Unobserved Heterogeneity and Finite Dependence

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- Suppose the sampling period, S, falls short of the time horizon T.
- Rather than express $u_{jt}(x)$ as a sum to T, we express u_{jt} as a sum to S and then use the value function at $S+1$:

$$
\begin{aligned}
u_{jt}(x) \quad = \quad & u_{1t}(x) + \psi_{1t}(x) - \psi_{jt}(x) \qquad (1) \\
& + \sum_{\tau=t+1}^{S} \sum_{x_\tau=1}^{X} \beta^{\tau-t} \left\{ \begin{array}{l} [u_{1\tau}(x_\tau) + \psi_{1t}(x_\tau)] \times \\ [\kappa_{\tau-1}(x_\tau|x,1) - \kappa_{\tau-1}(x_\tau|x,j)] \end{array} \right\} \\
& + \sum_{x_{S+1}=1}^{X} \beta^{S-t} V_{S+1}(x_{S+1})] \left[\kappa(x_{S+1}|x,1) - \kappa(x_{S+1}|x,j)\right]
\end{aligned}
$$

- Since the CCPs and state transitions are identified up to S, the utility flows would be exactly identified if $V_{S+1}(x)$ was known.
- However $V_{S+1}(x)$ is endogenous and depends on CCPs that occur after the sample ends.

Finite Dependence

Definition

- This complication is overcome when a property called **finite dependence** holds.
- A pair of choices $\{i, j\}$ exhibits *ρ-period dependence* at (t, x_t) if there exist a pair of sequences of decision weights:

$$\{\omega_{k\tau}(t, x_\tau, i)\}_{(k,\tau)=(1,t+1)}^{(J,t+\rho)} \text{ and } \{\omega_{k\tau}(t, x_\tau, j)\}_{(k,\tau)=(1,t+1)}^{(J,t+\rho)}$$

  such that for all $x_{t+\rho+1} \in \{1, \ldots, X\}$:

$$\kappa_{t+\rho+1}(x_{t+\rho+1} | t, x_t, i) = \kappa_{t+\rho+1}(x_{t+\rho+1} | t, x_t, j)$$

- Finite dependence:
  1. can be tested without specifying utilities.
  2. holds in most published empirical applications.
  3. extends to games by conditioning on the player as well.

- If there is finite dependence for (t, x_t, i, j), then:

$$u_{jt}(x_t) + \psi_j[p_t(x_t)] - u_{it}(x_t) - \psi_i[p_t(x_t)] =$$

$$\sum_{(k,\tau,x_\tau)=(1,t+1,1)}^{(J,t+\rho,X)} \beta^{\tau-t} \left\{ \begin{array}{l} u_{k\tau}(x_\tau) \\ +\psi_k[p_\tau(x_\tau)] \end{array} \right\} \left[\begin{array}{l} \omega_{k\tau}(t, x_\tau, i)\kappa_\tau(x_\tau | t, x_t, i) \\ -\omega_{k\tau}(t, x_\tau, j)\kappa_\tau(x_\tau | t, x_t, j) \end{array} \right] \tag{2}$$

- To apply finite dependence in estimation:
  - estimate the transition terms $f_{jt}(x_{t+1}|x)$
  - form $\kappa_\tau(x_\tau|t, x_t, j)$ terms with transitions and weights $\omega_{k\tau}(t, x_\tau, j)$
  - estimate the CCPs $p_t(x_t)$
  - plug CCPs into $\psi_j[p_t(x_t)]$ terms
  - estimate utility $u_{jt}(x_t)$ terms using (2) using Minimum Distance

Simple Examples of Finite Dependence

Terminal choices

- Terminal choices and renewal choices are widely assumed in structural econometric applications of dynamic optimization problems and games.
- A *terminal choice* ends the evolution of the state variable with an *absorbing state* that is independent of the current state.
- If the first choice denotes a terminal choice, then:

$$f_{1t}(x_{t+1}|x) \equiv f_{1t}(x_{t+1})$$

for all $(t, x) \in \mathbb{T} \times \mathbb{X}$ and hence:

$$\sum_{x_{t+1}=1}^{X} f_{1,t+1}(x_{t+2}) f_{jt}(x_{t+1}|x_t) = f_{1,t+1}(x_{t+2})$$

- Setting $\omega_{k\tau}(t, x, i) = 0$ for all (x, i) and $k \neq 1$, Equation (2) implies:

$$\begin{aligned} & u_{1t}(x_t) + \psi_1[p_t(x_t)] - u_{jt}(x_t) - \psi_j[p_t(x_t)] \\ = \; & \sum_{x_{t+1}=1}^{X} \beta \left\{ u_{1,t+1}(x_{t+1}) + \psi_1\left[p_{t+1}(x_{t+1})\right]\right\} f_{jt}(x_{t+1}|x_t) \end{aligned}$$

Simple Examples of Finite Dependence

Renewal choices

- Similarly a *renewal choice* yields a probability distribution of the state variable next period that does not depend on the current state.
- If the first choice is a renewal choice, then for all $j \in \{1, \ldots, J\}$:

$$
\begin{aligned}
\sum_{x_{t+1}=1}^{X} f_{1,t+1}(x_{t+2}|x_{t+1}) f_{jt}(x_{t+1}|x_t) &= \sum_{x_{t+1}=1}^{X} f_{1,t+1}(x_{t+2}) f_{jt}(x_{t+1}|x_t) \\
&= f_{1,t+1}(x_{t+2}) \sum_{x_{t+1}=1}^{X} f_{jt}(x_{t+1}|x_t) \\
&= f_{1,t+1}(x_{t+2}) \qquad (3)
\end{aligned}
$$

- In this case Equation (2) implies:

$$
\begin{aligned}
& u_{1t}(x_t) + \psi_1[p_t(x_t)] - u_{jt}(x_t) - \psi_j[p_t(x_t)] \\
= & \sum_{x=1}^{X} \beta \{u_{1,t+1}(x) + \psi_1[p_{t+1}(x)]\} [f_{jt}(x|x_t) - f_{1t}(x|x_t)]
\end{aligned}
$$

Simple Examples of Finite Dependence

An example of 2-period finite dependence

- How does finite dependence work when $\rho > 1$?
- Consider the following model of labor supply and human capital.
- In each of T periods an individual chooses:
  - $d_{2t} = 1$ to work
  - $d_{1t} = 1$ to stay home.
- She accumulates human capital, x_t, from working. If:
  - $d_{1t} = 1$ then $x_{t+1} = x_t$.
  - $d_{2t} = 1$ and $t > 1$ then $x_{t+1} = x_t + 1$.
  - $d_{j=2,t=1} = 1$ then

$$x_2 = \begin{cases} 2 \text{ with probability } 0.5 \\ 1 \text{ with probability } 0.5 \end{cases}$$

- Summarizing, human capital only increases with work, by a unit, except in the first period, when it might jump to two.

Simple Examples of Finite Dependence

Establishing finite dependence in the labor supply example

- When $t > 1$, work one period out of the next two, and:
  - set $\omega_{1,t+1}(t, x_\tau, 2) = 1$, implying $\omega_{2,t+1}(t, x_\tau, 2) = 0$
  - set $\omega_{2,t+1}(t, x_\tau, 1) = 1$, implying $\omega_{1,t+1}(t, x_\tau, 1) = 1$
  - to attain 1-period dependence with $x_{t+2} = x_t + 1$.
- When $t = 1$ after:
  - staying home at $t = 1$ (that is $d_{11} = 1$), work for the next two periods; equivalently set $\omega_{k\tau}(t, x, j)$ so that:

$$\omega_{2,2}(1, 0, 1) = \omega_{2,3}(1, 1, 1) = 1$$

  - working at $t = 1$ (that is $d_{21} = 1$), work in period 2 only if human capital increases one unit at $t = 1$; equivalently set $\omega_{k\tau}(t, x, j)$ so that:

$$\omega_{1,2}(1, 2, 2) = \omega_{2,2}(1, 1, 2) = \omega_{1,3}(1, 2, 2) = 1$$

  - to attain 2-period dependence with $x_3 = 2$.

Simple Examples of Finite Dependence

Nonstationary search model

- Consider a simple search model in which all jobs are temporary, lasting only one period.
- Each period $t \in \{1, \dots, T\}$ an individual may:
  - stay home by setting $d_{1t} = 1$
  - or apply for temporary employment setting $d_{2t} = 1$.
- Job applicants are successful with probability λ_t, time varying job offer arrival rates.
- Experience $x \in \{1, \dots, X\}$ increases by one unit with each period of work, up to X, and does not depreciate.
- Current utility $u_{jt}(x_t)$ depends on choices, time and experience.

- For all (t, x_t) with $x_t < X$ set:
  - $d_{1t} = 1$ (stay home) and then "apply for employment" with weight:

$$\begin{aligned} \lambda_t / \lambda_{t+1} &= \omega_{k=2,t+1}(t, x_t, i = 1) \\ &= 1 - \omega_{k=1,t+1}(t, x_t, i = 1) \end{aligned}$$

  - $d_{2t} = 1$ (seek work) and then stay home:

$$\omega_{k=1,t+1}\left(t, x_t, j = 2\right) = \omega_{k=1,t+1}\left(t, x_t + 1, j = 2\right) = 1$$

  - to attain one-period dependence since:

$$\kappa_3(x_{t+3}|t, x_t, 1) = \kappa_3(x_{t+3}|t, x_t, 2) = \begin{cases} 1 - \lambda_t \text{ for } x_{t+3} = x_t \\ \lambda_t \text{ for } x_{t+3} = x_t + 1 \end{cases}$$

- Note that if $\lambda_t > \lambda_{t+1}$ then $\omega_{2,t+1}(t, x_t, 1) > 1$ and $\omega_{1,t+1}(t, x_t, 1) = 1 - \lambda_t / \lambda_{t+1} < 0$.

- In principle these results also apply to games.
- Aside from games with terminal actions, one-period dependence is rare, because a rival's response typically depends on choices made by everyone in the previous period.
- Suppose player interdependencies only arise through payoffs and let:

$$x_t \equiv \left(x_t^{(1)}, \ldots, x_t^{(N)}\right) \quad d_t \equiv \left(d_t^{(1)}, \ldots, d_t^{(N)}\right)$$

$$d_t^{(n)} \equiv \left(d_{1t}^{(n)}, \ldots, d_{Jt}^{(n)}\right)$$

$$\begin{aligned} F_t\left(x_{t+1} \left| x_t, d_t \right.\right) &\equiv \prod_{n=1}^{N} F_t^{(n)}\left(x_{t+1}^{(n)} \left| x_t^{(n)}, d_t^{(n)} \right.\right) \\ &\equiv \prod_{n=1}^{N} \sum_{j=1}^{J} f_{jt}^{(n)}\left(x_{t+1}^{(n)} \left| x_t^{(n)} \right.\right) \end{aligned}$$

- A key feature of the incomplete information games settings we consider is that at t, when the players other than n collectively choose $d_t^{(\sim n)}$, they condition on (and can respond to) $d_{t-1}^{(n)}$, but not on $d_t^{(n)}$.
- Our approach to determining finite dependence in games exploits this feature in the following way:

1. Find the set of weight sequences that induce the other players matching up the weight distributions of $x_{t+\rho+1}^{(\sim n)}$, conditional on x_t, satisfying:

$$\kappa_{t+\rho+1}(x_{t+\rho+1}^{(\sim n)}|x_t, i) = \kappa_{t+\rho+1}(x_{t+\rho+1}^{(\sim n)}|x_t, j) \tag{4}$$

2. With one last choice of weight pairs at $t+\rho$, set $\omega_{k,t+\rho}^{(n)}\left(x_{t+\rho}, i\right)$ and $\omega_{k,t+\rho}^{(n)}\left(x_{t+\rho}, j\right)$ to line up the joint distribution of the states of everyone, and incorporating the restrictions that give (4).

- First suppose that for each $x \in \mathcal{A}_\tau$, there is an action $d^{(n)}(x)$ yielding some fixed $\overline{x}^{(n)} \in X^{(n)}$ for sure.
- For example assume a renewal state denoted by $\overline{x}^{(n)} \in \mathcal{X}^{(n)}$ can reached from any $x^{(n)} \in \mathcal{X}^{(n)}$ in one period with certainty.
- In this case the joint distribution across the two paths is aligned in $\tau + 1$ if there exists a weight sequence satisfying:

$$\kappa_{\rho+1}^{(\sim n)}(x_{\rho+1}^{(\sim n)}|x_t, 1) = \kappa_{\rho+1}^{(\sim n)}(x_{\rho+1}^{(\sim n)}|x_t, 2) \tag{5}$$

Distributional Assumptions about the Unobserved Variables

Motivating example: Rust's (1987) bus engine revisited

- What if we want to relax assumption that the distribution of unobserved variables is known?
- Then we must place identifying assumptions on the way systematic payoffs are parameterized.
- Recall Mr. Zurcher decides whether to replace the existing engine ($d_{1t} = 1$), or keep it for at least one more period ($d_{2t} = 1$).
- Bus mileage advances 1 unit ($x_{t+1} = x_t + 1$) if Zurcher keeps the engine ($d_{2t} = 1$) and is set to zero otherwise($x_{t+1} = 0$ if $d_{1t} = 1$).
- Transitory iid choice-specific shocks, ϵ_{jt} are Type 1 Extreme value.
- Zurcher sequentially maximizes expected discounted sum of payoffs:

$$E\left\{\sum_{t=1}^{\infty}\beta^{t-1}\left[d_{2t}(\theta_1 x_t + \theta_2 s + \epsilon_{2t}) + d_{1t}\epsilon_{1t}\right]\right\}$$

Motivating Example

ML Estimation when CCP's are known (infeasible)

- To show how the EM algorithm helps, consider the infeasible case where $s \in \{1, \ldots, S\}$ is unobserved but $p(x, s)$ is known.
- Let π_s denote population probability of being in unobserved state s.
- Supposing β is known the ML estimator for this "easier" problem is:

$$\{\hat{\theta}, \hat{\pi}\} = \arg\max_{\theta, \pi} \sum_{n=1}^{N} \ln \left[\sum_{s=1}^{S} \pi_s \prod_{t=1}^{T} l(d_{nt} | x_{nt}, s, p, \theta) \right]$$

where $p \equiv p(x, s)$ is a string of probabilities assigned/estimated for each (x, s) and $l(d_{nt} | x_{nt}, s_n, p, \theta)$ is derived from our representation of the conditional valuation functions and takes the form:

$$\frac{d_{1nt} + d_{2nt} \exp(\theta_1 x_{nt} + \theta_2 s + \beta \ln [p(0, s)] - \beta \ln [p(x_{nt} + 1, s)]}{1 + \exp(\theta_1 x_{nt} + \theta_2 s + \beta \ln [p(0, s)] - \beta \ln [p(x_{nt} + 1, s)])}$$

- Maximizing over the sum of a log of summed products is computationally burdensome.

- The EM algorithm is a computationally attractive alternative to directly maximizing the likelihood.
- Denote by $d_n \equiv (d_{n1}, \ldots, d_{nT})$ and $x_n \equiv (x_{n1}, \ldots, x_{nT})$ the full sequence of choices and mileages observed in the data for bus n.
- At the m^{th} iteration:

$$
\begin{aligned}
q_{ns}^{(m+1)} &= \Pr\left\{ s \,\middle|\, d_n, x_{n,} \theta^{(m)}, \pi_s^{(m)}, p \right\} \\
&= \frac{\pi_s^{(m)} \prod_{t=1}^{T} l(d_{nt} | x_{nt}, s, p, \theta^{(m)})}{\sum_{s'=1}^{S} \pi_{s'}^{(m)} \prod_{t=1}^{T} l(d_{nt} | x_{nt}, s', p, \theta^{(m)})}
\end{aligned}
$$

$$
\pi_s^{(m+1)} = N^{-1} \sum_{n=1}^{N} q_{ns}^{(m+1)}
$$

$$
\theta^{(m+1)} = \arg\max_{\theta} \sum_{n=1}^{N} \sum_{s=1}^{S} \sum_{t=1}^{T} q_{ns}^{(m+1)} \ln[l(d_{nt} | x_{nt}, s, p, \theta)]
$$

Motivating Example

Steps in our algorithm when s is unobserved and CCP's are unknown

Our algorithm begins by setting initial values for $\theta^{(1)}$, $\pi^{(1)}$, and $p^{(1)}(\cdot)$:

Step 1 Compute $q_{ns}^{(m+1)}$ as:

$$q_{ns}^{(m+1)} = \frac{\pi_s^{(m)} \prod_{t=1}^{T} l\left[d_{nt} \middle| x_{nt}, s, p^{(m)}, \theta^{(m)}\right]}{\sum_{s'=1}^{S} \pi_s^{(m)} \prod_{t=1}^{T} l\left(d_{nt} \middle| x_{nt}, s', p^{(m)}, \theta^{(m)}\right)}$$

Step 2 Compute $\pi_s^{(m+1)}$ according to:

$$\pi_s^{(m+1)} = \frac{\sum_{n=1}^{N} q_{ns}^{(m+1)}}{N}$$

Step 3 Update $p^{(m+1)}(x, s)$ using one of two rules below

Step 4 Obtain $\theta^{(m+1)}$ from:

$$\theta^{(m+1)} = \arg\max_{\theta} \sum_{n=1}^{N} \sum_{s=1}^{S} \sum_{t=1}^{T} q_{ns}^{(m+1)} \ln\left[l\left(d_{nt} \middle| x_{nt}, s_n, p^{(m+1)}, \theta\right)\right.$$

- Take a weighted average of decisions to replace engine, conditional on x, where weights are the conditional probabilities of being in unobserved state s.

Step 3A Update CCP's with:

$$p^{(m+1)}(x,s) = \frac{\sum_{n=1}^{N}\sum_{t=1}^{T} d_{1nt} q_{ns}^{(m+1)} I(x_{nt}=x)}{\sum_{n=1}^{N}\sum_{t=1}^{T} q_{ns}^{(m+1)} I(x_{nt}=x)}$$

- Or in a stationary infinite horizon model use identity from model that likelihood returns CCP of replacing the engine:

Step 3B Update CCP's with:

$$p^{(m+1)}(x_{nt},s_n) = l(d_{nt1}=1|x_{nt},s_n,p^{(m)},\theta^{(m)})$$

First Monte Carlo

Finite horizon renewal problem

- Suppose $s \in \{0, 1\}$ equally weighted.
- There are two observed state variables
  1. total accumulated mileage:

$$x_{1t+1} = \begin{cases} \Delta_t \text{ if } d_{1t} = 1 \\ x_{1t} + \Delta_t \text{ if } d_{2t} = 1 \end{cases}$$

  2. permanent route characteristic for the bus, x_2, that systematically affects miles added each period.
- We assume $\Delta_t \in \{0, 0.125, \ldots, 24.875, 25\}$ is drawn from:

$$f(\Delta_t | x_2) = \exp\left[-x_2(\Delta_t - 25)\right] - \exp\left[-x_2(\Delta_t - 24.875)\right]$$

and x_2 is a multiple 0.01 drawn from a discrete equi-probability distribution between 0.25 and 1.25.

First Monte Carlo

Finite horizon renewal problem

- Let θ_{0t} be an aggregate shock (denoting cost fluctuations say).
- The difference in current payoff from retaining versus replacing the engine is:

$$u_{2t}(x_{1t}, s) - u_{1t}(x_{1t}, s) \equiv \theta_{0t} + \theta_1 \min\{x_{1t}, 25\} + \theta_2 s$$

- Denoting the observed state variables by $x_t \equiv (x_{1t}, x_2)$, this translates to:

$$\begin{aligned} v_{2t}(x_t, s) - v_{1t}(x_t, s) &= \theta_{0t} + \theta_1 \min\{x_{1t}, 25\} + \theta_2 s \\ &\quad + \beta \sum_{\Delta_t \in \Lambda} \left\{ \ln \left[\frac{p_{1t}(0, s)}{p_{1t}(x_{1t} + \Delta_t, s)} \right] \right\} f(\Delta_t | x_2) \end{aligned}$$

First Monte Carlo

Table 1 of Arcidiacono and Miller (2011, page 1854)

MONTE CARLO FOR THE OPTIMAL STOPPING PROBLEM[a]

		s Observed			*s* Unobserved		Time Effects	
	DGP	FIML	CCP	Ignoring *s* CCP	FIML	CCP	*s* Observed CCP	*s* Unobserved CCP
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
θ_0 (intercept)	2	2.0100	1.9911	2.4330	2.0186	2.0280		
		(0.0405)	(0.0399)	(0.0363)	(0.1185)	(0.1374)		
θ_1 (mileage)	−0.15	−0.1488	−0.1441	−0.1339	−0.1504	−0.1484	−0.1440	−0.1514
		(0.0074)	(0.0098)	(0.0102)	(0.0091)	(0.0111)	(0.0121)	(0.0136)
θ_2 (unobs. state)	1	0.9945	0.9726		1.0073	0.9953	0.9683	1.0067
		(0.0611)	(0.0668)		(0.0919)	(0.0985)	(0.0636)	(0.1417)
β (discount factor)	0.9	0.9102	0.9099	0.9115	0.9004	0.8979	0.9172	0.8870
		(0.0411)	(0.0554)	(0.0591)	(0.0473)	(0.0585)	(0.0639)	(0.0752)
Time (minutes)		130.29	0.078	0.033	275.01	6.59	0.079	11.31
		(19.73)	(0.0041)	(0.0020)	(15.23)	(2.52)	(0.0047)	(5.71)

[a]Mean and standard deviations for 50 simulations. For columns 1–6, the observed data consist of 1000 buses for 20 periods. For columns 7 and 8, the intercept (θ_0) is allowed to vary over time and the data consist of 2000 buses for 10 periods. See the text and the Supplemental Material for additional details.

Second Monte Carlo

Structure

- Entrants pay startup cost to compete in the market, but not incumbents.
- Paying startup cost now transforms entrant into incumbent next period.
- Declining to compete in any given period is tantamount to exit.
- When a firm exits another firm potentially enters next period.
- There are two sources of dynamics in this model:
    1. An entrant depreciates startup cost over its anticipated lifetime.
    2. Becoming an incumbent reduces the probability of other firms entering the market, and hence increases expected profits.

Second Monte Carlo

Two observed state variables

- Each market has a permanent market characteristic, denoted by x_1, common to each player within the market and constant over time, but differing independently across markets, with equal probabilities on support $\{1, \ldots, 10\}$.
- The number of firm exits in the previous period is also common knowledge to the market, and this variable is indicated by:

$$x_{2t} \equiv \sum_{h=1}^{I} d_{1,t-1}^{(h)}$$

- This variable is a useful predictor for the number of firms that will compete in the current period.
- Intuitively, the more players paying entry costs, the lower the expected number of competitors.

Second Monte Carlo

Unobserved (Markov chain state) variables, and price equation

- The unobserved state variable $s_t \in \{1, \ldots, 5\}$ follows a first order Markov chain.
- We assume that the probability of the unobserved variable remaining unchanged in successive periods is fixed at some $\pi \in (0, 1)$, and that if the state does change, any other state is equally likely to occur with probability $(1 - \pi)/4$.
- We generated also price data on each market, denoted by w_t, with the equation:

$$w_t = \alpha_0 + \alpha_1 x + \alpha_2 s_t + \alpha_3 \sum_{h=1}^{I} d_{1t}^{(h)} + \eta_t$$

where η_t is distributed as a standard normal disturbance independently across markets and periods, revealed to each market after the entry and exit decisions are made.

Second Monte Carlo

Utility and number of firms and markets

- The flow payoff of an active firm i in period t, net of private information $\epsilon_{2t}^{(i)}$ is modeled as:

$$U_2\left(x_t^{(i)}, s_t^{(i)}, d_t^{(-i)}\right) = \theta_0 + \theta_1 x + \theta_2 s_t + \theta_3 \sum_{h=1}^{I} d_{1t}^{(h)} + \theta_4 d_{1,t-1}^{(i)}$$

- We normalize exit utility as $U_1\left(x_t^{(i)}, s_t^{(i)}, d_t^{(-i)}\right) = 0$
- We assume $\epsilon_{jt}^{(i)}$ is distributed as Type 1 Extreme Value.
- The number of firms in each market in our experiment is 6.
- We simulated data for $3,000$ markets, and set $\beta = 0.9$.
- Starting at an initial date with 6 entrants in the market, we ran the simulations forward for twenty periods.

Second Monte Carlo

Table 2 of Arcidiacono and Miller (2011, page 1862)

MONTE CARLO FOR THE ENTRY/EXIT GAME[a]

	DGP (1)	s_t Observed (2)	Ignore s_t (3)	CCP Model (4)	CCP Data (5)	Two-Stage (6)	No Prices (7)
Profit parameters							
θ_0 (intercept)	0	0.0207	−0.8627	0.0073	0.0126	−0.0251	−0.0086
		(0.0779)	(0.0511)	(0.0812)	(0.0997)	(0.1013)	(0.1083)
θ_1 (obs. state)	0.05	−0.0505	−0.0118	−0.0500	−0.0502	−0.0487	−0.0495
		(0.0028)	(0.0014)	(0.0029)	(0.0041)	(0.0039)	(0.0038)
θ_2 (unobs. state)	0.25	0.2529		0.2502	0.2503	0.2456	0.2477
		(0.0080)		(0.0123)	(0.0148)	(0.0148)	(0.0158)
θ_3 (no. of competitors)	−0.2	−0.2061	0.1081	−0.2019	−0.2029	−0.1926	−0.1971
		(0.0207)	(0.0115)	(0.0218)	(0.0278)	(0.0270)	(0.0294)
θ_4 (entry cost)	−1.5	−1.4992	−1.5715	−1.5014	−1.4992	−1.4995	−1.5007
		(0.0131)	(0.0133)	(0.0116)	(0.0133)	(0.0133)	(0.0139)
Price parameters							
α_0 (intercept)	7	6.9973	6.6571	6.9991	6.9952	6.9946	
		(0.0296)	(0.0281)	(0.0369)	(0.0333)	(0.0335)	
α_1 (obs. state)	−0.1	−0.0998	−0.0754	−0.0995	−0.0996	−0.0996	
		(0.0023)	(0.0025)	(0.0028)	(0.0028)	(0.0028)	
α_2 (unobs. state)	0.3	0.2996		0.2982	0.2993	0.2987	
		(0.0045)		(0.0119)	(0.0117)	(0.0116)	
α_3 (no. of competitors)	−0.4	−0.3995	−0.2211	−0.3994	−0.3989	−0.3984	
		(0.0061)	(0.0051)	(0.0087)	(0.0088)	(0.0089)	
π (persistence of unobs. state)	0.7			0.7002	0.7030	0.7032	0.7007
				(0.0122)	(0.0146)	(0.0146)	(0.0184)
Time (minutes)		0.1354	0.1078	21.54	27.30	15.37	16.92
		(0.0047)	(0.0010)	(1.5278)	(1.9160)	(0.8003)	(1.6467)

[a]Mean and standard deviations for 100 simulations. Observed data consist of 3000 markets for 10 periods with 6 firms in each market. In column 7, the CCP's are updated with the model. See the text and the Supplemental Material for additional details.