Dynamic Demand for Differentiated Products with Fixed-Effects Unobserved Heterogeneity

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MOTIVATION

- In many markets, **consumer demand can be dynamic**:

  Consumers' preferences depend on their past decisions.

- **Sources of dynamics in demand** include:

  Storability; durability; habits; switching costs; adoption costs; learning

- Dynamics generate **state dependence** in consumers' decisions, and **differences between short-run and long-run responses**.

- The estimation of **dynamic structural demand models** using **consumer-level panel data** tries to measure these causal effects and use them for counterfactual policy analysis and welfare evaluation.

  e.g., taxes; new products; mergers; cost pass-through

IDENTIFICATION CHALLENGES

- Disentangling **true dynamics** (causal effect of past decisions) versus **spurious dynamics** (persistent unobserved heterogeneity (UH)).
- Two important issues with **short panels** (with fixed T):
  - **Incidental parameters problem**
  - **Initial conditions problem**
- **ALL the applications** of dynamic discrete choice structural models have considered **Random Effects (RE)** models (finite mixture).
- Potential misspecification of parametric restrictions on distribution of UH can introduce **substantial biases in the estimates of the model parameters**.

PURPOSE OF THIS PAPER

- I apply and extend recent developments to study identification & estimation of **dynamic demand for differentiated product** with **Fixed Effects** consumer heterogeneity.

1. **Identification of structural parameters** in Logit & Nested Logit: Switching costs; Depretiation (depletion) of durable (storable) products; Price sensitivity.

2. Conditional Maximum Likelihood (CML) estimation of these parameters **does not suffer the curse of dimensionality**.

3. Illustrate these results with an **empirical application** using NIELSEN consumer scanner data from Chicago-Kilts Center.

RELATED LITERATURE

1. Long empirical literature on dynamic demand models in marketing / IO. In most papers, consumers are not forward-looking.

2. **Dynamic structural models of demand for differentiated products**. Seminal papers:

   - Erdem, Imai, & Keane (2003): Random effects model.
   - Hendel & Nevo (2006): Only observed heterogeneity.

3. **Dynamic discrete choice models with Fixed Effect unobserved heterogeneity and short panels**.

   - Honoré & Kyriazidou (2000, 2019); Honoré & Tamer (2006); Honoré & Weidner (2021).
   - Aguirregabiria, Gu, & Luo (2021)

4. **Other recent methods**: Kong, Dubé, & Daljord (2022); Berry & Compiani (2022). Parameteric models for persistent Unobs. heter.

OUTLINE

1. MODEL
2. IDENTIFICATION OF STRUCTURAL PARAMETERS
3. ESTIMATION
4. EMPIRICAL APPLICATION

1. MODEL

MODEL: DECISION & STATE VARIABLES

- J **products** indexed by j; consumers by i, calendar time by t.

- **Consumer Decision variable**:
  $y_{it} = 0$ means "no purchase"; $y_{it} = j > 0$ means "purchase j"

- State variables that **depend the consumer's choices**:
  $\ell_{it} =$ brand choice in last purchase
  $$\ell_{i,t+1} = 1\{y_{it} = 0\}\ell_{it} + 1\{y_{it} > 0\}y_{it}$$
  $d_{it} =$ time duration since last purchase.
  $$d_{i,t+1} = 1 + 1\{y_{it} = 0\}d_{it}$$

- State variables that **DO NOT depend on the consumer's choices**:
  prices, advertising, other time-varying product characteristics.
  $\mathbf{p}_{it} = (p_{it}(j) : j = 1, 2, ..., J)$

MODEL: CONSUMER PREFERENCES

- Consumers maximize expected & discounted intertemporal utility:

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \delta_i^s \; U_{i,t+s} \right]$$

δ_i **is unrestricted**: a component of the FE Unobserved Heterogeneity.

- Utility has four components:

$$U_{it} = b_i \left(y_{it}, \ell_{it}, d_{it} \right) + m_i \left(y_{it}, \mathbf{p}_{it} \right) - sc_i \left(y_{it}, \ell_{it} \right) + \varepsilon_{it}(y_{it})$$

$b_i \left(y_{it}, \ell_{it}, d_{it} \right)$ = utility from consumption of branded product.

$m_i \left(y_{it}, \mathbf{p}_{it} \right)$ = utility from consumption of composite product.

$sc_i \left(y_{it}, \ell_{it} \right)$ = switching cost / habits.

$\varepsilon_{it}(y_{it})$ = i.i.d. Logit / Nested Logit shock.

UTILITY: CONSUMPTION BRANDED PRODUCT

$$b_i\left(y_{it}, \ell_{it}, d_{it}\right) \equiv \begin{cases} \alpha_i(\ell_{it}) + \ln(c_{it}) & \text{if} \quad y_{it} = 0 \\ \alpha_i(j) + \ln(c_{it}) & \text{if} \quad y_{it} = j > 0 \end{cases}$$

- $\alpha_i(j)$ = flow utility for consumer i from consuming brand j.
- We can see $\alpha_i(j)$ as a combination of product & consumer characteristics, observable and unobservable to the researcher.

$$\alpha_i(j) = \mathbf{x}_j' \, \boldsymbol{\beta}_i^x + \boldsymbol{\xi}_j' \, \boldsymbol{\beta}_i^\xi$$

- $\boldsymbol{\alpha}_i \equiv (\alpha_i(1), \alpha_i(2), ..., \alpha_i(J))$ are the fixed effects for consumer i.

UTILITY: CONSUMPTION BRANDED PRODUCT [2/2]

- A **fundamental measurement problem** in this literature is that the researcher does not observe (with enough high frequency) a consumer's amounts of consumption c_{it} and inventory i_{it}.

- Here I follow a similar approach as in Erdem, Imai, & Keane (2003) and assume a consumption rule:

$$c_{it} \;=\; \begin{cases} \lambda^{dep}(\mathbf{w}_i, \ell_{it}) \; i_{it} & \text{if} \quad y_{it} = 0 \\ i_{it} & \text{if} \quad y_{it} > 0 \end{cases}$$

where $\lambda^{dep}(\mathbf{w}_i, j) \in (0,1)$ is an exogenous consumption rate that may vary across products, and across consumers according to observable characteristics $\mathbf{w}_i$.

- Together with the standard transition rule for inventories, we have:

$$\ln(c_{ht}) \;=\; constant \;-\; \beta^{dep}(\mathbf{w}_i, j) \; d_{it}$$

with $\beta^{dep}(\mathbf{w}_i, j) = -\ln(1 - \lambda^{dep}(\mathbf{w}_i, j))$.

UTILITY FROM COMPOSITE GOOD

$$m_i\left(y_{it}, \mathbf{p}_{it}\right) \;=\; \gamma(\mathbf{w}_i)\;\left(\mu_i - \sum_{j=1}^{J} p_{it}(j)\; 1\{y_{it} = j\}\right)$$

- μ_i = consumer's disposable income.
- $\gamma(\mathbf{w}_i)$ = marginal utility of the composite good, e.g., $\gamma(\mathbf{w}_i) = \mathbf{w}_i'\,\gamma$
- Identification results extend to the case of **nonlinear in consumption** but linear in parameters utlity from the composite good:

$$\gamma_1\left(\mu_i - \sum_{j=1}^{J} p_{it}(j)\; 1\{y_{it} = j\}\right) + \gamma_2\left(\mu_i - \sum_{j=1}^{J} p_{it}(j)\; 1\{y_{it} = j\}\right)^2$$

UTILITY: SWITCHING COSTS

$$sc_i\left(y_{it},\ell_{it}\right) = \sum_{k=1}^{J}\sum_{j\neq k} 1\{\ell_{it}=k \ \& \ y_{it}=j\}\ \beta^{sc}(\mathbf{w}_i,k,j)$$

- $\beta^{sc}(\mathbf{w}_i,k,j) =$ cost of switching from brand k to brand j.

UTILITY: LOGIT IDIOSYNCRATIC SHOCKS

- $\varepsilon_{it}(j)$'s are i.i.d. over (i,t,j) type I extreme value distributed.
- I provide identification & estimation results for Nested Logit version.

COMPLETE UTILITY FUNCTION

- Putting together the different components:

$$U_{it} = \begin{cases} \alpha_i(\ell_{it}) - \beta^{dep}(\ell_{it}) \; d_{it} + \varepsilon_{it}(0) & \textit{if} \quad y_{it} = 0 \\ \\ \alpha_i(j) + \gamma_i \left(\mu_i - p_{it}(j)\right) - \beta^{sc}(\ell_{it}, j) + \varepsilon_{it}(j) & \textit{if} \quad y_{it} = j > 0 \end{cases}$$

- We use $\mathbf{x}_{it} = (\ell_{it}, d_{it})$, and:

  $u_{\boldsymbol{\alpha}_i}(y_{it}, \mathbf{x}_{it}, \mathbf{p}_{it}) =$ utility excluding unobservable logit shocks.

MODEL: STOCHASTIC PROCESS FOR PRICES

- $p_{it}(j)$ has two components: persistent, $z_{it}(j)$; and transitory, $e_{it}(j)$.

$$p_{it}(j) \;=\; \rho\left(z_{it}(j),\; e_{it}(j)\right)$$

where $\rho(.)$ is a known function.

- Define $\mathbf{z}_{it} \equiv (z_{it}(j) : j = 1, 2, ..., J)$ and $\mathbf{e}_{it} \equiv (e_{it}(j) : j = 1, 2, ..., J)$.

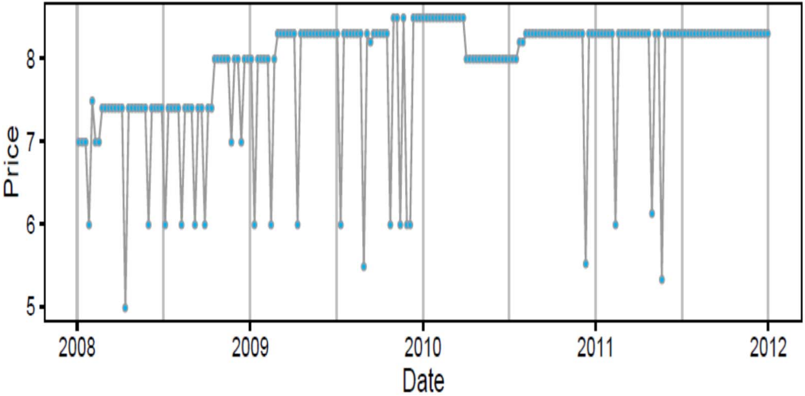
- **ASSUMPTION 1:**

(i) $\mathbf{z}_{it}$ follows a first order Markov process.

(ii) **Conditional independence of transitory component of prices:** Conditional on $\mathbf{z}_{it}$, $\left(\mathbf{e}_{i,t+1}, \mathbf{z}_{i,t+1}\right)$ does not depend on $\mathbf{e}_{it}$.

EXAMPLE: HI-LO PRICING

Figure: Weekly time series: price of laundry detergent product (Tide liquid 70oz)



EXAMPLE: HI-LO PRICING (2/2)

- Many supermarket products: evolution of weekly prices is characterized by the alternation between a regular price and a promotion price. See Hitsch, Hortacsu, & Lin (2019).

- Stochastic process for price:

$$p_{it}(j) = (1 - e_{it}(j)) \; z_{it}^{reg}(j) + e_{it}(j) \; z_{it}^{pro}(j)$$

- $z_{it}^{reg}(j) =$ Regular price (follows Markov chain.)

- $z_{it}^{pro}(j) =$ Promotion price (follows Markov chain.)

- $e_{it}(j) =$ Dummy variable for "promotion for product j in market i at period t". Satisfies the Conditional Independence Assumption 1(ii).

STOCHASTIC PROCESS FOR PRICES & IDENTIFICATION

- The stochastic process of prices is not needed for the identification of the parameters β^{sc} and β^{dep}.

- However, it plays a **key role in the identification of the price parameter** γ in a FE forward-looking model.

- Both $\mathbf{z}_{it}$ and $\mathbf{e}_{it}$ affect a consumer's current utility, but expected future utility (the continuation value) depends on $\mathbf{z}_{it}$ but not on $\mathbf{e}_{it}$.

- This exclusion restriction is key in the identification of γ.

- Given data on prices and a specification of the $\rho(.)$ function, it is possible to identify the two components $\mathbf{z}_{it}$ and $\mathbf{e}_{it}$.

CONSUMER DYNAMIC DECISION PROBLEM

- The decision problem of consumer i at period t is:

$$y_{it} = argmax_{j \in \mathcal{Y}} \left\{ u_{\boldsymbol{\alpha}_i}(j, \mathbf{x}_{it}, \mathbf{p}_{it}) + \varepsilon_{it}(j) + v_{\boldsymbol{\alpha}_i}(f_x(j, \mathbf{x}_{it}), \mathbf{z}_{it}) \right\}$$

- $f_x(j, \mathbf{x}_{it})$ = value of $\mathbf{x}_{i,t+1}$ given state $\mathbf{x}_{it}$ and decision $y_{it} = j$.
- $v_{\boldsymbol{\alpha}_i}(f_x(j, \mathbf{x}_{it}), \mathbf{z}_{it})$ = *continuation value function*.
- $P(j|\mathbf{x}_{it}, \mathbf{z}_{it}, \mathbf{e}_{it}, \boldsymbol{\alpha}_i)$ = **Conditional Choice Probability (CCP)**.
- Model implies:

$$\log P\left(j|\mathbf{x}_{it}, \mathbf{z}_{it}, \mathbf{e}_{it}, \boldsymbol{\alpha}_i\right) =$$

$$= u_{\boldsymbol{\alpha}_i}(j, \mathbf{x}_{it}, \mathbf{p}_{it}) + v_{\boldsymbol{\alpha}_i}(f_x(j, \mathbf{x}_{it}), \mathbf{z}_{it}) - \sigma_{\boldsymbol{\alpha}_i}(\mathbf{x}_{it}, \mathbf{z}_{it}, \mathbf{e}_{it})$$

where $\sigma_{\boldsymbol{\alpha}_i}(\mathbf{x}_{it}, \mathbf{z}_{it}, \mathbf{e}_{it})$ be the log of the denominator in the Logit CCP function (i.e, log of sum of exponentials of utilities).

2. IDENTIFICATION OF STRUCTURAL PARAMETERS

SUFFICIENT STATISTICS APPROACH

- I follow Aguirregabiria, Gu, & Luo (2021) who consider a sufficient statistic - conditional likelihood approach in the spirit of Cox (1958), Rasch (1960).

- Let $\boldsymbol{y}_i = \{\ell_1, d_1, y_1, y_2, ..., y_T\}$ be an individual's observed history; and let $\widetilde{\mathbf{z}}_i \equiv (\mathbf{z}_{i1}, \mathbf{z}_{i2}, ..., \mathbf{z}_{iT})$ and $\widetilde{\mathbf{e}}_i \equiv (\mathbf{e}_{i1}, \mathbf{e}_{i2}, ..., \mathbf{e}_{iT})$.

- $\boldsymbol{\theta}$ is the vector of structural parameters: $\beta^{sc}(.)$, $\beta^{dep}(.)$, and γ

- Probability of $\mathbf{y}_i$ conditional on history of prices $\widetilde{\mathbf{z}}_i$, $\widetilde{\mathbf{e}}_i$ and $\boldsymbol{\alpha}_i$ is:

$$\mathbb{P}\left(\mathbf{y}_i | \widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i, \boldsymbol{\alpha}_i, \boldsymbol{\theta}\right) =$$

$$\mathrm{p}^*(\ell_{i1}, d_{i1} | \boldsymbol{\alpha}_i) \prod_{t=2}^{T} \frac{\exp\{u_{\alpha_i}(y_{it}, \mathbf{x}_{it}, \mathbf{p}_{it}) + v_{\alpha_i}(y_{it}, \mathbf{x}_{it}, \mathbf{z}_{it})\}}{\sum_{j=0}^{J} \exp\{u_{\alpha_i}(j, \mathbf{x}_{it}, \mathbf{p}_{it}) + v_{\alpha_i}(j, \mathbf{x}_{it}, \mathbf{z}_{it})\}}$$

SUFFICIENT STATISTICS APPROACH (2/2)

- This log-probability has the following structure:

$$\log \mathbb{P}\left(\mathbf{y}_i | \widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i, \boldsymbol{\alpha}_i, \boldsymbol{\theta}\right) \;=\; \mathbf{s}(\mathbf{y}_i, \widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i)' \; \mathbf{g}(\boldsymbol{\alpha}_i) + \mathbf{c}(\mathbf{y}_i, \widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i)' \; \boldsymbol{\theta}$$

- This structure has several important implications.

1. $\mathbf{s}(\mathbf{y}_i, \widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i)$ is a sufficient statistic for $\boldsymbol{\alpha}$.

2. If the elements in the vector $\left[\mathbf{s}(\mathbf{y}_i, \widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i))', \mathbf{c}(\mathbf{y}_i, \widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i)'\right]$ are linearly independent, then CMLE implies the identification of $\boldsymbol{\theta}$.

A MORE INTUITIVE DESCRIPTION

- For every parameter in the vector $\boldsymbol{\theta}$, say θ_k, there exist two choice histories, say A and B, such that $\mathbf{s}(A) = \mathbf{s}(B)$ and $\mathbf{c}(A) - \mathbf{c}(B)$ is a vector where all the elements are zero except element k that is one.

- Under these conditions, we have that:

$$\theta_k \;=\; \log \mathbb{P}(A) - \log \mathbb{P}(B),$$

- Parameter θ_k is identified from the log odds ratio of histories A and B.

IDENTIFICATION OF β^{sc} AND γ

- For $k, j \geq 1$ with $k \neq j$, and any two natural numbers n_1 and n_2, consider the following choice histories ($\mathbf{0}_n$ = vector of n zeros):

$$A = (k,\ \mathbf{0}_{n_1},\ j,\ \mathbf{0}_{n_2},\ k,\ \mathbf{0}_{n_2},\ j)\,; B = (k,\ \mathbf{0}_{n_1},\ k,\ \mathbf{0}_{n_2},\ j,\ \mathbf{0}_{n_2},\ j)$$

- And the following condition on the history of prices:

  $\mathbf{z}_{it}$ is constant from period $n_1 + 2$ to $n_1 + 2n_2 + 4$

- Under these conditions, we have that:

$$\log \mathbb{P}(A) - \log \mathbb{P}(B) =$$

$$- \widetilde{\beta}^{sc}(k, j) - \gamma\ \left(e_{n_1+2}(j) - e_{n_1+3}(j) - e_{n_1+2}(k) + e_{n_1+3}(k)\right)$$

IDENTIFICATION OF β^{sc} AND γ (2/2)

$$\log \mathbb{P}(A) - \log \mathbb{P}(B) =$$

$$- \widetilde{\beta}^{sc}(k,j) - \gamma \ \left(e_{n_1+2}(j) - e_{n_1+3}(j) - e_{n_1+2}(k) + e_{n_1+3}(k) \right)$$

- This equation shows that:
  1. A change between periods $n_1 + 2$ and $n_1 + 3$ in the transitory component of the price of product j or k identifies parameter γ.
  2. The switching cost parameter $\widetilde{\beta}^{sc}(k,j)$ is identified from histories where this transitory component is constant.

IDENTIFICATION OF β^{dep}

- **ASSUMPTION 2.** *For any product j, there is a value of duration d_j^* – which can vary across products – such that $\beta^{dep}(j, n) = \beta^{dep}(j, d_j^*)$ for any duration $n \geq d_j^*$.* ■

- **PROPOSITION.** *For any product j and any duration n, define the pair of histories:*

$$A_{j,n} = (j,\ \mathbf{0}_{n-1},\ j,\ \mathbf{0}_{n+1}) \qquad \text{and} \qquad B_{j,n} = (j,\ \mathbf{0}_{n},\ j,\ \mathbf{0}_{n}).$$

If $d_j^* \leq (T-1)/2$, then d_j^* is identified from the following expression:

$$d_j^* \;=\; max\{n \;:\; \log \mathbb{P}(A_{j,n}) - \log \mathbb{P}(B_{j,n}) \neq 0\} \qquad ■$$

IDENTIFICATION OF β^{dep} (2/2)

- Then, for $n = d_j^* - 1$, we have that:

$$\log \mathbb{P}(A_{j,n}) - \log \mathbb{P}(B_{j,n}) \; = \; -\beta^{dep}(j, d_j^*) + \beta^{dep}(j, d_j^* - 1)$$

- The (local) depreciation rate $\beta^{dep}(j, d_j^*) - \beta^{dep}(j, d_j^* - 1)$ is identified.

- If $\beta^{dep}(j, d)$ is a linear function, i.e., $\beta^{dep}(j, d) = \overline{\beta}_j^{dep} \, d$, then the product-specific depreciation rate $\overline{\beta}_j^{dep}$ is identified.

3. ESTIMATION

CML ESTIMATION

- Let represent a sufficient statistic for $\boldsymbol{\alpha}_i$ as a binary indicator that combines the condition $\mathbf{y}_i \in \{A \cup B\}$, and restrictions on prices, that we represent as $r(\widetilde{\mathbf{z}}_\mathbf{i}, \widetilde{\mathbf{e}}_\mathbf{i}) = \mathbf{0}$. That is:

$$s_i = 1\{\mathbf{y}_i \in A \cup B \text{ and } r(\widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i) = \mathbf{0}\}$$

- There are many of these binary sufficient statistics. Let index them by $m \in \{1, 2, ..., M\}$. Then, the *conditional log-likelihood function* is:

$$\mathcal{L}(\boldsymbol{\theta}) = \sum_{m=1}^{M} \sum_{i=1}^{N} 1\{\mathbf{y}_i \in A^m \cup B^m\} \; 1\{r^m(\widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i) = \mathbf{0}\}$$

$$\log \left(\frac{\exp\{c^m(\mathbf{y}_i, \widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i)' \boldsymbol{\theta}\}}{\exp\{c^m(A^m, \widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i)' \boldsymbol{\theta}\} + \exp\{c^m(B^m, \widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i)' \boldsymbol{\theta}\}} \right)$$

CML ESTIMATION (2/2)

- Imposing exactly the restrictions on prices typically implies loosing a substantial amount of observations.

- To deal with this issue, we follow the Kernel weighting in Honore & Kyriazidou (2000)

- The Kernel Weighted conditional log-likelihood function is:

$$\mathcal{L}^{KW}(\boldsymbol{\theta}) = \sum_{m=1}^{M}\sum_{i=1}^{N} 1\{\mathbf{y}_i \in A^m \cup B^m\}\ K\left(\frac{r^m(\widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i)}{b_N}\right)$$

$$\log\left(\frac{\exp\{c^m(\mathbf{y}_i, \widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i)'\boldsymbol{\theta}\}}{\exp\{c^m(A^m, \widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i)'\boldsymbol{\theta}\} + \exp\{c^m(B^m, \widetilde{\mathbf{z}}_i, \widetilde{\mathbf{e}}_i)'\boldsymbol{\theta}\}}\right)$$

4. EMPIRICAL APPLICATION

DATA

- NIELSEN scanner data from Chicago-Kilts center.
- Period 2006-2019. Current estimates using only years 2017-2018.
- More than 40k participating households all over US.
- Rich demographics ($\mathbf{w}_i$): ZIP code, income, age, education, occupation, race, family size, family composition, type of residence,
- Data on every shopping trip.
- Product: Laundry detergent

ESTIMATION OF DEMAND PARAMETERS

Fixed Effects provide precise enough estimates ($N = 19,776$).

Estimates of Structural Parameters

Parameter	FE Kernel W. CML Estimate	FE Kernel W. CML (s.e.)	RE (2 types) + $\mathbf{w}_i'\alpha(j)$ Estimate	RE (2 types) + $\mathbf{w}_i'\alpha(j)$ (s.e.)
γ *Price*	1.7392	(**0.3018**)	1.155	(**0.1221**)
β^{sc}(*habits*) *Brand 1*	0.3804	(**0.0290**)	0.7551	(**0.0101**)
β^{sc}(*habits*) *Brand 2*	0.2556	(**0.0573**)	0.6695	(**0.0110**)
β^{sc}(*habits*) *Brand 3*	0.2388	(**0.0591**)	0.7360	(**0.0162**)
β^{dep}(*linear*) *Brand 1*	0.0597	(**0.0112**)	-0.0089	(**0.0040**)
β^{dep}(*linear*) *Brand 2*	0.0611	(**0.0118**)	-0.0161	(**0.0046**)
β^{dep}(*linear*) *Brand 3*	0.0692	(**0.0172**)	-0.0208	(**0.0072**)
Hausman test (p-value)	0.0000			

ESTIMATION OF DEMAND PARAMETERS

Hausman test clearly rejects the Random Effects model.

Estimates of Structural Parameters				
	FE Kernel W. CML		**RE (2 types) + $\mathbf{w}_i'\alpha(j)$**	
Parameter	*Estimate*	*(s.e.)*	*Estimate*	*(s.e.)*
γ *Price*	1.7392	(0.3018)	1.155	(0.1221)
$\beta^{sc}(habits)$ *Brand 1*	0.3804	(0.0290)	0.7551	(0.0101)
$\beta^{sc}(habits)$ *Brand 2*	0.2556	(0.0573)	0.6695	(0.0110)
$\beta^{sc}(habits)$ *Brand 3*	0.2388	(0.0591)	0.7360	(0.0162)
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$\beta^{dep}(linear)$ *Brand 3*	0.0692	(0.0172)	-0.0208	(0.0072)
Hausman test (p-val)	**0.0000**			

ESTIMATION OF STRUCTURAL PARAMETERS

Random Effects model over-estimates habits parameters.

Estimates of Structural Parameters

	FE Kernel W. CML		RE (2 types) + $\mathbf{w}_i'\alpha(j)$	
Parameter	*Estimate*	*(s.e.)*	*Estimate*	*(s.e.)*
γ *Price*	1.7392	(0.3018)	1.155	(0.1221)
$\beta^{sc}(habits)$ *Brand 1*	**0.3804**	(0.0290)	**0.7551**	(0.0101)
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ESTIMATION OF STRUCTURAL PARAMETERS

Random Effects model provides wrong sign for duration dependence.

Estimates of Structural Parameters				
	FE Kernel W. CML		**RE (2 types) + $\mathbf{w}_i'\alpha(j)$**	
Parameter	*Estimate*	*(s.e.)*	*Estimate*	*(s.e.)*
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Hausman test (p-value)	0.0000			

ESTIMATION OF DEMAND PARAMETERS

Random Effects model under-estimates price-sensitivity of demand.

Estimates of Structural Parameters

	FE Kernel W. CML		**RE (2 types) + $\mathbf{w}_i'\alpha(j)$**	
Parameter	*Estimate*	*(s.e.)*	*Estimate*	*(s.e.)*
γ *Price*	**1.7392**	(0.3018)	**1.155**	(0.1221)
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Hausman test (p-value)	0.0000			

CONCLUSIONS / EXTENSIONS

- This paper presents a Fixed Effects dynamic panel data model of demand for different products where consumers are forward looking.

- Some relevant extensions:

1. Identification of aggregate price elasticities (i.e., AME) following recent results.

2. Consumer purchases of multiple units (for inventory).

3. Dynamics from state variables other than ℓ_{it} and d_{it}.

4. Combining this dynamic demand model with dynamic model of price competition.