DSE 2022 AUS: ESTIMATING MODELS WITH HYPERBOLIC DISCOUNTING

Marc Chan
Department of Economics
University of Melbourne

15 December 2022

Roadmap

- Provide an introduction of hyperbolic discounting models, assuming some prior knowledge of dynamic programming techniques (which have been covered in previous DSE lectures).
- Consistent with the DSE theme, focus on the econometric underpinnings of such models (an open area for research) and then discuss empirical applications.
- This is a topic that spans several fields in micro- and macroeconomics, and in theoretical and empirical economics, with vastly different focal points.
- Due to time limitation, the pedagogical approach is more specific than broad.
    - Also as an excuse for my limited knowledge; I hope that the econometric focus will provide a useful perspective in this broad area.

Roadmap

- Heavily draw on recent papers, which cover formal identification and estimation of such models in a dynamic discrete choice setting.
  1. Abbring, Daljord and Iskhakov (2019): “Identifying Present-Biased Discount Functions in Dynamic Discrete Choice Models”
  2. Wang, Weiergraeber and Xiao (2022): “Identification of Dynamic Discrete Choice Models with Hyperbolic Discounting using a Terminating Action” (if time allows)

- The backward recursion has similarities in algorithm and notations with the ones covered in earlier DSE lectures, offering uniformity.

- Discuss a recent empirical application and the practical considerations and econometric issues involved in the empirical setting.
  1. Chan (2017): “ Welfare Dependence and Self-Control: An Empirical Analysis”

- This lecture is still work in progress, aiming to spark students’ interest in DSE models.

Understanding the dynamics of structural models

- Dynamic structural models take a stance on what agents are maximizing and their information sets.
  - See handbook chapter Abbring and Heckman (2007) for a discussion.

A modeller ponders upon the following difficult issues:

1. How do agents compare the present with the future?
2. What information do they act on?
3. What are their beliefs?
4. Are these constructs stable over time?
5. Is the model correctly specified?
6. Are there simpler models that can do the same job?

Understanding the dynamics of structural models

- Geometric discounting (on time-separable preferences) forms the backbone of such models but is tightly specified.

- Policy counterfactuals, especially those that are time dependent, will depend on this feature.

- But perhaps, when thinking about hyperbolic discounting models, it is useful to think of it as an avenue through which we can develop better tools in the future to deal with these difficult issues.
  - Related questions: Why do we need a dynamic model? What is the empirical content of such models?

Hyperbolic discounting

- Hyperbolic discounting factors are $\beta(\neq 1)$ and δ.

$$U_t(u_t, u_{t+1}, \cdots, u_T) \equiv u_t + \beta\delta \sum_{t'=t+1}^{T} \delta^{t'-t-1} E u_{t'}$$

- Abbring, Daljord and Iskhakov (2019) derive conditions for identification of sophisticated, quasi-hyperbolic time preferences in a finite horizon, dynamic discrete choice model under a set of economically motivated exclusion restrictions.
  - Will heavily draw on their work in discussions below.

- Identification is reduced to characterizing of the zero set of two bivariate polynomial moment conditions.
  - Their approach builds on Abbring and Daljord (2019) for geometric-discounting models.
  - Skip the huge literature on geometric-discounting models.

Hyperbolic discounting: historical motivation

- The literature on hyperbolic (present-biased) time preference has been motivated by evidence of preference reversals.
  - Subjects who prefer 1 apple today to 2 apples tomorrow tend to prefer 2 apples one year and one day from now to 1 apple one year from now (Thaler 1981)
- They are the defining feature of time-inconsistent preferences and have been considered as the core of an identification strategy.
- Historical motivation aside, if we consider geometric discounting as a tightly specified model, it is natural to think about the possibility of relaxing this model while not losing its empirical content.
- Hence today's approach is "technical" in that we focus more on the econometrics than behavioral implications (although they are not in conflict with each other).

Primitives

- Standard setup. Time $t = 1, \dots, T$. Action d_t from $D = \{1, \dots, K\}$. Vectors of state variables x_t and $\epsilon_t = \{\epsilon_{1,t}, \dots, \epsilon_{Kt}\}$.
- Observable states x_t have finite support χ (J elements) and evolve as controlled (d_t) first order Markov, with transition distribution Q_k if $k \in D$ is chosen.
- Utility shocks $\epsilon_{k,t}$ independent from x_t, prior states and choices, over time, and across choices, and have type 1 extreme value distributions.
- If agent chooses k in period t and state x, she collects a utility flow $u_{k,t}(x) + \epsilon_{k,t}$. Normalize $u_{K,t}(x) = 0$.

Choices and equilibrium

- Present-biased time preferences are time-inconsistent. Value functions do not follow from a standard dynamic program.

- Can think about the values as summarizing the pay-offs to players in a Stackelberg-like game played between selves in different periods.

- Let $\tilde{\sigma}_t\colon \chi \times \mathbb{R}^K \to D$ be an arbitrary choice strategy and $\tilde{\boldsymbol{\sigma}}_t = \{\tilde{\sigma}_t\}_{\tau=t}^T$ a strategy profile. The current choice-specific value function (regulates choices) for t<T is

$$w_{k,t}(x;\tilde{\boldsymbol{\sigma}}_{t+1}) = u_{k,t}(x) + \beta\delta \int v_{t+1}(x';\tilde{\boldsymbol{\sigma}}_{t+1})dQ_k(x'|x)$$

with terminal value $w_{k,T}(x) = u_{k,T}(x)$.

Choices and equilibrium

$$w_{k,t}(x;\tilde{\boldsymbol{\sigma}}_{t+1}) = u_{k,t}(x) + \beta\delta \int v_{t+1}(x';\tilde{\boldsymbol{\sigma}}_{t+1})dQ_k(x'|x)$$

- The stream of all future utilities are discounted *geometrically* by δ via the perceived long-run value function:

$$v_{t+1}(x;\tilde{\boldsymbol{\sigma}}_{t+1}) = \mathbb{E}_{\epsilon_{t+1}}\Big[u_{\tilde{\sigma}_{t+1}(x,\epsilon_{t+1}),t+1}(x) + \epsilon_{\tilde{\sigma}_{t+1}(x,\epsilon_{t+1}),t+1} + \delta \int v_{t+2}(x';\tilde{\boldsymbol{\sigma}}_{t+2})dQ_{\tilde{\sigma}_{t+1}(x,\epsilon_{t+1})}(x'|x)\Big]$$

for $t+1<T$, with terminal value $v_T(x;\tilde{\boldsymbol{\sigma}}_T) = \mathbb{E}_{\epsilon_T}\left[u_{\tilde{\sigma}_T(x,\epsilon_T),T}(x) + \epsilon_{\tilde{\sigma}_T(x,\epsilon_T),T}\right]$

- The future selves (with strategies $\tilde{\boldsymbol{\sigma}}_{t+1}$) have present bias, which are in conflict with the current self's time-consistent long-run preferences.

Choices and equilibrium

- For a <u>sophisticated</u> agent, her perceptions of her future strategies are correct in equilibrium, i.e., she correctly perceives her future selves are present-biased.

- In the intrapersonal equilibrium, her selves use a *perception perfect strategy* (O'Donoghue and Rabin 1999): a strategy profile $\boldsymbol{\sigma}_t^*$ such that each σ_t^* is a best response to the perceived future strategy profile $\boldsymbol{\sigma}_{t+1}^*$.

$$\sigma_t^*(x, \epsilon_t) = \arg\max_{k \in \mathcal{D}} \{w_{k,t}(x; \boldsymbol{\sigma}_{t+1}^*) + \epsilon_{k,t}\}$$

And $w_{k,T}(x; \sigma_{T+1}^*) = w_{k,T}(x)$.

- By backward recursion, a perception perfect strategy exists and is unique.

Identification with known β, δ

- Given primitives $Q_1, \dots, Q_K; \beta; \delta; u_{1,t}, \dots, u_{K-1,t}; t = 1, \dots T$, the model implies conditional choice probabilities:

$$p_{k,t}(x) = \Pr(d_t = k | x_t = x) = \mathbb{E}_{\epsilon_t}[\mathbb{1}\{\sigma_t^*(x, \epsilon_t) = k\}]$$

- Focus on identification of $\beta; \delta; u_{1,t}, \dots, u_{K-1,t}$ from the choice probabilities (observed) and state transitions $Q_1, \dots, Q_K$ (observed).

- Through the value contrasts ($w(.;.)$) we have

$$\ln\left(\frac{p_{k,t}(x)}{p_{K,t}(x)}\right) = w_{k,t}(x; \boldsymbol{\sigma}_{t+1}^*) - w_{K,t}(x; \boldsymbol{\sigma}_{t+1}^*)$$

$$-\ln(p_{K,t}(x)) = \ln\left(\sum_{k \in \mathcal{D}} \exp\left[w_{k,t}(x; \boldsymbol{\sigma}_{t+1}^*) - w_{K,t}(x; \boldsymbol{\sigma}_{t+1}^*)\right]\right)$$

Hotz and Miller (1993); Arcidiacono and Miller (2011)

Identification with known β, δ

- By backward induction, $u_{1,t}, \dots, u_{K-1,t}$ are identified given $Q_1, \dots, Q_K; \beta; \delta; p_{k,t}(x)$. (Theorem 1) To see this, observe:

Rust (1994); Magnac and Thesmar (2002)

$$
\begin{aligned}
& w_{k,t}(x; \boldsymbol{\sigma}^*_{t+1}) - w_{K,t}(x; \boldsymbol{\sigma}^*_{t+1}) \\
& = u_{k,t}(x) - u_{K,t}(x) + \beta\delta \int v_{t+1}(x'; \boldsymbol{\sigma}^*_{t+1}) \left[dQ_k(x'|x) - dQ_K(x'|x) \right], \ k \in \mathcal{D}
\end{aligned}
$$

$u_{K,t}(x) = 0$, $v_{t+1}(.;.)$ is known by backward recursion, $Q_k(.|.)$ is observed.

- Simplify to:

$$
\ln\left(\frac{p_{k,t}(x)}{p_{K,t}(x)}\right) = u_{k,t}(x) + \beta\delta \left[\boldsymbol{Q}_k(x) - \boldsymbol{Q}_K(x) \right] \boldsymbol{v}_{t+1}(\boldsymbol{\sigma}^*_{t+1})
$$

- $\boldsymbol{v_t}(\boldsymbol{\sigma^*_t})$: $J \times 1$ vector that stacks $v_t(x; \boldsymbol{\sigma^*_t})$, $x \in \chi$
- $\boldsymbol{Q_k}(x)$: $1 \times J$ vector that stacks $Q_k(x'|x)$, $x' \in \chi$

Concentrate identification on β, δ

$$\ln\left(\frac{p_{k,t}(x)}{p_{K,t}(x)}\right) = u_{k,t}(x) + \beta\delta\left[\boldsymbol{Q}_k(x) - \boldsymbol{Q}_K(x)\right]\boldsymbol{v}_{t+1}(\boldsymbol{\sigma}^*_{t+1})$$

- The key step is to recursively express $\boldsymbol{v}_{t+1}(.)$ as a function of $\boldsymbol{v}_{t+2}(.)$, and so on. Recall the definition of long-run value $v(.;.)$ (with δ):

$$\begin{aligned} v_{t+1}(x;\tilde{\boldsymbol{\sigma}}_{t+1}) = \mathbb{E}_{\epsilon_{t+1}}\Big[& u_{\tilde{\sigma}_{t+1}(x,\epsilon_{t+1}),t+1}(x) + \epsilon_{\tilde{\sigma}_{t+1}(x,\epsilon_{t+1}),t+1} \\ &+ \delta\int v_{t+2}(x';\tilde{\boldsymbol{\sigma}}_{t+2})dQ_{\tilde{\sigma}_{t+1}(x,\epsilon_{t+1})}(x'|x)\Big] \end{aligned}$$

- Combining with the choice-specific value $w(.;.)$ (with $\beta\delta$), this yields

$$\begin{aligned} & v_{t+1}(x;\boldsymbol{\sigma}^*_{t+1}) \\ &= \mathbb{E}_{\epsilon_{t+1}}\left[\max_{k\in\mathcal{D}}\left\{w_{k,t+1}(x;\boldsymbol{\sigma}^*_{t+2}) + \epsilon_{j,t+1}\right\} + \delta(1-\beta)\boldsymbol{Q}_{\sigma^*_{t+1}(x,\epsilon_{t+1})}(x)\boldsymbol{v}_{t+2}(\boldsymbol{\sigma}^*_{t+2})\right] \end{aligned}$$

- <u>Important:</u> If $\beta = 1$, the correction term $\delta(1-\beta)Qv_{t+2}$ = 0, and we get the standard dynamic program.

Concentrate identification on β, δ

- The Emax can be simplified as functions of the McFadden surplus, yielding

$$v_{t+1}(x;\boldsymbol{\sigma}^*_{t+1})$$
$$= m_{t+1}(x) + w_{K,t+1}(x;\boldsymbol{\sigma}^*_{t+2}) + \delta(1-\beta)\mathbb{E}_{\epsilon_{t+1}}\left[\boldsymbol{Q}_{\sigma^*_{t+1}(x,\epsilon_{t+1})}(x)\boldsymbol{v}_{t+2}(\boldsymbol{\sigma}^*_{t+2})\right]$$

where $m_{t+1}(x) = \mathbb{E}_{\epsilon_{t+1}}\left[\max_{k\in\mathcal{D}}\left\{w_{k,t+1}(x;\boldsymbol{\sigma}^*_{t+2}) - w_{K,t+1}(x;\boldsymbol{\sigma}^*_{t+2}) + \epsilon_{k,t+1}\right\}\right] = -\ln(p_{K,t+1}(x))$

- We also have

$$w_{K,t+1}(x;\boldsymbol{\sigma}^*_{t+2}) = \beta\delta\boldsymbol{Q}_K(x)\boldsymbol{v}_{t+2}(\boldsymbol{\sigma}^*_{t+2})$$

$$\mathbb{E}_{\epsilon_{t+1}}\left[\boldsymbol{Q}_{\sigma^*_{t+1}(x,\epsilon_{t+1})}(x)\boldsymbol{v}_{t+2}(\boldsymbol{\sigma}^*_{t+2})\right] = \sum_{k\in\mathcal{D}} p_{k,t+1}(x)\boldsymbol{Q}_k(x)\boldsymbol{v}_{t+2}(\boldsymbol{\sigma}^*_{t+2})$$

which yields the following **key equation**:

$$v_{t+1}(x;\boldsymbol{\sigma}^*_{t+1}) = m_{t+1}(x) + \delta\left[\beta\boldsymbol{Q}_K(x) + \underline{(1-\beta)\overline{\boldsymbol{Q}}_{t+1}(x)}\right]\boldsymbol{v}_{t+2}(\boldsymbol{\sigma}^*_{t+2})$$

Concentrate identification on β, δ

Key equation:

$$v_{t+1}(x; \boldsymbol{\sigma}^*_{t+1}) = m_{t+1}(x) + \delta \left[\beta \boldsymbol{Q}_K(x) + \underline{(1-\beta)\overline{\boldsymbol{Q}}_{t+1}(x)} \right] \boldsymbol{v}_{t+2}(\boldsymbol{\sigma}^*_{t+2})$$

where $\overline{\boldsymbol{Q}}_{t+1}(x) = \sum_{k \in \mathcal{D}} p_{k,t+1}(x) \boldsymbol{Q}_k(x)$

- $\bar{Q}_{t+1}(x)$ is the expected state transition probability distribution under strategy σ^*_{t+1} in state x.
- This **mixture** represents an expectation over how the choices of presented-biased future selves control future state transitions.
  - Their choices are in conflict with the current self's long-term preferences.
- If $\beta = 1$, the correction term disappears.

Concentrate identification on β, δ

Key equation:

$$v_{t+1}(x; \boldsymbol{\sigma}^*_{t+1}) = m_{t+1}(x) + \delta \left[\beta \boldsymbol{Q}_K(x) + \underline{(1-\beta)\overline{\boldsymbol{Q}}_{t+1}(x)}\right] \boldsymbol{v}_{t+2}(\boldsymbol{\sigma}^*_{t+2})$$

where $\overline{\boldsymbol{Q}}_{t+1}(x) = \sum_{k \in \mathcal{D}} p_{k,t+1}(x) \boldsymbol{Q}_k(x)$

- Define the $J \times J$ matrix of probability **mixtures**

$$\boldsymbol{Q}^{pb}_t(\beta) = \beta \boldsymbol{Q}_K + (1-\beta)\overline{\boldsymbol{Q}}_t$$

where $\overline{\boldsymbol{Q}}_t$ stacks $\overline{\boldsymbol{Q}}_t(x)$ and $\boldsymbol{Q}_K$ stacks $\boldsymbol{Q}_K(x)$

- We obtain a recursive expression for $\boldsymbol{v}_{t+1}(\boldsymbol{\sigma}^*_{t+1})$ in vector notation:

$$\boldsymbol{v}_{t+1}(\boldsymbol{\sigma}^*_{t+1}) = \boldsymbol{m}_{t+1} + \delta \boldsymbol{Q}^{pb}_{t+1}(\beta) \boldsymbol{v}_{t+2}(\boldsymbol{\sigma}^*_{t+2})$$

Concentrate identification on β, δ

$$\boldsymbol{v}_{t+1}(\boldsymbol{\sigma}^*_{t+1}) = \boldsymbol{m}_{t+1} + \delta \boldsymbol{Q}^{pb}_{t+1}(\beta)\boldsymbol{v}_{t+2}(\boldsymbol{\sigma}^*_{t+2})$$

- Completing the recursion until T gives a function of discount factors and data:

$$\boldsymbol{v}_{t+1}(\boldsymbol{\sigma}^*_{t+1}) = \boldsymbol{m}_{t+1} + \sum_{\tau=t+2}^{T} \delta^{\tau-t-1} \left(\prod_{r=t+1}^{\tau-1} \boldsymbol{Q}^{pb}_{r}(\beta) \right) \boldsymbol{m}_{\tau}$$

- Substituting back into $\ln\left(\frac{p_{k,t}(x)}{p_{K,t}(x)}\right) = u_{k,t}(x) + \beta\delta \left[\boldsymbol{Q}_k(x) - \boldsymbol{Q}_K(x)\right] \boldsymbol{v}_{t+1}(\boldsymbol{\sigma}^*_{t+1})$ we obtain:

$$\ln\left(\frac{p_{k,t}(x)}{p_{K,t}(x)}\right) = u_{k,t}(x) + \beta\delta \left[\boldsymbol{Q}_k(x) - \boldsymbol{Q}_K(x)\right] \left[\boldsymbol{m}_{t+1} + \sum_{\tau=t+2}^{T} \delta^{\tau-t-1} \left(\prod_{r=t+1}^{\tau-1} \boldsymbol{Q}^{pb}_{r}(\beta) \right) \boldsymbol{m}_{\tau}\right]$$

expressed as a difference in current utility and continuation values.

Exclusion restrictions

- Intuition for exclusion restrictions typically deliver a *variable* that affects continuation values, but not the current period's utility.

- Such an excluded variable typically imply more than two exclusion restrictions on states, which further restricts the (finite) identified set of discount factors.

- For simplicity, let $x_{a,1}, x_{a,2} \in \chi$ and $x_{b,1}, x_{b,2} \in \chi$ be two pairs of states such that $x_{a,1} \neq x_{a,2}$ and $x_{b,1} \neq x_{b,2}$.

- The exclusion restrictions, indexed by a and b, are

$$u_{k,t}(x_{a,1}) = u_{k,t}(x_{a,2}) \text{ and } u_{k,t}(x_{b,1}) = u_{k,t}(x_{b,2})$$

$$\text{for some } k \in \mathcal{D}/\{K\} \text{ and some } t < T - 1$$

Exclusion restrictions

- Taking differences we get a bivariate polynomial system of order $T-t$ in β and δ:

$$\ln\left(\frac{p_{k,t}(x_{a,1})}{p_{K,t}(x_{a,1})}\right) - \ln\left(\frac{p_{k,t}(x_{a,2})}{p_{K,t}(x_{a,2})}\right) =$$

$$\beta\delta\left[\boldsymbol{Q}_k(x_{a,1}) - \boldsymbol{Q}_K(x_{a,1}) - \boldsymbol{Q}_k(x_{a,2}) + \boldsymbol{Q}_K(x_{a,2})\right]\left[\boldsymbol{m}_{t+1} + \sum_{\tau=t+2}^{T} \delta^{\tau-t-1}\left(\Pi_{r=t+1}^{\tau-1}\boldsymbol{Q}_r^{pb}(\beta)\right)\boldsymbol{m}_\tau\right]$$

$$\ln\left(\frac{p_{k,t}(x_{b,1})}{p_{K,t}(x_{b,1})}\right) - \ln\left(\frac{p_{k,t}(x_{b,2})}{p_{K,t}(x_{b,2})}\right) =$$

$$\beta\delta\left[\boldsymbol{Q}_k(x_{b,1}) - \boldsymbol{Q}_K(x_{b,1}) - \boldsymbol{Q}_k(x_{b,2}) + \boldsymbol{Q}_K(x_{b,2})\right]\left[\boldsymbol{m}_{t+1} + \sum_{\tau=t+2}^{T} \delta^{\tau-t-1}\left(\Pi_{r=t+1}^{\tau-1}\boldsymbol{Q}_r^{pb}(\beta)\right)\boldsymbol{m}_\tau\right]$$

- The identified set is the zero set of the two moment conditions.

- Estimation can be done by minimum distance, for example.

Identification of β, δ

Assumption 1. Either $\frac{p_{k,t}(x_{a,1})}{p_{K,t}(x_{a,1})} \neq \frac{p_{k,t}(x_{a,2})}{p_{K,t}(x_{a,2})}$, or the following rank condition holds

$$[\boldsymbol{Q}_k(x_{a,1}) - \boldsymbol{Q}_K(x_{a,1}) - \boldsymbol{Q}_k(x_{a,2}) + \boldsymbol{Q}_K(x_{a,2})]\, \boldsymbol{m}_{t+1} \neq 0,$$

and either $\frac{p_{k,t}(x_{b,1})}{p_{K,t}(x_{b,1})} \neq \frac{p_{k,t}(x_{b,2})}{p_{K,t}(x_{b,2})}$, or the following rank condition holds

$$[\boldsymbol{Q}_k(x_{b,1}) - \boldsymbol{Q}_K(x_{b,1}) - \boldsymbol{Q}_k(x_{b,2}) + \boldsymbol{Q}_K(x_{b,2})]\, \boldsymbol{m}_{t+1} \neq 0.$$

Finally, we need some terminal conditions.

Assumption 2. $w_{K,T}(x) = 0$ and $v_T(x) = m_T(x)$ for all $x \in \mathcal{X}$.

- Given the rank conditions and terminal conditions hold, the identified set is discrete with no more than $(T-t)^2$ points. (Theorem 2)

see also Abbring and Daljord (2019)

Relation to preference reversals

- Lab approach: use contrasts between observed choices from menus of Sooner-Smaller (SS) and Larger-Later (LL) rewards.

- Recall that the long-run value function is

$$\boldsymbol{v}_{t+1}(\boldsymbol{\sigma}^*_{t+1}) = \boldsymbol{m}_{t+1} + \sum_{\tau=t+2}^{T} \delta^{\tau-t-1} \left(\prod_{r=t+1}^{\tau-1} \boldsymbol{Q}^{pb}_r(\beta) \right) \boldsymbol{m}_\tau$$

$$\boldsymbol{Q}^{pb}_t(\beta) = \beta \boldsymbol{Q}_K + \underline{(1-\beta)\overline{\boldsymbol{Q}}_t}$$

$$\overline{\boldsymbol{Q}}_{t+1}(x) = \textstyle\sum_{k\in\mathcal{D}} p_{k,t+1}(x) \boldsymbol{Q}_k(x)$$

- The correction term broadly represents the expected preference reversals of future selves by the current self.

- In observational data, it is less clear how exactly this maps into preference reversals (except that preference reversals are impossible under geometric discounting).

Illustration: three-period model

- Assume binary choice and two exclusion restrictions. In period T-2, the moment conditions are:

$$\Delta \ln(p_{1,T-2}(x_a)) = \beta\delta\Delta\boldsymbol{Q}_1(x_a)\left[\boldsymbol{m}_{T-1} + \delta\boldsymbol{Q}^{pb}_{T-1}\boldsymbol{m}_T\right]$$
$$\Delta \ln(p_{1,T-2}(x_b)) = \beta\delta\Delta\boldsymbol{Q}_1(x_b)\left[\boldsymbol{m}_{T-1} + \delta\boldsymbol{Q}^{pb}_{T-1}\boldsymbol{m}_T\right]$$

where, e.g.,

$$\Delta \ln(p_{1,T-1}(x_a)) = \ln\left(\frac{p_{1,T-1}(x_{a,1})}{p_{2,T-1}(x_{a,1})}\right) - \ln\left(\frac{p_{1,T-1}(x_{a,2})}{p_{2,T-1}(x_{a,2})}\right)$$
$$\Delta\boldsymbol{Q}_1(x_a) = [\boldsymbol{Q}_1(x_{a,1}) - \boldsymbol{Q}_K(x_{a,1}) - \boldsymbol{Q}_1(x_{a,2}) + \boldsymbol{Q}_K(x_{a,2})]$$

- This gives, for example:

$$\Delta \ln(p_{1,T-2}(x_a)) = \beta\delta\Delta\boldsymbol{Q}_1(x_a)\boldsymbol{m}_{T-1} + \underline{\beta\delta^2\Delta\boldsymbol{Q}_1(x_a)\overline{\boldsymbol{Q}}_{T-1}\boldsymbol{m}_T} + \beta^2\delta^2\Delta\boldsymbol{Q}_1(x_a)\left[\overline{\boldsymbol{Q}}_{T-1} - \boldsymbol{Q}_2\right]\boldsymbol{m}_T$$

- Identification of β, δ relies on a higher order interaction term, $\beta\delta^2$.

Alternative identification approaches

- Hyperbolic discounting models are undoubtedly more difficult to identify than geometric discounting models.
  - E.g., Longer time periods are required to identify β, δ.

- There are alternative ways to impose restrictions for identification.

- Wang, Weiergraeber and Xiao (2022) study identification for sophisticated and naïve agents with finite horizon, stationary flow utility, and presence of a terminating action.
  - The source of exclusion restriction is different than Abbring, Daljord and Iskhakov (2019).

- For the sophisticated agent, observing the final period is not required.
  - If the final period is observed, OLS estimators are available.

- Provide a quick technical walkthrough below, focussing on the econometrics.

Baseline case: sophisticated agent with final 3 periods of data

- Assume stationary flow utility $u_{k,t}(x) = u_k(x)$. In addition, let $u_K(x) \neq 0$ be the utility for terminating action K, which involves no continuation value (i.e., the model ends once K is chosen. (Skip some notations.)

- Baseline case: observe final three periods of data, $\{\boldsymbol{p}_T, \boldsymbol{p}_{T-1}, \boldsymbol{p}_{T-2}\}$. We have

$$\phi_{kK}(\boldsymbol{p}_T) = \boldsymbol{w}_{k,T} - \boldsymbol{w}_{K,T} = \boldsymbol{u}_k - \boldsymbol{u}_K$$

$$\begin{aligned}\phi_{kK}(\boldsymbol{p}_{T-1}) &= \boldsymbol{u}_k - \boldsymbol{u}_K + \beta\delta\boldsymbol{Q}_k\boldsymbol{v}_T(\tilde{\boldsymbol{p}}_T) \\ &= \boldsymbol{u}_k - \boldsymbol{u}_K + \beta\delta\boldsymbol{Q}_k\boldsymbol{v}_T(\boldsymbol{p}_T) \\ &= \phi_{kK}(\boldsymbol{p}_T) + \beta\delta\boldsymbol{Q}_k\left(-log\boldsymbol{p}_{KT} + \boldsymbol{u}_K\right)\end{aligned}$$

where $\tilde{\boldsymbol{p}}_T = \boldsymbol{p}_T$ because there is no future period to discount.

- The model primitives are $\beta, \delta, \boldsymbol{u}_K$.

Baseline case: sophisticated agent with final 3 periods of data

- Assuming $J \times J$ state transition matrix $\boldsymbol{Q}_k$ is full rank, invert the equation to get

$$\boldsymbol{u}_K = log(\boldsymbol{p}_{KT}) + \frac{1}{\beta\delta}\boldsymbol{Q}_k^{-1}\Delta\phi_{kK}(\boldsymbol{p}_T)$$

where $\Delta\phi_{kK}(\boldsymbol{p}_{T-1}) = \phi_{kK}(\boldsymbol{p}_{T-1}) - \phi_{kK}(\boldsymbol{p}_T)$

- Then, for a sophisticated agent in period T-2,

$$\begin{aligned} & \phi_{kK}(\boldsymbol{p}_{T-2}) \\ = \quad & \boldsymbol{u}_k - \boldsymbol{u}_K + \beta\delta\boldsymbol{Q}_k\boldsymbol{v}_{T-1}(\boldsymbol{p}_{T-1}) \\ = \quad & \boldsymbol{u}_k - \boldsymbol{u}_K + \beta\delta\boldsymbol{Q}_k\left(-log\boldsymbol{p}_{KT-1} + \boldsymbol{u}_K + \underline{\delta(1-\beta)}\bar{\boldsymbol{Q}}_{T-1}(-log(\boldsymbol{p}_{KT}) + \boldsymbol{u}_K)\right) \end{aligned}$$

- Finally, take difference in log-odds <u>**across time T-1 and T-2**</u> to cancel out flow utilities. We obtain a **system of J equations with unknowns** β, δ:

$$\Delta\phi_{kK}(\boldsymbol{p}_{T-1}) = \beta\delta\boldsymbol{Q}_k\left(-log(\boldsymbol{p}_{KT-1}) + log(\boldsymbol{p}_{KT}) + \delta(1-\beta)\bar{\boldsymbol{Q}}_{T-1}\frac{1}{\beta\delta}\boldsymbol{Q}_k^{-1}\Delta\phi_{kK}(\boldsymbol{p}_T)\right)$$

Baseline case: sophisticated agent with final 3 periods of data

$$\Delta\phi_{kK}(\boldsymbol{p}_{T-1}) = \beta\delta\boldsymbol{Q}_k\left(-log(\boldsymbol{p}_{KT-1}) + log(\boldsymbol{p}_{KT}) + \delta(1-\beta)\bar{\boldsymbol{Q}}_{T-1}\frac{1}{\beta\delta}\boldsymbol{Q}_k^{-1}\Delta\phi_{kK}(\boldsymbol{p}_T)\right)$$

- This system is linear in δ and $\beta\delta$:

$$\Delta\phi_{kK}(\boldsymbol{p}_{T-1}) = \begin{bmatrix} A & B \end{bmatrix} \times \begin{bmatrix} \delta\beta \\ \delta \end{bmatrix} \equiv \Omega(\boldsymbol{p}_{T-2}, \boldsymbol{p}_{T-1}, \boldsymbol{p}_T) \times \begin{bmatrix} \delta\beta \\ \delta \end{bmatrix}$$

$$A \equiv \boldsymbol{Q}_k(log(\boldsymbol{p}_{KT}) - log(\boldsymbol{p}_{KT-1}) - \bar{\boldsymbol{Q}}_{T-1}\boldsymbol{Q}_k^{-1}\Delta\phi_{kK}(\boldsymbol{p}_T))$$

$$B \equiv \boldsymbol{Q}_k\bar{\boldsymbol{Q}}_{T-1}\boldsymbol{Q}_k^{-1}\Delta\phi_{kK}(\boldsymbol{p}_T)$$

- Assuming $\Omega(.,.,.)$ has full column rank (rank 2), β and δ are identified.

- Can use OLS for estimation.

Other settings

- Sophisticated agent and final periods not observed:
  - β, δ are (locally) identified in a system of J nonlinear equations, using 4 consecutive periods of data.

- Naïve agent: current self erroneously think that future selves are time-consistent. (Note: backward recursion is slightly different.)
  - β, δ are (locally) identified using last 3 periods of data.

Empirical application: welfare time limits

- In practice, it is tricky to find direct evidence of preference reversal using standard observational data.
  - Additional complications involve unobserved heterogeneity, selection, etc.

- Nevertheless, the identification results rely on the nonlinearity of the backward recursion problem based on economically motivated restrictions.

- Briefly talk about Chan (2017), who estimates a hyperbolic discounting model of labour supply and welfare participation with heterogenous time preference parameters.

- Exclusion restrictions are constructed from variations in behaviour induced by time limits in a welfare reform experiment.

Empirical application: welfare time limits

- The model blends multiple sources of identification:
    1. Exclusion restrictions (via a variable) that changes future continuation values but not the utility flow.
    2. Stationary flow utility.
    3. 12 periods of quarterly data.
    4. "Last period" sometimes observed (problem becomes effectively static after exhausting time limit).
    5. Randomized control trial – control group <u>not</u> subject to time limits.
    6. Structure of welfare program (nonlinear BC) regulates work and welfare choice combinations.

- Model specification is reasonably flexible.
    - And with (unobserved) heterogeneity, technology (wage), endogenous initial conditions.

- Different versions of the model are estimated and compared.
    - Geometric discounting, hyperbolic (sophisticated, partially naïve), etc.

- Validation exercises, e.g., estimate model on treatment group and validate using control group; vice versa (doesn't work because discount factors cannot be identified).

Some empirical background and motivation

- Examines “welfare dependence”, broadly/vaguely defined as social welfare programs carrying perverse incentives that lead to dependence on the programs, e.g.,
  - Labour supply disincentives.
  - “Poverty trap” due to reduction of earnings or income.
  - Current participation in the program increases the probability of future participation.
  - Time-inconsistent preferences (Fang and Silverman 2009).
- Under time-inconsistency, there will be scope for paternalistic policies.
  - “People do not always choose whatever is best for them.” (note: by a long-run criterion.)
- The program studied was AFDC/TANF (cash welfare) for poor female heads of family with at least one child under the age of 18.
- A policy experiment called FTP operated in Florida in 1994-1999, which imposed a 24/36-month time limit on the treatment group.

Welfare time limit as an exclusion restriction

- Assume wage uncertainty and liquidity constraints.
- Become ineligible for welfare after the cumulative periods of welfare use, M (the exclusion variable), reaches the time limit $\bar{M}$.
  - If receive welfare now, forgo one potential future period of welfare use.
  - "Option value" creates an incentive to preserve welfare use for the future.
- Consider two otherwise identical individuals who have different values of M (e.g., m_1 and m_2, where $m_1 < m_2 < \bar{M}$).
- The utility flow is the same, yet the choice patterns are different due to one being closer to reaching $\bar{M}$ than the other.
- The choice patterns reveal how the individual discounts the future.
  - No changes in behaviour if not forward-looking.

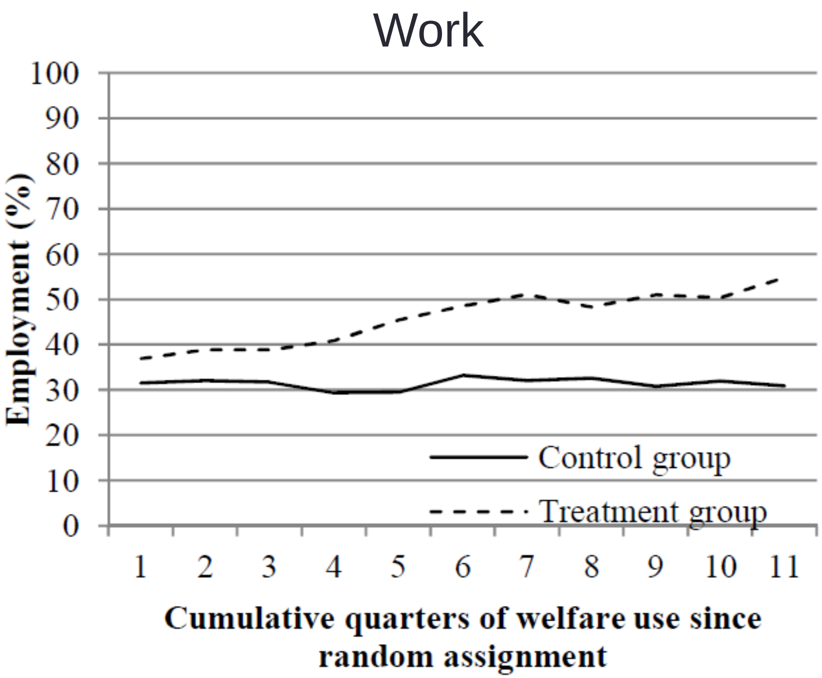
Summary statistics of control and treatment groups

	Control Group		Treatment Group		Difference[a]
Variable	Mean	Std. dev.	Mean	Std. dev.	
Highest grade completed	11.1	1.5	11.0	1.6	-0.1
Race (white=1, %)	44.8	49.7	45.0	49.7	0.2
Number of children under 18	2.1	1.0	2.0	1.0	-0.1
Age of youngest child (years)	5.1	4.3	4.9	4.1	-0.2
Ever worked full-time 6 months or more (%)	58.5	49.3	57.6	49.5	-0.9
On welfare for at least 36 of past 60 months prior to random assignment (%)	32.9	47.0	29.8	45.8	-3.1
During the last two years prior to random assignment:					
Total months of welfare receipt	14.1	9.0	13.7	9.2	-0.4
Total quarters of employment	2.0	2.5	2.0	2.4	0.0
Total earnings ($)	2878.0	5707.4	2682.0	5106.2	-196.0
During the sample period after random assignment:					
Fraction of time employed (%)	40.0	35.3	45.6	35.6	5.6 ***
Fraction of time using welfare (%)	48.3	34.1	44.7	32.2	-3.6 **
Fraction of time choosing:					
No work, no welfare (%)	24.9	30.5	27.2	32.1	2.3 *
Work, no welfare (%)	26.8	30.4	28.1	29.7	1.3
No work, welfare (%)	35.1	32.6	27.2	27.5	-8.0 ***
Work, welfare (%)	13.2	17.5	17.5	21.0	4.3 ***
Earnings (workers only, $)[b]	644.4	443.9	646.5	433.7	-
Number of individuals	1079	-	1060	-	-

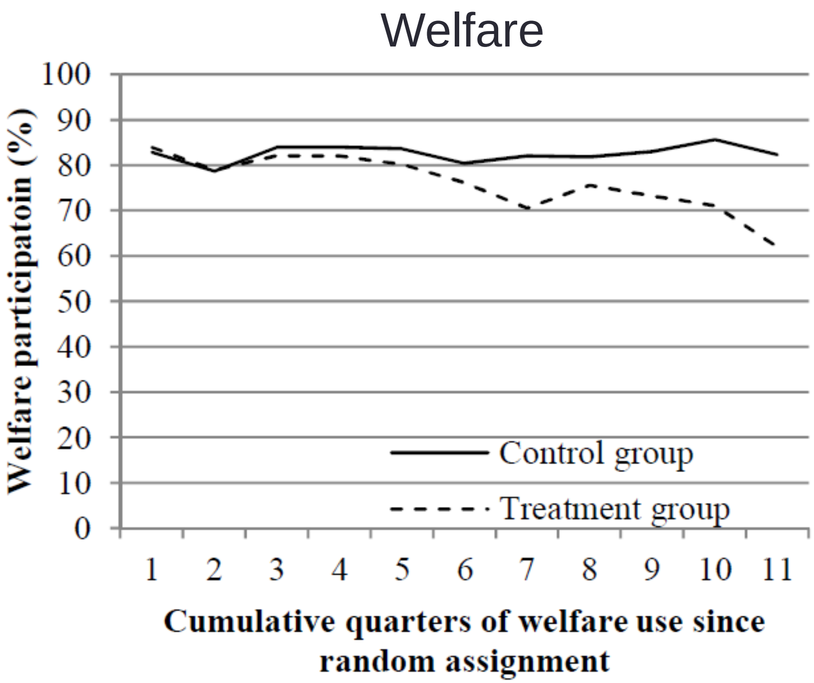
a Two-tailed t-test on difference in means between control and treatment group individuals. *, Significant at the 10 percent level; **, significant at the 5 percent level; ***, significant at the 1 percent level.

b Based on all worker observations in the sample. Earnings expressed at a monthly level.

Work and welfare use pattern conditional on exclusion variable M



(a) Employment.



(b) Welfare use.

Note: sample outcomes among individuals who used welfare last period.
M is defined to be 0 at the initial period of the experiment.

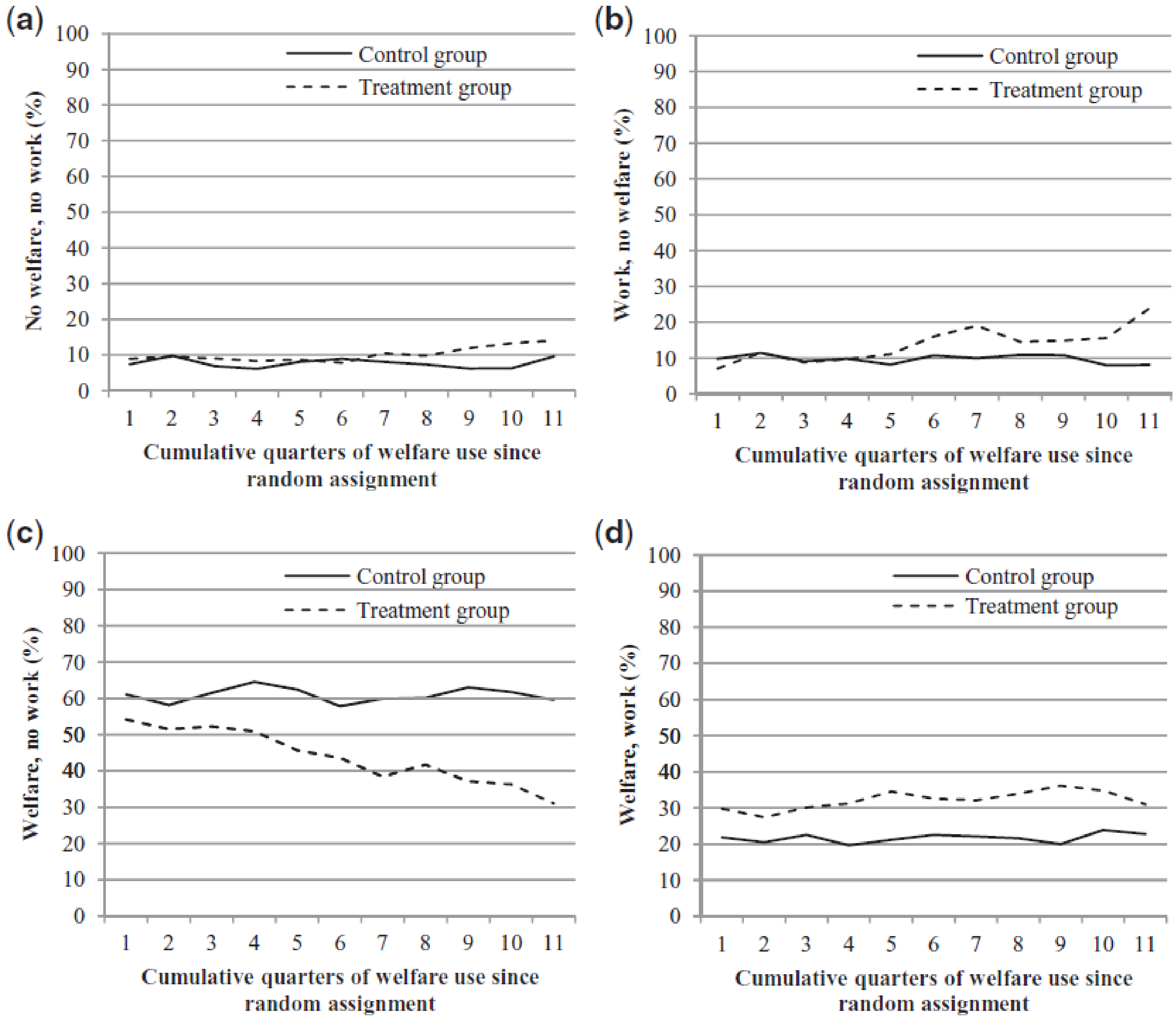
Work and welfare combinations by exclusion variable M

No work +
No welfare

work +
No welfare

No work +
welfare

work +
welfare



Note: sample outcomes among individuals who used welfare last period.
M is defined to be 0 at the initial period of the experiment.

Sample transition rates

Outcome in the Previous Period	Outcome in the Current Period: Work (%)	Welfare (%)	No Work, no Welfare (%)	Work, no Welfare (%)	No Work, Welfare (%)	Work, Welfare (%)	Number of Observations
Control group:							
No work, no welfare	11.9	8.0	81.8	10.2	6.4	1.6	2565
Work, no welfare	84.9	9.5	11.4	79.1	3.7	5.8	2824
No work, welfare	15.2	88.7	9.0	2.3	75.8	12.9	4594
Work, welfare	75.5	64.7	4.5	30.8	20.0	44.7	1706
Treatment group:							
No work, no welfare	12.0	7.3	82.2	10.5	5.8	1.5	2799
Work, no welfare	86.1	8.9	10.8	80.3	3.0	5.9	2872
No work, welfare	20.4†	84.1†	12.5†	3.4†	67.2†	16.9†	3513
Work, welfare	79.0†	71.0†	4.1	24.9†	16.9	54.1†	2152

a Numbers in the choice distribution may be subject to rounding error. One period is defined as one quarter. †, Significant at the 1 percent level in the two-tailed t-test on difference in means between control and treatment group individuals.

- Transition rates are different between the control and treatment groups.
- Also model work and welfare combinations; richer variation in practice.

Outline of model

- Three levels of labour supply, and whether to participate in welfare.
  - With state dependence in preferences.

- Experience accumulation. Can study the work commitment problem (Fang and Silverman 2004)
  - Agents underestimate the future value of human capital accumulation.
  - The model should also deal with the initial conditions problem; pre-random-assignment experience are not observed. (i.e., initial experience is subject to heterogenity)

- Unobserved heterogeneity:
  1. 2*2=4 unobserved types of individuals.
  2. Two “ability” types (wage equation).
  3. Two “preference” types (in both the utility function and time preference parameters).
  4. The ability and preference types can be correlated.
  5. The probability of each type depends on some initial conditions (important).

Hyperbolic discounting and commitment

- The discount factor δ_i and hyperbolic discounting parameters (present-bias β_i and naivety $\tilde{\beta}_i$) are heterogeneous.

- These estimates will determine the severity of self-control and commitment problems.
  - Work commitment problem.
  - Welfare trap.

- People may choose a commitment device that allow them to restrict their future budget sets.

- Policies evaluated using the “long-run criterion” (O’Donoghue and Rabin 1999; Fang and Silverman 2009)

Backward recursion

- Recall the continuation long-run value function given an arbitrary continuation strategy profile κ_{it}^{+} is:

$$v_{it}(\boldsymbol{S}_{it},\epsilon_{it};\kappa_{it}^{+},\boldsymbol{q})=u_{ik_\kappa t}+\delta_i E_t v_{i,t+1}(\boldsymbol{S}_{ik_\kappa,t+1},\epsilon_{i,t+1};\kappa_{i,t+1}^{+},\boldsymbol{q})$$

- For a perceived continuation strategy profile $\tilde{\kappa}_{it}^{+} \equiv \{\tilde{\kappa}_{ia}\}_{a=t}^{T}$, the strategy in period a is (a function of naivety factor $\tilde{\beta}_i$):

$$\tilde{\kappa}_{ia}(\boldsymbol{S}_{ia},\epsilon_{ia},\boldsymbol{q})\equiv \operatorname{argmax}_{d_{ia}\in D}\sum_{k=1}^{5} d_{ika}\left(u_{ika}+\tilde{\beta}_i\delta_i E_a v_{i,a+1}(\boldsymbol{S}_{ik,a+1},\epsilon_{i,a+1};\tilde{\kappa}_{i,a+1}^{+},\boldsymbol{q})\right)$$

- Given $E_t v_{i,t+1}(.,.;\tilde{\kappa}_{it+1}^{+},.)$, solve for $\tilde{\kappa}_{it}$. Then, construct $v_{it}(.,.;\tilde{\kappa}_{it}^{+},.)$.

- Note that, as a by-product, the perceived choice-specific value function is

$$\tilde{\bar{V}}_{ikt}(w_{it},\boldsymbol{S}_{it},\boldsymbol{q})\equiv \bar{u}_{it}(k;w_{it},\boldsymbol{S}_{it},\boldsymbol{q})+\tilde{\beta}_i\delta_i E_t v_{i,t+1}(\boldsymbol{S}_{ik,t+1},\epsilon_{i,t+1};\tilde{\kappa}_{i,t+1}^{+},\boldsymbol{q})$$

Backward recursion

- We can then form the (perceived) choice probabilities:

$$\tilde{P}_{ikt}(w_{it}, \boldsymbol{S}_{it}, \boldsymbol{q}) \equiv \frac{\exp(\tilde{\tilde{V}}_{ikt}(w_{it}, \boldsymbol{S}_{it}, \boldsymbol{q})/\sigma_{ci})}{\sum_{j=1}^{5} \exp(\tilde{\tilde{V}}_{ijt}(w_{it}, \boldsymbol{S}_{it}, \boldsymbol{q})/\sigma_{ci})}$$

- Then, compute expectation $E_{t-1} v_{it}(.,.,\tilde{\kappa}_{it}^{+},.)$ as:

$$E_{t-1} v_{it}(\boldsymbol{S}_{it}, \boldsymbol{\epsilon}_{it}; \tilde{\kappa}_{it}^{+} | w_{it}, \boldsymbol{q}) = eu + \sigma_{ci} \ln \left(\sum_{k=1}^{5} \exp(\tilde{\tilde{V}}_{ikt}(w_{it}, \boldsymbol{S}_{it}, \boldsymbol{q})/\sigma_{ci}) \right)$$

$$+ (1 - \tilde{\beta}_i)\delta_i \sum_{k=1}^{5} \tilde{P}_{ikt}(w_{it}, \boldsymbol{S}_{it}, \boldsymbol{q}) E_t v_{i,t+1}(\boldsymbol{S}_{ik,t+1}, \boldsymbol{\epsilon}_{i,t+1}; \tilde{\kappa}_{i,t+1}^{+}, \boldsymbol{q})$$

- This completes the recursion of the perceived long-run value function.

Backward recursion

- When the individual makes (actual) decisions, she believes that her future selves will follow the perceived continuation strategy profile $\tilde{\kappa}^{+}_{it+1}$. The current self's optimal strategy is

$$\kappa^{*}_{it}(\boldsymbol{S}_{it}, \epsilon_{it}, \boldsymbol{q}) \equiv \operatorname{argmax}_{d_{it} \in D} \sum_{k=1}^{5} d_{ikt} \left(u_{ikt} + \beta_i \delta_i E_t v_{i,t+1}(\boldsymbol{S}_{ik,t+1}, \epsilon_{i,t+1}; \tilde{\kappa}^{+}_{i,t+1}, \boldsymbol{q}) \right)$$

which is not used in backward recursion. (Note the β_i instead of $\tilde{\beta}_i$.)

- Her optimal decisions are compared with actual data and used for the likelihood function.

Heterogeneous hyperbolic discounting model (baseline model)

	Work (α_h)		Welfare (α_a)		Work and welfare (α_{ha})		Log wage	
Intercept	−798.19	(42.19)	−1024.63	(54.09)	147.11	(23.95)	6.02	(0.02)
Lagged work (γ_h)	505.10	(30.42)						
Lagged welfare (γ_a)			906.28	(47.02)				
Lagged work × lagged welfare (γ_{ha})					−59.70	(16.50)		
More than one child	54.18	(22.52)	−26.89	(22.88)	83.12	(15.68)		
Grade 12 or above	−59.18	(21.72)	−69.88	(23.99)	48.43	(15.21)	0.17	(0.02)
Race (non-white=1)	59.61	(25.47)	37.16	(27.44)	15.61	(16.18)	−0.12	(0.02)
Post-RA experience (ω_0)[a]							0.09	(0.01)
Post-RA experience × Grade 12 or above (ω_1)[a]							−0.01	(0.01)
Preference type-2 intercept	−386.70	(39.66)	−229.61	(35.28)	25.40	(31.88)		
Wage type-2 intercept (μ_w)							−0.84	(0.01)
	Discount factor (logistic coef.)		Present bias factor		Naivety factor			
Intercept	2.53	(0.42)	0.48	(0.11)	−0.36	(0.17)		
Race (non-white=1)	−1.39	(0.41)	0.33	(0.13)	0.20	(0.18)		
Grade 12 or above	−0.08	(0.20)	−0.02	(0.07)	1.07	(0.35)		
More than one child	−0.29	(0.22)	−0.22	(0.08)	0.34	(0.25)		
Preference type-2 intercept	0.74	(0.40)	0.20	(0.15)	0.54	(0.35)		
	Quadratic utility (α_y)[b]		Std. dev utility shock (σ_c)[c]		Std.dev wage Shock (σ_w)[c]			
Intercept	−4.06	(0.24)	5.80	(0.05)	−0.45	(0.01)		
Race (non-white=1)	−0.45	(0.18)	−0.02	(0.02)	−0.04	(0.01)		
Grade 12 or above	0.63	(0.20)	−0.05	(0.02)	−0.02	(0.01)		
More than one child	0.31	(0.15)	−0.06	(0.02)				
Other parameters								
FT work utility (α_{h2})	−781.04	(148.01)						
FT work × lagged work (γ_{h2})	798.20	(141.18)						
Enhanced service (B_{AS})	94.68	(32.08)						
Type probabilities (logistic coef.)			Wage type 2		Preference type 2			
Intercept			1.60	(0.15)	−2.44	(0.80)		
Wage type-2 dummy					3.30	(0.80)		
Ever had 6+ months of FT work			−0.27	(0.12)	−0.15	(0.17)		
Work experience in 2 years prior to RA[a]			−0.30	(0.04)	−0.82	(0.09)		
On welfare 36+ of the past 60 mths prior to RA			−0.02	(0.12)	−0.59	(0.19)		
Race (non-white=1)			−0.48	(0.13)	0.19	(0.23)		
Grade 12 or above			−0.34	(0.13)	−0.79	(0.22)		
More than one child					0.73	(0.23)		

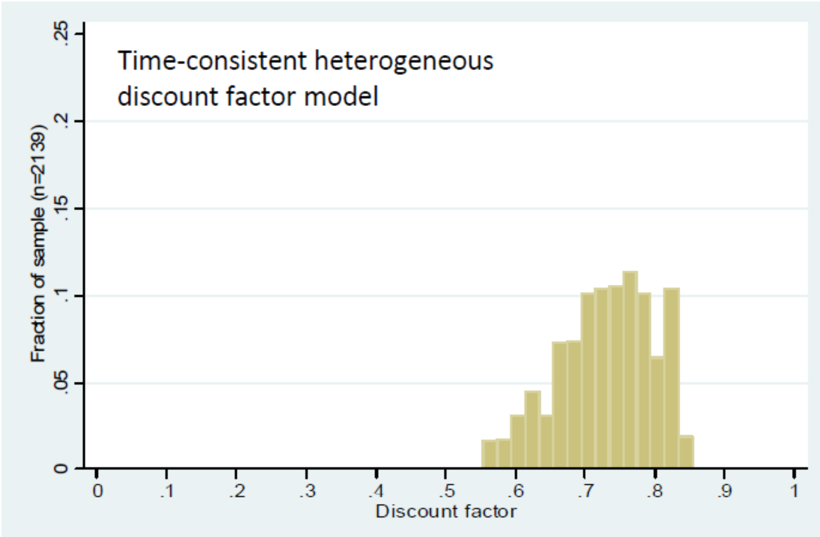
Notes: Number of observations = 23,025, log-likelihood = −28,219.15. Standard errors are given in parentheses. RA, random assignment.

[a] Cumulative periods of employment expressed in half-year intervals.

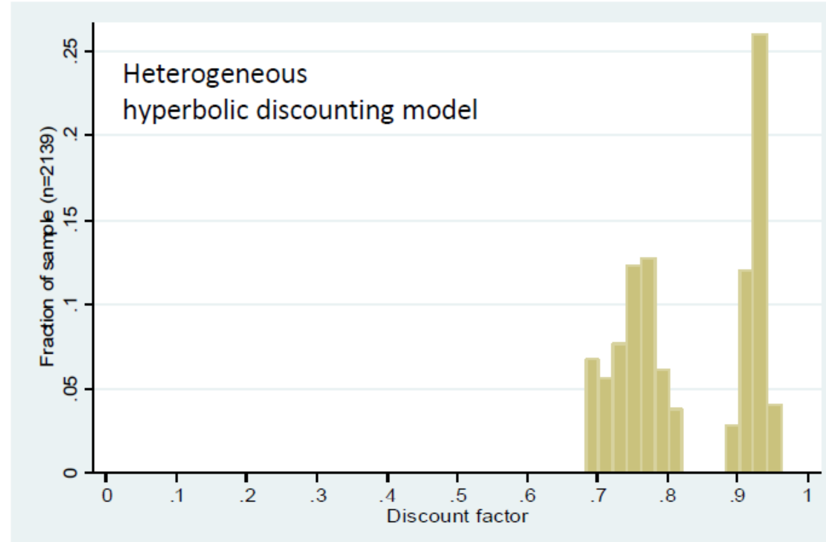
[b] Reported estimates and standard errors are multiplied by 10,000

[c] Transformation $\sigma = e^{Xa}$, where a is the vector of coefficients reported in the table.

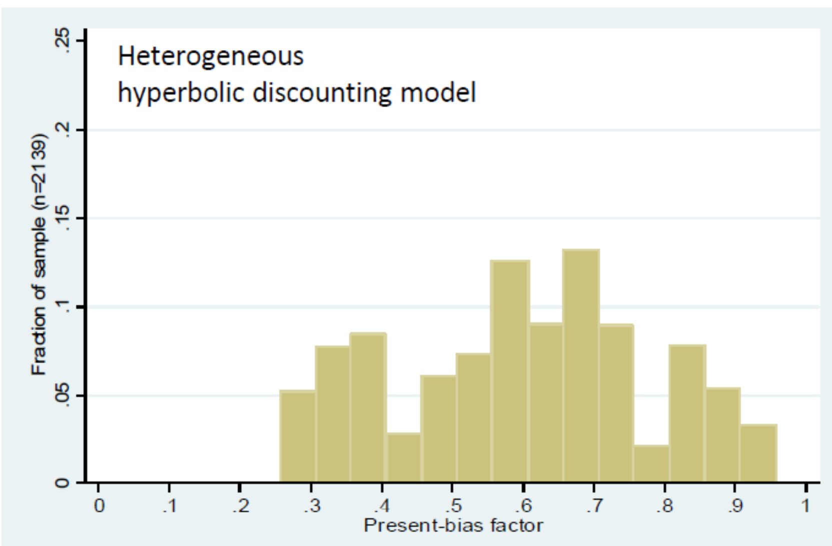
Time preference estimates



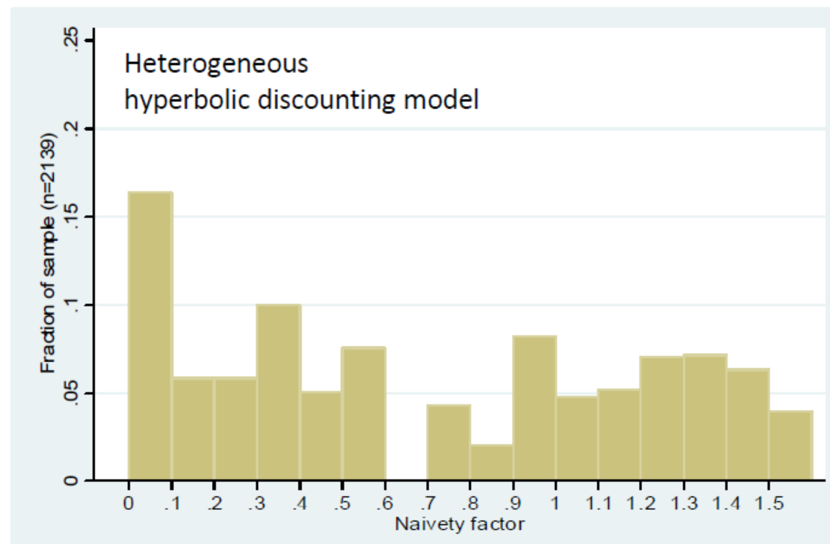
(a) Discount factor (mean=0.73, s.d.=0.07).



(b) Discount factor (mean=0.83, s.d.=0.09).



(c) Present-bias factor (mean=0.59, s.d.=0.17).



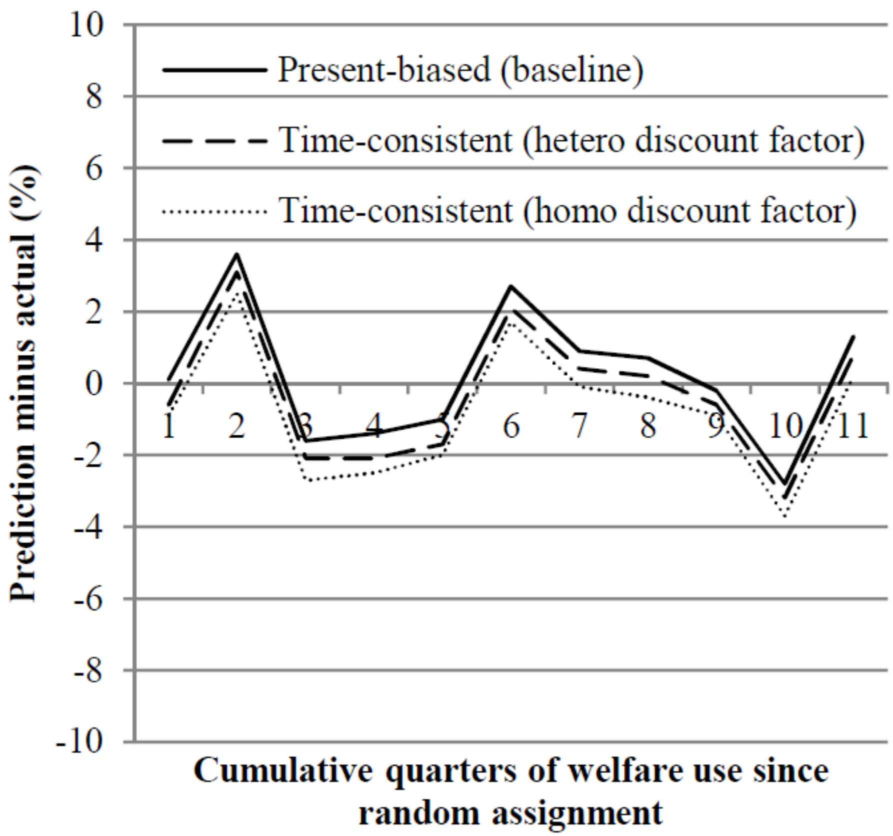
(d) Naivety factor (mean=0.68, s.d.=0.54).

Interpretation of the time preference estimates

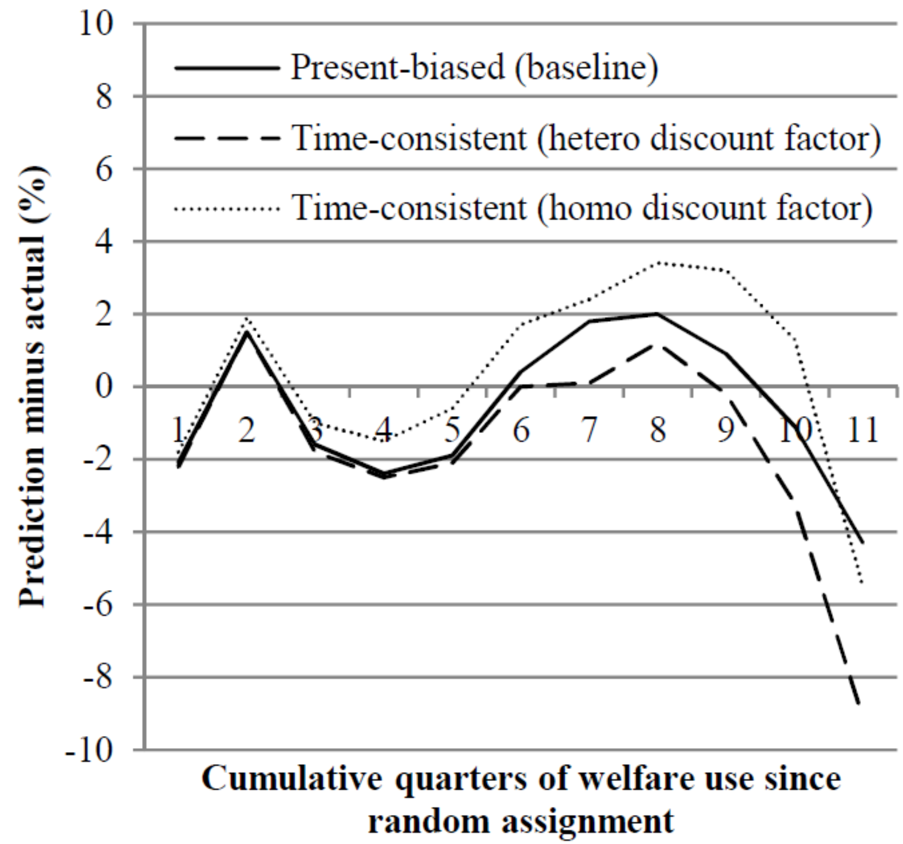
Around one-third of the individuals have a naivety factor exceeding one. Among these individuals, the mean of the discount factor, present bias factor, and naivety factor are 0.78, 0.59, and 1.30, respectively. The perceived aggregate discount factors are then 1, 1.01, 0.79, etc. (*i.e.* 1, $\tilde{\beta}\delta$, $\tilde{\beta}\delta^2, \ldots$), and the actual aggregate discount factors are 1, 0.46, 0.36, etc. (*i.e.* 1, $\beta\delta$, $\beta\delta^2, \ldots$). While the current self is unduly orientated towards the immediate payoff, she perceives that her future selves will favor immediate and near-immediate payoffs similarly and only discount longer term payoffs. Around one-sixth of the individuals have a very small naivety factor—for these individuals, the current self perceives that their future selves are (almost) myopic.[37]

- Individuals generally perceive themselves as time-inconsistent, so they may adopt a commitment device.

Goodness-of-fit by exclusion variable M



(a) Control group.



(b) Treatment group.

Income support and other static policy interventions

	Baseline	Expand welfare 1.112×	Expand EITC 1.32×	Work subsidy in welfare \$70	Income support \$50	
	(1)	(2)	(3)	(4)	(5)	
Work (%)	44.7	−0.4	0.9	1.0	−3.3	(41.4)
Welfare (%)	32.4	1.9	−0.7	1.0	−5.4	(27.0)
No work, no welfare (%)	32.5	−1.3	−0.2	−0.7	7.7	(40.2)
Work, no welfare (%)	35.1	−0.6	0.9	−0.3	−2.4	(32.7)
No work, welfare (%)	22.7	1.6	−0.7	−0.3	−4.4	(18.3)
Work, welfare (%)	9.6	0.3	−0.0	1.3	−0.9	(08.7)
APDV Earnings (\$)	390.3	−4.8	8.9	3.7	−25.7	(364.5)
APDV Net gov. expenditure (\$)	205.4	16.7	3.1	12.6	−14.0	(191.4)
APDV Utility (\$)	–	12.1	12.1	12.1	12.1	
Prefers new policy (%)[a]	–	99.3	100.0	99.1	90.7	

"income support": a payment to people who do not work and do not receive welfare.

Imposing dynamic sanctions on income support

	Original income support ($50)	Remove income support	Add sanction ($50): Dynamic type-1	Add sanction ($50): Dynamic type-2
	(0)	(1)	(2)	(3)
Work (%)	41.4	3.3	1.2	2.2
Welfare (%)	27.0	5.4	0.3	5.0
No work, no welfare (%)	40.2	−7.7	−1.3	−6.4
Work, no welfare (%)	32.7	2.4	1.0	1.4
No work, welfare (%)	18.3	4.4	0.1	4.2
Work, welfare (%)	8.7	0.9	0.2	0.8
APDV Earnings ($)	364.5	25.7	11.4	14.9
APDV Net gov. expenditure ($)	191.4	14.0	−1.3	14.3
APDV Utility ($)	–	−12.1	0.4	−12.7
Prefers amendment (%)[a]	–	9.3	17.5	3.9
Prefers amendment when it is revenue-neutral (%)[b]	–	0.0	69.2	0.0

- "type-1" sanction: ineligible for income support payment if **worked last period**.
  - Sanction makes people less "tempted" to return to idle if worked last period. (i.e., remain attached to work)
- "type-2" sanction: ineligible for income support payment if **not worked last period**.

Imposing dynamic sanctions on income support: time-consistent model

	Original income support ($50)	Remove income support	Add sanction ($50) Dynamic type-1	Dynamic type-2
	(0)	(1)	(2)	(3)
Work (%)	41.5	3.7	0.7	3.2
Welfare (%)	25.9	5.9	0.5	5.2
No work, no welfare (%)	41.0	−8.5	−1.0	−7.5
Work, no welfare (%)	33.1	2.6	0.5	2.3
No work, welfare (%)	17.5	4.8	0.3	4.3
Work, welfare (%)	8.4	1.1	0.2	0.9
APDV Earnings ($)	367.8	28.4	6.8	22.5
APDV Net gov. expenditure ($)	185.8	17.5	0.3	15.9
APDV Utility ($)	–	−21.4	−2.5	−19.1
Prefers amendment (%)[a]	–	0.0	0.0	0.0
Prefers amendment when it is revenue-neutral (%)[b]	–	0.0	0.0	0.0

- No one prefers a sanction regime in a time-consistent model.

Imposing dynamic sanctions on income support: (by degree of present bias in baseline model)

	Baseline model (Heterogeneous hyperbolic discounting)				Time-consistent model (heterogeneous discount factor)			
Present bias factor in the baseline model	0–0.4 (most present-biased)	0.4–0.6	0.6–0.8	0.8–1.0 (least present-biased)	0–0.4 (most present-biased)	0.4–0.6	0.6–0.8	0.8–1.0 (least present-biased)
Work (%)	1.6	1.3	0.9	1.1	0.7	0.8	0.5	0.8
Welfare (%)	0.0	0.3	0.3	0.4	0.4	0.4	0.6	0.6
No work, no welfare (%)	−1.5	−1.5	−1.0	−1.3	−1.0	−1.1	−0.9	−1.2
Work, no welfare (%)	1.5	1.2	0.7	0.9	0.5	0.6	0.3	0.6
No work, welfare (%)	−0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4
Work, welfare (%)	0.1	0.2	0.1	0.2	0.1	0.2	0.2	0.2
APDV earnings ($)	15.5	13.5	7.8	10.5	7.2	9.0	4.7	7.3
APDV net gov. expenditure ($)	−2.4	−2.0	0.1	−1.5	0.0	−1.0	1.8	−0.5
APDV utility ($)	3.5	1.0	−0.5	−2.2	−2.5	−2.6	−1.4	−3.1
Prefers sanction (%)[a]	24.8	13.8	24.8	1.6	0.0	0.0	0.0	0.0
Fraction of individuals in subgroup (%)	19.7	28.5	34.1	17.8	19.7	28.5	34.1	17.8

Note: in a “type-1” sanction, ineligible for income support payment if worked last period.

Conclusions

- Introduced hyperbolic discounting models, focussing on the econometric underpinnings of such models.
- Discussed identification using different approaches.
- Presented an empirical application in relation to practical considerations when estimating such models.
- Students could keep asking themselves the following questions:

1. How do agents compare the present with the future?
2. What information do they act on?
3. What are their beliefs?
4. Are these constructs stable over time?
5. Is the model correctly specified?
6. Are there simpler models that can do the same job?