Deep Learning and Heterogeneity

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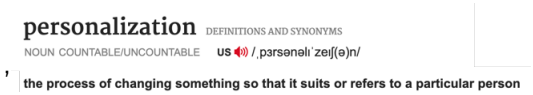
"If deep learning were a policy maker, personalized economic policies would be as finely tuned as your Netflix recommendations intriguing in theory, but sometimes leaving you with odd choices and a sense of existential uncertainty."

OpenAI API
GPT-4, August 2023

Personalization



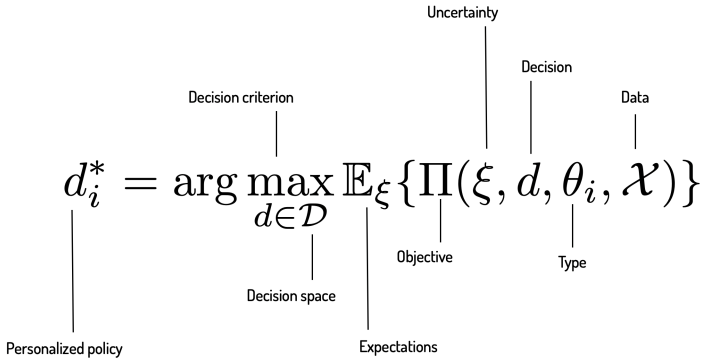
Personalization

$$d_i^* = f(\mathsf{stuff}_i)$$

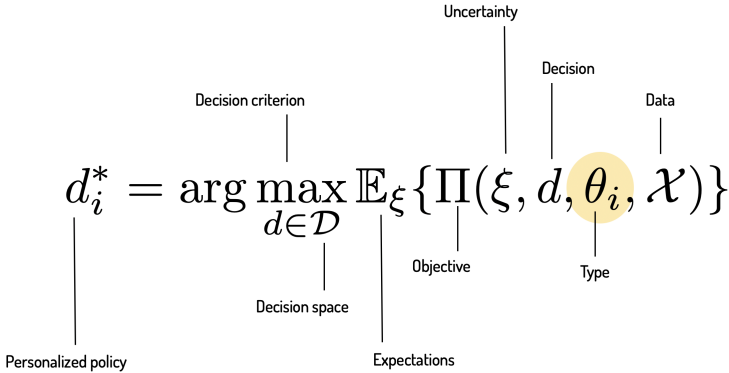
Personalization

$$d_i^* = \arg\max_{d \in \mathcal{D}} \mathbb{E}_\xi \{\Pi(\xi, d, \theta_i, \mathcal{X})\}$$

Personalization



Personalization



Structural Heterogeneity

- A fundamental building block of personalization is the construct of individual heterogeneity.
- Accounting for such heterogeneity is also relevant for
  - Accuracy (Bias)
  - Inference (Variance)
  - Decisions, Policy Design and Evaluation, Targeting, Segmentation and Personalization
- This presentation is about the **practice** of estimating and using heterogeneity measures in structural economic models with the goal of designing personalized policies

Agenda

- Framework
- Applications
- Discussion

The typical structural model.

$$\ell(\boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{\theta}_i)$$

A parametric per-observation loss function for a structural model.

The typical cheat.

$$\ell(\boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{\theta})$$

The easiest way to deal with heterogeneity is to ignore it.

The not-so new idea.

$$\ell(\boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{\theta}(\boldsymbol{X}))$$

We suggest projecting $\boldsymbol{\theta}_i$ on $\boldsymbol{x}_i$. Old idea, used quite often.

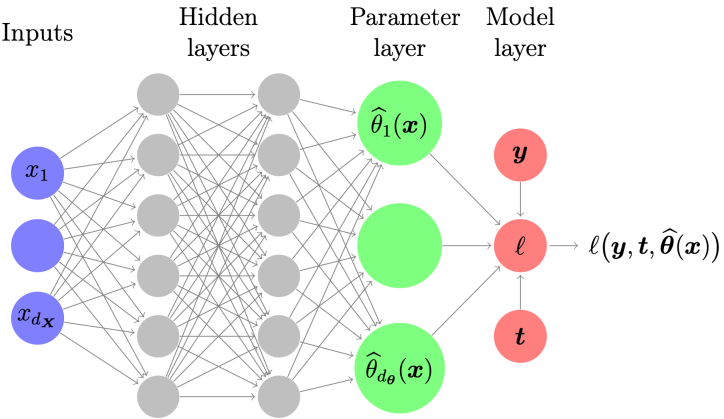
Pretty much the same idea as interaction except that θ is a function..

The new idea.

$$\ell(\boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{\theta}_{\mathrm{DNN}}(\boldsymbol{X}))$$

More specifically, we are proposing that the parameter functions be treated as a **Deep Neural Network** (DNN).

The Framework



The key idea is that the deep learning is targeted towards the parameters. The model is still parametric and defined by economic structure.

The estimator.

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta} \in \mathcal{F}_{\mathrm{DNN}}} \frac{1}{n} \sum_i \ell\left(\boldsymbol{y}_i, \boldsymbol{t}_i, \boldsymbol{\theta}(\boldsymbol{x}_i)\right)$$

Implementing an estimator is then relatively straightforward.

We can use standard tools such as **Tensorflow** or **Torch**.

Pieces of the puzzle...

$$\left[\theta_i - \widehat{\theta}_{\mathrm{DNN}}\left(\boldsymbol{x}_i\right)\right] = \underbrace{\left|\theta_i - \theta\left(\boldsymbol{x}_i\right)\right|}_{\text{approximation}} + \underbrace{\left|\theta\left(\boldsymbol{x}_i\right) - \theta_{\mathrm{DNN}}\left(\boldsymbol{x}_i\right)\right|}_{\text{bias}} + \underbrace{\left|\theta_{\mathrm{DNN}}\left(\boldsymbol{x}_i\right) - \widehat{\theta}_{\mathrm{DNN}}\left(\boldsymbol{x}_i\right)\right|}_{\text{variance}}$$

Is this approximation a good idea?

$$\beta(\boldsymbol{x}_i) = \beta(\quad)$$

Data is aplenty. Really.

Any unobserved heterogeneity will have to be orthogonal to *all* observed heterogeneity.

What about the other pieces?

$$\underbrace{\left|\theta_i - \theta\left(\boldsymbol{x}_i\right)\right|}_{\text{approximation}} \leq \underbrace{\epsilon \text{ [as information}(\boldsymbol{x}_i) \to \infty]}_{\text{assumption}}$$

$$\underbrace{\left|\theta\left(\boldsymbol{x}_i\right) - \theta_{\mathrm{DNN}}\left(\boldsymbol{x}_i\right)\right|}_{\text{bias}} = \ ?$$

$$\underbrace{\left|\theta_{\mathrm{DNN}}\left(\boldsymbol{x}_i\right) - \widehat{\theta}_{DNN}\left(\boldsymbol{x}_i\right)\right|}_{\text{variance}} = \ ?$$

We got this.

$$\underbrace{\left|\theta_i - \theta\left(\boldsymbol{x}_i\right)\right|}_{\text{approximation}} \leq \underbrace{\epsilon\ [\text{as information}(\boldsymbol{x}_i) \to \infty]}_{\text{assumption}}$$

$$\underbrace{\left|\theta\left(\boldsymbol{x}_i\right) - \theta_{\text{DNN}}\left(\boldsymbol{x}_i\right)\right|}_{\text{bias}} \leq \underbrace{\epsilon\ [\text{as complexity (DNN)} \to \infty]}_{\text{proof}}$$

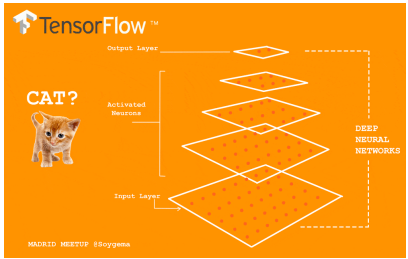
$$\underbrace{\left|\theta_{\text{DNN}}\left(\boldsymbol{x}_i\right) - \widehat{\theta}_{\text{DNN}}\left(\boldsymbol{x}_i\right)\right|}_{\text{variance}} \leq \underbrace{\epsilon\,[\text{as sample size}(n) \to \infty]}_{\text{proof}}$$

Result

$$\|\widehat{\boldsymbol{\theta}}_{\mathrm{DNN}} - \boldsymbol{\theta}\|_2^2 = O_{\mathbb{P}}\left(n^{-p/(p+d_C)} \log^8 n\right)$$

The rates are fast enough for us to do useful things with the estimators.

Why DNNs?



Universal Approximators

Can approximate any smooth function.

Scalability

Infrastructure allows for true scalability (up or down!)

Structural Compatibility

Global estimators that can be embedded into a structural model.

Interpretation maintained!

Structural Compatibility

- Consider a utility function

$$U_i = \alpha + \beta t_i + \varepsilon_i$$

- Where t_i is some treatment (for our example, say a targeted price)
- Then a "structural" choice with heterogeneity and the usual EVTII error gives

$$\mathbb{P}\left(y_i = 1 | \mathbf{x}_i, t_i\right) = \frac{\exp\left(\alpha_i + \beta_i t_i\right)}{1 + \exp\left(\alpha_i + \beta_i t_i\right)}$$

- Changing this to

$$\mathbb{P}\left(y_i = 1 | \mathbf{x}_i, t_i\right) = \frac{\exp\left(\alpha_{\mathsf{DNN}}\left(\mathbf{x}_i\right) + \beta_{\mathsf{DNN}}\left(\mathbf{x}_i\right) t_i\right)}{1 + \exp\left(\alpha_{\mathsf{DNN}}\left(\mathbf{x}_i\right) + \beta_{\mathsf{DNN}}\left(\mathbf{x}_i\right) t_i\right)}$$

- retains the structural interpretation *completely*.
  - $\beta\left(x\right)$ is still the price effect!
  - Can still use usual tricks for
    - WTP, Elasticity, Surplus, ...

Inference Objects

$$\boldsymbol{\mu}_0 = \mathbb{E}\Big[\boldsymbol{H}(\boldsymbol{X}, \boldsymbol{\theta}_0(\boldsymbol{X}); \boldsymbol{t}^*)\Big]$$

We usually estimate structural models because we wish to compute some economic objects.

These could me measures, counterfactuals or policy interventions.

Generic Influence function

$$\boldsymbol{\psi}(\boldsymbol{y}, \boldsymbol{t}, \boldsymbol{x}, \boldsymbol{\theta}, \boldsymbol{\Lambda}) = \boldsymbol{H}\left(\boldsymbol{x}, \boldsymbol{\theta}(\boldsymbol{x}); \boldsymbol{t}^*\right) - \boldsymbol{H}_{\boldsymbol{\theta}}(\boldsymbol{x}, \boldsymbol{\theta}(\boldsymbol{x}); \boldsymbol{t}^*)\boldsymbol{\Lambda}(\boldsymbol{x})^{-1}\boldsymbol{\ell}_{\boldsymbol{\theta}}(\boldsymbol{w}, \boldsymbol{\theta}(\boldsymbol{x})).$$

where

$$\boldsymbol{\Lambda}(\boldsymbol{x}) = \mathbb{E}[\boldsymbol{\ell}_{\boldsymbol{\theta}\boldsymbol{\theta}}(\boldsymbol{y}, \boldsymbol{t}, \boldsymbol{\theta}(\boldsymbol{x})) \mid \boldsymbol{X} = \boldsymbol{x}]$$

Applies to any smooth function and can be computed *automatically* using standard AD technology!

Estimator

$$\widehat{\boldsymbol{\mu}} = \frac{1}{S}\sum_{s=1}^{S}\widehat{\boldsymbol{\mu}}_s, \quad \widehat{\boldsymbol{\mu}}_s = \frac{1}{|\mathcal{S}_s|}\sum_{i\in\mathcal{S}_s}\boldsymbol{\psi}\left(\boldsymbol{w}_i, \widehat{\boldsymbol{\theta}}_s\left(\boldsymbol{x}_i\right), \widehat{\boldsymbol{\Lambda}}_s\left(\boldsymbol{x}_i\right)\right)$$

$$\widehat{\boldsymbol{\Psi}} = \frac{1}{S}\sum_{s=1}^{S}\widehat{\boldsymbol{\Psi}}_s, \quad \widehat{\boldsymbol{\Psi}}_s = \frac{1}{|\mathcal{S}_s|}\sum_{i\in\mathcal{S}_s}\left(\boldsymbol{\psi}\left(\boldsymbol{w}_i, \widehat{\boldsymbol{\theta}}_s\left(\boldsymbol{x}_i\right), \widehat{\boldsymbol{\Lambda}}_s\left(\boldsymbol{x}_i\right)\right) - \widehat{\boldsymbol{\mu}}\right)^2$$

Where S is the number of cross-fitting folds.

CLT

$$\sqrt{n}\widehat{\boldsymbol{\Psi}}^{-1/2}\left(\widehat{\boldsymbol{\mu}}-\boldsymbol{\mu}_0\right)=\sum_{i=1}^{n}\boldsymbol{\Psi}^{-1/2}\boldsymbol{\psi}\left(\boldsymbol{w}_i,\boldsymbol{\theta}_0\left(\boldsymbol{x}_i\right),\boldsymbol{\Lambda}\left(\boldsymbol{x}_i\right)\right)/\sqrt{n}+o_p(1)\rightarrow_d\mathcal{N}\left(\mathbf{0}_{d_\mu},\boldsymbol{I}_{d_{\boldsymbol{\mu}}}\right)$$

Provides all relevant inference objects.

Illustration #1: The Linear Model

- Consider (a scalar t)

$$\mathbb{E}[Y \mid \boldsymbol{T} = \boldsymbol{t}, \boldsymbol{T}\boldsymbol{x}] = \alpha(\boldsymbol{x}) + \beta(\boldsymbol{x})t$$

- So that

$$\ell(y, \boldsymbol{t}, \boldsymbol{\theta}(\boldsymbol{x})) = (y - \alpha(\boldsymbol{x}) + \beta(\boldsymbol{x})t)^2/2$$

- Then

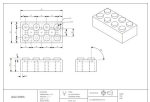
$$\psi(\boldsymbol{w}, \boldsymbol{\theta}, \boldsymbol{\Lambda}) = H\left(\boldsymbol{x}, \boldsymbol{\theta}(\boldsymbol{x}); t^*\right) + \frac{\dot{H}_1(\boldsymbol{x})\left(\lambda_2(\boldsymbol{x}) - \lambda_1(\boldsymbol{x})t\right) + \dot{H}_2(\boldsymbol{x})\left(\lambda_0(\boldsymbol{x})t - \lambda_1(\boldsymbol{x})\right)}{\lambda_2(\boldsymbol{x})\lambda_0(\boldsymbol{x}) - \lambda_1(\boldsymbol{x})^2}\Big(y - \alpha(\boldsymbol{x}) - \beta(\boldsymbol{x})t\Big)$$

Illustration #1: Linear Model

- If we are interested in the **ATE** then
  - $H = \beta$ and $H_1 = 0, H_2 = 1$
  - $\lambda_0 = 1, \lambda_1 = t, \lambda_2 = t^2$
- For a binary treatment we also have that $\lambda_2 = t^2 = t$ and that $\mathbb{E}(t|x) = \pi(x)$ [propensity score]
- Then (exactly as in DML),

$$\psi(\boldsymbol{w}, \boldsymbol{\theta}, \boldsymbol{\Lambda}) = \boldsymbol{\beta}(\boldsymbol{x}) + \frac{(t - \pi(\boldsymbol{x}))}{\pi(\boldsymbol{x}) - \pi(\boldsymbol{x})^2}\Big(y - \alpha(\boldsymbol{x}) - \beta(\boldsymbol{x})t\Big).$$

- For continuous treatment we will need $\mathbb{E}(t|x)$ and $\mathbb{E}\left(t^2|x\right)$ to fill in Λ



Applications

Individual Targeting

- Based on work with Günter Hitsch and Walter Zhang
- Decision to mail catalogs for one of the worlds target apparel companies
- Catalogs cost about $1 per mailing on a base of 120MM+ customers
- **Q**: Do we target customers with an intervention?
- **A**: Are the incremental sales large? — Calculate break-even treatment effect

$$\text{ROI} > 0 \Leftrightarrow \frac{m \cdot \mathbb{E}(Y(1) - Y(0)) - c}{c} > 0$$
$$\Leftrightarrow \mathbb{E}(Y(1) - Y(0)) > \frac{c}{m}$$

- If

$$Y = \alpha(\mathbf{X}) + \beta(\mathbf{X})\, t + \varepsilon$$

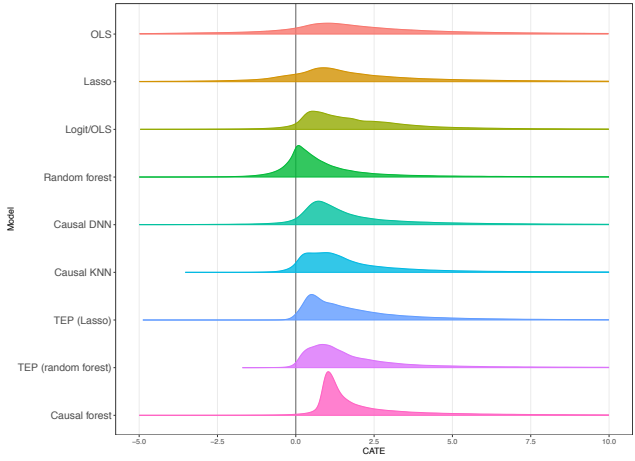
- then

$$\mathbb{E}(Y(1) - Y(0)) = \beta(\mathbf{X})$$

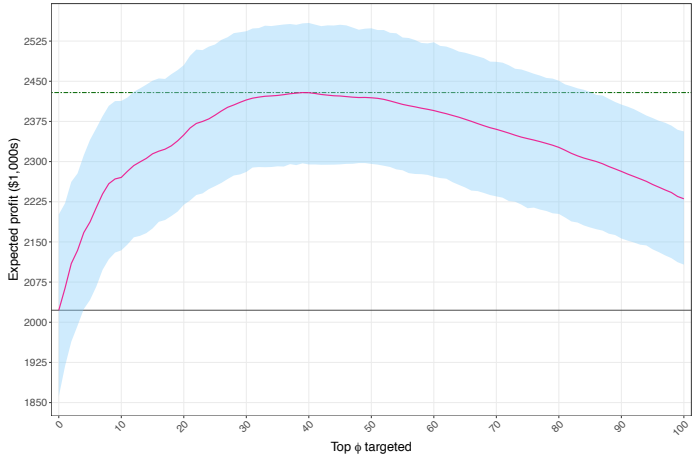
- And the optimal policy is simple..

$$d(\mathbf{x}_i) = \mathbb{I}\left(\beta(\mathbf{x}_i) > \frac{c}{m}\right)$$

Individual Targeting



Individual Targeting



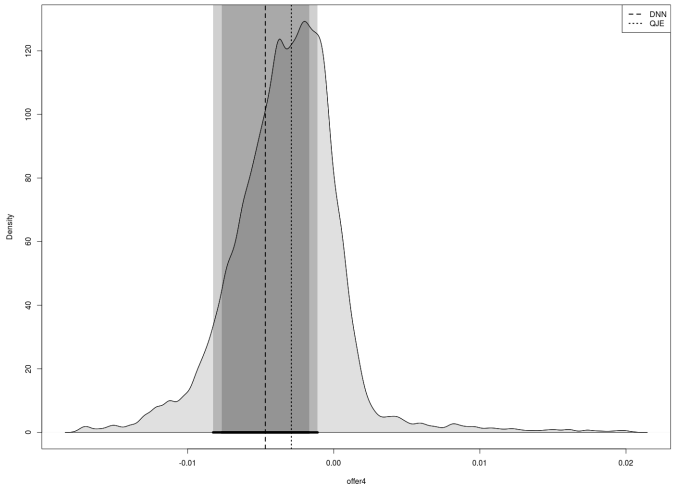
Content of Advertising

- **Bertrand, Karlan, Mullainathan, Shafir, Zinman (QJE, 2010)**:
  - Advertising for shortish-term loans in South Africa
  - Original questions: does advertising content matter? How much?
  - $Y = \{0, 1\}$ Applied for a loan (scalar in this case)
  - Policy/Treatment $\boldsymbol{T} = \mathbf{12}$ ad characteristics, randomly assigned (based on $\boldsymbol{x}$)
  - $T_1 =$ interest rate, directly compute valuation
  - Other qualities (photos, tables, uses) can be then valued
  - $\boldsymbol{X} = 11$ individual characteristics, some discrete
  - Economic Model for relationship of $\boldsymbol{T} \leftrightarrow Y$, enriched w/ heterogeneity in $\boldsymbol{X}$

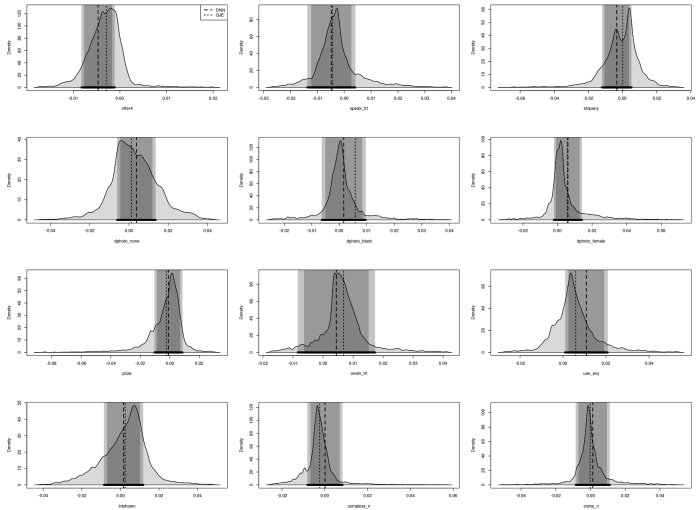
Marginal Effects Table

Variable	QJE	DNN ME	95% CI		$\widehat{Pr}\left(\hat{\beta}(x) > 0\right)$	Coef. of Variation
Interest rate offer	-0.0029	-0.0047	-0.0083	-0.0011	0.1337	1.0211
We speak your language	-0.0043	-0.0048	-0.0137	0.0041	0.2533	2.0542
Special rate for you	0.0001	-0.0034	-0.0120	0.0053	0.5001	4.4506
No photo	0.0013	0.0038	-0.0060	0.0136	0.5723	3.4931
Black photo	0.0058	0.0016	-0.0064	0.0096	0.5402	5.1348
Female photo	0.0057	0.0060	-0.0021	0.0141	0.6820	2.3375
Cell phone raffle	-0.0023	-0.0009	-0.0104	0.0085	0.4812	17.0059
Example loan shown	0.0068	0.0044	-0.0084	0.0173	0.8631	1.9379
No loan use mentioned	0.0059	0.0108	0.0009	0.0207	0.7499	1.0936
Interest rate shown	0.0025	0.0017	-0.0085	0.0119	0.6289	7.9903
Loss comparison	-0.0024	0.0001	-0.0081	0.0083	0.2342	89.3606
Competitors rate shown	-0.0002	0.0013	-0.0085	0.0111	0.4107	9.6790

Offer Rate Coefficient



Same for the Other Estimates



Beyond Marginal Effects: Optimal Offers

- Assume the firm wishes to maximize profits:

$$\max_{r=\text{rate}} \pi(r) := \max_{r=\text{rate}} L\left[rG\left(r\right)\right]\left[1 - D\left(r\right)\right]$$

  - $L =$ expected dollar loan amount, normalize to 1 (doesn't impact the rate)
  - $r =$ the interest rate offered
  - $G(r) =$ the probability of acceptance (depends on $\theta(\boldsymbol{x}_i)$)
  - $[1 - D\left(r\right)] =$ probability of non-default on the loan
- Then it is straightforward to show that

$$\frac{\partial \pi}{\partial r} = \left(r\dot{G}\left(r\right)\beta_r + G\left(r\right)\right)\left[1 - D\left(r\right)\right] - rG\left(r\right)\dot{D}\left(r\right)\delta = 0$$

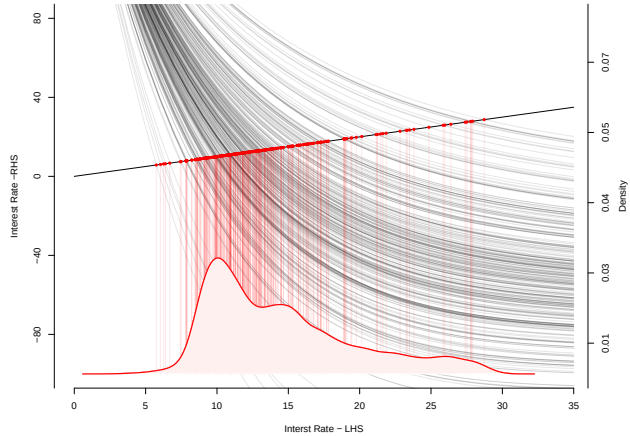
Beyond Marginal Effects: Optimal Offers

- There will a unique fixed point since the denominator of the RHS is decreasing in r for $\beta_r < 0$ (remember those 13%?) and$\delta > 0$
- Therefore

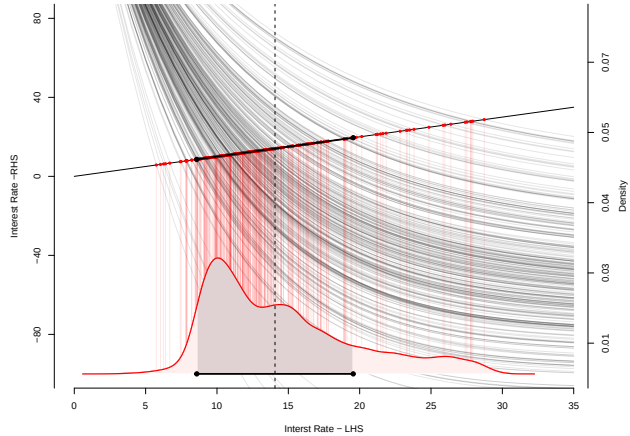
$$r^* = \frac{1 + r^* \left(1 - G\left(r^*\right)\right) \beta_r}{D\left(r^*\right) \delta}$$

- Even if we don't have it in closed form, r^* is a smooth function of $\boldsymbol{\theta}(\boldsymbol{x})$
- We can do inference on any $\boldsymbol{\mu}_0 = \mathbb{E}[\boldsymbol{H}(\boldsymbol{X}, \boldsymbol{\theta}, r^*)]$
- Impossible without
  1. Taking heterogeneity seriously
  2. Our new methods and ideas

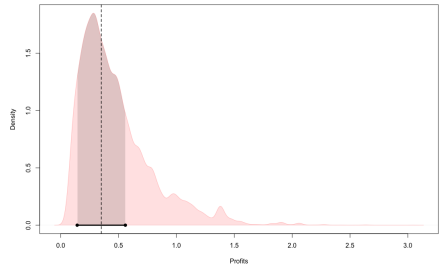
Optimal Offers



Optimal Offers



Profit from Personalized Offers



- Inference is automatic because π is just another function:

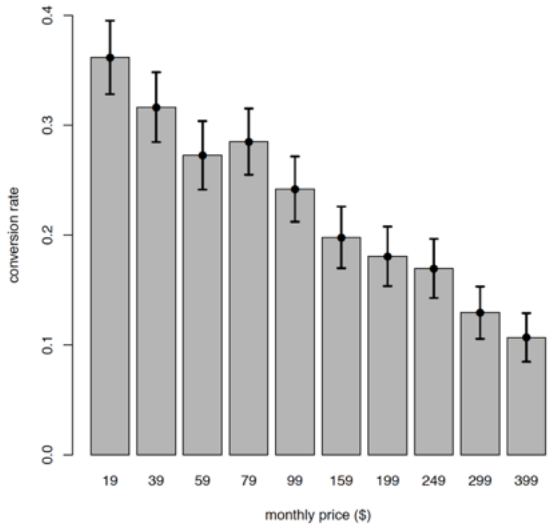
$$\mu = \mathbb{E}[\pi(r^*)] = \mathbb{E}[\pi(r^*(\boldsymbol{\theta}(\boldsymbol{x})))]$$

- We cannot write the IF down in closed form, but we can evaluate it:
- $\widehat{\mu} = \$0.35$, with a 95% confidence interval of $(\$0.1421, \$0.5586)$
- Incremental 5.7% in expected profits over the optimal (uniform) interest rate

Personalized Pricing

- Work with JP Dube
- Online recruiting platform to match jobseekers and potential employees
- Customers are potential employers that pay a monthly subscription rate to avail a stream of matched resumes for posted jobs
- Base price for "starter" firm (small business < 50 employees) was \$99/month
- Customers required to register details about firm, job descriptions etc before they can reach paywall
- i.e. we obtain set of features, x_i, for each new firm i
- Goal: use framework described to improve pricing at the firm

Pricing Experiment



The ML trap

- Let's assume that we are interested in learning a demand function

$$y = f(p)$$

- So that we can find optimal prices

$$p^* = \arg\max_p (p - c) f(p)$$

- We decide to use Random forests to obtain $\widehat{f}(p)$
- Plugging in and optimizing gives us...

$$p^* =$$

The ML trap

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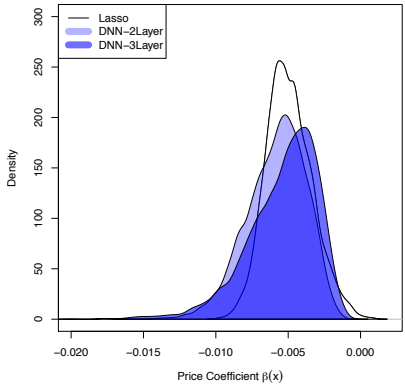
$$p^* = \infty$$

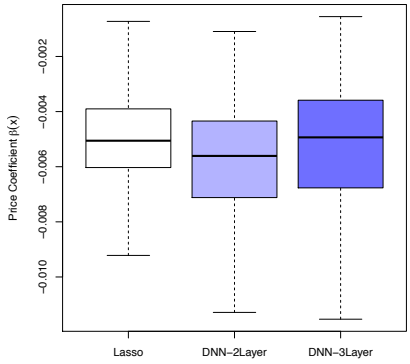
Personalized Pricing

- Note that our approach has the following ingredients:
  - A model of consumer behavior and heterogeneity parameter(s) (Ψ_i)
  - A selection procedure that identifies the relevant sparse subset of x_i and estimates parameters (Θ)
  - A set of decision variables and rules that optimize the firms objective, (π)
- Given $\Psi_i = \Psi(x_i; \Theta)$ the pricing problem is straight-forward

$$p_i^* = c + \left[\nabla_p \bar{q}(p_i^*, x_i; \Theta)\right]^{-1} \bar{q}(p_i^*, x_i; \Theta)$$

Price Effects



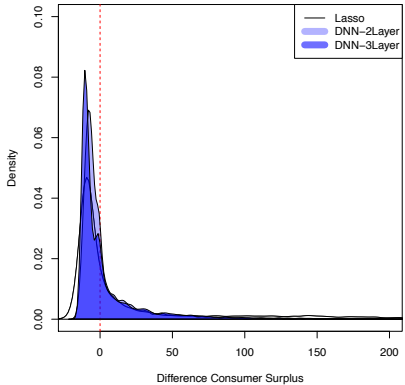


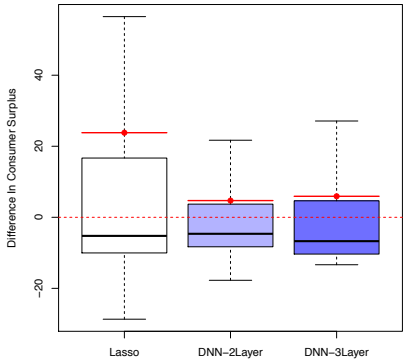
Pricing

Predicted versus Realized Outcomes in November 2015 Experiment

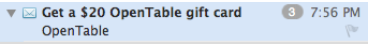
	Control ($99)	Test ($249)	Test (Personalized Pricing)
Sample size	1,360	1,430	2,485
Mean conversion	.23	.15	.15
	(.21, .25)	(.13, .17)	(.13, .16)
Mean revenue per consumer ($)	22.57	37.79	41.59
	(20.36, 24.77)	(33.15, 42.42)	(37.49, 45.7)
Posterior mean conversion	.26	.15	.14
	(.23, .29)	(.13, .18)	(.12, .17)
Posterior mean revenue per consumer ($)	25.5	38.37	41.05
	(23.26, 28.31)	(32.04, 44.9)	(33.78, 48.78)

Consumer Surplus





Personalized Messages



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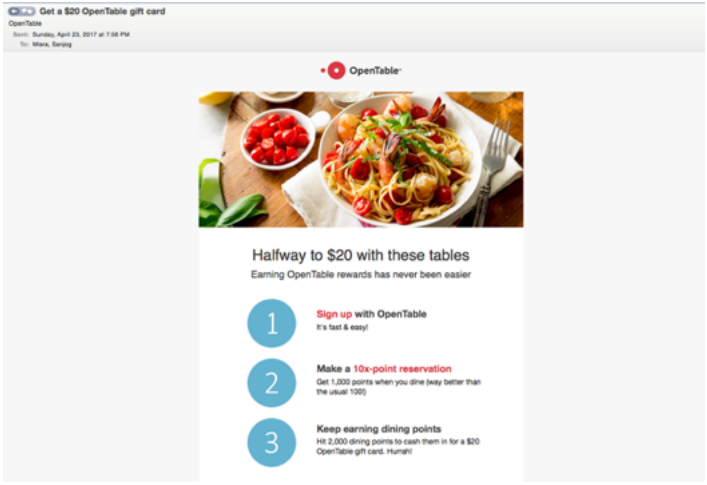
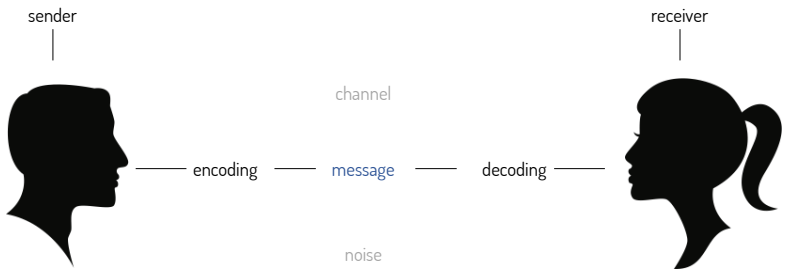


Messages




SNAP

Recertification

SAR 7 ELIGIBILITY STATUS REPORT

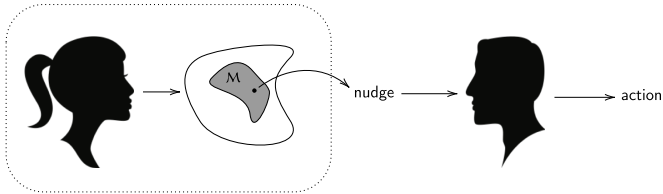
REPORT MONTH

- SNAP participants often fail to re-certify at the 6 month mark (over 50%)
- Simply need to fill out a form.
- Note: No interview at the 6-month mark in CA.

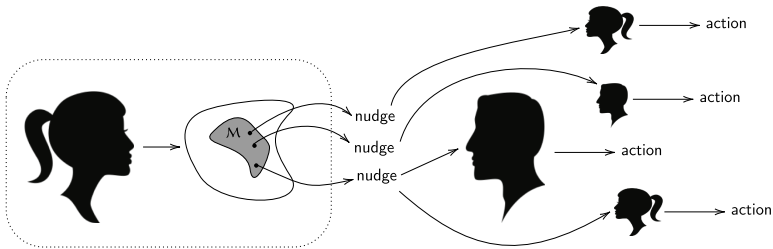
a [nudge] message

Hi [name], GetCalFresh here! To keep getting CalFresh, fill out your Semi-Annual Report (SAR 7) as soon as you can. It's required even if you have no changes to report. Do it online at: [link]

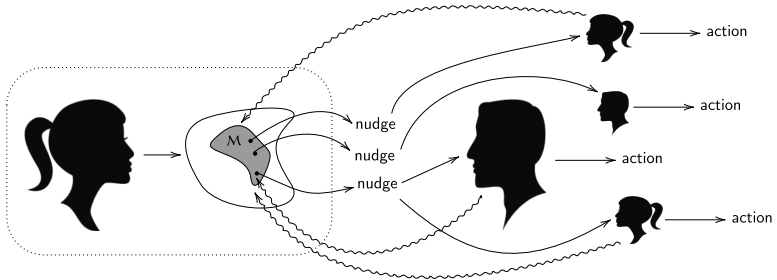
Designing Context



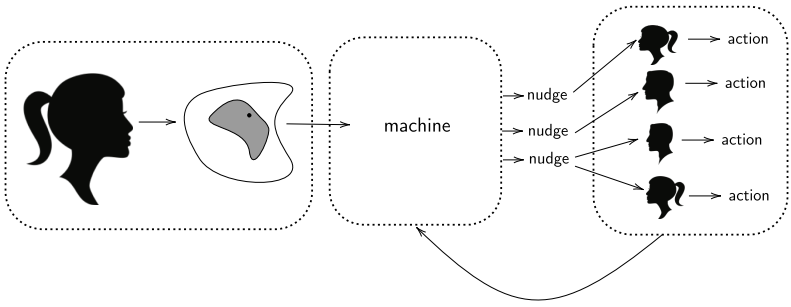
Personalization



Heterogeneity and beliefs



Personalized Algorithmic Messages



Application: SNAP Recertification

STATE OF CALIFORNIA - HEALTH AND HUMAN SERVICES AGENCY

CALIFORNIA DEPARTMENT OF SOCIAL SERVICES
CALIFORNIA DEPARTMENT OF HEALTH CARE SERVICES

WORK PAY$

SAR 7 ELIGIBILITY STATUS REPORT

REPORT MONTH ____________

TO KEEP YOUR BENEFITS COMING ON TIME, PLEASE SIGN THE FORM AFTER ________ 1st AND RETURN IT BY ________ 5th
(SUBMIT MONTH) (SUBMIT MONTH)

____
CASE NUMBER HERE

NEED HELP? *(County Specific instructions w/county url)*
Worker Name:
Worker Phone:
County:
Street address:
City, State, Zip Code
BAR CODE:

[DIST. ID HERE]

Check the box if you would like to STOP getting any of the following: ☐ **STOP my CalWORKs** ☐ **STOP my CalFresh** ☐ **STOP my Medi-Cal**

1. **Has anyone moved into or out of your home (including newborns) or did you move in with someone else since you last reported?** ☐ **Yes** ☐ **No** (If yes, complete the section below)

Date of Move (mm/dd/yy)	Name (First, Middle, Last)	Date Of Birth	Relationship To You	Regularly Purchase And Prepare Food Together?
☐ In ☐ Out / /		/ /		☐ YES ☐ NO
☐ In ☐ Out / /		/ /		☐ YES ☐ NO
☐ In ☐ Out / /		/ /		☐ YES ☐ NO

2. **Have there been any changes to your address since you last reported?** ☐ **Yes** ☐ **No** (If yes, complete the section below)
   New Address:____________________ Date Moved: ____________
   Mailing Address (if different than above)____________________

3. **If you have moved since you last reported please fill out the section below:**

Your rent or mortgage per month now?	If paid separately, your property taxes and home insurance per month now?
$	$

Do you have utility costs that are not included in your rent or mortgage payment? If so, check which ones:
☐ Phone ☐ Trash ☐ Water ☐ Electric/Gas ☐ Other heating or cooling costs

4. **CalWORKs only: Is anyone in your home:**
   A. **Running from an outstanding warrant?**
   B. **Found by a court to be in violation of probation or parole?**
   ☐ **Yes** ☐ **No** (If yes, complete the section below)

Name of person	A or B from above	In what state was the warrant issued, or did violation happen?	Date of warrant or violation

5. **Medical Costs: If anyone who gets CalFresh and is 60 years old or older, or disabled, had an increase in medical costs please complete the section below and attach proof:**

Who had the change?	Amount of increase:
	$

6. **Child Support: Did anyone who gets CalFresh have a change in the amount of child support they have to pay since they last reported?** ☐ **Yes** ☐ **No** If yes, complete the section below and **attach proof**.
   What was the amount paid in the **Report Month?** $__________.
   Who paid support?____________________
7. **Dependent Care: If anyone who gets CalFresh and either works, is looking for work, or is going to school, had an increase in out-of-pocket dependent care costs since they last reported, please complete the section below and attach proof:**
   What was the amount paid out-of-pocket in the **Report Month?** $__________
   Who paid: ____________________ **List dependent(s):** ____________________
8. **Did anyone: Get, buy, sell, trade or give away any property, land, homes, cars, bank accounts, money, payments (such as lottery/casino winnings, back benefits from social security), or other property items since last reported?**
   ☐ **Yes** ☐ **No** (If yes, complete the section below and **attach proof.** If you need more space, attach a separate piece of paper).

Who?	Type of Property?	When?	Amount/Value?	☐ Bought ☐ Sold ☐ Gave Away ☐ Spent ☐ Got as a gift ☐ Traded ☐ Won ☐ Other

SAR 7 (12/14) ELIGIBILITY STATUS REPORT - FOR CASH AID AND CALFRESH - REQUIRED FORM - SUBSTITUTES PERMITTED PAGE 1 OF 2

9. **Did anyone get income from employment in the Report Month?** ☐ **Yes** ☐ **No** (If yes, complete the section below and **attach proof**). The **Report Month** is listed at the top of the first page. List each job for each person who works. If you need more space attach a separate piece of paper. Examples include babysitting, salary, self-employment, sick pay, tips, etc. **If you lost your job, attach proof.**

	Job #1	Job #2	Job #3
Name of person who got income:			
Source of income/Employer name:	Self-employed, check here ☐	Self-employed, check here ☐	Self-employed, check here ☐
How often paid:	☐ Weekly ☐ Biweekly ☐ Other ☐ Monthly ☐ Twice monthly	☐ Weekly ☐ Biweekly ☐ Other ☐ Monthly ☐ Twice monthly	☐ Weekly ☐ Biweekly ☐ Other ☐ Monthly ☐ Twice monthly
Gross amount of income they got in the report month:	$ DATE(S) RECEIVED:	$ DATE(S) RECEIVED:	$ DATE(S) RECEIVED:
Hours worked per month:			

10. **Will there be any changes to your income from employment in the next six months (including income listed in #9)?**
    ☐ **Yes** ☐ **No** (If yes, explain here and **attach proof**). Examples: Stopping or starting a job; increase or decrease of income; changes in hours; quitting a job or going on strike; change in how often you are paid.
11. **Did anyone get money from any other source in the Report Month:** ☐ **Yes** ☐ **No** (If yes, complete the section below and **attach proof.**) The **Report Month** is listed at the top of the first page. Examples include: Social Security, Unemployment Compensation, Veteran's Benefits, State Disability Insurance (SDI), Child/Spousal Support, Worker's Compensation, Loans/Gifts, Earned/Unearned Housing, Utilities, Food, etc. If you no longer get money from a source you previously reported, **attach proof**.

Name	Source of income	One time payment or monthly	How much
			$
			$
			$

12. **Will there be any changes to money received from any other source in the next six months (including money listed in #11)?**
    ☐ **Yes** ☐ **No** (If yes, explain here and **attach proof**). Examples of changes: An increase or decrease in income or benefits, or if you will start or stop getting income or benefits.
13. **CalWORKs only: Have any of the following happened to anyone in your home since you last reported?** ☐ **Yes** ☐ **No**
    (If yes, check below and **attach proof**):
    ☐ Family Change (Married, divorced, separated, entered into a California Registered Domestic Partnership (RDP), have a non-California Domestic Partnership (DP), ended a DP or RDP, became pregnant, or is no longer pregnant?)
    ☐ Job/Employment (Start, stop, quit a job, started a business or went on strike?)
    ☐ Disability (Became disabled or recovered from a disability or major illness?)
    ☐ Immigration (Citizenship or immigration status change, or got a new card, form, or letter from USCIS (INS)?)
    ☐ Insurance (Started, stopped, or changed health, dental, or life insurance benefits, including MEDICARE?)
    ☐ Custody (Any change in the amount of time you care for/have custody of your children?)
    ☐ In-Home Support Services (Started or stopped getting services?)
    ☐ School Attendance
    For Age 18 or older student - started or stopped school/college? (You may be able to claim costs for books, school transportation, etc.)
    ☐ Someone paid for all of my housing, food, clothing or utility costs. (please explain) ____________________
    ☐ Other____________________

Please read carefully, sign, and date.

By signing this form:
- I understand and certify, under penalty of perjury, that all my answers on this report are correct and complete to the best of my knowledge.
- I understand the penalties for fraud are as follows: I may be sent to prison for up to 20 years and fined up to $250,000. I may have to pay back benefits if I was not eligible to them. The first time I break the rules on purpose I will not be able to get CalFresh for one year; the second time two years; and after the third time I will not be able to get CalFresh again.
- I understand and agree to give copies of all documents needed to complete my semi-annual report.
- I understand that in some instances, I may be asked to give consent to the County to make whatever contacts are necessary to determine eligibility.

CERTIFICATION - FRAUD WARNING

I UNDERSTAND THAT: If on purpose I do not report all facts or give wrong facts about my income, property, or family status to get or keep getting aid or benefits, I can be legally prosecuted. I may also be charged with committing a felony if more than $950 in Cash Aid, and/or CalFresh is wrongly paid out as a result of such an action. I have received a copy of the Instructions and Penalties for the SAR 7 Eligibility Status Report for Cash Aid and CalFresh.

YOU MUST SIGN AND DATE THIS REPORT AFTER THE LAST DAY OF THE REPORT MONTH OR IT WILL BE CONSIDERED INCOMPLETE.
I declare under penalty of perjury under the laws of the United States and the State of California that the facts contained in this report are true and correct and complete.

WHO MUST SIGN BELOW: **For Cash Aid**: You and your aided spouse, registered domestic partner, or the other parent (of cash-aided children) if living in the home. **For CalFresh**: The head of household, a responsible household member, or the household's authorized representative.

SIGNATURE OR MARK	DATE SIGNED	HOME PHONE	CONTACT/CELL PHONE
		( )	( )

SIGNATURE OF SPOUSE, REGISTERED DOMESTIC PARTNER, OR OTHER PARENT OF CASH AIDED CHILD(REN)	DATE SIGNED	SIGNATURE OF WITNESS TO MARK, INTERPRETER, OR OTHER PERSON COMPLETING FORM	DATE SIGNED

SAR 7 (12/14) ELIGIBILITY STATUS REPORT - FOR CASH AID AND CALFRESH - REQUIRED FORM - SUBSTITUTES PERMITTED PAGE 2 OF 2

Control Message (Code for America)

Hi [name], GetCalFresh here! To keep getting CalFresh, fill out your Semi-Annual Report (SAR 7) as soon as you can. It's required even if you have no changes to report. Do it online at: [link]

- Better than status quo by around 13%.(communicated to me)

Featurizing messages

- We borrow ideas from Context free grammars (Chomsky)
- The architect chooses the grammar: $G = \langle \mathcal{F}, V, S, R \rangle$
  - $\mathcal{F}$ is a finite set of terminal symbols (the content)
  - V is a finite set of variables (nonterminals)
  - S are start symbols
  - R is a finite relation $V \to (V \cup \mathcal{F})^*$ (a set of rules)
- Note that

$$\mathbf{F} \in \boldsymbol{\mathcal{F}} = \bigcup_{k \in K} \mathcal{F}_k$$

- with

$$m \Leftrightarrow \mathbf{F} \equiv \{F_1, F_2, \ldots, F_K\}$$

- and

$$F_k \in \mathcal{F}_k$$

- We can then use the optimal $\mathbf{F}^*$ in conjunction with the grammar to define nudges.

Featurizing messages

- Context free grammars (Chomsky)

Featurizing messages

- Consider the simple example

Featurizing messages

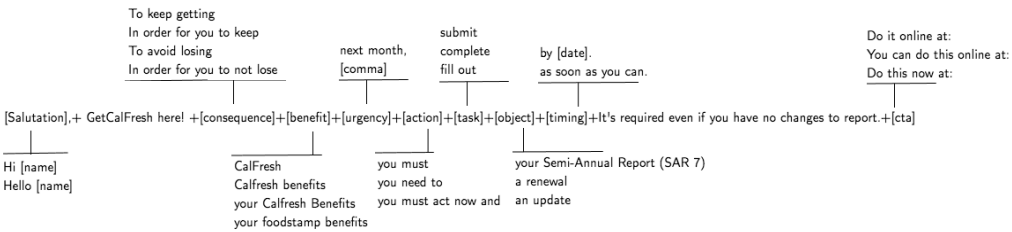
- Messages have structure or grammar (defined by architect)
- We exploit this grammar to construct features that are modular.
- These feature-sets are bijective to the message
  - 101 : John got on an airplane to Boston
  - 010 : Mary caught a ride to Chicago
- Think conjoint with a bit more structure

Marketing Messages

Featurization

[salutation] [benefit] [task] [object] [timing]
Hi [name], GetCalFresh here! To keep getting CalFresh, fill out your Semi-Annual Report (SAR 7) as soon as you can.
It's required even if you have no changes to report. Do it online at: [link]
[cta]

Featurization



General Model of Behavior

- We will assume that participants undertake an action (e.g. open emails) if the expected utility/value of doing so is positive.
- Consider a utility function

$$U_i = \alpha_i + \mathbf{F}'\beta_i + \varepsilon_i$$

- Then a "structural" choice with heterogeneity and the usual EVTII error gives

$$P\left(\text{response} = 1 | \mathbf{F}, \theta_i\right) = \frac{1}{1 + \exp\left(-\left(\alpha_i + \mathbf{F}'\beta_i\right)\right)}$$

  - where the β_i are the sensitivity of a person to message features $\mathbf{F}$
  - and $\theta_i = \{\alpha_i, \beta_i\}$
  - Note the individual subscript i

Structural Compatibility

- We will move from

$$P\left(\text{response} = 1|\mathbf{F}, \theta_i\right) = \frac{1}{1 + \exp\left(-\left(\alpha_i + \mathbf{F}'\beta_i\right)\right)}$$

- to...

$$P\left(\text{response} = 1|\mathbf{F}, \mathbf{x}_i\right) = \frac{1}{1 + \exp\left(-\left(\alpha_{\text{DNN}}\left(\mathbf{x}_i\right) + \mathbf{F}'\beta_{\text{DNN}}\left(\mathbf{x}_i\right)\right)\right)}$$

- We assume the types are captured by a vector of characteristics $\left(\mathbf{x}_i\right)$
- As before this retains the structural interpretation *completely*.
  - $\beta\left(x\right)$ is still the effect of $\mathbf{F}$!
  - Can still use usual (logit) tricks for ME, welfare, elasticity etc.

Model of Behavior

- to Recap
- We assume the types are captured by a vector of characteristics $(\mathbf{x}_i)$
- We assume that participants undertake an action (e.g. open emails) if the expected utility/value of doing so is positive.
- And under these and standard assumptions

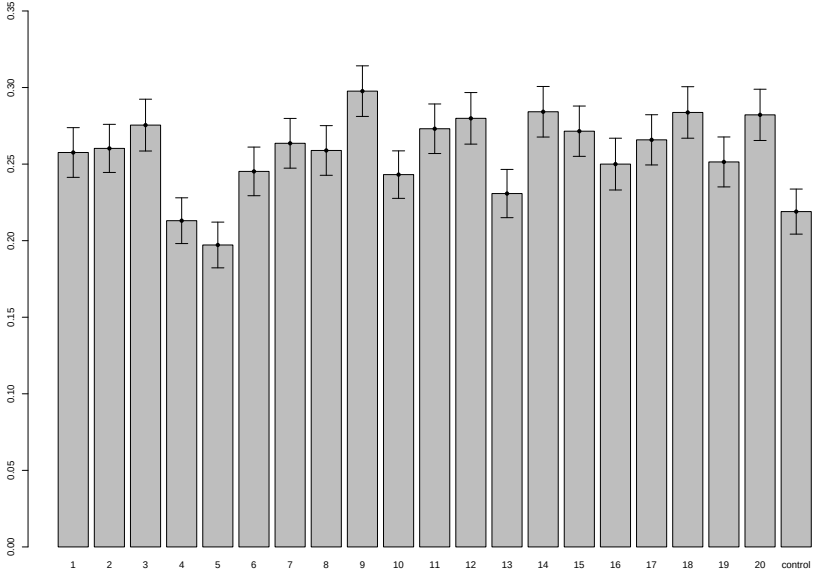
$$P\left(\text{response} = 1 | \mathbf{F}, \mathbf{x}_i\right) = \frac{1}{1 + \exp\left(-\left(\alpha_{\text{DNN}}\left(\mathbf{x}_i\right) + \mathbf{F}'\beta_{\text{DNN}}\left(\mathbf{x}_i\right)\right)\right)}$$

- Now we need to estimate $\theta\left(\mathbf{x}\right)$...

Experiment: Design and Data

- Q: Data? A: Experiments
- The first experiment was conducted in October, 2020
  - Message Space: $\mathcal{M} = \prod_{k=1}^{K} |\mathcal{F}_k| = \mathbf{13824}$
  - Design: $\mathcal{D} = 1 + \sum_{k=1}^{K} (|\mathcal{F}_k| - 1) = \mathbf{19}$
- We chose to test 20 messages ($N \approx 732$ each) + control
  - Chosen by Federov exchange algorithm.
- Total sample was $N = 15380$
- Other data: X_i (characteristics)

Experiment: Results



Results

Feature	$\mathbb{E}[\beta(x)]$	SE	t-stat	95%CI (lower)	95%CI (upper)
Intercept	-1.2903	0.0856	-15.071	-1.4581	-1.1225
Salutation: "Hi [name],"	–				
Salutation: "Hello [name],"	0.0593	0.0360	1.6453	-0.0113	0.1299
Consequence: "To keep getting "	–				
Consequence: "In order for you to keep "	0.1852	0.0447	4.1396	0.0975	0.273
Consequence: "To avoid losing "	0.1643	0.0499	3.2927	0.0665	0.2621
Consequence: "In order for you to not lose "	0.0480	0.0511	0.941	-0.0520	0.1481
Benefit: "CalFresh"	–				
Benefit: "CalFresh benefits"	-0.0069	0.0504	-0.1368	-0.1056	0.0918
Benefit: "your CalFresh benefits"	0.1012	0.0504	2.0073	0.0024	0.2000
Benefit: "your food stamp benefits"	-0.0411	0.0506	-0.8126	-0.1402	0.0580
Urgency: "next month,"	–				
Urgency: ","	-0.1392	0.0360	-3.8677	-0.2097	-0.0686
Action: " you must"	–				
Action: " you need to"	-0.0164	0.0450	-0.3636	-0.1046	0.0719
Action: " you must act now and"	-0.0266	0.0487	-0.5458	-0.1220	0.0688
Action: ""	-0.019	0.0531	-0.3587	-0.1231	0.0850
Task: "submit "	–				
Task: "complete "	0.0925	0.0452	2.0485	0.0040	0.1810
Task: "fill out "	0.1497	0.0449	3.3362	0.0617	0.2376
Object: "your Semi-Annual Report (SAR 7) "	–				
Object: "a renewal "	0.0382	0.0425	0.9003	-0.0450	0.1215
Object: "an update "	0.0545	0.0441	1.236	-0.0319	0.1408
Timing: "by Oct. 11."	–				
Timing: "as soon as you can."	-0.0082	0.0360	-0.2281	-0.0789	0.0624
CTA: "Do it online at:"	–				
CTA: "You can do this online at:"	0.0698	0.0466	1.4977	-0.0216	0.1612
CTA: "Do this now at:"	0.0985	0.0442	2.2281	0.0119	0.1851

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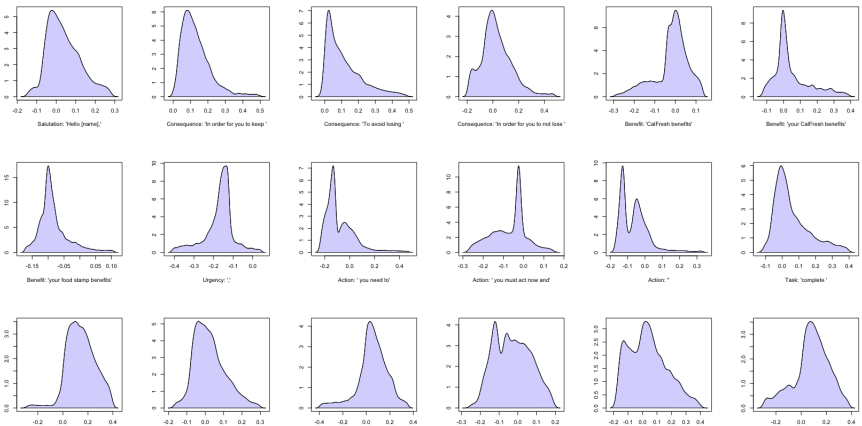
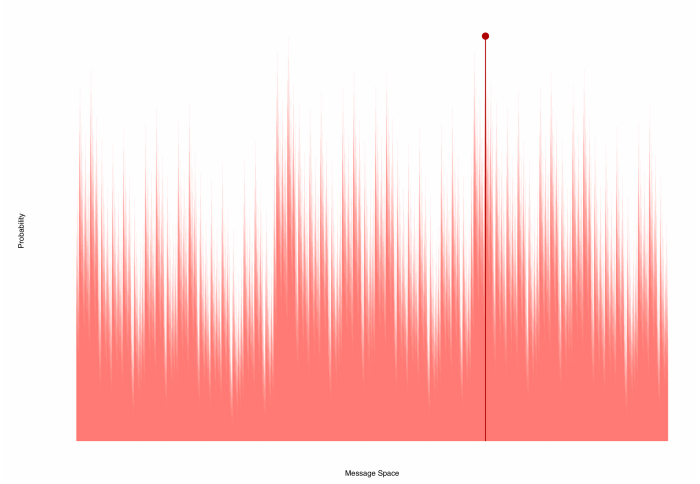
Heterogeneity



Heterogeneity



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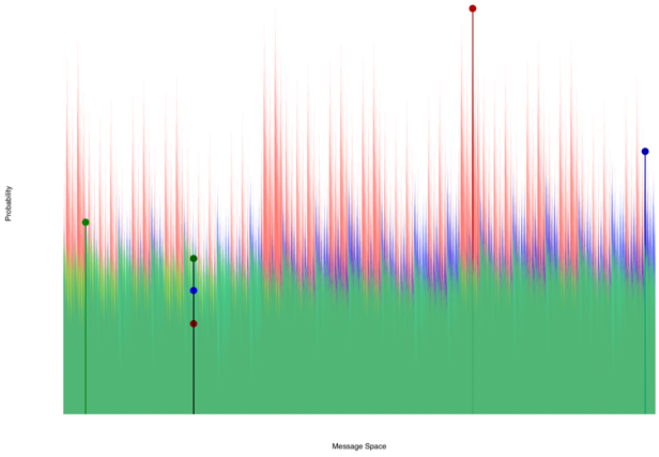
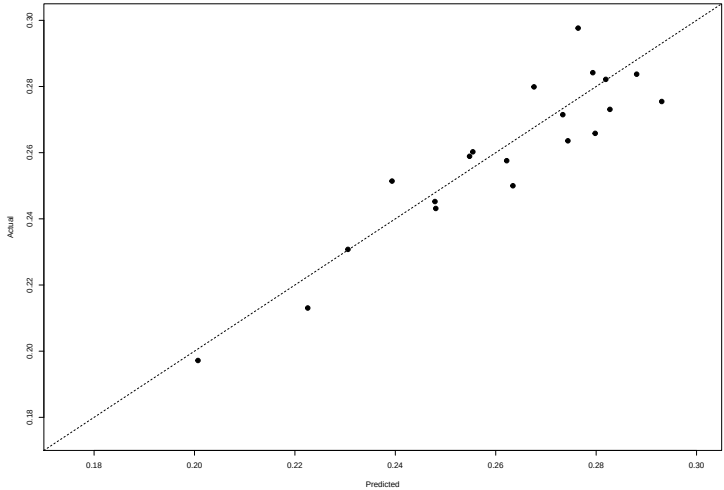


Honest Validation

- We implement a validation scheme that is *honest*.
  - The **individuals** in the validation exercise are **held out**.
  - The **message** being evaluated (control) is **held out**.
- Recall that the messages tested were on the convex hull of the design space.
- The control message lies at (close to) the edge of that space...

Honest Validation



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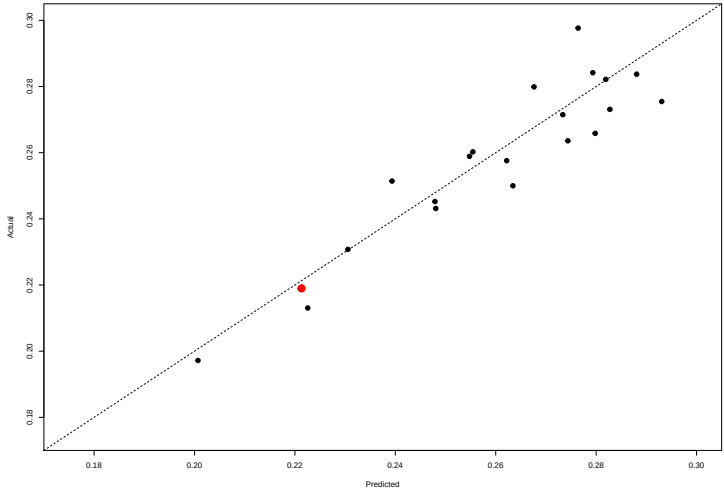
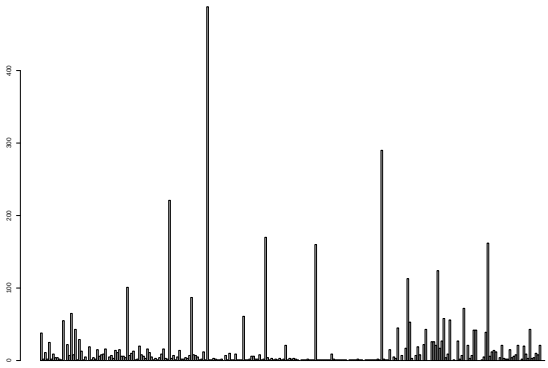


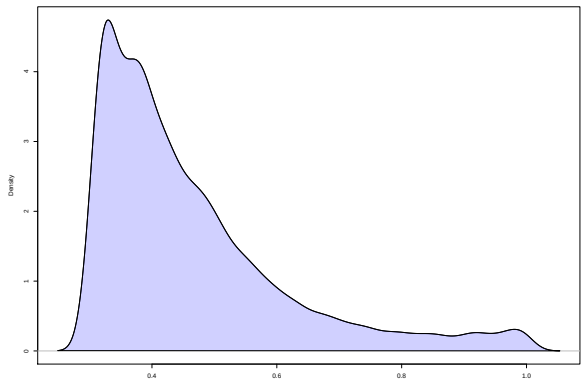
November Validation Experiment

- We implemented a second experiment ($N = 7773$) in November 2020.(*)
- We allocated individuals to four arms
  - Algorithmic (50%)
  - Control (10%)
  - Random (30%)
  - Optimal (10%)
- Messages were constructed based on the parameters estimated.
  - 252 Unique algorithmic messages

Algorithmic Nudges



Expectations...



Predictions

	(Raw) Prediction
Algorithmic	0.4654
Control	0.2373
Optimal	0.3557
Random	0.2678

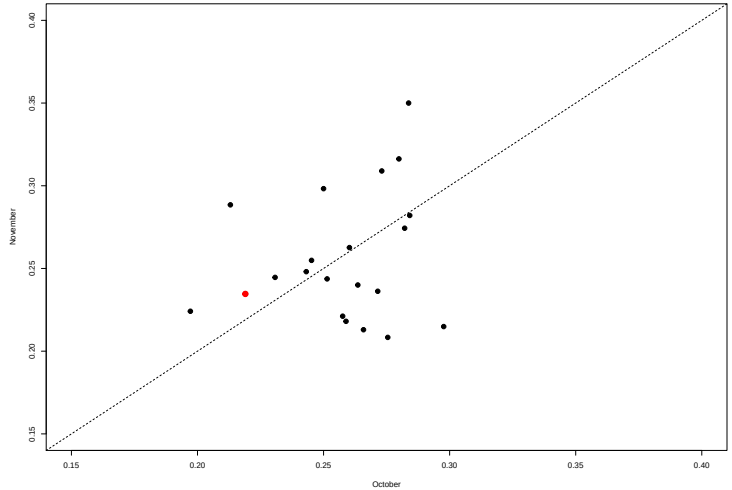
- Predictions are aggressive
- Do not account for noise/uncertainty
- Election day!!!
- Distribution of recipients not in our control.
- Doomsday Validation.

Validation

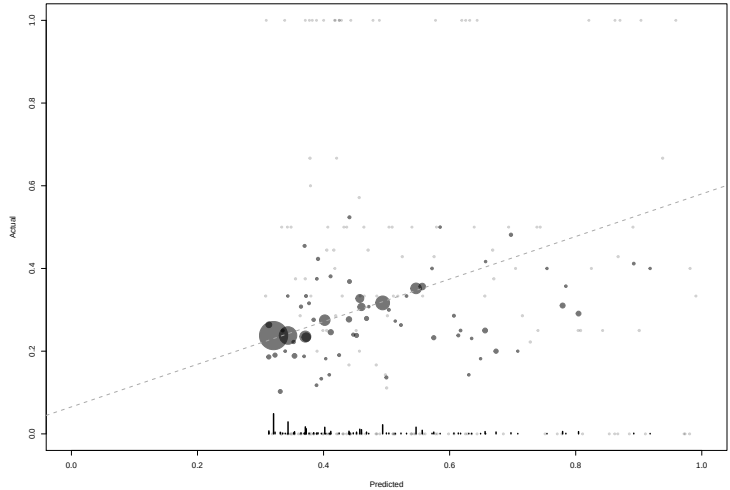
	(Raw) Prediction	Actual
Algorithmic	0.4654	0.2756
Control	0.2373	0.2346
Optimal	0.3557	0.2593
Random	0.2678	0.2559

- **Validated improvement from control to algorithmic is 17.47%!**

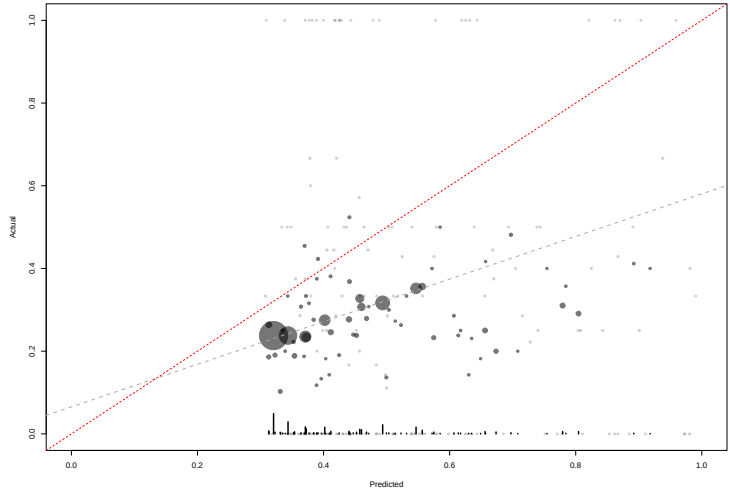
(in)Stability of preferences



Did it work?



Did it work?



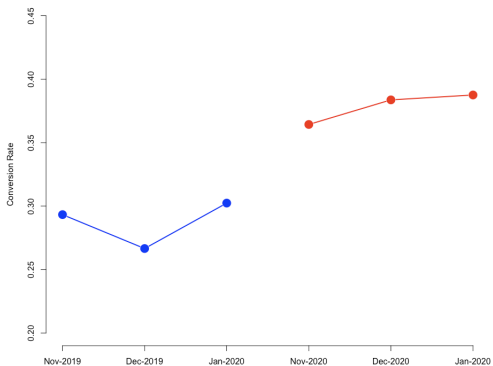
Español?

Hola [name], somos GetCalFresh! Para mantener sus beneficios de CalFresh el mes que viene, tiene que completar una renovación antes del [date]. Es necesario aun si su situación no ha cambiado. Hagalo ahora en: [link]

- We tested control vs. a single optimal message
- Translation done by Code for America team
- Results: $\mu_{\text{control}} = 0.3049$ vs. $\mu_{\text{optimal}} = 0.3690$ (sig at $\alpha = 0.05$)

Continuing performance

- Continued testing the solution
  - December and Jan exhibited decent results ($\approx$+30%)
  - Feb not as great - not sure why
- Year over year results look great!



Valuing Algorithmic Nudges

- **Idea:** Mullainathan and Shafir (2013) articulate a "bandwidth tax" that suggests that poor individuals more likely to fail to undertake high value activities.
- Estimator using validation (November)

$$\Delta = \left[\frac{\sum_{i \in T_{\text{alg}}} \widehat{B}_i \times 1\left[t_i = \text{alg}\right]}{\Pr\left(t_i = \text{alg}\right)} \right] - \left[\frac{\sum_{i \in T_{\text{control}}} \widehat{B}_i \times 1\left[t_i = \text{control}\right]}{\Pr\left(t_i = \text{control}\right)} \right]$$

- $\widehat{B}_i$ taken as max SNAP benefit conditional on family size.

$$\$\Delta = \$193,503.8/\text{month}$$

- Assuming they stay on SNAP for next **6 months** and using **12 cohorts** we get

$$\$\Delta_{\textbf{year}} \approx \$13.93 \text{ million}$$

- **For** $(\textbf{Income} = 0)$

$$\$\Delta_{\textbf{year}} \approx \$7.09 \textbf{ million}$$

Valuing Personalization

- To scale this consider that there are 41,500,000 SNAP recipients in the US (13% of US Population)
  - Assume $220 per recipient and 12 months on SNAP
  - Assume same attrition rates as in CA sample
- The total incremental disbursement from algorithmic nudges (relative to control) will be

$$\$\Delta_{\text{year}} \approx \$4.49 \textbf{ Billion}$$

The total incremental disbursement from algorithmic nudges (relative to optimal) will be

$$\$\Delta_{\text{year}} \approx \$1.75 \textbf{ Billion}$$

Discussion: Who does personalization help?

- If we want to answer the question we need to consider
- Consider the set of characteristics that describe the population (x)
- And let it have some distribution in the population $f(x)$ [before any intervention]
- Now consider two interventions (control,algorithmic)
- Now as: Is $f(x|\text{Alg}, y = 1)$ or $f(x|\text{Ctrl}, y = 1)$ closer to the $f(x)$?
- Think of the implication...

We can make this formal...

- Let f be the distribution of recipient characteristics $\mathbf{x}$. Or $f(\mathbf{x})$
- Denote by f_t the distribution among those who take-up under that policy. Or $f_t(\mathbf{x}|y=1)$
- A policy t is deemed fair if the ϕ-divergence between f and f_t is is zero. (FLM 2020)
- We use

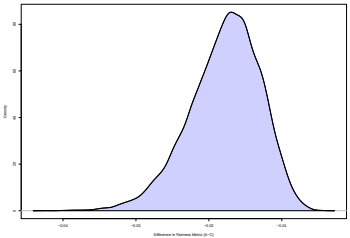
$$\begin{aligned}\mathbb{D}_{\mathsf{KL}}(f \parallel f_t) &= \int f(x) \ln \frac{f(x)}{f_t(\mathbf{x}|y=1)} dx \\ &= \int \ln\left(\frac{P(y=1)}{P(y=1|x)}\right) f(x)\, dx\end{aligned}$$

- The differential impact is simply

$$\Delta(f_t, f_{t'}|f) = [\mathbb{D}_{\mathsf{KL}}(f \parallel f_t) - \mathbb{D}_{\mathsf{KL}}(f \parallel f_{t'})]$$

- Measure is inspired by the targeting literature (Finkelstein and Notowidigdo 2019; Bertrand, Mullainathan, and Shafir 2004; Mani et al. 2013; Mullainathan and Shafir 2013).

Differential Impact



- The algorithmic nudge policy is fairer than the control nudge.
- ... and the optimal uniform nudge.

$$\mathbb{D}_{\mathsf{KL}}\left(f \parallel f_{\mathsf{alg}}\right) < \mathbb{D}_{\mathsf{KL}}\left(f \parallel f_{\mathsf{opt}}\right) < \mathbb{D}_{\mathsf{KL}}\left(f \parallel f_{\mathsf{control}}\right)$$

- Implies that algorithmic nudges are "fairer".

Subgroup Analysis: Income

- We consider the large subgroup where $Income = 0$
- This is 53.74% of the sample in October and 50.79% in November
- The benefits of the algorithmic approach are stark

$$\begin{aligned}\Delta &= \mu_{\text{alg}} - \mu_{\text{control}} \\ &= 0.2539 - 0.1980 \\ &= 0.0559\end{aligned}$$

- Difference is significant at the $\alpha = .05$ level
- Note that the sample take-up average is 0.2640.
- For $Income > 0$ we have $\Delta = 0.2985 - 0.2720 = 0.0265$ (ns)

Discussion: Measuring Welfare

- As policies become more personalized we need to re-visit the issue of aggregation of welfare.
- Consider the pricing study I discussed earlier...
- Atkinson (1970) proposes

$$W(\mathbf{p}) = \left[\frac{1}{N} \sum \mathbb{E}\left(V(p, x_i) \right)^r \right]^{1/r}$$

- Different values of r reflect different levels of inequality aversion

Discussion: Measuring Welfare

Table 8: Consumer Welfare and Data based Pricing

(a) Comparing Theoretically Optimal Pricing Policies

Measure	r	$S_r\left(\mathbf{p}^{pers}\right)$	$S_r\left(\mathbf{p}^{unif}\right)$	$\Delta = S_r\left(\mathbf{p}^{pers}\right) - S_r\left(\mathbf{p}^{unif}\right)$	$\%\Delta S_r\left(\mathbf{p}\right)$
Harmonic Mean	-1	46.8255	33.6011	13.2244	39.36
Geometric Mean	0	58.2786	57.5773	0.70127	1.22
Arithmetic Mean	$+1$	71.4094	95.2247	-23.8153	-25.01

(b) Implemented Personalized vs. Optimal Uniform

Measure	r	$S_r\left(\mathbf{p}^{pers}\right)$	$S_r\left(\mathbf{p}^{unif}\right)$	$\Delta = S_r\left(\mathbf{p}^{pers}\right) - S_r\left(\mathbf{p}^{unif}\right)$	$\%\Delta S_r\left(\mathbf{p}\right)$
Harmonic Mean	-1	50.1969	33.6011	16.5958	49.39
Geometric Mean	0	67.3144	57.5773	0.7371	16.91
Arithmetic Mean	$+1$	93.2841	95.2247	-1.9406	-2.04

(c) Implemented Personalized vs. Implemented Uniform Pricing Policies

Measure	r	$S_r\left(\mathbf{p}^{pers}\right)$	$S_r\left(\mathbf{p}^{unif}\right)$	$\Delta = S_r\left(\mathbf{p}^{pers}\right) - S_r\left(\mathbf{p}^{unif}\right)$	$\%\Delta S_r\left(\mathbf{p}\right)$
Harmonic Mean	-1	50.1969	43.4767	6.7202	15.46
Geometric Mean	0	67.3144	68.1889	-0.8745	-1.28
Arithmetic Mean	$+1$	93.2841	105.3496	-12.0656	-11.45

Discussion

- **Policy:**
  - Definitions of market need to be reconsidered
  - Suggests direct implications for policies around data and privacy.
  - Need to consider the welfare and fairness "losses" as the price of privacy.
  - Solutions such as differential privacy can be adopted but at some cost (especially to those that are truly vulnerable)
- **Generalization:**
  - Nothing special about the contexts we have studied.
  - The fame-work applies quite generally
- **Implications**
  - Personalization can be "good"
  - Consider distributive impact of decisions