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Deriving an estimate of the optimal reserve price: An application to British Columbian timber sales

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Abstract

In this paper, I use a simple game-theoretic model of behaviour at oral, ascending-price auctions within the independent private-values paradigm to put structure upon data from a sample of timber sales held in the province of British Columbia, Canada where, to a first approximation, the independent private-values paradigm appears appropriate. I then estimate several different empirical specifications and use the methods of Vuong to decide which model is closer to the truth than the others. Under different assumptions concerning the seller's valuation of the timber, estimates of the optimal reserve price are calculated.

Key words: Oral; Ascending-price auctions; English auctions; Independent private-values paradigm; Optimal reserve price; Timber sales
JEL classification: C1; C5; C7; D4; L1; Q2

1. Introduction

Over the past 35 years, economists have made considerable progress in understanding factors influencing prices realized from goods sold at auction. For example, they have found that the seller's expected revenue depends upon the type of auction employed, the rules that govern bidding, the number of potential bidders, the information available to potential bidders, and the attitudes of

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bidders toward risk. From a policy-maker's perspective, however, one of the most important problems involves choosing the selling mechanism that obtains the most profit for the seller. To a large extent, the structure of the optimal selling mechanism depends upon the informational environment. In one model of an informational environment, first developed by Vickrey (1961), it is assumed that each potential bidder knows only his own valuation for the object, but not those of his opponents, and that each valuation is an independent and random draw from a common distribution of valuations which is known to all of the potential bidders at the auction. When potential bidders are risk neutral, Vickrey showed that four quite different auction formats – the oral, ascending-price (English); the first-price, sealed-bid; the second-price, sealed-bid (Vickrey); and the oral, descending-price (Dutch) auctions – garner the same expected revenue for the seller. This result, which is known as the 'Revenue Equivalence Proposition' (REP), is of considerable interest to both economists and policy makers. Given the REP, one question that arises naturally is can one still improve upon the structure of the four auction formats? Within Vickrey's environment, which is often referred to as the independent private-values paradigm (IPVP), Riley and Samuelson (1981) have shown that devising a selling mechanism which maximizes the seller's expected gain involves choosing the reserve price, the minimum price that must be bid, optimally.

The literature concerned with mechanism design has often been criticized as lacking practical value because the optimal selling mechanism depends upon random variables whose distributions are typically unknown to the designer. In the past, because the distributions of valuations have been unknown, calculating the optimal reserve price, the optimal selling mechanism, for a real-world auction was impossible.

At auctions within the IPVP, the equilibrium bidding strategies of potential bidders are increasing functions of their valuations. For example, at the English auctions considered below, the dominant bidding strategy for bidders who lose the auction is to bid their valuations. Thus, in principle, it is possible to estimate the underlying probability law of valuations using the empirical distribution of bids from a crosssection of auctions.

At least three potential problems exist with such an approach. First, as shown below, the winning bid does not reveal complete information concerning the winner's true valuation. Second, in the presence of a reserve price, the empirical distribution of observed bids represents a truncated sample of data: only those potential bidders whose valuations exceed the reserve price will choose to bid. Finally, as in all empirical studies of auctions, the joint distribution of bidding and non-participation depends upon the number of potential competitors, but finding a measure of potential competition is often impossible, and when it can be done, the specific proxy is often inaccurate.

Below, I develop an empirical framework that allows me to estimate the probability law of valuations using the empirical distribution of bids from a truncated and potentially incomplete sample of data in the absence of a measure of potential competition. I implement this approach to derive an estimate of the optimal reserve price for auctions of timber harvesting rights in the province of British Columbia, Canada using information from a sample of timber sales held by the British Columbia Forest Service between 1984 and 1987. This exercise is of considerable practical value because, as discussed in detail below, the IPVP appears to capture well the important aspects of timber sales in British Columbia. Moreover, understanding how to obtain the most from the sale of province-owned (Crown) timber is an extremely important task in British Columbia because about 90% of all timber is owned by the Crown. In addition, forest products have historically accounted for over 20% of the province's manufactured output and almost 10% of the entire provincial product; British Columbia has historically accounted for about 25% of North American lumber production.

This paper is in five more sections. In Section 2, I describe the environment within which British Columbian timber sales are held and the data available concerning these sales, while in Section 3, I specify a theoretical model of bidding at English auctions (one of the auction formats used to sell timber in British Columbia) within the IPVP. In Section 4, I link the theoretical model of the English auction to data available concerning actual timber sales to construct the empirical specifications, while in Section 5, I present my empirical results and use them to derive estimates of the optimal auction under different assumptions concerning the seller's valuation for timber. In Section 6, I summarize and conclude the paper. I describe the construction of the data set and derive formulae too detailed for the body of the paper in an appendix.

2. Timber sales in British Columbia

In British Columbia, the *Forest Act* of 1979 permits small businesses to acquire the right to harvest timber on Crown land. The Minister of Forests sets aside a portion of each year's allowable cut to sell to eligible loggers and sawmillers through a series of public auctions held under the Small Business Forest Enterprise Program (SBFEP). Several criteria for eligibility exist. For example, to be eligible a person must be an independent logger (in Category 1) or a mill owner (in Category 2) over 19 years of age with at least 2 years of experience. In addition, a registration fee, which amounted to \$100 per annum in 1987, must be paid. Registrants can participate at auctions anywhere in the province, but over 90% of all sales in any particular Forest District involve only bidders from that district. During the period considered in this paper (1984–1987), the program also prohibited any registrant from holding more than two SBFEP sales at any one time. Below, I shall focus upon sales to loggers (Category 1 sales) because they comprise the bulk of timber sold under the SBFEP.

Table 1
Maximum likelihood estimates for probit model of auction choice

Variable	Price	Volume	Upset	Distance	Winter	Spring	Summer	Autumn
Estimate	−0.004	−0.022	−0.165	0.004	0.379	0.173	0.429	0.323
Standard error	0.015	0.013	0.179	0.003	0.712	0.720	0.656	0.660

Sample size = 222; Log-likelihood function = −141.922.

Although both English and first-price, sealed-bid auctions have been used to sell the right to harvest timber on Crown land, for reasons that will be made clear below, I shall focus solely upon English auctions. The reader should note that the choice of mechanism by the Ministry of Forests appears unrelated to such economic variables as log prices. In Table 1, I present the results of estimating a simple Probit model of mechanism choice where the dependent variable equalled zero if an English auction was used and one if a first-price, sealed-bid auction was used. The covariates included the observables 'average' log prices, 'average' minimum acceptable bids, volume of timber, distance to nearest timber processing facility, and dummy variables for the season of the sale.[1] None of these covariates has a significant coefficient estimate and jointly the estimates are insignificant at size 0.01, supporting the notion of random assignment.

The type of bidding admitted at the English auctions is quite simple. Essentially, the Ministry of Forests assigns a minimum price per cubic metre of timber harvested, (or 'scaled'). This minimum price, which is often referred to as the 'upset' rate, will vary across species and depends upon past lumber prices. The upset rate is known in advance to all potential bidders. Bidders may then verbally tender an additional amount per cubic metre of timber harvested, called the 'bonus' bid. Bonus bids are uniform across species. Although the auction rules in British Columbia vary slightly across Forest Districts, bidders are typically required to tender increments of no less than \$0.01 per cubic metre of timber harvested. The total amount bid is called the 'stumpage rate', and it will vary across species as it is the sum of the species-specific upset rate and the uniform bonus bid.

Each potential bidder has a considerable amount of information concerning the timber for sale. For example, from the timber-cruise report and other supporting documents he can obtain detailed information concerning the location of the

[1] Several different species potentially exist for each sale, and each species potentially has a different log price (upset rate, the minimum price per cubic metre that must be bid). The 'averages' presented are weighted averages of the log prices (upset rates) where the weights were determined by the proportion that species made up of the total volume.

timber, the surrounding terrain, and access to the timber by roads.[2] Typically, timber sold under the SBFEP is in areas that have quite well-developed road networks, so road-construction costs are usually negligible. A statistically unbiased estimate of the volume of standing timber by species and grade (known as the 'cruised' volume) is also available from the Forest Service. Errors may exist in the cruised-volume estimates, but potential bidders can and do inspect sales themselves. In any case, there is no reason to believe that any one potential bidder has more information than the others concerning the volume or quality of timber for sale.

For 129 sales of timber at English auction, I observe the district, year, and month in which the sale was held; the volume of timber by species; the upset rate by species; the distance to the nearest timber-processing facility; the number of actual bidders; and the final recorded bonus bids for each of the bidders. From other sources, which are described in detail in the appendix, I have derived measures for the price of timber and the number of potential bidders.

3. Bidding at English auctions within the IPVP

In this section, I outline the equilibrium bidding strategy at English auctions within the IPVP and describe how to calculate the optimal reserve price.[3] I begin by assuming that auctions can be modelled as non-coöperative games. To specify a game one must identify the players, characterize the information each player has, describe the strategies available to each player, describe how each player is rewarded, and characterize the equilibrium.

I consider auctions at which a single seller wishes to dispose of one object to $\mathscr{N}$ potential bidders (players). The ith player is assumed to know his valuation v_i $(i = 1, \ldots, \mathscr{N})$, but not those of his opponents. Heterogeneity in valuations is modelled as a continuous random variable V having probability density function $\phi(v)$ and cumulative distribution function $\Phi(v)$ that have support upon $[\underline{v}, \bar{v}]$. The valuations of players are assumed to be independent draws from $\Phi(v)$. Together, the above assumptions constitute the IPVP.[4]

The strategies available to the players are their bids. The seller is assumed to have valuation v_0 contained in the interval $[\underline{v}, \bar{v}]$ and to impose this valuation in the form of a known minimum price that must be bid. Bidding at English

[2] The timber-cruise report is a document prepared for the Ministry of Forests in which the timber for sale is described.

[3] For more details than are provided below, see the survey by McAfee and McMillan (1987).

[4] In models of Dutch or first-price, sealed-bid auctions, the assumption that $\Phi(v)$ is common knowledge to all of the potential bidders is invoked so that a Bayesian–Nash equilibrium can be calculated. This assumption is unnecessary to calculate the equilibrium at English or Vickrey auctions within the IPVP.

auctions can be modelled in several different ways. Milgrom and Weber (1982) have described one approach which involves assuming the sale price is measured on a thermometer initially set at v_0. Players with valuations below v_0 do not bid. As the thermometer rises, remaining players drop out at their valuations. The winner is the player with the highest valuation, and he pays the second-highest valuation.[5] Formally, for players who bid against at least one opponent and who do not win the auction, the dominant bidding strategy $\sigma(v)$ given valuation v is

$$\sigma(v) = v, \quad v_0 \leqslant v \leqslant \bar{v}.$$

Notice that risk aversion with respect to winning the auction does not affect the equilibrium bidding strategy: any utility function that is increasing in the net return will yield the same strategy.

The only behavioural assumptions of the model are that potential bidders bid independently and that losers tell the truth. By focusing upon English auctions, one can avoid the stronger common-knowledge-of-Φ assumption needed to solve for the equilibrium bid function at Dutch or first-price, sealed-bid auctions. Of course, this strength derives from the IPVP assumption. Within other paradigms (e.g., the common value paradigm or affiliated private-values paradigm), one would need to employ the common-knowledge-of-Φ assumption to solve the game.

Anecdotal evidence suggests that English auctions are typically used in environments where little information useful to all of the bidders concerning the value of the object is revealed in the course of the auction. Sales of oil and gas leases, for example, are not undertaken using English auctions because the proprietory information of any particular bidder concerning the probability of discovering oil (and thus the value of the lease) would be revealed in the course of bidding. That English auctions are used to sell timber lends support to the IPVP assumption.

In the case of British Columbian timber sales, several other factors suggest that the IPVP is a good approximation to the environment within which loggers bid for timber. For example, log prices are generally fixed to loggers either by contract or by list prices at sawmills. Moreover, during the period considered in this analysis (1984–1987) considerable price stability existed, so any asymmetries in expectations concerning future prices are unlikely to have been important. Thus, timber prices will be assumed fixed and the same to all potential bidders. Also, because each potential bidder knows a considerable amount about the timber to be sold, any asymmetries of information concerning the volume and quality of timber are surely minimal. Note too that because the payment scheme used by the Forest Service involves payment for the volume of timber removed from the forest (also known as 'scaled volume'), harvesters are perfectly insured by the government against any errors in the timber-cruise estimates. Therefore, a natural

[5] Note that if all save one player drop out at v_0, then the winner pays v_0, while if all drop out the object goes unsold.

explanation for differences in bidding behaviour is differences in harvesting costs which are likely individual-specific effects. Below, I shall assume that, for a given sale, these differences are random and independent across potential bidders as well as being identically distributed.

Because the winner is the bidder with the highest valuation and because he pays what his nearest opponent would have been willing to pay, the equilibrium pay-off to the ith player is $(v_i - v_{(2:\mathcal{N})})$ if he wins and zero otherwise. Here $v_{(i:\mathcal{N})}$ denotes the ith highest-order statistic for a sample of size $\mathcal{N}$ from the distribution of V. Note that the winning bid does not reveal complete information concerning the winner's true valuation.

One question that arises naturally in this context is can one improve upon the structure of the English auction within this paradigm? Riley and Samuelson (1981) have shown that within the IPVP the selling mechanism which maximizes the seller's expected gain is a selling mechanism where a reserve price ρ is chosen optimally. This ρ is different from the seller's valuation v_0. In fact, Riley and Samuelson have calculated that the optimal reserve price ρ^* satisfies

$$\rho^* = v_0 + \frac{[1 - \Phi(\rho^*)]}{\phi(\rho^*)}. \tag{3.1}$$

Thus, for example, suppose that V is distributed uniformly upon the interval $[0, 1]$, then ρ^* solves

$$\rho^* = v_0 + \frac{1 - \rho^*}{1} \quad \text{or} \quad \rho^* = \frac{v_0 + 1}{2}.$$

When v_0 equals zero, ρ^* equals one-half, and when v_0 equals the expectation of V, $\mathscr{E}[V]$, which is one-half, then ρ^* equals three-quarters. Clearly, to calculate ρ^* requires information concerning $\Phi(v)$, the distribution of latent heterogeneity.

To uncover the distribution of valuations, I shall use information from the distribution of bids. An attractive feature of using data from English auctions, when viewed through the lens of the Milgrom and Weber (1982) thermometer model of the English auction, is that, for those who bid, their tenders map out the valuation distribution.[6] Note, however, that the number of actual bidders (participants) N at an auction is random and endogenous, and typically less

[6] This assumption is an important identifying assumption which is not strictly met in the data used below. For example, bidders are required to increase bids by at least \$0.01 m^{-3} or in increments of \$100 for a sale involving 10,000 m^3 of timber, the average volume in my data. Thus, the continuous price feature of the thermometer model is only approximately true for the data-generating process. It is also possible that such discrete jumps in bids can cause the maximum valuations of some participants to be skipped, and for these 'bidders' not to appear in the data. To incorporate these realities into an empirical specification, economic models of bidding increments and the decision to cry out must be specified. Such tasks are, however, beyond the scope of this paper, and are left as topics for future research.

than $\mathscr{N}$: only those potential bidders with valuations exceeding the reserve price (however that is chosen) participate. Thus, the observed bid distribution is a truncated one. In addition, as noted above, the winning bid is not fully revealing concerning the winner's true valuation.

4. Empirical models

In order to uncover $\Phi(v)$, the distribution of latent valuations discussed in Section 3, I must map the observed data into a stochastic specification for V, derive the implications of this structure for the data-generating process, propose methods for estimating this data-generating process, and write down the exact specifications to be estimated. I break the description of this work into four subsections.

4.1. Mapping the observed data into a stochastic specification

To develop an empirically tractable model of bidding for timber within the environment described in Section 2 using the theoretical model described in Section 3, several additional assumptions must be made. First, I assume that only one stand of timber is to be auctioned, and on that stand at most k different species of timber exist. Letting p_j denote the price of species j (measured in dollars per cubic metre) and q_j denote the volume of species j (measured in cubic metres of timber), I assume next that a logger's valuation of a sale v depends upon total revenues $\sum_{j=1}^{k} p_j q_j$ and total harvesting costs. Total harvesting costs are assumed to depend upon the total volume of timber felled, but not to vary with the species composition. That is, holding stem diameters and stem densities constant, it costs the same to fell Douglas fir timber as it does Sitka spruce timber. In addition, such harvesting costs are also assumed to depend upon the distance to the nearest timber-processing facility (sawmill). Transportation costs are assumed to depend only upon the total volume, and not upon the species composition. As with felling, the argument here is that different species of timber do not have different transportation costs because they involve essentially the same amount of effort when being transported to a sawmill. Letting q denote the total volume of timber to be harvested (where q equals $\sum_{j=1}^{k} q_j$) and d denote the distance in kilometres to the nearest timber-processing facility, total harvesting costs are denoted $C(q, d)$. I assume that timber prices are known perfectly and are the same to all potential bidders. Thus, for any particular logger, the value of a timber sale v is then

$$v = \sum_{j=1}^{k} p_j q_j - C(q, d).$$

Introducing the weights $\{\lambda_j\}_{j=1}^k$ where λ_j equals (q_j/q), one can write v as

$$v = \left(\left(\sum_{j=1}^{k} p_j\lambda_j\right) - a\right)q,$$

where a equals $(C(q,d)/q)$ which denotes average harvesting costs for the sale. I assume that variations in average harvesting costs across bidders can be modelled as a continuous random variable A having probability density function $f(a)$ and cumulative distribution function $F(a)$.[7]

Conditional upon d, $\{p_j\}_{j=1}^k$, and $\{q_j\}_{j=1}^k$ bidding will depend upon how expensive it is for loggers to harvest. In the case of British Columbian timber sales, loggers must bid a non-negative bonus b above the species-specific upset rates set by the Ministry of Forests $\{u_j\}_{j=1}^k$. Hence, the species-specific stumpage rates $\{s_j\}_{j=1}^k$ tendered to the Crown must satisfy

$$s_j = u_j + b \geqslant u_j, \quad j = 1,\ldots,k.$$

What makes the bidding problem simple in the case of British Columbian timber sales is the fact that the bonus bid b must be the same across all species, so only one decision exists, the choice of b.

In general, the ith losing participant at an English auction will bid up to the point where zero profit obtains

$$\sum_{j=1}^{k}(p_j - a_i - s_{ji})q_j = \sum_{j=1}^{k}(p_j - a_i - u_j - b_i)q_j = 0. \tag{4.1}$$

Dividing both sides of (4.1) by q implies that the bonus bid b_i is a function β of the average harvesting cost a_i for participant i,

$$b_i = \beta(a_i) = \sum_{j=1}^{k}(p_j - u_j)\lambda_j - a_i = \hat{a} - a_i,$$

where $\hat{a}$ equals $\sum_{j=1}^{k}(p_j - u_j)\lambda_j$ and is known to all potential bidders. An English auction ends when the bidder with the lowest average harvesting costs (the highest valuation) bids just over the final offer of his opponent who has the second-lowest average harvesting costs (the second-highest valuation). Letting $\{a_{(i:\mathcal{N})}\}_{i=1}^{\mathcal{N}}$ denote the $\mathcal{N}$ average harvesting costs indexed in ascending order,

[7] Note that $\phi(v)$ is related to $f(a)$ by

$$\phi(v) = f(a)\left|\frac{\mathrm{d}a}{\mathrm{d}v}\right| = \frac{f\left(\sum_{j=1}^{k} p_j\lambda_j - v/q\right)}{q}.$$

the winning bonus bid w is

$$w = \beta(a_{(2:\mathcal{N})}) = \hat{a} - a_{(2:\mathcal{N})}.$$

Of course, when a bidder faces no opponents his winning bonus bid is zero.

4.2. Data-generating process of bids

The bonus-bidding rule defined above is a monotonically decreasing function of A over a relevant region: a bidder continues either until he wins or until zero profit obtains. The lower is a bidder's a (the higher is his v), the longer he remains at the auction. Because the bonus-bidding rule is a function of a random variable, it too is a random variable and its distribution is related to $F(a)$ (and $\Phi(v)$).

Central to the empirical specification for this study is the joint density of bidding and non-participation.[8] To derive this density, first ignore the reserve price and admit non-positive bids. For an ordered sample of average costs $\{A_{(i:\mathcal{N})}\}_{i=1}^{\mathcal{N}}$, the joint density of 'bids', where $B_{(i:\mathcal{N})}$ equals $(\hat{a} - A_{(i:\mathcal{N})})$, is

$$\begin{aligned} g_{\mathbf{B}_{(\mathcal{N})}}(b_{(1:\mathcal{N})}, b_{(2:\mathcal{N})}, \ldots, b_{(\mathcal{N}:\mathcal{N})}) &= \mathcal{N}! \prod_{i=1}^{\mathcal{N}} f(a_{(i:\mathcal{N})}) \\ &= \mathcal{N}! \prod_{i=1}^{\mathcal{N}} f(\hat{a} - b_{(i:\mathcal{N})}), \end{aligned}$$

where $\mathbf{B}_{(\mathcal{N})}$ denotes the $\mathcal{N}$-vector of ordered bids. Introducing a reserve price and admitting the fact that any observed bid must be greater than or equal to zero and less than $\hat{a}$ requires that this density be manipulated further. Three different regimes exist: more than one bidder, one bidder, and no bidders.

In the first, the number of actual bidders N has realization n which is two or greater. In this case, the winning bid $B_{(1:\mathcal{N})}$ (or W) equals $(\hat{a} - A_{(2:\mathcal{N})})$ with $A_{(2:\mathcal{N})}$ being less than $\hat{a}$. Integrating out the non-positive 'bids' and admitting that $A_{(1:\mathcal{N})}$ is less than $(\hat{a} - w)$ yields the following joint density of bidding and non-participation:

$$\begin{aligned} &\mathcal{N}! \int_{\hat{a}}^{\infty} f(a_{(\mathcal{N}-n+1:\mathcal{N})}) \int_{a_{(\mathcal{N}-n+1:\mathcal{N})}}^{\infty} f(a_{(\mathcal{N}-n+2:\mathcal{N})}) \ldots \int_{a_{(\mathcal{N}-1:\mathcal{N})}}^{\infty} f(a_{(\mathcal{N}:\mathcal{N})}) \\ &\quad \times \prod_{i=2}^{n} f(a_{(i:\mathcal{N})}) \int_{0}^{a_{(2:\mathcal{N})}} f(a_{(1:\mathcal{N})}) \, \mathrm{d}a_{(1:\mathcal{N})} \, \mathrm{d}a_{(\mathcal{N}:\mathcal{N})} \\ &\quad \ldots \mathrm{d}a_{(\mathcal{N}-n+2:\mathcal{N})} \mathrm{d}a_{(\mathcal{N}-n+1:\mathcal{N})} \end{aligned}$$

[8] I am grateful to a referee and an associate editor for forcing me to make explicit the discussion which follows, and to an editor, Cheng Hsiao, for providing the clue which resolved my simulation results with the conjectures made by the referee and the associate editor.

$$= \frac{\mathcal{N}!}{(\mathcal{N}-n)!}[1-F(\hat{a})]^{(\mathcal{N}-n)}\prod_{i=2}^{n} f(\hat{a}_{(i:\mathcal{N})}) \cdot F(a_{(2:\mathcal{N})})$$

$$= \binom{\mathcal{N}}{n}[1-F(\hat{a})]^{(\mathcal{N}-n)} n! \prod_{i=2}^{n} f(\hat{a}-b_{(i:\mathcal{N})}) \cdot F(\hat{a}-w). \tag{4.2}$$

As mentioned above, when only one bidder shows up at the auction, the dominant strategy for that person is to submit the minimum acceptable bid, in the case of timber a bonus bid of zero. In this case, $(\mathcal{N}-1)$ bidders have costs greater than $\hat{a}$, and one has costs less than $\hat{a}$. The binomial probability of such an event is

$$\mathcal{N}[1-F(\hat{a})]^{\mathcal{N}-1}F(\hat{a}). \tag{4.3}$$

The probability of no one bidding, when all costs are greater than $\hat{a}$, is

$$[1-F(\hat{a})]^{\mathcal{N}}. \tag{4.4}$$

Collecting (4.2)–(4.4), the contribution to the likelihood function of observed bids and non-participants is

$$\{[1-F(\hat{a})]^{\mathcal{N}}\}^{D_0}\{\mathcal{N}[1-F(\hat{a})]^{\mathcal{N}-1}F(\hat{a})\}^{D_1}$$
$$\times\left\{\binom{\mathcal{N}}{n}[1-F(\hat{a})]^{(\mathcal{N}-n)} n! \prod_{i=2}^{n} f(\hat{a}-b_{(i:\mathcal{N})}) \cdot F(\hat{a}-w)\right\}^{(1-D_0-D_1)} \tag{4.5}$$

where I have introduced the indicator variables

$$D_0 = \begin{cases} 1 & \text{if } N=0, \\ 0 & \text{otherwise} \end{cases}$$

and

$$D_1 = \begin{cases} 1 & \text{if } N=1, \\ 0 & \text{otherwise} \end{cases}$$

to signal when the random number of actual bidders N takes on a specific value, in this case, either zero or one.

Accurate and reliable measures of $\mathcal{N}$ are notoriously difficult to obtain. While every effort was made to get a useful measure of $\mathcal{N}$ for this study, measurement error in $\mathcal{N}$ is still possible. This can result in mismeasuring the number of non-participants $(\mathcal{N}-n)$ in the term $[1-F(\hat{a})]^{(\mathcal{N}-n)}$. Thus, one may want to focus upon the conditional (truncated) distribution of costs since in such a specification $(\mathcal{N}-n)$ would be absent.

To see this, consider the joint distribution of the number of actual bidders N and the ordered average costs for those bidders

$$g_{\mathbf{A}_{(N)},N}(a_{(1:\mathcal{N})}, a_{(2:\mathcal{N})}, \ldots, a_{(n:\mathcal{N})}, n)$$
$$= \binom{\mathcal{N}}{n}[1-F(\hat{a})]^{(\mathcal{N}-n)} n! \prod_{i=1}^{n} f(a_{(i:\mathcal{N})}).$$

This density can be factored as follows:

$$g_{\mathbf{A}_{(N)}|N}(a_{(1:\mathcal{N})}, a_{(2:\mathcal{N})}, \ldots, a_{(n:\mathcal{N})}|n)g_N(n)$$
$$= n!\prod_{i=1}^{n} \frac{f(a_{(i:\mathcal{N})})}{F(\hat{a})} \binom{\mathcal{N}}{n} [1 - F(\hat{a})]^{(\mathcal{N}-n)} F(\hat{a})^n,$$

where

$$g_N(n) = \binom{\mathcal{N}}{n} [1 - F(\hat{a})]^{(\mathcal{N}-n)} F(\hat{a})^n$$

is the binomial probability mass function for the number of actual bidders N. This implies that the density of the average costs for bidders, conditional upon the number of actual bidder N being n, is

$$g_{\mathbf{A}_{(N)}|N}(a_{(1:\mathcal{N})}, a_{(2:\mathcal{N})}, \ldots, a_{(n:\mathcal{N})}|n) = n!\prod_{i=1}^{n} \frac{f(a_{(i:\mathcal{N})})}{F(\hat{a})}.$$

Recognizing that the winning bidder does not fully reveal his costs yields the following conditional density function for observed bids:

$$n!\prod_{i=2}^{n} \frac{f(\hat{a} - b_{(i:\mathcal{N})})}{F(\hat{a})} \cdot \frac{F(\hat{a} - w)}{F(\hat{a})}. \tag{4.6}$$

Estimates derived from specification (4.6) will be less efficient than those derived from (4.5), but they will be robust to errors in $\mathcal{N}$. Because specification (4.6) is conditional upon realized actual competition, n can be treated as constant. Moreover, as the research of Andersen (1970) and Vuong (1983) has shown, the conditional maximum-likelihood estimator of parameters in specification (4.6) is consistent and distributed normally, asymptotically. Within the IPVP, specification (4.6) is quite robust. For it avoids the Bayesian–Nash assumption, is unaffected by the attitudes of bidder toward risk, and does not require information concerning $\mathcal{N}$. The use of conditional maximum-likelihood estimation is perhaps the most important methodolgical contribution of the paper. In all structural econometric analyses of auctions, a measure of potential competition is required. At English auctions within the IPVP, I have shown that one does not necessarily need a measure of $\mathcal{N}$. This would not be true for first-price, sealed-bid auctions within the IPVP. Hence, my exclusive focus upon English auctions in this paper.

4.3. *Methods of estimating the empirical models*

To estimate the optimal reserve price, one must recover information concerning the latent unobserved variable A. A natural way to proceed in recovering an estimate of A's distribution $F(a)$ would be to examine the empirical distribution of bonus bids and then to map back to the distribution of A. For example, in the absence of covariates and reserve prices and given a large enough sample, one could perform this exercise non-parametrically. However, the presence of a reserve price complicates matters because parts of the valuation distribution are unobserved. Because it is potentially necessary to have the entire valuation distribution in order to calculate the optimal reserve price, an alternative method to non-parametric estimation must be sought. I have chosen to estimate $f(a)$ using parametric methods. Implicit in any parametric assumption concerning $f(a)$ is an assumption concerning how a differential equation in A behaves locally. I extend this local behaviour into regions that I cannot observe in order to get estimates of $F(a)$, and consequently of $\Phi(v)$. Parametric models also provide a parsimonious framework within which to introduce observed heterogeneity across sales into the empirical specifications. In this case, at least fives types of observed heterogeneity (to be discussed below) appear important.

To employ the parametric approach, I follow Paarsch (1992) by assuming that $f(a)$ comes from a particular family of flexible distributions which can be characterized up to some unknown parameter vector $\boldsymbol{\theta}$

$$f(a) = f(a; \boldsymbol{\theta}).$$

Thus, the parameter vector $\boldsymbol{\theta}$ will imbed itself in (4.5) and (4.6). Using the methods of maximum likelihood or conditional maximum-likelihood, I can then back out estimates of $\boldsymbol{\theta}$.

4.4. *Specifications to be estimated*

In order to write down an exact specification for the empirical model of an English auction considered above, I must specify the five types of observed heterogeneity mentioned above and describe precisely how they affect harvesting costs and bonus bidding. I must also choose a family for $F(a)$.

The five types of observed heterogeneity are as follows: First, upset rates vary across timber sales. Second, log prices can vary across species, and they have varied somewhat over the period considered, so bidding can vary systematically across sales with different species compositions and in different time periods. Third, the volume of timber varies across sales, and this can affect average harvesting costs.[9] Fourth, the distance to the nearest timber processing facility varies from sale to sale. Finally, because logging is a regional industry (less than 10% of all timber sales in any particular Forest District involve bidders from outside of that district), the level of potential competition can vary from district to district, as well as over time, since no SBFEP registrant can hold more than two SBFEP sales at one time. In short, the number of potential bidders can vary across sales.

[9] There is an absence of information concerning stem-diameters and stem densities in this data set. Hence, in the work that follows, I assume constant stem-diameter and constant stem-density distributions across sales.

The following specifications admit the five types of observed heterogeneity discussed above. To introduce upset-rate and timber-price variation into the above framework, I allow upset rates and timber prices to vary across sales. For a sample of $t = 1, \ldots, T$ sales, I denote the upset rates and timber prices for species j at sale t by u_{jt} and p_{jt}, respectively, yielding

$$\hat{a}_t = \sum_{j=1}^{k} (p_{jt} - u_{jt})\lambda_{jt},$$

where λ_{jt} equals (q_{jt}/q_t) is the weight for species j at sale t. I assume that the ex ante total harvesting-cost function $C(d,q)$ depends upon d and q in the following way:

$$C(d,q) = \gamma_0 + \gamma_{q1}q + \gamma_{q2}q^2 + \gamma_{q3}q^3 + \gamma_{dq}dq.$$

Thus, felling costs are cubic in q, but transportation costs are linear in the product of volume and to distance to the nearest mill. Cubic felling costs admit an optimal volume for sales. Separability between felling and transportation costs is reasonable because these two activities are distinct. Proportionality of transportation costs is reasonable within this environment because the timber is relatively close to timber processing facilities and many firms exist to transport the logs. Average harvesting costs for the tth sale a_t will depend upon d_t and q_t according to

$$a_t = \gamma_{q1} + \gamma_{q2}q_t + \gamma_{q3}q_t^2 + \gamma_0 q_t^{-1} + \gamma_{dq}d_t. \tag{4.7}$$

A changing number of potential bidders across auctions is also admitted, denoted $\mathcal{N}_t$, and discussed extensively in Section A.6 of the appendix.

A number of ways of introducing randomness into (4.7) exist. For example, in earlier work (viz., Paarsch, 1989, 1991), I assumed that γ_{q1} followed a Weibull distribution, while the remaining γ's were unknown constants to be estimated. When both γ_{q3} and γ_{q2} equal zero, γ_{q1} can be interpreted of as a random marginal harvesting cost. Of course, an alternative would be to assume that the fixed costs of harvesting γ_0 are random and that γ_{q1} is a parameter to be estimated. Ex ante, none of these alternatives appears preferable, although some of them are more computationally parsimonious than others. While some of the alternatives may nest others, most do not. Thus, I use the methods of Vuong (1989) to decide upon the empirical specification that is closer to the truth than the others.

In general, I assume that the randomness in a 'parameter' γ follows a Weibull distribution which has cumulative distribution function

$$F_\gamma(c) = 1 - \exp(-\delta_1 c^{\delta_2}), \quad c > 0,\ \delta_1 > 0,\ \delta_2 > 0.$$

For more on this distribution, see Johnson and Kotz (1970). I use the methods of maximum likelihood and conditional maximum-likelihood to estimate the parameter vector $\theta = (\gamma', \delta')'$ where δ denotes the vector of parameters of the random parameter distribution and γ denotes the vector of remaining parameters of the cost function.

5. Empirical results and estimates of the optimal reserve price

Using the data, which have been described briefly in Section 2 and which are described in complete detail in the appendix, I estimated several different empirical specifications. As space precludes me from presenting all of my empirical work, in this section I focus upon the two most promising specifications. In particular, I consider empirical models in which either γ_{q1} or γ_0 follows the Weibull law.[10] I then use the procedures of Vuong (1989) to decide between the two specifications.

Parameter estimates for the random γ_{q1} specifications corresponding to (4.5) and (4.6) are presented in Table 2. In column (4.5a) of Table 2, I present the least restrictive parameter estimates of specification (4.5). Note that the estimate of γ_0, which is the fixed costs of harvesting, is \$3891.87. Note also that the estimate of γ_{q3} is small, but that this estimate is relatively imprecise. The restriction that γ_{q3} equal zero cannot be rejected having a p-value of less than 0.60. The estimate of γ_{q2} presented in column (4.5b) is significantly negative which implies increasing returns-to-scale in the harvest of SBFEP timber. Moreover, the cost advantage for large sales is substantial. In particular, according to these estimates the average cost of harvesting falls about \$8 per cubic metre between the sample average sale (having about $10{,}000\,\mathrm{m}^3$) and the largest sale in the sample (having about $50{,}000\,\mathrm{m}^3$), an amount more than the average bonus bid (see Table 4).[11]

In column (4.6a) of Table 2, I present the least-restrictive parameter estimates of specification (4.6) in which the assumption that the number of actual bidders or the number of potential bidders is observed is relaxed. Again, γ_{q3} is estimated to be small, and the estimate is imprecise as well as insignificant. As in column (4.5b), increasing returns-to-scale in harvesting appear present in the estimate of γ_{q2} presented in column (4.6b), but these scale economies are about one half of those estimated in specification (4.5). In Fig. 1, I present a graph of the expected average cost function for specification (4.6b) of Table 2 (where the expectation is taken over γ_{q1}) assuming 37.8 km, the sample average distance to the mill, for

[10] In some of my work, I also considered mixtures of Weibull distributions of the form

$$f_\gamma(c) = \pi f_{\gamma 1}(c) + (1-\pi) f_{\gamma 2}(c), \quad 0 \leqslant \pi \leqslant 1,$$

where $f_{\gamma 1}(c)$ and $f_{\gamma 2}(c)$ are different Weibull probability density functions; see Everitt and Hand (1981) for additional details concerning mixed distributions. However, when estimating such specifications, I found that they continually attempted to collapse to one-branch Weibull specification.

[11] One cannot rule out that an optimal size of a sale exists based upon the above evidence. For example, the optimal scale could exist at $90{,}000\,\mathrm{m}^3$, but is unobserved in the sample considered here.

Table 2
Maximum likelihood estimates: Weibull γ_{ql} specification

Specification	(4.5a)	(4.5b)	(4.6a)	(4.6b)
γ_0	3891.8712	3786.8913	3762.3312	3973.1016
	(859.1012)	(795.0515)	(1802.1011)	(1581.2231)
γ_{q2}	−0.0002	−0.0002	−0.0001	−0.0001
	(0.0002)	(0.0001)	(0.0002)	(0.0001)
γ_{q3}	0.0000	0	0.0000	0
	(0.0000)	—	(0.0000)	—
γ_{dq}	−0.1132	−0.1139	0.0502	0.0505
	(0.2306)	(0.2307)	(0.0788)	(0.0735)
δ_1	0.3686	0.3661	6.9335	6.7643
	(0.0432)	(0.0428)	(1.6236)	(1.6864)
δ_2	3.3230	3.3524	4.3119	4.0827
	(0.1748)	(0.1551)	(0.6511)	(0.4999)
LLF	−1319.1231	−1319.1875	372.8813	372.7174

The estimates for δ_1 and δ_2 are for cost in hundreds of dollars. White (1982) standard errors are presented in parentheses beneath each estimate.

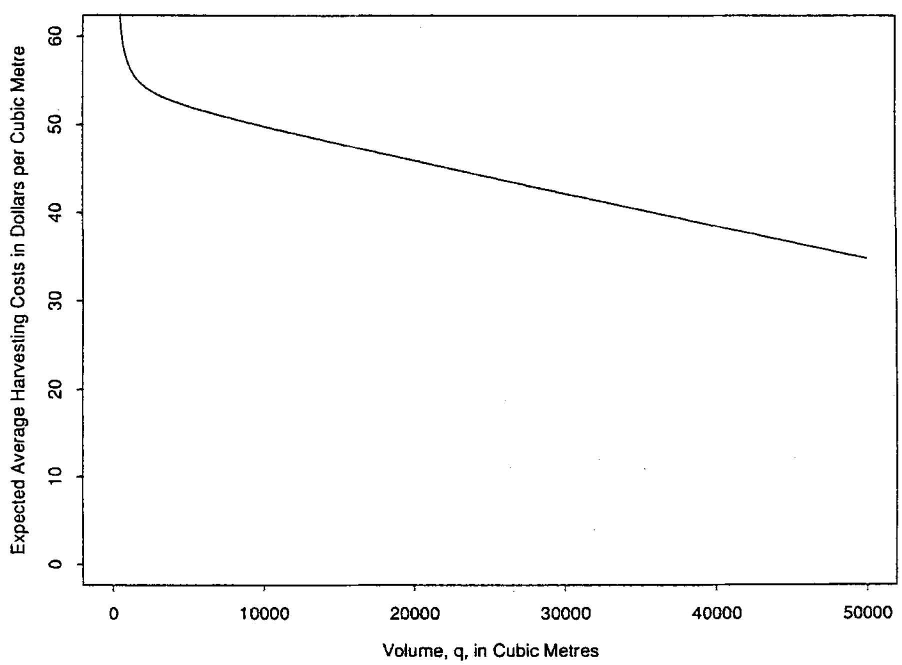

Fig. 1. Expected average harvesting costs for specification (4.6b) of Table 2.

d. Assuming that 37.8 km is a reasonable distance to the nearest mill, this figure suggests that the Forest Service could enhance the total stumpage price received considerably, simply by increasing the volume of a sale. Predictions beyond about 50,000 m^3, the maximum volume for a sale in this sample, cannot be made with any certainty however. The fixed costs of harvesting for this specification are slightly lower than those presented in column (4.5b) too. Because specification (4.6) requires the least amount of precision in the collected data, these results are probably more robust. Thus, my preferred estimates for this model of randomness are presented in column (4.6b).

Of course, a random γ_{q1} is not the only way in which the randomness could be introduced, so I considered many other ways in which to introduce the randomness, the most promising of which was through γ_0. In Table 3, I present my parameter estimates for the random γ_0 specifications of (4.5) and (4.6).

Table 3 warrants several remarks. First, the estimates of γ_{q1} are between \$13 and \$16 per cubic metre of timber harvested, and these are reasonable. Secondly, whereas the estimates of γ_{q2} are consistently negative in Table 2, some of the estimates of γ_{q2} in Table 3 are positive.

But of the two specifications which is preferred? I calculated Vuong's (1989) test statistic of the γ_{q1} specification versus the γ_0 specification. This statistic was 1.7643, which is distributed standard normal under the null hypothesis and has a p-value of less than 0.0764. Such evidence suggests that the γ_{q1} specification is closer to the true specification than is the γ_0 specification.

Table 3
Maximum likelihood estimates: Weibull γ_0 specification

Specification	(4.5a)	(4.5b)	(4.6a)	(4.6b)
γ_{q1}	15.4301	15.9029	13.8118	13.0995
	(0.4761)	(0.5285)	(5.3871)	(5.1873)
γ_{q2}	0.0005	0.0001	−0.0001	−0.0001
	(0.0001)	(0.0000)	(0.0001)	(0.0000)
γ_{q3}	−0.0000	0	0.0000	0
	(0.0000)	—	(0.0000)	—
γ_{dq}	−0.0028	0.0303	0.0126	0.0062
	(0.0139)	(0.0154)	(0.0653)	(0.0619)
δ_1	0.1062	0.1058	0.1421	0.0918
	(0.0076)	(0.0079)	(0.2331)	(0.2110)
δ_2	0.7243	0.7352	2.7408	2.5961
	(0.0280)	(0.0298)	(0.7257)	(0.6587)
LLF	−1376.5812	−1382.2613	407.6854	406.9981

The estimates for δ_1 and δ_2 are for cost in hundreds of dollars. White (1982) standard errors are presented in parentheses beneath each estimate.

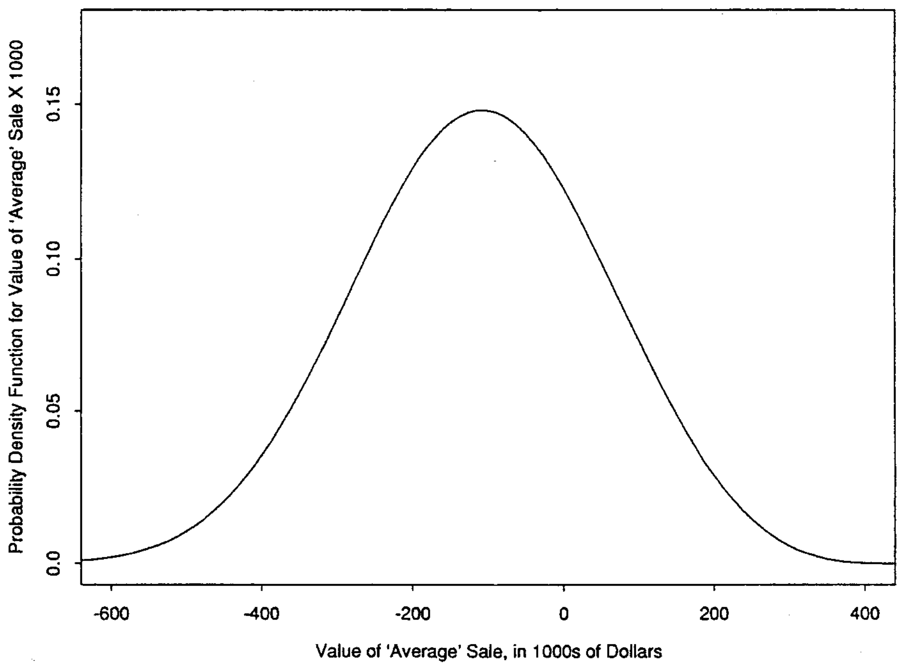

Fig. 2. Probability Density Function of 'Average Sale'- Table 2, Column (4.6b) Estimates.

Using the parameter estimates from column (4.6b) of Table 2 in conjunction with the sample means of the covariates, I calculated an estimate of the probability density function of valuations evaluated at the 'average' covariates of sales in the sample $\hat{\phi}(v; \bar{d}, \bar{p}, \bar{q}, \bar{u})$, where $\hat{\phi}(v; \bar{d}, \bar{p}, \bar{q}, \bar{u})$ is derived from $\hat{f}(a; \bar{d}, \bar{p}, \bar{q}, \bar{u})$ using the formula in footnote 7. This is presented in Fig. 2. Note that a little more than one-half of the mass is negative, a fact consistent with anecdotal evidence concerning the type of sales encountered in the SBFEP. In particular, it is often claimed that the major Tree-Farm-Licence holders (those who make up the other kind of land tenure in British Columbia, harvest about 90% of the timber each year, and supply the SBFEP with most of its timber) give up only marginal timber for disposition in SBFEP. Thus, SBFEP sales are typically of marginal profitability, but by using an auction the government finds the most efficient producer, and that firm earns some rent.

Assuming different values for v_0, the seller's valuation of the timber, I can also estimate the optimal reserve price ρ^* using $\hat{\phi}(v; d, p, q, u)$ evaluated at the sample covariates. For example, if v_0 equals zero, then the empirical distribution of the optimal reserve price estimates (the $\hat{r}^*$s) evaluated at each sampled covariate is presented in the top left panel of Fig. 3. (I present a derivation of the asymptotic distribution of R^* in Section A.8 of the appendix.) Given v_0, a little over 40% of

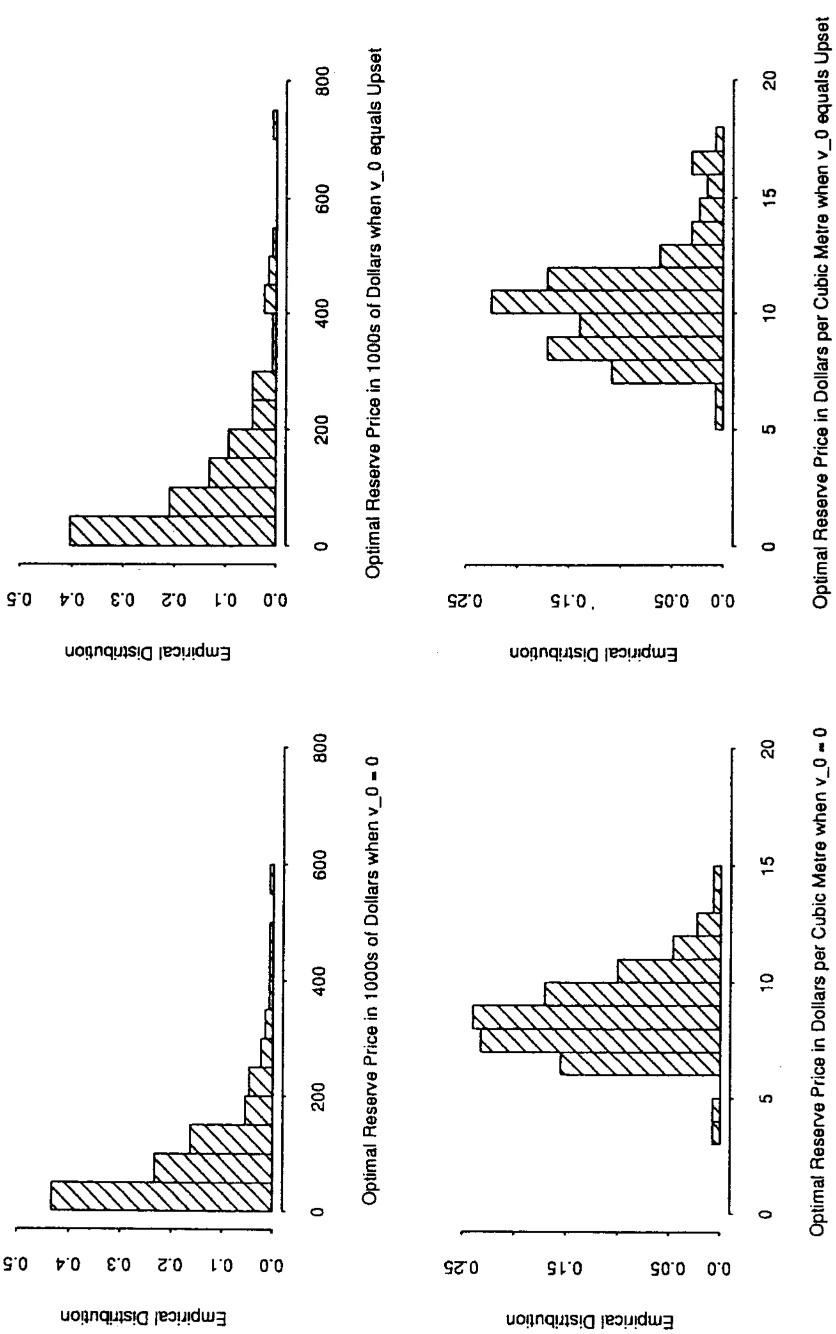

Fig. 3. Histograms of optimal Reserve price, per site and per cubic metre of timber Harvested.

the $\hat{r}^*$s were within two standard errors of the actual total upset price. The sample mean of the $\hat{r}^*$s is \$92,281.13, but the sample average of the total upset price ($\sum_{j=1}^{k} u_j q_j$) is \$26,891.18, suggesting that the Forest Service was too lenient in the setting of the reserve price for timber.

Because there is considerable heterogeneity with respect to the volume of timber to be harvested, in the bottom left panel, I present the empirical distribution of the $\hat{r}^*$s divided by the sample volume for each sale. The upset rate sample average is \$2.39 per cubic metre, while the average optimal reserve price per cubic metre is \$8.59.

In the top right panel, I present the empirical distribution of the $\hat{r}^*$s when I assume that the government set the total upset price equal to its valuation of the timber (i.e., v_0 equals $\sum_{j=1}^{k} u_j q_j$), while in the lower right panel, I present the empirical distribution of the $\hat{r}^*$s divided by the sample volume. The sample mean of the $\hat{r}^*$s in this case is \$112,729.90, while the average optimal reserve price per cubic metre is \$10.43. A little less than 40% of the $\hat{r}^*$s were within two standard errors of the actual total upset price.

Suppose that the sample total upset price is, in fact, the optimal reserve price. What then would be the revealed seller's valuation? In this case, the average seller's valuation over the sample covariates is $-\$89,948.16$; the median seller's valuation over the sample covariates, on the other hand, is $-\$62,792.45$. Only one of the revealed seller's valuation estimates is positive.

6. Summary and conclusion

In this paper, I have used a simple game-theoretic model of behaviour at English auctions within the IPVP to put structure upon data from a sample of timber sales held in the province of British Columbia where, to a first approximation, the IPVP appears appropriate. Estimates of several different empirical specifications were presented and the methods of Vuong (1989) were used to select a preferred specification. The preferred empirical specification in this study avoids the Bayesian–Nash assumption, is unaffected by the attitudes of bidder toward risk, and does not require information concerning the number of potential competitors $\mathcal{N}$. The use of conditional maximum-likelihood estimation is perhaps the most important methodological contribution of the paper. In all structural econometric analyses of auctions, a measure of potential competition is required. At English auctions within the IPVP, I have shown that one does not necessarily need such a measure.

Under different assumptions concerning the seller's valuation of the timber, estimates of the optimal reserve price were calculated. Given the robustness of the empirical specification, within the IPVP, we have considerable confidence in these optimal reserve-price estimates. The estimates suggest that the previous practice of imposing relatively low upset rates was suboptimal. Using my preferred empirical specification, I estimate that the 'average' optimal upset rate is somewhere between \$8.59 and \$10.43 m^{-3} (in 1987 dollars) instead of \$2.39, although the standard errors on particular estimates are often so large as to encompass the existing stumpages rates about forty percent of the time. It is, however, interesting to note that in April of 1994, the Ministry of Forest almost doubled the stumpage rates (in 1994 dollars) that it charges to firms. Note also that the cost structure of a representative firm suggests that the Forest Service could enhance the stumpage rates which it garners by increasing the volumes per site for sale from 10,000 to 50,000 m^3, or more. Predictions beyond about 50,000 m^3, the maximum volume for a sale in this sample, cannot be made with any certainty however.

Appendix

In this appendix, I document the development of the data set used, describing the sources from which the data were taken as well as the transformations used in obtaining the final data set. The important descriptive statistics are presented in Table 4. I also present the derivation of the asymptotic distribution of the optimal reserve-price estimator.

A.1. Bonus bids

For data concerning the bonus bids tendered at auctions, I travelled to nine Forest Districts in British Columbia, and searched the District records on file for a sample of timber sales. The data set covers auctions held between January 1984 and December 1987 inclusive. The districts considered are the Arrow, Campbell River, Kamloops, Kootenay Lake, Lillooet, Merritt, Port Alberni,

Table 4
Sample descriptive statistics – English auctions. Sample size = 129, January 1987 CPI = 1.0

Variable	Mean	SD	Minimum	Maximum
Winning Bonus Bid	6.89	7.01	0.00	28.16
'Average' upset	2.39	1.59	0.30	10.07
'Average' stumpage	9.29	7.36	0.30	31.87
'Average' price	46.89	6.69	34.14	67.49
Actual bidders	3.29	2.00	1.00	9.00
Potential bidders	92.39	31.88	27.00	185.00
Total cruised volume	10140.04	9720.55	130.00	53300.00
Conversion factor[a]	126.75	91.95	0.00	217.10
Haul distance	37.80	28.37	1.00	136.00

[a] Conversion factors apply only to interior sales. Zeros apply to coastal sales, of which there are 44.

Prince George East, and Prince George West. I chose these particular districts because the SBFEP is well-established in each.

I searched as many files as time permitted at each district. I eliminated sales for which files were unavailable during the collection period. There are potentially several reasons why files might be unavailable, but the most common in my case was that employees of the Ministry of Forests were using them in their work and could not release them. Sales were *not* selected by species composition or by the number of actual bidders at the auction. For each sale, a copy of the original bid record for the auction was made. In all, the data set contains information concerning 129 English auctions.

A.2. Upset rates

For each species in a sale, the Ministry of Forests calculates a minimum acceptable price per cubic metre which is the upset rate. Data concerning these rates were retrieved from the Harvest Database maintained by the provincial Ministry of Forests in Victoria, British Columbia, Canada.

A.3. Cruised timber volumes

Data concerning the volume of standing timber on each sale by species were also retrieved from the Harvest Database. These data are derived from information contained in the timber-cruise report.

A.4. Percentage small/large log composition and lumber recovery factors

Unlike on the coast, no log market exists in the interior of British Columbia. Consequently, valuing the timber on any sale in the interior, using market prices for logs, is impossible. Moreover, log prices from the coastal log market are unlikely to form a useful proxy since interior timber is quite different from coastal timber. One way of circumventing this problem is to convert a market price for interior lumber into a price index for logs.[12] I discuss the creation of this index separately in Section A.7. Here I simply describe the variables used.

The volume of lumber to be recovered from timber sales in the interior is calculated by using the percentage of small and large logs estimated on the sale, as well as by the lumber recovery factors (LRFs) for each type of log by species. These too are contained in the timber-cruise report.

Small and large logs are defined by diameter, with small logs having diameters less than 30 cm and large logs having diameters greater than 30 cm. The volume of timber to be recovered differs by log size and species, and so too do the LRFs. These data were also retrieved from the Harvest Database.

[12] I shall discuss one potential lumber price below.

A.5. Log and lumber prices

To value coastal sales, I used log prices from the Vancouver Log Market. These data were provided by the Council of Forest Industries. I considered seven different species: balsam, fir, hemlock, pine, red cedar, spruce, and yellow cedar. Some of the sales included timber for species such as alder, aspen, and poplar which I assumed were to be used as firewood. I ignored these species, since their volumes are negligible (less than 2% on average).

The lumber price series I have chosen to use in valuing interior timber is the real average monthly price per thousand board feet (1M) for one box car of Spruce–Pine–Fir (SPF), Western, Kiln-Dried (KD), 2 × 4s, Standard and Better (Std & Btr), Random Lengths (R/L), and is taken from the trade publication *Madison's Canadian Lumber Reporter*, weekly issues, 1984–1987.

This price series is listed as 'less 5 & 2%' discounts, and is FOB mill. Moreover, it is quoted in nominal US dollars. To convert the series into Canadian dollars I used the Canadian/US exchange rate series B3400 from the CANSIM database.

I converted all nominal data (bonus bids and upset rates as well as log and lumber prices) into real terms by dividing by the Canadian Consumer Price Index (CPI) setting January 1987 to one.

A.6. Number of potential bidders

All participants in the SBFEP must be registered at a district office, and their names and addresses are kept in the Small Business Forest Enterprise Program Registry. Eligible registrants may bid at any auction in the province, but over 90% of all sales in a particular district involve only bidders from that district. In my sample, all bidders are from the district in which the sale was held, and in a few cases from an adjacent district in the same region. For example, because the Prince George East and West districts border one another, four registrants from the East district bid at West auctions.

At any auction, only a subset of the potential bidders participates. There are at least two reasons for this: First, conditional upon timber prices and upset rates as well as the cost structures of SBFEP registrants, it is unprofitable to bid. Second, no registrant in the SBFEP can hold more than two SBFEP sales at one time.

In creating the variable 'number of potential bidders', I wanted to include those bidders who chose not to participate, but to exclude those who were ineligible. I have assumed that only those registrants from the home district holding fewer than two SBFEP sales at the time of a sale are potential bidders at that auction.

A.7. Data transformations

Other than converting the nominal data into real Canadian dollars, few transformations of the data described above were required. The main exception is log prices for the interior. Unlike the market on the coast, no formal log market exists for interior timber. Since some of the analysis rests upon the variable p, a proxy is required. I have chosen to convert the price for 1000 board feet (Mftb) of Spruce–Pine–Fir 2 × 4s (SPF) into such a proxy.

Suppose there are k different species indexed $j = 1, \ldots, k$. I assume that this SPF price applies across all species. Such an assumption is not as restrictive as it may sound since the bulk of interior wood is either fir, pine, or spruce. Moreover, a great deal of timber in the interior is small in diameter, so 2 × 4s would be a likely use for the logs.

The SPF prices must be converted into the appropriate units. The units of timber prices, upset rates, and bonus bids ($\{p_j\}_{j=1}^k$, $\{u_j\}_{j=1}^k$, and b) are dollars ($) per cubic metre ($m^3$), while those for timber volumes ($\{q_j\}_{j=1}^k$) are m^3. The units of SPF, on the other hand, are \$/Mftb. When divided by 1000, they are \$/ftb. I require a conversion factor, the units of which are ftb/m^3. The lumber recovery factors discussed in Section A.4 (which, for small and large logs, respectively, I denote by $\{\mathrm{LRF}_{sj}\}_{j=1}^k$ and $\{\mathrm{LRF}_{lj}\}_{j=1}^k$) are measured in ftb/$m^3$. Define q to be $\sum_{j=1}^k q_j$ and introduce the weights $\{\lambda_j\}_{j=1}^k$ where λ_j equals (q_j/q). Now the weights $\{\lambda_j\}_{j=1}^k$ are pure numbers as are the proportions of small and large logs (denoted ω_s and $(1-\omega_s)$). Thus, the conversion factor κ,

$$\kappa = \sum_{j=1}^{k} [\omega_s \mathrm{LRF}_{sj} + (1-\omega_s)\mathrm{LRF}_{lj}]\lambda_j, \tag{A.1}$$

has units ftb/m^3. The log price index I use for interior sales is

$$\frac{\mathrm{SPF}}{1000} \cdot \kappa = \frac{\$}{\mathrm{ftb}} \cdot \frac{\mathrm{ftb}}{\mathrm{m}^3} = \frac{\$}{\mathrm{m}^3}. \tag{A.2}$$

A.8. Asymptotic distribution of optimal reserve price estimator

The optimal reserve price can be viewed as a parameter ρ^* that satisfies an equation of the form

$$\begin{aligned}\rho^* &= v_0 + \frac{[1-\Phi(\rho^*;\mathbf{Z},\theta^0)]}{\phi(\rho^*;\mathbf{Z},\theta^0)} \\ &= \Omega(\rho^*; v_0, \mathbf{Z}, \theta^0),\end{aligned}$$

where v_0 is the seller's valuation, $\mathbf{Z}$ is a vector of observed covariates, and θ^0 is the true value of the unknown parameter vector θ. Consider an estimator $\mathbf{H}$ of θ that is consistent and distributed normally asymptotically. Thus,

$$\sqrt{T}(\mathbf{H} - \theta^0) \xrightarrow{\mathrm{d}} \mathrm{N}(\mathbf{0}, \mathbf{V}),$$

where $\mathbf{V}/T$ is the variance–covariance matrix of $\mathbf{H}$. Suppressing v_0 and $\mathbf{Z}$ for notational parsimony, R^*, an estimator of ρ^*, solves

$$\Psi(R^*, \mathbf{H}) = R^* - \Omega(R^*; \mathbf{H}) = 0.$$

Expand $\Psi(R^*, \mathbf{H})$ in a Taylor's series about $(\rho^*, \boldsymbol{\theta}^0)$, so

$$\Psi(R^*, \mathbf{H}) = 0 = \Psi(\rho^*, \boldsymbol{\theta}^0) + \Psi_{R^*}(\rho^*, \boldsymbol{\theta}^0)(R^* - \rho^*) + \nabla_{\mathbf{H}}\Psi(\rho^*, \boldsymbol{\theta}^0)'(\mathbf{H} - \boldsymbol{\theta}^0) + U.$$

Ignoring U, as it will be negligible in the neighbourhood of $(\rho^*, \boldsymbol{\theta}^0)$, one obtains

$$(R^* - \rho^*) = \frac{-\nabla_{\mathbf{H}}\Psi(\rho^*, \boldsymbol{\theta}^0)'(\mathbf{H} - \boldsymbol{\theta}^0)}{\Psi_{R^*}(\rho^*, \boldsymbol{\theta}^0)} \equiv \mathbf{m}'(\mathbf{H} - \boldsymbol{\theta}^0).$$

Thus,

$$\sqrt{T}(R^* - \rho^*) \xrightarrow{\text{d}} \mathrm{N}(\mathbf{0}, \mathbf{m}'\mathbf{V}\mathbf{m}).$$

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