Entry, Exit, and Technological Progress in Markov-Perfect Duopoly

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- Characterize and compute Markov-perfect equilibria in dynamic oligopoly game with
  - Sunk costs of entry,
  - Stochastic firm-specific technological progress,
  - Stochastic aggregate profitability, and
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    - Firms make exit decisions simultaneously with potentially mixed strategies.
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- Enabling insight, *Fake Backwards Induction* (FBI)
  - The game has no terminal nodes and is not a directed game.
  - We know that firms' expected payoffs equal zero whenever they both choose a positive probability of exit. These are *fake terminal nodes*.
  - We use backward induction to calculate equilibrium payoffs beginning with the fake terminal nodes.
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- Results:
  - Equilibrium existence and uniqueness
  - Fast equilibrium calculation using *contraction mappings*

Why bother?

- Oligopolies create and implement technological change.
- Tradeoffs relative to Ericson and Pakes (1995) and Doraszelski and Satterthwaite (2010).
  - Cost, technological progress depends only on continuation, not investment.
  - Benefit, equilibrium uniqueness and fast computation.
- Tradeoffs relative to Abbring, Campbell, Tilly and Yang (2018)
  - Cost, limited to duopolies.
  - Benefit, persistent post-entry heterogeneity.
- Complementarity with Ericson and Pakes (1995) and Doaszelski and Satterthwaite (2010): Use as a starting point for calculation using homotopy methods (de Vos, 2022).

Model Primitives

- Discrete time t, with each period subdivided into game stages.
- Firms discount future payoffs with $\beta \in [0, 1)$.
- Zero, one or two firms serve the market at any one time.
- $K_t \in \mathcal{K} \equiv \{1, 2, \ldots, \check{k}\}$ is the firm of interest's technology type. $X_t \in \{0\} \cup \mathcal{K}$ the its rival's type.
- Period t payoffs from production, $\pi(k, x, Y_t)$.
- $Y_t \in \mathcal{Y}$ is the exogenous state. This follows a Markov process.
- After production, potential entrants sequentially choose between staying out (zero payoff) or paying the sunk cost $\varphi(x, Y_t)$. Entrants start with $k = 1$. Entry cannot bring the number of firms above two.
- After entry, incumbents and new entrants make simultaneous continuation decisions. Exiting firms receive zero payoff, and continuation costs $\kappa(Y_t) \geq 0$.
- Any active firms draw new values of k and x from identical independent Markov processes with transition matrix Π.

Economically-Relevant Assumptions

Assumption (Monotone and Bounded Profits)

1. *For all* $k \in \{1, \ldots, \check{k} - 1\}$, *all* $x \in \{0\} \cup \mathbb{K}$, *and all* $y \in \mathcal{Y}$, $\pi(k, x, y) \leq \pi(k + 1, x, y)$;
2. *for all* $(k, x) \in \mathbb{K} \times \mathbb{K}$ *and all* $y \in \mathcal{Y}$, $\pi(k, x, y) \leq \pi\,(k, x - 1, y)$; *and*
3. *there is a* $\check{\pi} < \infty$ *such that for all* $(k, x) \in \mathbb{K} \times (\{0\} \cup \mathbb{K})$ *and* $y \in \mathcal{Y}$, $|\pi(k, x, y)| \leq \check{\pi}$.

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$$\Pr\left[K_{t+1} \geq k' \,\middle|\, K_t = k\right] \leq \Pr\left[K_{t+1} \geq k' \,\middle|\, K_t = k+1\right].$$

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Assumption (No Technology Regress)

Π *is upper diagonal.*

Markov Strategies and Equilibrium

- A Markov strategy is
  - an *entry rule* $a_E : (\{0\} \cup \mathbb{K}) \times \mathcal{Y} \to \{0,1\}$, paired with
  - a *survival rule* $a_S : \mathbb{K} \times (\{0\} \cup \mathbb{K}) \times \mathcal{Y} \to [0,1]$.
- In a *symmetric* Markov-perfect equilibrium, all players follow the same Markov strategy.
- Post-entry value

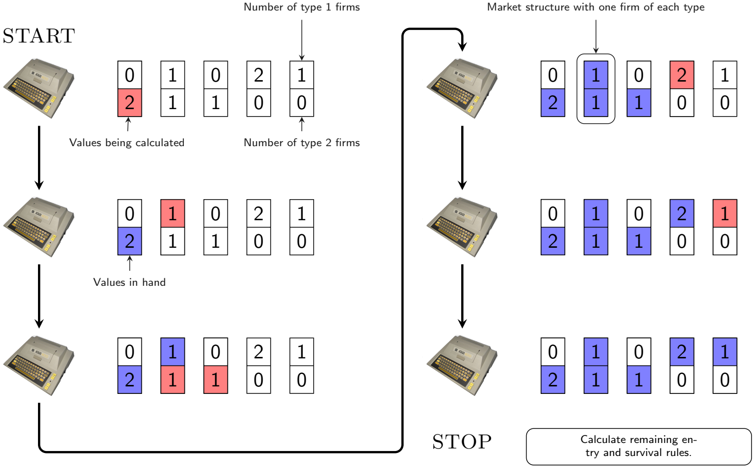
$$v_E(k,x,y) = \begin{cases} a_S(k,0,y)\,(v_S(k,0,y) - \kappa(y)) & \text{if } x = 0, \\ a_S(k,x,y) \times (a_S(x,k,y) v_S(k,x,y) & \\ \quad + (1 - a_S(x,k,y))\, v_S(k,0,y) - \kappa(y)) & \text{if } x > 0. \end{cases}$$

- Post-survival value

$$v_S(k,x,y) = \begin{cases} \beta \mathbb{E}\left[\pi(K',0,Y') + a_E(K',Y') v_E(K',1,Y') \right. & \\ \quad \left. + (1 - a_E(K',Y')) v_E(K',0,Y') \,\middle|\, Y = y, K = k \right] & \text{if } x = 0, \\ \beta \mathbb{E}\left[\pi(K',X',Y') + v_E(K',X',Y') \,\middle|\, Y = y, K = k, X = x \right] & \text{if } x > 0. \end{cases}$$

- In a *natural* Markov-perfect equilibrium,
  - all players maximize their payoffs, and
  - $a_S(l,h,y) > 0$ implies that $a_S(h,l,y) = 1$ for $l < h$.

The Case of $\check{k} = 2$.



Step 1: Calculation of $v_S(2,2,\cdot)$ and $v_E(2,2,\cdot)$

	Survive	Exit
Survive	$v_S(2,2,c)-\kappa(y)$, $v_S(2,2,c)-\kappa(y)$	$v_S(2,0,c)-\kappa(y)$, 0
Exit	0, $v_S(2,0,c)-\kappa(y)$	0, 0

- If $v_S(2,0,c) \geq v_S(2,2,c)$, then

$$v_S(2,2,y) = \beta\mathbb{E}\left[\pi(2,2,Y') + \max\{0, v_S(2,2,Y') - \kappa(Y')\} \,\middle|\, Y = y\right].$$

- This defines a contraction mapping with unique fixed point $v_S(2,2,\cdot)$.

Step 2: Calculation of $v_S(1,2,\cdot)$, $v_E(1,2,\cdot)$, $a_E(2,\cdot)$, and $a_S(1,2,\cdot)$)

- In a natural MPE, the post survival value of the type 1 firm satisfies

$$v_S(1,2,c) = \beta\Big(\Pi_{11}\mathbb{E}\left[\pi(1,2,Y') + \max\{0, v_S(1,2,Y') - \kappa(Y')\} \,\middle|\, Y = y\right] + \Pi_{12}\mathbb{E}\left[\pi(2,2,Y') + v_E(2,2,Y') \,\middle|\, Y = y\right]\Big).$$

- Since $v_E(2,2,\cdot)$ is known from Step 1, the right-hand side defines a contraction mapping with fixed point $v_S(1,2,\cdot)$.
- This firm's post entry value equals $v_E(1,2,y) = \max\{0, v_S(1,2,y) - \kappa(y)\}$
- For this firm, the entry and survival rules must be

$$a_E(2,y) = \mathbb{1}\{v_E(1,2,y) > \varphi(2,y)\}$$
$$a_S(1,2,y) = \mathbb{1}\{v_S(1,2,y) > \kappa(y)\}$$

Step 3: Calculation of $v_S(2,0,\cdot)$, $v_E(2,0,\cdot)$, $v_S(2,1,\cdot)$, and $v_E(2,1,\cdot)$

- In a natural MPE, the post survival value of the type 2 monopolist satisfies

$$\begin{aligned} v_S(2,0,y) = \beta\mathbb{E}\Big[&\pi(2,0,Y') + a_E(2,Y')\left(v_S(2,1,Y') - \kappa(Y')\right) \\ &+ \left(1 - a_E(2,Y')\right)\max\Big\{0, v_S(2,0,Y') - \kappa(Y')\Big\}\,\Big|\, Y = y\Big], \end{aligned}$$

- and the value of a type 2 duopolist facing a type 1 competitor satisfies

$$\begin{aligned} v_S(2,1,y) = \beta\Big(&\Pi_{11}\mathbb{E}[\pi(2,1,Y') + a_S(1,2,Y')(v_S(2,1,Y') - \kappa(Y')) \\ &+ \left(1 - a_S(1,2,Y')\right)\max\{0, v_S(2,0,Y') - \kappa(Y')\}\,|\, Y = y] \\ &+ \Pi_{12}\mathbb{E}[\pi(2,2,Y') + v_E(2,2,Y')\,\Big|\, Y = y]\Big). \end{aligned}$$

- We calculated $v_E(2,2,\cdot)$ in Step 1 and $a_E(2,\cdot)$ and $a_S(1,2,\cdot)$. The right-hand sides together define a contraction mapping with these objects as its fixed point.

Step 4: Calculation of $v_S(1,1,\cdot)$, $v_E(1,1,\cdot)$, and $a_E(1,\cdot)$

- This step is analogous to Step 1, with the additional complication that both firms could survive and enter the market structures $(2,1)$ or $(2,2)$.
- The reduced-form survival game resembles that from Step 1.
- Create a contraction mapping with $v_S(1,1,\cdot)$ as its fixed point. Use this to calculate the entry rule $a_E(1,\cdot)$.

Step 5: Calculation of $v_S(1, 0, \cdot)$ and $v_E(1, 0, \cdot)$

- This step is analogous to Step 3.
- Given the entry rule components $a_E(1, \cdot)$ and and the continuation values already in hand, we define a contraction mapping with $v_S(1, 0, \cdot)$ as its fixed point.

Step 6: Calculation of $a_S(2,0,\cdot)$, $a_S(1,0,\cdot)$, $a_S(2,1,\cdot)$, $a_E(0,\cdot)$, $a_S(2,2,\cdot)$ and $a_S(1,1,\cdot)$

- The values in hand determine optimal entry behavior into an empty market and optimal survival behavior when a firm does not face an identical rival.
- When no symmetric pure-strategy Nash equilibrium exists to the reduced-form survival game, unique mixed strategies $a_S(2,2,\cdot)$ and $a_S(1,1,\cdot)$ keep firms facing an identical rival indifferent between survival and exit.

What must be proven?

- Given the calculated payoffs *and the restrictions of a natural equilibrium*, the calculated strategies cannot be improved by a one-shot deviation.
- To show that the calculated candidate equilibrium is in fact an equilibrium, we must verify that the calculated survival rule dictates continuation of a type 2 firm whenever the type 1 firm chooses a positive probability of survival.
- For this, we prove that the candidate $v_S(k, x, y)$ weakly increases with k and weakly decreases with x.
- To establish equilibrium uniqueness, we show that $v_S(k, x, y)$ must weakly increase with k and weakly decrease with x in *any* natural MPE. This implies that the Bellman equations we use are *necessary* conditions for equilibrium payoffs.

Generalization and Results

- We extend the algorithm to allow for $\check{k} > 2$.
- We establish that the extended algorithm indeed calculates a natural MPE.
- We establish that the natural MPE is unique.

Computation Time on My Laptop

- Set $\check{k} = 30$.
- Y_t contains two components,
  - a persistent demand state that follows a Markov chain with 201 points of support.
  - a continuously-distributed conditionally-independent shock to entry and continuation costs.
- There are $186,930$ possible combinations of the demand state with the two firms' productivity values.
- With $\beta = 0.995$, average computation time is 37.6 seconds in Matlab.
- Set $\beta = 0.94$, average computation time falls to 23.4 seconds.