Endogenous Human Capital Formation and Labor Supply Elasticities

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Motivation

- **Labor supply** models are an important component in optimal tax design.
- **Modern labor supply models do not treat wages as exogenous**
- Wages are driven by human capital investment, resulting in:
    - Labor supply elasticities that vary by age
    - Different optimal tax design.
- **Two types of human capital investment models:**
    - Learning-by-Doing (LBD) – Work vs. Leisure
    - On-the-Job training (OJT) – Productive time, Learning time, Leisure

Question

- When are Wage Offers Revealed? Compare Wolpin vs. Rust Model Timing.
- Analyzing the effects of endogenous human capital formation:
- Changes in the Elasticity of labor supply by age.
- Do LBD and OJT models predict different patterns?
  - [Blandin and Peterman, 2019] – Yes
  - [da Costa and Santos, 2018] – No

This Study

- Solve and estimate both LBD and OJT models for the U.S. data.
- Improves upon previous work by incorporating:
    - Discrete labor supply with continuous consumption
    - Flexible timing for revelation of wage shocks

Model Characteristics

- **Four education groups** : Dropouts, high school graduates, some college, and college graduates.
- **Unobserved heterogeneity** :

  In initial human capital.

  In preference for leisure.
- **Continuous-Discrete choice set**: Continuous consumption and discrete leisure.
- **Two wage shocks**: One in the beginning of period and another after labor supply decision is made.
- **Two types of endogenous human capital** : Learning-By-Doing or On-the-Job training.

Model Specification (Utility)

Utility function $u(c, l) = \log(c) + \nu(l) + \lambda \phi(l)$

Where $\nu(l_t) = \frac{\theta}{\sigma}. l^{\sigma}$ is the utility from leisure and $\phi(l)$ is the taste shock for leisure. $\phi(l)$ follows an independent multivariate extreme value distribution associated with leisure and scale parameter λ.

Model Timing

Model Specification (Time Allocation)

We define h_t as the labor supply such that:

- h is total time net of leisure $(1 - l)$ in Learning-by-Doing (LBD) model.
- h is total time net of leisure and training time $(1 - l - i)$ in On-the-Job Training (OJT) model.

Model Specification

Human Capital

Human Capital Production Function

$$k' = \delta k + \alpha i^{\gamma} k^{\nu}$$

k is human capital at time t, Depending on the model being LBD or OJT, i is labor supply (h) or training, respectively.

Model Specification Wage Settings

Wage is proportional to human capital and incorporates two different wage shocks:

$$w = kRe^{\epsilon_W + \epsilon_R}$$

Where R is the price of human capital, ϵ_W and ϵ_R are two independently distributed wage shocks, differing in timing. Each follows a normal distribution:

ϵ_W is observed at the beginning of the period.

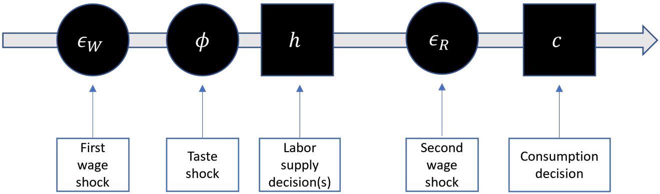
ϵ_R is observed after the work decision is made, before the consumption decision.

Wage shock $\quad \sigma_T^2 = \sigma_W^2 + \sigma_R^2$

Timing

Utility Function

We assume each period starts with an i.i.d. wage shock ϵ_W. Then, individuals decide about labor supply, knowing the taste shock ϕ_l. In the OJT model, they next decide about training i (conditional on l). Then second wage shock ϵ_R hits. Lastly, consumption decision is made.



Model Specification Transfers

Transfers: $UB(k)$ is an unemployment benefit. SS is a fixed amount that transferred to individuals at the social security allowance age. $Tax(.)$ aggregates all taxes, including income tax, unemployment benefit tax, social security tax, and capital gain tax.

$$TR = UB(k) + SS - Tax(UB, SS, w, h, a)$$

Model Specification

Unemployment Benefits

Unemployment Benefit: UB is positive only when $h = 0$ and is zero otherwise.

$$UB\left(b\right) = \begin{cases} 0 & \text{if } h > 0 \\ bk & \text{if } h = 0 \end{cases}$$

where we set argument b for each education group separately.

Model Specification (Assets)

Asset transition

$$a' = (1 + r)\,(a + wh - P - c + TR - Med)$$

r is the interest rate (set at 5%), TR_t is the total transfers, and Med_t is the medical expenditure.

Model Timing

Model Specification

Age and Taste for Leisure

To capture the rapid drop in labor supply at ages 60+ we need to let the taste for leisure vary with age. That is:

$$\theta^* = \theta + \mathrm{I}\left[t > 62\right]\left(t - 62\right)^{1.5}\bar{\theta}$$

where θ^* is taste for leisure in utility function.

Model Specification

Bequest Function

Bequest: We follow [Imai and Keane, 2004] using the following bequest function:

$$B(a') = \begin{cases} 3.\log(a' + \omega) - 1 - 3.\log(\omega) & \textit{if } \; a' > 0 \\ \left(\dfrac{a' - \omega}{\omega}\right)^3 & \textit{if } \; a' \leq 0 \end{cases}$$

Backward induction in LBD Backward induction in OJT

Solution

Our problem is a continuous-discrete type. Consumption is a continuous decision, whereas leisure (and training in OJT) is a discrete variable:

$$c \in R \quad l \in L = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$$

In OJT:

$$i \in I = \left\{0, \frac{1}{10}, ..., \frac{9}{10}, 1 = \bar{I}\right\}$$

Solution

The optimal consumption equalizes the marginal gain in the utility of consumption and the marginal expected disutility due to a decrease in the next period's asset:

$$\frac{du}{dc} = -\beta \frac{dV'}{da'} \frac{da'}{dc} \quad \rightarrow \quad \frac{1}{c\lambda} = -\beta \frac{dV'}{da'} \left(- \left(1 + r \right) \right)$$

$$\rightarrow c = \left(\frac{dV'}{da'} \lambda \right)^{-1} \quad \rightarrow 0 = \left(\frac{dV'}{da'} \lambda \right)^{-1} - c \quad if \quad t < T$$

Hence, the problem reduces to finding the root(s) at each point:

$(l, a, k, \phi\left(l\right), \epsilon_{WEK}, \epsilon_{R}$ and i in OJT)

Iterative Algorithm (Initials)

We can recover the information regarding consumption from already calculated future periods and use them as initial values for consumption.

Initial values:

c^0: Consumption $S^0 = -1$: determines direction of a change

$d^0 = 1$: step-size. $\overrightharpoon{\Delta} = 1$, $\overleftarrow{\Delta} = 1/2$: Change in step-size

Iterative Algorithm

In iteration i we determine direction and the size in next iteration until F^i is smaller than a pre-set limit, $\bar{F}$:

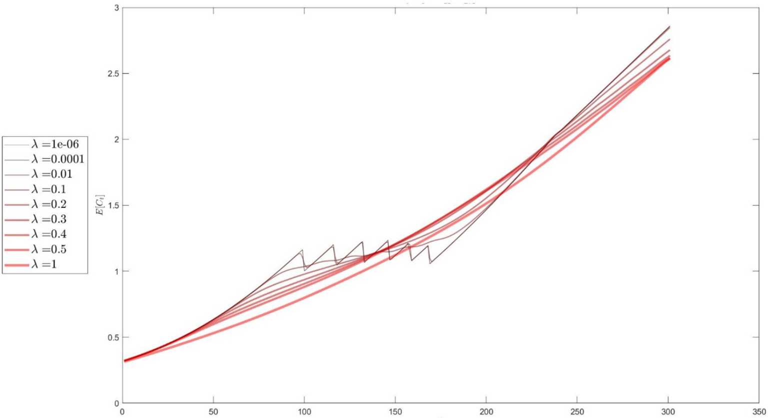
1. Calculate $F^i = \left(\frac{dV'}{da'(c^i)}\lambda\right)^{-1} - c^i$
2. Stopping criteria: if $F^i <= \bar{F} \rightarrow$ stop and return c^{i-1}
3. Determining the direction
   - if $F^i > 0 \ \rightarrow S^i = 1$
   - if $F^i < 0 \ \rightarrow S^i = -1$

Iterative Algorithm

4. If we keep moving the same direction, we keep the step-size , but if we change the direction we decrease step by multiplying the step-size by $\overleftarrow{\Delta}$
   - if $s^i = s^{i-1} \rightarrow d^i = d^{i-1}\overrightarrow{\Delta}$
   - if $s^i \neq s^{i-1} \rightarrow d^i = d^{i-1}\overleftarrow{\Delta}$
5. $c^i = c^{i-1} + d^i$
6. Repeat the process

Taste Shock & Non-monotonicity in Policy Function

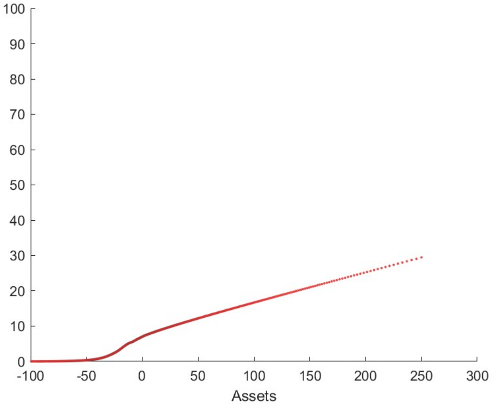
Non-monotonicity in the value function: We smooth by increasing the scale of taste shocks (our estimate of λ is between .2 and .3).



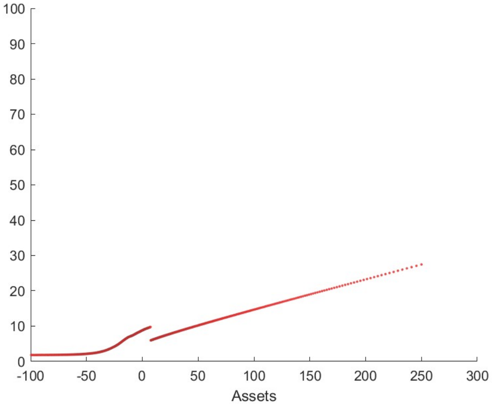
Finding Optimum Consumption

In what follows, we show how the algorithm converges to the optimal level of consumption for each asset level, given labor supply, wage shocks, taste shock, and human capital at the end of the current period.

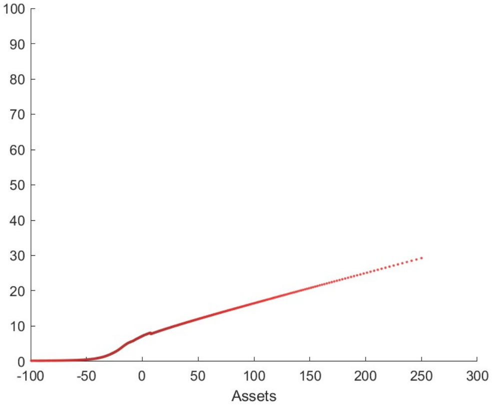
Finding Optimum Consumption (Iter=1)



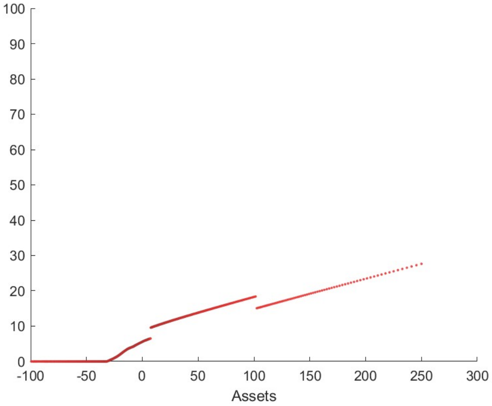
Finding Optimum Consumption (Iter=2)



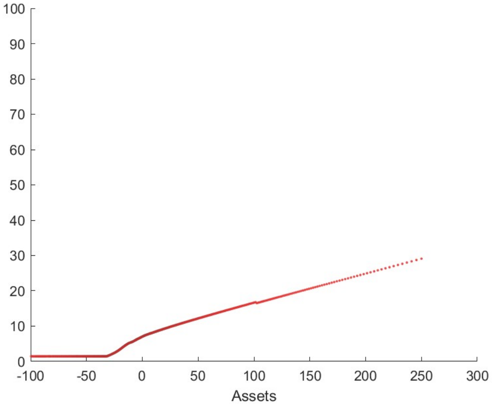
Finding Optimum Consumption (Iter=3)



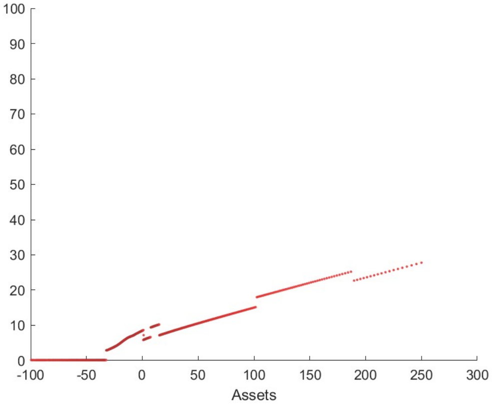
Finding Optimum Consumption (Iter=4)



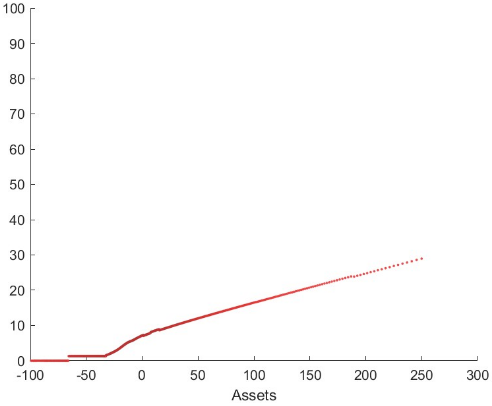
Finding Optimum Consumption (Iter=5)



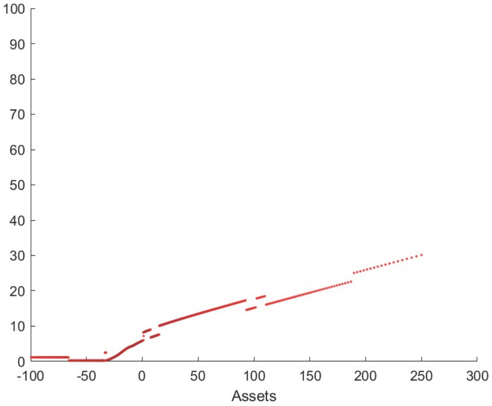
Finding Optimum Consumption (Iter=6)



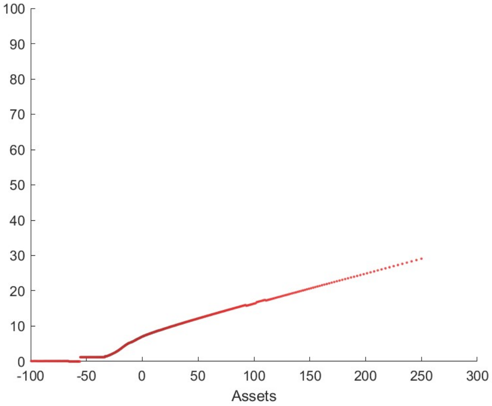
Finding Optimum Consumption (Iter=7)



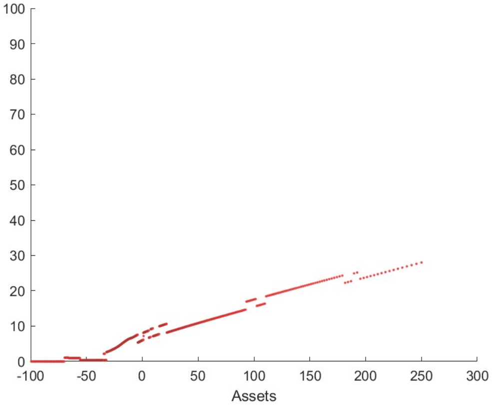
Finding Optimum Consumption (Iter=8)



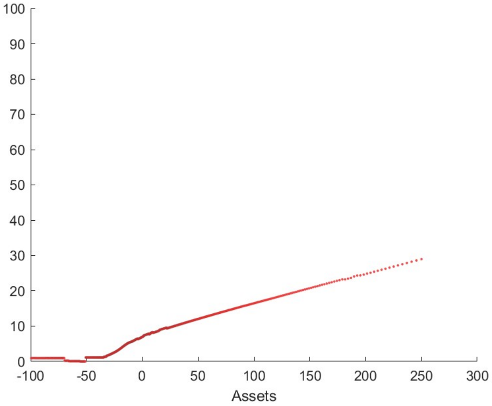
Finding Optimum Consumption (Iter=9)



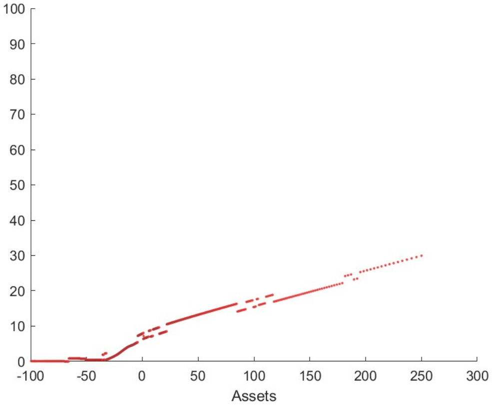
Finding Optimum Consumption (Iter=10)



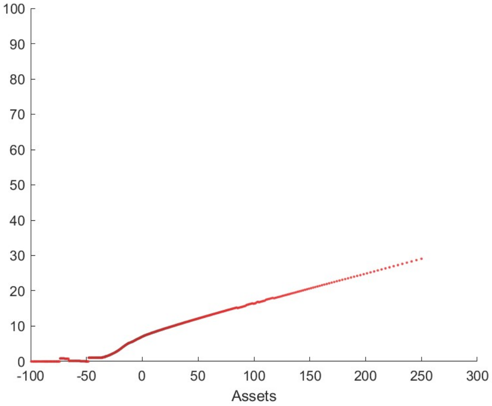
Finding Optimum Consumption (Iter=11)



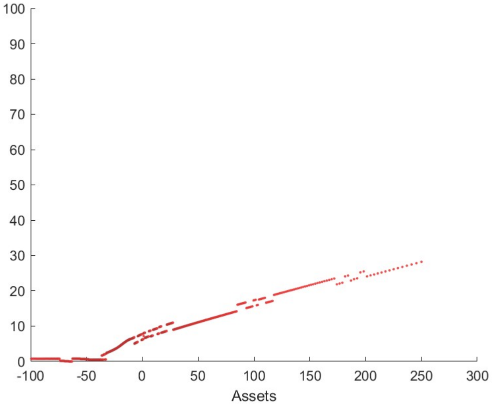
Finding Optimum Consumption (Iter=12)



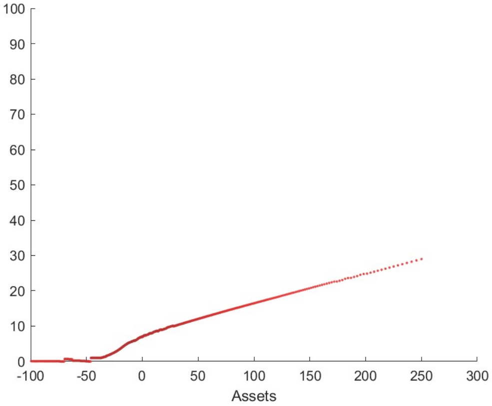
Finding Optimum Consumption (Iter=13)



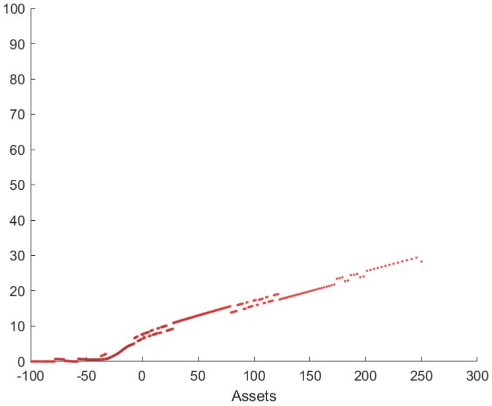
Finding Optimum Consumption (Iter=14)



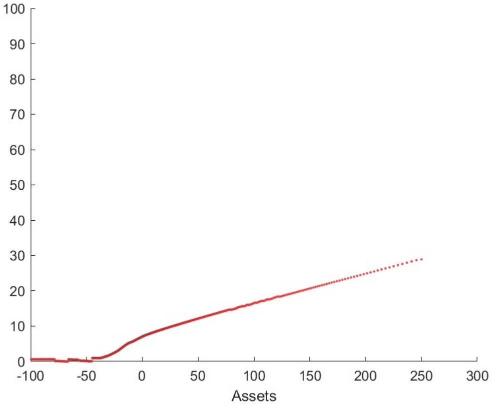
Finding Optimum Consumption (Iter=15)



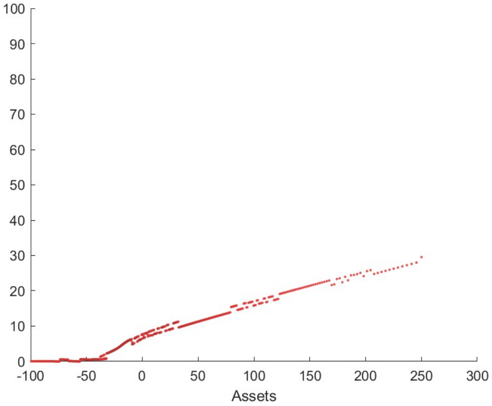
Finding Optimum Consumption (Iter=16)



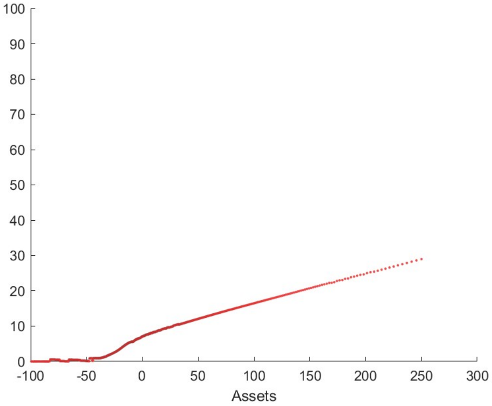
Finding Optimum Consumption (Iter=17)



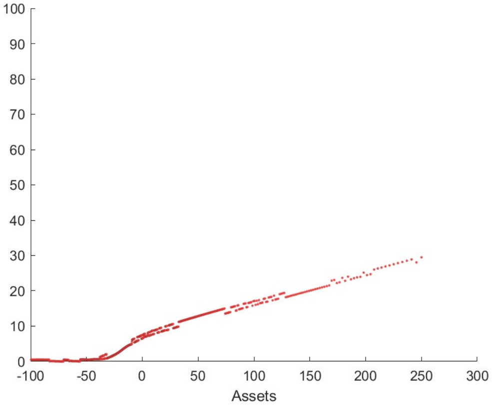
Finding Optimum Consumption (Iter=18)



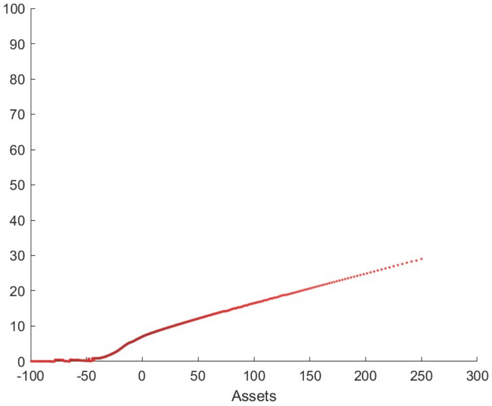
Finding Optimum Consumption (Iter=19)



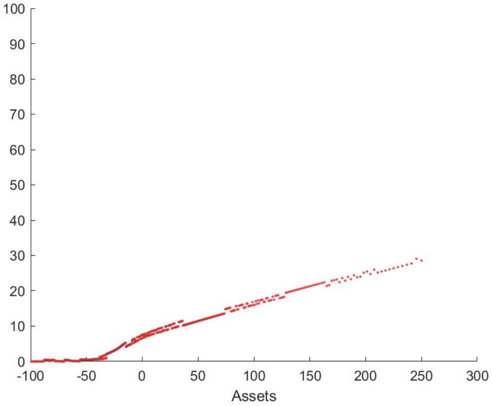
Finding Optimum Consumption (Iter=20)



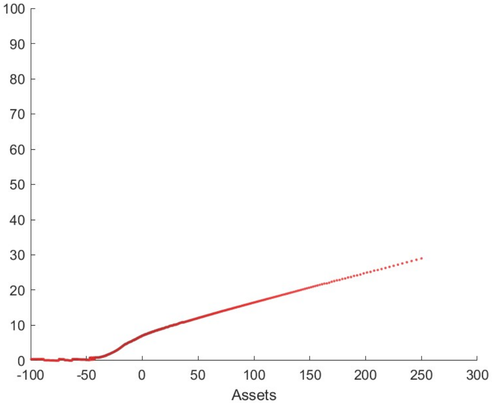
Finding Optimum Consumption (Iter=21)



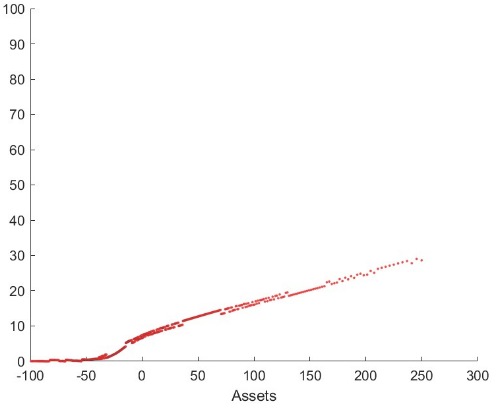
Finding Optimum Consumption (Iter=22)



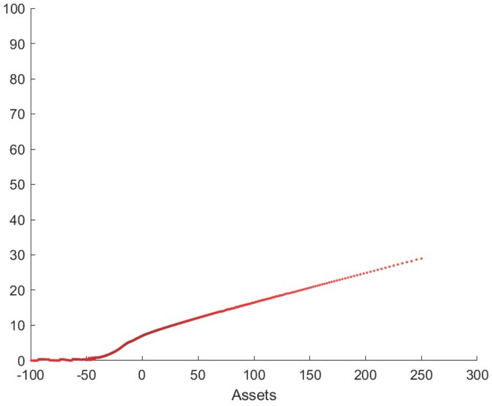
Finding Optimum Consumption (Iter=23)



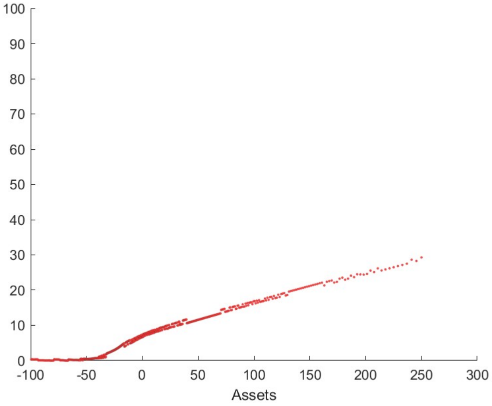
Finding Optimum Consumption (Iter=24)



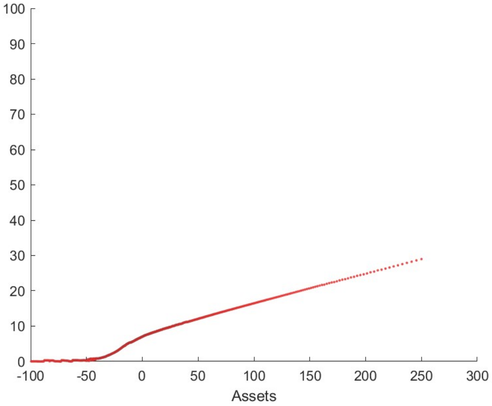
Finding Optimum Consumption (Iter=25)



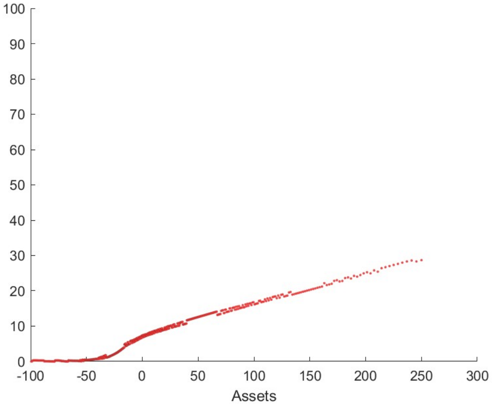
Finding Optimum Consumption (Iter=26)



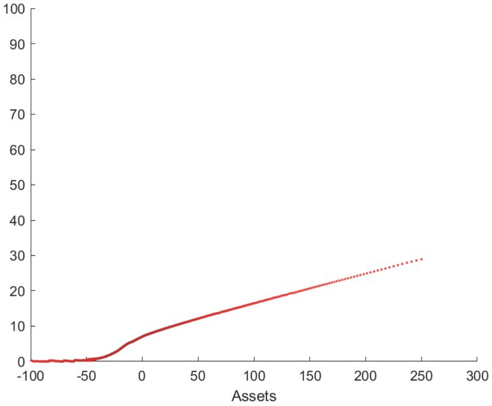
Finding Optimum Consumption (Iter=27)



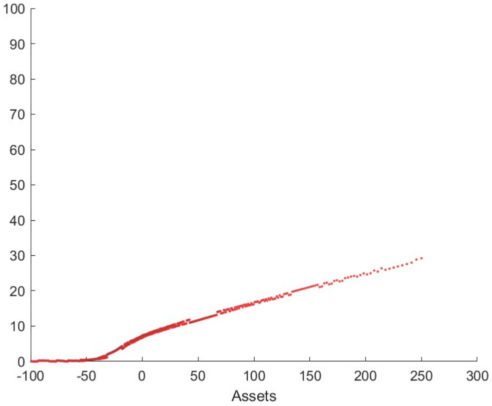
Finding Optimum Consumption (Iter=28)



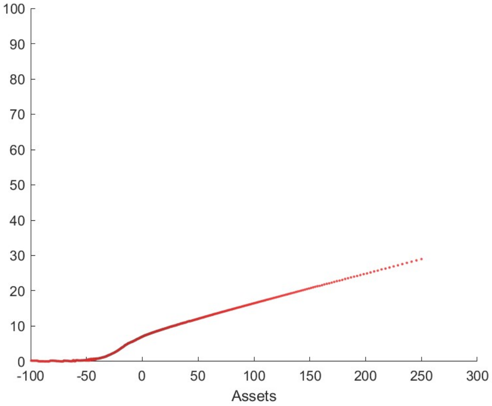
Finding Optimum Consumption (Iter=29)



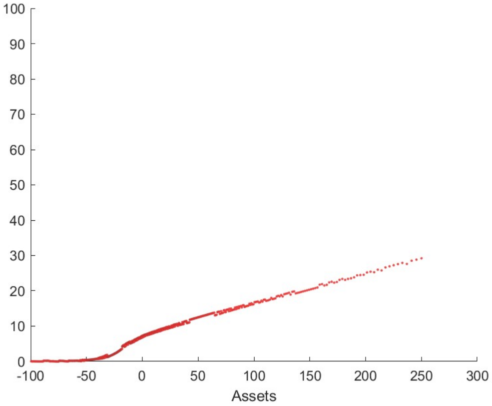
Finding Optimum Consumption (Iter=30)



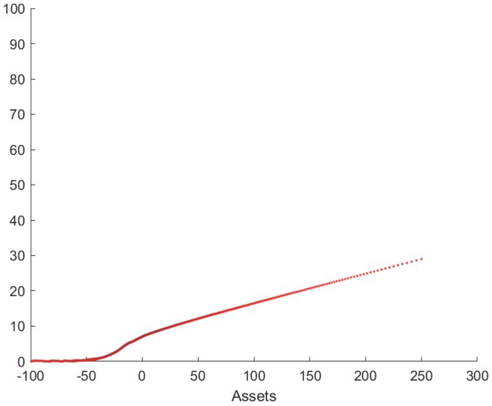
Finding Optimum Consumption (Iter=31)



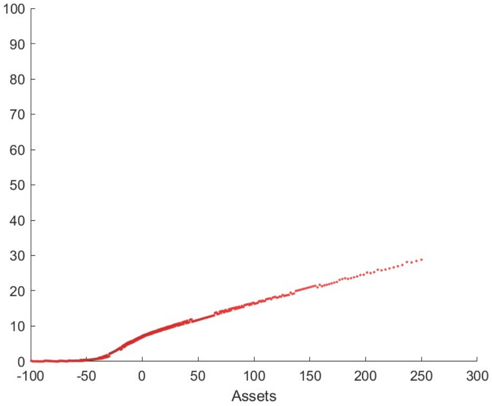
Finding Optimum Consumption (Iter=32)



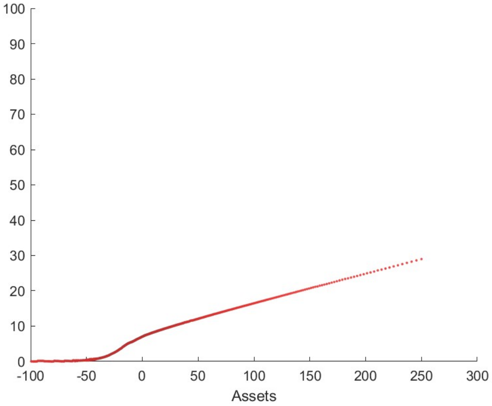
Finding Optimum Consumption (Iter=33)



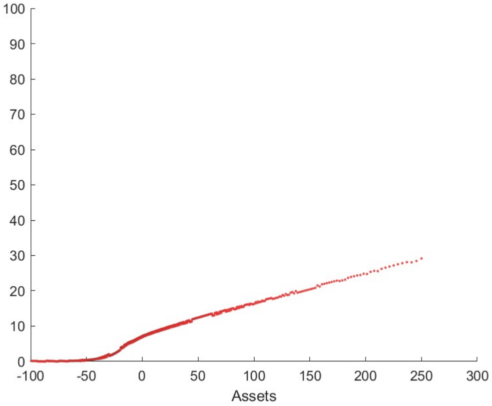
Finding Optimum Consumption (Iter=34)



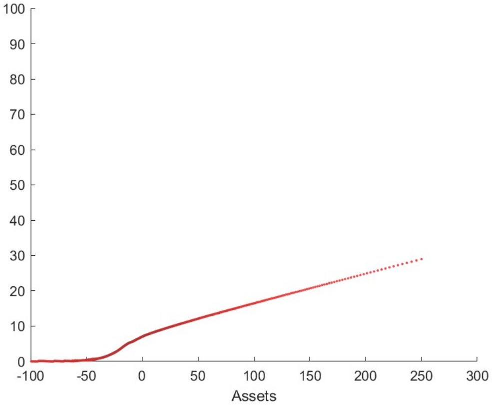
Finding Optimum Consumption (Iter=35)



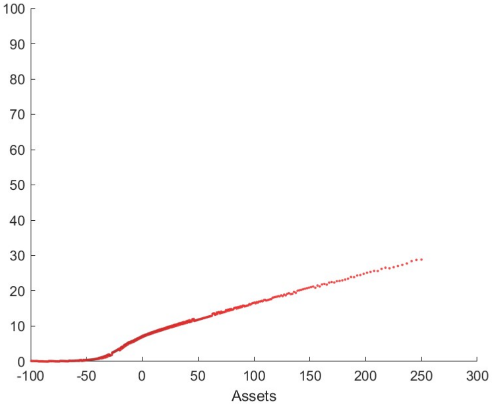
Finding Optimum Consumption (Iter=36)



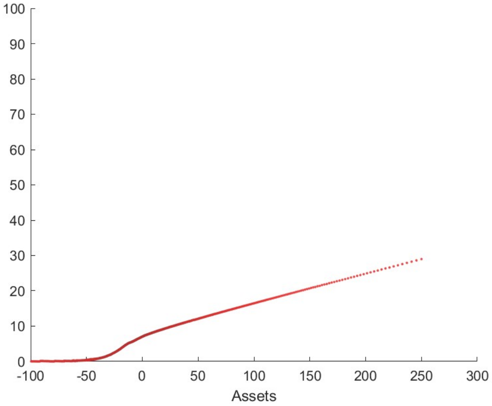
Finding Optimum Consumption (Iter=37)



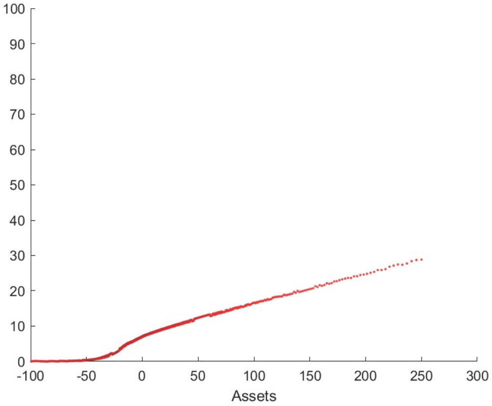
Finding Optimum Consumption (Iter=38)



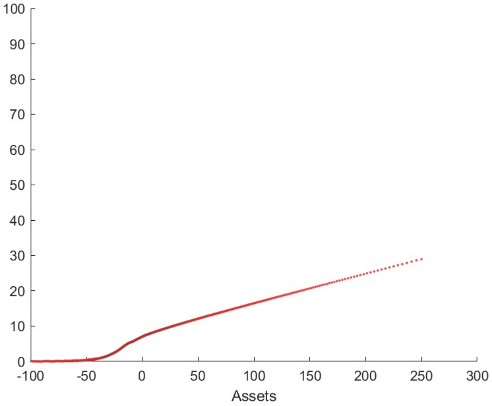
Finding Optimum Consumption (Iter=39)



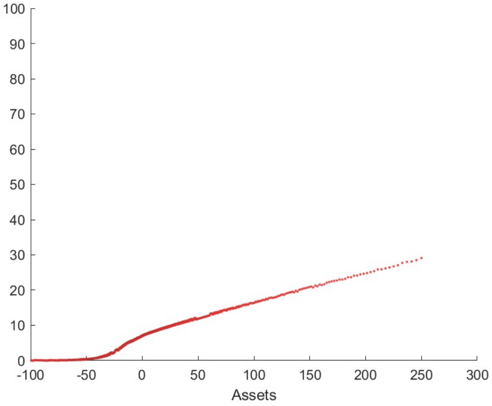
Finding Optimum Consumption (Iter=40)



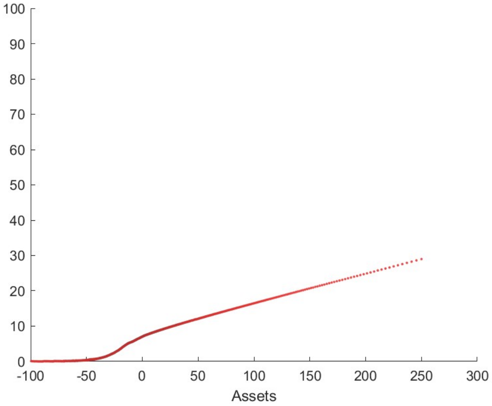
Finding Optimum Consumption (Iter=41)



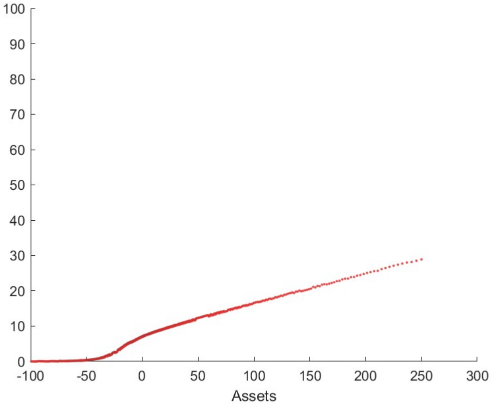
Finding Optimum Consumption (Iter=42)



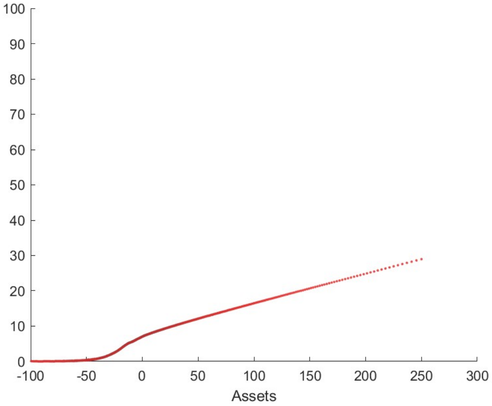
Finding Optimum Consumption (Iter=43)



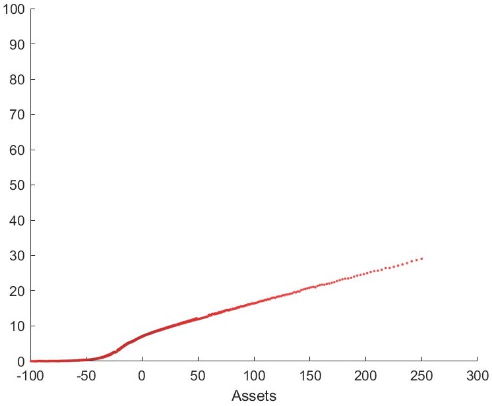
Finding Optimum Consumption (Iter=44)



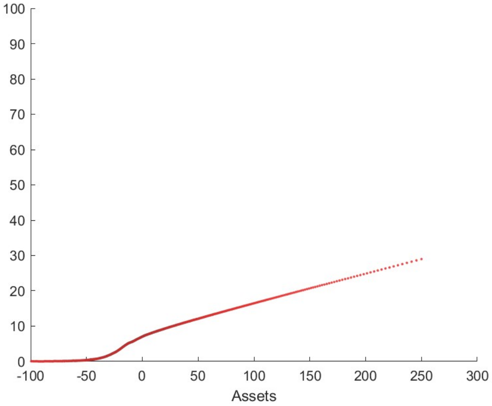
Finding Optimum Consumption (Iter=45)



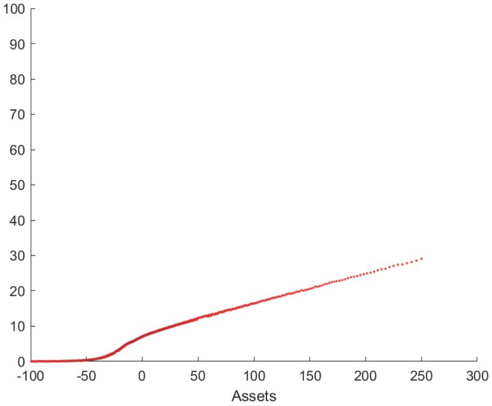
Finding Optimum Consumption (Iter=46)



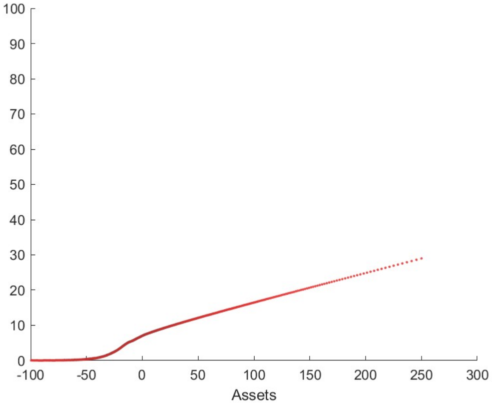
Finding Optimum Consumption (Iter=47)



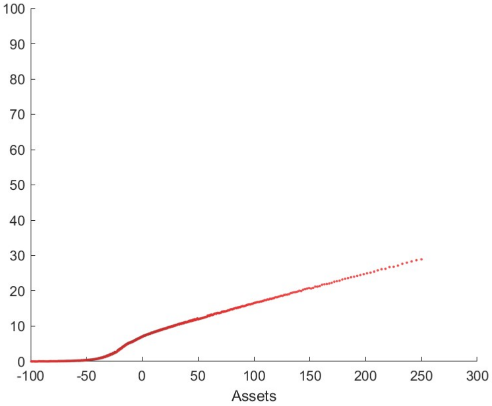
Finding Optimum Consumption (Iter=48)



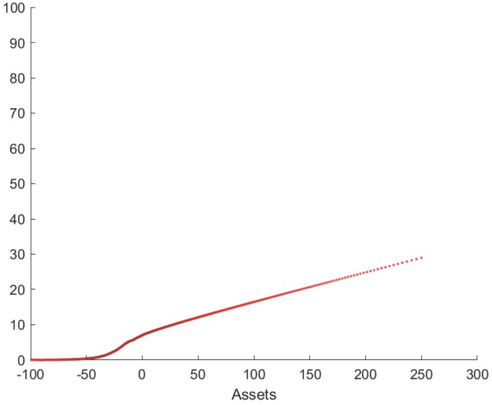
Finding Optimum Consumption (Iter=49)



Finding Optimum Consumption (Iter=50)



Finding Optimum Consumption (Iter=51)



Back to first iteration Repeat

Data

Consumer Expenditure Survey

To calculate the data moments required for our analysis, we utilize CPS, CES and PSID datasets. In total with four levels of education we fit: Annual data from 2011 to 2016, by age:

- Average consumption (household equivalence scale is squared root of HH size).
- Standard deviation of consumption.

Data

Current Population Survey

Annual data from 2012 to 2019, by age:

- Average Employment rate.
- Average annual hours conditional on work.
- Median full-time hourly wage.
- Standard deviation of log of the hourly wage rate.
- Transition rate: probability of working conditional on working last year.
- Transition Rate: probability of working conditional on not working last year.

Data

Panel Study of Income Dynamics

Biannual data from 2009 to 2019:

by experience

- Average Employment rate.
- Average annual hours conditional on work.
- Median full-time hourly wage.

by lag in two-year intervals

- Wage persistence (correlation of log(wage) residuals conditional on age polynomial)

Estimation Method

Estimation is by method of simulated moments (MSM). Solving both models as discussed , we simulate a sample of 5,000 individuals, and estimate the model by MSM, minimising the following:

$$\text{Objective} \qquad \sum_{g} \left(\frac{\hat{y}_g - y_g}{se_{y,g}} \right)^2$$

where, g refers to a particular group for which we compare the moments between data and the model. y_g is the moment calculated in the data, $\hat{y}_g$ is the moment based on simulations, and $se_{y,g}$ is the standard error in the data.

Estimation Method

We have 344, 345, 334, and, 331 moments for dropouts, high school graduates, some college, and college graduates, respectively. In total, we estimate 18 different parameters for each education group in LBD. To estimate the OJT models, we constrain our estimations to only four parameters in human capital function and fixing all the other 14 parameters at their LBD values.

Human Capital Parameters   Utility and Shocks Parameters   Unobserved Heterogeneity Parameters

Human Capital Parameters

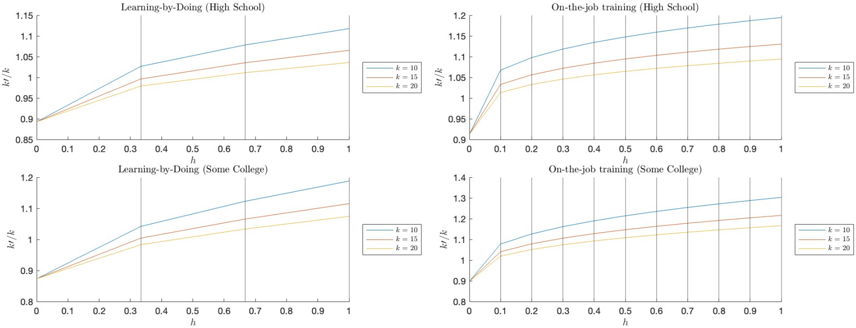
Estimation

	LBD				OJT			
	Dropout	HS	SC	College	Dropout	HS	SC	College
δ	0.92	0.90	0.89	0.84	0.94	0.92	0.91	0.86
α	0.39	0.60	0.83	0.95	0.50	0.73	0.98	1.00
γ	0.35	0.49	0.60	0.30	0.22	0.27	0.37	0.22
ν	0.36	0.41	0.41	0.50	0.39	0.43	0.47	0.55

Human Capital function $k' = \delta k + \alpha i^{\gamma} k^{\nu}$

Human Capital | Utility and Shocks | Heterogeneity

Change in Human Capital



Utility, Bequest and Wage shock Parameters

Estimation

	LBD				OJT			
	Dropout	HS	SC	College	Dropout	HS	SC	College
σ	0.60	0.64	0.64	0.87	0.60	0.64	0.64	0.87
λ_V	0.20	0.30	0.30	0.20	0.20	0.30	0.30	0.20
ϕ_0	0.60	0.73	0.73	0.82	0.60	0.73	0.73	0.82
$\bar{\theta}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ω	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ζ	0.25	0.16	0.16	0.10	0.25	0.16	0.16	0.10

Utility function $u(c,l) = \log(c) + \frac{\theta}{\sigma}.\ l^{\sigma} + \lambda\phi(l)$

Wage shock $\sigma_T^2 = \sigma_W^2 + \sigma_R^2$

Change in taste for leisure by age $\theta^* = \theta + \mathrm{I}[t > t_{RET}](t - t_{RET})^{1.5}\bar{\theta}$

Bequest Function $B(a') = \begin{cases} 3.\log(a' + \omega) - 1 - 3.\log(\omega) & \text{if } a' > 0 \\ \left(\frac{a' - \omega}{\omega}\right)^3 & \text{if } a' \leq 0 \end{cases}$

Parameters Ruling Heterogeneities

Estimation

	LBD				OJT			
	Dropout	HS	SC	College	Dropout	HS	SC	College
θ_1	0.50	0.31	0.31	0.22	0.50	0.31	0.31	0.22
θ_2	1.00	0.95	0.95	0.82	1.00	0.95	0.95	0.82
θ_3	1.03	1.47	1.47	1.62	1.03	1.47	1.47	1.62
$Pr\ (\theta_1)$	0.73	0.62	0.62	0.33	0.73	0.62	0.62	0.33
$Pr\ (\theta_2)$	0.20	0.29	0.29	0.56	0.20	0.29	0.29	0.56
$k_{0,1}$	2.20	9.40	13.00	16.21	4.00	14.80	23.80	23.41
$k_{o,2}$	12.10	14.40	18.00	21.60	13.90	19.80	28.80	28.80
$Pr\ (k_0^1)$	0.10	0.10	0.10	0.04	0.10	0.10	0.10	0.04

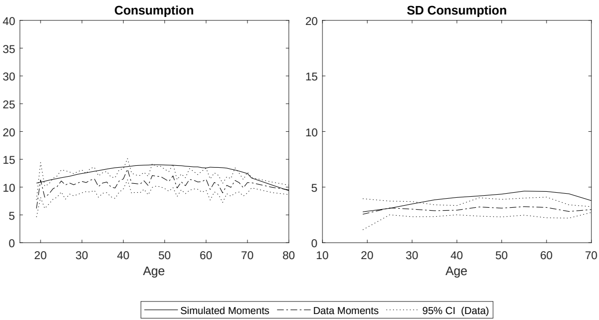
Utility function $u(c,l) = \log(c) + \frac{\theta_\kappa}{\sigma}.\ l^\sigma + \lambda\phi(l) \qquad \kappa \in \{1,2,3\}$

Human Capital function (first period) $k_1 = \delta k_{0,\pi} + \alpha i^\gamma k^\nu \qquad \pi \in \{1,2\}$

Human Capital | Utility and Shocks | Heterogeneity

Consumption's Fits

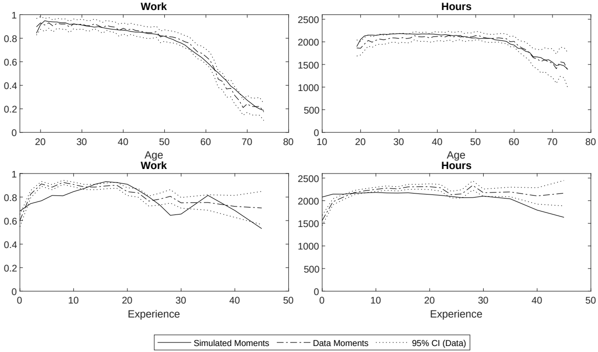
High School (LBD)



Dropout (LBD) HS (LBD) SC (LBD) College (LBD)
Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Labor Supply's Fits

High School (LBD)

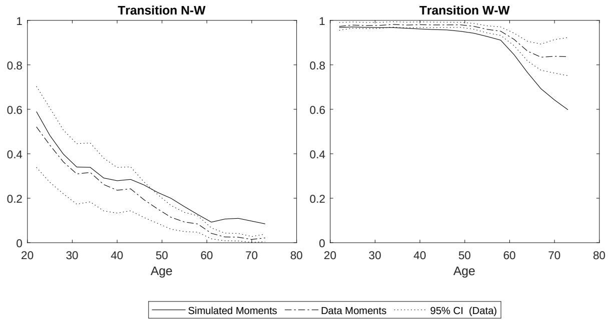


Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Transition to Work's Fits

High School (LBD)



Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Wage's Fits

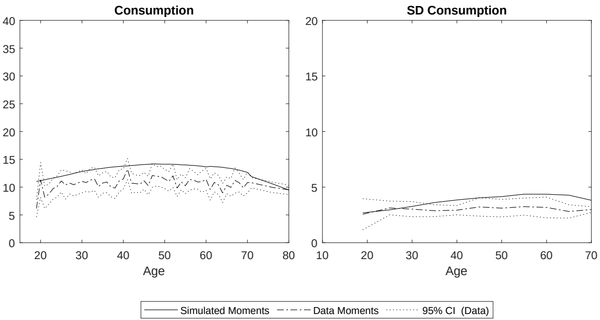
High School (LBD)

Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Consumption's Fits

High School (OJT)

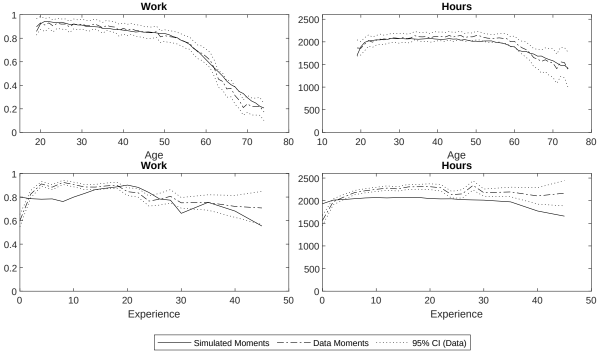


Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Labor Supply's Fits

High School (OJT)

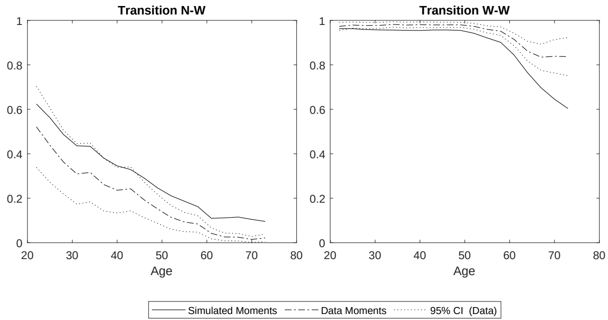


Dropout (LBD) | HS (LBD) | SC (LBD) | College (LBD)

Dropout (OJT) | HS (OJT) | SC (OJT) | College (OJT)

Transition to Work's Fits

High School (OJT)



Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

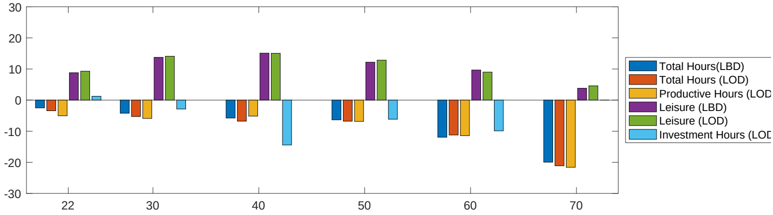
Wage's Fits

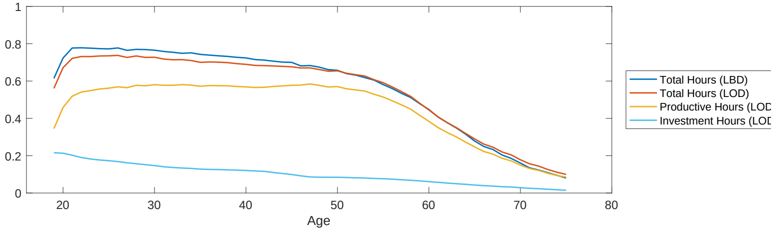
High School (OJT)

Dropout (LBD) | HS (LBD) | SC (LBD) | College (LBD)

Dropout (OJT) | HS (OJT) | SC (OJT) | College (OJT)

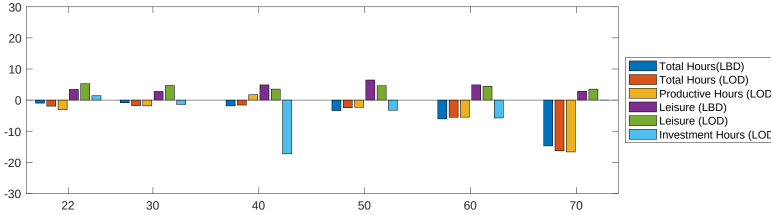
Responses (High School - Anticipated & Permanent [-10%])

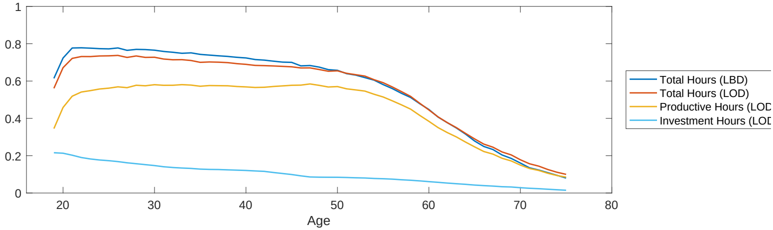




Dropout HS SC College

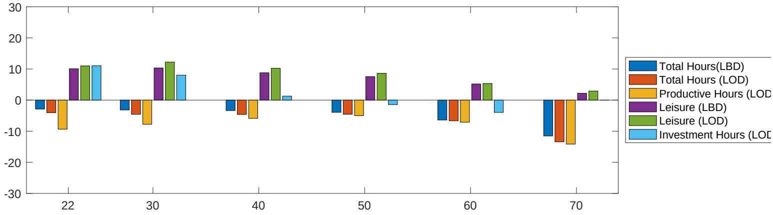
Responses (High School - Unanticipated & Permanent [-10%])

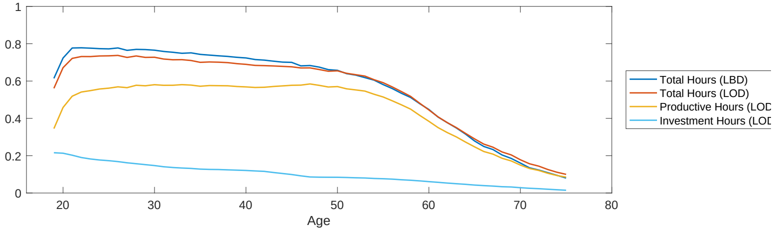




Dropout HS SC College

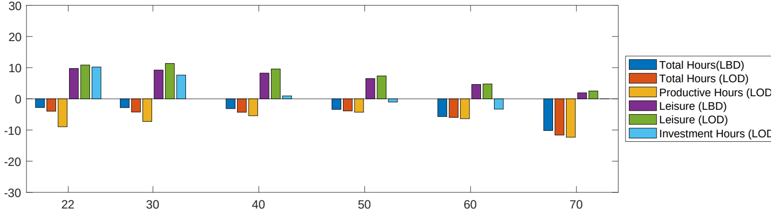
Responses (High School - Anticipated & Transitory [-10%])

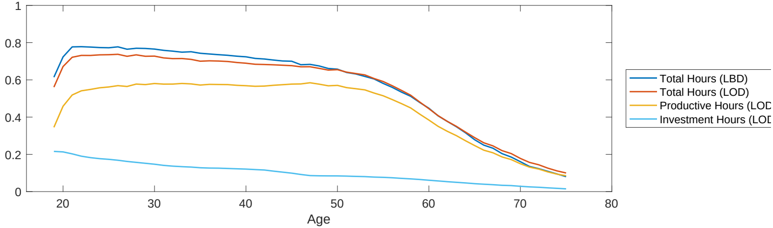




Dropout HS SC College

Responses (High School - Unanticipated & Transitory [-10%])





Dropout HS SC College

Labor Supply Response (%Δ)

Responding to -10% Anticipated Permanent Wage Shocks

Education	Model	22	30	40	50	60	70
Dropout	LBD	-6.76	-9.10	-8.63	-6.69	-25.12	-25.09
	OJT	-5.51	-6.92	-8.40	-7.71	-25.90	-31.54
HS	LBD	-2.51	-4.21	-5.76	-6.35	-11.94	-19.95
	OJT	-3.42	-5.28	-6.77	-6.78	-11.22	-21.13
SC	LBD	-1.74	-2.50	-4.64	-6.19	-10.27	-17.58
	OJT	-3.22	-3.76	-6.73	-6.43	-11.09	-19.43
College	LBD	-6.82	-0.64	-5.18	-5.99	-11.05	-14.57
	OJT	-6.04	-1.64	-4.48	-5.23	-11.24	-16.49

Anticipated Permanent Unanticipated Permanent Anticipated Transitory Unanticipated Transitory

Labor Supply Response (%Δ)

Responding to -10% Unanticipated Permanent Wage Shocks

Education	Model	22	30	40	50	60	70
Dropout	LBD	-3.40	-2.69	-2.75	-2.19	-14.92	-19.87
	OJT	-2.36	-1.89	-2.54	-2.44	-13.82	-22.92
HS	LBD	-0.98	-0.85	-1.87	-3.36	-6.04	-14.72
	OJT	-1.94	-1.76	-1.59	-2.44	-5.52	-16.31
SC	LBD	-0.78	-0.25	-0.46	-2.02	-4.51	-10.35
	OJT	-1.59	-0.82	-1.11	-2.02	-4.35	-13.19
College	LBD	-6.82	0.98	-0.82	-2.27	-7.38	-12.53
	OJT	-5.96	0.86	-0.27	-1.64	-7.33	-13.61

Anticipated Permanent | Unanticipated Permanent | Anticipated Transitory | Unanticipated Transitory

Labor Supply Response (%Δ)

Responding to -10% Anticipated Transitory Wage Shocks

Education	Model	22	30	40	50	60	70
Dropout	LBD	-4.44	-5.51	-6.03	-4.80	-12.37	-16.67
	OJT	-5.68	-5.57	-6.44	-6.98	-13.19	-20.91
HS	LBD	-2.88	-3.17	-3.36	-3.94	-6.40	-11.53
	OJT	-4.03	-4.58	-4.62	-4.54	-6.66	-13.43
SC	LBD	-2.57	-2.56	-2.48	-3.01	-3.81	-9.31
	OJT	-4.20	-3.83	-3.71	-3.56	-4.27	-10.68
College	LBD	-5.60	-1.53	-3.18	-4.80	-9.40	-12.26
	OJT	-4.89	-1.91	-3.38	-4.57	-9.54	-13.15

Anticipated Permanent | Unanticipated Permanent | Anticipated Transitory | Unanticipated Transitory

Labor Supply Response (%Δ)

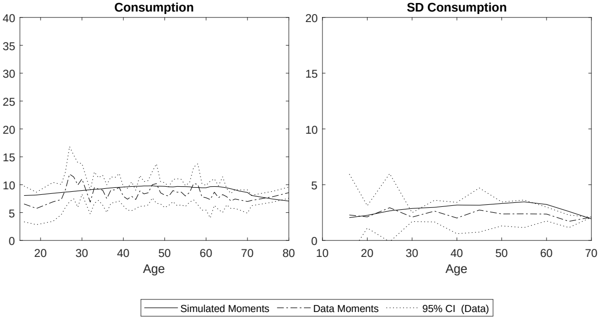
Responding to -10% Unanticipated Transitory Wage Shocks

Education	Model	22	30	40	50	60	70
Dropout	LBD	-4.13	-4.64	-5.09	-4.06	-11.13	-15.57
	OJT	-5.24	-5.26	-5.74	-6.17	-11.52	-17.72
HS	LBD	-2.77	-2.83	-3.15	-3.38	-5.69	-10.16
	OJT	-3.98	-4.25	-4.31	-3.88	-5.96	-11.63
SC	LBD	-2.45	-2.28	-2.13	-2.37	-3.41	-7.13
	OJT	-4.01	-3.42	-3.32	-2.61	-3.78	-9.61
College	LBD	-5.60	-1.38	-2.81	-4.16	-8.54	-11.70
	OJT	-4.89	-1.46	-3.20	-4.18	-8.67	-12.76

Anticipated Permanent | Unanticipated Permanent | Anticipated Transitory | Unanticipated Transitory

Consumption's Fits

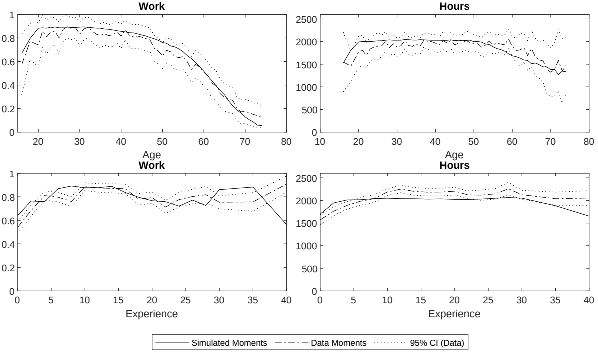
Dropouts (LBD)



Dropout (LBD) HS (LBD) SC (LBD) College (LBD)
Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Labor Supply's Fits

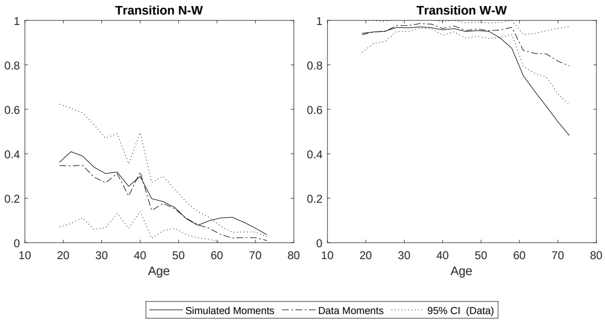
Dropouts (LBD)



Dropout (LBD) HS (LBD) SC (LBD) College (LBD)
Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Transition to Work's Fits

Dropouts (LBD)



Dropout (LBD) HS (LBD) SC (LBD) College (LBD)
Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Wage's Fits

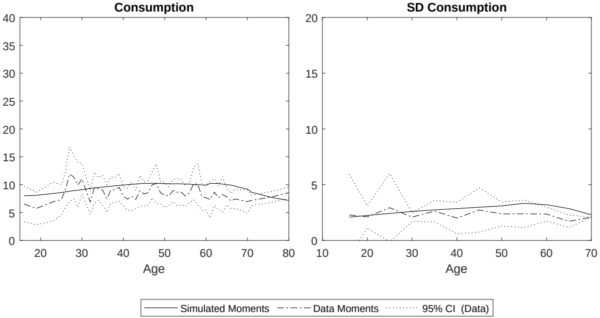
Dropouts (LBD)

Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Consumption's Fits

Dropouts (OJT)

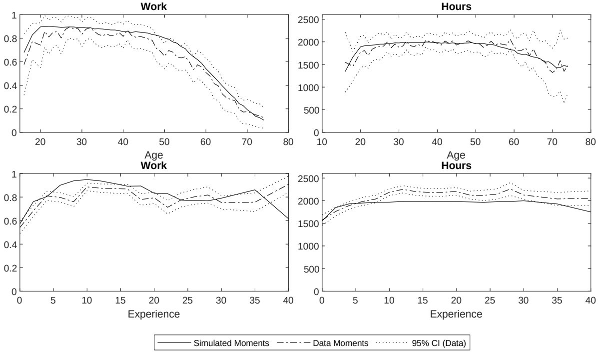


Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Labor Supply's Fits

Dropouts (OJT)

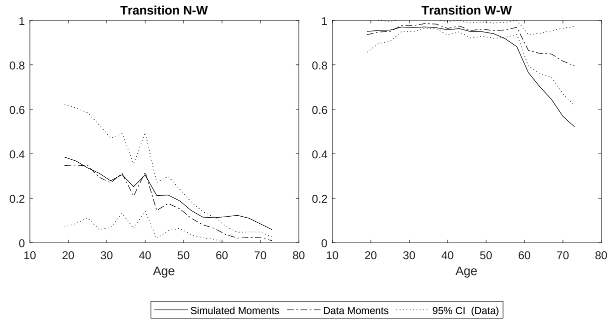


Dropout (LBD) | HS (LBD) | SC (LBD) | College (LBD)

Dropout (OJT) | HS (OJT) | SC (OJT) | College (OJT)

Transition to Work's Fits

Dropouts (OJT)

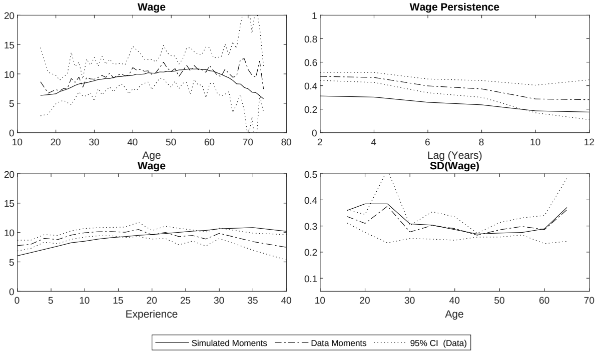


Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Wage's Fits

Dropouts (OJT)

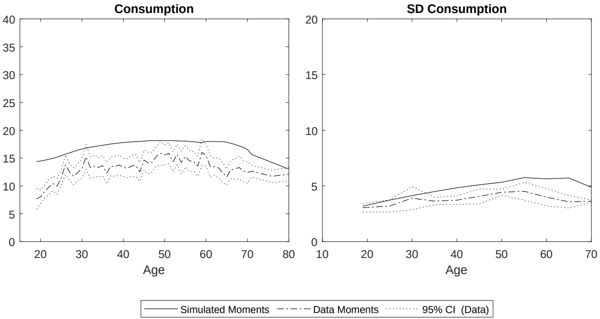


Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Consumption's Fits

Some College (LBD)

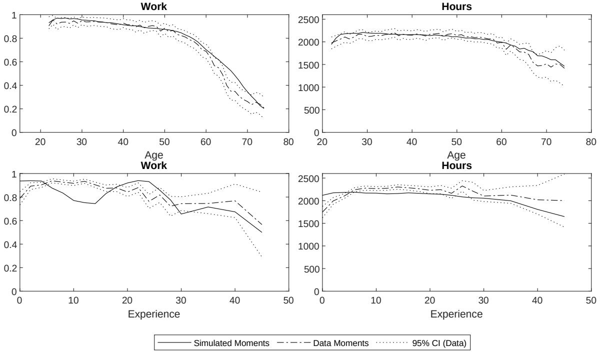


Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Labor Supply's Fits

Some College (LBD)

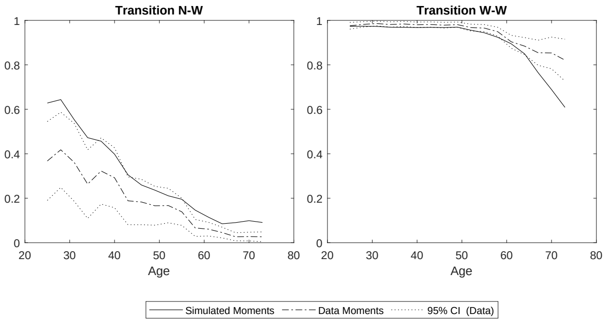


Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Transition to Work's Fits

Some College (LBD)



Dropout (LBD) | HS (LBD) | SC (LBD) | College (LBD)

Dropout (OJT) | HS (OJT) | SC (OJT) | College (OJT)

Wage's Fits

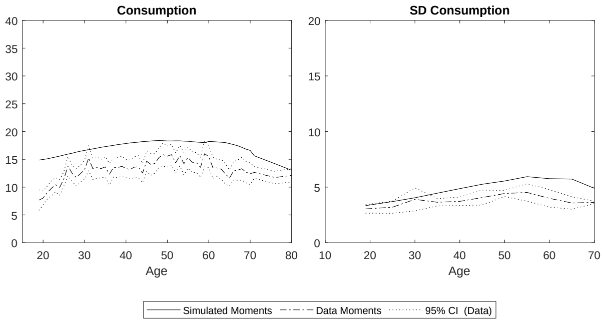
Some College (LBD)

Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Consumption's Fits

Some College (OJT)

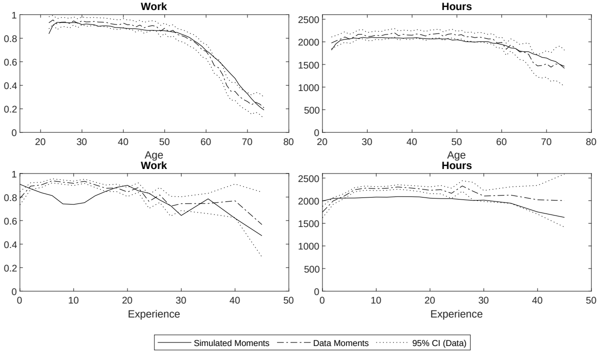


Dropout (LBD) | HS (LBD) | SC (LBD) | College (LBD)

Dropout (OJT) | HS (OJT) | SC (OJT) | College (OJT)

Labor Supply's Fits

Some College (OJT)

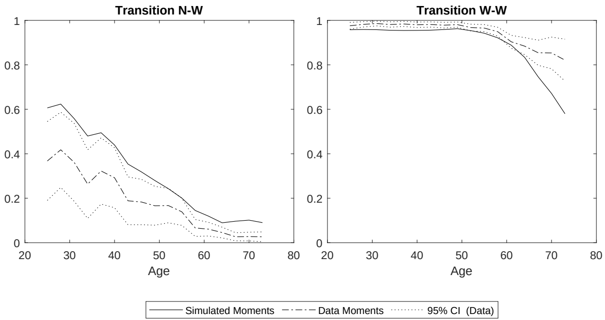


Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Transition to Work's Fits

Some College (OJT)



Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Wage's Fits

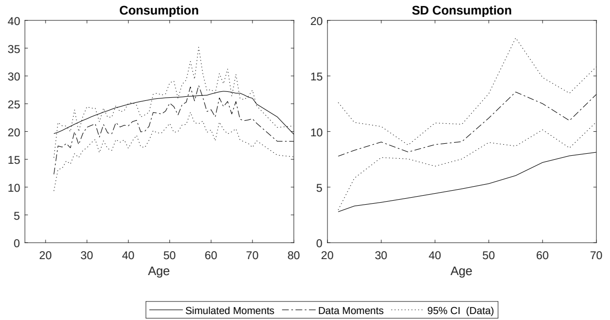
Some College (OJT)

Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Consumption's Fits

College (LBD)

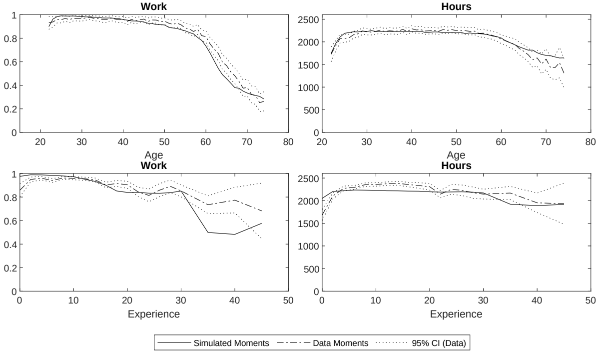


Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Labor Supply's Fits

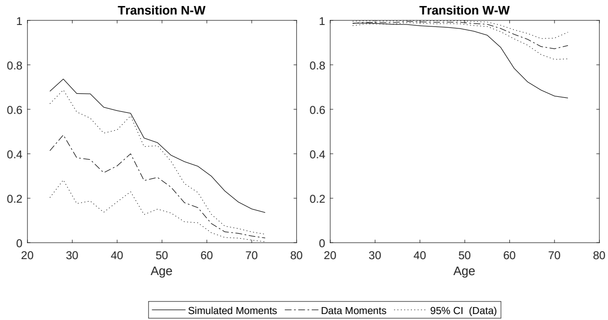
College (LBD)



Dropout (LBD) HS (LBD) SC (LBD) College (LBD)
Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Transition to Work's Fits

College (LBD)

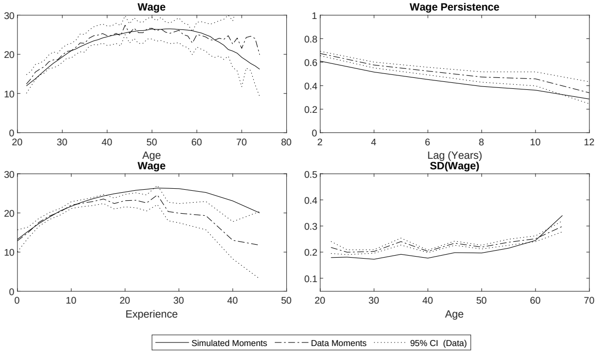


Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Wage's Fits

College (LBD)

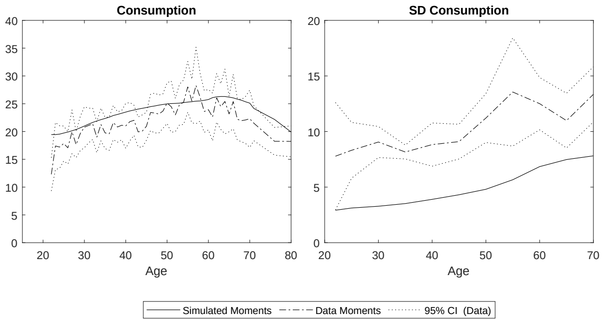


Dropout (LBD) | HS (LBD) | SC (LBD) | College (LBD)

Dropout (OJT) | HS (OJT) | SC (OJT) | College (OJT)

Consumption's Fits

College (OJT)

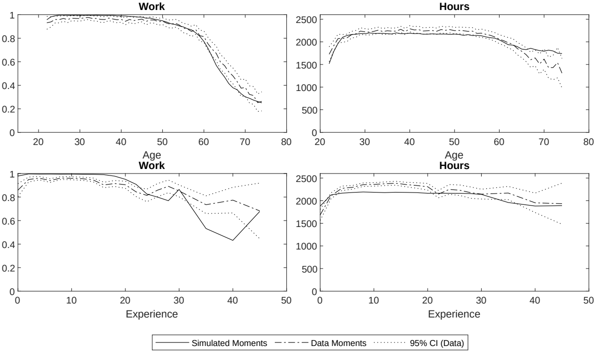


Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Labor Supply's Fits

College (OJT)

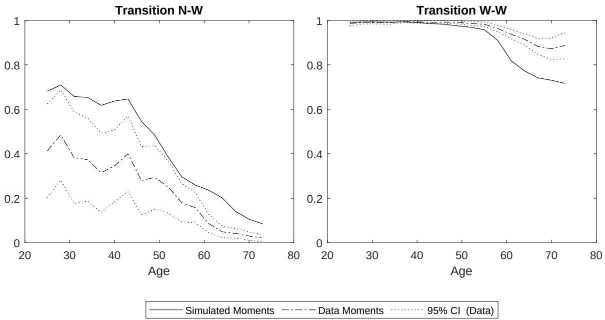


Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Transition to Work's Fits

College (OJT)

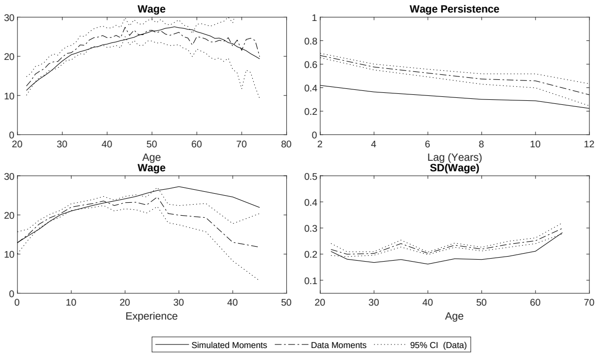


Dropout (LBD) HS (LBD) SC (LBD) College (LBD)

Dropout (OJT) HS (OJT) SC (OJT) College (OJT)

Wage's Fits

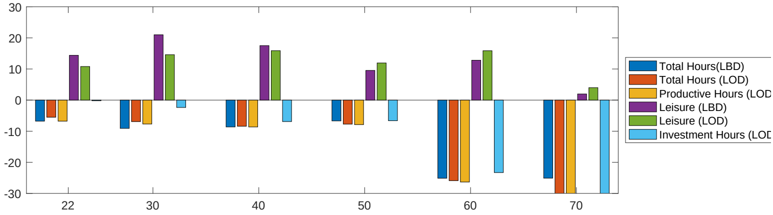
College (OJT)

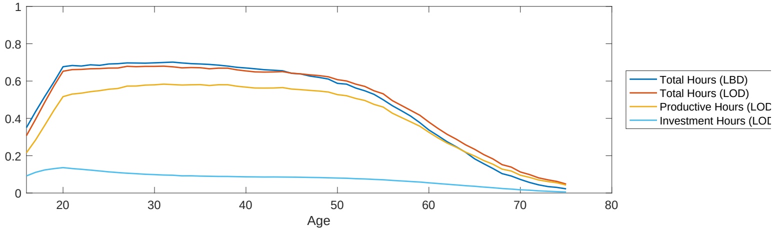


Dropout (LBD) | HS (LBD) | SC (LBD) | College (LBD)

Dropout (OJT) | HS (OJT) | SC (OJT) | College (OJT)

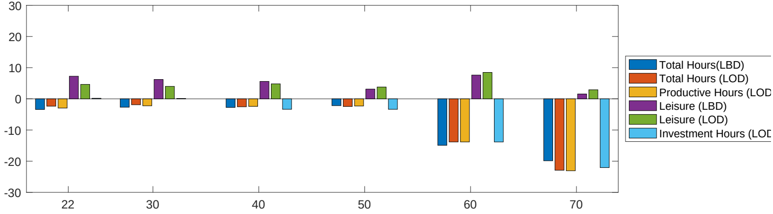
Responses (Dropout-Anticipated & Permanent [-10%])

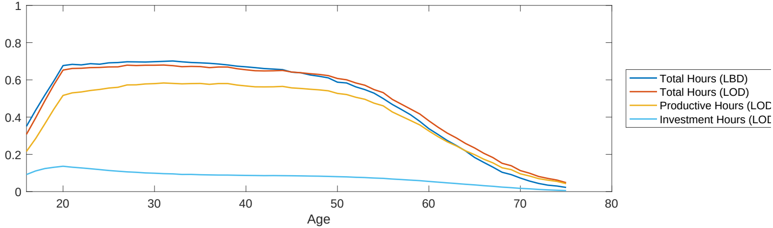




Dropout HS SC College

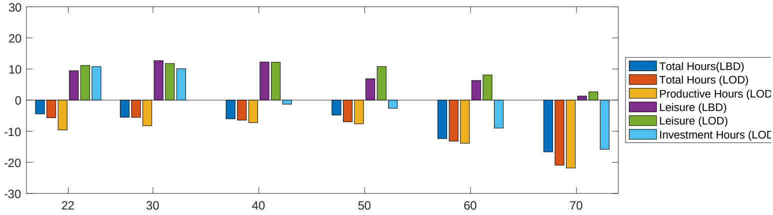
Responses (Dropout-Unanticipated & Permanent [-10%])

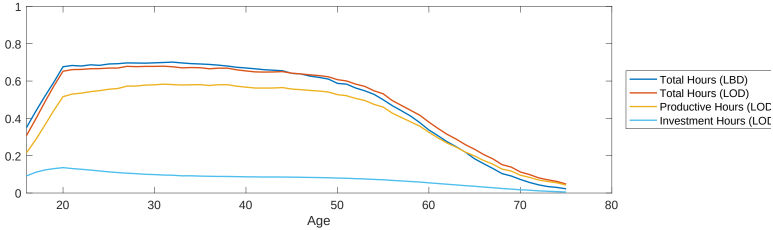




Dropout HS SC College

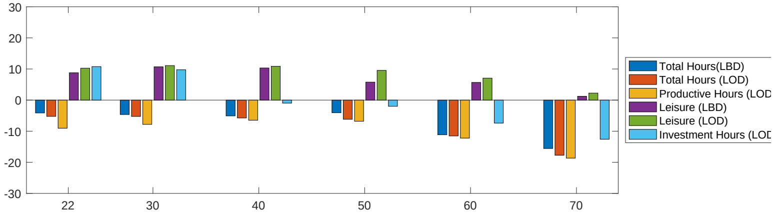
Responses (Dropout-Anticipated & Transitory [-10%])

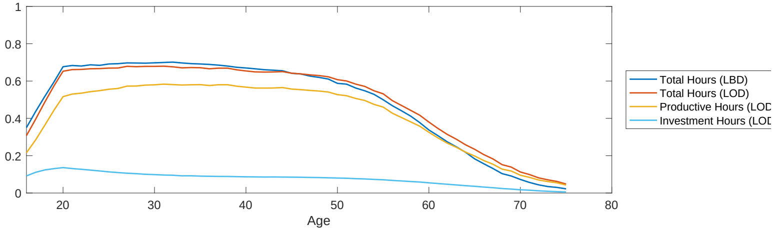




Dropout HS SC College

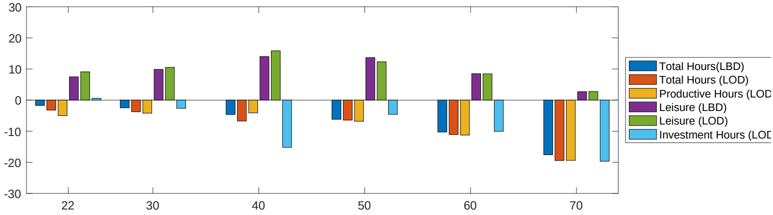
Responses (Dropout-Unanticipated & Transitory [-10%])

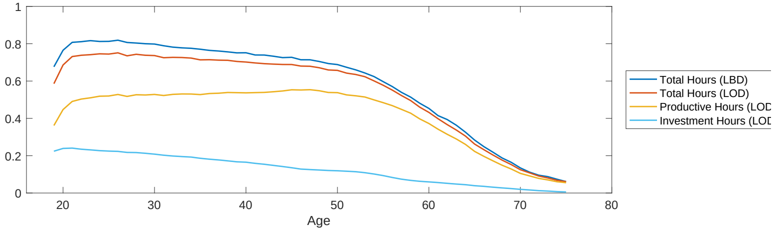




Dropout HS SC College

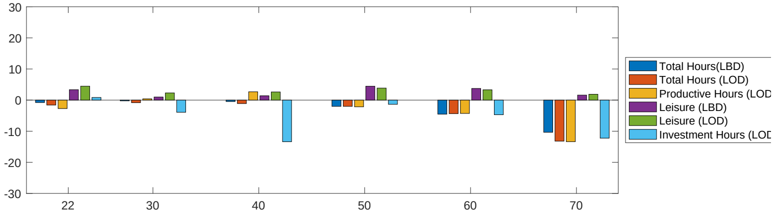
Responses (Some College-Anticipated & Permanent [-10%])

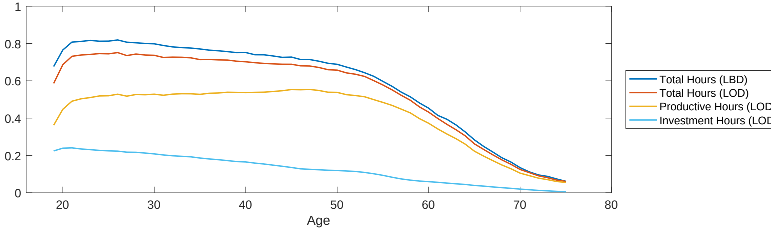




Dropout HS SC College

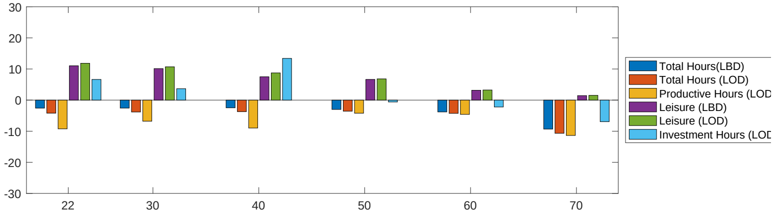
Responses (Some College-Unanticipated & Permanent [-10%])

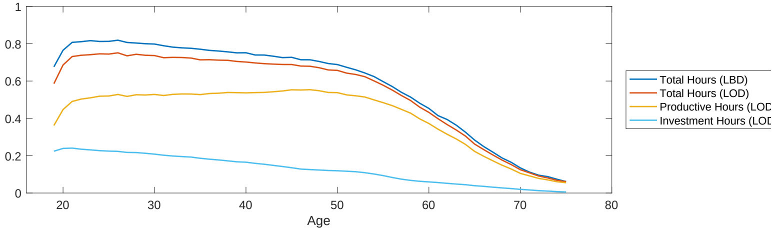




Dropout HS SC College

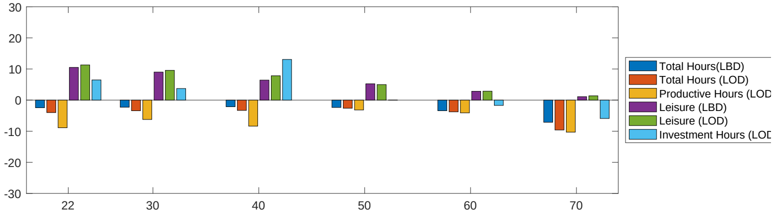
Responses (Some College-Anticipated & Transitory [-10%])

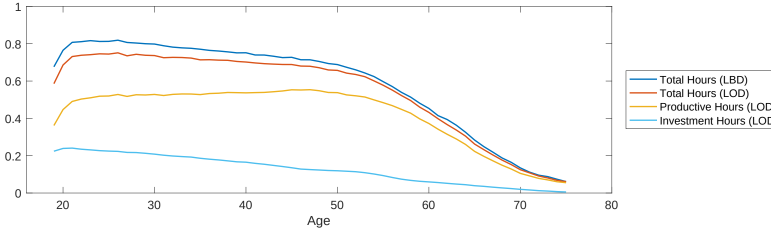




Dropout HS SC College

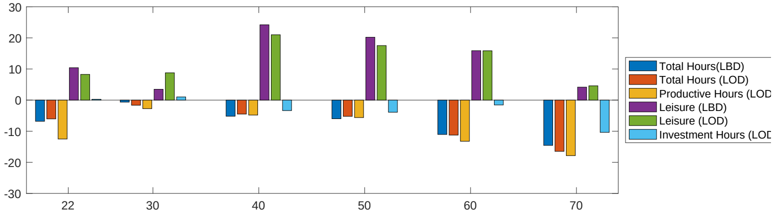
Responses (Some College-Unanticipated & Transitory [-10%])

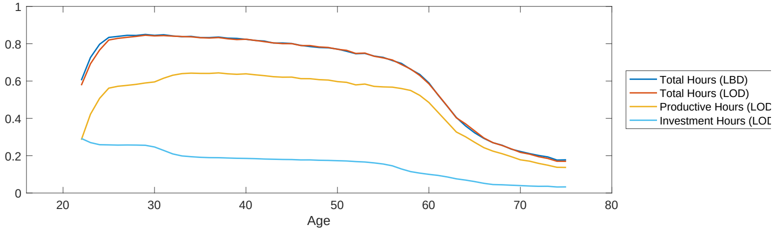




Dropout HS SC College

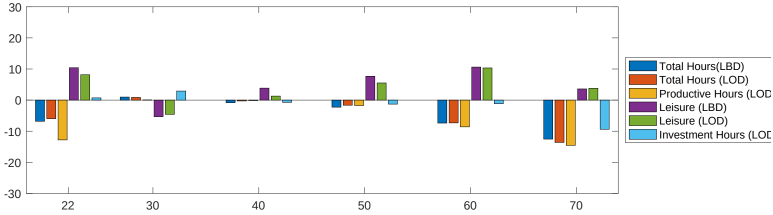
Responses (College-Anticipated & Permanent [-10%])

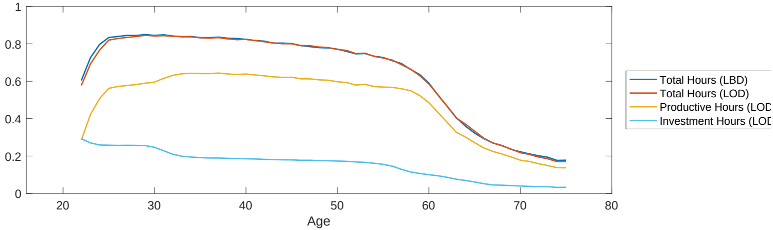




Dropout HS SC College

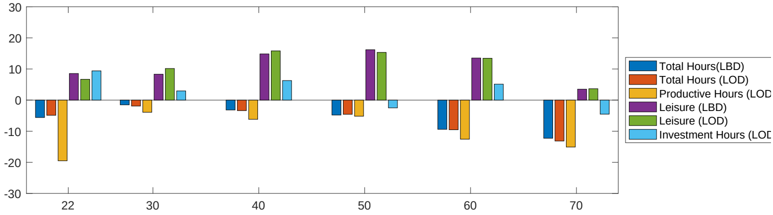
Responses (College-Unanticipated & Permanent [-10%])

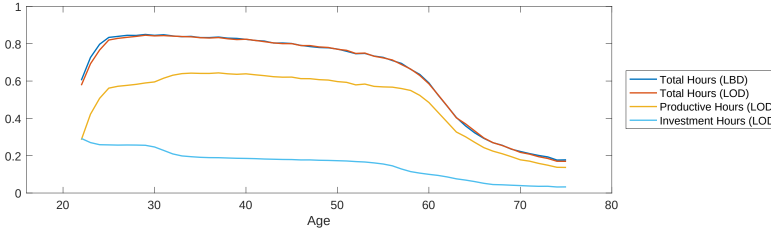




Dropout HS SC College

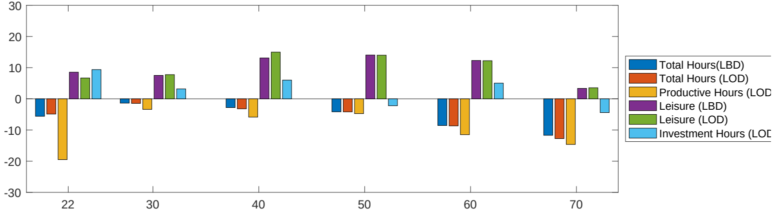
Responses (College-Anticipated & Transitory [-10%])

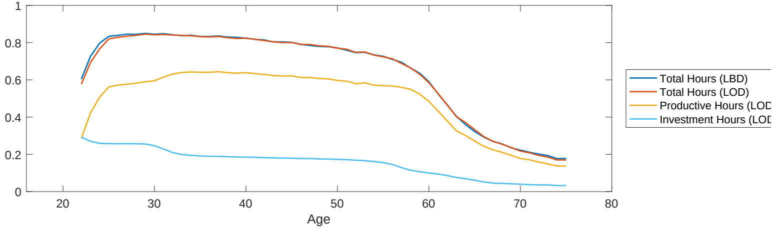




Dropout HS SC College

Responses (College-Unanticipated & Transitory [-10%])





Dropout HS SC College

Bellman's Equation in LBD

Model Specification

Utility: As explained we define the current utility as a function of consumption (c) and leisure (l):

$$\text{Utility function} \qquad u(c,l) = \log(c) \ + \frac{\theta}{\sigma} l^{\sigma} + \lambda.\phi(l)$$

Where $\phi(l)$ is a taste shock that only affects the leisure, and λ is the scale parameter. We normalize the current utility by the scale parameter:

$$\text{Normalized utility function} \qquad \hat{u}(c,l) = \frac{u(c,l)}{\lambda} = \frac{\log(c)}{\lambda} + \frac{\theta}{\sigma.\lambda} l^{\sigma} + \phi(l)$$

Bellman's Equation in LBD (Model Specification)

Utility at the end of the period, when all of the shocks (in the current period) are revealed, can be defined by $\overline{u}$:

End of Period u
$$\overline{u}\left(c, l, a, k, \phi\left(l\right), \epsilon_W, \epsilon_R\right) =$$
$$S.\left\{\hat{u}\left(c, l\right) + \beta V'\left(a', k'\right)\right\} + \left(1 - S\right).\beta.B\left(a'\right)$$

Where S is the survival probability and $B\left(a'\right)$ is the bequest as a function of the asset at the end of the current period. As mentioned, the asset evolves according to the following motion equation:

Bellman's Equation in LBD

Model Specification

Defining the transition functions:

Assets evolution	$a' = (1+r).\{a + w.h - P - c + TR\}$
	s.t. $h = 1 - l$
Wage function	$w = k.e^{\epsilon_W}.e^{\epsilon_R}$
Human capital evolution	$k' = \delta.k + \alpha.h^{\gamma}.k^{\nu}$

Bellman's Equation in LBD

Model Specification

We assume the decision toward consumption is made after the revelation of all three shocks (first wage shock, taste shock, and the second wage shock). Hence, maximizing $\overline{u}$ provides the maximum utility Z that can be achieved, given the states (a and k), shocks ($\phi(l)$, ϵ_W, and ϵ_R), and leisure decision:

$$Max(U) \qquad Z(l, \phi(l), a, k,\ \epsilon_W, \epsilon_R) = \max_{c} \overline{u}\ (c, l, a, k, \phi(l), \epsilon_W, \epsilon_R)$$

Bellman's Equation in LBD Model Specification

Moving backward in the current period, second wage shock (ϵ_R) hits. Hence this shock is not revealed at the time individuals decide about leisure. Integrating the ϵ_R out, we have:

$$X\left(l, a, k, \phi\left(l\right), \epsilon_W\right) = \int Z\left(l, \phi\left(l\right), a, k,\ \epsilon_W, \epsilon_R\right).f\left(\epsilon_R\right).d\epsilon_R$$

X provides the expected maximum utility for each state a and k, leisure level l, and the taste shock ϕ.

Bellman's Equation in LBD — Model Specification

Maximizing over X and integrating out the taste shock provides the utility given each state and first wage shock:

$$U(a, k, \epsilon_W) = \int \left(\max_{l} X(l, a, k, \phi(l), \epsilon_W) \right) .g(\phi(l)) .\phi(l)$$

While we can assume the first wage shock as state variable, we integrate it out as its value does not regulate the moving from one period to another and it is governed by random chance.

EMax function $\quad V(a, k) = \int U(a, k, \epsilon_W) .f(\epsilon_W) .\epsilon_W$

Bellman's Equation in OJT

Model Specification

Utility: We define the current utility as a function of consumption (c) and leisure (l):

$$\text{Utility function} \qquad u(c, l) = \log(c) + \frac{\theta}{\sigma} l^{\sigma} + \lambda.\phi(l)$$

Where $\phi(l)$ is a taste shock that only affects the leisure, and λ is the scale parameter. We normalize the current utility by the scale parameter:

$$\text{Normalized utility function} \qquad \hat{u}(c, l) = \frac{u(c, l)}{\lambda} = \frac{\log(c)}{\lambda} + \frac{\theta}{\sigma.\lambda} l^{\sigma} + \phi(l)$$

Bellman's Equation in OJT

Model Specification

Utility at the end of the period, when all of the shocks (in the current period) are revealed, can be defined by $\overline{u}$:

End of Period u
$$\overline{u}\left(c, l, a, k, \phi\left(l\right), \epsilon_W, \epsilon_R\right) =$$
$$S.\left\{\hat{u}\left(c, l\right) + \beta V'\left(a', k'\right)\right\} + \left(1 - S\right).\beta.B\left(a'\right)$$

Where S is the survival probability and $B\left(a'\right)$ is the bequest as a function of the asset at the end of the current period. As mentioned, the asset evolves according to the following motion equation:

Bellman's Equation in OJT
Model Specification

Defining the transition functions:

Assets evolution	$a' = (1+r).\{a + w.h - P - c + TR\}$
	s.t. $h = 1 - l$
Wage function	$w = k.e^{\epsilon_W}.e^{\epsilon_R}$
Human capital evolution	$k' = \delta.k + \alpha.h^{\gamma}.k^{\nu}$

Bellman's Equation in OJT   Model Specification

We assume the decision toward consumption is made after the revelation of all three shocks (first wage shock, taste shock, and the second wage shock). Hence, maximizing $\overline{u}$ provides the maximum utility Z that can be achieved, given the states (a and k), shocks ($\phi(l)$, ϵ_W, and ϵ_R), and leisure decision:

$$Max(U) \qquad Z\left(l, i, a, k, \ \epsilon_W, \epsilon_R, \phi(l)\right) = \max_{c} \overline{u} \ \left(c, l, i, a, k, \phi(l), \epsilon_W, \epsilon_R\right)$$

Bellman's Equation in OJT

Model Specification

Moving backward in the current period, second wage shock (ϵ_R) hits. Hence this shock is not revealed at the time individuals decide about leisure. Integrating the ϵ_R we have:

Integrating out the taste shock:

$$X\left(l, i, a, k, \phi\left(l\right), \epsilon_W\right) = \int Z\left(l, i, a, k, \phi\left(l\right),\ \epsilon_W, \epsilon_R\right).f\left(\epsilon_R\right).d\epsilon_R$$

X provides the expected maximum utility for each state (a, k), decisions about leisure l and training i and the taste shock ϕ.

Bellman's Equation in OJT Model Specification

We assume deciding about training is conditional on leisure decision, and hence should be made after leisure decision: $Max(U)$

$$W\left(l,a,k,\phi\left(l\right),\epsilon_W\right)=\max_i X\left(l,i,a,k,\phi\left(l\right),\epsilon_W\right)$$
$$\text{s.t. } 0\leq i\leq \min\left(\bar{I},\left(1-l\right)\right)$$

maximizing i (conditional on l among other variables) also determines the optimal level of h, (productive hours at work). Maximizing over W and integrating out the taste shock provides the utility given each state and first wage shock.

Integrating ϵ_R out:

$$U\left(a,k,\epsilon_W\right)=\int\left(\max_l W\left(l,a,k,\phi\left(l\right),\epsilon_W\right)\ \right).g\left(\phi\left(l\right)\right).\phi\left(l\right)$$

Bellman's Equation in OJT

Model Specification

Integrate the first wage shock out as its value does not regulate the moving from one period to another, and it is governed by random chance.

$$\text{EMax function} \qquad V(a,k) = \int U(a,k,\epsilon_W).f(\epsilon_W).\epsilon_W$$

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