# Life-Cycle Fertility, Human Capital, and Family Polices: A Discrete-Continuous Choice Framework

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# This Paper

- How do family leave policies affect life-cycle labor supply and fertility behavior?
- We formulate and estimate a dynamic model that accounts for:
  1. tradeoff between maternal time spent raising offspring and working
  2. family leave polices and their variation over time and states
  3. taxation policies and their variation over time and states
- We develop a new discrete-continuous choice model framework.
- We show that the model is semi-parametrically identified.
- We develop a three-stage estimation strategy based on the identification results.
- We estimate the model with PSID data to capture dynamics of the life-cycle and solve for the policy functions with the estimated parameters perturbed by the counterfactual policy innovations.

# Introduction

## Our Approach

- Our work joins a handful of studies that recognize the dynamic interactions between female labor supply and fertility by modeling and estimating the sequential determination of these joint events with panel data (Hotz and Miller,1988; Francesconi, 2002; Keane and Wolpin 2010; Adda, Dustman and Stevens, 2011, Wang 2022)
- The latter two also conduct counterfactual policy simulations:
  - Keane and Wolpin investigate changes to the welfare system;
  - Adda et al. simulate the effects of increasing child allowances.
- We evaluate the effect of job protection, paid leave, and other changes in parental leave and taxation policies in the USA.
- We will conduct counterfactual simulations on several policies:
  1. Alternative design of parental leave policies in terms of generosity and nonlinear eligibility.
  2. Pay for expenditure on offspring.
  3. Provision of child care.
  4. Pay women a wage to bear children.
  5. Retrain mothers who quit the labor force when they reenter it.

# Quasi-Experimental Variation in Policies in the USA

## Leave Policies

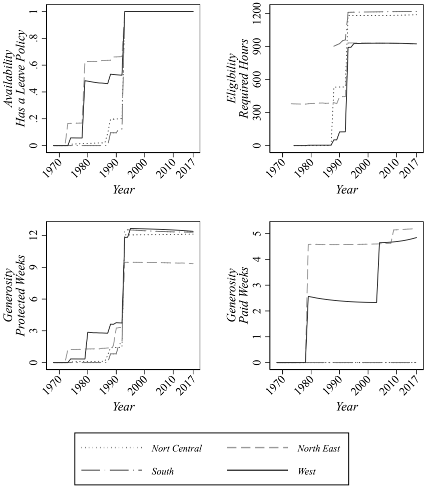


Figure: Variation in Leave Policies over Time and Across Regions

# Quasi-Experimental Variation in Policies in the USA

## Taxation Polices

- Variation in tax and welfare policies over time due to the six major reforms.
- Variation across states: we also group all states and D.C. into *low*, *medium* and *high* income tax groups based on the average income tax rate from 1978 to 2017.
  - *Low*: (e.g. Florida, Texas) average income tax $< 2\%$ (including zero)
  - *Medium*: (e.g. Illinois, Virginia) average income tax $\geq 2\%$ and $< 5\%$
  - *High*: (e.g. California, Wisconsin) average income tax $\geq 5\%$
- The tax/welfare policy variation across states (*low, medium, high*) and over time (*six major reforms*) creates 21 tax and welfare policy regimes in the data.

# The Life-Cycle Labor Market Consequences of Fertility

## Implementation

- We first use a standard event-study specification where $t = 0$ denotes the year in which an individual has their first child
- The event study runs for $t = -3, -2, \cdots, 10$, and separately for men and women:

$$Y_{ist} = \sum_{j \neq -1} \alpha_j \mathbf{1}\{j = t\} + \sum_k \beta_k \mathbf{1}\{k = age_{is}\} + \sum_y \gamma_y \mathbf{1}\{y = s\} + \sum_{i=1}^{5} \theta X_{is} + \nu_{ist}$$

- where $Y_{ist}$ is the outcome of interest (earnings, hours worked, participation rate and wage rate) for individual $i$ in year $s$ and at event time $t$,
  - $\sum_{j \neq -1} \alpha_j \mathbf{1}\{j = t\}$ are event time dummies
  - $\sum_k \beta_k \mathbf{1}\{k = age_{is}\}$ are age dummies
  - $\sum_y \gamma_y \mathbf{1}\{y = s\}$ are year dummies
  - $X_{is}$ is a vector of controls including education, race, marital status, and state fixed effects

# The Life-Cycle Labor Market Consequences of Fertility

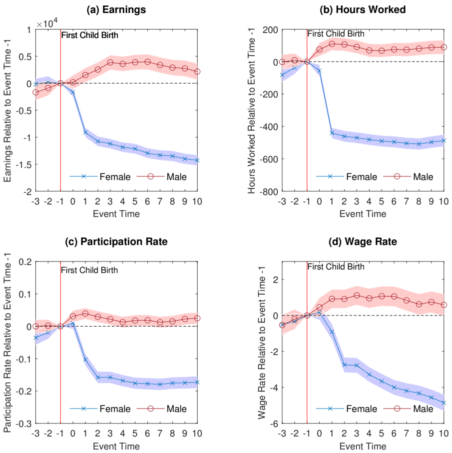


Figure: Motherhood Labor Market Penalty

Notes: event study coefficients; regressions run separately for men and women age 20 to 45. Base period is $t = -1$, coefficients measure the impact of having a child relative to the year immediately before the first child's birth. Earnings and wages are in real dollars of year 2015.

# The Life-Cycle Labor Market Consequences of Fertility

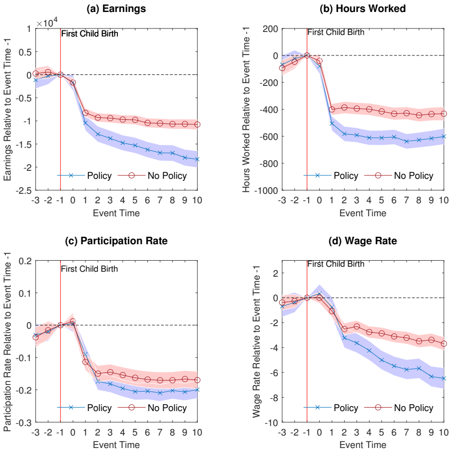


Figure: The Effect of Job-Protected Parental Leave on Women's Labor Market Outcomes upon Motherhood

Notes: event study coefficients; regressions run separately for women with and without protected parental leave entitlement, ages 20 to 45. Base period is $t = -1$, coefficients measure the impact of having a child relative to the year immediately before the first child's birth. Earnings and wages are in real dollars of year 2015.

# The Life-Cycle Labor Market Consequences of Fertility

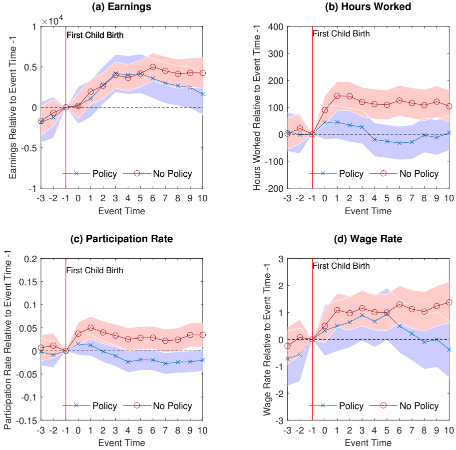


Figure: The Effect of Job-Protected Parental Leave on Men's Labor Market Outcomes upon Fatherhood

Notes: event study coefficients; regressions run separately for men with and without protected parental leave entitlement, ages 20 to 45. Base period is $t = -1$, coefficients measure the impact of having a child relative to the year immediately before the first child's birth. Earnings and wages are in real dollars of year 2015.

# Model

## A Life-cycle Model of Fertility and Female Labor Supply

- Women choose whether to work, how many hours to supply, and whether to have a child (tax, welfare, and leave policies affect their choices).
- Household gross income = the woman's and her spouse's labor income + household non-labor income + income replacement (if paid leave available).
- Wages are determined by human capital accumulated in the labor market, which depreciates.
  - Protected leave polices guarantee that women are paid at the same rate they were being paid before going on maternity leave
  - Protects against declines in wages due to loss of human capital.
- Women have preferences over consumption, leisure and births.
- Partnership status (marriage or cohabiting) and partner type are not choices.
  - However, distributions are endogenous to the woman's choices.
  - Women's partners are assumed to work full-time if they supply any labor.
- Women smooth their financial resources over time.

# Model
## Environment

- $t \in \{0, 1, \ldots, T\}$: a woman's age in years beyond adolescence.
- $\tau_t$: calendar year when the female is of age $t$
- $0 < T^F < T$: last fertile age
- $\pi \in \{\pi_0, \pi_1, \ldots, \pi_{\rho_r}\}$: policy environment to which she is exposed
  - $r \in \{0, 1, \ldots, \rho_r\}$: index of the policy environment $\pi_r$
- Aggregate expectations
  - Women are surprised by changes in policy $\pi_t$
  - But they have perfect foresight over the other aggregate objects $\underline{\omega}_t$

# Model

## Choices

- Consumption (continuous), $c_t \in \mathbf{R}_+$,
- Hours worked in the labor force (continuous)
  - $h_t \in [0, 1]$, $d_t \equiv \mathbf{1}\{h_t > 0\}$
- A woman of fertile age $(t \leq T^F)$ also makes fertility choices
  - $b_t \in \{0, 1\}$, $= 1$ if she decides to have a child.
- Hence, every period in her fertile years she chooses one of four discrete alternatives $k \in \{1, \ldots, 4\}$:
  1. neither to work nor to have a child $(k = 1)$,
  2. to work and not to have a child $(k = 2)$,
  3. to have a child and not to work $(k = 3)$,
  4. both to work and to have a child $(k = 4)$.
- Once she has passed her fertility age she only decides whether to work or not, i.e. $k \in \{1, 2\}$ for $t > T^F$.
- $d_{kt} \in \{0, 1\}$: indicator for whether she chooses alternative $k$ at $t$.

# Model

## Individual Characteristics, Human Capital and Children: I

- $z_t$: women's individual characteristics (include age, race, education, partnership status)
- $z'_t$: her partner's characteristics
- $x_t$: state variables of her problem
- $[h_{t-1}, \ldots, h_{t-\rho_w}]$: her human capital defined as the vector of recent work hours
  - Given by recent work hours, which captures depreciation
- $n_t \in \mathbf{Z}_+$, $\underline{a}_t \in [0, 17]^{n_t}$: number of kids under age 18 and their ages
  - Their laws of motion are intuitive and endogenous to the woman's birth decisions
  - Due to taxation and nurturing time needs, her choices are affected by her underage children $n_t$
  - Due to her preferences her choices are affected by their ages

# Model

## Individual Characteristics, Human Capital and Children: II

- Total available time is normalized to one.
- A woman's available time for working is limited by her children's nurturing time demands.
- Let $\phi_s$ be the time cost of nurturing a child of age $s$.
  - It is constant at $\phi > 0$ after age $\rho_c < 18$.
  - Falls to zero once a child reaches adulthood at age 18.
- $\varsigma_t$: children's nurturing time demands for a woman of age $t$:

$$\varsigma_t \equiv \sum_{s=0}^{\rho_c} \phi_s b_{t-s} + \phi \sum_{s=\rho_c+1}^{17} b_{t-s}$$

- Hence, the time available for work is endogenously constrained by her previous fertility choices: $h_t \in [0, 1 - \varsigma_t]$
- Her leisure $l_t \in [0, 1]$ is the residual time net of child nurture demands and labor:

$$l_t = 1 - h_t - \varsigma_t$$

# Model

## Partnership and Separation

Partnership dynamics are given stochastically by the following distribution conditional on state variables:

- $G(z'_t, m_t | z_{t-1}, z'_{t-1}, x_{t-1})$ which describes
  - $m_t$: partnership status
  - $z'_t$: partner's characteristics

  as a function of hers and her partner's characteristics, as well as other state variables such as tax and leave policies, human capital and birth history.

$G(\cdot)$ includes the following processes:

1. **Partnership status** (single, married, cohabitating) if single last period
2. **Partner type** (education) if transitioned into a partnership
3. **Marriage** if cohabiting. There's no transition from married to cohabiting
4. **Separation** if in a partnership (married, cohabitating)

# Model

## Leave Policies: I

- There are two types of leave policies:
  1. protected
  2. paid

- They target different parts of women's labor income paths.

- **Protected leave**:
  - targets future income
  - guarantees access the same job she had before taking time off
  - operationalized as covering losses in wage-relevant human capital

- **Paid leave**:
  - targets current income
  - provides time off paid at a fraction of her wages
  - $\iota(\pi) \in [0, 1]$: is the *replacement rate*

# Model

## Leave Policies: II

- $\ell_t = \{\ell_{1t}, \ell_{2t}\} \in \mathbf{R}^2_+$: vector of leave take-up
  - $\ell_{1t}$: protected take-up
  - $\ell_{2t}$: paid take-up
- We operationalize protected leave as preventing wage falls.
  - It prevents wage declines by crediting the worker with additional wage-relevant human capital if she reduces her labor supply following birth.
  - It gives rise to two measures of human capital.
    1. $h_t$: **actual human capital** accumulated at $t$, the hours a woman works in period $t$.
    2. $h^*_t$: **wage-equivalent human capital** accumulated at $t$, the hours that will determine the woman's wage rate.

$$h^*_t \equiv h_t + \ell_{1t}$$

# Model

## Leave Policies: III

- Conditional on having a child, a woman is granted the vector $\bar{h}_t \in \mathbf{R}_+^2$ of protected and paid leave

$$\bar{h}_t = b_t \cdot \kappa(\pi, h_{t-1}) \cdot H([h_{t-1}, \ldots, h_{t-\rho_w}])$$

which depends on her recent work history, where

1. $\kappa \in [0,1)^2$: captures the policy's generosity and eligibility criteria
2. $H \in (0,1)$: base hours. Let $H_t \equiv H([h_{t-1}, \ldots, h_{t-\rho_w}])$.
   - It depends on the intensity her recent labor attachment
   - It is used to compute how much leave will be granted
   - It determines her reduction in hours in response to a birth $(H_t - h_t)$
3. Leave hours granted at $t$ can only be used in the current period $t$

- Protected leave take-up:

$$\ell_{1t} = (H_t - h_t)\mathbf{1}\{0 \leq H_t - h_t \leq \bar{h}_{1t}\}$$

- Paid leave take-up:

$$\ell_{2t} = (H_t - h_t)\mathbf{1}\{0 \leq H_t - h_t \leq \bar{h}_{2t}\} + \bar{h}_{2t}\mathbf{1}\{H_t - h_t > \bar{h}_{2t}\}$$

# Model

## Budget Constraint I

- **Female** wages

$$w(x_t) = \omega_t \mu \exp\left\{ z_t' B_{r,3} + \sum_{s=1}^{\rho_w} (\delta_{r,1s} h^*_{t-s} + \delta_{r,2s} d_{t-s}) \right\}$$

  - $\omega_t$: aggregate trend in labor efficiency for women
  - $\mu$: fixed individual-specific productivity
  - $r$ indexes the policy environment which affects the wage paramenters

- **Male** labor participation and wages:
  - Probability of labor participation $Prob[d_t' = 1 | z_t, z_t', d_{t-1}', x_t]$
  - Provided a male works, he works full time hours $h'$
  - Hourly wage $w_t'$ is a function of potential experience:

$$\ln w_t' = \ln \omega_t' + \ln \mu' + B(z_t') \ln(t - 18)$$

  - Male labor income:

$$e'(x_t) \equiv w_t' d_t' h'$$

# Model

## Budget Constraint II

- Non-labor income, $e_t^{NL}$, follows an AR(1) process:

$$\ln e_t^{NL} = B^e \ln e_{t-1}^{NL} + B^{NL} X_t + \omega_t^{NL} + u_t^e$$

  - $B^e$: persistence of non-labor income
  - $X_t$: age, race, education, partnership status, and partner's education
  - $\omega_{\tau_t}^{NL}$: aggregate component of non-labor earnings
  - $u_t^e$: idiosyncratic innovation distributed $N(0, \sigma_e^2)$

- **Gross household income**:

$$W_k(h_t, x_t) \equiv w(x_t)h_t + \iota(\pi)w(x_t)\ell_{2t} + e'(x_t) + e^{NL}(x_t)$$

- **Net household income**:

$$\Upsilon_k(h_t, x_t) = W_k(h_t, x_t) - \left(\pi_{k0}^{tax}(x_t) + \pi_{k1}^{tax}(x_t) W_k(h_t, x_t)^{1-\pi_{k2}^{tax}(x_t)}\right)$$

  - $\pi_{k0}^{tax}$, $\pi_{k1}^{tax}$: intercept and slope of the policy
  - $\pi_{k2}^{tax}$: progressivity of the policy

# Model

## Lifetime Utility

$$-E\left\{\left[\sum_{t=0}^{T_R}\sum_{k\in C_t}\beta^t d_{kt}\exp\left(-\alpha c_t - u_k\left(h_t, x_t\right) - h_t\xi_t - \epsilon_{kt}\right)\right] - \left[\sum_{t=T_R+1}^{T}\beta^t\exp\left(-\alpha c_t\right)\right]\right\}$$

- $T_R$: retirement age, only consumption smoothing choices are made after
- $C_t$: discrete choice set, indexed by $t$ as women can only have births during their fertile age $(t \leq T_F)$
- Flow utility function is CARA with absolute risk aversion $\alpha$
- $u_k\left(h_t, x_t\right)$: captures preferences for births and leisure. It allows for preference interactions:
  - between the age distribution of the households' underage children and current births
  - between past and present leisure (or labor supply, alternatively)
- $\xi_t$: idiosyncratic disturbance to the marginal disutility of working

# Model

## Consumption Smoothing

- Individuals have access to a contingent claims market for consumption goods that they use to smooth consumption (a la Margiotta & Miller, 2000)
- In this environment, aside from the current policy state $\pi_t$ and aggregate wage and non-labor income effects $\underline{\omega}_t$, aggregate effects are transmitted through interest rates
- $\lambda_t$: value of a consumption unit discounted back $t$ periods
- The law of motion for savings $s_t$:

$$E_t[\lambda_{t+1} s_{t+1} | h_t, b_t, x_t] + \lambda_t c_t \leq \lambda_t \left( s_t + \underbrace{\Upsilon_k(h_t, x_t)}_{\text{net income}} \right)$$

# Model

## Optimal Choices I

- Define an index of household capital value at year $t$ sequentially as:

$$A_t(x_t) \equiv \sum_{k \in C_t} p_{kt}(x_t) \exp\left(\frac{-\overline{u}_k(x_t)}{B_t}\right) E\left[\exp\left(\frac{-\varepsilon^*_{kt}}{B_t}\right) \Big| x_t\right]$$
$$\times \left[\int \left(\int A_{t+1}(x_{t+1}) g_{kh}(x_{t+1}|x_t) dx_{t+1}\right) q_k(h|x_t) dh\right]^{1-\frac{1}{B_t}}$$

  - $p_{kt}(x_t) \equiv E\left[d^o_{kt} | x_t\right]$: conditional choice probability (CCP) of $k$ at $t$
  - $\overline{u}_k(x_t)$: per-period utility expected over $\xi$

$$\overline{u}_k(x_t) \equiv E[u_k(h_k(x_t, \xi_t), x_t) | x_t] + E[h_k(x_t, \xi_t) \xi_t | x_t] + \rho E[\Upsilon_k(h_k(x_t, \xi_t), x_t) | x_t]$$

  - $\varepsilon^*_{kt}$: truncated variable that takes on the value of $\varepsilon_{kt}$ only when $d_{kt} = 1$
  - $g_{kh}$: density function of the future state given choice $k$ and hours $h_{kt}$
  - $q_k(h|x_t)$: conditional continuous choice density (CCD) of choosing hours $h$ given state $x_t$ and discrete choice $k$
  - $B_t$: bond price at calendar year $\tau(t)$
  - $A_{T^R+1}(x_{T^R+1}) \equiv 1$

# Model

## Optimal Choices II

- At each age before retirement the optimal discrete choices $\underline{d}_t^o$ maximize:

$$\sum_{k \in C_t} d_{kt} \left[ \overline{u}_k(x_t) - (B_t - 1) \ln \left[ \int \left( \int A_{t+1}(x_{t+1}) g_{kh}(x_{t+1}|x_t) dx_{t+1} \right) q_k(h|x_t) dh \right] + \epsilon_{kt} \right]$$

- The optimal hours $h_t^o$ of a working woman satisfy the FOC for $k \in \{2, 4\}$:

$$\begin{aligned} -\xi_t = \; & \rho w(x_t) \left( 1 + \iota(\pi) \frac{\partial \ell_{2t}}{\partial h_t} \right) \left[ 1 - \frac{(1 - \pi_{2k}^{tax}(x_t)) \pi_{1k}^{tax}(x_t)}{W_k(h_{kt}, x_t)^{\pi_{2k}(x_t)}} \right] + \frac{\partial u_k(h_{kt}, x_t)}{\partial h_t} \\ & - \frac{(B_t - 1)}{\int A_{t+1}(x_{t+1}) g_{kh}(x_{t+1}|x_t) dx_{t+1}} \times \int \left[ \frac{\partial A_{t+1}(x_{t+1})}{\partial h_t} + \frac{A_{t+1}(x_{t+1})}{g_{kh}(x_{t+1}|x_t)} \frac{\partial g_{kh}(x_{t+1}|x_t)}{\partial h_t} \right] g_{kh}(x_{t+1}|x_t) dx_{t+1} \end{aligned}$$

# Identification

## Perfect Foresight Case

- Assume that the data is a long panel:
  - in each policy regime $\pi$ a synthetic panel can be constructed to string together comprehensive histories of the life cycle fertility and labor supply for each demographic group
- Our data is certainly a long panel as we have 50 years of data
- Let $x_{t+1+s}^{(kh,1)}$ for $s = 0, ..., T - t$ be the (perfectly anticipated) value of the state vector at $t + 1 + s$ given:
  - current state $x_t$
  - following the choice of action $k$ and hours $h_{kt}$ in period $t$
  - and the choice of alternative 1 (and hence zero hours) from period $t + 1$ to $t + 1 + s$

# Identification

## Semiparametic Identification

- Let $x_{t+1}^{(kh)}$ (or simply $x_{t+1}^{(k)}$ if $k \in \{1,3\}$) denote the evolution of the state into $t+1$ given discrete choice $k$ and hours choice $h_{kt}$.

- The index of household capital $A_t$ can be written as:

$$A_t\left(x_t\right) = \prod_{s=0}^{T-t}\left[p_{1t+s}(x_{t+s}^{(1)})\Gamma\left(\frac{B_{t+s}+1}{B_{t+s}}\right)^{B_{t+s}}\exp\left\{-\overline{u}_1(x_{t+s}^{(1)})\right\}\right]^{\chi_t(s)}$$

where $\chi_t(s)$ is a cumulative discount factor

- The ex-ante conditional value function of the perfect-foresight version of the problem for choice $k$ is given by:

$$\begin{aligned} V_k\left(x_t\right) &= \overline{u}_k(x_t) - (B_t-1)\sum_{s=0}^{T-t-1}\chi_{t+1}(s)\ln\Gamma\left(\frac{B_{t+1+s}+1}{B_{t+1+s}}\right)^{B_{t+1+s}} \\ &\quad - (B_t-1)\ln\int\left(\prod_{s=0}^{T-t-1}\left[p_{1t+1+s}(x_{t+1+s}^{(kh,1)})\exp\left\{-\overline{u}_1(x_{t+1+s}^{(kh,1)})\right\}\right]^{\chi_{t+1}(s)}\right)q_k(h|x_t)dh \end{aligned}$$

# Identification

## Observational Equivalence

- For $k \in \{2,3,4\}$ the log-odds ratio relative to alternative $k = 1$ can be written as:

$$
\begin{aligned}
\ln\left(\frac{p_{kt}(x_t)}{p_{1t}(x_t)}\right) =& \overline{u}_k(x_t) - \overline{u}_1(x_t) \\
& - (B_t - 1)\ln\int\left(\prod_{s=0}^{T-t-1}\left[p_{1t+1+s}(x_{t+1+s}^{(kh,1)})\exp\left\{-\overline{u}_1(x_{t+1+s}^{(kh,1)})\right\}\right]^{\chi_{t+1}(s)}\right) q_k(h|x_t)dh \\
& + (B_t - 1)\sum_{s=0}^{T-t-1}\chi_{t+1}(s)\left[\ln p_{1t+1+s}(x_{t+1+s}^{(1,1)}) - \overline{u}_1(x_{t+1+s}^{(1,1)})\right]
\end{aligned}
$$

- Therefore $\overline{u}_k(x_t)$ is identified up to the normalizing constant defined as:

$$
\begin{aligned}
& \overline{u}_1(x_t) + (B_t - 1)\ln\int\left(\prod_{s=0}^{T-t-1}\left[p_{1t+1+s}(x_{t+1+s}^{(kh,1)})\exp\left\{-\overline{u}_1(x_{t+1+s}^{(kh,1)})\right\}\right]^{\chi_{t+1}(s)}\right) q_k(h|x_t)dh \\
& + (B_t - 1)\sum_{s=0}^{T-t-1}\chi_{t+1}(s)\overline{u}_1(x_{t+1+s}^{(1,1)})
\end{aligned}
$$

# Identification

## Marginal Utility

- Decompose the observed state variables into pure demand shifters $z_t$ and the rest by letting $x_t = (\widetilde{x}_t, z_t)$.

- Assuming there exists at least one demand side instrument $\widetilde{z}_t$ with at least two points in its support $\widetilde{z}_1, \widetilde{z}_2$:

$$\frac{\partial u_k\left(h_{kt}, \widetilde{x}_t\right)}{\partial h_t} + \rho w(x_t)\left(1 + \iota(\pi)\frac{\partial \ell_{2t}}{\partial h_t}\right)\left[1 - \frac{(1-\pi_{2k}^{tax}(x_t))\pi_{1k}^{tax}(x_t)}{W_k(h_{kt}, x_t)^{\pi_{2k}(x_t)}}\right] =$$

$$(B_t - 1)\sum_{s=0}^{T-t-1}\chi_{t+1}(s)E\left[\frac{1}{p_{1t+1+s}(x_{t+1+s}^{(kh,1)})}\frac{\partial p_{1t+1+s}(x_{t+1+s}^{(kh,1)})}{\partial h_t} - \frac{\partial \overline{u}_1(x_{t+1+s}^{(kh,1)})}{\partial h_t}\middle| \widetilde{z}_j, \widetilde{x}_t, h_{kt}\right]$$

  for $j = 1, 2$

- There are two equations with two unknowns.
  - Therefore for every value $h_{kt}$ of hours worked observed in the data and every $\widetilde{x}_t$, then $\frac{\partial u_k\left(h_{kt}, \widetilde{x}_t\right)}{\partial h_t}$ and $\rho$ are identified.

# Identification

## Utility Level

- For $k \in \{2,3,4\}$ and any $\widetilde{h}$, current utility $u_k(\widetilde{h}, \widetilde{x})$ can be expressed as:

$$
\begin{aligned}
u_k(\widetilde{h}, \widetilde{x}_t) &= \int \left\{ u_k(h^o, \widetilde{x}) + \int_{h^o}^{\widetilde{h}} \frac{\partial u_k(h, \widetilde{x}_t)}{\partial h} dh \right\} q_k(h^o | \widetilde{x}_t) dh^o \\
&= E\left[ u_k\left(h^o, \widetilde{x}_t\right) | \widetilde{x}_t \right] + \int \left[ \int_{h^o}^{\widetilde{h}} \frac{\partial u_k\left(h, \widetilde{x}_t\right)}{\partial h} dh \right] q_k(h^o | \widetilde{x}_t) dh^o
\end{aligned}
$$

- From our definition of $\overline{u}_k$:

$$
\begin{aligned}
E\left[ u_k\left(h^o, \widetilde{x}_t\right) | \widetilde{x}_t \right] = \int \Big\{ & \overline{u}_k\left(x_t\right) - E\left[ h_k\left(x_t, \xi_t\right) \xi_t | x_t \right] \\
& - \rho E[\Upsilon_k(h_k\left(x_t, \xi_t\right), x_t) | x_t] \Big\} dF\left(z_t | \widetilde{x}_t\right)
\end{aligned}
$$

- Therefore $u_k(h_t, \widetilde{x}_t)$ is identified since $\overline{u}_k(x_t)$, $\frac{\partial u_k(h, \widetilde{x}_t)}{\partial h}$, and $\rho$ are identified, and $\Upsilon_k$ is known.

# Estimator

## Outline

- Estimation of the model proceeds in three stages
- **First stage**: we estimate
  1. conditional choice probabilities (CCPs)
  2. conditional continuous choice density functions (CCDs)
  3. the transition functions of partnership dynamics
  4. the wage and earnings equations,
  5. the tax-transfer functions.
  6. the child-nurturing equation

  These first-stage estimates account for the quasi-experimental variation across states and over time
- **Second stage**: we combine the Euler equation of optimal hours worked with the first-stage estimates and with demand-side instruments (i.e. policy variation) to estimate the marginal utility of leisure
- **Third stage**: we use a pseudo-maximum likelihood estimator to estimate the remaining parameters of the model

# First Stage Estimates

## CCPs Fit over the Life Cycle

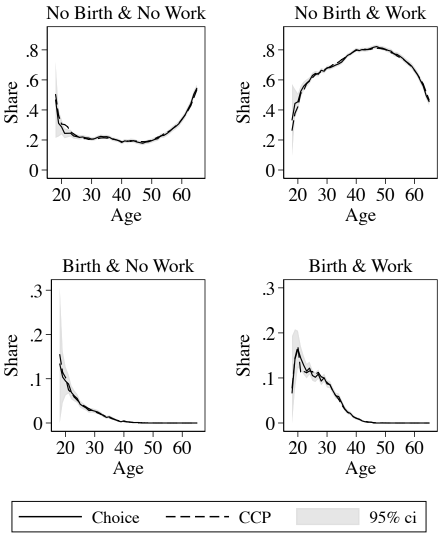


Figure: Conditional Choice Probabilities over the Life Cycle

# First Stage Estimates

## CCPs Fit over Time

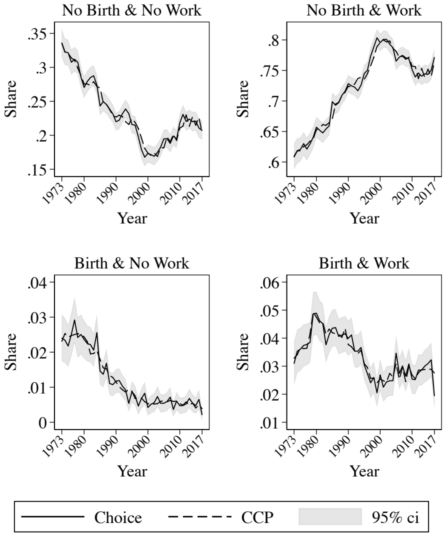


Figure: Conditional Choice Probabilities over Time

# First Stage Estimates

## CCPs and Leave Policy

Table: CCPs by Leave Regime

| | Shares | | | Mean CCPs | | |
|---|---|---|---|---|---|---|
| Choice | No Leave | Only Protected | Paid Available | No Leave | Only Protected | Paid Available |
| No work, no birth | 0.274 | 0.206 | 0.214 | 0.275 | 0.205 | 0.214 |
| Work, no birth | 0.667 | 0.759 | 0.739 | 0.666 | 0.760 | 0.739 |
| No work, birth | 0.019 | 0.006 | 0.010 | 0.019 | 0.007 | 0.010 |
| Work, birth | 0.040 | 0.029 | 0.037 | 0.040 | 0.028 | 0.037 |

Notes: we do not include leave regime indicators in the CCPs. Instead we control for the features of each policy (eligibility, generosity, reimbursement). Each leave regime groups several policies observed in the data.

# First Stage Estimates

## CCPs and Tax Policy

Table: CCPs by Tax Progressivity

| Choice | Shares | | | Mean CCPs | | |
|---|---|---|---|---|---|---|
| | Low | Medium | High | Low | Medium | High |
| No work, no birth | 0.207 | 0.241 | 0.271 | 0.207 | 0.240 | 0.276 |
| Work, no birth | 0.757 | 0.709 | 0.678 | 0.756 | 0.710 | 0.671 |
| No work, birth | 0.007 | 0.013 | 0.020 | 0.007 | 0.013 | 0.020 |
| Work, birth | 0.030 | 0.036 | 0.031 | 0.030 | 0.036 | 0.033 |

Notes: We include all features of the tax policies in the CCPs, including their progressivity.

# First Stage Estimates

## CCDs Fit Overall

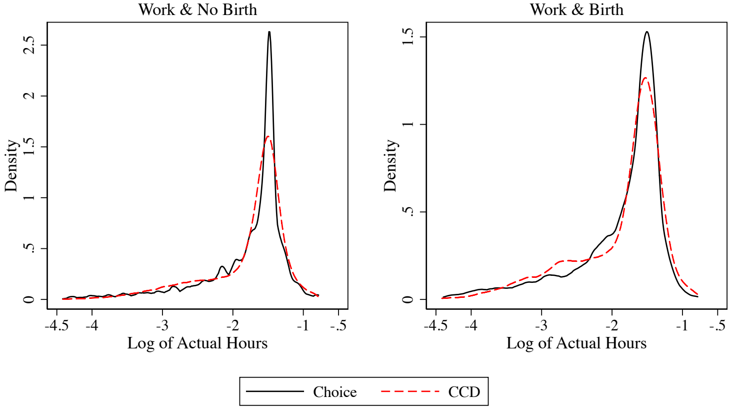


Figure: Conditional Continuous Choice Densities

# First Stage Estimates

## CCDs Fit over the Life Cycle

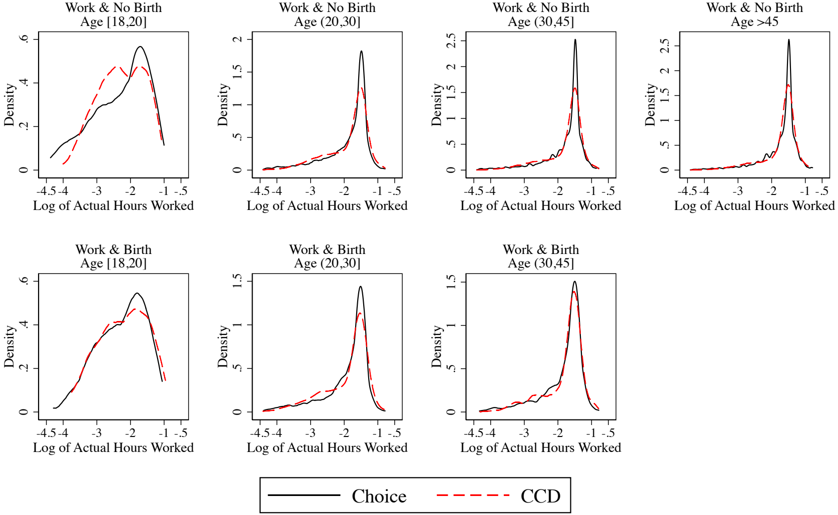


Figure: CCDs over the Life Cycle

# First Stage Estimates

## CCDs and Leave Policy

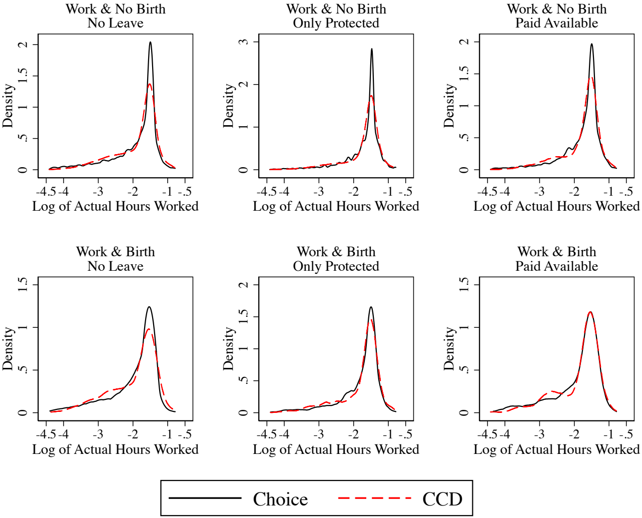


Figure: CCDs by Leave Regime

# First Stage Estimates

## CCDs and Leave Policy

Table: Average Hours and CCDs by Leave Regime

| | Mean Hours (Weighted) | | | Mean CCDs | | |
|---|---|---|---|---|---|---|
| Choice | No Leave | Only Protected | Paid Available | No Leave | Only Protected | Paid Available |
| Work, no birth | 0.176 | 0.199 | 0.187 | 0.176 | 0.199 | 0.188 |
| Work, birth | 0.165 | 0.187 | 0.175 | 0.165 | 0.189 | 0.178 |

Notes: Hours are weighted by the total amount of hours in a year (365*24). A full-time job is approximately 0.237 weighted hours. CCDs are obtained from a mixture of two normals. We do not include leave regime indicators in the CCDs. Instead we control for the features of each policy (eligibility, generosity, reimbursement). Each leave regime groups several policies observed in the data.

# First Stage Estimates

## CCDs and Tax Policy

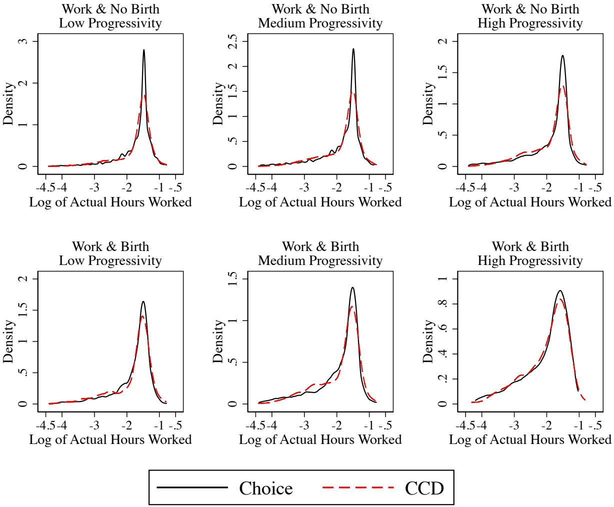


Figure: CCDs by Progressivity of Tax Regime

# First Stage Estimates

## CCDs and Tax Policy

Table: Average Hours and CCDs by Progressivity of Tax Regime

| | Mean Hours (Weighted) | | | Mean CCDs | | |
|---|---|---|---|---|---|---|
| Choice | Low | Medium | High | Low | Medium | High |
| Work, no birth | 0.201 | 0.184 | 0.177 | 0.200 | 0.185 | 0.177 |
| Work, birth | 0.188 | 0.172 | 0.160 | 0.189 | 0.173 | 0.160 |

Notes: Hours are weighted by the total amount of hours in a year (365*24). A full-time job is approximately 0.237 weighted hours. CCDs are obtained from a mixture of two normals. Each tax regime groups several policies observed in the data.

# First Stage Estimates

## CCDs and Labor Attachment

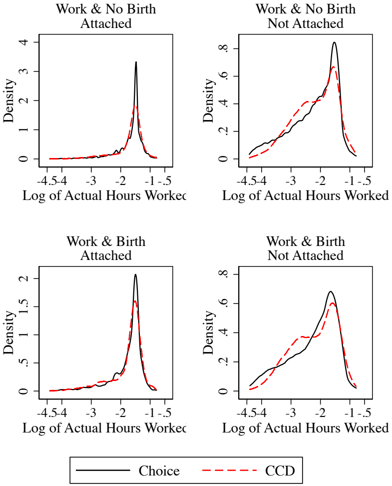


Figure: CCDs by Labor Attachment

Notes: Labor attachment ≡ labor participation in all of the last four years.

# First Stage Estimates

## CCDs and Fertility

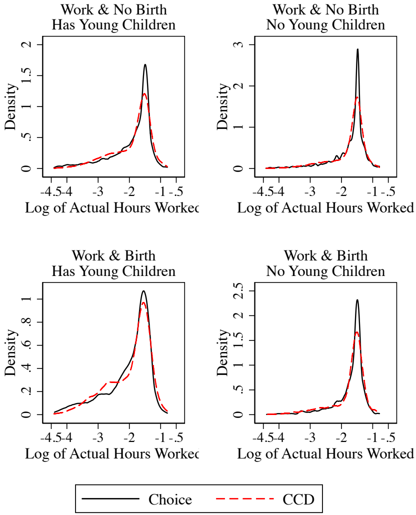


Figure: CCDs by Whether Woman Has Young Children

# First Stage Estimates

## CCDs and Partnership Status

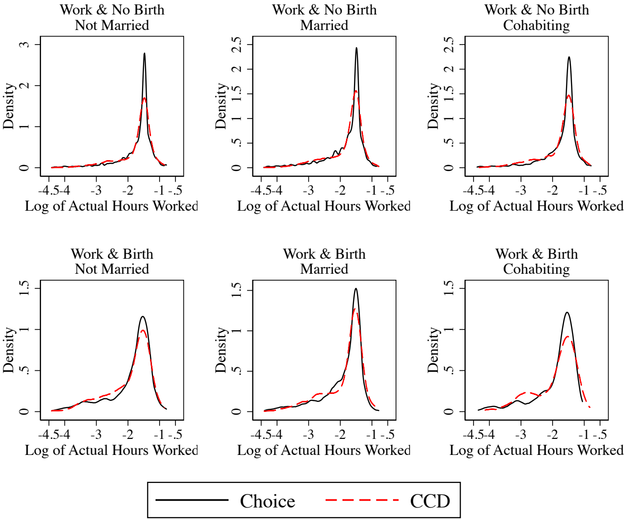


Figure: CCDs by Partnership Status

# Model: Motherhood Career Penalty