if the robot alter ego chooses not to submit any positive bids at the times during the auction where the corresponding human bidder submitted bids.

Counterfactual 2: actual vs simulated auction outcomes

Bidder, Auctions	Win rates		Mean profits				High bids			
			All auctions		Auctions won		All auctions		Auctions won	
	Act	CF	Act	CF	Act	CF	Act	CF	Act	CF
1 163	20.2	23.9	207 (4)	340 (5)	1023 (34)	1422 (34)	5671 (6)	5253 (7)	5821 (29)	5609 (27)
3 35	37.1	20.0	282 (15)	222 (17)	760 (49)	1112 (128)	6182 (21)	5439 (29)	6330 (50)	5461 (222)
5 146	5.5	3.4	16.1 (0.5)	13.8 (0.5)	294 (23)	402 (29)	5146 (7)	4586 (9)	5955 (64)	5642 (101)
6 163	13.5	19.0	33 (0.7)	83 (1.4)	246 (10)	435 (11)	5708 (5)	5205 (6)	5925 (30)	6011 (25)
8 315	10.8	10.8	23 (0.4)	52 (0.6)	212 (10)	478 (8)	4935 (3)	4048 (4)	5168 (32)	4766 (24)
9 323	9.9	8.0	144 (2)	220 (3)	1449 (55)	2732 (62)	5448 (3)	4754 (3)	5528 (25)	5474 (23)
10 227	12.3	11.9	68 (2)	80 (1)	550 (33)	673 (27)	4879 (5)	4199 (6)	5566 (30)	5279 (27)
11 361	9.7	8.9	29 (0.3)	49 (0.5)	301 (8)	554 (12)	5344 (3)	4595 (4)	5707 (20)	5404 (29)
14 167	15.6	16.2	46 (1)	112 (2)	292 (10)	694 (13)	4690 (7)	4038 (8)	5270 (46)	4312 (35)

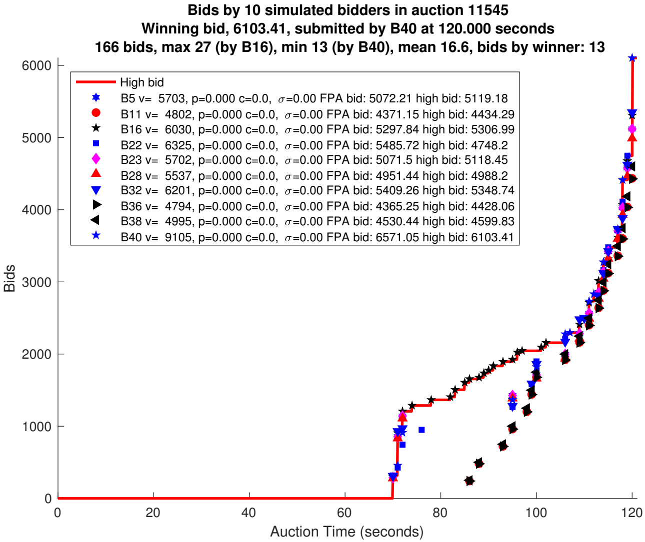
Counterfactual 2: actual vs simulated auction outcomes

Bidder, Auctions	Win rates		Mean profits				High bids			
			All auctions		Auctions won		All auctions		Auctions won	
	Act	CF	Act	CF	Act	CF	Act	CF	Act	CF
15 44	6.8	4.5	4.9 (0.5)	44.5 (5.1)	72 (14)	981 (295)	4664 (21)	3921 (29)	6370 (517)	4184 (697)
16 158	15.2	17.7	95 (2)	165 (3)	624 (29)	931 (21)	5428 (6)	4930 (7)	5368 (39)	5036 (33)
17 181	14.9	16.0	59 (1)	128 (2)	398 (13)	796 (30)	5528 (5)	5097 (6)	5781 (27)	5611 (31)
23 148	21.6	29.0	103 (3)	224 (4)	479 (30)	772 (19)	5383 (5)	4956 (7)	5504 (24)	5124 (20)
28 225	15.6	13.3	122 (2)	100 (1)	783 (23)	752 (15)	5641 (4)	5143 (5)	6158 (22)	5707 (24)
32 86	33.7	26.7	48 (2)	135 (4)	141 (9)	506 (23)	5828 (9)	5408 (13)	5982 (26)	5748 (35)
36 132	6.0	7.6	15 (0.6)	83 (3)	242 (29)	1094 (133)	5675 (6)	4998 (9)	6506 (139)	5948 (66)
47 203	5.4	7.9	9.2 (0.4)	25.5 (0.5)	170 (25)	323 (13)	5661 (4)	5271 (5)	5955 (95)	5711 (45)
58 45	28.9	26.7	567 (27)	597 (27)	1961 (118)	2240 (119)	5854 (26)	5432 (33)	6500 (80)	6056 (87)

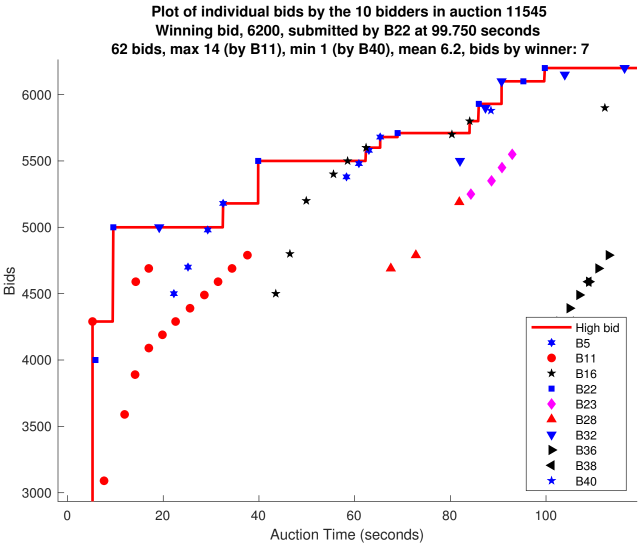
Counterfactual exercise 3: all bidders are frictionless robots

- The third counterfactual is similar to the second one, i.e. *all bidders' bids are those that are "recommended" by the bidding robots, i.e. the estimated DP bidding strategies.* But now we use *frictionless bidding strategies* i.e. the DP solution that conditions on the estimated value of each bidder, v, but sets $c = p = \sigma = 0$.
- We no longer condition on the times each bidder submitted their bids during the two minute auction, but instead allow the frictionless robot bidders decide how often and when to bid.
- That is, we assume that each bidder *always pays attention* and *faces no cost of bidding* and there *are no random shocks* affecting the decision whether to bid or not.
- In general we expect far more bidding activity in the frictionless counterfactual simulations. *Will all this bidding and learning enable bidders to approximate the "clock model" for ascending bid auctions?*
- Will losing bidders keep bidding until they reach their valuation v and then stop? If so, this auction should approximate the *static second price auction outcome.*

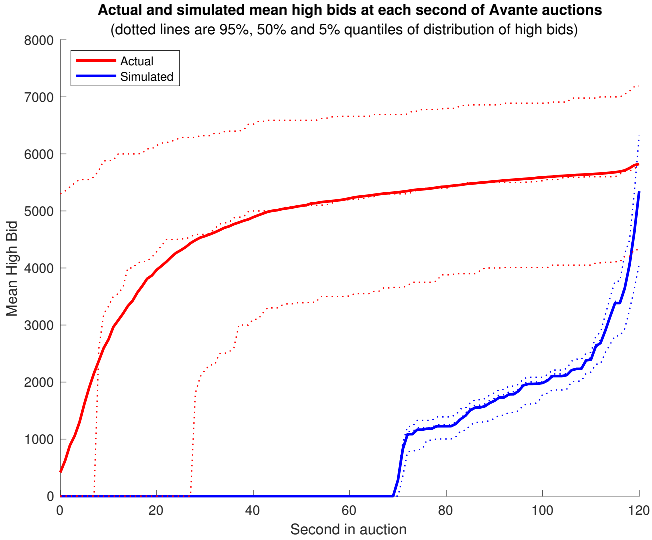
Simulated frictionless bidding outcome for auction 11545



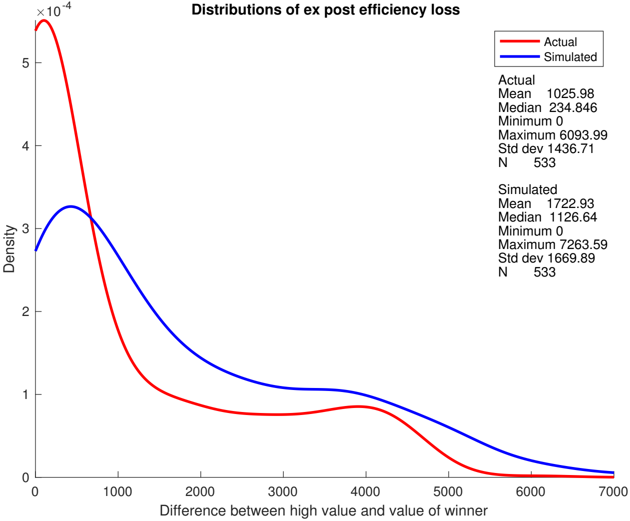
Actual outcome for auction 11545



Actual vs frictionless bid trajectories



Actual vs frictionless auction efficiency losses



Counterfactual 3: actual vs simulated auction outcomes

Bidder, Auctions	Win rates		Mean profits				High bids			
			All auctions		Auctions won		All auctions		Auctions won	
	Act	CF	Act	CF	Act	CF	Act	CF	Act	CF
1 163	20.2	22.0	207 (4)	432 (7)	1023 (34)	1957 (41)	5671 (6)	5169 (5)	5821 (29)	5387 (21)
3 35	37.1	31.4	282 (15)	582 (33)	760 (49)	1850 (123)	6182 (21)	5326 (18)	6330 (50)	5611 (55)
5 146	5.5	4.1	16.1 (0.5)	42.9 (1.9)	294 (23)	1044 (171)	5146 (7)	4718 (5)	5955 (64)	5109 (97)
6 163	13.5	8.6	33 (0.7)	78 (1.7)	246 (10)	912 (29)	5708 (5)	5098 (4)	5925 (30)	5206 (54)
8 315	10.8	7.6	23 (0.4)	71 (1.0)	212 (10)	930 (31)	4935 (3)	4523 (2)	5168 (32)	4837 (30)
9 323	9.9	32.2	144 (2)	783 (4)	1449 (55)	2433 (16)	5448 (3)	5057 (2)	5528 (25)	5544 (7)
10 227	12.3	13.2	68 (2)	137 (2)	550 (33)	1034 (19)	4879 (5)	4536 (4)	5566 (30)	5076 (18)
11 361	9.7	9.4	29 (0.3)	125 (1.4)	301 (8)	1332 (37)	5344 (3)	4852 (2)	5707 (20)	5158 (22)
14 167	15.6	15.0	46 (1)	118 (2)	292 (10)	787 (24)	4690 (7)	4300 (5)	5270 (46)	4875 (25)

Counterfactual 3: actual vs simulated auction outcomes

Bidder, Auctions	Win rates		Mean profits				High bids			
			All auctions		Auctions won		All auctions		Auctions won	
	Act	CF	Act	CF	Act	CF	Act	CF	Act	CF
15 44	6.8	4.5	4.9 (.5)	36.2 (5)	72 (14)	796 (376)	4664 (21)	4313 (16)	6370 (517)	4369 (616)
16 158	15.2	16.4	95 (2)	179 (3)	624 (29)	1091 (35)	5428 (6)	4937 (4)	5368 (39)	4862 (29)
17 181	14.9	15.5	59 (1)	213 (4)	398 (13)	1375 (37)	5528 (5)	5064 (4)	5781 (27)	5420 (22)
23 148	21.6	13.5	103 (3)	166 (4)	479 (30)	1225 (56)	5383 (5)	4848 (4)	5504 (24)	5137 (31)
28 225	15.6	16.0	122 (2)	267 (3)	783 (23)	1668 (26)	5641 (4)	5146 (3)	6158 (22)	5580 (15)
32 86	33.7	9.3	48 (2)	80 (3)	141 (9)	864 (66)	5828 (9)	5133 (6)	5982 (26)	5068 (91)
36 132	6.0	6.8	15 (0.6)	147 (5)	242 (29)	2156 (165)	5675 (6)	5162 (4)	6506 (139)	5720 (68)
47 203	5.4	4.9	9.2 (0.4)	54.2 (1.5)	170 (25)	1100 (84)	5661 (4)	5065 (3)	5955 (95)	5117 (75)
58 45	28.9	33.3	567 (27)	710 (28)	1961 (118)	2130 (88)	5854 (26)	5189 (19)	6500 (80)	5650 (44)

Summary of Counterfactual Auction Simulations

- Case 1: 4029 counterfactual auction/bidder-specific simulations where each bidder's bids at the times they bid are replaced by those of their estimated DP bidding strategy
- Case 2: 533 counterfactual auctions where all bidders' bids are given by their DP strategies, but at the times they actually bid
- Case 3: 533 "frictionless bidding" simulations where DP strategies with $c = p = \sigma = 0$ but the estimated v are computed for each bidder and the number and times at which bidders bid is determined by the strategies.

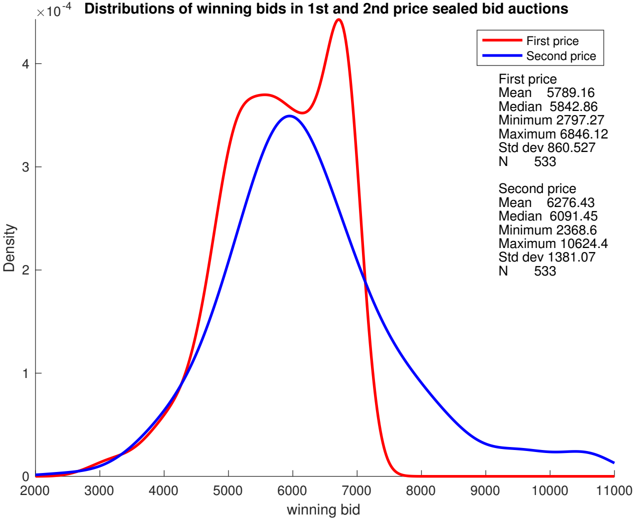
Case Auctions	Values of Winners		Win rates		Mean profits				High bids			
					All auctions		Auctions won		All auctions		Auctions won	
	Act	CF	Act	CF	Act	CF	Act	CF	Act	CF	Act	CF
1 533	6424 (3)	6685 (5)	13.2	7.8	79.5 (.1)	96.9 (.1)	600 (2)	1247 (5)	5398 (0.2)	4899 (0.3)	5824 (2)	5438 (4)
2 533	6424 (3)	6443 (3)	13.2	13.2	79.5 (0.1)	123.3 (0.1)	600 (2)	932 (2)	5398 (0.2)	4814 (0.3)	5824 (2)	5511 (2)
3 533	6424 (3)	6989 (4)	13.2	13.2	79.5 (.1)	217.4 (0.2)	600 (2)	1643 (2)	5398 (0.2)	4910 (0.2)	5824 (2)	5345 (1)

Empirical Auction Design Under Bounded Rationality

- Actual expected revenue per auction and counterfactual expected revenues under alternative assumptions and auction mechanisms
- Standard errors of mean revenues are shown below in parentheses

Korean auction Actual	Korean auction with frictions	Korean auction without frictions	First price sealed bid	Second price sealed bid
5824 (2)	5511 (2)	5345 (1)	5789 (2)	6276 (3)

Distribution of winning bids: first vs second price auctions



Conclusions

- We have analyzed a unique new data set on dynamic informationally restricted auctions invented by a Korean rental car company.
- We have shown that early bidding in these auctions is very prevalent and appears to reflect an attempt by bidders to learn the value of the high bid in the auction in order to win without overpaying.
- However we have suggested that this behavior may be inconsistent with the predictions of a perfect Bayesian equilibrium model of bidding in these auctions.
- In a 2 bidder, 2 period example, we showed there is no informative PBE: the only PBE is an uninformative equilibrium in which both bidders wait to the last period to submit their bids.
- In an *uninformative equilibrium* (which always exists) there is no early bidding and the outcome is the same as the equilibrium in a static first price sealed bid auction.

Conclusions

- In order to explain the bidding behavior we observe we developed a new dynamic model of rationally inattentive bidding that relaxes the assumption that bidders use PBE strategies.
- Instead we assume that experienced bidders have rational expectations of the stochatic process governing the high bid in the auction.
- Bidders solve dynamic programs to maximize their expected payoff from the auction, given their beliefs about the stochastic process governing the highest bid during the auction.
- We solve these dynamic programs and show that they can produce the early bidding behavior we observe.
- Early bidding enables bidders to learn the value of the high bid and to minimize the amount they ned to pay to win the auction.
- Our *theory* predicts that the dynamic Korean auctions generate lower expected revenues than the rental company could earn in a static first price sealed bid auction.

Conclusions

- However our *empirical analysis* has revealed prevalent *early overbidding* that pushes up winning prices in these auctions above what our model would predict, even when we adopt a "fixed effects" approach to estimation to estimate 4029 bidder-specific 4-parameter structural models that best predicts the actual bids for each bidder/auction pair.
- We ascribe the discrepancy to *bounded rationality* on the part of the bidders. When we take this into account, our empirical conclusion about the revenue-maximizing auction format is reversed: the Korean auction generates higher revenue than would a static sealed bid auction format.
- This is contingent on the assumption that while these bidders find it difficult to bid rationally in the dynamic Korean auction, they could bid rationally in a static sealed bid first price auction format.
- Further testing and investigation is needed to determine if the latter assumption is valid, or whether there is some "rational explanation" for the high early bidding we observe in these auctions.

Conclusions

- Recall the design of the Korean auction was motivated by suspected collusion and the informational restrictions, particularly suppressing bidder identities, is consistent with advice of auction experts of effective measures to thwart collusion.
- But the use of a dynamic auction while suppressing not just the identities bu the bids of other bidders is unqiue and has not been suggested by auction experts.
- It is not clear that the Korean auction is less susceptible to collusion compared to simpler static auction mechanisms that convey even less information, e.g. static second price auction with reserve price.
- If there is a significant common value or affliation in bidder values, then the it linkage principle argues for releasing more information, so a dynamic auction would be preferred to a static one.
- In FCC auctions, there is a need for *activity rules* to prevent *informational free-riding* in auctions where the high bid is communicated to bidders. Could the Korea auction be a clever way to discourage informational free-riding without activity rules?