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A theory of jump bidding in ascending auctions

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Abstract

Jump bidding is a commonly observed phenomenon that involves bidders in ascending auctions submitting bids higher than required by the auctioneer. Such behavior is typically explained as due to irrationality or to bidders signaling their value. We present field data that suggests such explanations are unsatisfactory and construct an alternative model in which jump bidding occurs due to strategic concerns and impatience. We go on to examine the impact of jump bidding on the outcome of ascending auctions in an attempt to resolve some policy disputes in the design of ascending auctions.

JEL classification: D44; C90

Keywords: Auction theory; Ascending auctions; Jump bidding

1. Introduction

The recent popular field use of ascending auctions for such things as auctioning spectrum licenses and other government assets has been justified by claims that they will yield highly efficient outcomes. This claim is derived in part from the well-known proof that, in single unit private value settings, it is a dominant strategy for all bidders to stay in the auction until their value is reached and then drop out. The result is that the winner is invariably the bidder with the highest value. The problem with generalizing this result to field auctions is that this proof only holds for the clock version of the ascending auction, yet the clock version of the ascending auction is not common among field implementations.

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The rules of an ascending clock auction involve all bidders beginning the auction being "in" and then watching as a continuous clock raises the price of the object. The only decision a bidder has to make is when to exit the auction irrevocably. There are four main differences in the auction rules between the typical field versions of the ascending auction and the clock version. The first is the use of a sequential property right in which a bidder places a bid to become the standing high bidder and maintains that right until someone chooses to submit a higher bid to displace them as high bidder. This structure leads to the second difference which is that bidders must now choose when and how much to bid at various points in the auction rather than only when to drop out. This expanded strategy space in turn leads to a difference in information sets. In the clock auction, a bidder knows how many other bidders are still "in" while in non-clock auctions a bidder only knows that he or someone else is the standing high bidder. A third common difference is the use of a discrete price space rather than a continuous one. The fourth difference is the use of a minimum required bid increment.

These changes to the auction structure invalidate the use of the clock auction strategy of bidders remaining in the auction just until their value is reached at a base level because this does not constitute a completely specified strategy in the non-clock setting. It is, however, commonly thought that the analog to the clock auction strategy, namely a bidder being willing to bid the minimum increment until his value is reached, and then ceasing to bid further (sometimes called "straightforward bidding") is still a dominant strategy or at least a Nash equilibrium strategy when the clock is removed. Statements asserting this can be found throughout the auction literature and typically take the form of simply asserting that there is no difference in the equilibrium strategy between the clock and non-clock auctions, as in the following from Cox et al. (1982, p. 2 and 8):

> This is the '... progressive auction, in which bids are freely made and announced until no purchaser wishes to make any further higher bid' Vickrey (1961), p. 14 ... Therefore, the strategy of remaining in the bidding competition as long as the bid on the floor does not exceed the bidder's value for the object, and of dropping out as soon as it does exceed value, is a dominant strategy.

As well as the following from McAfee and McMillan (1987, p. 702 and 708):

> In the English auction, the price is successively raised until only one bidder remains. This can be done by having an auctioneer announce prices, or by having bidders call the bids themselves ... the dominant strategy is to remain in the bidding until the price reaches the bidder's own valuation.

We do not intend to single out the authors of these quotations as particular offenders in this regard, but rather our intent is to show that this mistake is very easy to make even by the most careful and rigorous researchers. The frequency with which this mistake is made has led to a general belief in the existence of a folk theorem that the clock strategy translates cleanly to the non-clock setting. We have even found one paper, Kamecke (1998) that also asserts that this is a popular belief. He attempts to verify it, but the ultimate result of Kamecke (p. 407) is that the dominant strategy criterion is "not very effective in many English auction models." We will show that in a standard symmetric independent private values environment, straightforward bidding is not even typically part of a Nash equilibrium in the non-clock ascending auction, much less a dominant strategy.

In addition to constructing the equilibria of non-clock ascending auctions, we intend to analyze the properties of the equilibria with an eye towards the design of field auctions. Demange et al.

(1986) and Milgrom (2000) have previously been used as theoretical justifications for the field use of ascending auctions as both show that if bidders were to bid straightforwardly in multiple unit auctions, the efficient outcome would be approximately achieved. Neither paper, however, shows that straightforward bidding is an equilibrium strategy. The problem with using these papers as a basis for the design of field auctions is that if bidders do not bid straightforwardly, their results do not apply. The evidence indicates quite strongly that bidders do not, as shown by field data reported in Börgers and Dustmann (in press), Plott and Salmon (2004), and Easley and Tenorio (in press). The ability of ascending auctions to generate efficient outcomes is therefore in doubt because we have no theoretical basis to claim that efficient outcomes can be achieved in the absence of straightforward bidding.

Therefore, we will be particularly interested in determining what "nice" properties of ascending auctions are maintained in the presence of equilibrium based jump bidding. This is important beyond the mere tying up of theoretical loose ends because existing prior claims that deviation from straightforward bidding is detrimental to auction performance. This claim has first appeared in Cybernomics (2000) and has been made at least twice more in Banks et al. (2003) (BOPRS) and Porter et al. (2003). As stated in BOPRS (p. 312), the claim is that "jump bidding is encouraged by impatient bidders who desire to speed up the pace of the auction but sacrifice price and efficiency." This claim is based on experiments with single and multiple unit ascending auctions found in Coppinger et al. (1980) and McCabe et al. (1991). Because this claim has been made specifically in regard to the design of field ascending auctions, it is important to inquire as to its validity. The design and analysis of the model in this paper will provide a theoretical investigation in regard to the internal consistency of the claim while a follow-up paper, Isaac et al. (2005), will test the issue empirically.

Section 2 contains an overview of the background of both field data and prior theoretical explanations of jump bidding. Section 3 develops a model intended to account for jump bidding, and Section 4 contains the results of analyzing the model. Section 5 concludes.

2. Background

2.1. Field data

As a means of motivating the design of our theoretical model, we first present data from field auctions as a way of developing some stylized facts concerning the nature of jump bidding in field auctions. Table 1 shows the key characteristics of jump bidding behavior in the types of auctions of interest here. It contains the percentage of bids that were jump bids for various points in the auctions across all 41 of the spectrum license auctions conducted by the US Federal Communications Commission (FCC) up to August 1, 2002 and from the 3G spectrum auction in the UK. There are three different types of auctions listed for the FCC. Two of the types exist because of changes in FCC rules. The FCC changed the minimum possible bid increment (5 percent for #s 4–18 and 10 percent for #s 20–43), and they changed from allowing bidders to bid any amount above the minimum they wished (#s 4–18) to forcing bidders to bid integer multiples of the increment (#s 20–43). The third group of FCC auctions were either single unit auctions or auctions with only a few items for which the bidders likely possessed no interrelated values. For the two cases for which this can be done, bids are separated out as being jumps or big jumps. Jumps are any bids greater than the minimum required while big jumps are bids above the minimum by some small threshold to get an idea of the size of the jumps. In the FCC auctions, the criterion of a big jump is a bid that is at least 1.15× the minimum required while in the

Table 1
Percent of bids during first though last 10 percent of rounds of these auctions that were jump bids and percent of total that were non-trivial jump bids (i.e., bids that were at least 1.15× the minimum increment for FCC auctions or 1.5 percentage points above the minimum required in the UK auction)

	FCC				UK	
	4–18		20–43	Singles	Jumps	Big jumps
	Jumps	Big jumps	Jumps	Jumps		
10	27	9	1	21	24	6
20	40	10	1	23	21	8
30	38	8	6	18	22	8
40	38	9	1	18	28	11
50	40	9	2	11	24	12
60	39	11	3	7	20	7
70	41	9	3	6	20	3
80	41	9	6	8	30	2
90	44	9	6	17	33	4
100	35	9	2	16	14	0

Data given are in percentage.

UK case the criterion is that the bid be at least 1.5 percentage points greater than the minimum required.[1]

The pattern of jump bids across all of these auctions is quite consistent. Jump bids occur with virtually a uniform distribution throughout the course of the auctions, and most of the bids are only above the minimum required by a small amount. In many cases, the jump bids may be only above the minimum required by a few hundred or a few thousand dollars on bids denominated in millions. By comparing the FCC 4–18 column and the 20–43 column, we can also see a comparative static result that increasing the minimum increment decreases the amount of jump bidding.

While much of our field data and motivation is derived from multiple unit auctions, we will be working in a single unit two-bidder context for the theory in this paper for three primary reasons. The first is tractability. The second is that before investigating the effects of jump bidding in the multiple unit context, it is important to begin with a solid foundation and understanding of the single unit case. Third, virtually every *n* bidder single item auction case eventually becomes a two-bidder case, and thus we are working with the most important subcase.

2.2. Prior theory

Bidding in non-clock ascending auctions is discussed in several prior papers with some providing testable theories of behavior. One possible model of the behavior in these auctions is straightforward bidding (SFB). As described in Plott and Salmon (2004), Börgers and Dustmann (in press) and elsewhere, SFB bidders always bid the minimum amount required. The empirical

[1] The difference in the definition of a "big jump" between the two data sets is for convenience in analyzing the two. They are approximately the same because most of the bid increments used in the FCC auctions were 10%, and a big jump according to this second definition would have been one that was 10 percent × 1.15 = 11.5 percent or 1.5 percent points more. Two sets of auctions in the table lack any delineation between jumps and big jumps because in many of those auctions bidders were forced to bid in multiples of the increment, so any jump is a big jump.

data shown above categorically reject this aspect of SFB.[2] Even in the FCC auctions with large bid increments, jump bidding exists.

The most common and persistently mentioned explanation for jump bidding is signaling as detailed in Avery (1998) and Daniel and Hirshleifer (1997) (DH).[3] The models in these papers derive results in which bidders can signal their value on the first bid of the auction by placing a very large jump bid. The auction ends immediately if other bidders perceive that their own values are not high enough to compete with the signaled value. Otherwise, another bidder bids back to end the auction, in the DH case or straightforward bidding ensues in the Avery case. Avery develops this result in the context of affiliated values while DH does so with private values and costly bidding. These versions of signaling models can very clearly be rejected by the data. What is observed in every one of our data sets is the occurrence of jump bids that are relatively small yet persistent until near the end of the auction. The auctions usually continue much longer than 2–3 bids.[4]

Further, most of the jump bids in these auctions are observed to be of relatively modest size. In the case of the UK auction where the opening prices were around £100 million and the final prices were £4–6 billion, the notion that a bid of 1–2 percentage points above the minimum required bid would have any effect of intimidating other bidders seems unlikely. It seems even less likely that a bidder would continue making such minor jumps in an attempt to "warn off" competitors after seeing such warnings fail for 100 rounds. This indicates that something else is motivating these jumps bids. Similar arguments can be made for the FCC auctions. We do not argue that there are no cases in which bidders try to send signals to other bidders through their bidding behavior, but only that this explanation seems insufficient to explain the patterns observed in most auctions such as the ones we described.[5]

There have been a number of other possible motivations/explanations for jump bidding mentioned in the literature, in Rothkopf and Harstad (1994) and McCabe et al. (1991) in particular, but there are no developed models for them with testable predictions. Rothkopf and Harstad is a more general paper on the effects of minimum bid increments on ascending auctions. Their paper contains a sketch of a proof intended to show that jump bidding is not optimal on non-increasing value distributions such as the uniform distribution. Their paper also contains another common explanation for jump bidding, which is irrationality on the part of the bidders. While possible and perhaps probable in some cases, we do not find this to be a compelling explanation for the prevalence of the phenomenon.

We will compact all other explanations into three categories. The first such category is the standard "flat-maximum" argument that suggests that if several possible bids yield about the same expected value as a bid at the minimum increment, it might be reasonable to expect that

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[2] However, as shown in Plott and Salmon (2004), such an incorrect model can still be useful in analyzing some multiple unit ascending auctions.

[3] Easley and Tonorio also develop jump bidding as being motivated by signalling concerns but their model is developed to deal with a very different type of auction than discussed here. In their model bidders are uncertain as to whether other bidders might "find" the auction, and the nature of jump bidding is dependant upon this uncertainty.

[4] While it is technically feasible to extend these signalling models to allow for multiple jump bids as is done in DH, we believe that such notions can be rejected for field use due to the fact that such equilibria are Pareto dominated by the single jump equilibria and that they are behaviorally implausible due to the difficulty of the inferences involved.

[5] The "trailing digits" phenomenon, discussed in Salmon (2004), in early FCC auctions of bidders signalling their collusive intent by encoding messages in the last three digits of their bids is clearly a case of bidders sending signals through their bids. This is an isolated phenomenon, though, as well as a very different signaling methodology than outlined in Avery and DH.

a bidder would not worry about making “mistakes” by, for example, jumping rather than not. While this is certainly a reasonable possibility, we will not be explicitly capturing this in our model.

The second category is impatience, by which we mean some desire to have the auction close sooner rather than later. Although it is obvious in a single unit setting how a jump bid will serve to end an auction earlier, it may not be as obvious how one in something like the FCC or UK multiple unit auctions that have a simultaneous closing rule would do so. In the UK case, if one bidder puts in a large jump bid on license C, for example, he must still wait for bidding on the other four licenses to catch up, which might take several bids per license. Depending on the distribution of bids over the ensuing rounds that could take a while. We would point out that while the impact on the speed of the auction may be weaker in this cases, the bidder who placed the jump on the C license did insure that fewer rounds of bidding on that license were necessary and perhaps saved 2–4 rounds of bidding in the auction because of it.

The third possible motivation for jump bidding is strategic bidding. This explanation is commonly overlooked in other discussions of jump bidding and is perhaps least understood. Consider two bidders A and B with values $v_{\mathrm{A}} = 11$ and $v_{\mathrm{B}} = 10$ and a minimum increment of 2. If we assume that the first bidder must bid 2 and straightforward bidding ensues after that, the bidder who bids first will win the auction at a price of 10. Note that in the case when B bids first, this outcome is inefficient.

Consider A’s decision in the case where B bids first and B has just bid 6. If A is not forced to bid 8, he could choose to bid 9, 10 or 11. Note that if A bids 8, he will lose. On the other hand, were he to bid 9, B would not bid back as that would require a bid of 11. This is an example of what we will refer to as “notch” bidding as it represents an attempt by a bidder to catch the other bidder inside the notch of the increment to keep them from bidding again.

While notch bidding is generally seen as an end-of-auction phenomenon, similar strategic concerns can impact early auction behavior. Again consider the case of bidder A who has a value of 11 but assume the minimum increment is now 1. In a two-bidder auction, SFB now involves one bidder always bidding even numbers while the other bids odd numbers. Bidder A might have a preference to bid even numbers rather than odd in such a scenario. This might be from a belief that he could make his last possible bid at 10 instead of 11, and gain surplus of 1 rather than 0. If bidder B has bid 2, A might jump up to 4 instead of bidding back at the minimum increment to 3 to get on an even path. Alternatively with large bid increments, there may be certain paths that lead to catching greater numbers of possible opponents on notches and/or make it more likely that a bidder can win with a lower bid. A bidder could place a small jump early in an auction to get onto one of these paths and may be forced to repeatedly make small jumps to stay on it. To determine if these effects can or will occur in equilibrium requires the development of a formal model, which is done in the next section.

3. Theory

For non-clock English auctions, the optimal bidding strategy is significantly more complex to derive than in the standard clock case. In order to examine the issue, we will construct a general dynamic model of bidding in ascending auctions that will allow us to investigate how strategic concerns and impatience might lead to jump bidding. This approach is quite different than previous investigations into ascending auctions. To our knowledge, no one has previously solved the complete problem like this, and as we will show, the results seem to confound the conventional wisdom that has been conjectured as true for many years.

Consider an ascending auction with two bidders. We will define p_0 as the initial or starting price for the auction. A bidder will be allowed to begin the auction by bidding any amount $b_1 \geq p_0 + m$, where m is the minimum allowable bid increment. At any time t during the auction, the bidder will be able to bid any amount $b_t \geq b_{t-1} + m$, where b_{t-1} is the opponent's previous bid or current price in the auctions. We will assume that the values of the bidders, v_i, are distributed according to some CDF $F(v)$ on integers in the range $[\alpha, \beta]$. Bidders can have a discount rate of $\delta \in [0, 1]$ where $\delta = 1$ implies perfect patience and $\delta < 1$ implies impatience.[6] In this context, the general Bellman equation defining the optimal bid a bidder will choose at any point in the auction is defined by Eq. (1):

$$\max_{b_t} W(b_t|b_{t-1}) = \begin{bmatrix} (v_i - b_t)Pr(b_t + m > v_j | v_j \geq b_{t-1}) \\ +\delta(1 - Pr(b_t + m > v_j | v_j \geq b_{t-1}))E[W(b_{t+2}|b_{t+1})] \end{bmatrix}$$
$$\text{st}\, b_t \geq b_{t-1} + m. \tag{1}$$

This equation defines the bidder's expected surplus for bidding b_t as being equal to his surplus if he wins, $v_i - b_t$, times the probability of winning, plus the discounted expected continuation value, $E[W]$, if the bidder does not win with the current bid. To simplify notation, the fact that the bidder will cease bidding if $b_{t-1} + m > v_i$ has been left out of Eq. (1). This implies that a bidder will win not only if his bid $b_t + v_j$ but if his bid plus the minimum increment is greater than his opponent's value, $b_t + m > v_j$ as j will only wish to bid back if j's value is greater than or equal to the minimum of what he must bid.

The manner in which beliefs about the probability of winning are updated is a key detail of our approach. We have conditioned the probability of winning only on the most recent bid made by the opponent. We are specifically not conditioning the expectations of a bidder on the previous bid path or the equilibrium strategy. Whether the current price is 10 as a result of several rounds of straightforward bidding or from a single jump from an opening price of 0 makes no difference to our bidders' decisions from that point on. We are, therefore, assuming that the only information conveyed is that it indicates j's value is at least as high as the bid. The potential fact that the bid was a jump bid and that perhaps only a bidder with one specific value would make that size jump will be ignored. The reason for this is to remove the possibility of signaling equilibria because the implications of signaling equilibria have been derived elsewhere.[7] Also, we believe that this manner of updating beliefs is more in keeping with the real inference ability of bidders and should be more likely to capture the important behavioral effects at issue.

The value functions for this problem are inherently highly discontinuous and would be so even if we used a continuous value and price space. This is due to the presence of the minimum increment that we must include to model real auctions. The problem is that small increases in a person's bid might cause him to switch between winning the auction with a surplus to losing when a slightly larger increase could have allowed him to win again. Further, the solutions to the problem we find lack most of the smoothness and monotonicity properties that are desirable in

[6] This δ should in no way be confused with the person's traditional discount rate for savings or investment decisions. This is seen as purely a parameter of time preference.

[7] The equilibria we do derive could still be described as "signalling equilibria" as bidders are signalling some information about their values with each bid. We use the term, however, to distinguish between equilibria in which bidders are jump bidding for the express purpose of signalling information about their value, as in Avery and DH, versus cases in which bidders jump bid for other purposes and as a side effect signal information about their value.

deriving analytical solutions. Consequently, analytical methods for solving this problem will be ineffective.

We instead solve the problem using an exhaustive backward induction algorithm. It works by starting with a price, p, equal to the top of the value distribution, β, and then finds the best response a bidder would make for every possible value in [max(p_0, α), β] assuming that β was the last bid made by the opponent. If the current price is equal to β, then no bidder would wish to bid further as doing so involves incurring a negative surplus. Then the optimal responses are found assuming that the current price is $\beta - 1$ then $\beta - 2$ and so forth. At each step back in price, the decision for bidder i of what amount to bid in response to observing a current price of p is made knowing how bidder j would respond to the chosen bid for any value bidder j might have and also how bidder i will respond to j's response and so on until the auction is concluded. Note that p and b_{t-1} are used somewhat interchangeably at times. We use p as an attempt to make it clear that we are just iterating back through possible prices, not modeling an actual auction in progress. When we model a bidder as choosing a best response, he is considering $b_{t-1} = p$ or that this p was the most recent bid of his opponent. We are solving for a symmetric equilibrium, so solving for the best response of bidder i when he has a value v_i also yields the best response bidder j would have if j possessed the same value.

For each price level, p, the best response bid of bidder i for a given value, v_i, is computed by finding the $h \in \{0, 1, 2, 3, \ldots)$ that solves the following:

$$\max_h V[p + m + h|v_i] \tag{2}$$

where

$$V[p + m + h|v_i] = \sum_{k=b_{t-1}}^{v_i+m} \Phi(k, b_{t-1} + m + h) Pr(v_j = k|v_j \geq b_{t-1}) \tag{3}$$

and

$$\Phi(k, b_{t-1} + m + h) = \begin{cases} \delta^{t^*(k|b_{t-1}+m+h)-1}(v_i - p^*(k|b_{t-1} + m + h)) \text{ if} & \text{win} \\ 0 & \text{else.} \end{cases} \tag{4}$$

V is the expected value to a bidder with a value of v_i of bidding some amount $b_t = b_{t-1} + m + h$, assuming that $v_j \geq b_{t-1}$. If $h = 0$, then the bidder is bidding the minimum required. If V is less than zero for all possible h, then the bidder withdraws from the auction. Since we are using a backward induction algorithm, when considering any price $p = b_{t-1}$, the algorithm has already solved for the best response of any bidder possessing any value when facing a price above b_{t-1}. Thus, for any b_t bidder i might choose in response to p, the rest of the path of the auction is already determined given a specific value for the opponent. Therefore, it is possible to determine whether or not bidder i will win with a bid of b_t, the price at which he would win, p^*, and the number of bids required to get to that price, t^*. We use that deterministic structure to find the expected value of placing any bid b_t by summing over all possible values the opponent might have such that bidder i might win and finding the discounted surplus expected to be received by the bidder in each case times the probability of that case occurring. The indicator function Φ is a function that matches up the value of the opponent, k, the value of the bidder and his proposed bid with the portion of the equilibrium strategy already derived to determine if a bidder with a value v_i who bids $b_{t-1} + m + h$ will win against a bidder with value k along the ensuing bid path.

V can be calculated for every possible choice of h (which implies a specific b_t), making it easy to compare the expected utility from any bid choice and find the one that delivers the maximum utility. For most of our results, we will assume that bidders can bid only integers. We will also discuss what happens when values are integers but bidders are allowed to bid in non-integers. The discretization plus the assumption of a maximum possible value, β, allow for the use of a finite horizon backward induction algorithm.

Due to the fact that this is a finite game, the solution algorithm will deliver a Bayes–Nash equilibrium of the game as the strategy will be a best response to itself given the belief structure. This equilibrium will not be unique, and to deal with this issue systematically we have chosen to find the equilibrium that involves the least amount of jump bidding to bias our results against being able to find such equilibria. The multiplicity of equilibria is due to the possibility that a bidder might be indifferent between several possible bids, and in this case, our algorithm always chooses the lowest bid in the tied set. Further, because the beliefs in this model are not being updated with all possible information, the equilibria we find will not pass standard perfection criteria. Our methodology does however guarantee the main and important qualities of a perfect Bayesian equilibrium as it ensures that strategies are sequentially rational with respect to the bidders' beliefs or that all players are playing a best response to their beliefs at every possible subgame. Our approach will also ensure that these beliefs are being updated in a sensible and consistent manner. The only departure from a perfect Bayesian equilibrium is that the beliefs are not updated according to Baye's rule using the information from the equilibrium strategy profile and all prior path information. The equilibria we will find are similar in spirit to Markov Perfect equilibria for repeated games, and the methods we use are similar to those developed in Maskin and Tirole (1988).

4. Results

We will present examples using the discrete uniform and normal distributions over the integers on the range [1, 100]. In the case of the normal, we have set the mean of the distribution to be equal to 50.5 and tried three different standard deviations: 15, 22.5 and 30. To maintain the [1, 100] range, we truncate the normal distribution to that range by adding all the weight in the tails onto the bounds and discretize it by assigning all weight in the range $(x-0.5,\ x+0.5)$ to the integer x. When looking at the results from this setting, we are primarily interested in looking at the degree to which strategic concerns can motivate jump bidding. Consequently, most of our results from the normal distribution will assume perfect patience or $\delta=1$, although we also will discuss a few sets of results under the assumption of $\delta<1$.

In the case of uniformly distributed values, strategic effects are not an issue and we instead use this setting to look at the effects of impatience. This allows us to separate the two possible motivations for jump bidding to get an idea of how both impact bidding behavior. Our examples from the uniform distribution below assume three different discount rates: $\delta \in \{0.99, 0.95, 0.90\}$. If $\delta=1$ then the equilibrium our algorithm would find is straightforward bidding, although others do exist. For each of these three cases and the three cases of the different normal distributions, we have computed the equilibrium strategies assuming four different minimum increments: $m=\{1, 3, 7, 10\}$. While these cases by no means exhaust the full space of possible value distributions, discount rates and increments, they should be enough to allow for a reasonable characterization of the types of effects that impatient and strategic jumping have on behavior.

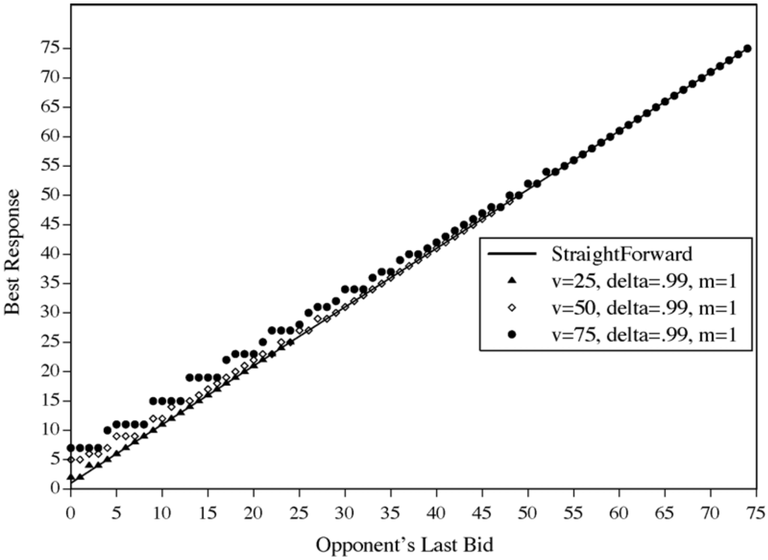

Fig. 1. Depiction of complete equilibrium strategies for bidders with values of 25, 50 and 75 assuming values are uniformly distributed, $\delta = 0.99$ and $m = 1$.

4.1. Sample bid functions

The full equilibrium strategy from any of these examples is a 100×101 matrix with each cell corresponding to an ordered pair of (v_i, p) containing the bid that a person with value of v_i would choose if he had just seen another bidder bid p. Including these matrices in the paper is not feasible, but we can demonstrate the implications of their characteristics. We can show two unambiguous results: (1) jump bidding occurs in equilibrium and (2) the jumps are of moderate size and will occur deep into auctions. The other characteristics of the bidding strategies and the auction outcomes vary depending upon the parameters involved.

Fig. 1 shows what three of the elements of one of these equilibria look like. It contains the complete strategies for bidders with values of 25, 50 and 75 assuming that values are uniformly distributed, $\delta = 0.99$ and $m = 1$. The way to read the graph is to note that the x-axis represents the price just bid by the opponent. The y-axis contains the bid that is a best response to that price. Consider, for example, an auction with two bidders who we will call A and B, both of whom happen to have a value of 75. Assume bidder A sees a $p_0 = 0$. This graph shows that A would place a bid of 7. Bidder B would now see a $b_{t-1} = 7$ and would best respond by bidding 11. Bidder A would then respond to a $b_{t-1} = 11$ by bidding 15, and the rest of the path of the auction could be constructed similarly.

The solid line in the graph depicts straightforward bidding or what a bidder's strategy would look like if he always chose to bid up by the minimum increment. Notice that the pattern of bidding in the graph illustrates that for low prices, bidders will generally place modest jump bids. As the prices rise, they jump less and less until eventually each bidder passes a price such that all future bids are straightforward. We will call the price at which they irrevocably commit to straightforward bidding as the bidders' STPrice, and we will measure how close this is to their value by using the statistic of STPrice/value.

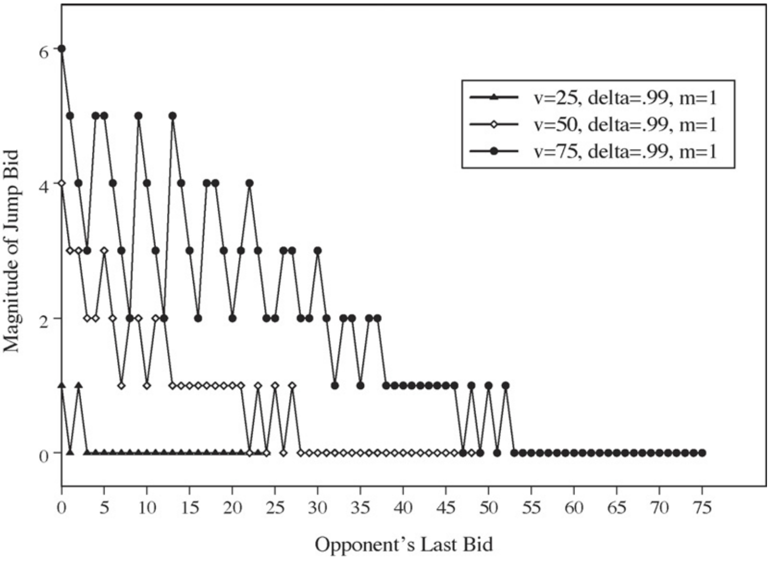

Fig. 2. Portion of equilibrium bids that are above minimum increment assuming the values are uniformly distributed, $\delta = 0.99$ and $m = 1$.

Another view of the same data can be seen in Fig. 2. This graph is constructed by taking the same strategies and displaying the portion of each bid that is above the minimum required. We can again see that the behavior approaches straightforward bidding, which is represented by the x-axis on this graph. This view of the data makes it clear that the size of the jumps do not decrease monotonically with the last bid made by the opponent. Fig. 3 is the analogous graph

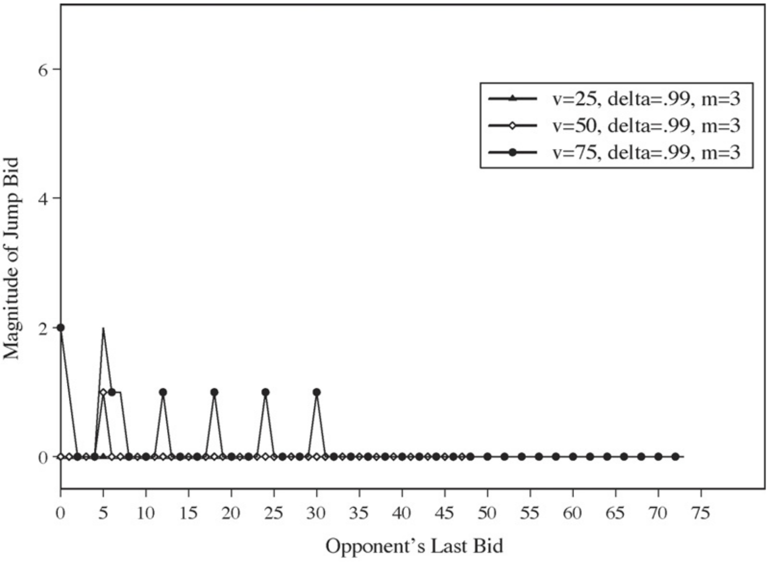

Fig. 3. Portion of equilibrium bids that are above minimum increment assuming the values are uniformly distributed, $\delta = 0.99$ and $m = 3$.

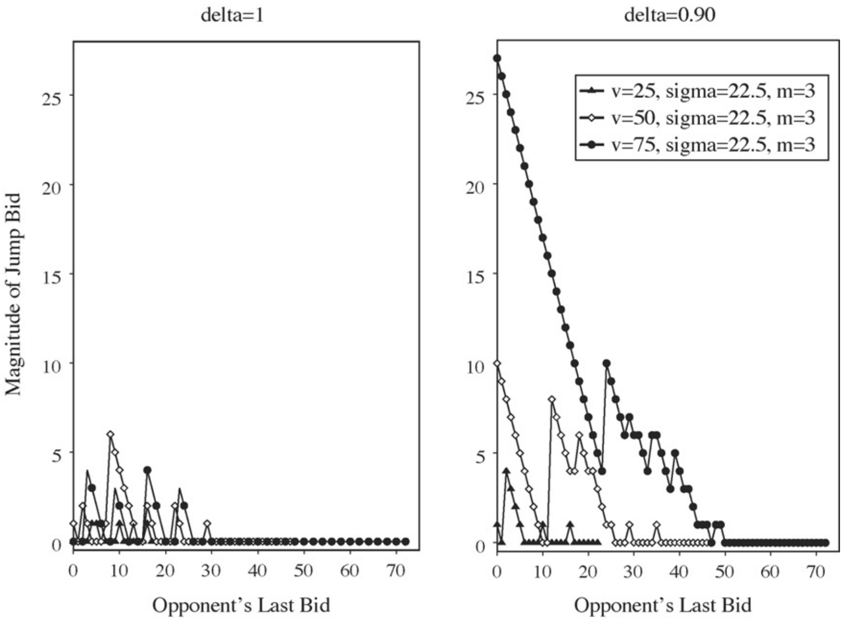

Fig. 4. Comparison of equilibrium bidding strategies between $\delta = 1$ and $\delta = 0.9$ assuming normally distributed values, $\sigma = 22.5$, $m = 3$.

under the assumption of uniformly distributed values and $\delta = 0.99$, but this time with a higher minimum bid increment of $m = 3$. There are far fewer jumps in this equilibrium, and bidding behavior approaches straightforward much faster (i.e., the STPrice of the bidders is lower). Both results are due to the fact that with a higher bid increment, the auction will end faster without the bidders having a need to force it along. The bidders respond to this fact by jumping less.

Fig. 4 shows an example set of strategies assuming normally distributed values and $m = 3$ and $\sigma = 22.5$. There are two panels comparing how equilibrium bidding strategies change according to the discount rate. The panel on the left assumes $\delta = 1$ while the one on the right assumes $\delta = 0.9$. The left panel shows that even with $\delta = 1$ or perfect patience, we see a non-trivial amount of jump bidding due to bid path effects. Notice too that the size of the jumps are not only not monotonically decreasing in the last bid of the opponent, but they are also not monotonically decreasing in v_i either. Moving to the panel on the right, the degree of jump bidding increases substantially.

4.2. Summary effects on bid functions

We have summarized the key details of the equilibrium strategies under each of our different parameter sets in Tables 2 and 3. Table 2 shows the number of elements of the 100×101 best response matrices that involve jumps as well as the average of the STPrice/value ratio for all values in the range [1, 100]. In the case of uniformly distributed values, the number of jumps drops as the size of the increment is increased and the STPrices of the bidders decrease as m rises. Both the number of jumps and the STPrices rise as bidders become more impatient.

Table 3 displays similar patterns for the normal distribution case with $\delta = 1$, but with some important differences. As m rises, the number of jump bids initially rises and then declines. The increment size leading to the least jumping is $m = 1$. The reason for this has to do with the fact that

Table 2
Description of how jump bidding changes, assuming the uniform distribution as the discount rate and minimum increment vary

Discount rate	Increment	Number of jumps	Average STPrice/value
0.99	1	2869	0.46
	3	611	0.20
	7	39	0.03
	10	14	0.01
0.95	1	3835	0.64
	3	2238	0.39
	7	535	0.18
	10	359	0.13
0.90	1	4191	0.73
	3	2862	0.48
	7	1218	0.30
	10	428	0.17

For each combination, we see the total number of elements from the best response matrix that is jump bids and the average STPrice/value ratio.

if $m=1$, there is no ability to catch someone on a notch, and the only benefit is from situations such as the one described above in which a bidder with an odd value prefers to be on an even bid path. As m rises, however, the ability to catch bidders on notches rises, and thus the incentive to jump up to bid paths that accomplish this rises, as does the incentive to jump pre-emptively or defensively to keep an opponent from gaining a more advantageous bid path. The number of jump bids declines monotonically as σ increases. This is logical since the distribution approaches a uniform as σ increases, and SFB is an equilibrium in that case with $\delta=1$. The STPrice follows the same pattern as the number of jumps.

In order to determine how impatience and path effects could combine to impact the strategies in an auction, we have computed the equilibrium strategies for the case of $\sigma=22.5$ and $\delta=0.9$

Table 3
This table describes how jump bidding changes, assuming the normal distribution as the standard deviation and minimum increment vary

σ	Increment	Number of jumps	Average STPrice/value
15	1	1030	0.42
	3	1671	0.48
	7	1348	0.38
	10	1223	0.31
22.5	1	659 (4173)	0.22 (0.73)
	3	1093 (2952)	0.42 (0.59)
	7	870 (1709)	0.35 (0.39)
	10	631 (1230)	0.27 (0.30)
30	1	181	0.06
	3	751	0.30
	7	601	0.32
	10	396	0.26

For each combination, we see the total number of elements from the best response matrix that are jump bids and the average STPrice/value ratio. Numbers in ( ) are under condition of $\delta=0.90$ while the rest assume $\delta=1$.

Table 4
Revenue comparison, assuming normally distributed values, between the expected revenue generated by straightforward bidding, St, and that generated by the equilibrium strategies assuming the standard deviation, minimum increment pair, Eq

m	$\sigma = 15$		$\sigma = 22.5$		$\sigma = 30$	
	St	Eq	St	Eq	St	Eq
1	42.57	42.35	39.51	39.33 (41.84)	38.75	38.59
3	42.49	41.93	39.47	38.89 (40.69)	38.72	38.23
7	42.13	41.90	39.29	39.01 (39.82)	38.67	38.27
10	41.68	41.95	39.97	38.89 (39.55)	38.65	38.28

Numbers in ( ) are under condition of $\delta = 0.90$ while the rest assume $\delta = 1$.

for each of the four bid increments. These results are listed in Table 3 in parentheses. As the bid increment rises, the number of jump bids rises as do the STPrices. The interesting thing to note is the disappearance of the non-monotonicity in the change in the number of jumps as m changes. The most jumps and highest STPrices occur for $m = 1$ and decline as the increment rises. We did not compute this for all normal cases and all discount rates, as the pattern should be the same as is demonstrated by this case.

It should be clear by now that far from being a necessarily irrational choice, or perhaps only reasonably explainable by signaling concerns, jump bidding can be seen as a natural result of even trivial levels of impatience and/or strategic concerns. It should also be clear that the general patterns of bidding we are observing in these equilibrium strategies match the stylized facts described from the field data. In both, we observe that jump bidding is a persistent phenomenon that is likely to be observed until very late in an auction and the amount of jump bidding falls as the bid increment rises. While we do not intend to suggest that this simple comparison serves as a thorough test of the theory, ours is the only theory among those discussed so far that can pass even this cursory comparison against the field data.

4.3. *Impact of jump bidding*

4.3.1. *Revenue*

Tables 4 and 5 contain revenue comparisons between equilibrium outcomes (Eq) and the outcomes of straightforward bidding (St). The numbers are full expected revenue calculations found by computing $\sum\sum R(i, j) f(i, j)$ where $R(i, j)$ is the revenue that would result from the value pair, and $f(i, j)$ is the joint probability of those two values being drawn. These two tables

Table 5
Revenue comparison, assuming uniformly distributed values, between the expected revenue generated by straightforward bidding, St, and that generated by the equilibrium strategies assuming the discount rate, minimum increment pair, Eq

m	$\forall\delta$	$\delta = 0.99$	$\delta = 0.95$	$\delta = 0.9$
	St	Eq	Eq	Eq
1	34.33	34.87	36.20	37.12
3	34.34	34.09	35.17	36.07
7	35.51	34.26	34.46	35.10
10	34.74	34.57	34.62	34.75

show that revenue is usually not very different between the equilibrium and SFB cases.[8] Under the assumption of normally distributed values with $\delta = 1$ (Table 4), SFB tends to lead to slightly more revenue, and the same holds for uniformly distributed values (Table 5), when the bid increment is high. However, as bidders get more impatient (i.e., δ decreases), more revenue tends to be generated by the equilibrium strategy. This effect is more pronounced with lower bid increments. The reason for this is that as the increment increases, jump bidding falls substantially. This leads to fewer bidders jumping of their own accord to a higher price than they have to in order to win the auction. While a higher increment can force them to do so, it would appear that it does not do enough of this to overcome the negative effects on revenue from the decrease in voluntary jumping. The implication of these results is that for patient bidders, allowing bidders to jump is approximately revenue neutral. If bidders are a little bit impatient, then allowing them to jump leads to a slight increase in revenue over not allowing them to jump so long as the bid increment is low.

4.3.2. Bidder utility

A simple revealed preference argument might suggest that as bidders could bid straightforwardly when allowed to jump and choose not to, they are definitely better off when allowed to jump. This argument is not correct due to the strategic nature of the problem. Partial results for uniformly distributed values can be found in Table 11, but we will omit presenting the full results to conserve space.[9] For all cases involving uniformly distributed values, the expected utility for bidders with virtually every possible value is greater when allowed to jump bid than when forced to bid straightforwardly. The effect is most pronounced for cases involving low bid increments and high discount rates (i.e., high in terms of impatience, not numerical value). For cases involving discount rates of 0.99 and large increments, equilibrium bidding is not much different from straightforward bidding, and neither are the expected utilities.

Cases involving normally distributed values with $\delta = 1$ are more interesting. When $\sigma = 15$, the equilibrium path still yields a higher expected value than the SFB path. For $\sigma = 22.5$ and 30, however, this no longer holds. At $\sigma = 22.5$, the SFB path has an almost imperceptible edge in expected value that becomes more pronounced when $\sigma = 30$. The reason for this is the existence of a prisoner's dilemma effect. When one bidder bids straightforwardly, a sophisticated bidder can force a bid path beneficial to himself and would therefore place jump bids. When both bidders are sophisticated, though, they have to place defensive jumps to keep from being forced onto disadvantageous paths themselves, sacrificing some expected utility but not as much as would be sacrificed by accepting the bad path.

On the other hand, if we look at the situation for more impatient bidders, such as the $\delta = 0.9$ and $\sigma = 22.5$ case we have used before, the bidders now become strictly better off by being allowed to jump. This is most pronounced when increments are low. It is also the case that for both uniformly and normally distributed values, bidders monotonically prefer higher bid increments on average as the average expected values for the bidders increase with the size of the increment. This last result appears to have been confirmed empirically in Lucking-Reiley (1999). During the field ascending auctions conducted for that study, the Lucking-Reily (p. 312) notes that "after feedback from bidders, I used larger minimum bid increments for the higher priced cards in auctions SE1 and

[8] Since the numbers are full population calculations, no tests of statistical significance for the observed differences are necessary. The focus is therefore on practical significance.

[9] Full results freely available upon request to the authors.

Table 6
Efficiency comparison, assuming normally distributed values, showing the percent of value pairs that result in an inefficient allocation out of the total number of pairs that could for both straightforward bidding, St and for equilibrium bidding, Eq, for each standard deviation and minimum increment pair

m	$\forall\sigma$	$\sigma = 15$		$\sigma = 22.5$	$\sigma = 30$
	# Possible	St (percent)	Eq (percent)	Eq (percent)	Eq (percent)
1	0	–	–	–	–
3	390	25.90	11.03	11.28 (11.79)	14.10
7	1122	27.81	12.39	15.15 (19.16)	20.32
10	1629	29.83	18.91	21.42 (20.63)	24.86

Numbers in ( ) are under condition of $\delta = 0.90$.

SE2," which suggests that the bidders involved preferred higher increments strongly enough to make the request.

4.3.3. *Efficiency*

Tables 6 and 7 contain an analysis of the effect of jump bidding on the efficiency of the auctions. For each bid increment, we note the number of value pairs that could possibly be inefficient along with the percentage of those cases that yield inefficient outcomes under both SFB and equilibrium bidding. The number of possibly inefficient cases rises with the increment as, for example, a bidder with a value of 40 cannot inefficiently outbid anyone if $m = 1$, but if $m = 7$ he can possibly inefficiently outbid bidders with values 41–46 by bidding 40. Both the number of possible value pairs that could be inefficient and the number that turn out to be inefficient under SFB do not change with either the standard deviation of the distribution or the level of impatience of the bidders. In all but one case, there are fewer value pairs that lead to inefficient outcomes in equilibrium when allowing jump bidding than would do so assuming SFB.

The main effect shown in the tables is that the predominant drag on efficiency is the introduction of a minimum bid increment greater than the minimum distance between bidder values. Allowing bidders to jump alleviates the impact of this. The other effect that becomes clear is that with a tighter distribution, lower σ, the ability of jump bidding to alleviate any inefficiency from the minimum increment is enhanced. Impatience on the part of the bidders also improves efficiency, at least in the uniform distribution cases. We represent efficiency in the manner we have because reporting typical efficiency numbers will not reveal as clearly the comparative statics at work. The reason for this is that the majority of the value pairs still lead to efficient outcomes, causing the overall efficiency to remain high, typically above 99.9 percent. We focus only on the

Table 7
Efficiency comparison, assuming uniformly distributed values, showing the number of value pairs that result in an inefficient allocation out of the total number of pairs that could for both straightforward bidding, St and for equilibrium bidding assuming the discount rate, minimum increment pair, Eq

m	$\forall\delta$	$\delta = 0.99$		$\delta = 0.95$	$\delta = 0.9$
	# Possible	St (percent)	Eq (percent)	Eq (percent)	Eq (percent)
1	0	–	–	–	–
3	390	25.90	16.41	8.97	9.49
7	1122	27.81	27.81	22.64	23.26
10	1629	29.83	30.03	28.18	26.34

Table 8
Characterization of jump bidding when bidders allowed to bid quarters and values distributed uniformly

Discount rate	Increment	Number of jumps	Average STPrice/value
1	1	12415	0.91
	3	13034	0.80
	7	10722	0.65
	10	9045	0.55

cases that could possibly be inefficient because this highlights the effects of jump bidding versus SFB.

We conclude from this that while the introduction of a minimum increment impairs the efficiency of an ascending auction, allowing jump bidding lessens this problem. While the magnitude of that improvement is not large, it is important to realize that the prior claim in the literature is that jump bidding moved efficiency in the opposite direction. We show that this claim is does not hold under the assumptions of our model.

4.4. Extensions

4.4.1. Finer price grid

There are many ways in which one might extend this basic model to look at different aspects of the problem. One issue of particular concern is the degree to which the discretization of the space affects the results. In particular, it might be reasonable to think that the prices should have a finer grid than values. For example, most bidders in an auction might round off their perceived values for items at dollars even though the auction allows them to bid in pennies. We have investigated this by increasing the resolution of our price grid, revealing a number of very interesting results.

The first result is that the number of jumps goes up dramatically. It is now the case that even perfectly patient bidders with uniformly distributed values will jump bid in equilibrium. Table 8 shows the relevant statistics on the number of bids that are jumps and the average STPrice/value for this case. It is important to note that the number of jumps in this case is in part so large because the best response matrix is now a 100×401 matrix, which is much larger than the prior case. Table 8 shows that the raw number of jump bids becomes quite large, and it turns out that the amount of jump bidding is greater even in terms of the percentage of elements of the best response matrix. The STPrices are also higher, which means that bidders jump bid much deeper into the auction. While we have only presented the numbers for this one case, the results for all other cases investigated show rises in the amount of jump bidding and higher STPrices as well.

The reason for the increase in jump bidding is due to the increased opportunity to engage in strategic jumping. For example, if a bidder would normally have placed a bid of 45 and the minimum increment is 3, he would have shut out bidders with values of 45, 46 and 47. Bidders with values 48 and up would have bid back. A bidder can now bid 45.25 and also exclude bidders with a value of 48 without incurring much of a cost. Virtually all jumps in the uniform $\delta = 1$ case are of this sort, and most of the increase in the other cases is due to similar issues. Tables 9 and 10 display revenue and efficiency results for the same cases of the uniform distribution as in the previous section assuming that bidders can bid quarters. While the results change a little, they still show that the effect from allowing jump bidding on revenue is approximately neutral while the effect on efficiency ranges from positive to neutral.

Table 9
Revenue comparison when allowing bidders to bid on quarters, assuming uniformly distributed values, between the expected revenue generated by straightforward bidding, St and that generated by the equilibrium strategies assuming the discount rate, minimum increment pair, Eq

m	$\forall\delta$ St	$\delta=0.99$ Eq	$\delta=0.95$ Eq	$\delta=0.9$ Eq
1	34.33	34.24	35.48	36.29
3	34.34	33.71	34.47	35.31
7	35.51	33.65	33.86	34.25
10	34.74	33.61	33.67	33.86

Table 10
Efficiency comparison when allowing bidders to bid quarters, assuming uniformly distributed values, showing the number of value pairs that result in an inefficient allocation out of the total number of pairs that could for both straightforward bidding, St and for equilibrium bidding assuming the discount rate, minimum increment pair, Eq

m	$\forall\delta$	$\delta=0.99$		$\delta=0.95$	$\delta=0.9$
	# Possible	St (percent)	Eq (percent)	Eq (percent)	Eq (percent)
1	0	–	–	–	–
3	390	25.90	15.13	10.00	23.59
7	1122	27.81	27.54	23.89	22.64
10	1629	29.83	30.01	28.91	26.40

Fig. 5 shows us in more detail exactly how this effect emerges. It displays the expected value functions for bidders for all bids that they could place at the start of the auction, $p=0$, with values of 2, 5 and 8 in a more limited case in which values are distributed on the integers between 1 and 10, $\delta=1$, and bidders are allowed to bid in pennies.[10] To simplify the calculations here, it is assumed that after this bid, straightforward bidding will follow. The results show that every integer is dominated by bidding ε above it, and this is what generates the incentive to jump bid by a small amount in cases where the price space has a finer resolution than the value space.

4.4.2. *Linear approximations of the strategies*

The equilibria we have constructed with our approach are rather complex and no one should expect actual bidders to be able to construct them exactly. It is important then to determine whether or not a bidder who is generally sensitive to the trade-offs involved in the equilibrium bid functions could use approximations of the actual strategies without sacrificing much expected utility.

To examine this issue, we have taken linear approximations of the real equilibrium strategies and recomputed all of the results on revenue, efficiency and bidder expected values, assuming that bidders play according to these approximations. The approximations are simplifications of the strategies as represented in Figs. 2–4. They are found by constructing a piece-wise linear version with the non-linear portion approximated by a line using the size of the jump bid the bidder would make facing a price of 0 as the y-intercept and the point at which they would begin bidding straightforwardly (STPrice) as the x-intercept. From that point on, straightforward bidding occurs. The essential result from analyzing the results is captured in Table 11 which

[10] The smaller range was chosen purely to facilitate full calculation of the expected value function, which is quite computationally intensive.

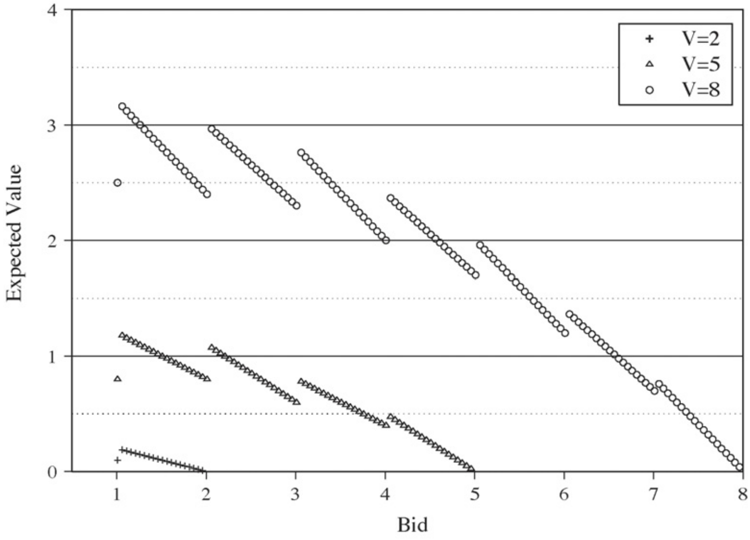

Fig. 5. Expected value functions for bidders with values of 2, 5 and 8 in the case that the values are uniformly distributed on integers in the range 1–10 and straightforward bidding will ensue after this bid.

displays the average expected value for bidders participating in an auction assuming uniformly distributed values under the cases of straightforward bidding, equilibrium bidding and bidding according to this approximation. For every scenario investigated, the equilibrium bidders are best off, followed closely by the approximate bidders with the straightforward bidders coming in third. The indication is that bidders do not lose much expected utility by deviating from the equilibrium bid functions to some degree and still do better than bidding straightforwardly.

To conserve space, we have not included the rest of the statistics on this case, but they work out about as one would expect. In both the uniform and normal cases, the expected revenue and

Table 11
Expected values of bidders for participating in auction assuming uniformly distributed values for the cases in which they are forced to bid straightforwardly (ST), able to bid according to the true equilibrium strategies (EQ) or their linear approximation (AP)

δ	m	EV[ST]	EV[EQ]	EV[AP]
0.99	1	14.56	15.54	15.33
	3	15.82	16.11	15.83
	7	16.18	16.37	16.20
	10	16.20	16.37	16.23
0.95	1	9.65	14.20	13.84
	3	13.69	14.73	14.46
	7	15.33	15.59	15.37
	10	15.69	15.94	15.70
0.90	1	6.50	13.37	12.94
	3	11.59	13.82	13.57
	7	14.36	14.79	14.54
	10	15.07	15.33	15.13

efficiency that result from the linear approximations are about the same as in the equilibrium case and almost always have the same relationship with the SFB results as the standard equilibrium results. The expected values of the bidders in the case of normally distributed values and discount rates of 1 are usually slightly under the expected values of SFB. Adding in impatience, however, leads to the approximate equilibrium of bidders doing better than SFB. All of this suggests that the general nature of our results should be fairly robust to bidders using some general approximation of the equilibrium strategies.

5. Conclusion

We have shown that neither irrationality nor signaling is required to generate jump bidding. We have also shown that when allowing for either impatience or strategic bidding, straightforward bidding is not generally part of the equilibrium set. Furthermore, when jump bidding is allowed, the auctions will be highly efficient and, in some cases, even marginally more efficient than if bidders were to bid straightforwardly. Instead of hurting revenue, allowing bidders to jump can increase revenue. Depending on the level of impatience of the bidders and the value distribution, allowing bidders to jump bid can also improve their expected utility from participating. This is an important result because, as shown in Ivanova-Stenzel and Salmon (2004), if bidders expect to obtain higher utility from participating in an auction, they are more likely to do so. One only need consider his own likelihood of participating in an auction for an object for which a substitute exists that is expected to sell for a price in the thousands to millions of dollars when the bidding starts at $1, the bid increment is $0.01 and the auctioneer does not allow jump bidding.

Our results confound prior speculation about what is an equilibrium and about the properties of equilibria in this context. We, therefore, view these theoretical results as a crucial link in our understanding of ascending auctions. In particular, our analysis reveals important implications for the design of ascending auctions that would be missed by only examining the clock model. The main issue overlooked by focusing only on the clock model are the trade-offs involved in the choice of different possible bid increments.

We speculate, though, that our results understate the effect of small bid increments and/or forcing bidders to bid straightforwardly on revenue and perhaps efficiency because we are not accounting for the possibility that bidders might choose to cease bidding early when the auctioneer allows for only a slow advance in prices. We noted above that bidders in the field experiments in Lucking-Reiley requested higher bid increments. In Shachat and Swarthout (2002), the authors find that sellers in an ascending procurement auction drop out of the auction earlier than they should, leading the buyers to pay more than they should have to. The design of those auctions did not allow jump bidding, and the authors state that "we conjecture that the tediousness of the English auction is responsible for the early exit behavior." If their conjecture is correct, then by not allowing bidders to jump bid, an auctioneer could have bidders exit the auction at lower prices than they would if they had been allowed to bid in a less tedious manner. This effect is consistent with the general nature of the results of our model but is impossible to explain in the standard clock model.

The current paper does not provide an empirical test of our theory. In a companion paper, Isaac et al., we provide a rigorous test of the model developed here and of the alternative models of jump bidding. That paper shows that the theory developed here is quite accurate in terms of predicting the outcomes of ascending auctions and obtains more support than the alternatives.

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