Dynamic Games: Problems and Prospects.

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Background: Static Empirical IO.

• Developed tools that enabled us to better analyze market outcomes conditional on the "state variables" of the problem.

• Common thread: incorporate the institutional background needed to make sense of the data used to analyzes market responses to environmental & policy changes.

• Focus was to incorporate

- **heterogeneity** (in plant productivity, products demanded, bidders and/or consumers) and,
- **equilibrium conditions** when we need to solve for variables that firms could change in response to the environmental change.

• Largely relied on earlier work by our game theory colleagues for the analytic frameworks.

In particular we most often assumed

• Each agent's actions affect all agents' payoffs, and at the "equilibrium" or "rest point"

- agents have correct perceptions, and
- the system is in some form of "Nash" equilibrium (policies such that no agent has an incentive to deviate).

• Our contribution was the development of an ability to adapt the analysis to the richness of different real world institutions.

To keep the model static we had to rule out circumstance where the static (q or p) choice has an independent effect on

- future costs [l.b.d., adjustment costs, networks,...]
- future demand [durable or experience goods, networks,...]
- future equilibrium choices [collusion, asymmetric information, ...]

• This also ruled out analyzing how the state variables might respond to changes in policy or environmental changes.

Dynamic analysis started analogously

• I.e. we took frameworks taken from our theory colleagues. that made assumptions which insured that the

1. state variables evolve as a Markov process
2. and the equilibrium is some form of Markov Perfection (no agent has an incentive to deviate at any value of the state variables).

Star and Ho (1969) and more directly Maskin and Tirole (1988) who use full information environments.

• We started as in static analysis, trying to adapt the dynamic framework to the richness of different real world institutions.

• Ericson and Pakes (1995) develop a framework for doing this.

- **States.**
  - $i \in \mathcal{Z}^+$ (could be multidimensional)
  - s_i will be the number of firms with efficiency level i,
  - $s = [s_i;\ i \in \mathcal{Z}^+]$ is the "industry structure" (a counting measure of the number of firms at each different efficiency level).
  - ζ aggregate state variable that evolves exogenously (perhaps as a Markov process). E.g. outside alternative.

- **Bellman Equation: "Capital Accumulation" Games**

$$V(i,s) = \max\{\phi, \pi(i,s) + \\ \sup_{(x \geq 0)} [-cx + \beta \sum V(i',s')pr(i',s'|x,i,s,\zeta)p(\zeta)]\}.$$

- $pr(i',s'|x,i,s,\zeta) = pr(i'|i,x,\zeta)q(s-e(i)|i,x,s,\zeta)$.

- $pr(i'|i,x,\zeta)$. This is a game where my own investment only affects my own state variables.

- $q(s-e(i)|i,x,s,\zeta)$ perception of where my competitors will be.

- $\mathcal{P} = \{p(i'|x,\zeta); x \in \mathcal{R}^+\}$, stochastically increasing in x for every ζ.
- $q[\cdot|i,s,\zeta]$ embodies the incumbent's beliefs about entry and exit.
- Many possible entry models; e.g. Must pay x_e $(> \beta\phi)$ to enter, and enters one period later at state $\omega_e \in \Omega^e \subset \mathcal{Z}^+$ with probability $p^e(\cdot)$. Only enters if the expected discounted value of future net cash flows from entering is greater than the cost of entry.

Dynamic Equilibrium.

1. Every agent chooses optimal policies given its perceptions on likely future industry structures
2. Those perceptions are consistent with the behavior of the agent's competitors.

Doraszelski and Satterwaite (2003) prove existence (to insure this we need random entry fees and exit costs), and E-P show that any equilibria

1. Is "computable", i; never more than $\overline{n}$ firms active & Only observe "i" on $\Omega = \{1, ..., K\} \Rightarrow$ need only compute equilibria for $(i, s) \in \Omega \times S$

$$S \equiv \{s = [s_1, ..., s_k] : \sum s_j \leq \overline{n} < \infty\} \;\Rightarrow\; \#S \leq K^N$$

2. Generates a homogeneous Markov chain for industry structures [for $\{s_t\}$], i.e. $Q(s'|s)$

$$Pr[s_{t+1} = s'|s^t] = Pr[s_{t+1} = s'|s_t] \equiv Q[s'|s_t].$$

3. And provide conditions on the primitives such that insure that any equilibrium $Q[\cdot|\cdot]$ is *ergodic*. [Picture].
   - R is frequently much smaller than S (and the divergence is greatest for large markets with many state variables).
   - In the limit the probabilities of being at the various points in R converges to an invariant measure. This invariant measure is often referred to as a "steady state" of the system, though "steady" seems to be a misnomer (as the state is not constant).

Brute Force Computation

Pakes and McGuire (1994, *RAND*). Important for understanding.

• The first algorithm we consider is a "backward solution" algorithm (the multiple agent analogue of what we do in single agent dynamic problems).

- In memory. Estimates of the value function and policies associated with each $(i, s) \in \Omega \times S$. Assume K and $\overline{n}$ known.
- Updating. *Synchronous*; i.e. it circles through the points in S in some fixed order and updates all estimates associated with *every* $s \in S$ at each iteration (here updating estimates at s involves updating estimates at each (i, s) that has $s_i > 0$).
- Convergence. The values and policies from successive iterations are the same. Converged policies and values satisfy all the properties of equilibrium values and policies (see below).

• Assume $i' = i + \nu - \zeta$ where $\mathcal{P}_v = \{P_\nu(\cdot|x)\}$, ζ exogenous.

Updating 1: Rewrite Bellman Equation.

$$V(i,s) = \max_{\chi \in \{0,1\}} \{[1-\chi]\phi + \qquad (1)$$

$$\chi\{\pi(i,s) - \sup_{x \geq 0}[-cx + \beta \sum_{\nu} w(\nu; i,s) p(\nu|x_1)]\}, \ \ \&$$

$$w(\nu;\ i,s) \equiv$$

$$\sum_{(\hat{s}_i',\zeta)} V(i+\nu-\zeta, \hat{s}_i' + e(i+\nu-\zeta)|w) q[\hat{s}_i'|i,s,\zeta]\mu(\zeta), \qquad (1a)$$

$$q[\hat{s}_i' = s_i^*|i,s,\zeta] \equiv Pr\{\hat{s}_i' = \hat{s}_i^*|i,s,\zeta, \text{equilibrium policies }\} \ \ (1b).$$

• $w(\nu; i,s)$ is the EDV of future net cash flow conditional on investment resulting in a particular value of ν, and the current state being (i,s) (it integrates out $\hat{s}_i'$, and ζ). It is all the agent needs to know, and generates a single agent problem.

Updating Rules.

- Calculate $w^{k-1}(\cdot|i,s)$ from the information in memory, i.e. from (x^{k-1}, V^{k-1}) (as in 1a),
- substitutes $w^{k-1}(\cdot)$ for $w(\cdot)$ in (1) and then solve the resultant *single agent* optimization problem for the j^{th} iteration's entry, exit and investment polices at (i,s).
- Incumbents solve for (χ^k, x^k) that

$$\max_{\chi\in\{0,1\}}\{[1-\chi]\phi + \chi \sup_{x\geq 0}[\pi(i,s) - cx + \beta \sum_{\nu} w^{k-1}(\nu;i,s)p(\nu|x)]$$

I.e. we solve the Kuhn-Tucker problem for investment conditional on continuing which if $\nu \in \{0,1\}, \zeta \in \{0,1\}$ is

$$\sum_{\nu} \frac{\partial p(v|x)}{\partial x} w(v;i,s) - c \leq 0,$$

with strict inequality iff $x = 0$. Then substitute the solution in the continuation value and determine whether it is greater than ϕ.

Potential entrants compute

$$V_e^k(s) = \beta \sum_{\zeta} w^{k-1}(\zeta; i_e, s + e(i_e))\mu(\zeta).$$

and set $\chi_e^k = 1 \Leftrightarrow V_e^k(s) > x_e$.

• Substitutes these policies and the w^{k-1} for the w, x and the max operator in (1), and labels the result $V^k(\cdot)$, and put $V^k(\cdot)$ in memory.

• Setting K. Start with the monopoly problem ($\overline{n} = 1$) and an oversized K; $\rightarrow$ a lowest i at which the monopolist remains active and a highest i at which the monopolist invests. $\rightarrow$ 1 and K in Ω.

• Setting $\overline{n}$. Set $\overline{n} = 2$ and do the iterative calculations again starting at $V^0(i_1, i_2) = V^*(i_1)$. Then set $\overline{n} = 3$ and set $V^0(i_1, i_2, i_3) = V^*(i_1, max(i_2, i_3))$. Continue until we reach an $\overline{n}$ that whenever $\overline{n} - 1$ firm's active there is never entry. This is $\overline{n}$.

Convergence & Equilibrium.

- At the end of the iteration calculate $||V^{k-1}(\cdot) - V^k(\cdot)||$ and $||x^{k-1}(\cdot) - x^k(\cdot)||$. If both are sufficiently small, stop. Else continue.

- **Equilibrium.** At fixed point each incumbent and potential entrant
  - uses, as its perceived distribution of the future states of its competitors, the actual distribution of future states of those competitors, and
  - chooses its policy to maximize its expected discounted value of future net cash flow given this distribution of the future of its competitors.

Computational Burden

The computational burden is (essentially) the product of three factors,

- the number of points evaluated at each iteration;
- the time per point evaluated;
- the number of iterations (value &/or policy function iterations).

Number of Points.

Since each of the $\overline{n}$ active firms can only be at K distinct states, the number of points we need to evaluate at each iteration, or

$$\#S \leq K^{\overline{n}}.$$

Exchangeability, of the value and the policy functions in the state variables of a firm's competitors implies that we do not need to differentiate between two vectors of competitors that are permutations of one another notation does not. Pakes (1993) shows that an upper bound for $\#S$ is given by the combinatoric

$$\binom{K+\overline{n}-1}{\overline{n}} << K^{\overline{n}}.$$

Burden per Point.

Determined by

- the cost of calculating the expected value of future states conditional on outcomes (of obtaining the $w^j(\cdot; i, s)$ from the information in memory).
- The cost of obtaining the optimal polices and the new value function given $w^j(\cdot; i, s)$.

• Think of this as individual firms playing against the rest. Assume that there is positive probability on each of κ points for each of the $m - 1$ active competitors of a given firm. Then we need to sum over κ^m possible future states and there are $\kappa \times m$ values of $w^j(\cdot)$ needed at that s. Average m should increase in $\overline{n}$, and κ should be determined by the nature of the state space per firm (it typically goes up exponentially in the number of state variables per firm).

Conclusion

It is clear that the computational burden of the model grows quickly in both the number of firms ever simultaneously active (it grows geometrically in this dimension) and the number of state variables per firm (it grows exponentially in this dimension). This is the problem known as "The Curse of Dimensionality" reappearing in games.

• Consequently pointwise algorithm has been used both; (i) as a tool for investigating theoretical issues where analytic solutions were not possible, and (ii) as a framework fo controlling for dynamic selection in static problems.

• Realistic empirical analysis of most applied problem could not use this. Two possible extensions

- Approximation techniques
- Use a different notion of equilibrium.

Approximation Techniques.

• A number are available. Each has their problems, but they compute equilibria with much less of a computational burden than the standard algorithm. I leave a discussion of them to others at the conference.

In order of appearance

- Deterministic approximation techniques. This starts in economics with the book by Judd(2004). It has now expanded with the use of various AI related tools.
- Stochastic Algorithm (Pakes and Mcguire, 2001)
- Continuous time algorithm. Doraszelski and Judd (2004).
- Oblivious equilibrium (mean field theory). Benkard, Van Roy, Weintraub, 2010. Approximate using moments of the distribution.

Alternative Equilibrium Concept

Premise: the complexity of Markov Perfection both

- limits our ability to do dynamic analysis of market outcomes, but also
- leads to a question of whether some other notion of equilibria will better approximate agents' behavior.

• What assumptions of MP might we relax? The initial frameworks made assumptions which insured that the

1. state variables evolve as a Markov process
2. and the equilibrium is some form of Markov Perfection (no agent has an incentive to deviate at any value of the state variables).

On the Markov Assumption.

Except in situations involving active experimentation to learn (where policies are transient), we are likely to stick with the assumption that states evolve as a time homogenous finite order Markov process. Reasons

- Convenience and fits the data well.
- Realism suggests information access and retention conditions limit the memory used.
- We can bound unilateral deviations (Ifrach and Weintraub, 2014), and have conditions which insure those deviations can be made arbitrarily small by letting the length of the kept history grow (White and Scherer, 1994).

On 2: Perfection.

The fact that Markov Perfect framework becomes unwieldily when confronted by the complexity of real world institutions, not only limits our ability to do empirical analysis of market dynamics

- it also raises the question of whether some other notion of equilibrium will better approximate agents' behavior.

The complexity issue implies that agents

- have access to and can retain a large amount of information (all state variables), and
- can either compute or learn an unrealistic number of strategies (one for each information set).

How demanding is this? Durable goods example.

Decrease the number of state variables by assuming agents only have access to a subset of the state variables.

- Since agents presumably know their own characteristics and these tend to be persistent, we would need to allow for asymmetric information: the "perfectness" notion would then lead us to a "Bayesian" Markov Perfect solution.

Is assuming "Bayesian MP" more realistic? It decreases the information access and retention conditions but increases the burden of computing the policies significantly. The additional burden results from the need to compute posteriors, as well as optimal policies; and the requirement that they be consistent with one another.

• I come back to whether one can learn the policies below, as the equilibrium concept I am going to use has a learning model embedded in it.

Abandon Perfection

Question. If we abandon Markov Perfection can we both

- better approximate agents' behavior and,
- enlarge the set of dynamic questions we are able to analyze.

• I start with strategies that are "rest points" to a dynamical system. This makes the job of defining a rest point much easier because

- strategies at the rest point likely satisfy a Nash condition of some sort; else someone has an incentive to deviate.
- However it still leaves opens the question: What is the form of the Nash Condition?

• There remains the question of how might firm's get to it, which I will turn to later.

What Conditions Can We Assume for the Rest Point at States that are Visited Repeatedly?

We expect (and I believe should integrate into our modelling) that

1. Agents perceive that they are doing the best they can at each of these points, and
2. These perceptions are at least consistent with what they observe.

Note. It might be reasonable to assume more than this: that agents (i) know and/or (ii) explore, properties of outcomes of states not visited repeatedly. I come back to this below.

Formalization of Assumptions.

• Denote the information set of firm i in period t by $J_{i,t}$. $J_{i,t}$ will contain both public (ξ_t) and private ($\omega_{i,t}$) information, so $J_{i,t} = \{\xi_t, \omega_{i,t}\}$.

• Assume $(J_{1,t}, \ldots J_{n_t,t})$ evolves as a finite state Markov process on $\mathcal{J}$ (or can be adequately approximated by one).

• Policies, say $m_{i,t} \in \mathcal{M}$, will be functions of $J_{i,t}$. For simplicity assume $\#\mathcal{M}$ is finite, and that it is a simple capital accumulation game (not necessary & not always appropriate), i.e. $\forall (m_i, m_{-i}) \in \mathcal{M}^n$, & $\forall\, \omega \in \Omega$

$$P_\omega(\cdot|m_i, m_{-i}, \omega) = P_\omega(\cdot|m_i, \omega),$$

• The public information, ξ, is used to predict competitor behavior and common demand and cost conditions (these evolve as an exogenous Markov process).

• A "state" of the system, is

$$s_t = \{J_{1,t}, \ldots, J_{n_t,t}\} \in \mathcal{S},$$

$\#\mathcal{S}$ is finite. $\Rightarrow$ any set of policies will insure that s_t will wander into a recurrent subset of $\mathcal{S}$, say $\mathcal{R} \subset \mathcal{S}$, in finite time, and after that $s_{t+\tau} \in \mathcal{R}$ w.p.1 forever.

• Note that the agents need not keep track of all of s_t, only $J_{i,t}$; i.e. $J_{i,t}$ is whatever management conditions on when forming dynamic policies.

• Let the agent's perception of the expected discounted value of current and future net cash flow were it to chose m at state J_i, be

$$W(m|J_i), \;\; \forall m \in \mathcal{M} \;\; \& \;\; \forall J_i \in \mathcal{J},$$

• and of expected profits be $\pi^E(m|J_i)$.

Our assumptions imply:

- Each agent choses an action which maximizes its perception of its expected discounted value, and
- For those states that are visited repeatedly (i.e. are in the recurrent class or $\mathcal{R}$), these perceptions are consistent with observed outcomes.

Formally

$$\textbf{A}. \quad W(m^*|J_i) \geq W(m|J_i), \;\; \forall m \in \mathcal{M} \;\; \& \;\; \forall J_i \in \mathcal{J},$$

$$\textbf{B}. \;\; \&, \; \forall J_i \;\; \text{which is a component of an} \;\; s \in \mathcal{R}$$

$$W(m(J_i)|J_i) = \pi^E(m|J_i) + \beta \sum_{J_i'} W(m^*(J_i')|J_i')p^e(J_i'|J_i),$$

where, if $p^e(\cdot)$ provides the empirical probability (the fraction of periods the event occurs)

$$\pi^E(m|J_i) \equiv \sum_{J_{-i}} E[\pi(\cdot)|J_i, J_{-i}]p^e(J_{-i}|J_i),$$

and

$$\left\{ p^e(J_{-i}|J_i) \equiv \frac{p^e(J_{-i}, J_i)}{p^e(J_i)} \right\}_{J_{-i}, J_i},$$

while

$$\left\{ p^e(J_i'|J_i) \equiv \frac{p^e(J_i', J_i)}{p^e(J_i)} \right\}_{J_i', J_i}. \;\; \spadesuit$$

"Experience Based Equilibrium"

• These are the conditions of a (restricted) EBE (Fershtman and Pakes, 2012). For earlier work with similar ideas in anonymous games with repeated interactions (agents draw randomly from an infinite number of competitors) see Fudenberg and Levine, 1993 ,on self confirming equilibria.

• Bayesian Perfect satisfy them, but so do weaker notions.

• We now turn to its : computation (which introduces an algorithm which enables firms to learn optimal. policies), overcoming multiplicity issues, and then to an empirical example.

• The empirical example will move to continuous state space and continuous controls, and introduce an alternative computational algorithm with different costs and benefits
(it does not have a learning interpretation, but is typically much faster to compute as it does not require discreetizing the state space).

Computational Algorithm.

• Asynchronous reinforcement learning algorithm (first used for dynamic games in Pakes and McGuire, 2001), earlier machine learning literature for single agent problems calls this "Q-learning".

• The fact that it is based on learning from realized data makes it a candidate to analyze perturbations to the environment (provided they are not large enough to induce experimentation), as well as to compute equilibrium to dynamic games.

• If there is more than one equilibria, the learning algorithm will pick one out. If algorithm is run many times from the same initial conditions, it will pick out a distribution of equilibria.

• Formally it circumvents the two sources of the curse of dimensionality in computing equilibrium.

Iterations

• The computation problem is now different for management then for the researcher as management just conditions on $J_{i,t}$ but the researcher must compute equilibria for $s_t = (J_{1,t} \dots J_{n_t})$. Here we deal with the computational problem for researchers. We come back to the firm's problem below.

• **Iterations are defined by**

- A location, say $L^k = (J_1^k, \dots J_{n(k)}^k) \in \mathcal{S}$: is the information sets of active agents .
- Objects in memory (i.e. M^k):
  (i) perceived evaluations, W^k,
  (ii) No. of visits to each point, h^k.

Must update (L^k, W^k, h^k)**.** Computational burden determined by; memory constraint, and compute time. I use a simple (not neccesarily optimal) structure to memory.

Update Location.

- Calculate "greedy" policies (policies are now "m") for each agent

$$m^*_{i,k} = arg \max_{m \in \mathcal{M}} W^k(m|J_{i,k})$$

- Take random draws on outcomes conditional on $m^*_{i,k}$:

- E.g.; if we invest in "payoff relevant" $\omega_{i,k} \in J_{i,k}$, draw $\omega_{i,k+1}$ conditional on $(\omega_{i,k}, m^*_{i,k})$.

- Use outcomes to update $L^k \rightarrow L^{k+1}$.

Update W^k.

• "Learning" interpretation: Simple case: assume agent observes the competitors' static controls m_{-i} (usually a price or bid) and initially we will assume the agent knows (perhaps through estimation) the primitives; $\pi_i(\cdot), p(\omega_{i,t+1}|\omega_{i,t}, m_{i,t})$. Can generalize and allow them not to be known.

• Its ex poste perception of what its value would have been had it chosen m is

$$V^{k+1}(J_{i,k}, m) =$$

$$\pi(\omega_{i,k}, m, m_{-i,k}, d_k) + \max_{\tilde{m} \in M} \beta W^k(\tilde{m}|J_{i,k+1}(m)),$$

where $J_i^{k+1}(m)$ is what the $k+1$ information would have been given m and *competitors actual play*.

Treat $V^{k+1}(J_{i,k})$ as a random draw from the possible realizations of $W(m|J_{i,k})$, and update W^k as in stochastic integration (Robbins and Monroe,1956)

$$W^{k+1}(m|J_{i,k}) =$$

$$\frac{1}{h^k(J_{i,k})} V^{k+1}(J_{i,k}, m) + \frac{(h^k(J_{i,k}) - 1)}{h^k(J_{i,k})} W^k(m|J_{i,k}),$$

or

$$W^{k+1}(m|J_{i,k}) - W^k(m|J_{i,k}) =$$

$$\frac{1}{h^k(J_{i,k})}[V^{k+1}(J_{i,k}, m) - W^k(m|J_{i,k})].$$

(other weights are more efficient as the early estimates of $V^{k+1}(J_{i,k}, m)$ are noisier than the later estimates, and it would be good to know how to use information on close states to update a given state ....)

Notes.

• If we have equilibrium valuations we tend to stay their, i.e. if * designates equilibrium

$$E[V^*(J_i, m^*)|W^*] = W^*(m^*|J_i).$$

So if it reaches equilibrium it tends to stay at equilibrium.

• As in all computational algorithms for dynamic games there is no guarantee that it will converge, but the learning interpretation provides a rational for using it regardless.

• In fact, the smoothing implicit in the draws tends to mitigate the convergence problem in pointwise algorithms, but it may take many iterations.

• Agents (not only the analyst) could use the algorithm to find equilibrium policies or adjust to perturbations in the environment.

- Formally algorithm has no curse of dimensionality.
  1. Computing continuation values: integration is replaced by averaging two numbers.
  2. algorithm eventually wanders into $\mathcal{R}$ and stays their, and $\#\mathcal{R} \leq \#\mathcal{J}$.

• Fershtman and Pakes (2012) also provide a test for equilibrium that has no curse of dimensionality.

• Still the number of states can grow large (typically grows linearly in the number of state variables).

• The stochastic approximation literature for single agent problems often augments this with functional form approximations (“TD learning” or Sutton and Barto,1998).

Multiplicity of REBE.

• Recall that $\mathcal{R}$ is the recurrent class (the points that are visited repeatedly).

• $\mathcal{R}$ contains both "interior" and "boundary" points. Points at which there are feasible strategies which can lead outside of $\mathcal{R}$ are boundary points. Interior points are points that can only transit to other points in $\mathcal{R}$ no matter which (feasible) policy is chosen.

• Our conditions only insure that perceptions of outcomes are consistent with the results from actual play at interior points. Perceptions of outcomes for some feasible (but in-optimal) policy at boundary points are not tied down by actual outcomes.

• "MPBE" are a special case of (restricted) EBE and they have multiplicity. Here differing perceptions at boundary points can support a (possibly much) wider range of equilibria.

Narrowing the Set of Equilibria.

• In any empirical appliction the data will rule out equilibria. m^* is observable, at least for states in $\mathcal{R}$, and this implies inequalities on $W(m|\cdot)$. With enough data $W(m^*|\cdot)$ will also be observable up to a mean zero error.

• Use external information to constrain perceptions of the value of outcomes outside of $\mathcal{R}$. If available use it.

• Asker, Fershtman, Jihye, and Pakes, (2020, *RAND*), allow firms to experiment with $m_i \neq m_i^*$ at boundary points. Leads to a stronger notion of, and test for, equilibrium (Boundary Consistency).

• Boundary consistency insures that perceptions are consistent with the results from **actual play** for each **feasible** action at boundary points (and hence on $\mathcal{R}$) if one played the strategies in memory.

Boundary Consistency.

• Let $B(J_i|\mathcal{W})$ be the set of actions at $J_i \in s \in \mathcal{R}$ which could generate outcomes which are not in the recurrent class (if it is not empty, J_i is a boundary point) and $B(\mathcal{W}) = \cup_{J_i \in \mathcal{R}} B(J_i|\mathcal{W})$. Then the extra condition needed to insure "Boundary Consistency" is:

• **Extra Condition.** Let τ index future periods, then $\forall (m, J_i) \in B(\mathcal{W})$

$$W(m^*|J_i) \geq$$

$$E\left[\sum_{\tau=0}^{\infty} \delta^{\tau} \pi\Big(m(J_{i,\tau}), m(J_{-i,\tau})\Big)\Big| J_i = J_{i,0}, \mathcal{W}\right],$$

where $E[\cdot|J_i, \mathcal{W}]$ takes expectations over future states starting at J_i using the policies generated by $\mathcal{W}$. ♠

Letting the data guide us.

• Need a candidate for J_i (the firm's information). Use any exogenous information plus data: i.e. an empirical analysis of the observable determinants of the dynamic controls (investment, exit,...).

• This should pick up the variables that both we (the researchers) observe and that management pays attention to when making its decision.

• Empirical question: are there variables that the economist does not observe and are serially correlated that impact managements decisions?

- If not we can use CCP techniques discussed in the course by Bob Miller, or Bajari Benkard and Levin (2007), Pakes Ostrovsky and Berry (2007).
- If there are we need to introduce method that allow for the impact of unobserved serially correlated state variables.

E.g.: Pharma Advertising In Dynamic Equilibrium

Background for Dubois Pakes.

- Pharma advertising is of two types
  - detailing,
  - DTC.

Only two developed countries allow DTC for perscription pharma (U.S. & New Zealand), and other countries place more restrictions on detailing.

- The argument for and against advertising.
  - *Against*; incentives for miss-information & returns largely a result of business stealing and so do not generate benefits to society.
  - *For*; make consumers aware that they can treat a condition before it becomes serious (particularly those that do not regularly see doctors), and providers aware of treatment alternatives.

- Question: are there more socially efficient ways to organize Pharma advertising?

Steps In The Analysis.

1. Estimate a demand system (we used BLP).
2. Recover the "quality" (or ξ) term that generates different market shares conditional on price.
3. Estimate a controlled Markov process for ξ;
   $\xi_{j,t} = \rho_\xi \xi_{j,t-1} + \beta_a \{a_{j,t} > 0\} log[a_{j,t-1}] + \mu_{j,t}$.
4. Estimate an equation for the determinants of expenditures on each type of advertising. Allow the residual for each equation to be serially correlated. Their values becomes unobserved state variables.
5. Given the state variables from 4, and estimates of parameters which are invariant to institutions, provide an algorithm for computing equilibria with continuous controls and states.
6. Compare computed policies to the data, and to the policies that would be generated by counterfactual institutions.

- Steps 1 to 3 standard. So I start with 4.

Advertising Model.

• $W(a|\xi_{i,t}, J_{i,t})$: the perceived EDV of returns conditional on $a \equiv (a_d, a_D)$

$$W(a|\xi_{i,t}, J_{i,t}) \equiv \mathcal{E}\left[\sum_{\tau=1}^{\infty} \beta^{\tau} \pi(\cdot)_{t+\tau} \Big| \xi_{i,t}, J_{i,t}\right],$$

where

- $\mathcal{E}(\cdot|\cdot)$ is the agent's expectations operator,
- $(J_{i,t}, \xi_{i,t})$ are the set of variables the firm conditions on when making its advertising decisions.

• Assuming the firm knows the demand system, production costs, and the process generating ξ, the goal of empirical work is to determine the

- observables in $J_{i,t}$, and
- costs and the process generating the impact of unobservables.

- Management maximizes. So the marginal return to advertising is

$$\mathcal{E}\Big[\sum_{\tau=1}^{\infty}(\beta\rho)^{\tau}\frac{\partial\pi(\cdot)_{t+\tau}}{\partial a_{h,t}}|J_{i,t},\xi_t\Big]$$

and we approximate with

$$\approx \theta_{0,h}\left(\left[\frac{\partial\pi(\cdot)_{t-1}}{\partial\xi_{t-1}}\right]\frac{\beta_{a,h}}{a_h}\right)^{\theta_{1,h}} exp[w_{h,t-1}\beta_{w,h}+\omega_{h,t}],$$

where $\omega_{h,t}\in J_t$ gives the impact of variables not in our data and

$$\omega_{h,t}=\rho_{\omega,h}\omega_{h,t-1}+\nu_{h,t},\ \ \text{for } h\in\{d,D\}$$

$$E[(\nu_{d,t},\nu_{D,t})'|J_t,\xi_t]=0 \text{ and } Var[\nu_{d,t},\nu_{D,t}]'=\Sigma_\nu.$$

- Notice that though the impact of competitors' advertising policy is partly captured by $\pi(\cdot)$, it may also directly enter the variables in w.

Accounting for Two Properties of the Data.

1. We need estimates of

$$\frac{\partial \pi(\cdot)_{t-1}}{\partial \xi_{t-1}} = \frac{\partial D(\cdot)_{t-1}}{\partial \xi_{t-1}}(p_{t-1} - c)$$

where

- p is list prices which are known to us, and
- c is *marginal cost minus rebates* & is not known.

• We estimate c from the advertising equation & compare the estimate to those from the f.o.c. from a static Nash pricing assumption.

2. Drugs often advertised in some but not all periods (Dube et. al 2022, advertising "pulsates"). Need model for zeroes to avoid selection bias.

Market	N	$a_{dt} > 0$	$a_{Dt} > 0$	$a_{Dt} > 0$ $\& a_{Dt-1} > 0$	$a_{dt} > 0$ $\& a_{dt-1} > 0$	$a_{Dt} > 0$ $\& a_{Dt-1} > 0$ $\& a_{dt-1} > 0$	$a_{dt} > 0$ $\& a_{dt-1} > 0$ $\& a_{Dt-1} > 0$
Antiasthma	1733	0.552	0.095	0.089	0.516	0.089	0.096
Antiulcer	1602	0.448	0.052	0.045	0.392	0.045	0.052
Anticholesterol	1088	0.492	0.142	0.125	0.449	0.117	0.131
Antidepressants	1592	0.417	0.072	0.067	0.367	0.062	0.065

• Introduce a fixed cost, $\{u_h \sim Exp(f_h)\}_{h\in\{d,D\}}$, & if the increment in the expected returns from advertising is less than u_h the firm does not advertise.

This implies

$$(i) \quad a_{h,t} = 0 \quad \Rightarrow \quad Pr\left\{ u_{h,t} \geq \theta_{0,h} + \theta_{1,h} log[\frac{\partial \pi(\cdot)_{t-1}}{\partial \xi_{t-1}}] + w_{t-1}\beta_{w,h} + \omega_{h,t} \right\};$$

$$(ii) \quad a_{h,t} > 0 \quad \Rightarrow \quad log[a_{h,t}] = \theta_{0,h} + \theta_{1,h} log[\frac{\partial \pi(\cdot)_{t-1}}{\partial \xi_{t-1}}] + w_{t-1}\beta_{w,h} + \omega_{h,t}$$

Recall that $a_{h,t} = 0 \Rightarrow \omega_{h,t} = \rho_{\omega,h}\omega_{h,t-1} + \nu_t$, so if

$$\underline{\nu}_{h,t} \equiv \theta_{0,h} + \theta_{1,h} log[\frac{\partial \pi(\cdot)_{t-1}}{\partial \xi_{t-1}}] + w_{t-1}\beta_{w,h} + \rho_{\omega,h}\omega_{h,t-1},$$

the rhs of these two equations is $\underline{\nu}_{h,t} + \nu_{h,t}$, so we have

$$E[\{a_{h,t} = 0\}|J_t] = Pr\left\{u_h \geq \nu_{h,t} + \underline{\nu}_{h,t}\right\} = \int_{u_h} F_{\nu_h}(u_h - \underline{\nu}_{h,t}) dF_{u_h} \equiv P_h(\underline{\nu}_{h,t})$$

where $(F_{\nu_h}(\cdot), F_{u_h}(\cdot))$ are distributions for (ν_h, u_h).

Compute $E[a_{h,t}|a_{h,t} > 0, J_{i,t}, \xi_{i,t}]$.

- Non-parametric $(F_\nu(\cdot), F_u(\cdot))$, $\underline{\nu}_{h,t} = P_h^{-1}(P_{h,t})$. Quasi first-differencing

$$log[a_{h,j,t}] - \rho_h log[a_{h,j,t-1}] = \theta_{0,h}(1-\rho_h)$$

$$+\theta_{1,h}\left[log\left(\frac{\partial D(\cdot)_{t-1}}{\partial \xi_{t-1}}(p_{t-1,j}-c_j)\right) - \rho_{\omega,h} log\left(\frac{\partial D(\cdot)_{t-2}}{\partial \xi_{t-2}}(p_{t-2,j}-c_j)\right)\right]$$

$$+\beta_{w,h}\left(w_{j,t-1} - \rho_{\omega,h} w_{j,t-2}\right) + M(P_{h,t,j}) + e_{h,j,t},$$

where $e_{h,t} = \nu_{h,t} - E[\nu_{h,t}|a_{h,t} > 0, J_t, \xi_t]$.

- Parameter heterogeneity
  - the parameters $\{\rho_{\omega,h}, \theta_{0,h}, \theta_{1,h}, \beta_{w,h}\}_{h\in\{d,D\}}$ differ across markets.
  - $\{c_j\}_j$ differs both across drugs in a given market, and since it includes rebates, between the branded and generic versions of the drug.

Estimation: determine variables in w_h, & parameters of costs & ω_h process (needed for equilibrium computation).

• Nested algorithm: for each value of $\{(\rho_{\omega,h}, \theta_{0,h}, \theta_{1,h}, \beta_{w,h})\}_{h\in\{d,D\}}$ and $M(\cdot)$, we concentrate out the $\{c_j\}_{j\in J}$ by finding the value that solves

$$0 = \sum_t \Big(log[a_{h,t}] - \rho_{\omega,h} log[a_{h,t-1}] - [\theta_{0,h}(1-\rho_{\omega,h}) \Big)$$

$$-\sum_t \left[\theta_{1,h} log\left(\frac{\partial D(\cdot)_t}{\partial \xi_t}(p_t - c)\right) - \rho_{\omega,h}\theta_{1,h} log\left(\frac{\partial D(\cdot)_{t-1}}{\partial \xi_{t-1}}(p_{t-1} - c)\right) + \beta_{w,h}\left(w_t - \rho_{\omega,h} w_{t-1}\right) + M(P_{h,t})\right].$$

• Since $e_{h,t}$ may be correlated with the right hand side variables, we tried estimating the outer loop estimated with and without instruments (prior period observables), and we needed the instruments.

• Tried several functional forms for $M(\cdot)$. A linear function of the log odds ratio (i.e. $P/(1-P)$) performed best[1].

[1] We We also tried to estimate the discount factor but the objective was flat about the 95% annual value we are using.

Marginal Costs.

Table: Mean Prices, Marginal Costs, and Markups by Market.

marketgeneric			Mean		
	Price	Marginal cost	Margin	Marginal cost	Margin
Antiasthma: Branded	2.01	0.22	0.86	1.49	0.36
Antiasthma: Generic	0.29	0.12	0.56	0.13	0.60
Anticholesterol: Branded	2.70	0.19	0.92	1.62	0.42
Anticholesterol: Generic	1.17	0.12	0.90	0.78	0.48
Antidepressants: Branded	5.01	0.35	0.92	3.52	0.34
Antidepressants: Generic	0.43	0.12	0.74	0.12	0.75
Antiulcer: Branded	2.99	0.00	1.00	2.17	0.33
Antiulcer: Generic	1.18	0.00	1.00	0.72	0.54

Columns 2 and 3 provide estimates from the advertising equation.
Columns 4 and 5 provide them from the static f.o.c.s.
Costs were constrained to be non-negative.

Summary of Empirical Results.

- Cost Estimates (from advertising equation vs from static Nash pricing).
  - Costs from the static analysis are noticeably higher.
  - Advertising Equation: branded markups between 86 and 100%. Highest branded markup from static f.o.c. was 42%.
  - Advertising Equation: markups that are higher for branded drugs. Static f.o.c: markups are higher for generic drugs.

- *Variables needed for policy function*
  - Observables: derivative of profits w.r.t. advertising; advertising of competitors, time to loss of exclusivity.
  - Highly significant positive serial correlation in seven of the eight equations. Need serial correlated unobserved state variable.
  - Parameters differ quite a bit across markets.

Table: GMM estimates of Detailing and DTC equation.

	Antiasthma	Anticholesterol	Antiulcer	Antidepressants
	GMM	GMM	GMM	GMM
	b/se	b/se	b/se	b/se
rho				
Constant	0.842***	0.597***	1.000***	0.625***
	(0.019)	(0.035)	(0.122)	(0.040)
theta1d				
Constant	1.158***	0.791***	0.355	1.258***
	(0.104)	(0.054)	(1.527)	(0.144)
betad				
Constant	-0.217	-0.485***	-0.555**	-0.508***
	(0.115)	(0.098)	(0.191)	(0.139)
betad2				
Constant	0.036***	0.041***	0.037	0.076***
	(0.010)	(0.009)	(0.145)	(0.009)
Md				
Constant	1.256***	0.273***	0.666**	0.542***
	(0.132)	(0.052)	(0.207)	(0.061)
rhoDTC				
Constant	0.015	0.157**	0.641***	1.000***
	(0.066)	(0.057)	(0.103)	(0.015)
theta1DTC				
Constant	0.147	0.453***	1.306	2.651
	(0.081)	(0.090)	(1.340)	(2.545)
betaD				
Constant	-0.140	-0.240	-1.802*	-0.574*
	(0.205)	(0.134)	(0.734)	(0.292)
betaD2				
Constant	-0.042***	0.054***	0.167	-1.020
	(0.006)	(0.013)	(0.104)	(0.919)
MD				
Constant	0.017***	0.400***	0.002*	0.720***

Recovering Estimates of Variances of ν and u.

• Recall that for $a_h = 0$ we require $u_h > \nu_h - \underline{\nu}_h$ where

$$\underline{\nu}_{h,t} \equiv \theta_{0,h} + \theta_{1,h} log[\frac{\partial \pi(\cdot)_{t-1}}{\partial \xi_{t-1}}] + w_{t-1}\beta_{w,h} + \rho_{\omega,h}\omega_{h,t-1}.$$

Assume $\nu_h \sim \mathcal{N}(0, \sigma^2_{\nu,h})$ and $u_h \sim exp[f]$, so $Var(u) = 1/f^2$. Then

$$P_h(\hat{\underline{\nu}}_{h,t}) = \int_\nu exp[-f_h \nu_{h,t} - f_h \hat{\underline{\nu}}_{h,t}] d\mathcal{N}_{\nu,h} = exp[-f_h \hat{\underline{\nu}}_{h,t} + f_h^2 \sigma^2_{\nu,h}/2],$$

where we have substituted $\hat{\underline{\nu}}_{h,t}$ for $\underline{\nu}_{h,t}$. Use a minimum χ^2 estimator.

• *Results.* Magnitude of variances similar across markets and between the two types of expenditures. Detailing preciesely estimated, DTC less so.

• Fixed cost variance larger in magnitude that the variance of the increment in ξ.

Table: Min $\sum_{j,t} \frac{\left(\hat{P}_{j,t}(\cdot) - exp[-f\underline{v} + f^2\sigma_\nu^2/2]\right)^2}{\hat{P}_{j,t}(\cdot)[1-\hat{P}_{j,t}(\cdot)] + var(\hat{P}_{j,t})}$ - Detailing

Market	f	(Std. Err.)	$1/f^2$	(Std. Err.)	σ_ν^2	(Std. Err.)
Antiasthma	.547	(.0082)	3.333	(.0998)	2.893	(.0177)
Antiulcer	.659	(.0138)	2.297	(.0964)	3.381	(.0217)
Anticholesterol	.866	(.0233)	1.332	(.0717)	3.849	(.0411)
Antidepressants	.43	(.0133)	5.399	(.3358)	2.589	(.0763)

Note: * for $p < .05$, ** for $p < .01$, and *** for $p < .001$.

Table: Min $\sum_{j,t} \frac{\left(\hat{P}_{j,t}(\cdot) - exp[-f\underline{v} + f^2\sigma_\nu^2/2]\right)^2}{\hat{P}_{j,t}(\cdot)[1-\hat{P}_{j,t}(\cdot)] + var(\hat{P}_{j,t})}$ - DTC

Market	f	(Std. Err.)	$1/f^2$	(Std. Err.)	σ_ν^2	(Std. Err.)
Antiasthma	.385	(.0083)	6.728	(.292)	3.903	(.0461)
Antiulcer	.625	(.208)	2.558	(1.7022)	1.238	(1.1409)
Anticholesterol	.644	(.0204)	2.409	(.1531)	4.186	(.0619)
Antidepressants	.37	(.0209)	7.291	(.823)	6.086	(.15)

Algorithm for Computing Equilibria.

- Take demand, costs, the processes generating ξ, $\{u_h \ \& \ \omega_h\}_{h \in \{d.D\}}$, and the observable variables the firms condition on, from above.
- Use these in an iterative algorithm to compute policy functions that satisfy the conditions of an experience based equilibrium for the current institutions, and then for counterfactual institutions.
- Both the states and the policy functions are continuous. Convergence occurs when the policies from adjacent iterations coincide.

Experience Based Equilibrium:

1. firm choses policies that maximize their perceptions of the EDV of future net cash flows conditional on the variables they use to determine their advertising expenditures, and
2. perceptions are consistent with outcomes at states that are visited repeatedly by the Markov process generated by the policies.

• Iterations are indexed by l (associated with policy functions), and we drop the firm index (j).

• Initial estimate of the policy function is the function estimated above.

• Iteration l. Simulate K sample paths for $\pi(\cdot)$ and $\{\omega_h\}$ for each firm

• Then compute their averages, $\{\pi^l(\cdot)_t\}_t$ and $\{\omega^l_{h,t}\}$.

• Use these averages to compute an estimate of the marginal returns to a_h as

$$\sum_{\tau=t}^{\infty}(\beta\rho)^{\tau}\frac{\partial\pi^l(\cdot)_{t+\tau}}{\partial\xi_{t+1}}\frac{\beta_{a,h}}{a_{h,t}} \approx \mathcal{E}\Big[\sum_{\tau=t}^{\infty}(\beta\rho)^{\tau}\frac{\partial\pi(\cdot)_{t+\tau}}{\partial\xi_{t+1}}\frac{\beta_{a,h}}{a_{h,t}}\Big)|J_t,\xi_t\Big].$$

• Currently we are simulating ξ from its initial value, as we want to compare it to the simulated ξ from our counterfactual. If we were just concerned with fit we would use the ξ from each period to see if our estimates of future returns fit well. We have not done the latter yet.

- Our iteration l estimate of $a^l_{h,t}$ is obtained as

$$a^l_{h,t} = \mathbb{P}[a^l_{h,t} = 0] + [1 - \mathbb{P}(a^l_{h,t} = 0)]a^{*,l}_{t,h},$$

where

$$\mathbb{P}[a^l_{h,t} = 0] = \exp(-f \cdot (\theta^l_{0,h} + \theta^l_{1,h} \log(\frac{\partial \widehat{\pi(\xi', J')}}{\partial \xi'}|_{\xi^{lk}_t}) + w_t \beta^l_{w,h} + \omega^l_{h,t}))$$

and

$$a^{*,l}_{t,h} = \sum_{\tau=t}^{\infty} (\beta\rho)^\tau \frac{\partial \pi^l(\cdot)_{t+\tau}}{\partial \xi_{t+1}} \beta_{a,h}.$$

- Update the policy function. Regress the dependent variable

$$log\left[\sum_{\tau=t}^{\infty}(\beta\rho)^{\tau}\frac{\partial\pi^{l=1}(\cdot)_{t+\tau}}{\partial\xi_{t+1}}\beta_{a,h}\right]-\omega_{h,t}^{l}\ \ (\equiv \log(a_{h,t}^{l*})-\omega_{h,j,t}^{l})$$

on

$$\approx \theta_{0,h}^{l+1}+\theta_{1,h}^{l+1}log\left(\frac{\partial\pi^{l}(\xi',J')}{\partial\xi'}\right)+w_t\beta_{w,h}^{l+1}$$

- Finally compute

$$Y^{l}\equiv\frac{1}{\sum_{j,t}1}\sum_{j,t}\left(\frac{W_{j,t}^{l}-W_{j,t}^{l-1}}{W_{j,t}^{l-1}}\right)^{2}.$$

- If $Y^{l}<10^{-5}$ stop. If $Y^{l}>10^{-5}$ proceed to iteration $l+1$.

Some details

- Use the first period of advertising to initiate the $\{\omega_h\}$ processes (eliminates the need to assume an initial condition).

- Two possibilities for terminal period's contribution:
  1. Simulate model out a hundred periods. This does not account for perceptions of differences between the future and the past.
  2. Use the last observed advertising, say T_j, to approximate for $\sum_{\tau=T_j}^{\infty}(\beta\rho)^{\tau}\frac{\partial\pi(\cdot)_{t+\tau}}{\partial\xi_{t+1}}\beta_{a,h}$. (eliminates the need to assume something for the firm's perceptions about $\pi(\cdot)$ after the sample ends).

Use of data

- makes for a smaller computational burden and does not require stationarity of the future but,
- it does generate an approximation error. The approximation error is; period specific, & mean zero given everything known up to the period.

Results (for Cholesterol only).

- Preliminaries.
  - Convergence took between three and six iterations. We are using the same random draws in each iteration, so convergence is only required for the policy function parameters.
  - Standard errors do not account for first stage variance in parameters.
  - higher for DTC than detailing (2.4 vs 1.3; and 4.2 vs 3.8)
  - Use of data for post-sample perceptions does better in final periods.
- Table
  - All three variables matter (though incorrect standard errors).
  - Profit derivative and time to loe always positive.
  - Other advertising differs in sign in the two markets

Fit Comparisons for Cholesterol

- We have yet to do this correctly.
  - We are using ξ_0 and simulating $\{\xi_t\}_{t=1}$. This is because we want to compare to an equilibrium without DTC and use the same procedure for both. For fit we would use the data's $\{\xi_t\}_t$ and compare estimated perceptions to those implied by the data
  - Now we are assuming initial $\omega = 0$. For fit to actual data could first period of advertising to initiate the $\{\omega_h\}$ processes.

Also

- The data is a random draw and we predict averages.
- The graphs compare for the non-zero actual data values, against the simulated averages. Simulations will average over zeros, so we expect the simulations to be a bit lower (more so for DTC).

Fit Results.

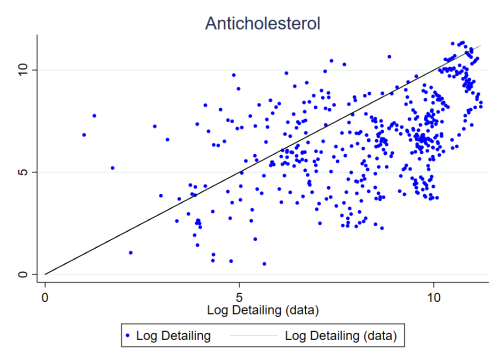
- Hard to evaluate given how we are doing this.

- Better for detailing than DTC.

- This was expected since both the variances in the u and in ν are higher for DTC than detailing (2.4 vs 1.3; and 4.2 vs 3.8).

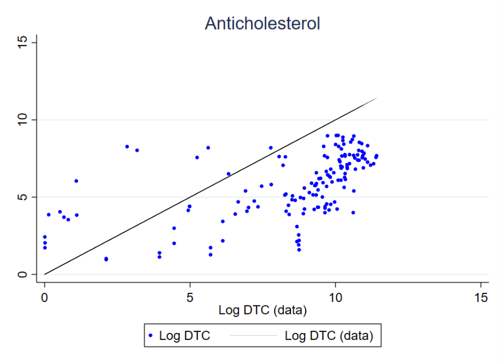
- The estimates that use the OOS simulation and those that do not only differ in the final few quarters where the one's that use the advertising approximation for OOS do a bit better.

Figure: Simulated against observed values (without zeros): Anticholesterol



Note: Simulated log Detailing against observed log Detailing on horizontal axis.

Figure: Simulated against observed values (without zeros): Anticholesterol



Note: Simulated log DTC against observed log DTC on horizontal axis.

Time Series for Three Drugs with the Largest Share.

- Detailing is predicted well, noticeable under-prediction of DTC. We are using averages, and the data is a random draw, so convexity of the profit and advertising functions will generate one difference (particularly for DTC).
- Hard to tell the difference between the two treatments of the value at the end of sample, but a bit of an advantage for using end of sample advertising (the red).
- Notice that advertising goes down at the end of the period, as these drugs are nearing the end of their patent lives.

Figure: LIPITOR

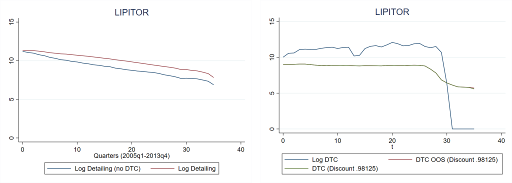
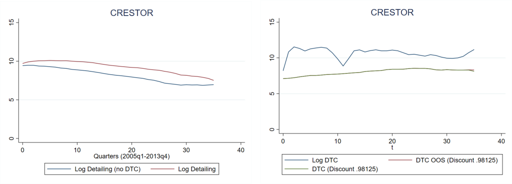
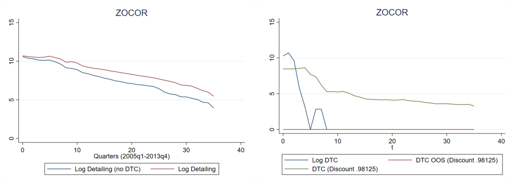


Figure: CRESTOR



Note: Simulated values with and without OOS. Quarters on horizontal axis (starting 2005q1).

Figure: Simulated Detailing and DTC against observed values: ZOCOR

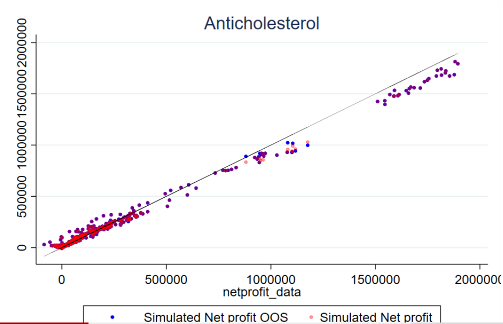


Note: Simulated values with and without OOS. Quarters on horizontal axis (starting 2005q1).

Net Profit Predictions.

• Profits are larger in magnitude than advertising, and they change only to the extent that the advertising changes quantity sold.
• From the table with business stealing effects, we know that large changes in advertising can lead to huge changes in revenue, but with good fits we expect small differences in profits

Figure: Simulated Net Profit against observed: Anticholesterol



Eliminating DTC: Two Comparisons.

1. Compare equilibrium outcomes when DTC is allowed to equilibrium outcomes when it is not allowed.
2. Compare outcomes to the outcomes when only the given firm stops DTC and the rest of its competitors continue with DTC to examine the relative importance of business stealing.

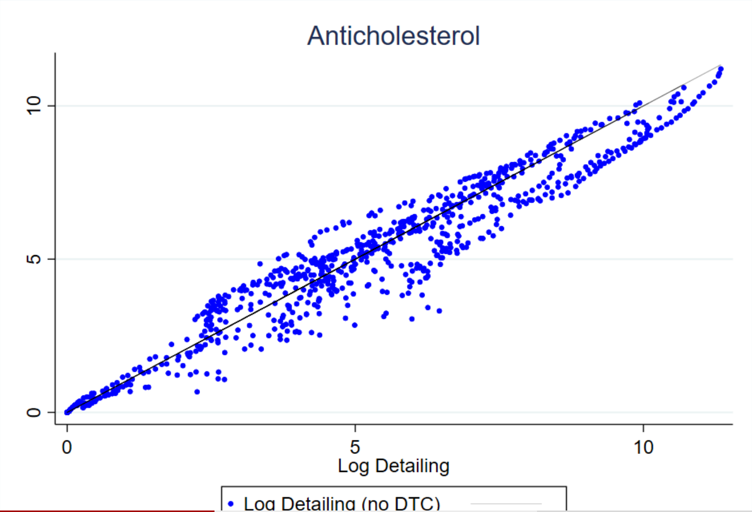
• In both cases we compare:

- detailing policies,
- net profit, and
- we will compare consumer surplus (not done yet).

• Will compare this to a revenue tax with proceeds going to advertising the ability of drugs to resolve conditions, but does not use brand names.

Detailing With and Without DTC

Figure: Simulated Detailing with (horizontal axis) and without DTC :



Quantiles of log detailing with and without DTC Anticholesterol market (in millions per quarter).

	5%	10%	25%	50%	75%	90%	95%
Log detailing	0.0	.20	2.7	5.0	7.1	8.9	9.9
Log detailing (no DTC)	0.0	.23	3.0	4.9	6.9	8.4	9.2

- Detailing Comparisons:
  - firms that do no or only a moderate amount when DTC is allowed, do more detailing when DTC is not allowed, while
  - firms that do a lot of detailing when DTC is allowed do less detailing when DTC is not allowed.

- Comparing distributions: a single crossing between quantiles just under median.

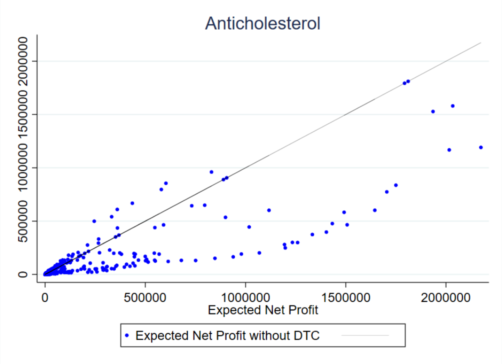
Net Profits With and Without DTC.

Table: With and without DTC for Anticholesterol market (in millions per quarter)

	5%	10%	25%	50%	75%	90%	95%
Net profit	.004	.23	.34	3.3	25	240	497
Net profit (no DTC)	.004	.23	.40	2.9	24	112	206

- Revenue changes mimic detailing changes but the magnitude of the revenue change is more than that of detailing changes, so
  - firms that did little detailing increase their detailing and their net profits, while
  - firms that do a lot of detailing, would decrease their detailing and their net profits
- The highest 10% of firms loose more than half of their profits.

Figure: Simulated Net Profit without DTC against with DTC.



Note: Simulated Net Profit with DTC on horizontal axis against without DTC.

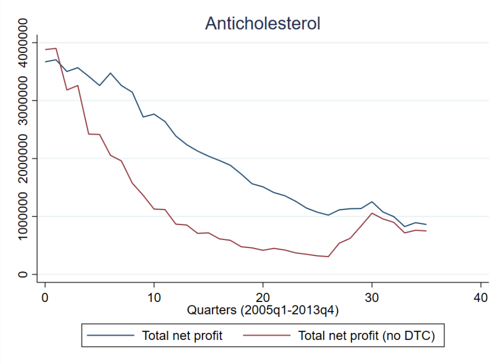
Possible Explanation.

• DTC and detailing are highly positively correlated. The effect of eliminating DTC on detailing depends on the amount of detailing a firm does when DTC is allowed.

• *Possible Explanation.* DTC influences what the patient asks the doctor for.

- Firms with a lot of DTC have an incentive to convince doctors that their drug is suitable.
- When there is no DTC the doctor is less subject to patient priors, and hence more open to all drugs. This increases the incentives for detailing of less well known drugs.

Figure: Simulated Total Net Profits: Anticholesterol



Note: Simulated Total Net Profits.

- Initially the impact grows but as patents moved toward the end of their patent lives they reduce DTC and the impact of earlier DTC dissipates.

Table: Mean and Standard Deviation of simulated log detailing and net profit with and without DTC for Anticholesterol market

	Mean	Standard Deviation
Log detailing	4.931	2.957
Log detailing (no DTC)	4.767	2.806
Net profit	85,953.352	262,246.781
Net profit (no DTC)	50,908.809	170,374.922
Subsample doing DTC in data		
Log detailing	7.993	2.160
Log detailing (no DTC)	7.353	1.984
Net profit	361,504.188	515,366.188
Net profit (no DTC)	171,704.859	335,847.281
Unilateral deviation to no DTC for drugs doing DTC		
Net profit	308,933.63	1,479,590.00
Net profit (no DTC)	54,540.87	280,365.94

Note: In 1,000 US$ per quarter .

Losses from cancelling DTC for firms who do DTC.

- They would save $235 million per quarter in detailing expenditures.
- However there is still a net profit loss of about $190 million per quarter.
- This implies that they would lose about 53% of their profits.
- The profit loss from unilaterally cancelling detailing would be even higher, about 82% (this is calculated from the actual advertising expenditures rather than the computed equilibrium expenditures).
- Caveats
  - A loss in profits might translate into a loss in R&D, hampering the development of new drugs to treat diseases.
  - We have not yet computed the dollar equivalent of the Consumer Surplus, to compare to the loss though we know it goes down
- That is it. Thanx. ♠