

Welfare Dependence and Self-Control: An Empirical Analysis

MARC K. CHAN

University of Melbourne

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A hyperbolic discounting model of labour supply and welfare participation with heterogeneous time preference parameters is estimated. Exclusion restrictions are constructed from variations in behaviour induced by time limits in a welfare reform experiment. We find that most individuals are time-inconsistent, and they exhibit varying degrees of present bias and perception of the commitment problem. Introducing a welfare component to the tax system can make individuals worse off by aggravating the commitment problem. Certain dynamic policy interventions carry sizeable commitment-related work incentives; for instance, a dynamic sanction triggered by past employment can be preferred by some individuals as a commitment device.

Keywords: Welfare dependence, Hyperbolic discounting, Time limits, Female labour supply, Welfare reform experiment, Discrete choice dynamic programming

JEL Codes: I3, C3, J2

1. INTRODUCTION

It is widely believed that social welfare programmes carry incentives that can lead to dependence on the programmes. The nature of dependence is, however, often subject to interpretation by researchers and policymakers. Welfare programmes can generate work disincentives, inducing individuals to receive less earnings and rely on welfare benefits as the main source of income. Moreover, if the work disincentive is strong, income can drop, creating a “poverty trap” that is considered undesirable by policymakers. Plant (1984) sees welfare dependence as a phenomenon that is related to the dynamics of welfare programme participation. He defines the welfare trap as one in which previous participation in the programme increases the probability of current participation, *ceteris paribus*. While he sees addiction to welfare as a potential cause of the welfare trap, the nature and implications of the addiction are not further investigated.¹

The goal of this article is to examine welfare dependence from a unified perspective, with an emphasis on its relationship with self-control. In recent years, models of (quasi-)hyperbolic

1. Plant discussed several causes of welfare dependence, but they were not empirically distinguished in the article. For related discussions on welfare dependence, see for instance Bane and Ellwood (1983) and Gottschalk and Moffitt (1994).

discounting have become popular as a convenient tool for studying problems of addiction and self-control (*e.g.* Laibson, 1997; O'Donoghue and Rabin, 1999).² These models assume that agents exhibit present-biased preferences, and the intertemporal optimization problem can be viewed to consist of many autonomous selves, one in each period, with each self having a disproportionate tendency of immediate gratification. The misalignment between short-run and long-run goals gives rise to problems of commitment and self-control, and creates room for the design of policy instruments for their rectification. From a time-consistent perspective, the behaviour of a present-biased individual is suboptimal, as she overemphasizes immediate rewards and overlooks future rewards. The structure of the welfare system may exacerbate problems of commitment and self-control. With time-inconsistency, the normative implications are strikingly different and lean towards paternalism. Policies may be available to correct for “internalities” that the individual inflicts upon herself (*e.g.* Gruber and Koszegi, 2001).³

We use administrative data from a welfare reform experiment to estimate a heterogeneous hyperbolic discounting model of labour supply and welfare participation.⁴ By allowing for heterogeneity in discounting and self-control, the analysis considerably extends current field evidence on the estimation of time preference parameters. Individuals can have different discount factors, present bias, and self-perception of present bias (“naivety”). As a result, some individuals may be less time-consistent than the others, and their behavioural response to a commitment device can be different.⁵ This general specification is made possible by variations in behaviour induced by welfare time limits in the experiment. The time limit sets an upper bar on the stock of periods that an individual can receive welfare. For those who have not reached the limit, this creates an intertemporal trade-off in which welfare participation reduces the stock of future eligible periods. The time limit can only affect current decisions if people are forward-looking; in other words, it provides a natural exclusion restriction that affects future but not current payoffs. This is valuable because recent studies have shown that exclusion restrictions are often central to the identification of time preference (Magnac and Thesmar, 2002; Fang and Wang, 2015).⁶

2. Early research in this area can be traced back to Strotz (1956), Phelps and Pollak (1968), and Pollak (1968). Recent applications include consumption decisions (*e.g.* Harris and Laibson, 2002), as well as retirement and savings (*e.g.* Diamond and Koszegi, 2003; Gustman and Steinmeier, 2012).

3. By contrast, there are no internalities when preference is time-consistent. The individual chooses a time-consistent plan that maximizes her stream of utility. Therefore, a policy intervention that attempts to remove Plant's welfare trap is likely to be distortionary by nature.

4. The model contains three sources of dynamics: (1) work experience increases human capital; (2) there is state dependence in that lagged work and welfare participation affect the current disutilities from work and welfare; (3) for those in the treatment group, the time limit induces behavioural dynamics involving the stock of remaining welfare eligibility. Other features include unobserved heterogeneity in skill and preference, and a piecewise linear budget constraint that incorporates welfare, earned income tax credit (EITC) and income tax.

5. A commitment device is broadly defined as an arrangement entered into by an agent that restricts her future choices (or budget sets), with the aim of satisfying goals that are otherwise difficult due to intra-personal conflict (*e.g.* Bryan *et al.*, 2010). An individual who is time-consistent (or thinks she is time-consistent) will never adopt a commitment device.

6. To our knowledge, Mahajan and Tarozzi (2011) and Fang and Wang (2015) are the only studies in the hyperbolic discounting literature that use exclusion restrictions. Neither of the studies use policy to form exclusion restrictions. The empirical application in Fang and Wang (2015) considers a stationary model of whether to undertake mammography, estimated with a two-period panel. The instantaneous utility payoff of mammography depends on the individual's health status and income (payoff-relevant state variables). The exclusion variables do not affect the instantaneous payoff, but affect the transition of the payoff-relevant state variables. These include a set of demographic characteristics such as race, marital status, education, age, and whether the individual's mother is alive at the age of 70 years. Mahajan and Tarozzi (2011) estimate a three-period model of insecticide-treated net adoption, using elicited beliefs on time preferences as exclusion restrictions. Earlier studies follow a parametric approach (*e.g.* Laibson *et al.*, 2007; Paserman, 2008; Fang and Silverman, 2009).

We also identify policies that restrict the budget set but alleviate the commitment problem substantially, such that they increase the “long-run”, or time-consistent, utility of a present-biased individual. Targeting interventions are thus possible because these policies can be voluntarily adopted by present-biased individuals. In the literature, such policies have been a theoretical possibility, but they are difficult to establish from empirical models. For instance, Fang and Silverman (2009) find that time limits can generate large effects on employment, but they are too crude as a commitment device to increase the long-run utility of present-biased individuals.⁷

The estimation results provide strong evidence for present-bias and considerable heterogeneity in discounting and self-control. By comparing with a time-consistent heterogeneous discount factor model, the likelihood ratio test rejects the null hypothesis of time-consistency at the 1% significance level. The distribution of the present bias factor has a mean of 0.59 and standard deviation (s.d.) of 0.17. Individuals generally perceive themselves as time-inconsistent.

The simulation results focus on a variety of static and dynamic policy interventions. One focus is an “income support” programme, which adds a welfare component to the tax system by introducing a \$50 subsidy to single mothers who neither work nor use welfare. The programme improves government expenditure and utility by substituting for the more expensive welfare programme and enhancing the insurance role of the transfer system. However, this comes at the expense of employment and the commitment problem.

Dynamic tax/sanctions and work subsidies are proposed to improve incentives. A dynamic sanction triggered by past employment satisfies the following criteria: (1) make benefit rules stricter, (2) save the government money, and yet (3) make people better off. It induces sizeable commitment-related work incentives and increase utility, especially among the most present-biased individuals. Around one-sixth of control group individuals prefer the dynamic sanction regime. Welfare time limits are found to be attractive to some present-biased individuals, and work subsidies provide extra utility to individuals by mitigating their self-control problem. A revenue-neutral reform package, which consists of a welfare benefit reduction and a work subsidy, is then considered as a utility-improving solution. The results suggest that one may exploit time-inconsistency so as to make programme design more desirable.

This article proceeds as follows. Section 2 provides a brief summary of the institutional background. Section 3 describes the model and identification. Section 4 discusses the basic features of the data. Section 5 presents estimation and policy simulation results. Section 6 gives concluding remarks. Additional results are provided in Online Appendix (Chan, 2016).

2. WELFARE REFORM AND FAMILY TRANSITION PROGRAMME

In the late 1990s, the U.S. implemented a reform that resulted in sweeping changes in the welfare system for female-headed families. The Aid to Families with Dependent Children (AFDC) programme was a federal programme that provided cash assistance to low-income single mothers with children under 18 years of age. Under the reform, AFDC was replaced by the Temporary Assistance for Needy Families (TANF) programme. The main goals of the reform were to reduce dependence on the welfare system, and to help single mothers achieve self-subsistence via

7. Fang and Silverman (2009) use data from NLSY79 to estimate a model with present-biased preference. They consider a relatively parsimonious specification; for instance, work preference and the wage equation are not separately distinguished, and employment and welfare are considered as mutually exclusive options. They also simulate the effects of workfare (*i.e.* work requirements for welfare recipients). However, unlike time limits, which unambiguously reduce the generosity of the welfare programme, the workfare scenarios that they examine involve a subsidy that compensates for lost home production.

promoting work. Time limits were arguably the most controversial component of the TANF programme. Prior to the reform, some states implemented pilot initiatives, which allowed them to deviate from federal AFDC rules. These initiatives provided valuable information on the effects of policy elements that anticipated the TANF programme.

Family Transition Programme (FTP) was the first welfare reform initiative in which some families reached their time limit and had their benefits cancelled.⁸ FTP was a policy experiment that operated from 1994 to 1999 in Escambia, a mid-sized county in western Florida. From May 1994 through October 1996, welfare applicants were randomly assigned to the control or treatment group. The control group was subject to the rules of the AFDC programme, which had no welfare time limits. The treatment group was subject to a time limit which, by default, restricted families to 24 months of welfare receipt within any 60-month period. For individuals who (1) had received AFDC for at least 36 of the 60 months prior to random assignment, or (2) were under 24 years old, had no high school diploma, and had little recent work history, they were restricted to a maximum of 36 months of welfare receipt within any 72-month period.⁹ There were two other key policy differences. The treatment group was subject to more generous financial incentives that encouraged work—the first \$200 of earnings were disregarded in the calculation of welfare benefits, and a benefit reduction rate of 50% was applied to all earnings in excess of \$200.¹⁰ Welfare recipients in the treatment group also received enhanced employment services, while those in the control group received conventional AFDC services.

Figure 1 plots the programme benefit functions and marginal tax rates (MTR) faced by single mothers with one to two children. The calculation of welfare benefits also include food stamps due to categorical eligibility in the program; see Section 3 and the Appendix for further details. The benefits and MTRs vary widely by the level of gross earnings and family size. For instance, in 1995, among one-child families with earnings less than \$500, the MTR is close to -20 percent if they do not use welfare, but can reach beyond +40 percent if they receive welfare.¹¹ The differences are still substantial at higher earnings. For one-child families, the net benefit amount remains positive at earnings below \$1,200; the break-even point is around \$1,400 for two-child families. Welfare benefits are more generous in the treatment group due to increased earnings disregards. This creates further variation in the benefit and MTR at the lower range of earnings.

8. Although a main feature of TANF was the 5-year time limit of federal cash assistance, there were few pilot initiatives that implemented a time limit policy. Exceptions include Iowa, which implemented a time limit on an individual basis in 1993; Connecticut, which implemented a 21-month limit in the Jobs First programme in 1996. Using the Connecticut initiative, Bitler *et al.* (2006) find that earnings disregards generate heterogeneous effects that are consistent with the predictions of static labour supply theory. They do not consider the time limit. Using FTP, Grogger and Michalopoulos (2003) test for reduced-form predictions of the effects of time limit. They find that the time limit caused a larger reduction in welfare use among families with young children, a phenomenon that is consistent with the behaviour of liquidity constrained, forward-looking individuals who face earnings uncertainty.

9. Staff members had no discretion in assigning the time limit. In the data, slightly more than half of the treatment group were subject to the default time limit. The treatment group was well informed by staff members regarding the dynamic mechanism of the time limit (Bloom *et al.*, 2000).

10. The asset limit for welfare eligibility in the treatment group was also higher; see Section 5.7.1 for further discussion.

11. Due to eligibility thresholds, the benefit functions exhibit a small discontinuity at \$1,097 (\$6) and \$1,376 (\$31) of earnings for one-child and two-child families, respectively. In a continuous hours model, this can be a potential problem in estimation because the optimal hours are usually obtained by first order conditions. Depending on the econometric specification, the likelihood function may not be continuous in parameters. However, our model has only three discrete levels of labour supply. Given the wage, the individual simply evaluates income at *discrete* points on the budget constraint, and makes the choice based on the utility and preference shock at each point. Thus, the discontinuities should not undermine estimation as it might in a continuous hours model. In addition, not many individuals have earnings as high as the discontinuity level—when the eligibility thresholds are removed, the log-likelihood (evaluated at the same parameter estimates) changes minimally (≈ 0.01).

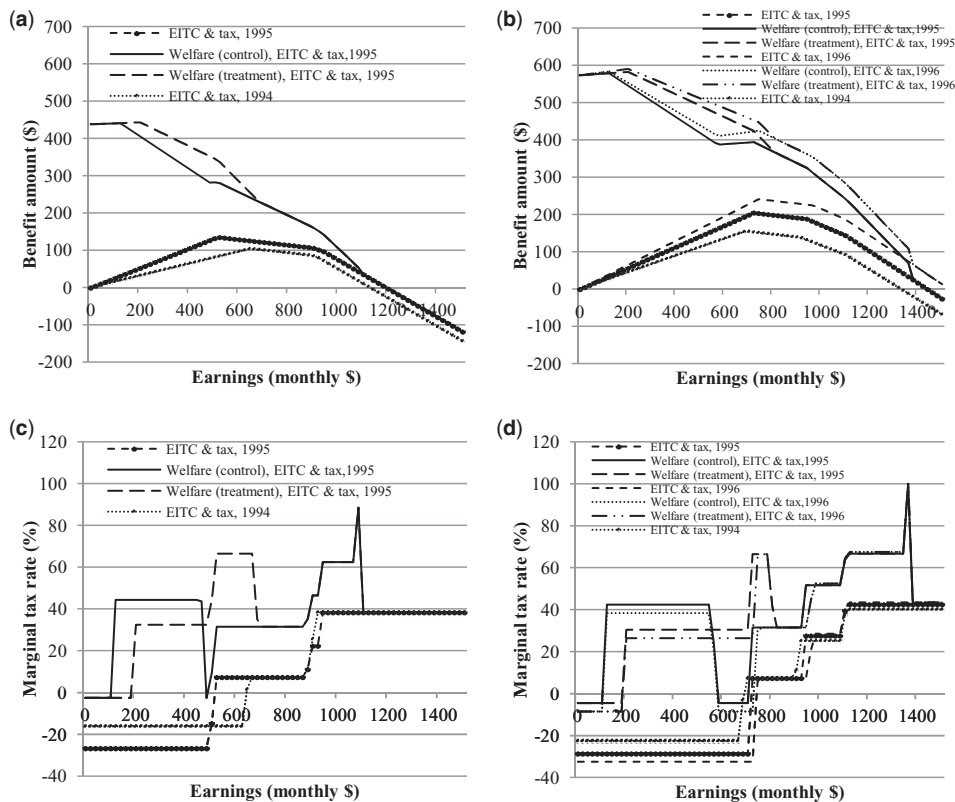


FIGURE 1

Programme benefit functions and marginal tax rates. (a) Benefit function, one child. (b) Benefit function, two children. (c) Marginal tax rate, one child. (d) Marginal tax rate, two children.

Another source of variation comes from the EITC expansion in the mid-1990s, which reduced the MTR at low earnings.

3. ECONOMIC MODEL

A discrete choice dynamic programming model is described below, and the estimation procedure is provided in the Appendix. In decision period t , individual i chooses her level of labour supply (h_{it}) and whether to participate in welfare (d_{ait}). The individual chooses one of the following elements of the choice set: (i) no work, no welfare ($h_{it}=0, d_{ait}=0$); (ii) part-time work, no welfare ($h_{it}=1, d_{ait}=0$); (iii) full-time work, no welfare ($h_{it}=2, d_{ait}=0$); (iv) no work, welfare ($h_{it}=0, d_{ait}=1$); (v) part-time work, welfare ($h_{it}=1, d_{ait}=1$). A full-time worker is assumed to work twice as many hours as a part-time worker.¹² Given the data structure, it is assumed that a decision period is a *quarter* (3 months). However, variables are expressed in monthly units unless

12. A full-time worker is assumed to work 160 h per month, which is normalized as $h_{it}=2$. Receiving welfare and working full-time is not considered as a choice in the model, as full-time workers are usually ineligible for welfare (e.g. Chan, 2013).

otherwise stated. To facilitate the discussion, the following dummy variables for employment and full-time work are defined: $d_{hit} = \mathbf{1}\{h_{it} > 0\}$, and $d_{hit}^{ft} = \mathbf{1}\{h_{it} = 2\}$. The “deterministic” part of the utility function has the following stylized form:

$$\begin{aligned} \bar{u}_{it} = & y_{it}(h_{it}, d_{ait}) + \alpha_{yi}[y_{it}(h_{it}, d_{ait})]^2 + \alpha_{hi}d_{hit} + \alpha_{h2}d_{hit}^{ft} + \alpha_{ai}d_{ait} + \alpha_{hai}d_{hit}d_{ait} \\ & + \gamma_h d_{hit}d_{hi,t-1} + \gamma_{h2}d_{hit}^{ft}d_{hi,t-1} + \gamma_a d_{ait}d_{ai,t-1} + \gamma_{ha}d_{hit}d_{ait}d_{hi,t-1}d_{ai,t-1}. \end{aligned} \quad (1)$$

The individual’s income (y_{it}) is determined by her choice of labour supply and welfare participation, as well as the shape of her budget constraint. The coefficient of the quadratic term of income is α_{yi} .¹³ Preferences for work and welfare participation are determined by parameters α_{hi} , α_{h2} , α_{ai} , and α_{hai} . There is state dependence in the preferences for work and welfare participation, which is determined by parameters γ_h , γ_{h2} , γ_a , and γ_{ha} . Income is determined by a piecewise-linear budget constraint¹⁴:

$$y_{it} = E_{it} + B_A(E_{it}, e_i, \mathbf{Z}_{Ai})d_{ait} + B_{AS}e_i d_{hit}d_{ai,t-1} + B_E(E_{it}, \mathbf{Z}_{Ei}) - T(E_{it}, \mathbf{Z}_{Ti}). \quad (2)$$

Income is equal to gross earnings E_{it} plus transfer benefits minus tax. Gross earnings is the product of the wage rate w_{it} and labour supply h_{it} , *i.e.* $E_{it} \equiv w_{it}h_{it}$. The programme benefit functions for welfare and EITC are denoted by $B_A(\cdot)$ and $B_E(\cdot)$, respectively. The welfare benefit amount varies by treatment status e_i due to FTP earnings disregards. The coefficient B_{AS} represents the effect of FTP employment service; if a treatment group individual uses welfare now, the cost of employment will be lower by B_{AS} dollars next period. The tax function $T(\cdot)$ consists of income and payroll tax. The functions are parameterized by “programme benefit rules” \mathbf{Z}_{Ai} , \mathbf{Z}_{Ei} , and \mathbf{Z}_{Ti} , which vary by family size. The Appendix contains formal definitions of the formulas.

The welfare benefit function, $B_A(\cdot)$, includes food stamps, which is an important source of income for this population. Despite being a separate federal programme, almost all welfare participants receive food stamp benefits, due to the fact that welfare participants are automatically granted food stamps and do not need to apply for it. Among single mothers who do not receive welfare, the food stamp participation rate among the eligible has historically been far below 100%.¹⁵ Following the convention in the literature, we incorporate food stamps into $B_A(\cdot)$, and assume that individuals do not receive food stamp benefits when they do not use welfare (*e.g.* Hoynes, 1996; Fang and Silverman, 2009; Keane and Wolpin, 2010).¹⁶

13. The marginal utility of income is assumed to be zero when $y_{it} > -1/2\alpha_{yi}$.

14. The individual is assumed to consume all her income each period. See Section 5.7.1 for a relaxation of this assumption. An exception among similar structural models is Blundell *et al.* (2016), who focus on a broader population. The literature finds extensive evidence that low-income single mothers have very little assets (*e.g.* Hurst and Ziliak, 2006). Moreover, there is very limited access to credit (*e.g.* Sullivan, 2008).

15. When individuals leave welfare or exhaust their welfare time limit, they also lose their food stamp benefits automatically. To continue receiving food stamps, these individuals need to separately apply for the program. Using a nationally representative sample from the Survey of Income and Program Participation, Chan (2013) documents that between 1992 and 1999, the proportion of single mothers receiving both welfare and food stamps dropped from around 30% to 10%, while the proportion of single mothers receiving food stamps only remained stable at around 15%. As a result, many individuals became “disconnected”—received neither welfare nor food stamps—during the period.

16. As a result, the estimates on disutilities related to non-welfare choices may appear larger—among individuals who do not receive welfare, some do so because they receive food stamps. This undermines counterfactuals that are directly related to food stamps. An alternative approach is to allow all individuals who do not use welfare to receive food stamp benefit $B_{Fit}(\cdot)$ (see formula in the Appendix). However, this approach is controversial because it implies universal food stamp participation. In Online Appendix, we present estimation results from this model, which adds $B_{Fit}(\cdot)$ to equation (2), and report the estimated distributions of time preference parameters.

The wage follows a log-normal distribution:

$$\ln w_{it} = X_{wi} \psi_w + \omega_0 \mathcal{E}_{it} + \omega_1 X_{wi}^{ed} \mathcal{E}_{it} + \mu_{wi} + \epsilon_{wit}. \quad (3)$$

The wage equation consists of a vector of covariates X_{wi} that includes the unit constant, race (=1 if non-white), and education (=1 if grade 12 or above). The wage rate is a function of post-random-assignment work experience, which is denoted by $\mathcal{E}_{it} \equiv \sum_{s=1}^{t-1} d_{his}$, $\mathcal{E}_{i1} = 0$. The marginal return of work experience depends on the individual's education ($X_{wi}^{ed} = 1$ if grade 12 or above). There is an unobserved permanent component μ_{wi} , which will be discussed below. The logarithm of the wage rate is subject to a normally distributed shock ϵ_{wit} , which has zero mean and s.d. σ_{wi} , and is independent of other shocks in the model.

Denote the index representation of the choice set by $k \in \{1, 2, 3, 4, 5\}$. The utility of alternative k is the sum of the “deterministic” part $\bar{u}_{it}(k)$ (equation (1)) and a choice-specific preference shock ϵ_{cikt} :

$$u_{ikt} = \bar{u}_{it}(k) + \epsilon_{cikt}. \quad (4)$$

The vector of choice-specific shocks, denoted ϵ_{cit} , is assumed to follow an i.i.d. extreme value distribution with means at Euler's constant and s.d. at $(\pi/\sqrt{6})\sigma_{ci}$, where $\pi/\sqrt{6} \approx 1.2825$ is a normalization constant.

3.1. Intertemporal optimization problem

The individual is assumed to have (β, δ) -preference, or quasi-hyperbolic preference, which is potentially time-inconsistent (e.g. Phelps and Pollak, 1968; Laibson, 1997; O'Donoghue and Rabin, 1999). The parameter δ is the (standard) discount factor, and β is called the present bias factor, which captures the individual's short-term impatience. In this model, utility is discounted by a factor of $1 - \beta\delta$ next period, then by a factor of $1 - \delta$ in each subsequent period. For instance, for an individual in period τ , the discounted stream of utility from period τ to τ' is

$$u_{ik_\tau\tau} + \beta_i \sum_{t=\tau+1}^{\tau'} \delta_i^{t-\tau} u_{ik_t t}, \quad (5)$$

where k_t denotes the individual's choice in period t , and $u_{ik_t t}$ is the utility in period t (given realizations of shocks) when the individual's choice is k_t .

When the present bias factor is equal to one, the model reduces to the standard model with exponential discounting, in which the individual exhibits time-consistent preference. When the present bias factor is strictly less than one, the individual exhibits present-biased preference, which is time-inconsistent. The rate of discounting is disproportionately larger next period than in subsequent periods, reflecting a tendency of immediate gratification. Since the rate of discounting is no longer independent of the delay period, the individual has imperfect ability to commit to a certain action. The intertemporal optimization problem can be viewed to consist of many autonomous selves, one in each period, that act in their own interests. When the current self makes decisions, she potentially takes into account of the strategic relationship between the current self and future selves. Therefore, the current self's perception of her future selves also matters in the decision-making process.

To illustrate the state space and law of motion of state variables, the intertemporal optimization problem of the time-consistent model is described first. Then, the quasi-hyperbolic discounting model is introduced as the baseline model of the analysis.

3.1.1. Time-consistent preference. The maximization problem can be written in the following recursive form:

$$V_{it}(S_{it}, \epsilon_{it}) \equiv \max_{d_{it} \in D} \sum_{k=1}^5 d_{ikt} (u_{ikt} + \delta_i E_t V_{i,t+1}(S_{i,t+1}, \epsilon_{i,t+1})). \quad (6)$$

The value function $V(\cdot)$ has two sets of state variables. The error space $\epsilon_{it} \equiv (\epsilon_{cit}, \epsilon_{wit})$ consists of preference and wage shocks that are integrated out by the expectation operator in each period of the backward recursion procedure.¹⁷ The deterministic part of the state space S_{it} is carried around explicitly as an argument in the expected value function, and evolves according to a law of motion. For control group members, $S_{it} = (d_{hi,t-1}, d_{ai,t-1}, \mathcal{E}_{it})$. Given the state variables, the individual chooses the best alternative in the choice set D . For control group members, the choice set is $D_0 \equiv \{d_{it} | \sum_{k=1}^5 d_{ikt} = 1\}$, where d_{it} is a five-dimensional vector with $d_{ikt} \in \{0, 1\}$ as the k th element.¹⁸

For treatment group members, the dynamic programming problem is more complicated due to the presence of welfare time limits. The individual becomes ineligible for welfare when the cumulative number of periods of welfare participation since random assignment (M_{it}) reaches the limit \bar{M}_i . The choice set is D_0 if the individual has not reached the limit, and it is $D_1 \equiv \{d_{it} | \sum_{k=1}^3 d_{ikt} = 1\}$ otherwise. The time limit introduces a state variable with the following law of motion:

$$M_{i,t+1} = M_{it} + d_{ait}, \quad \text{and} \quad M_{i1} = 0. \quad (7)$$

When the individual reaches the time limit and becomes ineligible for welfare, she becomes eligible again after \bar{m} periods of ineligibility (with m_{it} being the counter), and M_{it} is then reset to zero.¹⁹ Therefore, for treatment group members, $S_{it} = (d_{hi,t-1}, d_{ai,t-1}, \mathcal{E}_{it}, M_{it}, m_{it})$.

3.1.2. Present-biased preference. The solution of the intertemporal optimization problem is the equilibrium of an intrapersonal game that can be solved by backward recursion. The full description is given in the Appendix. The problem can be thought of as having multiple autonomous selves, with one self in each period. Each self optimizes her behaviour and is potentially aware that her future selves may act in their own interests. In the recursion procedure,

17. The recursive procedure is described in detail in the Appendix. When the individual solves the dynamic programming problem, she perceives the variables outside S_{it} and ϵ_{it} to remain unchanged over time.

18. Welfare remains part of the choice set when the benefit amount is zero (*e.g.* when the individual has a high wage that period). This avoids the likelihood function from becoming degenerate for some individuals who are recorded as having welfare but the benefit is zero.

19. The laws of motion are:

$$m_{i,t+1} = \begin{cases} m_{it} + 1 & \text{if } M_{it} = \bar{M}_i, \\ 0 & \text{if } M_{it} < \bar{M}_i, \end{cases} \quad (8)$$

$$m_{i1} = 0; \quad M_{i,t+1} = 0 \quad \text{and} \quad m_{i,t+1} = 0 \quad \text{if } m_{it} = \bar{m}. \quad (9)$$

We adopt the approach of Chan (2013) in modelling the periodic nature of the FTP time limit. The length of ineligibility is set at twelve periods (*i.e.* twelve quarters, or 36 months) to mimic FTP policy. The time horizon, defined as the number of periods remaining (measured in period 1) until the individual's youngest child reaches 18 years of age, is set at forty-two periods. Sensitivity analysis shows that the length of the time limit \bar{M}_i is much more important than \bar{m} or the time horizon in determining behaviour.

the analogy of the value function is the “continuation long-run utility” that, with a slight abuse of notation, is described as follows:

$$v_{it}(S_{it}, \epsilon_{it}; \delta_i, \tilde{\beta}_i) = u_{ikt} + \delta_i E_t v_{i,t+1}(S_{ik,t+1}, \epsilon_{i,t+1}; \delta_i, \tilde{\beta}_i). \quad (10)$$

The parameter $\tilde{\beta}_i$, called the “naivety factor”, is the individual’s perception of the present bias factor of her future selves. Two special cases—“sophisticated” and “naive” present-biased agents—can be made (*e.g.* Strotz, 1956; Pollak, 1968; O’Donoghue and Rabin, 1999; Fang and Silverman, 2004, 2009). An individual is said to be “sophisticated” if her perceived present bias factor is correct, *i.e.* $\tilde{\beta}_i = \beta_i$. She is said to be “naive” if she believes that her future selves are time-consistent, *i.e.* $\tilde{\beta}_i = 1$. In practice, an individual can be neither sophisticated nor naive, *i.e.* $\tilde{\beta}_i \neq \beta_i$ and $\tilde{\beta}_i \neq 1$.²⁰

Because the perception may reflect the individual’s awareness, confidence, or optimism about her self-control problem in the future, the behavioural implications can be different. For instance, a naive present-biased agent does not adjust her behaviour in response to her inability to commit, and she never adopts a commitment device. By contrast, a non-naive (or sophisticated) agent takes into account of the extra incentives generated by the commitment problem. She may adopt a commitment device that constrains the budget sets of her future selves.

The solution procedure first computes the continuation long-run utility in all periods using backward recursion. Then, at period t , the individual’s optimal decision is

$$\kappa_{it}^*(S_{it}, \epsilon_{it}) \equiv \operatorname{argmax}_{d_{it} \in D} \sum_{k=1}^5 d_{ikt} \left(u_{ikt} + \beta_i \delta_i E_t v_{i,t+1}(S_{ik,t+1}, \epsilon_{i,t+1}; \delta_i, \tilde{\beta}_i) \right). \quad (11)$$

Note that this decision is influenced by the present bias factor β_i .

3.2. Heterogeneity

The wage equation in equation (3) and the following parameters are subject to heterogeneity²¹:

$$\begin{aligned} \alpha_{hi} &= X_i \psi_h + \mu_{hi}; & \alpha_{ai} &= X_i \psi_a + \mu_{ai}; & \alpha_{hai} &= X_i \psi_{ha} + \mu_{hai}, \\ \delta_i &= 1 / (1 + \exp(-(X_i \psi_\delta + \mu_{\delta i}))); & \beta_i &= X_i \psi_\beta + \mu_{\beta i}; & \tilde{\beta}_i &= X_i \psi_{\tilde{\beta}} + \mu_{\tilde{\beta} i}. \end{aligned} \quad (12)$$

The other parameters that vary in the population are $\alpha_{yi} = X_i \psi_y$, $\sigma_{ci} = \exp(X_i \psi_{\sigma_c})$, and $\sigma_{wi} = \exp(X_{wi} \psi_{\sigma_w})$. The vector of covariates X_i includes the unit constant, family size (=1 if more than one child), race (=1 if non-white), and education (=1 if grade 12 or above). Unobserved heterogeneity enters into the model in the form of discrete types, so the unobserved permanent components follow a discrete probability distribution with finite points of support (*e.g.* Heckman and Singer, 1984). In the model there are two skill/wage types (wtype = 1, 2) and two preference types (ptype = 1, 2), leading to a total of four types of individuals. For a (wtype = q_w , ptype = q_p) individual, let her unobserved characteristics be represented by the vector $\mu_{q_w, q_p} \equiv$

20. See, for instance, O’Donoghue and Rabin (2001) who examine the theoretical implications of “partial naivety”, in which $\beta < \tilde{\beta} < 1$.

21. The logistic specification ensures that the discount factor lies in the unit interval. An individual is said to be future-biased (“preproportion”, a time-inconsistent tendency to accelerate unpleasant tasks for mid-term goals) if the present bias factor is larger than 1. This does not affect the validity of the backward recursion procedure because the discounting is still carried out as $1, \beta\delta, \beta\delta^2, \dots$, *i.e.* more distant future periods are still discounted relative to the present.

(μ_{wq_w}, μ_{pq_p}) , where $\mu_{pq_p} \equiv (\mu_{hq_p}, \mu_{aq_p}, \mu_{haq_p}, \mu_{\delta q_p}, \mu_{\beta q_p}, \mu_{\tilde{\beta} q_p})$. The unobserved characteristics of the first skill/wage type and first preference type are normalized to zero, *i.e.* $\mu_{w1} = 0$ and $\mu_{p1} = 0$.

For each individual, her probability associated with each unobserved type can depend on a vector of covariates \mathbf{x}_i . This includes demographic characteristics as well as her “initial conditions”, which are proxied by measures of work experience and welfare use during the pre-random-assignment period. These information allow the distribution of unobserved heterogeneity to be captured more precisely.²² Details of the specification are given in the Appendix.

3.3. Identification

This section first discusses the intuitive arguments for the identification of time preference parameters under time limits, followed by a discussion of other policy features. A more formal analysis is provided in Online Appendix.

The theoretical rationale for the identification of time preference is based on the following literature. In an early article, Rust (1994) shows that the discount factor in discrete choice dynamic programming models is generally underidentified. Magnac and Thesmar (2002) show that this issue can be resolved if exclusion restrictions are used. Fang and Wang (2015) show further that under general conditions, exclusion restrictions can also be used for the identification of the present bias factor and naivety factor in models with hyperbolic discounting. They focus on a particular type of exclusion restrictions, namely, exclusion variables that affect the transition probabilities of states over time, but do not affect the per-period utility function. Because the transition probabilities are affected by time preference parameters via an intertemporal trade-off, variations in such exclusion variables can provide a source of variation in transition probabilities that reveal how the individual discounts the future.

In the model, the key exclusion restriction is the number of remaining periods of welfare eligibility under the time limit (*i.e.* $\bar{M}_i - M_{it}$, which is denoted by \tilde{M}_{it}).²³ For instance, consider two treatment group individuals who have different positive numbers of periods of remaining eligibility, but are otherwise identical. The exclusion restriction does not affect their current payoffs. However, their conditional choice probabilities will be different, because the exclusion restriction affects the expected future component of the value function.

Note that even if there is no control group, time preference can still be identified using the above exclusion restriction; the control/treatment difference is not essential for identification. However, the control group can increase efficiency by generating additional variation in \tilde{M}_{it} . In the treatment group, \tilde{M}_{it} only varies over $\bar{M}_i, \bar{M}_i - 1, \dots, 1, 0$. Because control group individuals are not subject to time limits, effectively $\tilde{M}_{it} = \infty$ in the control group. This provides a nice source of variation that helps pin down the time preference parameters more precisely.

22. For instance, when there is unobserved heterogeneity in wage, those individuals who worked more during the pre-random-assignment period will (somewhat mechanically) be more likely to belong to a high-skill type. The vector \mathbf{x}_i includes a binary indicator of whether the individual has ever had more than 6 months of full-time work prior to random assignment; total periods of employment in the 2 years prior to random assignment (measured in half years); a binary indicator of whether the individual was on welfare for at least 36 of the 60 months prior to random assignment. Note that pre-random-assignment outcomes are called “initial conditions” because they are outcomes prior to the first decision period of the model. This definition does not apply in a broader model that covers the entire life cycle.

23. In the labour literature, few studies have used exogenous policy variations to form exclusion restrictions that identify the discount factor. One exception is Ferrall (2012). In his analysis, treatment group members in the Self-Sufficiency Project (SSP) will be eligible for a wage subsidy if the following one-time event occurs—find a full-time job and go off income assistance within 1 year following random assignment. Differences in behaviour between the control and treatment groups within that period allow the discount factor to be recovered.

In the analysis sample, there are multiple values of \tilde{M}_{it} which the researcher can exploit to infer about time preference. Treatment group individuals who face the default time limit ($\bar{M}_i = 8$) have a range of \tilde{M}_{it} between 1 and 8 during the sample period. For those who face the longer time limit ($\bar{M}_i = 12$), the range of \tilde{M}_{it} is between 1 and 12. If the model is time-consistent, the discount factor δ will be overidentified à la Magnac and Thesmar (2002). In the case of present-biased preference, more exclusions are needed for the identification of the discount factor δ , present bias factor β , and naivety factor $\tilde{\beta}$. With multiple values of \tilde{M}_{it} , there are sufficient exclusions to identify these parameters (Proposition 2, Fang and Wang, 2015).

The time limit generates the following behavioural incentives under present-biased preference. First, the individual underrates the option value of retaining welfare eligibility for future use. Secondly, the time limit changes the pre-existing work commitment problem, in which she underrates the benefit of work experience accumulation—this can generate two extra effects among non-naive individuals.²⁴ On the one hand, the imposition of a time limit alleviates the work commitment problem, which creates an incentive to work now (Fang and Silverman, 2004). On the other hand, she has an incentive to receive welfare now, as this can alleviate the work commitment problem next period by making the next-period self more constrained by the time limit.²⁵ Thus, data on employment and welfare use combinations can provide useful information on the magnitude of present bias and naivety.

The role of other policy features are described as follows. The policy structure in the data can generate independent relative variations in income y between the following choices and the base alternative. (1) Part-time (PT)/Full-time (FT) work, no welfare ($k = 2, 3$): the EITC and income tax formulas form a piecewise linear budget constraint with different marginal tax rates at different earnings. The formulas vary by family size. Moreover, the EITC expansion tends to change PT and FT income disproportionately. (2) No work, welfare ($k = 4$): the maximum welfare benefit varies by family size. (3) Work, welfare ($k = 5$): besides EITC and income tax, the welfare formula also contributes to the piecewise-linearity of the budget constraint. The benefits and marginal tax rates differ by family size.

These features allow the effect of policy interventions related to each choice to be identified.²⁶ In addition, the welfare earnings disregards differ between the control and treatment groups; this extra variation in the budget constraint is useful, but not essential for identification. All the above features are linked in the structural model via a normalization of income in the utility function and budget constraint, and formally defined benefit formulas.²⁷

24. Fang and Silverman (2004) discuss the theoretical implications of the work commitment problem and its relationship with the time limit. The option value is not discussed, partly due to the absence of uncertainty in their model.

25. Both effects reconcile with seemingly surprising results in Fang and Silverman (2004), who find that the imposition of a time limit will trigger some sophisticated individuals who were working in the absence of time limits to take welfare instead under a time limit. Also note a special case when there is only one period of eligibility left—if the individual receives welfare now, she will become ineligible for welfare in the future.

26. Although PT/FT work status is not observed in the data, PT/FT utility parameters can still be identified using the above policy variations in the budget constraint, and the distribution of *observed earnings* in the data. For more details, see the estimation section in the Appendix. Another approach, which is not pursued in this article, is to simply restrict FT disutility to be twice the PT disutility.

27. As a result, the strength of the relationship between income and choice is captured by the s.d. of the choice shock. The income effect, captured by α_{yi} , can be identified by parallel regions (e.g. shifts due to the EITC expansion) in the budget constraint. The model allows wages to vary with preferences via observables and unobserved types, which are identified by the panel structure of the data. See for instance Chan (2013), who discuss the overidentifying restrictions and assumptions on the identification of utility and wage parameters.

TABLE 1
Summary statistics of selected variables

Variable	Control group		Treatment group		Difference ^a
	Mean	Std. dev.	Mean	Std. dev.	
Highest grade completed	11.1	1.5	11.0	1.6	-0.1
Race (white = 1, %)	44.8	49.7	45.0	49.7	0.2
Number of children under 18	2.1	1.0	2.0	1.0	-0.1
Age of youngest child (years)	5.1	4.3	4.9	4.1	-0.2
Ever worked full-time 6 months or more (%)	58.5	49.3	57.6	49.5	-0.9
On welfare for at least 36 of past 60 months prior to random assignment (%)	32.9	47.0	29.8	45.8	-3.1
During the last 2 years prior to random assignment					
Total months of welfare receipt	14.1	9.0	13.7	9.2	-0.4
Total quarters of employment	2.0	2.5	2.0	2.4	0.0
Total earnings (\$)	2,878.0	5,707.4	2,682.0	5,106.2	-196.0
During the sample period after random assignment					
Fraction of time employed (%)	40.0	35.3	45.6	35.6	5.6***
Fraction of time using welfare (%)	48.3	34.1	44.7	32.2	-3.6**
Fraction of time choosing					
No work, no welfare (%)	24.9	30.5	27.2	32.1	2.3*
Work, no welfare (%)	26.8	30.4	28.1	29.7	1.3
No work, welfare (%)	35.1	32.6	27.2	27.5	-8.0***
Work, welfare (%)	13.2	17.5	17.5	21.0	4.3***
Earnings (workers only, \$) ^b	644.4	443.9	646.5	433.7	—
Number of individuals	1,079	—	1,060	—	—

^aTwo-tailed *t*-test on difference in means between control and treatment group individuals. *Significant at the 10% level; **significant at the 5% level; ***significant at the 1% level.

^bBased on all worker observations in the sample. Earnings expressed at a monthly level.

4. DATA

The analysis sample is constructed from data provided by Manpower Demonstration Research Corporation (MDRC), which administered the policy experiment. The MDRC data contains individuals who were randomly assigned between May 1994 and February 1995. It includes individuals' demographic characteristics at the time of random assignment, and administrative records on welfare participation (by month) and earnings (by quarter). Both welfare and earnings data are realigned as quarterly variables, with the first quarter defined as the quarter in which random assignment occurred.²⁸ The analysis sample consists of 2,139 single mothers, of whom 1,079 are in the control group and 1,060 are in the treatment group.²⁹ For each individual, the first twelve quarters of data following random assignment are typically used, which results in a total of 25,164 observations.³⁰

Table 1 reports the summary statistics. The *t*-tests on difference in means show no significant difference between the control and treatment groups in terms of demographic characteristics and

28. Data on welfare participation is aggregated to the quarterly level as follows. Let x_t be the cumulative months on welfare from the month of random assignment to the last month of quarter t . The recoded welfare participation variable in quarter t equals $\text{int}(x_t/3) - \text{int}(x_{t-1}/3)$, where $\text{int}(x_t)$ is the largest integer smaller than or equal to x_t .

29. The sample excludes individuals with missing demographic information, used zero months of welfare in the first 2 years following random assignment (they had little or no contact with the welfare programme; see Bloom *et al.* (2000)), or had extreme measures of long-term welfare exposure. Twenty-five individuals with college degrees were dropped.

30. There are 23,025 observations if quarter 1 is not counted. The panel is slightly unbalanced as a result of excluding an observation if the imputed age of the youngest child exceeds 18 or the individual has hit the time limit.

TABLE 2
Sample one-period transition rates by lagged choice

Outcome in the previous period	Outcome in the current period						Number of observations
	Work (%)	Welfare (%)	No work, no welfare (%)	Work, no welfare (%)	No work, welfare (%)	Work, welfare (%)	
Control group							
No work, no welfare	11.9	8.0	81.8	10.2	6.4	1.6	2,565
Work, no welfare	84.9	9.5	11.4	79.1	3.7	5.8	2,824
No work, welfare	15.2	88.7	9.0	2.3	75.8	12.9	4,594
Work, welfare	75.5	64.7	4.5	30.8	20.0	44.7	1,706
Treatment group							
No work, no welfare	12.0	7.3	82.2	10.5	5.8	1.5	2,799
Work, no welfare	86.1	8.9	10.8	80.3	3.0	5.9	2,872
No work, welfare	20.4*	84.1*	12.5*	3.4*	67.2*	16.9*	3,513
Work, welfare	79.0*	71.0*	4.1	24.9*	16.9	54.1*	2,152

Notes: Numbers in the choice distribution may be subject to rounding error. One period is defined as one quarter.

*Significant at the 1% level in the two-tailed *t*-test on difference in means between control and treatment group individuals.

pre-random-assignment behaviour. The average single mother in the sample is a high school dropout (grade 11) and has two children. Less than half of the individuals have ever worked full time for 6 months or more, and close to one-thirds have been on welfare for at least 3 years in the 5 years prior to random assignment. During the sample period following random assignment, there are significant differences in outcomes between both groups. The average fraction of time employed is 5.6 percentage points higher (45.6%), and the average fraction of time using welfare is 3.6 points lower (44.7%) among treatment individuals relative to control individuals. A closer look reveals that treatment group individuals spend substantially less time being a *non-working* welfare recipient (−8 percentage points); their time is shifted towards being a *working* welfare recipient or being “idle” (*i.e.* no work and no welfare). These summary statistics are consistent with the combined effects of the welfare time limit and expanded earnings disregards.

Table 2 reports the quarterly transition rates of sample choice outcomes by treatment status. There are notable differences in welfare use transitions. For instance, a treatment group individual who was a *non-working* welfare recipient last period (or quarter) is much less likely to stay in the same state than a control group individual (67.2% versus 75.8%). However, if she was a *working* welfare recipient last period, she is more likely to stay in the same state relative to a control group individual (54.1% versus 44.7%).

Figure 2 shows further evidence that the time limit generates systematic differences/divergence in welfare use dynamics. Among control group individuals who were welfare recipients last period, the employment and welfare participation rates remain steady around 30 and 80%, respectively, independent of the cumulative quarters of welfare use since random assignment. By contrast, cumulative welfare use matters greatly in the treatment group because of the time limit; between 1 and 11 cumulative quarters of welfare use, the employment rate increases from 38% to 55% and the welfare participation rate drops from 82% to 61%.³¹

31. The *t*-test of difference in means between the control and treatment groups is conducted at each level of cumulative quarters of welfare use. The difference in employment is statistically significant at the 1% level at all cumulative quarters of welfare use. The difference in welfare participation is statistically significant at the 1% level at 7, 9, 10, 11 cumulative quarters of welfare use. Note that the welfare participation rate increases between 7 and 8 cumulative quarters of welfare use because of sample differences. In the sample, treatment group individuals who are subject to the default time limit have a maximum of 7 cumulative quarters of welfare use. Further breakdown of choices by cumulative welfare use are shown in Figure A1.

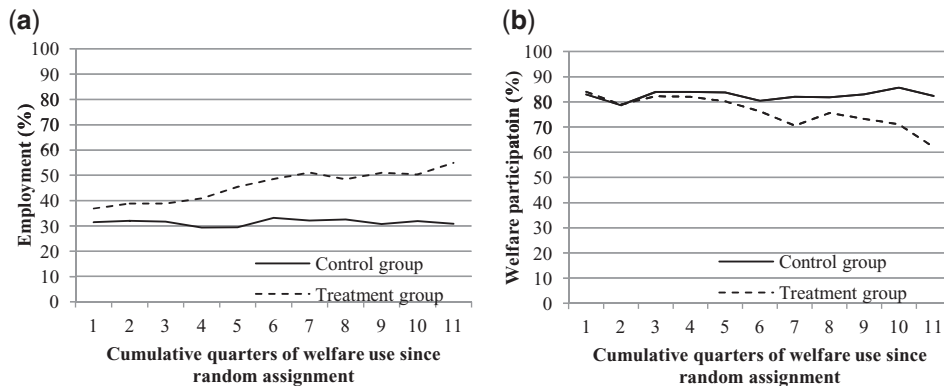


FIGURE 2

Sample outcomes by cumulative periods of welfare use (among individuals who used welfare last period).

(a) Employment. (b) Welfare use.

5. RESULTS

5.1. Main estimates

Table 3 reports the estimates from the baseline model (*i.e.* heterogeneous hyperbolic discounting). The initial non-pecuniary costs of work and welfare use are large, but they reduce substantially once the individual has engaged in the activity.³² The wage rate increases by 9% for every half-year increase in post-random-assignment work experience. There is unobserved heterogeneity in preferences and the wage equation, as indicated by the type-specific intercepts and distribution of types.³³ The initial pre-random-assignment conditions suggest that prior work experience increases the probability that the individual belongs to a high-skill type, and prior welfare exposure increases the probability that she belongs to a high welfare preference type. The model exhibits diminishing marginal utility of income; the estimated curvature, along with the s.d. of choice and wage shocks, differ slightly across demographic characteristics.³⁴ The coefficient on FTP enhanced service suggests that if a treatment group individual uses welfare now, the cost of employment will reduce by \$94.68 next period. The time preference parameters exhibit varying degrees of heterogeneity. Non-white individuals tend to have a lower discount factor and a higher present bias factor. Individuals with more education tend to have a higher naivety factor; individuals with multiple children tend to have a lower present bias factor.³⁵

32. If the individual did not work last period, the utility for PT work is -798.19 , and the utility for FT work is $-798.19 - 781.04 = -1579.23$. If she worked last period, the PT and FT utilities are $-798.19 + 505.10 = -293.09$ and $-1579.23 + 798.20 = -781.03$, respectively. The standard errors are larger for FT utilities (α_{h2} and γ_{h2}) than PT utilities.

33. There are two skill/wage types (1 (high), 2 (low)) and two preference types (1, 2), leading to four combinations. The probability of the preference type depends on the skill type. The unconditional type distribution (skill, preference) is as follows: 33.4% of individuals belong to (1, 1), 1.5% belong to (1, 2), 31.3% belong to (2, 1), 33.8% belong to (2, 2).

34. For a white single mother who is a high school dropout with one child, the coefficient on α_y is -0.000406 . This implies that the marginal utility of income is zero beyond \$1231. Of the workers (or 97.7% of all individuals) 93% in this category have earnings below this level.

35. The naivety factor measures an individual's perception of self-control. Recent studies in psychology suggest that several key dimensions of non-cognitive skills (self-esteem, locus of control, neuroticism, and generalized self-efficacy) are markers of the same higher-order concept—self-evaluation of one's own abilities and control (*e.g.* Judge *et al.*, 2002; see Cobb-Clark, 2015 for a recent review). The MDRC data contains two background questions on perceived control: (1) "I have little control over things that happen to me"; (2) "there is little I can do to change many of the

TABLE 3
Heterogeneous hyperbolic discounting model (baseline model)

	Work (α_h)	Welfare (α_a)	Work and welfare (α_{ha})	Log wage
Intercept	-798.19 505.10	(42.19) (30.42)	147.11 (23.95)	6.02 (0.02)
Lagged work (γ_h)				
Lagged welfare (γ_a)				
Lagged work \times lagged welfare (γ_{ha})		906.28 (47.02)	-59.70 (16.50)	
More than one child	54.18 (22.52)	-26.89 (22.88)	83.12 (15.68)	0.17 (0.02)
Grade 12 or above	-59.18 (21.72)	-69.88 (23.99)	48.43 (15.21)	-0.12 (0.02)
Race (non-white=1)	59.61 (25.47)	37.16 (27.44)	15.61 (16.18)	0.09 (0.01)
Post-RA experience (α_0) ^a				-0.01 (0.01)
Post-RA experience \times Grade 12 or above (α_1) ^a				-0.84 (0.01)
Preference type-2 intercept	-386.70 (39.66)	-229.61 (35.28)	25.40 (31.88)	
Wage type-2 intercept (μ_w)				
	Discount factor (logistic coef.)	Present bias factor	Naivety factor	
Intercept	2.53 (0.42)	0.48 (0.11)	-0.36 (0.17)	
Race (non-white=1)	-1.39 (0.41)	0.33 (0.13)	0.20 (0.18)	
Grade 12 or above	-0.08 (0.20)	-0.02 (0.07)	1.07 (0.35)	
More than one child	-0.29 (0.22)	-0.22 (0.08)	0.34 (0.25)	
Preference type-2 intercept	0.74 (0.40)	0.20 (0.15)	0.54 (0.35)	
	Quadratic utility (α_y) ^b	Std. dev utility shock (σ_c) ^c	Std. dev wage shock (σ_w) ^c	
Intercept	-4.06 (0.24)	5.80 (0.05)	-0.45 (0.01)	
Race (non-white=1)	-0.45 (0.18)	-0.02 (0.02)	-0.04 (0.01)	
Grade 12 or above	0.63 (0.20)	-0.05 (0.02)	-0.02 (0.01)	
More than one child	0.31 (0.15)	-0.06 (0.02)		
Other parameters				
FT work utility (α_{h2})	-781.04 (148.01)			
FT work \times lagged work (γ_{h2})	798.20 (141.18)			
Enhanced service (B_{AS})	94.68 (32.08)			
Type probabilities (logistic coef.)		Wage type 2	Preference type 2	
Intercept		1.60 (0.15)	-2.44 (0.80)	
Wage type-2 dummy			3.30 (0.80)	
Ever had 6+ months of FT work		-0.27 (0.12)	-0.15 (0.17)	
Work experience in 2 years prior to RA ^a		-0.30 (0.04)	-0.82 (0.09)	
On welfare 36+ of the past 60 mths prior to RA		-0.02 (0.12)	-0.59 (0.19)	
Race (non-white=1)		-0.48 (0.13)	0.19 (0.23)	
Grade 12 or above		-0.34 (0.13)	-0.79 (0.22)	
More than one child			0.73 (0.23)	

Notes: Number of observations = 23,025, log-likelihood = -28,219.15. Standard errors are given in parentheses. RA, random assignment.

^aCumulative periods of employment expressed in half-year intervals.

^bReported estimates and standard errors are multiplied by 10,000

^cTransformation $\sigma = e^{Xa}$, where a is the vector of coefficients reported in the table.

Table 4 reports the estimates from a time-consistent model with heterogeneous discount factors. Most coefficients remain similar to the baseline model. However, the discount factor estimates are different—there is more unobserved heterogeneity, and individuals with multiple children tend to have a lower discount factor. Overall, statistical tests provide evidence in support of the heterogeneous hyperbolic discounting model. In particular, the likelihood ratio test rejects the null hypothesis of time-consistency at the 1% significance level.³⁶

Figure 3 reports the predicted distributions of the time preference parameters. The results are generated as follows. From the model, each time preference parameter has sixteen possible values (*i.e.* preference type \times race \times HS degree \times (kids > 1) = $2 \times 2 \times 2 \times 2 = 16$). However, the individual's preference type is unobserved in the data; depending on her initial conditions, each individual can have a different probability associated with each preference type (see the Appendix). Therefore, for each individual, we can compute a weighted prediction of her time preference parameter, where the weights are her type probabilities (*i.e.* the unobserved types are “integrated out”). By repeating this for all individuals in the sample, the histogram can have more than sixteen points of support. Figure 3(a) is generated from the time-consistent model; the discount factor has a mean of 0.73 and s.d. of 0.07. Figures 3(b)–(d) are generated from the baseline model. The discount factor has a mean of 0.83 and s.d. of 0.09, and around half of the individuals have a discount factor lower than 0.9. Individuals are generally present-biased, although the hyperbolic discounting parameters are highly heterogeneous. The present bias factor has a mean of 0.59 and s.d. of 0.17, and the naivety factor has a mean of 0.68 and s.d. of 0.54. Around 95% of the individuals have a present bias factor lower than 0.9.

Around one-third of the individuals have a naivety factor exceeding one. Among these individuals, the mean of the discount factor, present bias factor, and naivety factor are 0.78, 0.59, and 1.30, respectively. The perceived aggregate discount factors are then 1, 1.01, 0.79, etc. (*i.e.* 1, $\tilde{\beta}\delta$, $\tilde{\beta}\delta^2$, ...), and the actual aggregate discount factors are 1, 0.46, 0.36, etc. (*i.e.* 1, $\beta\delta$, $\beta\delta^2$, ...). While the current self is unduly orientated towards the immediate payoff, she perceives that her future selves will favor immediate and near-immediate payoffs similarly and only discount longer term payoffs. Around one-sixth of the individuals have a very small naivety factor—for these individuals, the current self perceives that their future selves are (almost) myopic.³⁷

Figure 4 compares the goodness-of-fit of the welfare participation rate by cumulative periods (quarters) of welfare use since random assignment (M_{it}).³⁸ Predictions from the baseline model,

important things in my life”. Among low-education individuals, 46.4% agree or strongly agree to at least one of the above questions. Among high-education individuals, the proportion is 30.3%. Among white and non-white individuals, the proportions are 37.2 and 39.5%, respectively; among individuals with one child and multiple children, the proportions are 35.5 and 40.4%, respectively.

36. The log-likelihood of the baseline and time-consistent models are $-28,219.15$ and $-28,231.26$, respectively. The test statistic is $2 \times (28,231.26 - 28,219.15) = 24.22$ with an asymptotic chi-square distribution of ten degrees of freedom. The p -value is 0.007. In a further analysis, a hyperbolic discounting model assuming that all individuals are “sophisticated” ($\tilde{\beta}_i = \beta_i \forall i$) is also estimated. The log-likelihood is $-28,225.51$ (see Online Appendix). The likelihood ratio test rejects the null hypothesis of sophisticated agents at the 5% significance level (test statistic is $2 \times (28,225.51 - 28,219.15) = 12.72$ with 5 d.f. and p -value is 0.026). This suggests that the naivety factor is generally different from the present bias factor.

37. In the literature, the naivety factor has almost not been estimated so there are few benchmarks to refer to. In their online appendix, Fang and Wang (2015) find that the naivety factor estimate ranges between 1 and 1.25 depending on the specification. Note that when the naivety factor is presented in Figure 3(d), it is restricted to be non-negative. Around 13% of the individuals have $\tilde{\beta} < 0$ (with mean -0.14). A negative naivety factor appears nonsensical behaviourally, as the current self will perceive her future selves to dislike future payoffs. Empirically, to obtain such estimates from the data, some individuals would have to make choices in a way that reflect their extreme “pessimism” about the behaviour of their future selves.

38. The figure focuses on individuals who were welfare recipients last period (see also Figure 2). For more information regarding within-sample fit of the baseline model, see also Figure A2 and Table A1, which report the

TABLE 4
Time-consistent heterogeneous discount factor model

	Work (α_h)	Welfare (α_a)	Work and welfare (α_{ha})	Log wage
Intercept	-799.49 (37.40)	-1051.93 (46.24)	147.83 (23.36)	6.02 (0.02)
Lagged work (γ_h)	498.55 (29.96)			
Lagged welfare (γ_a)		892.91 (45.84)		
Lagged work \times lagged welfare (γ_{ha})			-44.94 (16.44)	
More than one child	30.83 (16.89)	-48.28 (12.31)	76.40 (15.05)	0.18 (0.02)
Grade 12 or above	-63.39 (16.64)	-54.88 (11.34)	36.94 (15.19)	-0.13 (0.02)
Race (non-white=1)	50.57 (16.25)	45.90 (10.99)	23.55 (15.13)	0.09 (0.01)
Post-RA experience (ω_0) ^a				-0.01 (0.01)
Post-RA experience \times grade 12 or above (ω_1) ^a				
Preference type-2 intercept	-387.90 (35.90)	-243.04 (23.68)	25.93 (31.89)	-0.83 (0.01)
Wage type-2 intercept (μ_w)				
Discount factor (logistic coef.)				
Intercept	0.63 (0.25)			
Race (non-white=1)	0.20 (0.17)			
Grade 12 or above	0.00 (0.16)			
More than one child	-0.44 (0.17)			
Preference type-2 intercept	2.17 (0.35)			
Quadratic utility (α_y) ^b				
Intercept	-4.10 (0.24)			
Race (non-white=1)	-0.35 (0.17)			
Grade 12 or above	0.58 (0.19)			
More than one child	0.28 (0.16)			
Other parameters				
FT work utility (α_{h2})	-780.44 (146.58)			
FT work \times lagged work (γ_{h2})	809.50 (140.08)			
Enhanced service (B _{AS})	80.99 (27.12)			
Type probabilities (logistic coef.):				
Intercept				
Wage type-2 dummy		1.60 (0.15)		
Ever had 6+ months of FT work		-0.28 (0.12)		
Work experience in 2 years prior to RA ^a		-0.29 (0.04)		
On welfare 36+ of the past 60 months prior to RA		-0.02 (0.12)		
Race (non-white=1)		-0.50 (0.13)		
Grade 12 or above		-0.32 (0.13)		
More than one child				
			Preference type 2	
			-1.97 (0.64)	
			2.88 (0.62)	
			-0.12 (0.17)	
			-0.74 (0.08)	
			-0.62 (0.19)	
			-0.01 (0.22)	
			-0.83 (0.22)	
			0.75 (0.22)	

Notes: Number of observations = 23,025, log-likelihood = -28,231.26. Standard errors are given in parentheses. RA, random assignment.

^aCumulative periods of employment expressed in half-year intervals.

^bReported estimates and standard errors are multiplied by 10,000.

^cTransformation $\sigma = e^{Xa}$, where a is the vector of coefficients reported in the table.

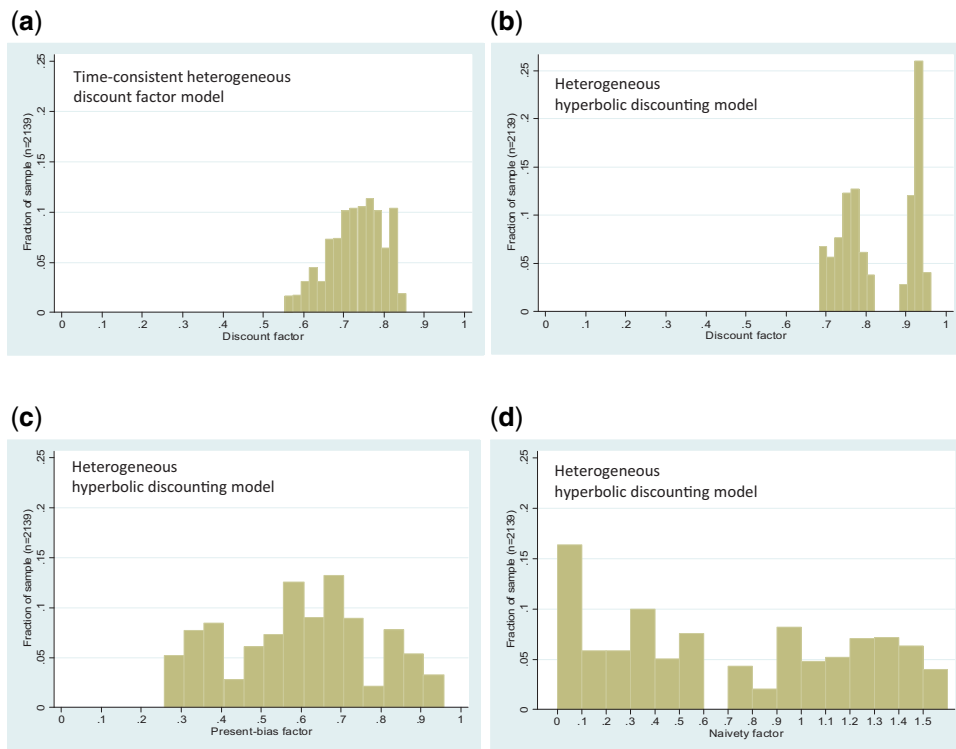


FIGURE 3

Heterogeneity in time preference parameters. (a) Discount factor (mean=0.73, s.d.=0.07). (b) Discount factor (mean=0.83, s.d.=0.09). (c) Present-bias factor (mean=0.59, s.d.=0.17).

time-consistent heterogeneous discount factor model, and time-consistent homogeneous discount factor model (see Online Appendix) are presented.³⁹ The prediction error typically lies within 3 percentage points. All models generate similar predictions within the control group. However, the models generate different predictions within the treatment group. For instance, relative to the baseline model, the time-consistent heterogeneous discount factor model tends to under-predict welfare participation among individuals who have high cumulative welfare use. In addition, relative to the baseline model, both time-consistent models predict steep decline in welfare participation at high cumulative periods of welfare use.

The above results considerably extend current field evidence on the estimation of time preference parameters. In the existing literature, more restrictive analysis have been pursued due to data and policy limitations. For comparison, key parameter estimates from recent empirical studies are reported below. Fang and Wang (2015) are the first to estimate the discount factor, present bias factor, and naivety factor simultaneously. Their baseline estimates are 0.68 (s.e. 0.12), 0.68 (s.e. 0.19), and 1.00 (s.e. 0.28), respectively. They use certain demographic characteristics as

simulated choice outcomes and transition rates, respectively. The model also provides a reasonably good fit to earnings; see Online Appendix for a comparison of actual versus simulated earnings distributions among workers.

39. By comparing both time-consistent models, the likelihood ratio test rejects the null hypothesis of homogeneous discount factor at the 10% significance level (test statistic is $2 \times (28,236.10 - 28,231.26) = 9.68$ with 5 d.f. and p -value is 0.085).

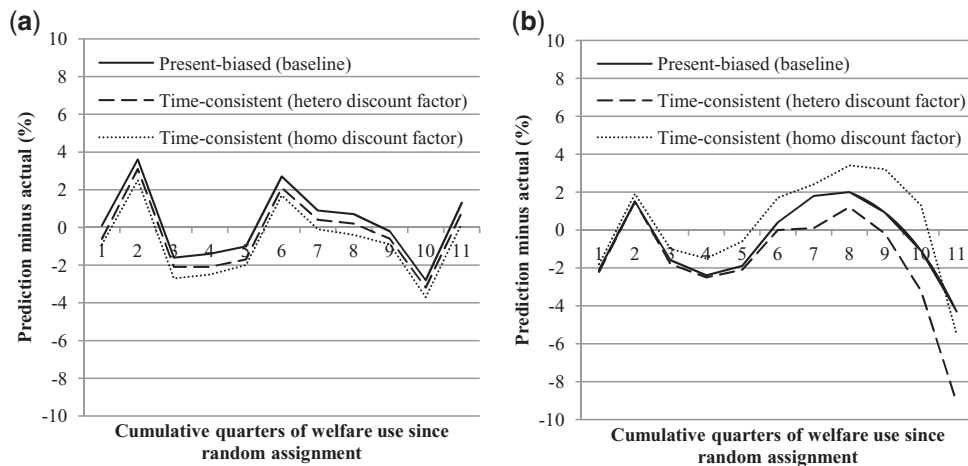


FIGURE 4

Goodness-of-fit of welfare participation by cumulative periods of welfare use (among individuals who used welfare last period). (a) Control group. (b) Treatment group.

exclusion variables, and the time preference parameters are assumed to be homogeneous. Ferrall (2012) is the first to estimate a structural model with a distribution of discount factors. He finds that 37% of the individuals have a discount factor of close to one, 18% have a discount factor of 0.44, and the rest are close to myopic.⁴⁰ Mahajan and Tarozi (2011) find that 40% of individuals in their sample are time-consistent, 50% are naive present-biased, and 10% are sophisticated present-biased. The discount factor estimate (all individuals) is 0.79, and the present bias factor estimate for naive and sophisticated agents are 0.97 and 0.55, respectively. Some parameters are not precisely estimated.⁴¹ Earlier studies usually adopt a simple specification on the discounting parameter(s) and do not have clear exclusion variables. For instance, in Fang and Silverman (2009), the sophisticated present-biased model yields the discount factor and present bias factor at 0.88 (s.e. 0.02) and 0.34 (s.e. 0.07), respectively.

5.2. Role of hyperbolic discounting parameters

Table 5 reports counterfactuals that involve changing the hyperbolic discounting parameters. The results reveal how present bias can affect the pattern and efficiency of intertemporal decisions. Column 1 represents the baseline scenario, in which individuals make decisions under the original discounting estimates. Column 2 assumes individuals are naive ($\tilde{\beta}_i = 1$). Column 3 assumes individuals are time-consistent ($\tilde{\beta}_i = \beta_i = 1$). Simulation is conducted for control group individuals under their policy environment. In all simulations, each decision period represents a quarter (3

40. He exploits particular policy features in the SSP for identification. There are four types of individuals, and each type of individual has a specific mixture proportion in geographical location and family size. The discount factor is different by type. The type proportions are 37% (1), 18% (2), 37% (3), 8% (4).

41. The 95% confidence intervals are: discount factor [0.00, 0.94], present bias factor for naive agents [0.92, 0.98], present bias factor for sophisticated agents [0.01, 0.73]. Because the present bias factor of naive agents is close to one, they are very similar to being time-consistent.

TABLE 5
Effects of hyperbolic discounting

	Baseline (1)	Becoming naïve (2)	Becoming time-consistent (3)
<i>All periods^a</i>			
Work (%)	44.7	44.3	48.6
Welfare (%)	32.4	32.9	50.3
No work, no welfare (%)	32.5	32.5	21.8
Work, no welfare (%)	35.1	34.5	27.9
No work, welfare (%)	22.7	23.2	29.6
Work, welfare (%)	9.6	9.8	20.7
APDV Earnings (\$)	390.3	385.4	392.5
APDV net gov. expenditure (\$)	205.4	206.7	286.6
APDV utility (difference from baseline, \$)	—	−1.0	+23.6
<i>At Period 1 (first quarter)</i>			
Work (%)	32.7	32.6	37.9
Welfare (%)	74.1	74.2	82.1
No work, no welfare (%)	13.3	13.3	8.1
Work, no welfare (%)	12.6	12.5	9.8
No work, welfare (%)	54.0	54.1	53.9
Work, welfare (%)	20.1	20.1	28.2
Earnings (\$)	165.6	165.2	177.4
Net gov. expenditure (\$)	417.1	417.1	450.0
Utility (difference from baseline, in \$)	—	−1.0	−61.2
<i>At the end of year 5</i>			
Work (%)	46.2	45.6	50.1
Welfare (%)	30.1	30.9	50.2
No work, no welfare (%)	32.6	32.6	20.5
Work, no welfare (%)	37.3	36.5	29.3
No work, welfare (%)	21.2	21.8	29.4
Work, welfare (%)	8.9	9.1	20.8
Earnings (\$)	463.1	455.1	464.6
Net gov. expenditure (\$)	164.6	167.1	253.0
Utility (difference from baseline, in \$)	—	−2.0	+43.0

Notes: Results are based on simulations from all control group individuals under the baseline policy environment. Column 2 sets the naivety factor to 1. Column 3 sets the present bias factor and naivety factor to 1. In all simulations, each decision period represents a quarter, but all variables are expressed in monthly units unless otherwise stated.

^aFrom period 1 to the end of the time horizon. APDV earnings and net government expenditure are based on a discount factor of 0.9 per annum. APDV utility is computed from the long-run criterion, based on the discount factors estimated from the model.

months), but all the variables are expressed in monthly units unless otherwise stated. The top panel reports average outcomes over the full horizon.⁴²

In the top panel, the amortized present discounted values (APDVs) of earnings, net government expenditure, and utility are computed. The present discounted values are computed using the “long-run criterion” (*e.g.* O’Donoghue and Rabin, 1999; Fang and Silverman, 2009). This criterion discounts all future flows using the discount factor only; therefore, it can be viewed as an evaluation from a fictitious time-consistent individual just prior to the decision-making sequence

42. The time horizon consists of forty-two periods. Twenty-five paths are simulated for each individual. In addition, in all simulation tables, we do not separately report PT and FT work because they are not the focus of the analysis. Nevertheless, in the baseline scenario, the model finds that 14.9% of individuals choose “PT work and no welfare”, and 20.2% choose “FT work and no welfare”. The relative proportion of PT versus FT workers remains stable in various counterfactual environments (partly due to the fact that no counterfactual policies in the article focus on influencing PT versus FT work).

from period 1.⁴³ The APDVs have been normalized to reflect average flow per period.⁴⁴ All APDV outcomes are reported as the average across individual values. Net government expenditure is defined as welfare benefits (including food stamps) plus EITC benefits minus tax. The table reports changes in utility between the counterfactual and baseline. The difference is expressed in monetary equivalents, which is defined as the amount of per-period lump sum income tax (applied in the counterfactual for all periods) such that the APDV utility (averaged across individuals) becomes the same as the baseline scenario.

In the baseline scenario, the average employment and welfare participation rates are 44.7 and 32.4%, respectively. Working welfare recipients constitute 30% of all welfare recipients, or 9.6% of all individuals in the control group. The average earnings and net government expenditure (expressed in monthly \$) are \$390.3 and \$205.4, respectively. When individuals become naive (column 2), they work less and use welfare more. There is a reduction in earnings and a smaller increase in government expenditure. From a time-consistent perspective, the individual is also worse off, with a slight reduction in utility.

Individuals have no commitment problem when they are time-consistent (column 3). The average employment and welfare participation rates increase to 48.6 and 50.3%, respectively. Because individuals no longer delay employment and welfare use (both incur large initial costs), the proportion of idle individuals reduces from 32.5% to 21.8%; however, many workers become welfare recipients to receive more government benefits. As a whole, earnings increase slightly, and net government expenditure increases substantially to \$286.6. There is a moderate improvement in utility of \$23.6.

The bottom panels report outcomes at period 1 and the end of year 5, respectively. Earnings growth becomes somewhat slower when individuals are naive. When individuals are time-consistent, the initial utility is \$61.2 lower than the baseline scenario due to the initial costs incurred by working welfare recipients. At the end of year 5, the utility is \$43 larger than the baseline scenario.

5.3. *Welfare, EITC, and income support*

Table 6 compares the effects of several counterfactual policies that improve utility by the same amount (which is equivalent to a \$12.1 per-period lump sum income transfer). Average outcomes among control group individuals are reported. Column 2 increases the maximum welfare benefit level by 11.2%. Employment drops by 0.4 percentage points and welfare use increases by 1.9 percentage points. Earnings reduce by \$4.8 and net government expenditure increases by \$16.7; therefore, the net income increases by \$11.9. Column 3 increases both the EITC phase-in rate and maximum benefit level by 32%. The net income change is \$12, but government expenditure increases only slightly (\$3.1) partly due to reduced welfare use. Column 4 encourages work in the welfare system by introducing a \$70 subsidy to all working welfare recipients. The increase in employment is similar to the EITC expansion (1.0 percentage point). The income change is slightly larger, but over two-thirds are due to increased government expenditure.

43. An alternative criterion is called the Pareto criterion (*e.g.* Goldman, 1979), which requires that all period selves to be better off. O'Donoghue and Rabin (1999) remark that this criterion is often too restrictive for policy analysis.

44. Let $PDV_i = \sum_{t=0}^{T-1} x_{it} \delta_t^i$ be the present discounted value of a stream of flows (x_{it}) from period 0 to $T-1$. $APDV_i$ is defined as the flow per period such that $PDV_i = APDV_i \sum_{t=0}^{T-1} \delta_t^i$; therefore, $APDV_i = PDV_i \frac{\delta_i^{T-1}}{\delta_i^0 - 1}$. The APDVs of earnings and net government expenditure are computed with an annualized discount factor of 0.9. The APDV of utility is computed with the individual's discount factor.

TABLE 6
Effects of static policy interventions

	Baseline	Expand welfare 1.112×	Expand EITC 1.32×	Work subsidy in welfare \$70	Income support \$50
	(1)	(2)	(3)	(4)	(5)
Work (%)	44.7	−0.4	0.9	1.0	−3.3 (41.4)
Welfare (%)	32.4	1.9	−0.7	1.0	−5.4 (27.0)
No work, no welfare (%)	32.5	−1.3	−0.2	−0.7	7.7 (40.2)
Work, no welfare (%)	35.1	−0.6	0.9	−0.3	−2.4 (32.7)
No work, welfare (%)	22.7	1.6	−0.7	−0.3	−4.4 (18.3)
Work, welfare (%)	9.6	0.3	−0.0	1.3	−0.9 (08.7)
APDV Earnings (\$)	390.3	−4.8	8.9	3.7	−25.7 (364.5)
APDV Net gov. expenditure (\$)	205.4	16.7	3.1	12.6	−14.0 (191.4)
APDV Utility (\$)	—	12.1	12.1	12.1	12.1
Prefers new policy (%) ^a	—	99.3	100.0	99.1	90.7

Notes: Results are based on simulations of all control group individuals from period 1 to the end of the time horizon. Numbers in columns 2–5 (except for numbers in parentheses, which represent raw outcomes) represent simple differences in outcomes from baseline. Column 2 increases the maximum welfare benefit level by 11.2%. Column 3 increases the EITC phase-in rate and maximum benefit level by 32%. Column 4 introduces a work subsidy of \$70 for welfare recipients. Column 5 introduces a subsidy of \$50 for all individuals who do not work and do not receive welfare.

^aFraction of individuals that prefer the new policy regime to the policy regime in column 1.

Column 5 adds a welfare component to the tax system (called “income support”) by introducing a \$50 subsidy to all idle individuals (*i.e.* single mothers who neither work nor use welfare). This programme not only redistributes income but also enhances the insurance role of the transfer system. The main drawback is sizeable work disincentives—employment and earnings reduce by 3.3 percentage points and \$25.7, respectively. However, the programme *reduces* government expenditure by \$14 and improves utility; many individuals substitute the income support programme for welfare, which provides more generous benefits but also entails large initial costs.

The last row of the table reports the fraction of individuals who will prefer the counterfactual policy regime if it is offered as a choice just prior to the decision-making sequence from period 1.⁴⁵ Time-consistent individuals will always prefer the regime that offers a more generous budget constraint. However, present-biased individuals may not prefer a counterfactual regime if it substantially worsens the commitment problem. Over 99% support the welfare expansions and 100% support the EITC expansion. By contrast, only 90.7% prefer the introduction of income support.

5.4. Time limits

Table 7 reports the effects on the control group under the following time limits: standard time limit; prowork time limit; benefit reduction time limit. For each policy, two different time limit lengths are reported. The lenient limits have the same effect on utility (columns 1, 3, 5). The stringent limits are chosen to have the closest possible effect on utility as the removal of a \$50 income support (columns 2, 4, 6).

Under the standard limit, any participation of welfare, regardless of the benefit amount received or work status, counts as one period towards the time limit. Because working welfare recipients are

45. For each regime, the continuation long-run utility in period 1 is computed. The individual prefers the regime that yields the highest continuation long-run utility.

TABLE 7
Effects of welfare time limits

	Standard limit		Prowork limit		Benefit reduction limit (\$100)	
	4-year	3-year	3.25-year	2.25-year	2.25-year	1.25-year
	(1)	(2)	(3)	(4)	(5)	(6)
Work (%)	1.3	2.1	1.4	2.6	0.9	1.4
Welfare (%)	-8.4	-12.9	-6.8	-11.5	-4.3	-6.2
No work, no welfare (%)	4.7	7.1	4.6	7.3	2.5	3.6
Work, no welfare (%)	3.7	5.8	2.2	4.2	1.8	2.6
No work, welfare (%)	-6.0	-9.2	-6.0	-9.9	-3.5	-5.0
Work, welfare (%)	-2.4	-3.7	-0.8	-1.6	-0.8	-1.2
APDV Earnings (\$)	13.5	23.9	9.9	21.0	9.0	15.1
APDV Net gov. expenditure (\$)	-40.3	-64.9	-35.3	-62.3	-29.3	-45.3
APDV Utility (\$)	-5.7	-11.4	-5.7	-12.9	-5.7	-13.1
Prefers new policy (%) ^a	5.0	5.4	5.4	1.0	5.6	1.0

Notes: Results are based on simulations of all control group individuals from period 1 to the end of the time horizon. Numbers represent simple differences in outcomes from baseline (no time limits). For each type of policy, two different time limit lengths are considered. APDV, amortized present discount value.

^aFraction of individuals that prefer the new policy regime to the baseline policy regime (no time limits).

discouraged, this contradicts a main goal of welfare reform, which is to help welfare recipients achieve self-subsistence via work. Under the proposed “prowork” limit, welfare participation does not count towards the time limit if the welfare recipient is working.⁴⁶ The law of motion in equation (7) becomes⁴⁷:

$$M_{i,t+1} = M_{it} + d_{ait}(1 - d_{hit}), \quad \text{and} \quad M_{i1} = 0. \quad (13)$$

If the individual reaches the standard or prowork time limit, she becomes ineligible for welfare for the rest of the time horizon. The benefit reduction limit is the same as the standard limit, except that it decreases the benefit amount instead of removing eligibility.

The results indicate that given the same effect on utility, the prowork limit improves employment by a larger degree than the standard limit, and has less radical effects on other outcomes. Fewer individuals exit welfare, and there is a smaller effect on earnings and net government expenditure. The benefit reduction limit has smaller effects than the standard and prowork limits.

The standard time limit is not strong enough as a commitment device to increase average utility. This finding is consistent with Fang and Silverman (2009)⁴⁸; however, the results also

46. The actual picture of welfare reform is more complicated. Under TANF, the states are required to have a substantial fraction of welfare recipients working, or else lose a portion of federal funding. They have a strong incentive to keep the working welfare recipients in TANF (often by imposing sanctions on non-working recipients). Although prowork time limits were never implemented, they are a better description of what most states actually implemented under TANF than the “standard” time limit (see Fang and Keane, 2004 for related evidence).

47. More generally, a period of welfare participation that involve working can be counted as a *fraction* of a period towards the time limit. For instance, the fraction can be the actual welfare benefit received divided by the maximum benefit amount. Usually the fraction is considerably lower than one because working welfare recipients often receive partial welfare benefits. In this general case, the prowork time limit is analogous to a “welfare account”, in which an individual is eligible for a fixed *dollar* sum of welfare benefits throughout the time horizon. See Laroque (2009) for a discussion of history-dependent tax and benefit schemes.

48. Because their model does not allow welfare recipients to work, prowork time limits are not feasible in their analysis.

TABLE 8
Amendments to income support programme

	Original income support (\$50)	Remove income support	Add sanction (\$50)		Add work subsidy (\$50)		
			Dynamic type-1	Dynamic type-2	Static	Dynamic type-1	Dynamic type-2
	(0)	(1)	(2)	(3)	(4)	(5)	(6)
Work (%)	41.4	3.3	1.2	2.2	2.8	2.1	0.8
Welfare (%)	27.0	5.4	0.3	5.0	-1.3	-1.0	-0.3
No work, no welfare (%)	40.2	-7.7	-1.3	-6.4	-1.5	-1.0	-0.5
Work, no welfare (%)	32.7	2.4	1.0	1.4	2.8	2.1	0.8
No work, welfare (%)	18.3	4.4	0.1	4.2	-1.3	-1.0	-0.4
Work, welfare (%)	8.7	0.9	0.2	0.8	0.1	0.0	0.1
APDV Earnings (\$)	364.5	25.7	11.4	14.9	19.2	15.0	4.8
APDV Net gov. expenditure (\$)	191.4	14.0	-1.3	14.3	15.1	12.2	2.6
APDV Utility (\$)	—	-12.1	0.4	-12.7	20.0	14.4	5.4
Prefers amendment (%) ^a	—	9.3	17.5	3.9	100.0	100.0	100.0
Prefers amendment when it is revenue-neutral (%) ^b	—	0.0	69.2	0.0	45.9	28.3	56.0

Notes: Results are based on simulations of all control group individuals from period 1 to the end of the time horizon. Numbers in columns 1–6 represent simple differences in outcomes from column 0. Column 0 introduces a subsidy of \$50 for all individuals who neither work nor receive welfare. Column 1 removes the subsidy. Column 2 sanctions the payment if the individual worked last period. Column 3 sanctions the payment if the individual did not work last period. Column 4 adds a subsidy to workers. Column 5 adds a subsidy to workers who also worked last period. Column 6 adds a subsidy to workers who did not work last period.

^aFraction of individuals that prefer the amendment to the original policy regime in column 0.

^bFraction of individuals that prefer the amendment, when it also includes a lump sum transfer that attains revenue-neutrality.

show that a moderate time limit regime is in fact attractive to some individuals. Around 5% of individuals prefer a 4-year standard limit to no time limits, and 5.4% prefer a 3.25-year prowork limit to no time limits.

5.5. *Dynamic interventions: amendments to income support*

A present-biased individual makes inefficient intertemporal choices from a time-consistent perspective. She tends to underestimate the future value of human capital accumulation, leading to low employment.⁴⁹ A targeted solution would, for instance, involve a *relative* increase in the expected future value of working versus not working. Note that traditional policy instruments such as EITC primarily affects decisions by changing the static budget constraint. These policies involve a relative increase in the current utility of working versus not working, but they do not necessarily involve a relative increase in the expected future value of working versus not working.

Several types of dynamic interventions are proposed below, using income support as an example. The previous results (Table 6) showed that the income support programme improved government expenditure and utility at the expense of employment and the commitment problem. Table 8 considers dynamic interventions that can improve its incentives. Let $b_H = \$50$ be the

49. As discussed in Online Appendix, there are two effects: “present-bias effect” and “sophistication effect”. When both effects work in opposite directions, the former usually dominates; indeed, in all the scenarios investigated by Fang and Silverman (2004), employment does not decrease when a time-consistent individual becomes present-biased (naïve or sophisticated).

benefit amount of income support. The leftmost column reports the average outcomes of the control group under the original income support programme. In columns 2 and 3, a dynamic sanction/tax component is added:

- Type-1 sanction: Income support is unavailable if the individual worked last period. The formula is $-b_H(1-d_{hit})(1-d_{ait})d_{hi,t-1}$.
- Type-2 sanction: Income support is unavailable if the individual did not work last period. The formula is $-b_H(1-d_{hit})(1-d_{ait})(1-d_{hi,t-1})$.

Both policies encourage employment from a static perspective (for those with the correct past work status). The type-1 sanction generates a dynamic work *disincentive* among all individuals—if an individual does not work now, she can avoid the sanction altogether next period. By contrast, the type-2 sanction generates a dynamic work incentive among all individuals.

Columns 4–6 introduce a work subsidy component:

- Static subsidy: The formula is $b_{WS}d_{hit}$, where $b_{WS} = \$50$ is the subsidy amount.
- Type-1 subsidy: Work subsidy is available if the individual worked last period. The formula is $b_{WS}d_{hit}d_{hi,t-1}$.
- Type-2 subsidy: Work subsidy is available if the individual did not work last period. The formula is $b_{WS}d_{hit}(1-d_{hi,t-1})$.

The type-1 subsidy generates a dynamic work *incentive* among all individuals—if an individual does not work now, the subsidy will be unavailable next period. The type-2 subsidy generates a dynamic work disincentive among all individuals.⁵⁰

Adding a type-1 sanction to the income support programme increases employment by 1.2 percentage points, in spite of the dynamic work disincentive involved. The type-1 sanction *increases* average utility by \$0.4; this suggests that the policy can alleviate the commitment problem sufficiently to make individuals better off on average.⁵¹ In fact, 17.5% of control group individuals prefer a type-1 sanction regime to no sanctions. Under the regime, the next-period self faces a higher cost of not working (thus less tempted to not to work) whenever the current self chooses to work. Thus, the regime provides a means by which the current self can commit her future selves to work.

The type-2 sanction increases employment by a larger degree (2.2 percentage points). However, it also substantially increases welfare use, resulting in an increase of government expenditure. Average utility reduces by \$12.7, and only 3.9% of individuals prefer a type-2 sanction regime to no sanctions. Under the regime, the next-period self faces a higher cost of not working whenever the current self *does not* work. Its commitment mechanism is thus different from the type-1 sanction.

The static and type-1 work subsidies increase employment by 2.8 and 2.1 percentage points, respectively. Due to the dynamic work disincentive, the type-2 subsidy has a much smaller effect on employment (0.8 percentage points). Unlike the type-1 sanction, all work subsidies increase government expenditure. The average utility increases in all three scenarios, and 100% of individuals prefer a work subsidy regime to no subsidies.

50. The features of the above sanctions and subsidies are summarized in Table A3. For a discussion of more complicated forms of the static work subsidy, see Keane (1995).

51. Note that there are individuals who are worse off. Note that if the sanction regime is not mandatory, individuals will self-select into the regime, implying that the increase in average utility will be higher than \$0.4.

TABLE 9
Income support programme, time-consistent heterogeneous discount factor model^a

	Original income support (\$50)	Remove income support	Add sanction (\$50)		Add work subsidy (\$50)		
			Dynamic type-1	Dynamic type-2	Static	Dynamic type-1	Dynamic type-2
	(0)	(1)	(2)	(3)	(4)	(5)	(6)
Work (%)	41.5	3.7	0.7	3.2	3.1	2.4	0.8
Welfare (%)	25.9	5.9	0.5	5.2	-1.3	-1.1	-0.3
No work, no welfare (%)	41.0	-8.5	-1.0	-7.5	-1.7	-1.2	-0.5
Work, no welfare (%)	33.1	2.6	0.5	2.3	3.0	2.3	0.7
No work, welfare (%)	17.5	4.8	0.3	4.3	-1.4	-1.1	-0.3
Work, welfare (%)	8.4	1.1	0.2	0.9	0.1	0.0	0.1
APDV Earnings (\$)	367.8	28.4	6.8	22.5	21.3	17.2	4.5
APDV Net gov. expenditure (\$)	185.8	17.5	0.3	15.9	14.7	11.8	2.6
APDV Utility (\$)	—	-21.4	-2.5	-19.1	14.0	9.9	4.3
Prefers amendment (%) ^a	—	0.0	0.0	0.0	100.0	100.0	100.0
Prefers amendment when it is revenue-neutral (%) ^b	—	0.0	0.0	0.0	20.7	22.3	41.7

Notes: Results are based on simulations of all control group individuals from period 1 to the end of the time horizon. Numbers in columns 1–6 represent simple differences in outcomes from column 0. Column 0 introduces a subsidy of \$50 for all individuals who neither work nor receive welfare. Column 1 removes the subsidy. Column 2 sanctions the payment if the individual worked last period. Column 3 sanctions the payment if the individual did not work last period. Column 4 adds a subsidy to workers. Column 5 adds a subsidy to workers who also worked last period. Column 6 adds a subsidy to workers who did not work last period.

^aFraction of individuals that prefer the amendment to the original policy regime in column 0.

^bFraction of individuals that prefer the amendment, when it also includes a lump sum transfer that attains revenue-neutrality.

Of course, work subsidies are naturally more popular than sanctions because benefits have increased. A fairer comparison should involve *revenue-neutral* policies. To carry out the comparison, each sanction/subsidy is counteracted by a per-period lump sum income transfer such that it becomes revenue-neutral. The last row of Table 8 reports the fraction of individuals that prefers the “neutralized” sanction/subsidy regime to the no-sanction/subsidy regime. When all the policy regimes are revenue-neutral, the most popular one is the dynamic type-1 sanction, which is preferred by 69.2% of individuals. This is followed by the static work subsidy (45.9%), dynamic type-2 subsidy (56%), and dynamic type-1 subsidy (28.3%). None of the individuals prefer a dynamic type-2 sanction.

5.6. Comparison with results from the time-consistent model

Table 9 reports several policy simulations from the time-consistent heterogeneous discount factor model.⁵² As in the previous table, the leftmost column reports the average outcomes of the control group under the original income support programme. Column 1 removes income support. Columns 2–6 add sanctions and work subsidies. Relative to the baseline model, removing income support leads to a larger change in employment and utility. The employment effect of the type-1 sanction becomes smaller and the effect of the type-2 sanction becomes bigger (0.7 and 3.2

52. For additional policy simulation results, see Table A2.

TABLE 10
Effects of a \$50 dynamic type-1 sanction by degree of present bias

Present bias factor in the baseline model	Baseline model (Heterogeneous hyperbolic discounting)				Time-consistent model (heterogeneous discount factor)			
	0–0.4 (most present- biased)	0.4–0.6	0.6–0.8	0.8–1.0 (least present- biased)	0–0.4 (most present- biased)	0.4–0.6	0.6–0.8	0.8–1.0 (least present- biased)
Work (%)	1.6	1.3	0.9	1.1	0.7	0.8	0.5	0.8
Welfare (%)	0.0	0.3	0.3	0.4	0.4	0.4	0.6	0.6
No work, no welfare (%)	–1.5	–1.5	–1.0	–1.3	–1.0	–1.1	–0.9	–1.2
Work, no welfare (%)	1.5	1.2	0.7	0.9	0.5	0.6	0.3	0.6
No work, welfare (%)	–0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4
Work, welfare (%)	0.1	0.2	0.1	0.2	0.1	0.2	0.2	0.2
APDV earnings (\$)	15.5	13.5	7.8	10.5	7.2	9.0	4.7	7.3
APDV net gov. expenditure (\$)	–2.4	–2.0	0.1	–1.5	0.0	–1.0	1.8	–0.5
APDV utility (\$)	3.5	1.0	–0.5	–2.2	–2.5	–2.6	–1.4	–3.1
Prefers sanction (%) ^a	24.8	13.8	24.8	1.6	0.0	0.0	0.0	0.0
Fraction of individuals in subgroup (%)	19.7	28.5	34.1	17.8	19.7	28.5	34.1	17.8

Notes: Results are based on simulations of all control group individuals from period 1 to the end of the time horizon. Numbers represent simple differences from no sanction on income support (Tables 8 and 9, column 0).

^aFraction of individuals in subgroup that prefer the sanction regime to no sanction.

percentage points, respectively). Because individuals are assumed to be time-consistent, the type-1 sanction no longer increases utility. The type-2 sanction leads to a larger reduction in utility. The work subsidies lead to larger increases in employment but smaller increases in utility.

In the time-consistent model, none of the individuals prefer to remove income support or impose sanctions, even when these policies are made revenue-neutral. In addition, the neutralized work subsidies become less popular than they are in the baseline model; for instance, only 20.7% of individuals prefer a static work subsidy regime. The difference in results between both models suggests that the work subsidies provide extra utility to individuals by mitigating their self-control problem.

Among the above policies, the dynamic type-1 sanction is the only policy that: (1) makes benefit rules stricter, (2) saves the government money, and yet (3) makes people better off. However, its aggregate effects are quite small, which can be due to substantial heterogeneity in the present bias factor. Table 10 investigates how the policy affects individuals with different degrees of present bias. Control group individuals are first partitioned into four subgroups according to their predicted present bias factor in the *baseline model* (see also Figure 3(c)): (1) 0–0.4 (most present-biased, 19.7%); (2) 0.4–0.6 (28.5%); (3) 0.6–0.8 (34.1%); (4) 0.8–1 (least present-biased, 17.8%). Policy effects from both the baseline and time-consistent models are then reported for each subgroup.⁵³

The baseline model shows that the policy effects are more dramatic among the most present-biased individuals. In this subgroup, the sanction increases employment by 1.6 percentage points, reduces government expenditure by \$2.4, and yet *increases* utility by \$3.5. One-fourth of the individuals in this subgroup prefer the sanction regime. As we turn to the less present-biased subgroups, the positive effect on utility dissipates and becomes negative. In the least present-biased subgroup, the sanction increases employment by 1.1 percentage points, reduces

53. In Table A5, we also report the simulated outcomes (under a \$50 income support) for each subgroup.

TABLE 11
Predicted work probabilities under dynamic sanction

	Work (%)			
	Baseline model (heterogeneous hyperbolic discounting)		Time-consistent model (heterogeneous discount factor)	
	Subgroup 1 (prefer sanction in baseline model)	Subgroup 2 (not prefer sanction)	Subgroup 1 (prefer sanction in baseline model)	Subgroup 2 (not prefer sanction)
If did not work last period				
No sanction on income support	26.2	11.8	25.1	11.7
Type-1 sanction on income support	26.3	11.5	24.8	11.3
Difference	+0.1	−0.3	−0.4	−0.5
If worked last period				
No sanction on income support	82.0	78.2	82.6	78.4
Type-1 sanction on income support	83.1	79.5	83.4	79.4
Difference	+1.1	+1.3	+0.7	+1.0

Notes: Results are based on simulations of all control group individuals during the sample period. Numbers may be subject to rounding error.

government expenditure by \$1.5, and *reduces* utility by \$2.2. Only 1.6% of the individuals in this subgroup prefer the sanction regime.

By contrast, the time-consistent model generates similar policy effects across the four subgroups. The employment effect ranges between 0.5 and 0.8 percentage points. In addition, all subgroups are subject to a reduction in utility (between \$1.4 and \$3.1). We also see that policy predictions from the baseline and time-consistent models converge in the least present-biased subgroup, and diverge in the most present-biased subgroup. This shows that present bias does generate qualitatively different policy implications.

Table 11 provides further evidence on differences in behavioural incentives generated by the baseline and time-consistent models.⁵⁴ Two policy scenarios are considered: (1) 50-dollar income support and no sanctions; (2) 50-dollar income support and a \$50 type-1 sanction. For each scenario, the table reports the predicted employment probabilities conditional on work status last period. The probabilities are reported separately for two pre-defined subgroups in the control group: (1) individuals who prefer the type-1 sanction under the baseline model (subgroup 1, 17.5%); (2) individuals who prefer no sanctions under the baseline model (subgroup 2, 82.5%).

As discussed previously, under a type-1 sanction, individuals who did not work last period are subject to a dynamic work disincentive, but their static budget constraint remains unaffected. Therefore, if there are no commitment-related incentives, the policy should reduce the employment probability among these individuals. As the table shows, this is indeed the case for the time-consistent model. Among subgroup-1 individuals who did not work last period, the type-1 sanction reduces the current employment probability by 0.4 percentage points (0.5 percentage points for subgroup 2).

The baseline model gives a somewhat different picture. Individuals may use the sanction as a commitment device, resulting in a commitment-related work incentive especially among

54. Gustman and Steinmeier (2012) compare policy effects in exponential and hyperbolic discounting models of consumption and retirement. The time preference parameters are chosen such that the individual yields approximately the same level of assets at the age of 62 years. Their results are mixed—while it is frequently difficult to distinguish behaviour between both models, there are policy simulations in which the effects are non-trivially different.

TABLE 12
Response to productivity and reform package

	Baseline model (heterogeneous hyperbolic discounting)			Time-consistent model (heterogeneous discount factor)			
	Permanent productivity $-10\%^a$	Standard error of productivity $+10\%^b$	Reform: $0.888 \times$ welfare and work subsidy of ^c \$47.5	Permanent productivity $-10\%^a$	Standard error of productivity $+10\%^b$	Reform: $0.888 \times$ welfare and work subsidy of ^c \$51.5	\$47.5
	(1)	(2)	(3)	(4)	(5)	(6a)	(6b)
Work (%)	-2.3	-1.0	3.2	-2.5	-1.1	3.8	3.6
Welfare (%)	2.0	0.9	-3.2	2.0	0.9	-3.7	-3.6
No work, no welfare (%)	0.8	0.4	-0.2	1.0	0.5	-0.3	-0.2
Work, no welfare (%)	-2.8	-1.2	3.4	-3.0	-1.4	4.1	3.8
No work, welfare (%)	1.5	0.6	-3.0	1.6	0.6	-3.5	-3.4
Work, welfare (%)	0.4	0.3	-0.2	0.4	0.3	-0.2	-0.2
APDV Earnings (\$)	-55.7	-7.9	23.5	-58.0	-8.7	28.1	26.3
APDV net gov. expenditure (\$)	17.4	2.1	0.0	17.6	2.1	0.0	-1.4
APDV utility (\$)	-14.9	-6.1	9.1	-11.0	-4.1	6.3	5.1
Prefers new policy (%)	—	—	64.1	—	—	47.7	44.6
Implied elasticity on work	0.52	-0.22	—	0.56	-0.23	—	—

Notes: Results are based on simulations of all control group individuals from period 1 to the end of the time horizon. Numbers represent simple differences from baseline.

^aThe mean of wage reduces by 10%.

^bStandard error of wage shock increases by 10%, keeping the mean of wage constant.

^cThe reform package reduces the maximum welfare benefit level by 11.2%, and introduces a subsidy to workers.

subgroup-1 individuals (who also prefer the sanction regime). In fact, among subgroup-1 individuals who did not work last period, the type-1 sanction will *increase* the current employment probability by 0.1 percentage points. For a subgroup-2 individual who did not work last period, there is a reduction of 0.3 percentage points.

Table 12 compares the response of the baseline and time-consistent models to productivity changes and a policy reform package in the control group: (1) the level of productivity (in the wage equation) drops permanently by 10%; (2) the standard error of productivity shock increases by 10%; (3) a reform package consisting of a 11.2% reduction in the maximum welfare benefit level and a static work subsidy of \$47.5 (or \$51.5).

In the baseline model, the productivity drop reduces employment by 2.3 percentage points, implying a wage elasticity of 0.52. Welfare use increases by 2 percentage points. When productivity becomes more volatile, employment reduces by 1 percentage point and welfare use increases by 0.9 percentage points. In both cases, more than one-fourth of the reduction in earnings can be offset by the increase in government expenditure. Relative to the baseline model, the time-consistent model is slightly more responsive to productivity changes; for instance, the productivity drop implies a wage elasticity of 0.56. In addition, utility reduces by a smaller amount, indicating that individuals' well-being is less susceptible to productivity changes in the time-consistent model.

The scale of the reform is chosen such that it is revenue-neutral in each model. In the baseline model, the reform increases employment by 3.2 percentage points and reduce welfare use by 3.2 percentage points. Average utility increases by \$9.1, and 64.1% of individuals prefer the reform regime. In the time-consistent model, a larger work subsidy is required to make the reform revenue-neutral (\$51.5 as opposed to \$47.5). Despite the more generous subsidy, utility increases by a smaller amount (\$6.3), and fewer individuals prefer the reform regime (47.7%). Again, these

TABLE 13
Savings at 4.5 years after random assignment

	Control group		Treatment group	
	Data ^a	Model ^b	Data ^a	Model ^b
Savings balance (mean)	164.5	135.0	172.9	152.3
Savings balance (std. dev.)	623.2	352.1	660.6	378.8
Fraction of individuals with:				
Any savings (%)	24.4	22.3	25.5	23.7
>\$100 savings (%)	14.3	19.0	15.7	20.6
>\$500 savings (%)	6.7	11.4	7.1	12.8
>\$1,000 savings (%)	3.7	4.3	3.4	4.9

^aBased on the follow-up survey administered around 4.5 years following random assignment. Control group consists of 654 respondents; treatment group consists of 638 respondents.

^bSimulated from the baseline model (heterogeneous hyperbolic discounting).

results suggest that one may exploit time-inconsistency so as to make programme design more desirable.

5.7. Sensitivity analysis

5.7.1. Model specification. The baseline model assumes that individuals spend all their income every period. Although this is a common assumption in models that involve the low-income population, in reality individuals with relatively high income in this subpopulation may be able to accumulate savings. In 1996, the federal poverty threshold for a single-adult family with two children was \$12,641 per year, or \$1,053 per month.⁵⁵ In the estimation sample, around 85% of workers (or 94% of individuals) have earnings below this level. Although there is no information on savings during the sample period, some information is available in MDRC's follow-up survey, which was separately administered 4–5 years after random assignment. Table 13 presents summary statistics on savings at 4.5 years after random assignment, among individuals who responded to the survey.⁵⁶ The average savings balance is \$164.5 and \$172.9 in the control and treatment group, respectively. The standard deviation is around \$650. In both groups, one-fourth of the individuals have a positive savings balance and 10% have more than \$500 of savings balance.

To investigate the impact of savings, the baseline model is modified by introducing a conventional budget equation with a minimal “consumption floor”. The model is not re-estimated, and is just used to evaluate what will happen when this budget equation is introduced. Let A_{it} be the opening savings balance at period t . The individual's utility is determined by her consumption c_{it} (instead of current income y_{it}). At each period, the individual's consumption c_{it} and saving s_{it} follow the budget constraints $c_{it} + s_{it} = A_{it} + y_{it}$ and $A_{i,t+1} = R s_{it}$, where R is the gross interest rate. It is assumed that the individual is only able to save if the sum of her savings balance A_{it} and current income y_{it} exceeds the federal poverty threshold \bar{y} . Otherwise, she spends all her income and savings to meet her needs. More specifically, the amount of saving is assumed to be $s_{it} = \max\{A_{it} + y_{it} - \bar{y}, 0\}$; given savings balance A_{it} and income y_{it} , her saving s_{it} will be completely determined by the budget equation. This saving is assumed to generate a future

55. The threshold is set by the U.S. Census Bureau at three times the cost of a minimum food diet in 1963, updated annually for inflation.

56. This is a measure of liquid assets. The survey question is: “Rounding to the nearest hundred, how much altogether do you have in savings?” For further discussions, see Bloom *et al.* (2000).

TABLE 14
Sensitivity analysis: allowing for saving

	Baseline model (heterogeneous hyperbolic discounting)			Time-consistent model (heterogeneous discount factor)		
	From (1), effect of adding			From (4), effect of adding		
	Allow for saving, baseline	Welfare asset limit \$1,000	Income support \$50	Allow for saving, baseline	Welfare asset limit \$1,000	Income support \$50
	(1)	(2)	(3)	(4)	(5)	(6)
Work (%)	-3.3	0.1	-3.1	-2.3	0.1	-3.6
Welfare (%)	-0.4	-0.4	-5.3	-0.9	-0.6	-5.8
No work, no welfare (%)	3.3	0.1	7.6	2.8	0.2	8.3
Work, no welfare (%)	-2.9	0.3	-2.2	-1.8	0.4	-2.5
No work, welfare (%)	-0.0	-0.2	-4.4	-0.5	-0.3	-4.7
Work, welfare (%)	-0.4	-0.2	-0.9	-0.4	-0.3	-1.1
APDV earnings (\$)	-25.2	1.3	-21.4	-8.0	2.1	-24.0
APDV net gov. expenditure (\$)	-4.6	-1.6	-13.4	-9.0	-2.4	-16.4
APDV utility (in \$)	34.8	-0.3	10.0	36.8	-0.0	19.1
Savings balance	123.4	-0.6	-10.6	163.7	-0.4	-14.7
Savings balance at 4.5 years	135.0	-0.0	-12.4	182.6	0.5	-19.2

Notes: Results are based on simulations of all control group individuals from period 1 to the end of the time horizon. Columns 1 and 4 report simple differences in outcomes from the baseline scenario in which savings are not allowed.

value of $\beta_i \delta_i (A_{i,t+1} + \alpha_y A_{i,t+1}^2)$ to the individual. The above stylised setup generates two opposing effects on labour supply. First, if the savings balance is larger, the individual will be less inclined to work due to the income effect. Secondly, because the act of saving entails future value to the individual, she may be more inclined to work now to generate savings in the future. Therefore, although saving is somewhat mechanically determined, the individual can still adjust it indirectly through changing her labour supply (which affects y_{it}).

Table 13 reports simulated savings at 4.5 years after random assignment from the modified baseline model (which is not re-estimated). The simulation assumes that $A_{i0} = 0$ and $R = 1$. Although savings data are not used for estimation, the simulation results match the data well. The average simulated savings balance is \$135.0 and \$152.3 in the control and treatment group, respectively. Therefore, as in actual data, the treatment group accumulates slightly more savings than the control group. Although the s.d. in simulation is lower than the actual value, the model predicts the fractions of individuals at key regions of savings reasonably well.

Table 14 compares the response of the baseline and time-consistent models when control group individuals are allowed to save. In the baseline model, employment and welfare use reduce by 3.3 and 0.4 percentage points, respectively. Due to the smoothing of consumption, utility increases by \$34.8; the average savings balance is \$123.4. In the time-consistent model, there is a smaller reduction in employment (2.3 percentage points). The average savings balance is higher at \$163.7, but utility increases by a similar amount.

Table 14 also reports the effects of two policies when saving behaviour is present. The first policy imposes an asset limit of \$1,000 to welfare eligibility. Because not many individuals accumulate substantial assets, the policy reduces welfare use by around 0.5 percentage points, and have a small effect on other outcomes. The second policy is a \$50 income support programme. Relative to an environment where individuals cannot save, the policy generates a slightly smaller effect on employment and other outcomes. The policy also reduces the savings balance by around 8%, suggesting that individuals rely less on their own savings to insure against risks.

Although our parsimonious approach matches savings data reasonably well, it does not preclude other approaches, in particular the conventional savings model. Nevertheless, the alternative model should generate predictions that are consistent with the following data patterns: (1) a large proportion of the individuals have no savings, and (2) savings are marginally higher in the treatment group, despite the large difference in asset limit tests between both groups.^{57,58} Due to data limitations and numerical complexity of the conventional savings model, this direction will be left for future research.

Results from two additional exercises are summarized below. The first exercise examines fertility. Among all demographic changes this is particularly important to budget analysis, because welfare and EITC benefits, as well as income tax deductions, depend on the number of children. Although changes in demographic characteristics are not observed during the sample period, in the MDRC follow-up survey 22.8% of the respondents indicated that they gave birth during the 4.5 years following random assignment. Assuming each individual gave birth once, this implies an average birth rate of 1.44% per period. In the simulation exercise, individuals are assumed to follow the above homogeneous law of motion in fertility, and the budget constraint and preferences are updated following child birth (Table A4).^{59,60} Employment and welfare use increase by 0.7 and 0.8 percentage points, respectively, as individuals who give birth move into welfare and some are at work simultaneously. The net government expenditure increases by around 6%. Despite the above differences, the effects of policy and productivity changes are robust to the inclusion of fertility behaviour.

The second exercise involves the time horizon. The time preference estimates may be sensitive to the length of the time horizon used in estimation. To address this concern, the baseline model is estimated using alternative lengths of the time horizon: thirty-two periods, twenty-seven periods, and twenty-two periods; the log-likelihoods are 28,219.87, 28,220.51, and 28,221.69, respectively. These specifications lead to only minor changes in the time preference estimates.⁶¹

57. Individuals in FTP can accumulate substantial assets and remain eligible. The asset limit is \$1,000 in AFDC and \$5,000 in FTP. The value of vehicle assets excluded from this definition increased from \$1,500 to \$8,150. In MDRC's follow-up survey, the fraction of individuals with more than \$1,000 of savings is 3.7 and 3.4% in the control and treatment group, respectively; the vehicle ownership rate is 60 and 59%, respectively.

58. A large policy difference in the asset limit test does not necessarily induce large differences in savings—as is the case with other policies, this should be empirically tested. Hurst and Ziliak (2006) discuss the theoretical predictions from consumption-saving models when asset limits are present. Using data from the Panel Study of Income Dynamics, they find that the substantial increases in asset limits during welfare reform had no economically or statistically significant effect on the savings of single mothers. The upper bound saving response to a \$1,000 increase in the liquid asset limit (which is \$1,000 under AFDC) is \$40, and the average ranges from –\$80 to \$10 depending on the specification.

59. Another demographic characteristic that can affect programme eligibility is marital status. In the MDRC survey, 17% of the respondents became married during the 4.5 years following random assignment. A similar law of motion can be specified in marital status. However, because the population of interest is single mothers, only the pre-marriage periods will be relevant in the simulation exercise; thus, this exercise is not pursued here.

60. Note that the model is not re-estimated due to data limitations, which reduces the power of this robustness check. A more ideal approach would be to allow the woman to expect child birth with some probability each period, and the probability can depend on some demographic characteristics. This requires addressing the problem of missing state variables, which is very challenging in likelihood-based estimation, but could be accommodated more easily in an indirect inference procedure (see for instance Gourieroux *et al.*, 1993). Another advantage of indirect inference is that it can combine data sources more easily.

61. As a referee pointed out, the length of the decision period may be crucial in determining the time preference estimates (and not just through the usual time aggregation). For the decision period length to be treated as an additional parameter of the problem, event history type data in some longitudinal surveys will be ideal for such analysis. Empirical articles on hyperbolic discounting predominantly use annual data for estimation. In this respect, it is encouraging to also detect time-inconsistency in our data (quarterly). Given the short sample period and the way the exclusion variable is constructed, it would be difficult to estimate the model with annual data.

TABLE 15
Performance of models estimated from subsamples

	Work (%)	Welfare (%)	No work, no welfare (%)	Work, no welfare (%)	No work, welfare (%)	Work, welfare (%)
Control group						
Data ^a	40.1	48.4	24.9	26.8	35.0	13.4
Baseline model	40.0	48.1	26.2	25.7	33.8	14.3
Model estimated with control group data	39.2	47.8	26.4	25.9	34.4	13.3
Model estimated with treatment group data ^b	37.3	53.2	24.9	22.0	37.9	15.3
Treatment group						
Data ^a	46.1	43.6	27.7	28.7	26.2	17.4
Baseline model	45.0	42.2	28.7	29.2	26.4	15.8
Model estimated with control group data ^b	43.6	44.6	27.0	28.4	29.5	15.2
Model estimated with treatment group data	46.0	42.8	28.2	29.0	25.8	17.0
Predicted effects of FTP earnings disregard ^c						
Baseline model	+0.9	+0.9	-0.5	-0.5	-0.4	+1.3
Model estimated with control group data	+0.8	+0.5	-0.3	-0.2	-0.5	+1.0
Model estimated with treatment group data	+1.3	+1.2	-0.4	-0.8	-0.8	+2.0
Predicted effects of FTP time limit ^c						
Baseline model	+0.8	-7.8	+4.3	+3.4	-5.1	-2.7
Model estimated with control group data	+0.2	-3.1	+1.9	+1.2	-2.1	-1.0
Model estimated with treatment group data	+3.2	-13.9	+4.9	+9.0	-8.1	-5.8

^aAverage outcome during the sample period.

^bOut-of-sample prediction using the validation sample.

^cAverage percentage-point effect during the sample period.

5.7.2. Estimation using subsamples and model validation. In this exercise, the model is estimated using the control or treatment group subsample only (see Online Appendix for estimation results). Due to the time limit, the treatment group's decision problem involves the exclusion variable \tilde{M}_{it} . This variable is absent in the control group's decision problem. The baseline model is first estimated using the treatment group subsample. The time preference parameters remain well identified, with the standard errors becoming somewhat larger. The discount factor estimates become higher. The present bias factor estimates also become somewhat higher, but substantial present bias still remain.

The baseline model was estimated unsuccessfully using the control group subsample, as all the time preference estimates had huge standard errors. A simpler model—the time-consistent heterogeneous discount factor model—is then estimated using the control group subsample. Relative to the time-consistent model based on the full sample, the discount factor estimates are considerably lower, there is no significant heterogeneity, and the standard errors can be 3–4 times as large. These results are consistent with Ferrall (2012), who finds that the standard error of the discount factor becomes much larger if only the data moments within the control group are used.⁶² However, unlike Ferrall (2012), the standard errors of other coefficients in the model do not increase substantially. Other coefficients remain well identified, including the ones that determine the curvature of the utility function.

Table 15 compares the predictive performance of the above models.⁶³ The upper panels in the table use control group and treatment group data as the validation sample, respectively.

62. Ferrall's (2012) approach is slightly different. The model is first estimated using moments from both the control and treatment groups. Then, based on the model estimates, standard errors are re-computed using moments from one of the groups only.

63. The exercise shares some similarities with Lise *et al.* (2005) and Todd and Wolpin (2006), in which the treatment group is used as a validation sample for a structural model that is estimated with the control group sample.

For comparison, predictions from the full-sample baseline model are also reported. The model estimated with control group data has good within-sample performance, but it under-predicts employment of the treatment group.⁶⁴ The model estimated with treatment group data also has good within-sample performance, but it under-predicts employment and over-predicts welfare use in the control group. The lower panels compare the predicted effects of FTP earnings disregard and FTP time limit from each model. Relative to the full-sample baseline model, the predicted policy effects from the control group model are smaller, while the predicted policy effects from the treatment group model are bigger. These differences are more dramatic in predictions involving the FTP time limit.

6. CONCLUSION

In this article, a hyperbolic discounting model of labour supply and welfare participation with heterogeneous time preference parameters was estimated. Exclusion restrictions were constructed from variations in behaviour induced by welfare time limits in a policy experiment. The model had the following major features that together determine welfare dependence: (1) time-inconsistent individuals were unable to fully commit to their plans; (2) dynamics due to state dependence in preferences and human capital accumulation; (3) labour supply disincentives generated by a piecewise-linear budget constraint.

We found that most individuals were time-inconsistent and had varying degrees of present bias, and many perceived themselves as time-inconsistent. Our estimates provided a more comprehensive piece of evidence on time preference than the existing literature. We examined conventional policies such as welfare and EITC, and controversial counterfactuals such as income support, which could make individuals worse off by worsening the commitment problem. We also found that time-inconsistency opened up an array of dynamic policy interventions for consideration. These interventions were important because they directly affected how individuals make efficient intertemporal decisions. Among them were various time limits (esp. prowork limits) and dynamic tax/sanctions or work subsidies. The results were evaluated against a number of sensitivity analysis, in particular shutting down time-inconsistency.

We also found that commitment-related incentives were more notable in certain types of policies. One example was the “type-1” dynamic sanction, which reduced income support benefits if the individual worked last period. The policy carried an additional work incentive in that it provided a means by which the current self can commit her futures to work. As a result, policy effects predicted by hyperbolic discounting and time-consistent models could differ in magnitude and qualitative patterns, especially among individuals with severe present bias.

To extend the literature, it would be helpful to collect further evidence in a broader setting than this policy experiment, using other policy variations as a source of identification. It may also be useful to validate estimates from data on elicited beliefs with observational data. The ideas of prowork time limit and dynamic sanction/subsidy are intriguing, but their properties should be further confirmed by actual behaviour, ideally from a policy experiment, before large-scale implementation. Along the lines of these policies, other commitment devices can also be developed. The revenue-neutrality of certain policies such as EITC expansion partly depends

Attanasio *et al.* (2011) and Ferrall (2012) assess the importance of control-treatment policy variations to the predictive performance of structural models. Keane and Wolpin (2007) use Texas as a holdout sample to validate the structural model in Keane and Wolpin (2010).

64. The out-of-sample predictions are generated by modifying the budget constraint such that it matches the policy environment in the other group. The coefficient of enhanced services is obtained from the estimate in the full-sample baseline model.

on increases in tax revenue at a later stage. However, the government may also exhibit time-inconsistent preference, which complicates the budget analysis. These are potentially fruitful directions for future research.

A. APPENDIX

A.1. Program benefit and tax formulas

The programme benefit and tax formulas in the budget constraint (equation (2)) are defined as follows. The net income for the determination of cash welfare (AFDC/FTP) benefits is

$$N_{Ait} = \max\{(E_{it} - b_{A2i})(1 - r_{Ai}), 0\}, \quad (14)$$

and the cash benefit amount is

$$\tilde{B}_{Ait} = \max\{b_{A1i} - N_{Ait}, 0\}. \quad (15)$$

According to programme rules, welfare participants are categorically eligible for food stamps, which means that food stamp benefits are automatically assigned to them. The net income for the determination of food stamp benefits is

$$N_{Fit} = \max\{0.8E_{it} + \tilde{B}_{Ait} - D_F, 0\}. \quad (16)$$

The food stamp benefit amount is

$$B_{Fit} = \max\{b_{Fi} - 0.3N_{Fit}, 0\} \quad (17)$$

if the gross and net income eligibility thresholds are satisfied, *i.e.* $E_{it} \leq e_i$ and $N_{Fit} \leq e_{Ni}$, otherwise $B_{Fit} = 0$. The total welfare benefit amount is $B_{Ait} = \tilde{B}_{Ait} + B_{Fit}$.

The programme benefit rule Z_{Ai} consists of the maximum cash benefit amount b_{A1i} , dollar earnings disregard b_{A2i} , percent earnings disregard r_{Ai} , maximum food stamp benefit amount b_{Fi} , eligibility thresholds e_i and e_{Ni} , and the sum of standard and shelter deduction $D_F = 184$. The benefit parameters b_{A1i} , b_{Fi} , e_i , and e_{Ni} vary by family size. The earnings disregards b_{A2i} and r_{Ai} vary by treatment status.⁶⁵

The EITC benefit amount is

$$B_{Eit} = \begin{cases} r_{E1i}E_{it}, & \text{if } E_{it} < b_{E1i}, \\ r_{E1i}b_{E1i}, & \text{if } b_{E1i} \leq E_{it} < b_{E2i}, \\ \max\{r_{E1i}b_{E1i} - r_{E2i}(E_{it} - b_{E2i}), 0\}, & \text{if } E_{it} \geq b_{E2i}. \end{cases} \quad (18)$$

The programme benefit rule Z_{Ei} consists of the tax credit phase-in rate r_{E1i} , phase-out rate r_{E2i} , lower bracket threshold b_{E1i} , and upper bracket threshold b_{E2i} . The rates and bracket thresholds all vary by family size.⁶⁶

The tax formula is

$$T_{it} = r_{T1} \max\{E_{it} - b_{T1} - b_{T2i}, 0\} + r_{T2}E_{it}. \quad (19)$$

The programme rule Z_{Ti} consists of federal income tax rate $r_{T1} = 0.15$, payroll tax rate $r_{T2} = 0.0765$ (half of the tax incidence), standard deduction as head of household b_{T1} , and personal exemption amount b_{T2i} . The exemption amount varies by family size.⁶⁷ Florida had no state income tax during the sample period.

65. For instance, in 1995, b_{A1i} was \$241, \$303, and \$364, b_{Fi} was \$214, \$306, and \$389, e_{Ni} was \$844, \$1059, and \$1276, and e_i was \$1097, \$1376, and \$1658 for single mothers with 1, 2, and 3 children, respectively. Welfare recipients in the control group had a \$120 and 33% disregard for the first four months of work (treated as one quarter in the model), and a \$90 disregard subsequently. Welfare recipients in the treatment group had a \$200 and 50% disregard.

66. For instance, in 1994, $Z_{Ei} = (0.236, 0.1598, 645, 916)$ for families with one child (monthly dollar amounts, same below) and $(0.3, 0.1768, 687, 916)$ for families with two or more children. In 1995, the benefit parameters were $(0.34, 0.1598, 513, 940)$ and $(0.36, 0.2022, 720, 940)$, respectively.

67. In 1995, the sum of deduction and exemption for a head of household with one dependent was \$895 (monthly amount), and increased by \$208 for each additional dependent.

A.2. Model estimation

The model is estimated by the method of maximum likelihood. In each period, the researcher observes the individual's employment status d_{hit} , welfare participation status d_{ait} , earnings E_{it} , covariates X_i and \mathbf{x}_i , treatment status e_i , programme benefit rules \mathbf{Z}_{Ai} , \mathbf{Z}_{Ei} , and \mathbf{Z}_{Ti} , and the vector of state variables \mathbf{S}_{it} . The parameters to be estimated are in the following parts of the model: preferences in utility ($\psi_h, \psi_a, \psi_{ha}, \psi_y, \alpha_{h2}, \gamma_h, \gamma_{h2}, \gamma_a, \gamma_{ha}, \psi_{\sigma_c}, \mu_{h2}, \mu_{a2}, \mu_{ha2}$), time preference ($\psi_\delta, \psi_\beta, \psi_{\tilde{\beta}}, \mu_{\delta 2}, \mu_{\beta 2}, \mu_{\tilde{\beta} 2}$), wage equation ($\psi_w, \omega_0, \omega_1, \psi_{\sigma_w}, \mu_{w2}$), budget constraint (B_{AS}), and type probability ($\phi_w, \phi_p, \phi_{wp}$).

In each iteration in the parameter space, computation of the likelihood for individual i consists of three nested loops. The inner loop computes the likelihood for each period t given the individual's type and the expected value function (or expected continuation long-run utility) obtained from the backward recursion procedure. The middle loop carries out the backward recursion procedure of the dynamic programming problem given the individual's type. In the outer loop, the likelihood is computed as the average of type-specific likelihoods, which are weighted by the individual's type probabilities.

The likelihood contribution in the inner loop is derived as follows. Let $\mathbf{q} = (\text{wtype}, \text{ptype}) \in \mathcal{Q} = \{(1, 1), (2, 1), (1, 2), (2, 2)\}$ denote the individual's type. Let \bar{V}_{ikt} denote the choice-specific value, *exclusive* of the choice-specific preference shock, for alternative k . In the time-consistent model, \bar{V}_{ikt} is⁶⁸:

$$\bar{V}_{ikt}(w_{it}, \mathbf{S}_{it}, \mathbf{q}) \equiv \bar{u}_{it}(k; w_{it}, \mathbf{S}_{it}, \mathbf{q}) + \delta_i E_t V_{i,t+1}(\mathbf{S}_{ik,t+1}, \boldsymbol{\epsilon}_{i,t+1}; \mathbf{q}). \quad (20)$$

In the hyperbolic discounting model, \bar{V}_{ikt} is:

$$\bar{V}_{ikt}(w_{it}, \mathbf{S}_{it}, \mathbf{q}) \equiv \bar{u}_{it}(k; w_{it}, \mathbf{S}_{it}, \mathbf{q}) + \beta_i \delta_i E_t v_{i,t+1}(\mathbf{S}_{ik,t+1}, \boldsymbol{\epsilon}_{i,t+1}; \tilde{\kappa}_{i,t+1}^+; \mathbf{q}). \quad (21)$$

The choice-specific value is conditional on wage w , state variables \mathbf{S} , type \mathbf{q} , and the expected value function or continuation long-run utility. The distributional assumption for the choice shock implies that the conditional choice probability in period t has the following form:

$$P_{ikt}(w_{it}, \mathbf{S}_{it}, \mathbf{q}) \equiv \frac{\exp(\bar{V}_{ikt}(w_{it}, \mathbf{S}_{it}, \mathbf{q})/\sigma_{ci})}{\sum_{j=1}^5 \exp(\bar{V}_{ijt}(w_{it}, \mathbf{S}_{it}, \mathbf{q})/\sigma_{ci})}. \quad (22)$$

Let $f_i(\cdot)$ denote the probability density function of wage. The likelihood contribution of individual i in period t , denoted L_{it} , is given as follows:

$$L_{it} = \begin{cases} \int P_{i1t}(w, \mathbf{S}_{it}, \mathbf{q}) f_i(w|\mathbf{S}_{it}, \mathbf{q}) dw & \text{if } d_{hit} = 0, d_{ait} = 0; \\ P_{i2t}(E_{it}, \mathbf{S}_{it}, \mathbf{q}) f_i(E_{it}|\mathbf{S}_{it}, \mathbf{q}) + P_{i3t}(E_{it}/2, \mathbf{S}_{it}, \mathbf{q}) f_i(E_{it}/2|\mathbf{S}_{it}, \mathbf{q}) & \text{if } d_{hit} = 1, d_{ait} = 0; \\ \int P_{i4t}(w, \mathbf{S}_{it}, \mathbf{q}) f_i(w|\mathbf{S}_{it}, \mathbf{q}) dw & \text{if } d_{hit} = 0, d_{ait} = 1; \\ P_{i5t}(E_{it}, \mathbf{S}_{it}, \mathbf{q}) f_i(E_{it}|\mathbf{S}_{it}, \mathbf{q}) & \text{if } d_{hit} = 1, d_{ait} = 1. \end{cases}$$

The wage is integrated out in the likelihood contribution using Gaussian–Hermite quadrature with five points (*e.g.* Butler and Moffitt, 1982; Swann, 2005).

Estimation and identification related to PT/FT work status (albeit not being the main focus of the article) are explained further below. For an individual who is observed to work but not receive welfare ($d_{hit} = 1, d_{ait} = 0$), she must either work part-time ($h_{it} = 1$) or full-time ($h_{it} = 2$). Because PT/FT work status is not observed, the likelihood contribution must combine both cases. According to the model, earnings from full-time work is twice as much as earnings from part-time work (see $E_{it} = w_{it} h_{it}$ in Section 3). If she works part-time, her wage w_{it} is just equal to observed earnings (E_{it}). The likelihood contribution is $P_{i2t}(E_{it}, \mathbf{S}_{it}, \mathbf{q}) f_i(E_{it}|\mathbf{S}_{it}, \mathbf{q})$. If she works full-time, her wage w_{it} is equal to half of observed earnings ($E_{it}/2$), so the likelihood contribution is $P_{i3t}(E_{it}/2, \mathbf{S}_{it}, \mathbf{q}) f_i(E_{it}/2|\mathbf{S}_{it}, \mathbf{q})$. As seen from above, because the likelihood contributions depend on $f_i(\cdot)$, the distribution of observed earnings contains information that can identify the PT/FT utility parameters.⁶⁹ Furthermore, the parameters can be identified by policy variations in the budget constraint (Section 3.3).

68. For notational simplicity, other control variables are not shown in the expression.

69. A stylized example is given as follows. Given the covariates and type, the wage distribution is log-normal (equation (3)). Among working welfare recipients, the observed earnings distribution will resemble this distribution (everyone works part-time according to the choice set). Yet among workers who are not on welfare, observed earnings will appear as a mixture distribution—for part-time workers, we observe $E_{it} = w_{it}$; for full-time workers, we observe $E_{it} = 2w_{it}$. The shape of the mixture will provide information on how many individuals choose full-time work versus part-time work, and consequently PT/FT work preference.

Let $\pi_{qi}(\mathbf{x}_i)$ denote the probability that an individual is of type $\mathbf{q} = (q_w, q_p)$. The probability is given as follows:

$$\pi_{qi}(\mathbf{x}_i) := \Pr(\text{wtype} = q_w, \text{ptype} = q_p | \mathbf{x}_i) = \Pr(\text{wtype} = q_w | \mathbf{x}_i) \Pr(\text{ptype} = q_p | \mathbf{x}_i, \text{wtype} = q_w) \quad (23)$$

$$= \frac{\exp((q_w - 1)\mathbf{x}_i \boldsymbol{\phi}_w)}{1 + \exp(\mathbf{x}_i \boldsymbol{\phi}_w)} \frac{\exp((q_p - 1)(\mathbf{x}_i \boldsymbol{\phi}_p + \phi_{wp}(q_w - 1))}{1 + \exp(\mathbf{x}_i \boldsymbol{\phi}_p + \phi_{wp}(q_w - 1))}. \quad (24)$$

The conditional probabilities in equation (23) have a logistic form.⁷⁰ Moreover, the probability of preference type (ptype) is a function of the skill/wage type (wtype). This is captured by parameter ϕ_{wp} . If ϕ_{wp} is positive, then being a type-2 skill person will increase the probability that she is a type-2 preference person.

Suppose individual i is observed in the data for T_i periods and there are N individuals in the sample. The log-likelihood function is⁷¹:

$$LL = \sum_{i=1}^N \ln \sum_{\mathbf{q} \in Q} \pi_{qi}(\mathbf{x}_i) \prod_{t=2}^{T_i} L_{it}. \quad (25)$$

The standard errors are computed using the BHHH algorithm (Berndt *et al.*, 1974).

A.3. Backward recursion of the dynamic programming problem

Time-consistent preference. The expectation of the value function in equation (6) is computed as follows. The first step involves integrating over the distribution of the choice-specific preference shocks. The distributional assumption implies that the integral has a closed form solution (*e.g.* Rust, 1987),

$$E_{t-1} V_{it}(\mathbf{S}_{it}, \boldsymbol{\epsilon}_{it} | w_{it}, \mathbf{q}) = eu + \sigma_{ci} \ln \left(\sum_{k=1}^5 \exp(\bar{V}_{ikt}(w_{it}, \mathbf{S}_{it}, \mathbf{q}) / \sigma_{ci}) \right), \quad (26)$$

where eu is Euler's constant and $\bar{V}_{ikt}(\cdot)$ is the choice-specific value defined in equation (20). The wage is then integrated out:

$$E_{t-1} V_{it}(\mathbf{S}_{it}, \boldsymbol{\epsilon}_{it} | \mathbf{q}) = \int E_{t-1} V_{it}(\mathbf{S}_{it}, \boldsymbol{\epsilon}_{it} | w, \mathbf{q}) f_i(w | \mathbf{S}_{it}, \mathbf{q}) dw. \quad (27)$$

The integration over the wage distribution is computed by Gaussian–Hermite quadrature with five points.

Present-biased preference. As in Fang and Silverman (2009), we focus on Markov strategies. Denote a Markov strategy as a mapping $\kappa_{it} : \mathcal{X}_i \mapsto D$, where \mathcal{X} is the state space and D is the choice set. The continuation strategy profile in period t is defined as the set of strategies from period t to the end of the time horizon \mathcal{T} , and is denoted by $\kappa_{it}^+ \equiv \{\kappa_{ia}\}_{a=t}^{\mathcal{T}}$. The continuation long-run utility given continuation strategy profile κ_{it}^+ is formally defined as

$$v_{it}(\mathbf{S}_{it}, \boldsymbol{\epsilon}_{it}; \kappa_{it}^+, \mathbf{q}) = u_{ik_{it}} + \delta_i E_i v_{i,t+1}(\mathbf{S}_{ik_{it},t+1}, \boldsymbol{\epsilon}_{i,t+1}; \kappa_{i,t+1}^+, \mathbf{q}), \quad (28)$$

where k_{it} is the individual's choice based on the strategy in period t .

The solution procedure first recursively solves for the continuation long-run utility that is defined upon the *perceived continuation strategy profile*. A perceived continuation strategy profile $\tilde{\kappa}_{it}^+ \equiv \{\tilde{\kappa}_{ia}\}_{a=t}^{\mathcal{T}}$ is defined as a continuation strategy profile such that the strategy in each period a , where $a \in \{t, t+1, \dots, \mathcal{T}\}$, is

$$\tilde{\kappa}_{ia}(\mathbf{S}_{ia}, \boldsymbol{\epsilon}_{ia}, \mathbf{q}) \equiv \arg\max_{d_{ia} \in D} \sum_{k=1}^5 d_{ika} \left(u_{ika} + \tilde{\beta}_i \delta_i E_a v_{i,a+1}(\mathbf{S}_{ik,a+1}, \boldsymbol{\epsilon}_{i,a+1}; \tilde{\kappa}_{i,a+1}^+, \mathbf{q}) \right). \quad (29)$$

Note that the perceived continuation strategy profile is influenced by the naivety factor $\tilde{\beta}_i$.

70. For instance, $\Pr(\text{wtype} = 1 | \mathbf{x}_i) = \frac{1}{1 + \exp(\mathbf{x}_i \boldsymbol{\phi}_w)}$ and $\Pr(\text{wtype} = 2 | \mathbf{x}_i) = \frac{\exp(\mathbf{x}_i \boldsymbol{\phi}_w)}{1 + \exp(\mathbf{x}_i \boldsymbol{\phi}_w)} = \frac{1}{1 + \exp(-\mathbf{x}_i \boldsymbol{\phi}_w)}$, which is the usual logistic form. Hence, type 1 is the “base category” relative to type 2. If an element in $\boldsymbol{\phi}_w$ is positive, this implies that a larger value in the corresponding element in \mathbf{x}_i will increase the probability that the individual is of type-2 skill.

71. The first period is not used directly for likelihood computation (*e.g.* Chan, 2013). This method of conditional maximum likelihood estimation does not require specifying the distribution of outcomes in period 1, but incurs a cost on statistical efficiency.

The recursion from period $t+1$ to t is described as follows. Using the perceived continuation strategy profile and its corresponding continuation long-run utility in period $t+1$, the perceived continuation strategy profile in period t can be solved recursively via equation (29). Then, applying equation (28), the corresponding continuation long-run utility in period t is

$$v_{it}(S_{it}, \epsilon_{it}; \tilde{\kappa}_{it}^+, \mathbf{q}) = u_{ik_{it}} + \delta_i E_t v_{i,t+1}(S_{ik_{it},t+1}, \epsilon_{i,t+1}; \tilde{\kappa}_{i,t+1}^+, \mathbf{q}). \quad (30)$$

The expectation of the continuation long-run utility in equation (30) is computed as follows. First, in a similar manner to equation (21), define the *perceived* choice-specific value \tilde{V}_{ikt} as

$$\tilde{V}_{ikt}(w_{it}, S_{it}, \mathbf{q}) \equiv \tilde{u}_{it}(k; w_{it}, S_{it}, \mathbf{q}) + \tilde{\beta}_i \delta_i E_t v_{i,t+1}(S_{ik,t+1}, \epsilon_{i,t+1}; \tilde{\kappa}_{i,t+1}^+, \mathbf{q}). \quad (31)$$

Then, given wage w_{it} and type \mathbf{q} , the expectation of the maximum value of the maximization problem in equation (29) has the following solution:

$$eu + \sigma_{ci} \ln \left(\sum_{k=1}^5 \exp(\tilde{V}_{ikt}(w_{it}, S_{it}, \mathbf{q}) / \sigma_{ci}) \right). \quad (32)$$

To obtain the expectation of the continuation long-run utility, the above expression is adjusted upward using a weighted sum of expected continuation long-run utilities in period $t+1$:

$$\begin{aligned} E_{t-1} v_{it}(S_{it}, \epsilon_{it}; \tilde{\kappa}_{it}^+ | w_{it}, \mathbf{q}) &= eu + \sigma_{ci} \ln \left(\sum_{k=1}^5 \exp(\tilde{V}_{ikt}(w_{it}, S_{it}, \mathbf{q}) / \sigma_{ci}) \right) \\ &+ (1 - \tilde{\beta}_i) \delta_i \sum_{k=1}^5 \tilde{P}_{ikt}(w_{it}, S_{it}, \mathbf{q}) E_t v_{i,t+1}(S_{ik,t+1}, \epsilon_{i,t+1}; \tilde{\kappa}_{i,t+1}^+, \mathbf{q}), \end{aligned} \quad (33)$$

where the weights $\tilde{P}_{ikt}(\cdot)$ are choice probabilities obtained from the following closed form solution:

$$\tilde{P}_{ikt}(w_{it}, S_{it}, \mathbf{q}) \equiv \frac{\exp(\tilde{V}_{ikt}(w_{it}, S_{it}, \mathbf{q}) / \sigma_{ci})}{\sum_{j=1}^5 \exp(\tilde{V}_{ijt}(w_{it}, S_{it}, \mathbf{q}) / \sigma_{ci})}. \quad (34)$$

The wage is then integrated out using Gaussian–Hermite quadrature with five points:

$$E_{t-1} v_{it}(S_{it}, \epsilon_{it}; \tilde{\kappa}_{it}^+, \mathbf{q}) = \int E_{t-1} v_{it}(S_{it}, \epsilon_{it}; \tilde{\kappa}_{it}^+ | w, \mathbf{q}) f_i(w | S_{it}, \mathbf{q}) dw. \quad (35)$$

This completes the recursion procedure from period $t+1$ to t .

When the individual makes decisions, she believes that her future selves will follow the perceived continuation strategy profile $\tilde{\kappa}_{i,t+1}^+$. The current self's optimal strategy is

$$\kappa_{it}^*(S_{it}, \epsilon_{it}, \mathbf{q}) \equiv \arg \max_{d_{it} \in D} \sum_{k=1}^5 d_{ikt} \left(u_{ik_{it}} + \beta_i \delta_i E_t v_{i,t+1}(S_{ik,t+1}, \epsilon_{i,t+1}; \tilde{\kappa}_{i,t+1}^+, \mathbf{q}) \right). \quad (36)$$

The current self's optimal strategy is not used in backward recursion.⁷² Her optimal decisions are compared with actual data and used for the construction of the likelihood function.

72. An exception is sophisticated agents, whose perception of the present bias factor is correct. As a result, the perceived continuation strategy profile coincides with the optimal strategy in equation (36) in each period.

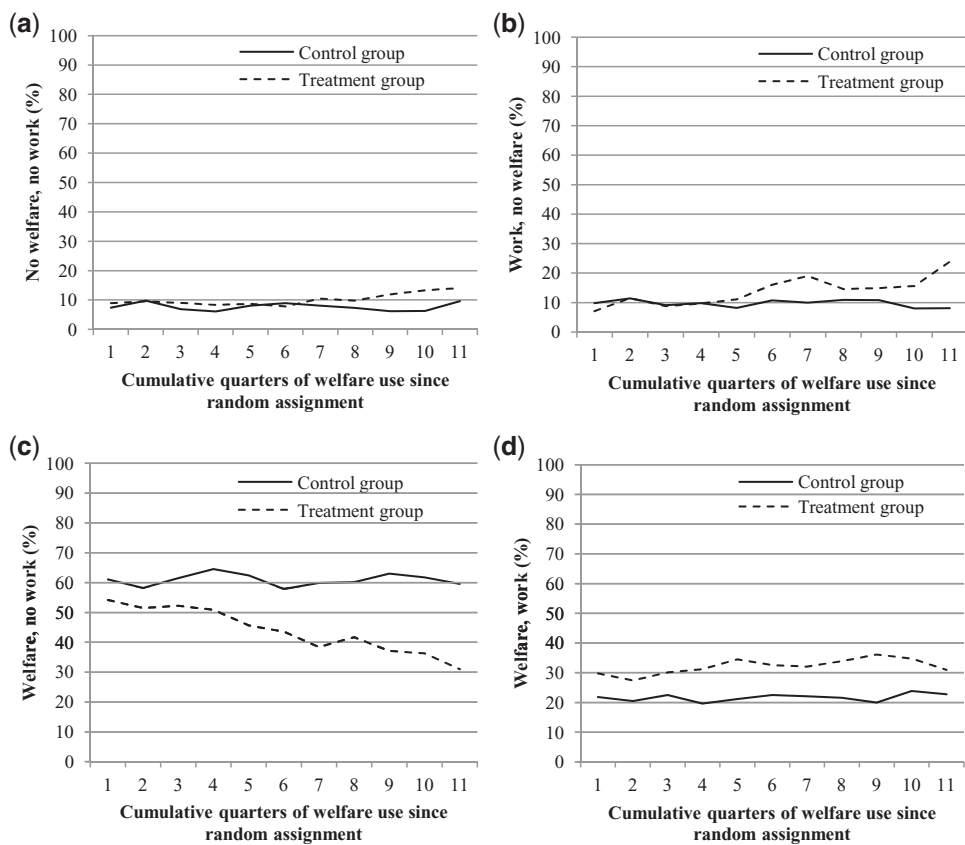


FIGURE A1

Sample choices by cumulative periods of welfare use (among individuals who used welfare last period). (a) No work, no welfare. (b) Work, no welfare. (c) No work, welfare. (d) Work, welfare.

TABLE A1
Simulated within-sample one-period transition rates

Outcome in the previous period	Outcome in the current period					
	Work (%)	Welfare (%)	No work, no welfare (%)	Work, no welfare (%)	No work, welfare (%)	Work, welfare (%)
Control group						
No work, no welfare	11.3	11.4	78.8	9.8	9.9	1.5
Work, no welfare	85.8	6.7	12.0	81.4	2.2	4.4
No work, welfare	17.0	86.7	11.2	2.2	71.8	14.9
Work, welfare	71.0	73.7	3.2	23.1	25.9	47.8
Treatment group						
No work, no welfare	11.0	10.4	79.9	9.7	9.0	1.4
Work, no welfare	86.1	6.6	11.9	81.5	2.0	4.6
No work, welfare	20.8	83.8	13.3	2.9	66.0	17.9
Work, welfare	76.7	72.3	3.1	24.6	20.2	52.1

Notes: Numbers in the choice distribution may be subject to rounding error. For sample outcomes, see Table 2.

TABLE A2
Policy changes, time-consistent heterogeneous discount factor model

	Baseline	Expand welfare	Expand EITC	Work subsidy in welfare	Income support	Standard time limit	Prowork time limit	Benefit reduction time limit
		1.112×	1.32×	\$70	\$50	3-year	2.25-year	1.25-year
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Work (%)	45.2	−0.5	1.1	1.0	−3.7	2.0	2.5	1.4
Welfare (%)	31.8	1.9	−0.8	1.0	−5.9	−12.5	−11.2	−6.6
No work, no welfare (%)	32.4	−1.2	−0.3	−0.7	8.5	7.0	7.2	3.9
Work, no welfare (%)	0.0	−0.7	1.2	−0.3	−2.6	5.5	4.0	2.7
No work, welfare (%)	22.3	1.7	−0.8	−0.3	−4.8	−9.0	−9.7	−5.3
Work, welfare (%)	9.5	0.3	−0.1	1.4	−1.1	−3.5	−1.5	−1.3
APDV earnings (\$)	396.2	−5.5	10.5	3.7	−28.4	21.2	19.4	15.2
APDV net gov. expenditure (\$)	203.2	16.7	2.6	12.3	−17.5	−63.6	−61.6	−47.4
APDV utility (\$)	—	9.5	9.8	7.2	21.4	−5.2	−7.3	−6.1
Prefers new policy (%) ^a	—	100.0	100.0	100.0	100.0	0.0	0.0	0.0

Notes: Results are based on simulations of all control group individuals from period 1 to the end of the time horizon. Numbers in columns 2–8 represent simple differences in outcomes from column 1.

^aFraction of individuals that prefer the new policy regime to the policy regime in column 1.

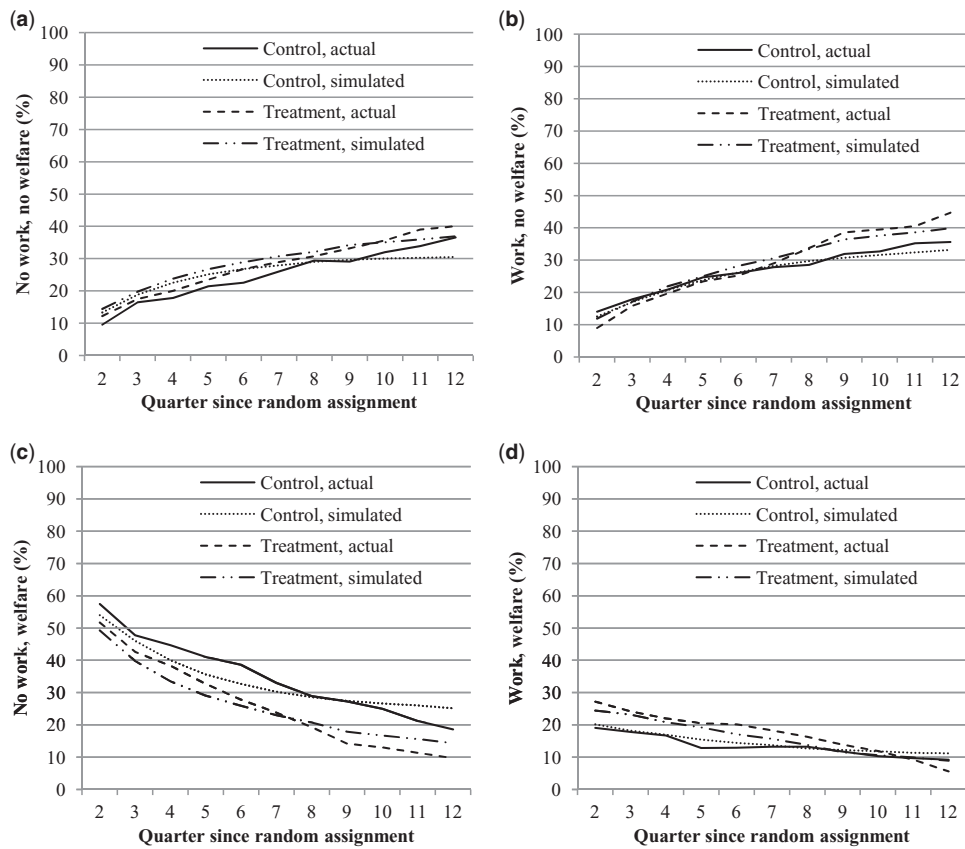


FIGURE A2

Actual versus simulated choices by period. (a) No work, no welfare. (b) Work, no welfare. (c) No work, welfare. (d) Work, welfare.

TABLE A3
Work incentives of tax/sanctions and work subsidies, time-consistent individuals

	Type of work incentive			
	For individuals who worked last period		For individuals who did not work last period	
	Static ^a	Dynamic ^b	Static ^a	Dynamic ^b
Work subsidy				
Dynamic type-1	Positive	Positive	No effect	Positive
Dynamic type-2	No effect	Negative	Positive	Negative
Static	Positive	Ambiguous	Positive	Ambiguous
Tax/sanction				
Dynamic type-1	Positive	Negative	No effect	Negative
Dynamic type-2	No effect	Positive	Positive	Positive
Static	Positive	Ambiguous	Positive	Ambiguous

^aThe current reward of working versus not working as given by the static budget constraint.

^bThe expected future reward of working versus not working under time-consistent preference.

TABLE A4
Sensitivity analysis: allowing for fertility

	No fertility	Allow for fertility		
	Baseline	Baseline	Income support \$50	Mean productivity -10%
	(1)	(2)	(3)	(4)
Work (%)	44.7	45.4	-3.2	-2.3
Welfare (%)	32.4	33.2	-5.4	2.0
No work, no welfare (%)	32.5	31.5	7.7	0.8
Work, no welfare (%)	35.1	35.3	-2.2	-2.8
No work, welfare (%)	22.7	23.2	-4.5	1.5
Work, welfare (%)	9.6	10.1	-0.9	0.5
APDV earnings (\$)	390.3	394.3	-24.8	-56.2
APDV net gov. expenditure (\$)	205.4	217.2	-16.0	18.0
APDV utility (\$)	—	—	11.3	-14.9
Prefers new policy (%) ^a	—	—	90.7	—

Notes: Results are based on simulations of all control group individuals from period 1 to the end of the time horizon. Numbers in columns 3 and 4 represent simple differences in outcomes from column 2.

^aFraction of individuals that prefer the new policy regime to the policy regime in column 2.

TABLE A5
Simulated outcomes under a \$50 income support by degree of present bias

Present bias factor in the Baseline model	Baseline model (heterogeneous hyperbolic discounting)				Time-consistent model (heterogeneous discount factor)			
	0–0.4 (most present- biased)	0.4–0.6	0.6–0.8	0.8–1.0 (least present- biased)	0–0.4 (most present- biased)	0.4–0.6	0.6–0.8	0.8–1.0 (least present- biased)
Work (%)	40.2	40.5	41.5	44.2	40.6	40.7	41.2	44.2
Welfare (%)	23.0	22.8	33.6	25.6	21.0	21.0	33.0	25.5
No work, no welfare (%)	43.3	43.3	36.6	39.0	44.4	44.4	37.1	39.2
Work, no welfare (%)	33.7	33.9	29.8	35.5	34.6	34.6	29.9	35.3
No work, welfare (%)	16.5	16.2	21.9	16.8	15.1	14.9	21.7	16.5
Work, welfare (%)	6.6	6.6	11.7	8.7	5.9	6.1	11.3	9.0
APDV earnings (\$)	382.9	378.2	334.0	380.9	392.2	383.8	333.4	380.9
APDV net gov. expenditure (\$)	183.1	141.2	273.3	124.2	171.0	133.0	270.6	123.9
Fraction of individuals in subgroup (%)	19.7	28.5	34.1	17.8	19.7	28.5	34.1	17.8

Note: Results are based on simulations of all control group individuals from period 1 to the end of the time horizon.

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Supplementary Data

Supplementary data are available at *Review of Economic Studies* online.

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