Search and Price Formation with Incomplete Information

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DSE Canberra, Dec 2022

Introduction

- Large, influential literature on search in dynamic general equilibrium models.
- Two questions remain largely unexplored:

1. How important is (in)complete information?
    - Growing evidence that complete information cannot explain real-world bargaining: Backus et al. (2020); Larsen (2021); Byrne et al. (2022)
    - Myerson and Satterthwaite (1983) rules out efficient trade in many of these settings.
2. How do multiple trading mechanisms affect search outcomes?
    - Examples: government procurement, financial markets, housing, online markets.
    - Mechanisms differ in trade probabilities, total surplus, and surplus division.

Implications for search outcomes

- Both assumptions interact with search in important ways.
- Trade efficiency affects search cost and duration.
  - Consider fitting model parameters to search outcomes, like time-on-market.
  - Lower efficiency $\rightarrow$ lower trade probability $\rightarrow$ more matches required $\rightarrow$ greater search intensity.
- Agents will shift across multiple mechanisms in response to changing economic conditions.

Our model

- We provide a framework to jointly study two-sided incomplete information and multiple trading mechanisms in a Diamond-Mortensen-Pissarides model.
- Agents search in either a bargaining mechanism or an auction mechanism, both featuring two-sided incomplete information and mechanism-specific search costs.
- Payoffs are driven by market tightness, or the buyer-to-seller ratio at each mechanism.
- Buyers and sellers sort into markets to satisfy a mechanism indifference condition.
- We estimate the model using housing transaction data from Sydney featuring both auctions and negotiations.
    - Valuation distributions and matching processes are identified from auction data.
    - Search parameters and shock processes are estimated from dynamic equilibrium conditions.

Mechanism models - preliminaries

- A set of $n \geq 1$ buyers attempt to trade with a seller.
- Buyer i has valuation v_i and an outside option $\mathcal{V}^B$ shared by all buyers.
- Assume $v_i - \mathcal{V}^B$ is i.i.d from a distribution F.
- Seller has valuation c and an outside option $\mathcal{V}^S$.
- Assume $c - \mathcal{V}^S$ is drawn from a distribution G independent from buyers' valuations.

Second-best under incomplete information

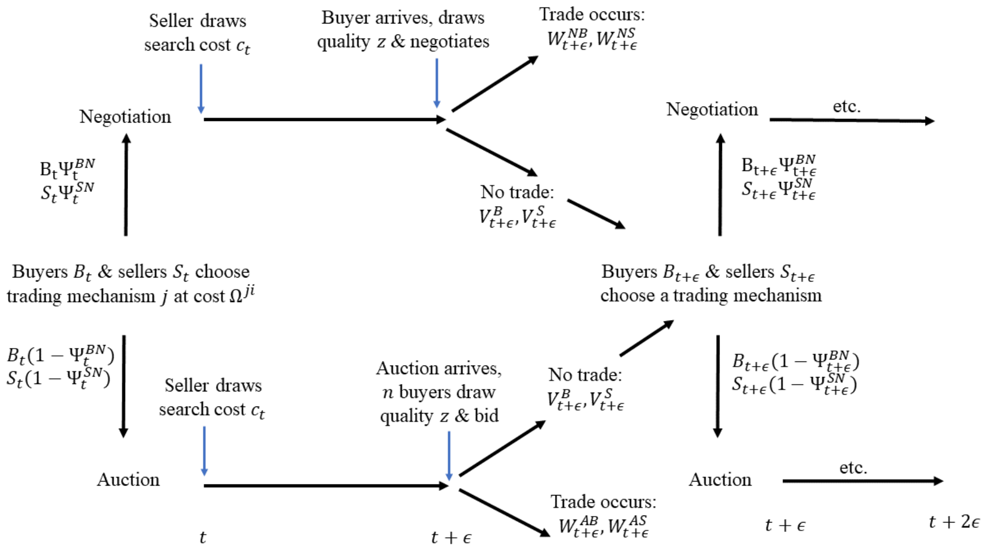
- We consider a mechanism design approach to bargaining.
- Myerson and Satterthwaite (1983) establish that ex-post efficient trade is impossible when agents have private information.
- They also characterize the second-best outcome (MS mechanism).
    - Direct mechanism maximizing total surplus while maintaining budget balance.
- Our framework assumes that the "rules of bargaining," as governed by regulation, contract, or social norms, implement the best expected outcome.
- Let $\mathcal{W}^{ji}$ be the expected payoff conditional on trade for $j \in \{N, A\}$ and $i \in \{B, S\}$.

▸ Mechanism details

Auction: second-price sealed-bid

- A seller with valuation c sets a reserve price R that solves $R = c + \frac{1-F(R)}{f(R)}$.
- The auction results in a sale if the highest buyer valuation exceeds the reserve price.
- Buyer i wins the auction if $v_i > \max\{v^{(n-1)}, R(c)\}$ and makes a payment of $P^A(\mathbf{v}, c) = \max\{v^{(n-1)}, R(c)\}$ to the seller.
- Let $\mathcal{W}^{ji,n}$ be the expected payoff conditional on trade for $j \in \{N, A\}$ and $i \in \{B, S\}$ when there are n bidders.

Model overview



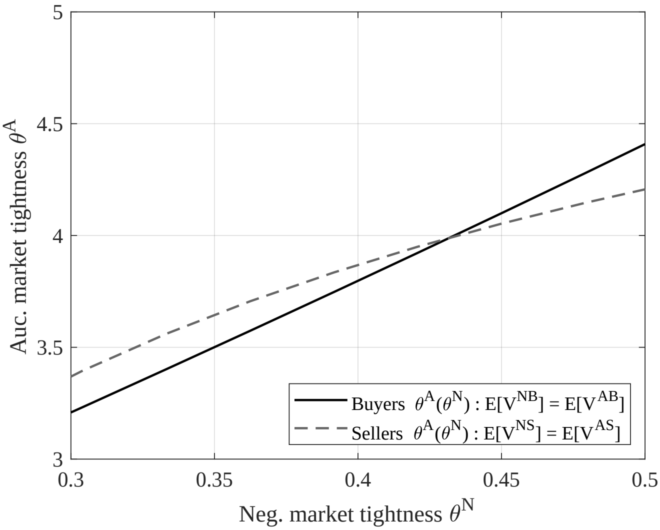
Institutional setting - Sydney housing market

- We apply the model to the greater Sydney metro area housing market.
- Housing is the largest asset held on most household balance sheets.
  - The Sydney housing market is estimated to be worth AUD$10 trillion.
- Homes are sold by bilateral negotiation and auction, both regulated under NSW law.
- Data from 14,482 auctions from large Sydney auction firm.
- All auctions are English auctions in which the seller can set a binding reserve price.
- Combined with transaction data for all properties in Sydney from 2011 - 2016.

Estimation

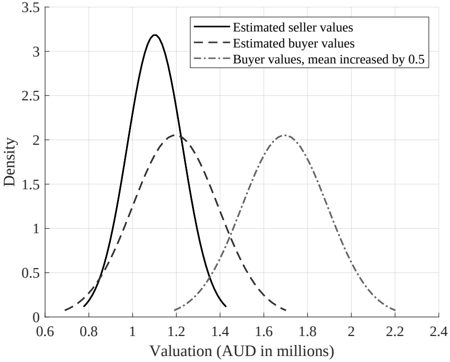
1. Estimate primitives of the transaction mechanisms using microdata. ▸ Details
   - Use structural econometric auction methods to estimate F and G.
   - Estimate arrival process of buyers to auction using observed N.
   - Using estimated $\hat{F}, \hat{G}$, estimate MS efficiency parameter $\hat{\eta}$, and then the arrival process at negotiation to match observed seller time-on-market.
2. Solve the full model for flow payoffs/search costs and other dynamic parameters. ▸ Details
   - Generate functional approximations for endogenous variables using micro estimates.
   - Solve for steady state flow utility parameters $\{\Omega^{ji}\}$, buyer and seller mass (B, S), and mechanism choice probabilities (Ψ^{BN}, Ψ^{SN}) using perturbation methods.
   - Estimate variance and persistence of shocks using Simulated Method of Moments.

Steady-state visualization

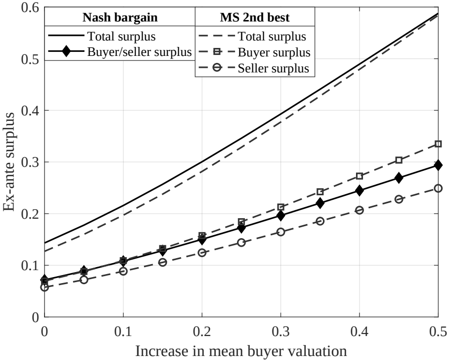


Results - valuation distributions and negotiation surplus

(a): Estimated valuation distributions and shift



(b): Negotiation ex-ante surplus



Steady State Parameterization

Table: **Models with Competing Auctions: Inc. Info vs Nash**

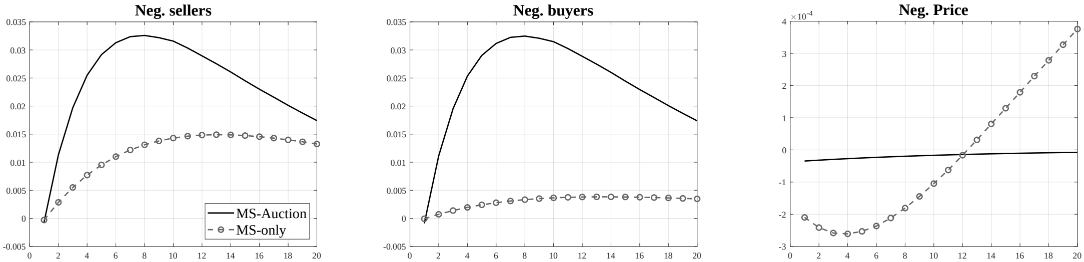
Model-Solution Parameters	MS	Nash
Auc. Sell. Search Cost ($\overline{\Omega}^{AS}$)	1.97%	1.97%
Neg. Sell. Search Cost ($\overline{\Omega}^{NS}$)	1.45%	1.08%
Auc. Buy. Search Cost ($\overline{\Omega}^{AB}$)	0.15%	0.13%
Neg. Buy. Search Cost ($\overline{\Omega}^{NB}$)	0.45%	0.46%
Buyer-to-seller ratio	4.58	3.52
Neg. buyer mass (Ψ^{BN})	0.64	0.56
Neg. seller mass (Ψ^{SN})	0.72	0.76

Notes: Costs of search are reported as percentage of the mean price for model. The flow utility is reported as a percentage of the mean price at auction.

▸ Shock estimates

Effect of two mechanisms: moving shock

- We turn off auctions and examine dynamic responses to a moving rate shock.
- Owners lose matches at higher rate → increase in total buyers and sellers.



Conclusion

- We study the effect of incomplete information and multiple mechanisms on search and price formation.
- Our results demonstrate the importance of trade mechanism efficiency in interpreting the role of search frictions in price formation.

Future work:

- Policy analysis: analyze the impact that multiple mechanisms have on policy outcomes.
  - Taxes: kinked tax schedule may drive agents away from high-price mechanism.
  - Information disclosure: how does the market respond to more precise signals of agents' private valuations?

APPENDIX SLIDES

Nash bargaining

- Nash bargaining is an efficient trading mechanism that assumes complete information and implements the first best outcome of ex-post efficient trade.
- Buyer bargaining weight $\psi \in [0,1]$, with allocation rule given by

$$Q^E(v,c) = \begin{cases} 1 & \text{if } v \geq c \\ 0 & \text{otherwise.} \end{cases}$$

- The price conditional on trade occurring is given by $P^E(v,c) = \psi v + (1-\psi)c$. Buyer and seller surplus are given by

$$\mathcal{W}^{EB,n} = \mathsf{Pr}(Q^E = 1|n) \cdot \mathbb{E}[v - P^E(v,c) \mid Q^E = 1, n] + \mathcal{V}^B$$

$$\mathcal{W}^{ES,n} = \mathsf{Pr}(Q^E = 1|n) \cdot \mathbb{E}[P^E(v,c) - c \mid Q^E(v,c) = 1, n] + \mathcal{V}^S$$

MS mechanism

- Define the a-weighted virtual type functions:

$$\Phi^a(v) = v - (1-a)\frac{1-F(v)}{f(v)}, \qquad \Gamma^a(c) = c + (1-a)\frac{G(c)}{g(c)}$$

- The allocation rule is given by

$$Q^\eta(v,c) = \begin{cases} 1 & \text{if } \Gamma^{1/\eta}(c) \leq \Phi^{1/\eta}(v) \\ 0 & \text{otherwise.} \end{cases}$$

where η captures distortion away from the first-best due to budget balance.

- Payoffs for buyers and sellers for allocation Q^N with optimally chosen ρ given by

$$\mathcal{W}^{BN} = \mathbb{E}[v - \Psi^0(v) \mid Q^N(v,c) = 1, n] + \mathcal{V}^B,$$

$$\mathcal{W}^{SN} = \mathbb{E}[\Gamma^0(c) - c \mid Q^N(v,c) = 1] + \mathcal{V}^S.$$

Go back

Estimation details - mechanisms

- Assume normal distribution for valuations for $i \in \{B, S\}$ at auction k:

$$V_k^i \sim \mathcal{N}\left(\zeta_\mu^i X_k^\mu + \alpha_\mu^i \eta_k,\ \zeta_\sigma^i X_k^\sigma + \alpha_\sigma^i \eta_k\right)$$

- Estimate unobserved housing quality η_k following Roberts (2013): $\underline{R} = m(\eta; X)$ for m known and strictly increasing in η, where $\underline{R}$ is seller commitment price.

- Assume the number of bidders is governed by finite Poisson mixture:

$$\gamma_n^A\left(\theta^A, \epsilon\right) = \sum_{i=1}^{I} w_i \frac{(c_i \theta^A \epsilon)^n e^{-c_i \theta^A \epsilon}}{n!},$$

  estimated using EM algorithm.

- Estimate per-week probability of sale in negotiation from transaction census data, and match arrival distribution of buyers to this probability. Go back

Estimation details - dynamics

- Parameters governing dynamics of shocks estimated via SMM
- Use weekly linearly detrended data (2005:I to 2016:XII) on auction price, negotiation price, negotiation TOM, auction sales share, and auction clearance rate
- All covariances and autocovariances up to a 4-week lag are used (75 moments) to estimate 8 structural parameters
- Use a 3-step Newey West estimator that solves $\widehat{\alpha} = \arg\min_{\alpha \in \mathcal{A}} \|M(\alpha|Y)\|_{\mathbf{W}_T}$, where $\mathbf{M}(\alpha|Y) := \frac{1}{sT-b}\sum_{t=1+b}^{sT} m_t(\alpha) - \frac{1}{T}\sum_{t=1}^{T} m_t(Y)$
- Other parameters (discount factor, probability auction held, probability buyer searchers, mobility rates) are calibrated to match long-run average mortgage rates, seller auction holding period, buyer TOM, and mobility data. Go back

Shock Parameter Estimates

Table: Shock Parameter Estimates

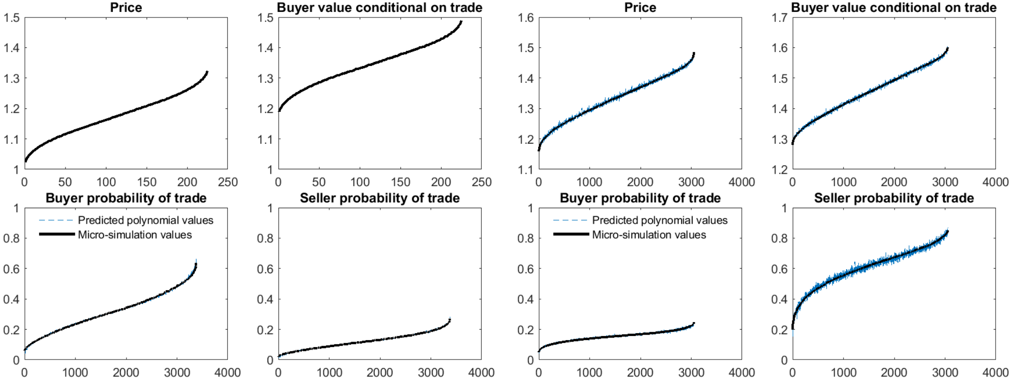
Parameter	Estimate	Parameter	Estimate
Flow utility shock (ρ_{r^H})	0.0232	Flow utility shock (σ_{r^H})	0.0336
	(0.0003)		(0.0008)
Moving shock (ρ_{α^m})	0.0769	AS search cost ($\sigma_{\Theta^{AS}}$)	1.9293×10^{-6}
	(0.0475)		(1.8401×10^{-6})
Discount factor shock (ρ_β)	0.9822	AP meas. error (σ_A)	0.0134
	(0.0450)		(0.0134)
Moving shock (σ_{α^m})	3.408×10^{-5}	NP meas. error (σ_N)	0.0253
	(1.3608×10^{-5})		(0.0253)
Discount factor shock (σ_β)	2.7998×10^{-5}		
	(3.784×10^{-5})		

Go back

Model fit - polynomial approximation

(a): Myerson–-Satterthwaite Mechanism Polynomial Approximation

(b): Auction Mechanism Polynomial Approximation



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