Multidimensional matching and labor market complementarity[1]

Esben Scriver Andersen[*] Young Jun Lee[**]

[*]Australian National University

[**]Università Bocconi

December 16, 2022

[1]This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 740369).

Contributions

Proposes an empirical framework for two-sided matching in the labor market.

Contributions

Proposes an empirical framework for two-sided matching in the labor market.

This paper contribute to the existing literature by

1. Showing existence and uniqueness of equilibrium without restricting alternative to be gross substitute.

Contributions

Proposes an empirical framework for two-sided matching in the labor market.

This paper contribute to the existing literature by

1. Showing existence and uniqueness of equilibrium without restricting alternative to be gross substitute.
2. Allowing matching to be multidimensional.

Contributions

Proposes an empirical framework for two-sided matching in the labor market.

This paper contribute to the existing literature by

1. Showing existence and uniqueness of equilibrium without restricting alternative to be gross substitute.
2. Allowing matching to be multidimensional.
3. Empirical finding: workers of different educational type can be complements.

Behavioural model

- Perturbed utility model (PUM)

Behavioural model

- Perturbed utility model (PUM)

$$\text{ARUM: } p_i(v) = Pr\left(\arg\max_j v_j + \varepsilon_j = i\right)$$

$$\text{PUM: } p(v) = \arg\max_{q \in \Delta^J} \sum_j q_j v_j + G(q)$$

Behavioural model

- Perturbed utility model (PUM)

$$\text{ARUM: } p_i(v) = Pr\left(\arg\max_j v_j + \varepsilon_j = i\right)$$

$$\text{PUM: } p(v) = \arg\max_{q\in\Delta^J} \sum_j q_j v_j + G(q)$$

$$\text{ARUM: } \varepsilon_j \sim \text{EV} \Rightarrow p_i(v) = \frac{\exp(v_i)}{\sum_j \exp(v_j)}$$

Behavioural model

- Perturbed utility model (PUM)

$$\text{ARUM: } p_i(v) = Pr\left(\arg\max_j v_j + \varepsilon_j = i\right)$$

$$\text{PUM: } p(v) = \arg\max_{q\in\Delta^J} \sum_j q_j v_j + G(q)$$

$$\text{ARUM: } \varepsilon_j \sim \text{EV} \Rightarrow p_i(v) = \frac{\exp(v_i)}{\sum_j \exp(v_j)}$$

$$\text{PUM: } G(q) = -\sum_j q_j \log q_j \Rightarrow p_i(v) = \frac{\exp(v_i)}{\sum_j \exp(v_j)}$$

Dimensions of matching market

1. Type of workers: $x \in \mathcal{X} = \{1, \dots, |\mathcal{X}|\}$,
2. Type of firms: $y \in \mathcal{Y} = \{1, \dots, |\mathcal{Y}|\}$.
3. Type of occupations: $z \in \mathcal{Z} = \{1, \dots, |\mathcal{Z}|\}$

Worker's problem

The worker of type x can either choose a pair of firm and occupation types $(y, z) \in \mathcal{Y} \times \mathcal{Z}$ or choose to become unemployed

Worker's problem

The worker of type x can either choose a pair of firm and occupation types $(y, z) \in \mathcal{Y} \times \mathcal{Z}$ or choose to become unemployed

The worker base its choice on the vector of optimal choice probabilities, $p^x\left(W_{x\cdot\cdot}\right) \in \triangle^{|\mathcal{Y}||\mathcal{Z}|+1}$, that maximize the worker's perturbed utility

$$G_x^*\left(U_x\left(W_{x\cdot\cdot}\right)\right) = \max_{p \in \triangle^{|\mathcal{Y}||\mathcal{Z}|+1}} \left\{ \sum_{y,z} p_{yz} u_{xyz}\left(w_{xyz}\right) + p_0 u_{x0}\left(w_{x0}\right) + G_x\left(p\right) \right\},$$

Firm's problem

The firm of type y chooses how many workers of each type to employ and how to allocate them across occupations, $(n^y_{xz})_{(x,z)\in\mathcal{X}\times\mathcal{Z}}$:

$$\max_{\left(n^y_{xz}\right)_{(x,z)\in\mathcal{X}\times\mathcal{Z}}} \left\{ F_y\left((n^y_{xz})_{(x,z)\in\mathcal{X}\times\mathcal{Z}}, r_y \right) - \sum_{x,z} w_{xyz} n^y_{xz} \right\}$$

Firm's problem

The firm of type y chooses how many workers of each type to employ and how to allocate them across occupations, $(n^y_{xz})_{(x,z)\in\mathcal{X}\times\mathcal{Z}}$:

$$\max_{(n^y_{xz})_{(x,z)\in\mathcal{X}\times\mathcal{Z}}} \left\{ F_y\left((n^y_{xz})_{(x,z)\in\mathcal{X}\times\mathcal{Z}}, r_y \right) - \sum_{x,z} w_{xyz} n^y_{xz} \right\}$$

Assumption 1

(a) F_y *exhibits constant returns to scale*

(b) $r_y = c_y \left(N_y - \sum_{x,z} n^y_{xz} \right)$*: Endogenous scarce managerial resource*

Firm's problem

The firm of type y chooses how many workers of each type to employ and how to allocate them across occupations, $(n^y_{xz})_{(x,z)\in\mathcal{X}\times\mathcal{Z}}$:

$$\max_{\left(n^y_{xz}\right)_{(x,z)\in\mathcal{X}\times\mathcal{Z}}} \left\{ F_y\left((n^y_{xz})_{(x,z)\in\mathcal{X}\times\mathcal{Z}}, r_y \right) - \sum_{x,z} w_{xyz} n^y_{xz} \right\}$$

Assumption 1

(a) F_y *exhibits constant returns to scale*

(b) $r_y = c_y \left(N_y - \sum_{x,z} n^y_{xz} \right)$*: Endogenous scarce managerial resource*

The firm of type y chooses its optimal vector of input composition, $q^y(W_{\cdot y \cdot}) \in \triangle^{|\mathcal{X}||\mathcal{Z}|+1}$

$$\bar{F}^*_y\left(W_{\cdot y \cdot}\right) = \max_{q\in\triangle^{|\mathcal{X}||\mathcal{Z}|+1}} \left\{ \bar{F}_y\left(q\right) - \sum_{x,z} w_{xyz} q_{xz} \right\}$$

Equilibrium

- Workers' and firms' choices
  1. Labor supply of match of type (x, y, z): $N_x \times p^x_{yz}(W_{x,\cdot\cdot})$,
  2. Labor demand for match of type (x, y, z): $N_y \times q^y_{xz}(W_{\cdot y \cdot})$.

Equilibrium

- Workers' and firms' choices
  1. Labor supply of match of type (x, y, z): $N_x \times p^x_{yz}(W_{x,\cdot\cdot})$,
  2. Labor demand for match of type (x, y, z): $N_y \times q^y_{xz}(W_{\cdot y \cdot})$.

- Equilibrium outcomes (μ^*, W^*)

$$\mu^*_{xyz} = N_x \times p^x_{yz}(W^*_{x\cdot\cdot}) = N_y \times q^y_{xz}(W^*_{\cdot y \cdot}), \qquad \forall\, (x, y, z) \in \mathcal{X} \times \mathcal{Y} \times \mathcal{Z}$$

Equilibrium

- Workers' and firms' choices
  1. Labor supply of match of type (x, y, z): $N_x \times p^x_{yz}(W_{x,\cdot\cdot})$,
  2. Labor demand for match of type (x, y, z): $N_y \times q^y_{xz}(W_{\cdot y \cdot})$.

- Equilibrium outcomes (μ^*, W^*)

$$\mu^*_{xyz} = N_x \times p^x_{yz}(W^*_{x\cdot\cdot}) = N_y \times q^y_{xz}(W^*_{\cdot y \cdot}), \qquad \forall (x, y, z) \in \mathcal{X} \times \mathcal{Y} \times \mathcal{Z}$$

Theorem 1

If $u_{xyz}(\cdot)$ is linear in w_{xyz} for all (x, y, z), an equilibrium (μ^*, W^*) uniquely exists.

Empirical application

Firm y: $\max_{q \in \triangle^{|\mathcal{X}||\mathcal{Z}|+1}} \left\{ \bar{F}_y(q) + \sum_{x,z} \pi_{xyz} q_{xz} - \sum_{x,z} w_{xyz} q_{xz} \right\}$

- $\bar{F}_y(q)$: similarity function (see Fosgerau and Nielsen (2021))

$$\begin{aligned}\bar{F}_y(q) &= -\sum_{x,z} q_{xz} m_{xz}(q^y) \\ &= -\sum_{x,z} q_{xz} \log(q_{xz}) + \lambda_{\mathcal{Z}} \sum_{x,z} q_{xz} \log\left(\frac{q_{xz}}{\sum_{z'} q_{xz'}}\right) + \lambda_{\mathcal{X}} \sum_{x,z} q_{xz} \log\left(\frac{q_{xz}}{\sum_{x'} q_{x'z}}\right)\end{aligned}$$

Counterfactual changes

	Logit model				Similarity model			
	Unskilled	Skilled	Medium	High	Unskilled	Skilled	Medium	High
Management	-0.00	-0.01	-0.01	-0.00	-0.21	-0.58	-0.69	-1.13
High	-0.00	-0.00	-0.01	2.21	0.36	0.59	1.94	5.49
Medium	-0.00	-0.01	-0.01	-0.00	-0.20	-0.57	-0.80	-0.85
Basic	-0.06	-0.11	-0.01	-0.00	0.06	0.28	-0.17	-0.58
Other	-0.02	-0.01	-0.00	-0.00	0.03	0.05	-0.02	-0.09
Missing	-0.01	-0.01	-0.00	-0.00	-0.23	-0.18	-0.12	-0.36

Table: Excess demand in the manufacturing sector, 1,000

Conclusion

We propose an empirical framework for two-sided matching in the labor market.

- allow alternatives to be complements.
- allow multidimensional matching
- find that complementarity is empirical relevant.

References

FOSGERAU, M. and NIELSEN, N. (2021). Similarity, perturbed utility and discrete choice.