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THE DIFFUSION OF WAL-MART AND ECONOMIES OF DENSITY

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THE DIFFUSION OF WAL-MART AND ECONOMIES OF DENSITY

BY THOMAS J. HOLMES¹

The rollout of Wal-Mart store openings followed a pattern that radiated from the center outward, with Wal-Mart maintaining high store density and a contiguous store network all along the way. This paper estimates the benefits of such a strategy to Wal-Mart, focusing on the savings in distribution costs afforded by a dense network of stores. The paper takes a revealed preference approach, inferring the magnitude of density economies from how much sales cannibalization of closely packed stores Wal-Mart is willing to suffer to achieve density economies. The model is dynamic with rich geographic detail on the locations of stores and distribution centers. Given the enormous number of possible combinations of store-opening sequences, it is difficult to directly solve Wal-Mart's problem, making conventional approaches infeasible. The moment inequality approach is used instead and works well. The estimates show the benefits to Wal-Mart of high store density are substantial and likely extend significantly beyond savings in trucking costs.

KEYWORDS: Economies of density, moment inequalities, dynamics.

1. INTRODUCTION

WAL-MART OPENED ITS FIRST STORE in 1962, and today there are over 3,000 Wal-Mart stores in the United States. The rollout of stores illustrated in Figure 1 displays a striking pattern. (See also a video of the rollout posted as Supplemental Material for this article (Holmes (2011)).²) Wal-Mart started in a relatively central spot in the country (near Bentonville, Arkansas) and store openings radiated from the inside out. Wal-Mart never jumped to some far-off location to later fill in the area in between. With the exception of store number 1 at the very beginning, Wal-Mart always placed new stores close to where it already had store density.

This process was repeated in 1988 when Wal-Mart introduced the supercenter format (see Figure 2). With this format, Wal-Mart added a full-line grocery store alongside the general merchandise of a traditional Wal-Mart. Again, the diffusion of the supercenter format began at the center and radiated from the inside out.

¹I have benefited from the comments of many seminar participants. In particular, I thank Glenn Ellison, Gautam Gowrisankaran, and Avi Goldfarb for their comments as discussants, Pat Bajari and Kyoo-il Kim for suggestions, and Ariel Pakes for advice on how to think about this problem. I thank the referees and the editor for comments that substantially improved the paper. I thank Junichi Suzuki, Julia Thornton Snider, David Molitor, and Ernest Berkas for research assistance. I thank Emek Basker for sharing data. I am grateful to the National Science Foundation under Grant 0551062 for support of this research. The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

²The video of Wal-Mart's store openings can also be seen at www.econ.umn.edu/~holmes/research.html.

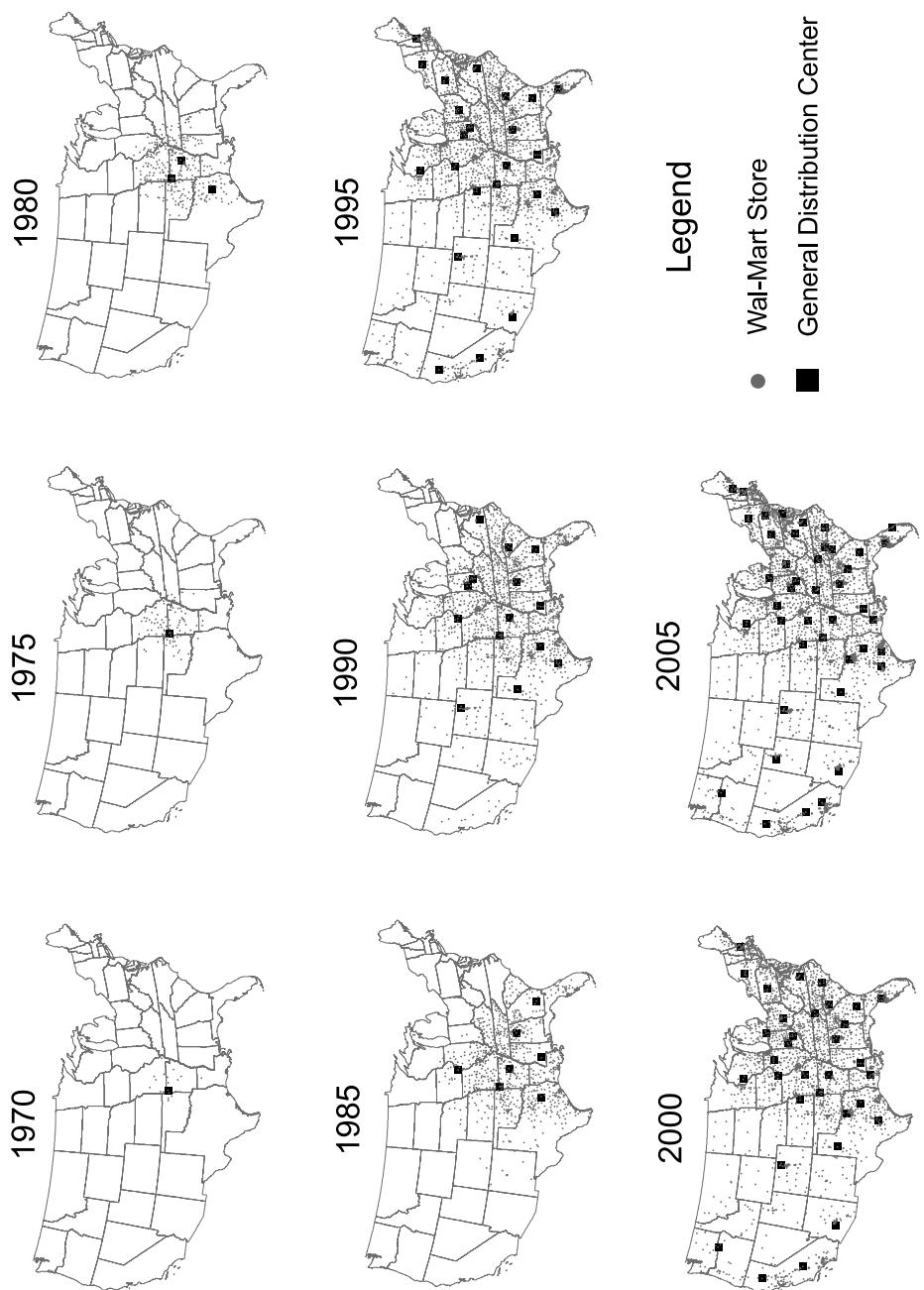


FIGURE 1.—Diffusion of Wal-Mart stores and general distribution centers.

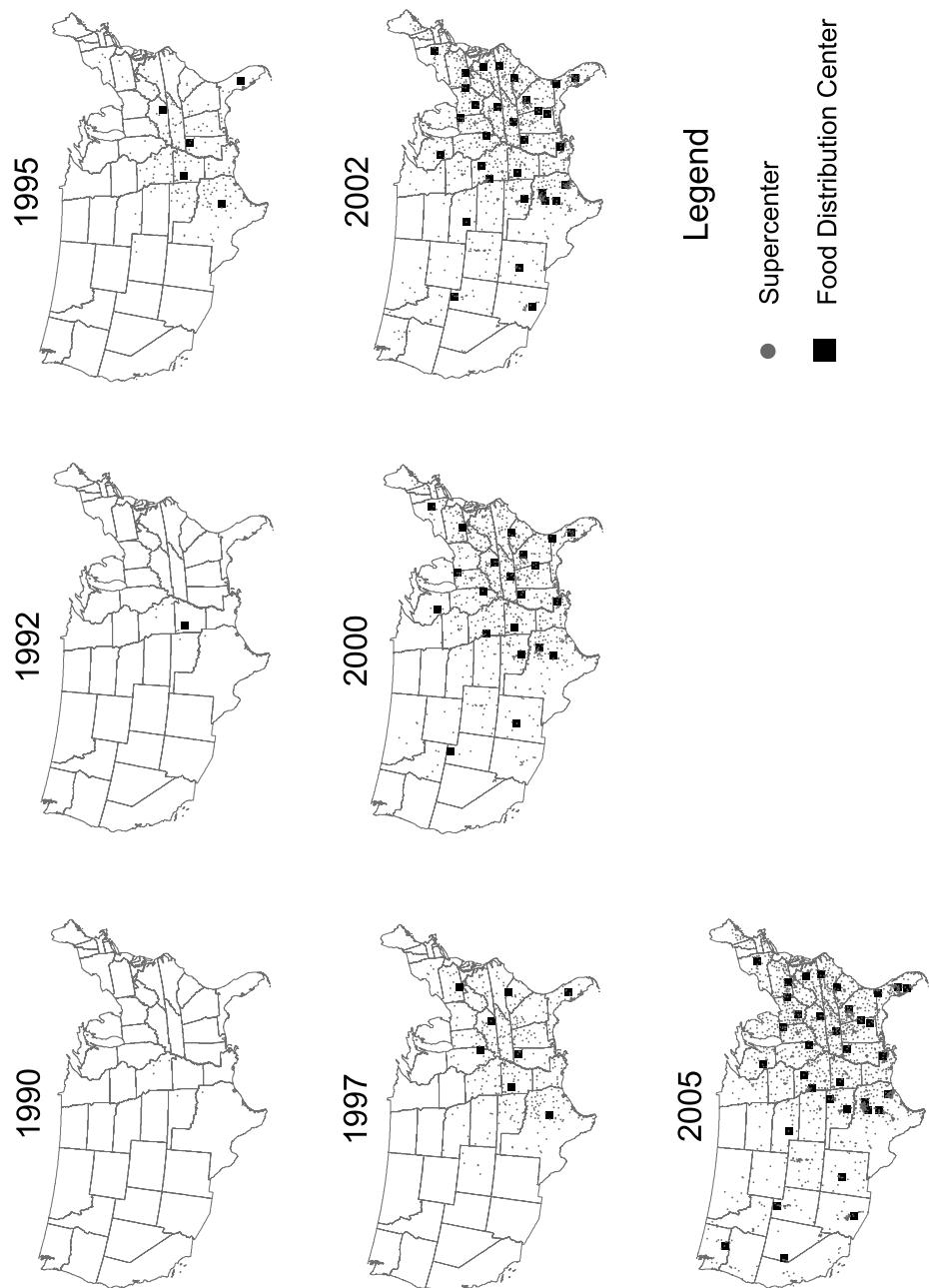


FIGURE 2.—Diffusion of supercenters and food distribution centers.

This paper estimates the benefits of such a strategy to Wal-Mart, focusing on the logistic benefits afforded by a dense network of stores. Wal-Mart is vertically integrated into distribution: general merchandise is supplied by Wal-Mart's own regional distribution centers; groceries for supercenters are supplied through its own food distribution centers.³ When stores are packed closely together, it is easier to set up a distribution network that keeps stores close to a distribution center, and when stores are close to a distribution center, Wal-Mart can save on trucking costs. Moreover, such proximity allows Wal-Mart to respond quickly to demand shocks. Quick response is widely considered to be a key aspect of the Wal-Mart model. (See Holmes (2001) and Ghemawat, Mark, and Bradley (2004).) Wal-Mart famously was able to restock its shelves with American flags on the very day of 9/11.

A challenge in estimating the benefits of density is that Wal-Mart is notorious for being secretive. I cannot access confidential data on its logistics costs, so it is not possible to conduct a direct analysis relating costs to density. Even if Wal-Mart were to cooperate and make its data available, the benefits of being able to quickly respond to demand shocks might be difficult to quantify with standard accounting data. Instead, I pursue an indirect approach that exploits revealed preference. Although density has a benefit, it also has a cost, and I am able to pin down that cost. By examining Wal-Mart's choice behavior of how it trades off the benefit (not observed) versus the cost (observed with some work), it is possible to draw inferences about how Wal-Mart values the benefits.

The cost of high store density is that when stores are close together, their market areas overlap and new stores cannibalize sales from existing stores. The extent of such cannibalization is something I can estimate. For this purpose, I bring together store-level sales estimates from ACNielsen and demographic data from the U.S. Census at a very fine level of geographic detail. I use this information to estimate a model of demand in which consumers choose among all the Wal-Mart stores in the general area where they live. The demand model fits the data well, and I am able to corroborate its implications for the extent of cannibalization with certain facts Wal-Mart discloses in its annual reports. Using my sales model, I determine that Wal-Mart has encountered significant diminishing returns in sales as it has packed stores closely together in the same area.

I write down a dynamic structural model of how Wal-Mart rolled out its stores over the period 1962–2005. The model is quite detailed and distinguishes the exact location of each individual store, the location of each distribution center, the type of store (regular Wal-Mart or supercenter), and the kind of distribution center (general merchandise or food). The model takes into account wage and land price differences across locations. The model takes into account that although there might be benefits of high *store density* to Wal-Mart, there

³ According to Ghemawat, Mark, and Bradley (2004), over 80 percent of what Wal-Mart sells goes through its own distribution network.

also might be disadvantages of high *population density*—beyond high wages and land prices—because the Wal-Mart model might not work so well in very urban locations.

Given the enormous number of different possible combinations of store-opening sequences, it is difficult to directly solve Wal-Mart's optimization problem. This leads me to consider a perturbation approach that rules out deviations from the chosen policy as being nonoptimal. When the choice set is continuous, a perturbation approach typically implies equality constraints (i.e., first-order conditions). Here, with discrete choice, the approach yields inequality constraints. To average out measurement error, I aggregate the inequalities into moment inequalities; for inference, I follow Pakes, Porter, Ho, and Ishii (2006).

Identification is partial, that is, there is a *set* of points satisfying the moment inequalities, rather than just a single point. There have been significant developments in the partial identification literature. (See Manski (2003) for a monograph treatment.) Much of the recent interest is driven by its application to game-theoretic models with multiple equilibria (e.g., Ciliberto and Tamer (2009)). The possibility of multiple equilibria is not an issue in the decision-theoretic environment considered here. Nevertheless, the moment inequality approach is useful because of the discrete choice nature of the problem. A concern with any partial identification approach is that the identified set may potentially be so wide as to be relatively uninformative. In practice, this concern should be ameliorated by the imposition of a large number of constraints that narrow the identified set to a tight region. This is the case here.

For my baseline specification with the full set of constraints imposed, I estimate that when a Wal-Mart store is closer by 1 mile to a distribution center, over the course of a year, Wal-Mart enjoys a benefit that lies in a tight interval around \$3,500. This estimate extends significantly beyond likely savings in trucking costs alone. Given the many miles involved in Wal-Mart's operations and its thousands of stores, the estimate implies that economies of density are a substantial component of Wal-Mart profitability.

An economy of density is a kind of economy of scale. Over the years, various researchers have made distinctions among types of scale economies and noted the role of density. For the airline industry, Caves, Christensen, and Tretheway (1984) distinguished an economy of density from traditional economies of scale as arising when an airline increases the frequency of flights on a given route structure (as opposed to increasing the size of the route structure, holding fixed the flight frequency per route). (See also Caves, Christensen, and Swanson (1981).) The analogy here would be Wal-Mart expanding by adding more stores in the same markets it already serves (as opposed to expanding its geographic reach and keeping store density the same). Roberts (1986) made an analogous distinction in the electric power industry. This paper differs from the existing empirical literature in three ways. First, there is rich micromodeling with an explicit spatial structure. I do not have lumpy market units (e.g.,

a metro area) within which I count stores; rather, I employ a continuous geography. Second, I explicitly model the channel of the density benefits through the distribution system, rather than leaving them as a “black box.” Third, rather than directly relate costs to density, I use a revealed preference approach as explained above. (See also Holmes and Lee (2009).)

There is a large literature on entry and store location in retail.⁴ There is also a growing literature on Wal-Mart itself.⁵ This paper is most closely related to the recent parallel work of Jia (2008).⁶ Jia estimated density economies by examining the site selection problem of Wal-Mart as the outcome of a static game with K-Mart. Jia’s paper features interesting oligopolistic interactions that my paper abstracts away from. My paper highlights (i) dynamics and (ii) cannibalization of sales by nearby Wal-Marts that Jia’s paper abstracts away from.

2. MODEL

A retailer (Wal-Mart) has a network of stores supported by a network of distribution centers. The model specifies how Wal-Mart’s revenues and costs in a period depend on the configuration of stores and distribution centers that are open in the period. It also specifies how the networks change over time.

There are two categories of merchandise: *general merchandise* (abbreviated by g) and *food* (abbreviated by f). There are two kinds of Wal-Mart stores: A *regular store* sells only general merchandise; a *supercenter* sells both general merchandise and food.

There is a finite set of locations in the economy. Locations are indexed by $\ell = 1, \dots, L$. Let $d_{\ell\ell'}$ denote the distance in miles between any given pair of locations ℓ and ℓ' . At any given period t , a subset $\mathcal{B}_t^{\text{Wal}}$ of locations have a Wal-Mart. Of these, a subset $\mathcal{B}_t^{\text{Super}} \subseteq \mathcal{B}_t^{\text{Wal}}$ are supercenters and the rest are regular stores. In general, the number of locations with Wal-Marts will be small relative to the total set of locations, and a typical Wal-Mart will draw sales from many locations.

Sales revenues at a particular store depend on the store’s location and its proximity to other Wal-Marts. Let $R_{jt}^g(\mathcal{B}_t^{\text{Wal}})$ be the general merchandise sales revenue of store j at time t given the set of Wal-Mart stores open at time t . If store j is a supercenter, then its food sales $R_{ji}^f(\mathcal{B}_t^{\text{Super}})$ analogously depend on the configuration of supercenters. The model of consumer choice from which this demand function will be derived will be specified below in Section 4. With this demand structure, Wal-Mart stores that are near each other will be regarded as substitutes by consumers. That is, increasing the number of nearby stores will decrease sales at a particular Wal-Mart.

⁴See, for example, Bresnahan and Reiss (1991) and Toivanen and Waterson (2005).

⁵Recent papers on Wal-Mart include Basker (2005, 2007), Stone (1995), Hausman and Leibtag (2007), Ghemawat, Mark, and Bradley (2004), and Neumark, Zhang, and Ciccarella (2008).

⁶See also Andrews, Berry, and Jia (2004).

I abstract from price variation and assume Wal-Mart sets constant prices across all stores and over time. In reality, prices are not always constant across Wal-Marts, but the company's every day low price (EDLP) policy makes this a better approximation for Wal-Mart than it would be for many retailers. Let μ denote the gross margin. Thus, for store j at time t , $\mu R_{jt}^g(\mathcal{B}_t^{\text{Wal}})$ is sales receipts less the cost of goods sold for general merchandise.

In the analysis, three components of cost will be relevant besides the cost of goods sold: (i) distribution costs, (ii) variable store costs, and (iii) fixed costs at the store level. I describe each in turn.

Distribution Costs

Each store requires distribution services. General merchandise is supplied by a general distribution center (GDC) and food is supplied by a food distribution center (FDC). For each store, these services are supplied by the closest distribution center. Let d_{jt}^g be the distance in miles from store j to the closest GDC at time t and analogously define d_{jt}^f . If store j is a supercenter, its distribution cost at time t is

$$\text{DistributionCost}_{jt} = \tau d_{jt}^g + \tau d_{jt}^f,$$

where the parameter τ is the cost per mile per period per merchandise segment (general or food) of servicing this store.⁷ If j carries only general merchandise, the cost is τd_{jt}^g .

The distribution cost is a fixed cost that does not depend on the volume of store sales. This would be an appropriate assumption if Wal-Mart made a single delivery run from the distribution center to the store each day. The driver's time is a fixed cost and the implicit rental on the tractor is a fixed cost that must be incurred regardless of the size of the load. To keep a tight rein on inventory and to allow for quick response, Wal-Mart aims to have daily deliveries even for its smaller stores. So there clearly is an important fixed cost component to distribution. Undoubtedly, there is a variable cost component as well, but for simplicity I abstract from it.

Variable Costs

The larger the sales volume at a store, the greater the number of workers needed to staff the checkout lines, the larger the parking lot, the larger the required shelf space, and the bigger the building. All of these costs are treated as variable in this analysis. It may seem odd to treat building size and shelving as a variable input. However, Wal-Mart very frequently updates and expands its

⁷More generally, the distribution costs for the two segments might differ. I constrain them to be the same because it greatly facilitates my estimation procedure later in the paper.

stores. So in practice, store size is not a permanent decision that is made once and for all; rather, it is a decision made at multiple points over time. Treating store size as a variable input simplifies the analysis significantly.

Assume that the variable input requirements at store j are all proportionate to sales volume R_j :

$$\text{Labor}_j = \nu^{\text{Labor}} R_j,$$

$$\text{Land}_j = \nu^{\text{Land}} R_j,$$

$$\text{Other}_j = \nu^{\text{Other}} R_j.$$

Wages and land prices vary across locations and across time. Let Wage_{jt} and Rent_{jt} denote the wage and land rental rate that store j faces at time t . Other consists of everything left out so far that varies with sales, including the rental on structure and equipment in the store (the shelving, the cash registers, etc.). The other cost component of variable costs is assumed to be the same across locations and the price is normalized to 1.

Fixed Costs

We expect there to be a fixed cost of operating a store. To the extent the fixed cost is the same across locations, it will play no role in the analysis of where Wal-Mart places a given number of stores. We are only interested in the component of fixed cost that varies across locations.

From Wal-Mart's perspective, urban locations have some disadvantages compared to nonurban locations. These disadvantages go beyond higher land rents and higher wages that have already been taken into account above. The Wal-Mart model of a big box store at a convenient highway exit is not applicable in a very urban location. Moreover, Sam Walton was very concerned about the labor force available in urban locations, as he explained in his autobiography (Walton (1992)).

To capture potential disadvantages of urban locations, the fixed cost of operating store j is written as a function $c(\text{Popden}_j)$ of the population density Popden_j of the store's location. The functional form is quadratic in logs,

$$(1) \quad c(\text{Popden}_j) = \omega_0 + \omega_1 \ln(\text{Popden}_j) + \omega_2 \ln(\text{Popden}_j)^2.$$

A supercenter is actually two stores, a general merchandise store and a food store, so the fixed cost is paid twice. It will be with no loss of generality in our analysis to assume that the constant term $\omega_0 = 0$, since the only component of the fixed cost that will matter in the analysis is the part that varies across locations.

Dynamics

Everything that has been discussed so far considers quantities for a particular time period. I turn now to the dynamic aspects of the model. Wal-Mart operates in a deterministic environment in discrete time where it has perfect foresight. The general problem Wal-Mart faces is to answer the following questions for each period:

Q1. How many new Wal-Marts and how many new supercenters should be opened?

Q2. Where should the new Wal-Marts and supercenters be put?

Q3. How many new distribution centers should be opened?

Q4. Where should the new distribution centers be put?

The main focus of the paper is on part Q2 of Wal-Mart's problem. The analysis conditions on Q1, Q3, and Q4 being what Wal-Mart actually did, and solves Wal-Mart's problem of getting Q2 right. Of course, if Wal-Mart's actual behavior solves the true problem of choosing Q1–Q4, then it also solves the constrained problem of choosing Q2, subject to Q1, Q3, and Q4 fixed at what Wal-Mart did.

Getting at part Q1 of Wal-Mart's problem—how many new stores Wal-Mart opens in a given year—is far afield from the main issues of this paper. In its first few years, Wal-Mart added only one or two stores per year. The number of new store openings has grown substantially over time, and in recent years they sometimes number several stores in 1 week. Presumably, capital market considerations have played an important role here. This is an interesting issue, but not one I will have anything to say about in this paper.

Problems Q3 and Q4 regarding distribution centers are closely related to the main issue of this paper. I will have something to say about this later in Section 7.

Now for more notation. To begin with, the discount factor each period is β . The period length is a year, and the discount factor is set to $\beta = .95$.

As defined earlier, $\mathcal{B}_t^{\text{Wal}}$ is the set of Wal-Mart stores in period t , and $\mathcal{B}_t^{\text{Super}} \subseteq \mathcal{B}_t^{\text{Wal}}$ is the set of supercenters. Assume that once a store is opened, it never shuts down. This assumption simplifies the analysis considerably and is not inconsistent with Wal-Mart's behavior because it rarely closes stores.⁸ Then we can write $\mathcal{B}_t^{\text{Wal}} = \mathcal{B}_{t-1}^{\text{Wal}} + \mathcal{A}_t^{\text{Wal}}$, where $\mathcal{A}_t^{\text{Wal}}$ is the set of new stores opened in period t . Analogously, a supercenter is an absorbing state, $\mathcal{B}_t^{\text{Super}} = \mathcal{B}_{t-1}^{\text{Super}} + \mathcal{A}_t^{\text{Super}}$ for $\mathcal{A}_t^{\text{Super}}$, the set of new supercenter openings in period t . A supercenter can open two ways. It can be a new Wal-Mart store that opens as a supercenter as well or it can be a conversion of an existing Wal-Mart store.

Let N_t^{Wal} and N_t^{Super} be the number of new Wal-Marts and supercenters opened at t , that is, the cardinality of the sets $\mathcal{A}_t^{\text{Wal}}$ and $\mathcal{A}_t^{\text{Super}}$. Choosing these

⁸Wal-Mart's annual reports disclose store closings that are on the order of two per year.

values is defined as part Q1 of Wal-Mart's problem. These are taken as given here. Also taken as given is the location of distribution centers of each type and their opening dates (parts Q3 and Q4 of Wal-Mart's problem).

There is exogenous productivity growth of Wal-Mart according to a growth factor ρ_t in period t . If Wal-Mart were to hold fixed the set of stores, and demographics also stayed the same, then from period 1 to period t , revenue and all components of costs would grow by (an annualized) factor ρ_t . As will be discussed later, the growth of sales per store of Wal-Mart has been remarkable. Part of this growth is due to the gradual expansion of its product line, initially from hardware and variety items to food, drugs, eyeglasses, tires, and so on. The part of growth due to food through the expansion into supercenters is explicitly modeled here, but expansion into drugs, eyeglasses, tires, and so on is not modeled explicitly. Instead, this growth is implicitly picked up through the exogenous growth parameter ρ_t . The role ρ_t plays in Wal-Mart's problem is like a discount factor.

A policy choice of Wal-Mart is a vector $a = (\mathcal{A}_1^{\text{Wal}}, \mathcal{A}_1^{\text{Super}}, \mathcal{A}_2^{\text{Wal}}, \mathcal{A}_2^{\text{Super}}, \dots)$ that specifies the locations of the new stores opened in each period t . Define a choice vector a to be feasible if the number of store openings in period t under policy a equals what Wal-Mart actually did, that is, N_t^{Wal} new stores in a period and N_t^{Super} supercenter openings. Wal-Mart's problem at time 0 is to pick a feasible a to maximize

$$(2) \quad \max_a \sum_{t=1}^{\infty} (\rho_t \beta)^{t-1} \left[\sum_{j \in \mathcal{B}_t^{\text{Wal}}} [\pi_{jt}^g - c_{jt}^g - \tau d_{jt}^g] + \sum_{j \in \mathcal{B}_t^{\text{Super}}} [\pi_{jt}^f - c_{jt}^f - \tau d_{jt}^f] \right],$$

where the operating profit for merchandise segment $e \in \{g, f\}$ at store j in time t is

$$\pi_{jt}^e = \mu R_{jt}^e - \text{Wage}_{jt} \text{Labor}_{jt}^e - \text{Rent}_{jt} \text{Land}_{jt}^e - \text{Other}_{jt}^e$$

and where d_{jt}^e is the distance to the closest distribution center at time t for merchandise segment e .

No explicit mention has been made about the presence of sunk costs. Implicitly, sunk costs are large, and that is why no store is ever closed once opened. Sunk costs can easily be worked into the model by having some portion of the present value of the fixed cost in equation (1) paid at entry rather than in perpetuity each period. This leaves the objective in equation (2) unchanged.

3. THE DATA

Five main data elements are used in the analysis. The first element is store-level data on sales and other characteristics. The second is opening dates for stores and distribution centers. The third is demographic information from the

TABLE I
SUMMARY STATISTICS OF STORE-LEVEL DATA^a

Store Type	Variable	N	Mean	Std. Dev.	Min	Max
All	Sales (\$millions/year)	3,176	70.5	30.0	9.1	166.4
Regular Wal-Mart	Sales (\$millions/year)	1,196	47.0	20.0	9.1	133.9
Supercenter	Sales (\$millions/year)	1,980	84.7	25.9	20.8	166.4
All	Employment	3,176	254.9	127.3	31.0	812.0
Regular Wal-Mart	Employment	1,196	123.5	40.1	57.0	410.0
Supercenter	Employment	1,980	333.8	91.5	31.0	812.0

^aEnd of 2005, excludes Alaska and Hawaii. Source: Trade Dimensions retail data base.

Census. The fourth is data on wages and rents across locations. The fifth is other information about Wal-Mart from annual reports.

The first data element comprising store-level variables was obtained from Trade Dimensions, a unit of ACNielsen. These data provide *estimates* of store-level sales for all Wal-Marts open at the end of 2005. These data are the best available and the primary source of market share data used in the retail industry. Ellickson (2007) recently used these data for the supermarket industry.

Table I presents summary statistics of annual store-level sales and employment for the 3,176 Wal-Marts in existence in the contiguous United States as of the end of 2005. (Alaska and Hawaii are excluded in all of the analysis.) Almost two-thirds of all Wal-Marts (1,980 out of 3,176) are supercenters that carry both general merchandise and food. The remaining 1,196 are *regular Wal-Marts* that do not have a full selection of food. The average Wal-Mart has annual sales of \$70 million. The breakdown is \$47 million per regular Wal-Mart and \$85 million per supercenter. The average employment is 255 employees.

The second data element is opening dates of the four types of Wal-Mart facilities. The table treats a supercenter as two different stores: a general merchandise store and a food store. The two kinds of distribution centers are general (GDC) and food (FDC). Table II tabulates opening dates for the four types of facilities by decade. Appendix A explains how this information was collected. Note that if a regular store is later converted to a supercenter, it has an opening date for its general merchandise store and a later opening date for its food store. This is called a *conversion*.

The third data element, demographic information, comes from three decennial censuses: 1980, 1990, and 2000. The data are at the level of the *block group*, a geographic unit finer than the Census tract. Summary statistics are provided in Table III. In 2000, there were 206,960 block groups with an average population of 1,350. I use the geographic coordinates of each block group to draw a circle of radius 5 miles around each block group. I take the population within this 5-mile radius and use this as my population density measure. Table III reports that the mean density in 2000 across block groups equals 219,000 people

TABLE II
DISTRIBUTION OF WAL-MART FACILITY OPENINGS BY DECADE AND OPENING TYPE^a

Decade Open	General Merchandise (Including Supercenters)			Food Store (Part of Supercenter)		General Distribution Centers		Food Distribution Centers	
	Opened This Decade		Cumulative	Opened This Decade		Cumulative	Opened This Decade		Cumulative
	Decade	Decade	Decade	Decade	Decade	Decade	Decade	Decade	Decade
1962–1969	15	15	0	0	1	1	0	0	0
1970–1979	243	258	0	0	1	2	0	0	0
1980–1989	1,082	1,340	4	4	8	10	0	0	0
1990–1999	1,130	2,470	679	683	18	28	9	9	9
2000–2005	706	3,176	1,297	1,980	15	43	26	35	

^aSource: See Appendix A.

within a 5-mile radius. The table also reports mean levels of per capita income, share old (65 or older), share young (21 or younger), and share black. The per capita income figure is in 2000 dollars for all the Census years, using the consumer price index (CPI) as the deflator.⁹

The fourth data element is information about local wages and local rents. The wage measure is the average retail wage by county from County Business Patterns, a data set from the U.S. Census Bureau. This measure is payroll divided by employment. I use annual data over the period 1977–2004. It is difficult to obtain a consistent measure of land rents at a fine degree of geographic detail over a long period of time. To proxy land rents, I use information about residential property values from the 1980, 1990, and 2000 decennial censuses. For each Census year and each store location, I create an index of property

TABLE III
SUMMARY STATISTICS FOR CENSUS BLOCK GROUPS^a

	1980	1990	2000
<i>N</i>	269,738	222,764	206,960
Mean population (1,000)	.83	1.11	1.35
Mean population density (1,000 in 5-mile radius)	165.3	198.44	219.48
Mean per capita income (thousands of 2000 dollars)	14.73	18.56	21.27
Share old (65 and up)	.12	.14	.13
Share young (21 and below)	.35	.31	.31
Share black	.10	.13	.13

^aSource: “Summary File 3, Census of Population and Housing” (U.S. Bureau of the Census (1980, 1990, 2000)).

⁹Per capita income is truncated from below at \$5,000 in year 2000 dollars.

values by adding up the total value of residential property within 2 miles of the store's location and scaling it so the units are in inflation-adjusted dollars per acre. See Appendix A for how the index is constructed. Interpolation is used to obtain values between Census years. The Census data are supplemented with property tax data on property valuations of actual Wal-Mart store locations in Iowa and Minnesota. As discussed in Appendix A, there is a high correlation between the tax assessment property valuations of a Wal-Mart site and the property value index.

The fifth data element is information from Wal-Mart's annual reports, including information about aggregate sales for earlier years. I also use information provided in the Management Discussion section of the reports on the degree to which new stores cannibalize sales of existing stores. The specifics of this information are explained below when the information is incorporated into the estimation.

4. ESTIMATES OF OPERATING PROFITS

This section estimates the components of Wal-Mart's operating profits. Section 4.1 specifies the demand model and Section 4.2 estimates it. Section 4.3 treats various cost parameters. Section 4.4 explains how estimates for 2005 are extrapolated to other years.

4.1. *Demand Specification*

A discrete choice approach to demand is employed, following common practice in the literature. I separate the general merchandise and food purchase decisions, and begin with general merchandise. A consumer at a particular location ℓ chooses between shopping at the "outside option" and shopping at any Wal-Mart located within 25 miles. Formally, the consumer's choice set for Wal-Marts is

$$\bar{\mathcal{B}}_{\ell}^{\text{Wal}} = \{j, j \in \mathcal{B}^{\text{Wal}} \text{ and } \text{Distance}_{\ell j} \leq 25\},$$

where $\text{Distance}_{\ell j}$ is the distance in miles between location ℓ and store location j . (The time subscript t is implicit throughout this subsection.)

If the consumer chooses the outside alternative 0, utility is

$$(3) \quad u_0 = b(\text{Popden}_{\ell}) + \text{LocationChar}_{\ell} \alpha + \varepsilon_0.$$

The first term is a function $b(\cdot)$ that depends on the Popden_{ℓ} at the consumer's location ℓ . Assume $b'(\text{Popden}) \geq 0$; that is, the outside option is better in higher population density areas. This is a sensible assumption because we would expect there to be more substitutes for a Wal-Mart in larger markets for the usual reasons. A richer model of demand would explicitly specify the alternative shopping options available to the consumer. I do not have sufficient

data to conduct such an analysis, so instead account for this mechanism in a reduced-form way.¹⁰ The functional form used in the estimation is

$$b(\text{Popden}) = \alpha_0 + \alpha_1 \ln(\underline{\text{Popden}}) + \alpha_2 (\ln(\underline{\text{Popden}}))^2,$$

where

$$(4) \quad \underline{\text{Popden}} = \max\{1, \text{Popden}\}.$$

The units of the density measure are thousands of people within a 5-mile radius. By truncating Popden at 1, $\ln(\text{Popden})$ is truncated at 0. All locations with less than 1,000 people within 5 miles are grouped together.¹¹

The second term of equation (3) allows demand for the outside good to depend on a vector LocationChar_ℓ of location characteristics that impact utility through the parameter vector α . In the empirical analysis, a location is a block group. The characteristics will include the demographic and income characteristics of the block group.

The third term of the outside good utility in equation (3) is a logit error term. Assume this is drawn independently and identically distributed (i.i.d.) across all consumers living in block group ℓ .

Next consider the utility from buying at a particular Wal-Mart $j \in \bar{\mathcal{B}}_\ell^{\text{Wal}}$. It equals

$$(5) \quad u_{\ell j} = -h(\text{Popden}_\ell) \text{Distance}_{\ell j} + \text{StoreChar}_j \gamma + \varepsilon_j$$

for $h(\text{Popden})$ parameterized by

$$h(\text{Popden}) = \xi_0 + \xi_1 \ln(\underline{\text{Popden}}).$$

The first term of equation (5) is the disutility of commuting $\text{Distance}_{\ell j}$ miles to the store from the consumer's home. The second term of equation (5) allows utility to depend on an additional characteristic StoreChar_j of store j . In the empirical analysis, this characteristic will be store age. In this way, the demand model will capture that brand-new stores have fewer sales, everything else being the same. The last term is the logit error ε_j .

The probability $p_{j\ell}^g$ a consumer at location ℓ shops at store j can be derived using the standard logit formula. The model's predicted general merchandise revenue for store j is then

$$(6) \quad R_j^g = \sum_{\{\ell | j \in \bar{\mathcal{B}}_\ell^{\text{Wal}}\}} \lambda^g \times p_{j\ell}^g \times n_\ell,$$

¹⁰This is in the spirit of recent work such as Bajari, Benkard, and Levin (2007), who estimated policy functions and equilibrium relationships directly.

¹¹This same truncation is applied throughout the paper.

where λ^g is spending per consumer. In words, there are n_ℓ consumers at location ℓ and a fraction $p_{j\ell}^g$ of them are shopping at store j where they will each spend λ^g dollars.

Spending on food is modeled the same way. The parameters are the same except for the spending λ^f per consumer. The formula for food revenue R_j^f at store j is analogous to (6). Note that when calculating food revenue, it is necessary to take into account that the set of alternatives for food $\bar{\mathcal{B}}_\ell^{\text{Super}}$ is, in general, different from the set of alternatives $\bar{\mathcal{B}}_\ell^{\text{Wal}}$ for general merchandise.

4.2. Demand Estimation

Recent empirical papers on demand typically use data sets with quantities directly determined from sales records. In these analyses, quantities are treated as being measured without error. Following Berry, Levinsohn, and Pakes (1995), demand models are estimated to perfectly fit these sales data, with unobserved product characteristics soaking up discrepancies. In contrast, the store-level sales information used here is estimated by Trade Dimensions, using proprietary information it has acquired. There is certainly measurement error in these estimates that needs to be incorporated into the demand estimation. For simplicity, I attribute all of the discrepancies between the model and the data to classical measurement error.

Given a vector ψ of parameters from the demand model, we can plug in the demographic data and obtain predicted values of general merchandise sales $R_j^g(\psi)$ for each store j from equation (6) and predicted values of food sales $R_j^f(\psi)$. Let R_j^{Data} be the sales volume in the data. Let η_j^{Sales} be the discrepancy between measured log sales and predicted log sales. For a regular store, this equals

$$\eta_j^{\text{Sales}} = \ln(R_j^{\text{Data}}) - \ln(R_j^g(\psi)).$$

For a supercenter, this equals

$$\eta_j^{\text{Sales}} = \ln(R_j^{\text{Data}}) - \ln(R_j^g(\psi) + R_j^f(\psi)).$$

Assume the η_j^{Sales} are i.i.d. normally distributed. The model is estimated using maximum likelihood, and the coefficients are reported in Table IV in the column labeled “Unconstrained.”

Getting right the extent to which new stores cannibalize sales of existing stores is crucial for the subsequent analysis. Fortunately, Wal-Mart has provided information that is helpful in this regard. Wal-Mart’s annual report

TABLE IV
PARAMETER ESTIMATES FOR DEMAND MODEL

Parameter	Definition	Unconstrained	Constrained (Fits Reported Cannibalization)
λ^g	General merchandise spending per person (annual in \$1,000)	1.686 (.056)	1.938 (.043)
λ^f	Food spending per person (annual in \$1,000)	1.649 (.061)	1.912 (.050)
ξ_0	Distance disutility (constant term)	.642 (.036)	.703 (.039)
ξ_1	Distance disutility (coefficient on $\ln(\underline{\text{Popden}})$)	−.046 (.007)	−.056 (.008)
α	Outside alternative valuation parameters		
	Constant	−8.271 (.508)	−7.834 (.530)
	$\ln(\underline{\text{Popden}})$	1.968 (.138)	1.861 (.144)
	$\ln(\underline{\text{Popden}})^2$	−.070 (.012)	−.059 (.013)
	Per capita income	.015 (.003)	.013 (.003)
	Share of block group black	.341 (.082)	.297 (.076)
	Share of block group young	1.105 (.464)	1.132 (.440)
	Share of block group old	.563 (.380)	.465 (.359)
γ	Store-specific parameters		
	Store age 2 + dummy	.183 (.024)	.207 (.023)
σ^2	Measurement error	.065 (.002)	.065 (.002)
N		3,176	3,176
Sum of squared error		205.117	206.845
R^2 (Likelihood)		.755 −155.749	.753 −169.072

for 2004 disclosed (Wal-Mart Stores, Inc. (1971–2006) (Annual Report 2004, p. 20)) the following information:

As we continue to add new stores domestically, we do so with an understanding that additional stores may take sales away from existing units. We estimate that comparative store

sales in fiscal year 2004, 2003, 2002 were negatively impacted by the opening of new stores by approximately 1%.

This same paragraph was repeated in the 2006 annual report with regard to fiscal year 2005 and 2006. This information is summarized in Table V.¹²

To define the model analog of cannibalization, first calculate what sales would be in a particular year for preexisting stores if no new stores were opened in the year and if there were no new supercenter conversions. Next calculate predicted sales to preexisting stores when the new store openings and supercenter conversions for the particular year take place. Define the percentage difference to be the *cannibalization rate* for that year. This is the model analog of what Wal-Mart is disclosing.

Table V reports the cannibalization rates for various years using the estimated demand model. The parameter vector is the same across years. What varies over time are the new stores, the set of preexisting stores, and the demographic variables. The demand model—estimated entirely from the 2005 cross-sectional store-level sales data—does a very good job fitting the cannibalization rates reported by Wal-Mart. For the years that Wal-Mart disclosed that the rate was “approximately 1%,” the estimated rates range from .67 to 1.43. It is interesting to note the sharp increase in the estimated cannibalization rate beginning in 2002. Evidently, Wal-Mart reached some kind of saturation point in 2001. Given the pattern in Table V, it is understandable that Wal-Mart has felt the need to disclose the extent of cannibalization in recent years.

TABLE V
CANNIBALIZATION RATES, FROM ANNUAL REPORTS AND IN MODEL^a

Year	From Annual Reports	Demand Model (Unconstrained)	Demand Model (Constrained)
1998	n.a.	.62	.48
1999	n.a.	.87	.67
2000	n.a.	.55	.40
2001	1	.67	.53
2002	1	1.28	1.02
2003	1	1.38	1.10
2004	1	1.43	1.14
2005	1	1.27	1.00 ^b

^aSource: Estimates from the model and Wal-Mart Stores, Inc. (1971–2006) (Annual Reports 2004, 2006).

^bCannibalization rate is imposed to equal 1.00 in 2005.

¹²Wal-Mart's fiscal year ends January 31, so the fiscal year corresponds (approximately) to the previous calendar year. For example, the 2006 fiscal year began February 1, 2005. In this paper, I aggregate years like Wal-Mart (February through January), but I use 2005 to refer to the year beginning February 2005.

In what follows, the estimated upper bound on the degree of density economies will be closely connected to the degree of cannibalization. The more cannibalization Wal-Mart is willing to tolerate, the higher the inferred density economies. The estimated cannibalization rates of 1.38, 1.43, and 1.27 for 2003, 2004, and 2005 qualify as approximately 1, but one may worry that these rates are on the high end of what would be consistent with Wal-Mart's reports. To explore this issue further, I estimate a second demand model where the cannibalization rate for 2005 is constrained to be exactly 1. The estimates are reported in the last column of Table IV. The goodness of fit under the constraint is close to the unconstrained model, although a likelihood ratio test leads to a rejection of the constraint. In the interests of being conservative in my estimate of a lower bound on density economies, I will use the constrained model throughout as the baseline model.

The parameter estimates reveal that, as hypothesized, the outside good is better in more dense areas and that utility decreases in distance traveled to a Wal-Mart. To get a sense of the magnitudes, Table VI examines how predicted demand in a block group varies with population density and distance to the closest Wal-Mart, with the demographic variables in Table III set to their mean values, and with only one Wal-Mart (2 or more years old) in the consumer choice set. Consider the first row, where the distance is zero and the population density is varied. The negative effect of population density on demand is substantial. A rural consumer right next to a Wal-Mart shops there with a probability that is essentially 1. At a population density of 50,000, this falls to .72 and falls to only .24 at 250,000. The model captures in a reduced-form way that in a large market, there tend to be many substitutes compared to a small market. A rural consumer who happens to live 1 mile from a Wal-Mart is unlikely to have many other choices. In contrast, an urban consumer who lives 1 mile from a Wal-Mart is likely to have many nearby discount-format stores to

TABLE VI
COMPARATIVE STATICS WITH DEMAND MODEL^a

Distance (Miles)	Population Density (Thousands of People Within a 5-Mile Radius)						
	1	5	10	20	50	100	250
0	.999	.989	.966	.906	.717	.496	.236
1	.999	.979	.941	.849	.610	.387	.172
2	.997	.962	.899	.767	.490	.288	.123
3	.995	.933	.834	.659	.372	.206	.086
4	.989	.883	.739	.531	.268	.142	.060
5	.978	.803	.615	.398	.184	.096	.041
10	.570	.160	.083	.044	.020	.011	.006

^aUses constrained model.

choose among and, in addition, have nearby substitute formats like a Best Buy, Home Depot, or shopping mall.

Next consider the effect of distance, holding population density fixed. In a very rural area, increasing distance from 0 to 5 miles has only a small effect on demand. This is exactly what we would expect. Raising the distance further from 5 to 10 miles has an appreciable effect, .98 to .57, but still much demand remains. Contrast this with higher density areas. At a population density of 250,000, an increase in distance from 0 to 5 miles reduces demand on the order of 80 percent.¹³ A higher distance responsiveness in urban areas is just what we would expect.

A few remarks about the remaining parameter estimates are in order. Recall that λ^g and λ^f are spending per consumer in the general merchandise and food categories. The estimates can be compared to aggregate statistics. For 2005, per capita spending in the United States was 1.8 in general merchandise stores (North American Industry Classification System [NAICS] 452) and 1.8 in food and beverage stores (NAICS 445) (in thousands of dollars). The aggregate statistics match the model estimates well ($\lambda^g = 1.9$ and $\lambda^f = 1.9$ in the constrained model; $\lambda^g = 1.7$ and $\lambda^f = 1.6$ in the unconstrained model), although for various reasons we would not expect them to match exactly. The only store characteristic used in the demand model (besides location) is store age. This is captured with a dummy variable for stores that have been open 2 or more years. This variable enters positively in demand, so everything else being the same, older stores attract more sales.

4.3. Variable Costs at the Store Level

In the model, required labor input at the store level is assumed to be proportionate to sales. In the data, on average there are 3.61 store employees per million dollars of annual sales. I use this as the estimate of the fixed labor coefficient, $\nu^{\text{Labor}} = 3.61$. In the empirical work below, I examine the sensitivity of the results to this calibrated parameter and to the other calibrated parameters.

To determine the cost of labor at a particular store, the coefficient ν^{Labor} is multiplied by average retail wage (annual payroll per worker) in the county where the store is located. Table VII reports information about the distribution of labor costs across the 2005 set of Wal-Mart stores. The median store faces a labor cost of \$21,000 per worker. Given $\nu^{\text{Labor}} = 3.61$, this translates into a labor cost of $3.61 \times 21,000$ per million in sales or, equivalently, 7.5 percent of sales. The highest labor costs can be found at stores in San Jose, California, where wages are almost twice as high as they are for the median store.

¹³The reader may note that the distance coefficient ξ_1 on $\ln(m)$ in Table IV is actually slightly negative. This effect is overwhelmed by the nonlinearity in the logit model combined with the impact of density on the outside utility. It is possible to rescale units so distance cost is constant or increasing in density and get the same implied demand structure.

TABLE VII
DISTRIBUTION OF VARIABLE INPUT COSTS^a

Estimated 2005 Labor Costs			
Quartile	Store Location	Annual Payroll per Worker (\$)	Wages as Percentage of Sales
Minimum	Pineville, MO	12,400	4.5
25	Litchfield, IL	19,300	7.0
50	Belleville, IL	21,000	7.6
75	Miami, FL	23,000	8.3
Maximum	San Jose, CA	37,900	13.7

Estimated 2005 Land Value–Sales Ratios			
Quartile	Store Location	Index of Residential Property Value per Acre (\$)	Land Value as Percentage of Sales
Minimum	Lincoln, ME	1,100	.0
25	Campbellsville, KY	32,100	1.2
50	Cleburne, TX	67,100	2.4
75	Albany, NY	137,300	5.0
Maximum	Mountain View, CA	1,800,000	65.0

^aPercentiles of distribution are weighted by sales revenue.

An issue that needs to be raised about the County Business Patterns wage data is measurement error. Dividing annual payroll by employment is a crude way to measure labor costs because it does not take into account potential variations in hours per worker (e.g., part time versus full time) or potential variations in labor quality. The empirical procedure used below explicitly takes into account measurement error.

Turning now to land costs, Appendix A describes the construction of a *property value index* for each store through the use of Census data. As discussed in the Appendix, this index, along with property tax data for 46 Wal-Mart locations in Minnesota and Iowa, is used to estimate a *land value to sales ratio* for each store. The distributions of this index and ratio are reported in Table VII. Perhaps not surprisingly, the most expensive location is estimated to be the Wal-Mart store in Silicon Valley (in Mountain View, California), where the ratio of the land value for the store relative to annual store sales is estimated to be 65 percent. The rental cost of the land, including any taxes that vary with land value, is assumed to be 20 percent of the land value. For the median store from Table VII (the Wal-Mart in Cleburne, Texas), this implies annual land costs of about half a percent of sales ($.5 \approx .2 \times 2.4$). It is important to emphasize that this rental cost is for the land, not structures. (Half of a percent of sales would be a very low number for the combined rent on land and struc-

tures and equipment.) The rents on structures and equipment are separated out because they should be approximately the same across locations, as least as compared to variations across stores in land rents. The cost of cinderblocks for walls, steel beams for roofing, shelving, cash registers, asphalt for parking lots, and so on are all assumed to be the same across locations.

I now turn to those aspects of variable costs that are the same across locations. I begin with cost of goods sold. Wal-Mart's gross margin over the years has ranged from .22 to .26. (See Wal-Mart's annual reports.) To be consistent with this, the gross margin is set equal to $\mu = .24$.

Over the years, Wal-Mart has reported *operating, selling, general, and administrative expenses* that are in the range of 16–18 percent of sales. Included in this range is the store-level labor cost discussed above, which is on the order of 7 percent of sales and has already been taken into account. Also included is the cost of running the distribution system, the fixed cost of running central administration, and other costs that I do not want to include as variable costs. I set the residual variable cost parameter $\nu^{\text{Other}} = .07$. Netting this out of the gross margin μ yields a net margin $\mu - \nu^{\text{Other}} = .17$. In the analysis, the breakdown between μ and ν^{Other} is irrelevant; only the difference matters.

The analysis so far has explained how to calculate the operating profit of store j in 2005 as

$$(7) \quad \pi_{j,2005} = (\mu - \nu^{\text{Other}})(R_{j,2005}^g + R_{j,2005}^f) \\ - \text{LaborCost}_{j,2005} - \text{LandRent}_{j,2005},$$

where the sales revenue comes from the 2005 demand model, and labor cost and land rent are explained above. The next step is to extrapolate this model to earlier years.

4.4. Extrapolation to Other Years

We have a demand model for 2005 in hand, but need models for earlier years. To get them, assume demand in earlier years is the same as in 2005 except for the multiplicative scaling factor ρ_t introduced above in the definition of Wal-Mart's problem in equation (2). For example, the 2005 demand model with no rescaling predicts that, at the 1971 store set and 1971 demographic variables, average sales per store (in 2005 dollars) is \$31.5 million. Actual sales per store (in 2005 dollars) for 1971 is \$7.4 million. The scale factor for 1971 adjusts demand proportionately so that the model exactly matches aggregate 1971 sales. Over the 1971–2005 period, this corresponds to a compound annual real growth rate of 4.4 percent. Wal-Mart significantly widened the range of products it sold over this period (to include tires, eyeglasses, etc.). The growth factor is meant to capture this. The growth factor calculated in this manner has leveled off in recent years to around 1 percent a year. Wal-Mart has also been expanding by converting regular stores to supercenters. This expansion

is captured explicitly in the model rather than indirectly through exogenous growth.

Demographics change over time, and this is taken into account. For 1980, 1990, and 2000, I use the decennial census for that year.¹⁴ For years in between, I use a convex combination of the censuses.¹⁵

5. PRELIMINARY EVIDENCE OF A TRADE-OFF

This section provides some preliminary evidence of an economically significant trade-off to Wal-Mart. Namely, the benefits of increased economies of density come at the cost of cannibalization of existing stores. This section puts to work the demand model and other components of operating profits compiled above.

Consider some Wal-Mart store j that first opens in time t . Define the *incremental sales* $R_{j,t}^{e,\text{inc}}$ of store j to be what the store adds to total Wal-Mart sales in segment $e \in \{g, f\}$ in its opening year t , relative to what sales would otherwise be across all other stores open that year. The incremental sales of store number 1 opening in 1962 equals $R_{j,1962}^e$ that year. For a later store j , however, the incremental sales are, in general, less than store j 's sales, $R_{j,t}^{e,\text{inc}} \leq R_{j,t}^e$, because some part of the sales may be diverted from other stores. Using the demand model, we can calculate $R_{j,t}^{e,\text{inc}}$ for each store.

Table VIII reports that the average annual incremental sales at opening in general merchandise across all stores equals \$36.3 million (in 2005 dollars throughout). Analogously, average incremental sales in food from new supercenters is \$40.2 million. (Note that conversions of existing Wal-Marts to supercenters count as store openings here.) To make things comparable across years, the 2005 demand model is applied to the store configurations and demographics of the earlier years with no multiplicative scale adjustment ρ_t . In an analogous manner, we can use equation (7) to determine the *incremental operating profit* of each store at the time it opens. The average annual incremental profit in general merchandise from a new Wal-Mart is \$3.1 million and in food from new supercenters is \$3.6 million. Finally, we can ask how far a store is from the closest distribution center in the year it is opened. On average, a new Wal-Mart is 168.9 miles from the closest regular distribution center when it opens, and a new supercenter is 137.0 miles from the closest food distribution center.

Incremental sales and operating profit can be compared to what sales and operating profit would be if a new store were a stand-alone operation. That is, what would sales and operating profits be at a store if it were isolated so

¹⁴I use 1980 for years before 1980 and 2000 for years after 2000.

¹⁵For example, for 1984 there is .6 weight on 1980 and .4 weight on 1990, meaning 60 percent of the people in each 1980 block group are assumed to be still around as potential Wal-Mart customers, and 40 percent of the 1990 block group consumers have already arrived. This procedure keeps the geography clean, since the issue of how to link block groups over time is avoided.

TABLE VIII
INCREMENTAL AND STAND-ALONE VALUES OF NEW STORE OPENINGS^a

	<i>N</i>	Incremental Sales	Operating Profit	Center Distance (Miles)	Incremental Distribution	Stand-Alone Operating Profit
Part A: General Merchandise (New Wal-Marts Including Supercenters)						
All	3,176	36.3	3.1	168.9	41.4	3.6
By state's Wal-Mart age at opening						
1–2	288	38.0	3.5	343.3	38.7	3.6
3–5	614	39.5	3.5	202.0	41.5	3.7
6–10	939	37.6	3.3	160.7	40.9	3.6
11–15	642	36.1	2.9	142.1	42.2	3.4
16–20	383	32.9	2.8	113.7	41.2	3.5
21 and above	310	29.5	2.4	90.2	44.4	3.6
Part B: Food (New Supercenters)						
All	1,980	40.2	3.6	137.0	44.8	4.0
By state's Wal-Mart age at opening						
1–2	202	42.4	3.9	252.9	43.9	3.9
3–5	484	42.7	4.0	171.2	44.7	4.1
6–10	775	41.0	3.6	113.5	45.3	4.0
11–15	452	36.7	3.2	95.3	45.3	3.9
16–20	67	30.1	2.8	94.0	38.6	3.5

^aAll evaluated at 2005 demand equivalents in millions of 2005 dollars.

that none of its sales is diverted to or from other Wal-Mart stores in the vicinity? Table VIII shows for the average new Wal-Mart, there is a big difference between stand-alone and incremental values, implying a substantial degree of market overlap with existing stores. Average stand-alone sales is \$41.4 million compared to an incremental value of \$36.3 million, approximately a 10 percent difference. Two considerations account for why the big cannibalization numbers found here are not inconsistent with the 1 percent cannibalization rates reported earlier in Table V. First, the denominator of the cannibalization rate from Table V includes all preexisting stores, including those areas of the country where Wal-Mart is not adding any new stores. Taking an average over the country as a whole understates the degree of cannibalization taking place where Wal-Mart is adding new stores. Second, stand-alone sales include sales that a new store would never get because the sales would remain in some existing store (but would be diverted to the new store if existing stores shut down).

Define the Wal-Mart Age of a state to be the number of years that Wal-Mart has been in the state.¹⁶ The remaining rows in Table VIII classify stores by the Wal-Mart age of their state at the store's opening. Those stores in the row labeled "1–2" are the first stores in their respective states. Those stores in the row labeled "21 and above" are opened when Wal-Mart has been in their states for over 20 years.

Table VIII shows that incremental operating profit in a state falls over time as Wal-Mart adds stores to a state and the store market areas increasingly begin to overlap. Things are actually flat the first 5 years at \$3.5 million in incremental operating profit for general merchandise, but it falls to \$3.3 million in the second 5 years and then to \$2.9 million and lower beyond that. An analogous pattern holds for food. This pattern is a kind of diminishing returns. Wal-Mart is getting less incremental operating profit from the later stores it opens in a state.

The table also reveals a benefit from opening stores in a state where Wal-Mart has been for many years. The incremental distribution center distance is relatively low in such states. It decreases substantially as we move down the table to stores opening later in a state. The very first stores in a state average about 300 miles from the closest distribution center. This falls to less than 100 miles when the Wal-Mart age of the state is over 20 years. There is a trade-off here: the later stores deliver lower operating profit but are closer to a distribution center. The magnitude of the trade-off is on the order of 200 miles for \$1 million in operating profit. This trade-off is examined in a more formal fashion in the next three sections, and the results are roughly on this order of magnitude.

6. BOUNDING DENSITY ECONOMIES: METHOD

It remains to pin down the parameters relating to density. There are three such parameters, $\theta = (\tau, \omega_1, \omega_2)$. The τ parameter is the coefficient on distance between a store and its distribution center. It captures the benefit of *store density*. The parameters ω_1 and ω_2 relate to how fixed cost varies with *population density* in equation (1).

The estimation task is spread over three sections. This section lays out the set identification method. Section 7 presents the baseline estimates and interprets them. After that, Section 8 tackles the issue of inference, addressing the specific complications that arise here. Section 8 also discusses the robustness of the findings to alternative specifications and assumptions.

¹⁶For the purposes of this analysis, the New England states are treated as a single state. Maryland, Delaware, and the District of Columbia are also aggregated into one state.

6.1. The Linear Moment Inequality Framework

My approach follows the partial identification literature initiated by Manski (2003). The contributions to this literature have been extensive. In my application, I follow Pakes, Porter, Ho, and Ishii (2006) (hereafter PPHI). In the first part of this section, I lay out the general linear moment inequality framework. In the second part, I map Wal-Mart's choice problem into the framework.

Let there be a set of M linear inequalities, with each inequality indexed by a ,

$$(8) \quad y_a \geq x'_a \theta, \quad a \in \{1, 2, \dots, M\}$$

for scalar y_a , 3×1 vector x_a , and parameter vector $\theta \in \bar{\Theta} \subseteq R^3$. It is known that at the true parameter $\theta = \theta^\circ$, equation (8) holds for all a . In what follows, a indexes deviations from the actual policy a° that Wal-Mart chose, y_a is the incremental operating profit from doing a° rather than a , and $x'_a \theta$ is the incremental cost. The revealed preference that Wal-Mart chose a° implies equation (8) must hold at the true parameter θ° for all deviations a .

Let $\{z_{a,k}, k = 1, 2, \dots, K\}$ be a set of K instruments for each a . Assume the instruments are nonnegative, $z_{a,k} \geq 0$. Hence, at the true parameter $\theta = \theta^\circ$,

$$z_{a,k} y_a \geq z_{a,k} x'_a \theta \quad \text{for all } a \text{ and } k.$$

Suppose the M observations $\{(y_a, x_a, z_{a,1}, z_{a,2}, \dots, z_{a,K}), a = 1, \dots, M\}$ are drawn randomly from an underlying population, and that the population averages $E[z_{a,k} y_a]$ and $E[z_{a,k} x_a]$ are well defined.

Assume x_a and $z_{a,k}$ are directly observed, but there is measurement error on y_a . In particular, we observe

$$\tilde{y}_a = y_a + \eta_a,$$

where $E[\eta_a | x_a, z_{a,k}] = 0$. Taking expectations, we obtain a set of K moment inequalities that are satisfied at the true parameter θ° , that is,

$$(9) \quad m_k(\theta) \geq 0 \quad \text{for } k \in \{1, 2, \dots, K\}$$

for $m_k(\theta)$ defined by

$$m_k(\theta) \equiv E[z_{a,k} \tilde{y}_a] - E[z_{a,k} x'_a] \theta.$$

The identified set Θ^I is the subset of points satisfying the K linear constraints in equation (9). Defining $Q(\theta)$ by

$$(10) \quad Q(\theta) = \sum_{k=1}^K (\min\{0, m_k(\theta)\})^2,$$

the identified set can equivalently be written as the points $\theta \in \Theta^I$ solving

$$0 = \min_{\theta \in \bar{\Theta}} Q(\theta).$$

(As an aside, taking the square in equation (10) is common in the literature, but I also consider an alternative formulation that leaves out the square, summing the absolute value of any deviations. An attractive feature of this linear version is that it can be minimized through linear programming; see footnote 22 in Section 8.)

Let $\tilde{m}_k(\theta)$ and $\tilde{Q}(\theta)$ be the sample analogs of $m_k(\theta)$ and $Q(\theta)$:

$$(11) \quad \begin{aligned} \tilde{m}_k(\theta) &\equiv \sum_{a=1}^M \frac{z_{ak}\tilde{y}_a}{M} - \sum_{a=1}^M \frac{z_{a,k}x'_a}{M}\theta, \\ \tilde{Q}(\theta) &\equiv \sum_{k=1}^K (\min\{0, \tilde{m}_k(\theta)\})^2. \end{aligned}$$

We can define an analog estimate of the identified set Θ^I as the set of θ that solves

$$(12) \quad \hat{\Theta}^I = \arg \min_{\theta \in \bar{\Theta}} \tilde{Q}(\theta).$$

If the sample moments are consistent estimates of the population moments, then $\hat{\Theta}^I$ is a consistent estimate of the identified set Θ^I .

If there were no measurement error, $\eta_a = 0$, a preferable estimation strategy would be to use the information content in the M disaggregated inequalities in equation (8), rather than aggregate to the K moment inequalities and lose the individual information. Bajari, Benkard, and Levin (2007) considered just such an environment. There is an error term in the first stage of their procedure but not in the second stage where they exploit inequalities derived from choice behavior. Their method minimizes the analog of equation (10) applied to the M disaggregated inequalities in equation (8). Here, with measurement error in the second stage, a disaggregated approach like this runs into problems. Consider a simple example where the right-hand-side variable is just a constant and it is known that $y_a \geq \theta$ (where θ is a scalar, for the example). Suppose we observe $\tilde{y}_a = y_a + \eta_a$ (i.e., there is measurement error on the left-hand-side variable). If we pick θ to minimize the disaggregated analog of equation (11), the solution is the set of θ that satisfies

$$(13) \quad \left\{ \theta \mid \theta \leq \min_a \{y_a + \eta_a\} \right\}.$$

That is, we require each individual inequality to hold. This works fine with no measurement error. If there is measurement error with full support, then

asymptotically the estimated set goes to minus infinity. In large samples, significantly negative outlier draws of η_a pin down the estimate. In contrast, by aggregating to moment inequalities, the measurement error averages out in large samples.

6.2. Applying the Approach to Wal-Mart

Looking again at Wal-Mart's objective function in equation (2) and noting the log-linear form in equation (1) of how the fixed cost c varies with population density, we can readily see that Wal-Mart's objective is linear in the parameters τ , ω_1 , and ω_2 , consistent with the structure above. Define y_a by

$$y_a \equiv \Pi(a^\circ) - \Pi(a)$$

for

$$(14) \quad \Pi(a) \equiv \sum_{t=1}^{\infty} (\rho_t \beta)^{t-1} \left(\sum_{j \in \mathcal{B}_t^{\text{Wal}}(a)} \pi_{jt}^g(a) + \sum_{j \in \mathcal{B}_t^{\text{Super}}(a)} \pi_{jt}^f(a) \right).$$

Thus, y_a is the increment in the present value of operating profit from implementing the chosen policy a° instead of a deviation a . (Note that equation (14) includes periods into the distant future that I do not have information on. In calculating y_a , these future terms are differenced out, because the deviations considered involve past behavior, not future.) Define the three-element vector x_a by

$$x_{1,a} \equiv \Delta D_a,$$

$$x_{2,a} \equiv \Delta C_{1,a},$$

$$x_{3,a} \equiv \Delta C_{2,a}.$$

The first element is the present value ΔD_a of the difference in distribution-distance miles between the two policies. This is calculated by substituting $d_{ji}^e(a)$ for $\pi_{jt}^e(a)$ in equation (14), $e \in \{f, g\}$. The second and third elements are the analogous summed present value differences of $\ln(\text{Popden})$ and $\ln(\text{Popden})^2$. (These are the present values of the differences in the first and second terms of the fixed cost c , given in (1).) Since a° solves the problem in equation (2), at the true parameter $\theta^\circ = (\tau^\circ, \omega_1^\circ, \omega_2^\circ)$,

$$y_a \geq x_a' \theta$$

must hold for each deviation a , just as in equation (8) above.

There are two categories of error in the analysis. The first comes from the demand estimation in the first stage. The second arises from measurement

error on the wages and land rents discussed in Section 4. Call this the second-stage error.

To begin to account for these two sources of error, let $R_{jt}^e(a)$ be the sales revenue under policy a at store j at time t for segment $e \in \{f, g\}$ at policy a , evaluated at the true demand parameter vector ψ° . Let $\hat{R}_{jt}^e(a)$ be the estimated value using $\hat{\psi}$ from the first stage. It is useful to initially isolate the second-stage error and then account for the first-stage error later. Evaluating at the true demand parameter ψ° , the observed operating profit at a particular segment e , store j , and time t equals

$$(15) \quad \tilde{\pi}_{jt}^e(a) = (\mu - \nu^{\text{Other}})R_{jt}^e(a) - (\text{Wage}_{jt} + \varepsilon_{jt}^{\text{Wage}})\nu^{\text{Labor}}R_{jt}^e(a) \\ - (\text{Rent}_{jt} + \varepsilon_{jt}^{\text{Rent}})\nu^{\text{Land}}R_{jt}^e(a),$$

where $\varepsilon_{jt}^{\text{Wage}}$ and $\varepsilon_{jt}^{\text{Rent}}$ are the measurement errors on wages and the rents alluded to earlier. The present value of the measurement error, analogous to (14), equals

$$(16) \quad \boldsymbol{\epsilon}_a \equiv \sum_{t=1}^{\infty} (\rho_t \beta)^{t-1} \left(\sum_{j \in \mathcal{B}_t^{\text{Wal}}(a)} (\varepsilon_{jt}^{\text{Wage}} \nu^{\text{Labor}} + \varepsilon_{jt}^{\text{Rent}} \nu^{\text{Land}}) R_{jt}^g(a) \right. \\ \left. + \sum_{j \in \mathcal{B}_t^{\text{Super}}(a)} (\varepsilon_{jt}^{\text{Wage}} \nu^{\text{Labor}} + \varepsilon_{jt}^{\text{Rent}} \nu^{\text{Land}}) R_{jt}^f(a) \right).$$

Define the differenced measurement error associated with deviation a to be

$$\eta_a \equiv \boldsymbol{\epsilon}_{a^\circ} - \boldsymbol{\epsilon}_a.$$

Finally, let

$$\tilde{y}_a \equiv y_a + \eta_a$$

be the observed differenced operating profits at deviation a , evaluated at the true demand vector ψ° . Assume the underlying second-stage measurement errors $\varepsilon_{jt}^{\text{Wage}}$ and $\varepsilon_{jt}^{\text{Rent}}$ are mean zero and independent of the other variables in the analysis. Then $E[\eta_a | x_a] = 0$, so that with the first-stage error ignored, the analysis maps directly into the linear moment inequality framework outlined above, for now putting aside the selection of instruments. So that the set estimate $\hat{\Theta}'$ defined by equation (12) is a consistent estimate of Θ' , it is necessary that the differenced measurement error η_a entering in the moment equalities average out when the number of stores N is large. Assuming the underlying store-level measurement errors $\varepsilon_{jt}^{\text{Wage}}$ and $\varepsilon_{jt}^{\text{Rent}}$ are independent across stores is sufficient but not necessary for this to be true.

To take into account the first-stage error, let $\hat{\tilde{y}}_a$ be defined the same way as \tilde{y}_a , but using the estimated demand parameter vector $\hat{\psi}$ instead of the true value ψ° . We can write it as

$$(17) \quad \hat{\tilde{y}}_a = \tilde{y}_a + [\hat{\tilde{y}}_a - \tilde{y}_a].$$

The error term in brackets arises because of the measurement error in store-level sales encountered in the demand estimation. It was assumed earlier that this store-level measurement error is drawn i.i.d. across stores. Hence, taking asymptotics with respect to the number of stores N , the estimate $\hat{\psi}$ is a consistent estimate of ψ° and $\hat{\tilde{y}}_a$ is a consistent estimate of \tilde{y}_a . Then $\hat{\Theta}^I$ constructed by using $\hat{\tilde{y}}_a$ instead of \tilde{y}_a is a consistent estimate of Θ^I .

It remains to describe the choice of deviations to consider and the selection of the instruments. Like Fox (2007) and Bajari and Fox (2009), I restrict attention to pairwise resequencing, that is, deviations a in which the opening dates of *pairs* of stores are reordered. For example, store number 1 actually opened in 1962 and number 2 opened in 1964. A pairwise resequencing would be to open store number 2 in 1962, store number 1 in 1964, and to leave everything else the same.

I define groups of deviations that share characteristics. An indicator variable for membership in the group plays the role of an instrument, so taking means over inequalities within each group creates a moment inequality for each group. The groups are chosen to be informative about the parameters and thus narrow the size of the identified set Θ^I . There are 12 different groups, which are formally defined in Table IX. Summary statistics for each group are reported in Table X.

Each of the 12 deviation groups is in one of three broad classifications. The first broad classification is “Store density decreasing” deviations. To construct these deviations, I find instances where Wal-Mart at some relatively early time period (call it t) is adding another store (call it j) near where it already has a large concentration of stores, and there is some other store location j' opened at a later period $t' > t$ that would have been far from Wal-Mart’s store network if it had opened at time t instead. In the deviation, Wal-Mart opens the farther-out store sooner (j' at t) and the closer-in store later (j at t'), and this decreases store density in the earlier period. Analogously, the second broad classification consists of “Store density increasing” deviations that go the other way. The third broad classification, “Population density changing” deviations, holds store density roughly constant by flipping stores opened in the same state. For these, the stores involved in the flip come from different population density locations.

Let χ_a^k be an indicator variable equal to 1 if deviation a is in group k defined in Table IX and equal to 0 otherwise. The group definitions depend only on store locations and opening dates, and these are all assumed to be measured without error, that is, there is no measurement error in χ_a^k . Moreover, the error

TABLE IX
DEFINITIONS OF DEVIATION GROUPS^a

Deviation Category	Deviation Group	Description (Store j' Flips with Store j)
Store density decreasing		Find the set of stores, $S = \{j, t_j \geq t_j^{\text{state}} + 10\}$. For each $j \in S$, find all j' , where (i) $t_{j'} \geq t_j + 3$, (ii) j' is in a different state than j , and (iii) $t_{j'} \leq t_{j'}^{\text{state}} + 4$. Take all of these and further classify by group on the basis of ΔD_a as follows:
	1	$-.75 \leq \Delta D_a < 0$
	2	$-1.50 \leq \Delta D_a < -.75$
	3	$\Delta D_a < -1.50$
Store density increasing		Find the set of stores, $S = \{j, t_j \leq t_j^{\text{state}} + 5\}$. For each $j \in S$, find all j' , where (i) $t_{j'} \geq t_j + 3$, (ii) j' is in a different state than j , and (iii) $t_{j'} \geq t_{j'}^{\text{state}} + 10$. Take all of these and further classify by group on the basis of ΔD_a as follows:
	4	$0 < \Delta D_a \leq .75$
	5	$.75 < \Delta D_a \leq 1.50$
	6	$1.50 < \Delta D_a$
Population density changing		Take pairs of stores (j, j') opening in the same state, where $t_j \leq t_{j'} + 2$. Classify based on Popden_j (in units of 1,000 people within 5-mile radius). Define density classes 1, 2, 3, and 4 by $\text{Popden}_j < 15$, $15 \leq \text{Popden}_j < 40$, $40 \leq \text{Popden}_j < 100$, and $100 \leq \text{Popden}_j$.
	7	j in class 4, j' in class 3
	8	j in class 3, j' in class 2
	9	j in class 2, j' in class 1
	10	j in class 1, j' in class 2
	11	j in class 2, j' in class 3
	12	j in class 3, j' in class 4

^aNotes: The table uses the following notation: t_j is the opening date of store j , t_j^{state} is the opening date of the first store in the state where j is located, ΔD_a is the present value of the increment in distribution distance miles (in 1,000s) from doing the actual policy a° instead of deviating and doing a . In words, to construct group 1, take the set of all stores opening when there is at least one store in their state that is 10 years old or more. For each such store, find alternative stores that open 3 or more years later in different states, where Wal-Mart has been in the different state no more than 4 years when the alternative store opens. Openings for general merchandise stores and food stores are considered two different opening events. In cases where a supercenter opens from scratch rather than as a conversion of an existing Wal-Mart, there are two opening events. In all the pairs considered, a general merchandise opening is paired with another general merchandise opening, and a food opening with another food opening.

η_a is mean zero conditional on χ_a^k , given the independence assumption already made about $\varepsilon_{jt}^{\text{Wage}}$ and $\varepsilon_{jt}^{\text{Rent}}$. Hence, χ_a^k is a valid instrument.

Let the set of *basic instruments* be defined by

$$z_{a,k} = \text{Weight}_a \times \chi_a^k$$

TABLE X
SUMMARY STATISTICS OF DEVIATIONS BY DEVIATION GROUP

Deviation Group	Brief Description of Group	Number of Deviations	Mean Values			
			$\Delta\bar{H}$	ΔD	ΔC_1	ΔC_2
			(Millions of 2005 Dollars)	(Thousands of Miles)	(log Popden)	(log Popden ²)
Store density decreasing						
1	$-.75 \leq \Delta D < 0$	64,920	-2.7	-.4	-.6	-3.0
2	$-1.50 \leq \Delta D < -.75$	61,898	-3.6	-1.1	-1.5	-9.0
3	$\Delta D < -1.50$	114,588	-4.7	-3.0	-3.4	-22.2
Store density increasing						
4	$0 < \Delta D \leq .75$	158,208	3.0	.3	-1.9	-17.2
5	$.75 < \Delta D \leq 1.50$	34,153	3.7	1.0	-3.6	-28.9
6	$1.50 < \Delta D$	16,180	5.9	2.1	-4.8	-37.7
Population density changing						
7	Class 4 to class 3	7,048	1.2	.0	3.2	31.1
8	Class 3 to class 2	10,435	3.7	.0	3.4	25.7
9	Class 2 to class 1	14,399	5.3	-.1	3.5	19.3
10	Class 1 to class 2	12,053	-2.4	.0	-3.4	-19.3
11	Class 2 to class 3	14,208	.6	-.1	-3.9	-29.4
12	Class 3 to class 4	14,877	2.5	.0	-4.6	-44.9
All	Weighted mean	522,967	-.2	-.6	-2.1	-15.6

for a weighting variable

$$\text{Weight}_a = \frac{1}{\sum_{t=1}^{t_a^{\text{First}}} (\rho_t \beta)^{t-1}},$$

where t_a^{First} is the first period that deviation a is different from a° . This rescales things to the present value at the point when the deviation actually begins.

Additional instruments are obtained by interacting the basic instruments with positive transformations of the x_a . Define $x_{i,a}^+ \equiv x_{i,a} - x_i^{\min}$, where $x_{i,a}$ is the i th element of x_a and $x_i^{\min} = \min_a x_{i,a}$. Analogously, $x_{i,a}^- \equiv x_i^{\max} - x_{i,a}$ for $x_i^{\max} = \max_a x_{i,a}$. Level 1 interaction instruments are obtained by multiplying the $x_{i,a}^+$ and $x_{i,a}^-$, of which there are six, by each of the 12 basic moments for a total of $72 = 6 \times 12$ level 1 interaction moments. Analogously, we can take the various second-order combinations, such as $x_{1,a}^+ x_{1,a}^+$, $x_{1,a}^+ x_{2,a}^+$, and so on, and multiply them times the basic instruments to create level 2 interaction moments, of which there are $252 = 21 \times 12$. In the full set of all three types of moment inequalities, there are $336 = 12 + 72 + 252$ restrictions.

Before moving on to the baseline results, I make a comment about optimization error. The above discussion models the choice a° as the true solution

to Wal-Mart's problem (2). For any deviation a that has higher measured discounted profit, the discrepancy is attributed to a combination of first-stage estimation error and second-stage measurement error on wages and land rentals. In principle, one could incorporate optimization error on the part of Wal-Mart as a further source of discrepancy (and PPHI discussed how to do this), but I have not done that here.

7. BOUNDING DENSITY ECONOMIES: BASELINE RESULTS

This section presents the baseline density economy estimates and provides a discussion of the results. The next section addresses issues of confidence intervals and robustness.

The complete set of all deviations in the 12 groups defined above consists of 522,967 deviations. Table X presents summary statistics by deviation group. In particular, it reports means of $\Delta\tilde{\Pi}_a$ (the \tilde{y}_a variable) as well as the means of ΔD_a , $\Delta C_{1,a}$, and $\Delta C_{2,a}$ (which make up the x_a vector). The variables are all rescaled by Weight_a before taking means, so present values are taken as of the point when the deviations begin.

Consider the Store density decreasing deviations, groups 1–3. These deviations open farther-out stores sooner and closer-in stores later. Thus, distribution miles are fewer when the actual policy is chosen instead of deviations in these groups (i.e., $\Delta D_a < 0$). We also see that by choosing the actual policy rather than these deviations, Wal-Mart is sacrificing operating profit. These losses average $-\$2.7$, $-\$3.6$, and $-\$4.7$, in millions of 2005 dollars, for groups 1, 2, and 3. We can see evidence of a trade-off across groups 1, 2, and 3 as the absolute value of mean $\Delta\tilde{\Pi}_a$ increases (the sacrifices in operating profit) and as the absolute value of mean ΔD_a increases (the savings in miles).

For the sake of illustration, suppose we only consider the deviations in group 1. Also, temporarily zero out the ω_1 and ω_2 coefficients on population density. Then using the information in Table X, the moment inequality for group 1 reduces to

$$E[\Delta\tilde{\Pi}_1] - \tau E[\Delta D_1] = -2.7 + \tau.4 \geq 0,$$

or $\tau \geq \$6.75$, where the units are in thousands of 2005 dollars per mile year. Freeing up ω_1 and ω_2 loosens the constraint. For example, suppose we plug in $\omega_1 = 4.28$ and $\omega_2 = -.50$ (this choice is explained shortly). Then the moment inequality from group 1 is instead

$$\begin{aligned} E[m_1] &= E[\Delta\tilde{\Pi}_1] - \tau E[\Delta D_1] - \omega_1 E[\Delta C_1] - \omega_2 E[\Delta C_2] \\ &= -2.7 + \tau.4 - 4.28(-.6) - (-.50)(-3.0) \\ &= -1.4 + \tau.4 \geq 0, \end{aligned}$$

TABLE XI
BASELINE ESTIMATED BOUNDS ON DISTRIBUTION COST τ^a

	Specification 1		Specification 2		Specification 3	
	Basic Moments (12 Inequalities)		Basic and Level 1 (84 Inequalities)		Basic and Levels 1, 2 (336 Inequalities)	
	Lower	Upper	Lower	Upper	Lower	Upper
Point estimate	3.33	4.92	3.41	4.35	3.50	3.67
Confidence thresholds						
With stage 1 error correction						
PPHI inner (95%)	2.69	6.37	2.89	5.40	3.01	4.72
PPHI outer (95%)	2.69	6.41	2.86	5.45	2.97	5.04
No stage 1 correction						
PPHI inner (95%)	2.84	5.74	2.94	5.11	3.00	4.62
PPHI outer (95%)	2.84	5.77	2.93	5.13	2.99	4.97

^aUnits are in thousands of 2005 dollars per mile year; number of deviations $M = 522,967$; number of store locations $N = 3,176$.

or $\tau \geq \$3.33$.¹⁷ This is substantially looser than when $\omega_1 = \omega_2 = 0$ is imposed.

Now turn to the general problem of bounding τ . Let $\underline{\tau}$ and $\bar{\tau}$ be the lower and upper bounds of τ in the identified set Θ^I . When a solution satisfying all of the sample moment inequalities exists, as is the case here, the estimates of these bounds are obtained through linear programs that impose the moment inequalities and the a priori restrictions $\omega_1 \geq 0$ and $\omega_2 \leq 0$. Table XI presents the results.

The first set of estimates imposes only the 12 basic moment inequalities. Table X contains all the information needed to do this. The estimated lower bound is in fact $\hat{\underline{\tau}} = \3.33 , and this is obtained when $\omega_1 = 4.28$ and $\omega_2 = -.50$, the values used above. In the solution to the linear programming problem, moment 1 is binding, as are moments 9 and 12, and the remaining inequalities have slack. The estimated upper bound is \$4.92.

By adding in interaction moments, additional restrictions are imposed, narrowing the identified set. The additional moments created when the basic moments $[Ey]_k - [Ex'\theta]_k \geq 0$ are multiplied by positive transformations of the x are analogous to the familiar moment conditions for ordinary least squares (OLS), $(y - x\theta)'x = 0$. With both level 1 and level 2 interactions included, the estimate of the identified set is narrowed to the extremely tight range of \$3.50–\$3.67. This case with the full set of interactions will serve as my baseline estimate.

¹⁷Because of rounding, there is a slight discrepancy in these two inequalities.

7.1. Discussion of Estimates

The parameter τ represents the cost savings (in thousands of dollars) when a store is closer to its distribution center by 1 mile over the course of a year. At the baseline estimate of τ in a tight range at \$3.50, if all 5,000 Wal-Mart stores (here, supercenters are counted as two stores) were each 100 miles farther from their distribution centers, Wal-Mart's costs would increase by almost \$2 billion per year.

To get a sense of the direct cost of trucking, I have talked with industry executives and have been quoted marginal cost estimates of \$1.20 per truck mile for “in-house” provision. If a store is 100 miles from the distribution center (200 miles round trip) and if there is a delivery every day throughout the year, then the trucking cost is $1.20 \times 200 \times 365 = \$85,400$, or \$.85 in thousands of dollars per mile year. Thus, the baseline estimated cost savings in a tight range around \$3.50 is approximately four times as large as the savings in trucking costs alone.¹⁸ The difference includes the valuations Wal-Mart places on the ability to quickly respond to demand shocks. My industry source on trucking costs emphasized the value of quick turnarounds as an important plus factor beyond savings in trucking costs.

A second perspective on the τ parameter can be obtained by looking at Wal-Mart’s choice of when to open a distribution center (DC). An in-depth analysis of this issue is beyond the scope of this paper, but some exploratory calculations are useful. Recall that Wal-Mart’s problem specified in (2) held DC opening dates fixed and considered deviations in store opening dates. Now hold store opening dates fixed and consider deviations in DC openings. Denote t_k^{open} to be the year DC k opens. Define $D_{k,t}^{\text{inc}}$ to be DC k ’s *incremental contribution* in year t to reduction in store distribution miles. This is how much higher total store distribution miles would be in year t if distribution center k were not open in that year. Assume there is a fixed cost ϕ_k of operating distribution center k in each year. Optimizing behavior implies that the following inequalities must hold for the opening year $t = t_k^{\text{open}}$:

$$(18) \quad \begin{aligned} \phi_k &\leq \tau D_{k,t}^{\text{inc}}, \\ \phi_k &\geq \tau D_{k,t-1}^{\text{inc}}. \end{aligned}$$

The first inequality says that the fixed cost of operating the distribution center in year t_k^{open} must be less than the distribution cost savings from it being open.¹⁹ Otherwise, Wal-Mart can increase profit by delaying the opening by a year.

¹⁸My distances are calculated “as the crow flies,” whereas the industry trucking estimate is based on the highway distance, which is longer. The discrepancy is small relative to the magnitudes discussed here.

¹⁹There is also a marginal cost involved with distribution, but assume this is the same across distribution centers, so shifting volume across distribution centers does not affect marginal cost.

TABLE XII
MEAN INCREMENTAL MILES SAVED AND STORES SERVED FOR DISTRIBUTION CENTERS
ACROSS ALTERNATIVE OPENING DATES INCLUDING ACTUAL

	1 Year Prior to Actual	Actual Year Opened	1 Year After Actual	2 Years After Actual
All distribution centers ($N = 78$)				
Mean incremental miles saved	4.4	5.8	6.7	7.1
Mean stores served	23.6	52.1	58.4	62.9
By type of DC				
Regional distribution centers ($N = 43$)				
Mean incremental miles saved	6.1	7.7	8.7	8.9
Mean stores served	37.1	68.6	76.1	79.0
Food distribution centers ($N = 35$)				
Mean incremental miles saved	2.3	3.4	4.3	5.0
Mean stores served	6.9	31.8	36.5	43.0

The second inequality states that the fixed cost exceeds the savings of opening it the year before (otherwise it would have been opened a year earlier). Now if $D_{k,t}^{\text{inc}}$ changes gradually over time, then equation (18) implies $\phi_k \approx \tau D_{k,t}^{\text{inc}}$ holds approximately at the date of opening. (Think of this as a first-order condition.)

Table XII reports the mean values of the $D_{k,t}^{\text{inc}}$ statistic across distribution centers. The statistic is reported for the year the distribution center opens, as well as the year before opening and the 2 years after opening. For example, to calculate this statistic in the year before opening, the given DC is opened 1 year early, everything else the same, and the incremental reduction in store miles is determined. I also report how the mean number of stores served varies when the DC opening date is moved up or pushed back. At opening, the mean incremental reduction in store miles of a distribution center is 5.8 thousand miles and the DC serves 52.1 stores. The later the DC opens, the higher the incremental reduction in store miles and the more stores served. This happens because more stores are being built around it.

If we knew something about the fixed cost, then the condition $\phi_k \approx \tau D_{k,t}^{\text{inc}}$ provides an alternative means of inferring τ . A very rough calculation suggests a ballpark fixed cost of \$18 million per year.²⁰ Since the mean value of $D_{k,t}^{\text{inc}}$ when DCs open equals 5.8 thousand miles, we back out an estimate of τ equal to

$$\hat{\tau} = \frac{\phi}{\text{mean } D_{k,t}^{\text{inc}}} = \frac{\$18 \text{ (\$million per year)}}{5.8 \text{ (thousands of miles)}} = \$3.10.$$

²⁰Distribution centers are on the order of 1 million square feet. Annual rental rates including maintenance and taxes are on the order of \$6 per square foot, so \$6 million a year is a rough approximation for the rent of such a facility. A typical Wal-Mart DC has a payroll of \$36 million. If a third of labor is fixed cost, then we have a total fixed cost of $\$18 = \$6 + \$12$ million.

This estimate is close to the baseline estimate from above on the order of \$3.50. It is encouraging that these two approaches—coming from two very different angles—provide similar results.

8. BOUNDING DENSITY ECONOMIES: CONFIDENCE INTERVALS AND ROBUSTNESS

To apply the PPHI method of inference to this application, two issues need to be confronted. First, there is correlation in the error terms across deviations when two deviations involve the same store. Second, first-stage demand estimation error needs to be taken into account. Section 8.1 explains how the two issues are addressed. Section 8.2 discusses the confidence intervals for the baseline estimates. Section 8.3 examines the robustness of the baseline results to alternative assumptions.

There are a variety of different alternative approaches for inference with moment inequalities in addition to PPHI, including Imbens and Manski (2004), Chernozhukov, Hong, and Tamer (2007), Romano and Shaikh (2008), and Andrews and Soares (2010). The main focus of this paper is on the sample analog estimates of the identified set, as presented in Section 7. PPHI complemented this focus, as they simulated the construction of sample analogs. (See also Luttmer (1999).) In the recent literature, a distinction is made between constructing a confidence interval for the identified set versus a confidence interval for the true parameter. The work below is in the first category; the confidence intervals are for extreme points of the identified set.

8.1. Procedure

To explain the PPHI method, it is useful to introduce additional notation. Let \tilde{w}_a be a vector that stacks the moment inequality variables for deviation a into a column vector that is $(K + 3K) \times 1$. The first K elements contain the $z_{a,k}\tilde{y}_a$ and the remaining $3K$ elements contain the $z_{a,k}x'_a$ in vectorized form, where again K is the number of moment inequalities. PPHI assumed the \tilde{w}_a are drawn independently and identically. (This will not be true here, but ignore this for now to explain what they do.) Let Σ be the variance–covariance matrix of the distribution of \tilde{w}_a . The sample mean of \tilde{w}_a over the M deviations equals

$$(19) \quad \bar{w} = \sum_{a=1}^M \frac{\tilde{w}_a}{M}$$

and it has a variance–covariance matrix equal to Σ/M . Take the sample variance–covariance matrix $\hat{\Sigma}$ as a consistent estimate of Σ .

PPHI proposed a way to simulate *inner* and *outer* confidence intervals that asymptotically bracket the true confidence interval for $\underline{\tau}$. Begin with the inner

approximation first. Consider a set of simulations of the moment inequality exercise indexed by s from $s = 1$ to S . For each simulation s , draw a random column vector $\boldsymbol{\varpi}^s$ with $K + 3K$ elements from the normal distribution with mean zero and variance–covariance $\hat{\Sigma}/M$. Then define

$$\mathbf{w}^s = \bar{\mathbf{w}} + \boldsymbol{\varpi}^s.$$

Put \mathbf{w}^s into moment inequality form, letting the first K elements be $\mathbf{w}^{1,s}$ and reshaping the remaining elements into a $K \times 3$ matrix $\mathbf{w}^{2,s}$. Analogous to equation (9), we then have K moment inequalities for each simulation s ,

$$(20) \quad \mathbf{w}^{1,s} - \mathbf{w}^{2,s} \theta \geq 0.$$

Define $\tilde{Q}^{\text{inner},s}(\theta)$ to be the analog of $\tilde{Q}(\theta)$ in equation (11). Define $\underline{\tau}^{\text{inner},s}$ by

$$\hat{\underline{\tau}}^{\text{inner},s} \equiv \inf\{\tau, (\tau, \omega_1, \omega_2) \in \arg \min \tilde{Q}^{\text{inner},s}(\theta)\},$$

which produces the analog estimate of $\underline{\tau}$ in the simulation. The inner α percent confidence interval for $\underline{\tau}$ is obtained by taking the $\frac{\alpha}{2}$ and $1 - \frac{\alpha}{2}$ percentiles of the distribution of the simulated estimates, $\{\hat{\underline{\tau}}^{\text{inner},s}, s = 1, \dots, S\}$.

The construction of the outer approximation is the same, with one difference. For each simulation s , reformulate the set of moment inequalities in equation (20) by adding a nonnegative vector ζ :

$$\mathbf{w}^{1,s} - \mathbf{w}^{2,s} \theta + \zeta \geq 0$$

for ζ defined by

$$\zeta \equiv \max\{0, \mathbf{w}^1 - \mathbf{w}^2 \hat{\theta}\}.$$

The vector ζ is the “slack” in the nonbinding moment inequalities evaluated at the actual data (as opposed to the simulated data) and at the estimate in the identified set $\hat{\theta} \in \hat{\Theta}^I$ containing the lower bound $\hat{\underline{\tau}}$. Adding the slack term makes it easier for any given point to satisfy the inequalities. PPHI showed that, asymptotically, the distribution of $\hat{\underline{\tau}}^{\text{outer},s}$ is stochastically dominated by the distribution of $\hat{\underline{\tau}}^{\text{inner},s}$ and that the distribution of $\hat{\underline{\tau}}$ lies in between. One thing to note is that Andrews and Guggenberger (2009) and Andrews and Soares (2010) forcefully advocated a criterion of uniform consistency for confidence intervals, and PPHI did not address that.

Two issues arise in applying the method here. First, PPHI’s assumption of independence does not hold here. There are $M = 523,000$ deviations used in the analysis, but these are derived from only $N = 3,176$ different store locations. If $\hat{\Sigma}/M$ is used to estimate the variance of $\bar{\mathbf{w}}$ as in PPHI, the amount of averaging that is taking place is exaggerated, since the measurement error is at

the store level. The issue is addressed by the following subsampling procedure (see Politis, Romano, and Wolf (1999)). Draw a *store location subsample* of size $b \ll N$ from the N store locations (here $b = \frac{N}{3}$). The *deviation subsample* consists of those deviations where both stores being flipped are in the store location subsample. Next calculate mean \bar{w}_b in the deviation subsample. Repeat, subsampling with replacement, and use the different subsamples to estimate $\text{var cov}(\bar{w}_b)$. Next use

$$\text{var cov}(\bar{w}_N) = \frac{b}{N} \text{var cov}(\bar{w}_b)$$

to rescale the estimate of $\text{var cov}(\bar{w}_b)$ into an estimate of $\text{var cov}(\bar{w}_N)$ (see Appendix B1 for a discussion). Use this rather than $\hat{\Sigma}/M$ to draw normal random variables ϖ^s in the PPHI procedure.

The second issue is variance from the first-stage demand estimation. Using equation (17), the left-hand-side term of the k th moment inequality can be written as

$$(21) \quad \bar{w}_k^1 = \sum_{a=1}^M \frac{z_{a,k} \tilde{y}_a}{M} + \sum_{a=1}^M \frac{z_{a,k} [\hat{\tilde{y}}_a - \tilde{y}_a]}{M}.$$

The second term is, in general, not zero because the first-stage estimate $\hat{\psi}$ of demand is different from the true parameter ψ° . Thus, the second term is an additional source of variance. To take this into account, I use a simulation procedure that is valid based on the following asymptotic argument. Appendix B2 shows, first, that for large N , the covariance between the two terms goes to zero relative to the variances of the two terms. Hence, the covariance can be ignored in large samples. It shows, second, that for large N , the variance of the second term is approximately what it would be with only first-stage variation in $\hat{\psi}$ and no second-stage variation. In the simulation procedure, for each subsample s , I take the subsample mean as above and plug this in for the first term. To get a draw for the second term, let $\hat{\Sigma}^\psi$ be the estimated variance–covariance matrix from the first-stage demand estimation. For each subsample s (again of size b), take a normal draw ψ_b^s centered at $\hat{\psi}$ with variance $\frac{N}{b} \hat{\Sigma}^\psi$ and use this to construct a $\hat{\tilde{y}}_a^s$. Then take an estimate of the mean of $z_{a,k} [\hat{\tilde{y}}_a^s - \hat{\tilde{y}}_a]$ to plug in for the second term of equation (21), where $\hat{\tilde{y}}_a^s$ uses ψ_b^s and $\hat{\tilde{y}}_a$ uses $\hat{\psi}$. Finally, use the different subsamples s to estimate $\text{var cov}(\bar{w}_b)$ as above. Splitting equation (21) into two pieces this way makes it possible to economize on deviations used to construct the second term.²¹ (These deviations are expensive because ψ varies.)

²¹To estimate the second term, a fixed set of 100 deviations is used for each of the 12 basic groups, and all variation is driven by differences in ψ^s . It would have been infeasible to calculate

The procedure can be summarized as follows. The basic PPHI approach simulates the distribution of extreme points of the identified set. It does this by calculating sample analogs of the identified set for simulated moments. It constructs an estimate of the variance–covariance matrix of the asymptotic normal distribution of the moments using the variance–covariance of the raw data. The procedure used here changes this last step in two respects. First, to take into account that two deviations involving the same store are not independent, a subsampling approach is used. In particular, subsamples of stores are drawn and the moment inequalities are constructed for each subsample. Second, to take into account first-stage estimation error of the demand model, draws are taken from the asymptotic distribution of the demand parameters. The simulated errors from each such draw are added to the means from a subsample draw just discussed, with adjustments made for subsample size. The distribution of this composite of two simulation draws is used to estimate the variance–covariance matrix needed for the PPHI procedure.

8.2. Estimates of Confidence Thresholds

Table XI reports confidence thresholds for the baseline estimates. One set of estimates takes into account first-stage demand error and the second set does not.²² The left-side 95 percent confidence thresholds of the lower bounds are reported (the 2.5 percentile points) and the right-side thresholds of the upper bound are reported (i.e., the 97.5 percentile points). The first thing to note is that the inner and outer PPHI thresholds (which asymptotically bracket the true thresholds) are similar. Henceforth, I focus on the outer threshold, the conservative choice. The second thing to note is that, like the point estimates, the PPHI thresholds narrow as we move across the table and add more constraints from interactions. Third, the point estimates of the bounds are relatively precise. For example, take Specification 3 where the point estimates of the lower and upper bounds are \$3.50 and \$3.67. The corresponding confidence thresholds are \$2.97 and \$5.04.²³ Fourth, taking into account the first-stage demand error makes a difference, particularly in Specification 1.

8.3. Robustness Results

I turn now to the robustness of the results to alternative assumptions and samples. The results are reported in Table XIII. All specifications in the table

this term for all the different deviations associated with subsample s , so the asymptotic argument making it valid to vary only ψ^s is very useful.

²²For the second set, the simulated draws over the demand parameter $\hat{\psi}$ that add to the variance in the second term of equation (21) are shut down.

²³I have also experimented by seeing how the results change if I leave the square out of the criterion function in equation (10). The results are very similar and are not reported.

TABLE XIII
ROBUSTNESS ANALYSIS OF ESTIMATED BOUNDS ON DISTRIBUTION COST τ^a

Specification	Number of Sample Deviations	Lower Bound		Upper Bound	
		Point Estimate	PPII Outer Thresh.	Point Estimate	PPII Outer Thresh.
Baseline	522,967	3.50	(2.97)	3.67	(5.04)
Split sample by opening date					
1962–1989	331,847	3.51	(2.92)	4.81	(7.19)
1990–2005	191,120	3.43	(1.85)	3.43	(3.90)
Vary operating profit parameters from first stage: net margin (baseline = .17)					
.15	522,967	2.62	(2.18)	3.05	(3.90)
.19	522,967	4.36	(3.78)	4.36	(6.22)
Labor cost factor					
20% higher	522,967	4.12	(3.64)	4.12	(5.66)
20% lower	522,967	2.80	(2.29)	3.38	(4.38)
Rent cost factor					
20% higher	522,967	3.60	(3.04)	3.79	(5.12)
20% lower	522,967	3.41	(2.89)	3.55	(5.03)
Exclude any deviations with change in supercenters	366,412	3.78	(2.66)	3.78	(5.61)
Include only deviations with change in supercenters	156,555	3.79	(1.52)	3.79	(4.72)
By supercenter opening					
1988–1997	59,693	.70	(.00)	.70	(3.48)
1998–2005	96,862	4.38	(2.48)	4.38	(5.39)

^aUnits are in dollars per mile year; all specifications include basic, level 1, and level 2 interactions (336 moment inequalities).

impose the full set of moment conditions (basic plus level 1 plus level 2). The top row is the baseline from Table XI.

The first exercise partitions the perturbations based on the opening date of the first store in the deviation pair. The first sample contains the deviations where the first store of the pair opens before 1990, the second sample where it opens 1990 and later. One thing to note is that for the later time period, τ is point identified. That is, there exists no θ vector that simultaneously solves the full set of 336 empirical moment inequalities, and in such a case, the point estimate of the lower and upper bounds minimizes the squared deviations in (11). The main take-away quantitative point here is that the estimate for the first time period $\tau \in [3.51, 4.81]$ is very similar to the estimate for the second time period $\tau = 3.43$ and to the baseline estimate $\tau \in [3.50, 3.67]$.

As already explained, the confidence intervals reported here incorporate error from the first-stage estimate of demand parameters, but there are other parameters estimated in the first stage that are not incorporated into the confidence intervals. A particularly important parameter is the net margin $\mu - \nu^{\text{other}}$ that equals variable profit per dollar of sale, excluding labor costs and land rents. It was argued that a plausible estimate for this is $\hat{\mu} - \hat{\nu}^{\text{other}} = .17$, and this is used in the baseline. Table XIII shows what happens when instead the estimate is lowered to .15 or raised to .19. Significant changes in this key parameter do lead to significant changes in the estimates. For example, when the parameter is lowered from .17 to .15, the estimated lower bound on τ falls from \$3.50 to \$2.62. Nevertheless, even this low-end estimate for τ is quite large relative to the \$.85 industry estimate discussed above that would be based purely on trucking considerations. Analogous to changes in $\mu - \nu^{\text{other}}$, Table XIII also reports the results with alternative labor cost and rental cost assumptions (increasing or decreasing each by a factor of 20 percent). The results for alternative labor costs are similar to the results for alternative $\mu - \nu^{\text{other}}$. The results for alternative rental costs are not much different from the baseline, because rents are a relatively small share of costs.

An issue that has been ignored up to this point is how to treat the distribution of groceries during the early phase of Wal-Mart's entry into the grocery business. Wal-Mart initially outsourced grocery wholesaling and then later built up its own grocery distribution system. In particular, it opened its first supercenter selling groceries in 1988, but did not open its first food distribution center until 1993, 5 years later, and likely did not become fully integrated until several years beyond that. For the baseline estimates, for lack of a better alternative, I assume that food distribution costs are constant before 1993 (i.e., invariant to store locations) and as of 1993 Wal-Mart is fully integrated with distances calculated according to Wal-Mart's own internal network. I expect there is some measurement error for grocery distribution distances during the early phase of Wal-Mart's supercenter business. This is a concern because my procedure yields inconsistent estimates of the identified set Θ^I when there is measurement error in the x variables.

To examine the robustness of the results to this issue, I first exclude all deviations that impact grocery distribution (i.e., any deviations involving supercenter opening dates). This eliminates 157,000 deviations. Estimating the model on the remaining 366,000 deviations, this "supercenter-excluded" estimate is $\tau = 3.78$, consistent with the baseline. Next I consider those deviations that do include changes in supercenter opening dates. The result of $\tau = 3.79$ is also similar to the baseline. Differences emerge when this subsample is broken up by opening date. In the later period when Wal-Mart is completely integrated into grocery wholesaling, the results are similar to the baseline, but for the early period when Wal-Mart was building its network, the point estimate is only $\tau = .70$. The measurement error during this period noted above is one potential explanation for this discrepancy. Also, the sample is small and the

PPHI confidence bounds are relatively far apart—0 at the bottom and 3.48 at the top.

9. CONCLUDING REMARKS

This paper examines the dynamic store location problem of Wal-Mart. Using the moment inequality approach, the paper is able to bound a technology parameter relating to the benefits Wal-Mart obtains when stores are close to distribution centers. The paper illustrates the power of this type of approach in getting a sensible analysis out of what would otherwise be complex and likely intractable.

Wal-Mart has attracted much attention, and various interest groups have attempted to slow its growth, for example, by trying to get local governments to use zoning restrictions to block entry of stores. These kinds of policies limit store density. The analysis here is not at the stage where it is possible to run a policy experiment to evaluate the welfare effects of limiting Wal-Mart's growth. Among other things, that would require uncovering how such limits would impact Wal-Mart's DC network, and (except briefly in Section 7) this has been held fixed in the analysis. Nevertheless, the estimates of this paper suggest any policy that would substantially constrain store density would result in significant cost increases.

Although the analysis is rich in many dimensions—notably in its fine level of geographic detail and in the way it incorporates numerous data objects—it has limitations. One is that all economies of density are channeled through the benefits of stores being close to distribution centers. Benefits can potentially emerge through other channels, including *management* (it is easier for upper-level management to oversee a given number of stores when the stores are closer together) and *marketing* (satisfied Wal-Mart customers might tell their friends and relatives on the other side of town about Wal-Mart—this benefits Wal-Mart only if it has a store on the other side of town).²⁴ A caveat, then, is that my estimate of τ may be picking up some economies of density from management and marketing. I have chosen to focus on *distribution* both because (i) I can measure it (i.e., the locations of distribution centers), but cannot measure management and marketing activities, and because (ii) my priors tell me distribution is very important for Wal-Mart. My findings in Section 7 regarding DC opening dates that corroborate my baseline findings are particularly helpful here. DC openings should be unrelated to management and marketing sources of economies of density.

Although the analysis allows for several varieties of measurement error, it simplifies by leaving out a structural error, that is, location-specific factors that

²⁴Neumark, Zhang, and Ciccarella (2008) quoted Sam Walton on the importance of the marketing benefit.

influence Wal-Mart's behavior but are not in the data. For example, an alternative explanation to density economies for the pattern of Wal-Mart's rollout is that there is simply something good about the Wal-Mart model in Bentonville, Arkansas, that deteriorates away from this point for whatever reason. This is not plausible, but an alternative possibility of unobserved heterogeneity across store locations *within* markets is plausible. In such a case, the diminishing returns from adding additional stores to the same market exhibited in Table VIII may be understated. (Locations picked last in a market may be worse in unobservable ways.) In this way, my results potentially underestimate density economies.

The analysis does not take explicit account of the location of competitors. If one region is empty of competition, while other regions are crowded with competitors, a chain might concentrate its stores in the empty region first and the crowded regions later. This is not a plausible explanation for the qualitative pattern of Wal-Mart's rollout, of starting in the center and going to the outside later. As of 1977, when Wal-Mart was still a regional operation with only 181 stores, the national leader K-Mart had over 1,000 stores distributed across all of the contiguous states. Although K-Mart's stores did not uniformly follow population, it is a good first approximation, and K-Mart continued its relatively uniform coverage as it doubled its store count over the next 5 years.²⁵ There was no void in the discount industry in the middle of the United States.

Even if the location of rivals is not the primary driver of the overall *qualitative* pattern, keeping track of competitor locations matters for the *quantitative* analysis, as it impacts the relative profitability of different sites. The analysis accounts for the presence of rivals only *implicitly*, allowing the outside alternative to get better in denser areas in a reduced-form way. Following the logic of Bresnahan and Reiss (1991), there tend to be more rivals in larger markets. To the extent the reduced form does not fully capture differences in rival presence across store locations, there is unobserved heterogeneity and the comments just made above apply. Finally, the analysis ignores the issue of preemption. This is a particularly interesting area for future research, as large density economies suggest possibilities for preemption might extend beyond the store level to the regional level.

APPENDIX A: DATA

The selected data and programs used in the paper are posted as Supplemental Material (Holmes (2011)) and also at my website. In particular, data on facility locations and opening dates are posted there. The store-level sales and employment data used in this paper can be obtained from Trade Dimensions.

²⁵A regression of K-Mart store counts and population at the state level (in logs and weighted by log population) has a slope of .9 and $R^2 = .86$ for both 1977 and 1983. This is a period over which the total store count increased from 1,089 to 1,941.

Facility Locations and Opening Dates

The data for Wal-Mart stores were constructed as follows. In November 2005, Wal-Mart made public a file which listed for each Wal-Mart store the address, store number, store type (supercenter or regular store), and opening date. These data were combined with additional information posted at Wal-Mart's website about openings through January 31, 2006 (the end of fiscal year 2006). The opening date mentioned above is the date of the original store opening, not the date of any later conversion to a supercenter. To get the date of supercenter conversions, I used two pieces of information. From announcements at Wal-Mart's website, I determined the dates of conversions taking place in 2001 and after. To get the dates of conversions taking place before 2001, I used data collected by Basker (2005) based on published directories.

The data on Wal-Mart's food distribution centers (FDCs) are based on reports that Wal-Mart is required to file with the Environmental Protection Agency (EPA) as part of a risk management plan.²⁶ (The freezers at FDCs use chemicals that are potentially hazardous.) Through these reports, all FDCs are identified. The opening dates of most of the FDCs were obtained from the reports. Remaining opening dates were obtained through a search of news sources.

Data on Wal-Mart general distribution centers (GDCs) that handle general merchandise were cobbled together from various sources including Wal-Mart's annual reports and other direct Wal-Mart sources, as well as Mattera and Purinton (2004) and various web and news sources. Great care was taken to distinguish GDCs from other kinds of Wal-Mart facilities such as import centers and specialty distribution centers such as facilities handling Internet purchases.

The longitude and latitude of each facility were obtained from commercial sources and manual methods.

In the analysis, I aggregate time to the year level, where the year begins February 1 and ends January 31 to follow the Wal-Mart fiscal year. January is a big month for new store openings (it is the modal month), and February and March are the main months for distribution center openings. New January-opened stores soon obtain distribution services from new February-opened DCs. To be conservative in not overstating the number of distribution store miles, I assume that the flow of services at a DC begins the year prior to the opening year. Put differently, in my analysis I shift down the opening year of each DC by 1 year.

Wage Data

The wages are the average retail wage in the county containing the store (payroll divided by March employment). The source is County Business Patterns, U.S. Bureau of the Census (1977, 1980, 1985, 1990, 1995, 1997, 1998,

²⁶The EPA data are distributed by the Right-to-Know Network at <http://www.rtknet.org/>.

2000, 2002, 2004) with interpolation in intervening years and years with missing values. For 2000 and beyond, the NAICS definition of the retail sector *does not* include eating and drinking establishments. For 1997 and earlier, the Standard Industrial Classification (SIC) code definition of retail *does* include eating and drinking, and these are subtracted out for these years.

Property Value Data

For each store location and each census year, I created an index of residential property value as follows. I identified the block groups within a 2-mile radius of each store and called this the store's neighborhood. Total property value in the neighborhood was calculated as the aggregate value of owner-occupied property plus 100 times monthly gross rents of renter-occupied property. This was divided by the number of acres in a circle with radius of 2 miles, and the consumer price index was used to convert it into 2005 dollars.

County property tax records were obtained from the Internet for 46 Wal-Mart locations in Minnesota and Iowa. (All stores in these states were searched, but only for these locations could the records be obtained.) Define the *land value–sales ratio* to be land value from the tax records as a percentage of the (fitted) value of 2005 sales for each store. For these 46 stores, the correlation of this value with the 2000 property value index is .71. I regressed the land value–sales ratio on the 2000 property value index without a constant term and obtained a slope of .036 (standard error of .003). The regression line was used to obtain fitted values of the land value–sales ratio for all Wal-Mart stores.

APPENDIX B1: VARIANCE–COVARIANCE OF THE SUBSAMPLE

I will show that $\text{var cov}(\bar{w}_N)$ decreases at rate $\frac{1}{N}$ (where N is the number of stores) rather than rate $\frac{1}{M}$ (where M is the number of deviations). To make the point, for simplicity consider the following simple case. Suppose there are only two periods, $t = 1, 2$, and that Wal-Mart has chosen to open $N/2$ stores in period 1, with store numbers $A_1 = \{1, 2, \dots, \frac{N}{2}\}$, and the remaining $N/2$ stores in period 2, with store numbers $A_2 = \{\frac{N}{2} + 1, \dots, N\}$. Consider all deviations where Wal-Mart instead opens store $k \in A_2$ in period 1 and delays the opening of a store $j \in A_1$ until period 2. There are $M = (\frac{N}{2})^2$ deviations, and let $a = (j, k)$ index them.

It is sufficient to look at the component of \bar{w}_N that is the mean of the \tilde{y} , that is,

$$\bar{w}^1 = \sum_{a=1}^M \frac{\tilde{y}_a}{M} = \sum_{a=1}^M \frac{y_a + \eta_a}{M},$$

where η_a is the difference in store-level measurement errors,

$$\eta_{j,k} = \varepsilon_j - \varepsilon_k.$$

We can write the measurement error component of the above as

$$\sum_{a=1}^M \frac{\eta_a}{M} = \frac{\sum_{j=1}^{N/2} \sum_{k=N/2+1}^N [\varepsilon_j - \varepsilon_k]}{\left(\frac{N}{2}\right)^2}.$$

It is straightforward to calculate that the variance is

$$\text{var}\left(\sum_{a=1}^M \frac{\eta_a}{M}\right) = \frac{1}{N} 4\sigma_\varepsilon^2.$$

APPENDIX B2: VARIANCE–COVARIANCE RELATED TO THE FIRST STAGE

I show here that (i) the variance of each of the two terms of equation (21) goes to zero at rate $1/N$, while the covariance goes to zero at a faster rate, and (ii) for large N , the variance of the second term is approximately equal to what the variance would be with just first-stage error.

To simplify the exposition, assume for each deviation a , we can write $\tilde{y}_a(\psi)$ as

$$\tilde{y}_a(\psi) = R_{a,1}(\psi)(1 + \varepsilon_{a,1}) - R_{a,2}(\psi)(1 + \varepsilon_{a,2}),$$

where $R_{a,1}(\psi)$ is the revenue of the store whose opening is being delayed due to deviation a and $R_{a,2}(\psi)$ is the revenue of the store being opened early. This is a simplification of equation (15), but it is sufficient. For now, assume there are only $M = N/2$ deviations (where again N is the number of stores) and that each store only appears in one deviation (I address the general case below). The $\varepsilon_{a,k}$ are all independent classical measurement errors, across a and $k \in \{1, 2\}$, that combine the measurement error of the wages and rents of the given store into one place. For each (a, k) there is a corresponding store j , and I make the substitution shortly.

Define w by

$$(22) \quad w = \sum_{a=1}^{N/2} \frac{\tilde{y}_a(\psi^\circ)}{N/2} + \sum_{a=1}^{N/2} \frac{\tilde{y}_a(\hat{\psi}) - \tilde{y}_a(\psi^\circ)}{N/2}.$$

I need to show that for large N , the covariance of these two terms goes to zero relative to their variances. Since $\tilde{y}_a(\psi^\circ)$ is nonrandom, the covariance of the

two terms can be expanded to

$$\begin{aligned}
& E \left[\sum \frac{R_{a,1}(\psi^\circ) \varepsilon_{a,1} - R_{a,2}(\psi^\circ) \varepsilon_{a,2}}{N/2} \right. \\
& \quad \times \sum \frac{R_{a,1}(\hat{\psi}) \varepsilon_{a,1} - R_{a,2}(\hat{\psi}) \varepsilon_{a,2} - R_{a,1}(\psi^\circ) \varepsilon_{a,1} - R_{a,2}(\psi^\circ) \varepsilon_{a,2}}{N/2} \Big] \\
& + E \left[\sum \frac{R_{a,1}(\psi^\circ) \varepsilon_{a,1} - R_{a,2}(\psi^\circ) \varepsilon_{a,2}}{N/2} \right. \\
& \quad \times \sum \frac{R_{a,1}(\hat{\psi}) - R_{a,2}(\hat{\psi}) - E[R_{a,1}(\hat{\psi}) - R_{a,2}(\hat{\psi})]}{N/2} \Big] \\
& \quad \left. - E \left[\sum \frac{R_{a,1}(\psi^\circ) \varepsilon_{a,1} - R_{a,2}(\psi^\circ) \varepsilon_{a,2}}{N/2} \right. \right. \\
& \quad \left. \left. \times \sum \frac{E[R_{a,1}(\hat{\psi}) \varepsilon_{a,1} - R_{a,2}(\hat{\psi}) \varepsilon_{a,2} - R_{a,1}(\psi^\circ) \varepsilon_{a,1} - R_{a,2}(\psi^\circ) \varepsilon_{a,2}]}{N/2} \right] \right].
\end{aligned}$$

The independence of the $\varepsilon_{a,1}$ and $\varepsilon_{a,2}$ (from stage 2) from the measurement error in store sales (from stage 1) implies that the second and third terms of this expanded expression are zero. The second term is zero, because independence implies

$$\text{cov} \left(\sum \frac{R_{a,1}(\psi^\circ) \varepsilon_{a,1} - R_{a,2}(\psi^\circ) \varepsilon_{a,2}}{N/2}, \sum \frac{[R_{a,1}(\hat{\psi}) - R_{a,2}(\hat{\psi})]}{N/2} \right) = 0,$$

the third term is zero, because $\varepsilon_{a,1}$ and $\varepsilon_{a,2}$ are mean zero. Hence, we can write the covariance between the two terms of w in (22) as

$$\begin{aligned}
& E \left[\sum \frac{R_{a,1}(\psi^\circ) \varepsilon_{a,1} - R_{a,2}(\psi^\circ) \varepsilon_{a,2}}{N/2} \right. \\
& \quad \times \sum ([R_{a,1}(\hat{\psi}) \varepsilon_{a,1} - R_{a,2}(\hat{\psi}) \varepsilon_{a,2}] \\
& \quad \left. - [R_{a,1}(\psi^\circ) \varepsilon_{a,1} - R_{a,2}(\psi^\circ) \varepsilon_{a,2}]) / (N/2) \right] \\
& = E \left[\sum_{a=1}^{N/2} ([R_{a,1}(\hat{\psi}) R_{a,1}(\psi^\circ) \varepsilon_{a,1}^2 + R_{a,2}(\hat{\psi}) R_{a,2}(\psi^\circ) \varepsilon_{a,2}^2] \right. \\
& \quad \left. - [R_{a,1}(\psi^\circ)^2 \varepsilon_{a,1}^2 - R_{a,2}(\psi^\circ)^2 \varepsilon_{a,2}^2]) / (N^2/4) \right]
\end{aligned}$$

$$= \frac{1}{N} E \left[\frac{\sum_{j=1}^N [R_j(\hat{\psi}) - R_j(\psi^\circ)] R_j(\psi^\circ) 4\epsilon_j^2}{N} \right],$$

where I have substituted j for (a, k) in the last step. The estimate $\hat{\psi}$ is a consistent estimate of ψ° , which implies the covariance above goes to zero at a faster rate than $1/N$.

It is immediate that the first term of (22) is of order $1/N$, so now turn to the variance of the second term,

$$\begin{aligned} & \text{var} \left[\sum_{a=1}^{N/2} \frac{\tilde{y}_a(\hat{\psi}) - \tilde{y}_a(\psi^\circ)}{N/2} \right] \\ &= \text{var} \left[\sum_{a=1}^{N/2} \frac{R_{a,1}(\hat{\psi}) - R_{a,2}(\hat{\psi})}{N/2} \right. \\ &\quad \left. + \frac{[R_{a,1}(\hat{\psi}) - R_{a,2}(\psi^\circ)]\epsilon_{a,1} - [R_{a,1}(\hat{\psi}) - R_{a,2}(\psi^\circ)]\epsilon_{a,2}}{N/2} \right]. \end{aligned}$$

The variable $\epsilon_j \epsilon_k$ is mean zero and independent of the variable $[R_j(\hat{\psi}) - R_j(\psi^\circ)][R_k(\hat{\psi}) - R_k(\psi^\circ)]$, and the variable ϵ_j (which depends on measurement error in wages and rents) is independent of $\hat{\psi}$ (which depends on measurement error of store-level sales). Together these imply that the variance of the second term of (22) equals

$$\text{var} \left[\sum_{a=1}^{N/2} \frac{R_{a,1}(\hat{\psi}) - R_{a,2}(\hat{\psi})}{N/2} \right] + \frac{1}{N} E \left[\sum_{j=1}^N \frac{[R_j(\hat{\psi}) - R_j(\psi^\circ)]^2 \epsilon_j^2}{N/4} \right].$$

The first term, which contains the variance just from the first stage, is of order $1/N$. The second term goes to zero faster than $1/N$.

Now return to the issue that an individual store may appear in multiple deviations. The same argument holds, as all of the terms involving a given store can be grouped together, and this will show up in the final terms as a scaling coefficient for store j .

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