Beached Assets?
Capital Turnover and Emissions in Shipping

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Research Question

How do emissions policies affect CO_2 emissions from maritime shipping **over time**, given that the **fleet** and its **operation** are **endogenous**?

- Fuel efficiency varies with ship **speed**, age, and **size**
- Speed shifts **supply** and affects **price**, which drive shipbuilding and scrapping
- I construct and estimate a dynamic model of the dry bulk shipping industry with endogenous speed, **entry**, and **exit**
- I evaluate a **fuel tax**, **speed limit**, and **entry subsidy**

Motivation

- Maritime shipping:
  - Carries $\sim$ 70% of global trade by value
  - Contributes a quarter of trade-related emissions
- Decarbonization is **urgent**
- Technological **lock-in** impedes rapid decarbonization
  - Difficult to upgrade and long-lived
- **Emissions policies** induce adjustments in:
  - Operation (short-run)
  - Turnover rate (mid- to long-run)

Contribution

The first *structurally estimated* model of maritime shipping *emissions* with *endogenous speed, entry, and exit*:

- Shipping emissions analyses assume exogenous price and entry/exit trends or take a broad macro-approach

  (e.g., Faber et al. 2020; Parry et al. 2022; Rutherford et al. 2020; Sheng et al. 2018)

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- Shipping investment models do not consider emissions

  (e.g., Kalouptsidi 2014; Campello et al. 2021; Gkochari 2015; Greenwood and Hanson 2015)

  - Additional heterogeneity over **size**
  - More detailed **fuel efficiency**
  - Richer exit process

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  - Additional heterogeneity over **size**
  - More detailed **fuel efficiency**
  - Richer exit process

- Entry/exit models of emissions study other industries

  (e.g., Cullen and Reynolds 2017; Fowlie et al. 2016; Ryan 2012; Scott 2014; Toyama 2019)

  - Shipping has an important **intensive margin**
  - I focus on the **time horizon** of responses

Preview of Results

- **Fuel tax** causes immediate, **persistent** emissions decrease
- Efficacy of a **speed limit diminishes** by over a half over a decade due to entry/exit effect
- Both initially **suppress exit** of older ships while reducing emissions
- **Entry subsidy** is more effective at **inducing exit**

Key Industry Characteristics

- Lock-in/persistence
  - Ships cost tens of millions of dollars
  - Ships last over 25 years
  - Upgrades are either minor or very difficult/expensive
- Heterogeneity in fuel efficiency/emissions
  - Age (wear)
  - Size
  - Operation (speed)

Heterogeneity Size and Vintage Size vs. Vintage

Emissions Policies

- Policy is set by the International Maritime Organization
  - Relies on consensus between nations and industry
  - Target of net-zero by 2050, with intermediate targets
- Policies I evaluate:
  - Fuel Tax
    - Typical first-best policy
    - Seems to be politically infeasible
  - Speed limit
    - Similar in practice to 2023 efficiency standard for existing ships (EEXI)
  - Entry subsidy
    - Induces exit of old ships

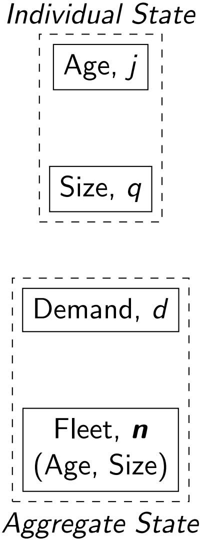
Dry Bulk Shipping

- One of three main sectors
- Transports iron ore, coal, grain, etc.
- Segmented by **size categories** due to infrastructure constraints
- **Competitive** market (HHI $= 0.003$) Distribution
- Operate like **taxis**
- **Global** market
- Likely applicable to the largest component of emissions

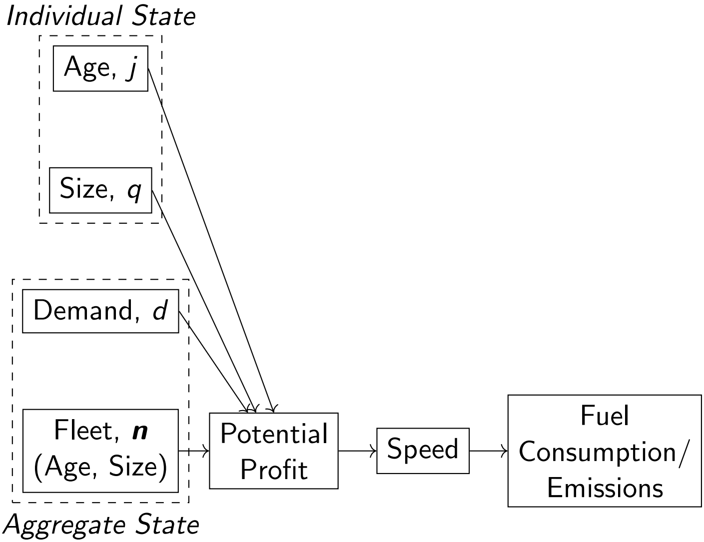
Model Environment

- Focus on one size category: **Handymax**
- **A ship = a firm** Distribution
- New ship = firm entry
- Scrapping a ship = firm exit
- No upgrading
- Time is discrete (period = month)
- Intratemporal operational choice is **speed**
- Emissions are directly proportional to fuel use
- Fuel efficiency depends on age and **size** Details Equations

Model Components



Model Components



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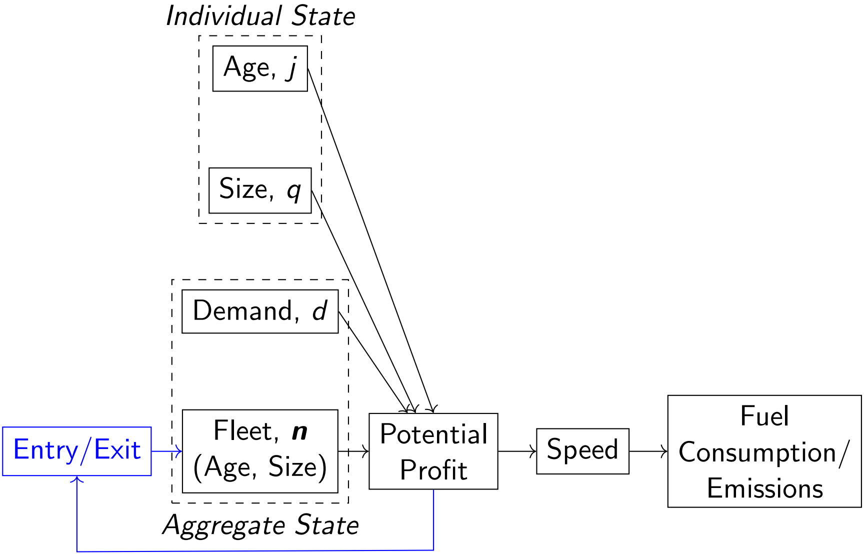

Incumbent Problem: Continue or Exit

$$V(j,q;d_t,\boldsymbol{n}_t) = \underbrace{\pi^*(j,q;d_t,\boldsymbol{n}_t)}_{optimal\ \ profit} + \beta E_{\phi_{it}} \max\{\underbrace{q\phi_{it}}_{scrap}, \underbrace{VC(j,q,d_t,\boldsymbol{n}_t)}_{continue}\}$$

where continuation value:

$$VC(j,q;d_t,\boldsymbol{n}_t) \equiv E_{d_{t+1},\boldsymbol{n}_{t+1}}[V(j+1,q;d_{t+1},\boldsymbol{n}_{t+1})|j,q,d_t,\boldsymbol{n}_t]$$

1. Observe aggregate state $(d_t,\boldsymbol{n}_t)$.
2. Observe individual scrap value draw $\phi_{it} \sim F_\phi$
3. Decide on exit (Exit if $q\phi_{it} > VC(j,q;d_t,\boldsymbol{n}_t)$)
4. Receive (optimal) period payoff: $\pi^*(j,q;d_t,\boldsymbol{n}_t)$
5. Exit decision is implemented

Scrap Price Exit Probability

Potential Entrant Problem: Enter or Not

- Large pool of identical, short-lived potential entrants for each size
- Each observes common:
  - Entry value: $VE(q, d_t, \boldsymbol{n}_t) \equiv \beta E_{d_{t+1}, \boldsymbol{n}_{t+1}}[V(0, q, d_{t+1}, \boldsymbol{n}_{t+1}) | d_t, \boldsymbol{n}_t]$
  - Entry cost: $\kappa_q(d_t, \boldsymbol{n}_t)$
- Free entry condition: $VE(q, d_t, \boldsymbol{n}_t) \leq \kappa_q(d_t, \boldsymbol{n}_t)$
- Potential entrants play symmetric mixed entry strategy
- Number of entrants $M_t^q \sim Poisson(\lambda_q(d_t, \boldsymbol{n}_t))$

Entry/Exit Equilibrium

Symmetric Markov Perfect Equilibrium:

- Firms use common stationary exit strategy (cutoff rule)
- All incumbents choose exit probability to maximize value function in every state
- In each state, either entrants have zero expected discounted profits or entry rate is zero (or both):

$$VE(q, d_t, \boldsymbol{n}_t) \leq \kappa_q(d_t, \boldsymbol{n}_t),$$

with equality if $\lambda_q(d_t, \boldsymbol{n}_t) > 0$

Static Equilibrium: Optimal Speed

- Owners rent ships to charterers on a voyage basis
- Ship speed s determined by charterer **voyage cost minimization**:

$$\min_{s} \underbrace{pq\frac{x}{s}}_{\text{charter}} + \underbrace{c\eta(j,q)xs^2}_{\text{fuel}} + \underbrace{F^c_{j,q}}_{\text{fixed}}$$

- p: charter/shipping price per mass-time
- x: voyage distance
- c: fuel price
- η: fuel consumption

Results

Static Equilibrium: Optimal Speed

- Owners rent ships to charterers on a voyage basis
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$$\min_{s} \underbrace{pq\frac{x}{s}}_{\text{charter}} + \underbrace{c\eta(j,q)xs^2}_{\text{fuel}} + \underbrace{F^c_{j,q}}_{\text{fixed}}$$

$$\Rightarrow s^*_{j,q} = \left(\frac{pq}{2c\eta(j,q)}\right)^{1/3}$$

- p: charter/shipping price per mass-time
- x: voyage distance
- c: fuel price
- η: fuel consumption

Results

Static Equilibrium

- Supply: $Q = \psi \sum_{j,q} n_{j,q} q s^*_{j,q}(p)$

  ψ: fraction of time travelling loaded

- Demand: $p = \bar{x} e^d Q^\gamma$

  $\bar{x}$: average distance per period

- Owner profit: $\pi^*(j, q; d, \boldsymbol{n}) = pq - F_{j,q}$

Eqbm. Profit Results

Data

	Data	Sample	Range	Frequency
Demand				
	Shipping contracts[2]	sample	2010-2022	daily
	Seaborne trade[1]		$<$2000-2022	monthly
Supply				
	Fleet, limited detail[1]	universe	2005, 2012-2022	annual
	Demolitions[1]	$\sim 4/5$	2006-2022	monthly
Value Function				
	New and used ship sales[1]	sample	2010-2022	monthly
Fuel Efficiency				
	Ship tracking[3]	universe	2019-2021	hourly
	Reported emissions[4]	EU trips	2019-2021	annual

Sources: [1]Clarksons Research, [2]Baltic Exchange, [3]Spire, [4]EU MRV

Two-Step Estimation Procedure

Exogenous objects:

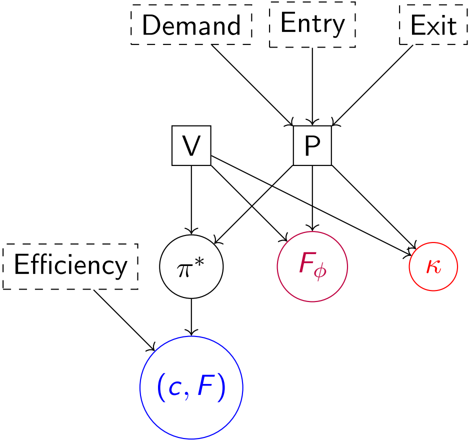
- Demand $\gamma, d_t, \rho, \sigma_\epsilon^2$

Equilibrium objects:

- Value functions V
- Transition probabilities P

Primitives:

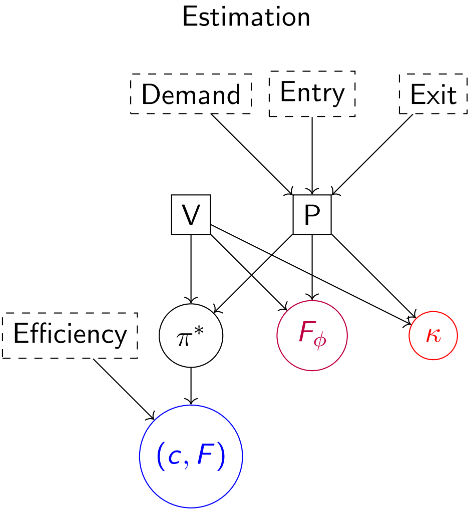
- Scrap value distribution F_ϕ
- Entry costs κ
- Optimal profits π^*
- Cost parameters $c, F_{j,q}$



Counterfactual Equilibrium

Primitives:

- Scrap value distribution F_ϕ
- Entry costs κ
- Cost parameters c, F



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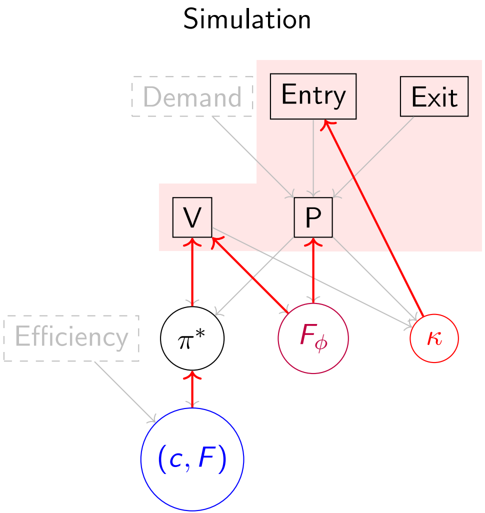
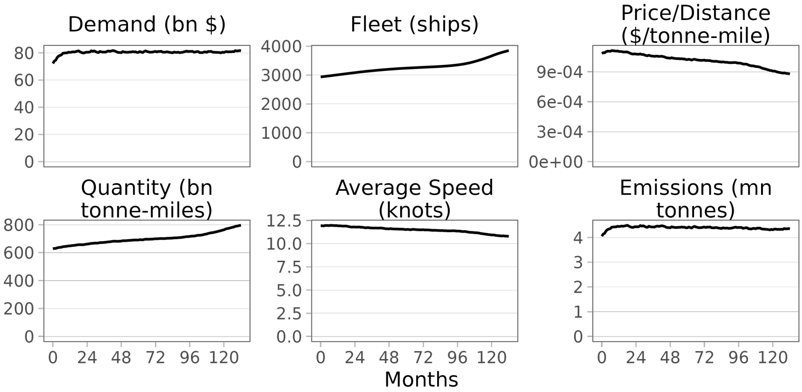


Counterfactuals

- Simulation:
  - Starting from Jan. 2012
  - Average of 4800 simulations
  - 1 year before policy, 10 years after

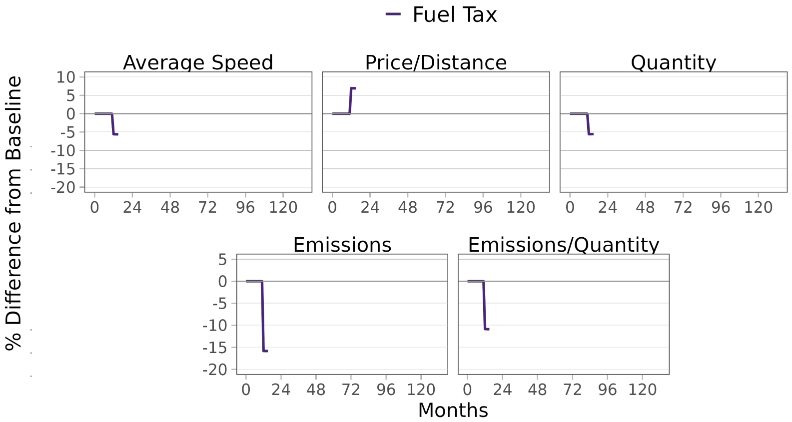
1. Fuel tax: $(1 + 0.2) \times c$
2. Speed limit: $s \leq 12.4$ knots
   - Calibrated to achieve equivalent immediate emissions reduction
3. Entry subsidy: $(1 - 0.05) \times \kappa(d_t, \boldsymbol{n}_t)$

Baseline Evolution



Entry/Exit

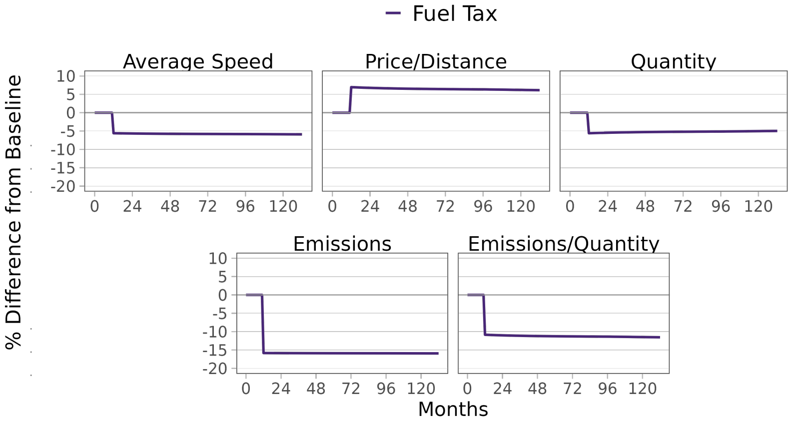
What is the *immediate* response to a **fuel tax**?



Slower ships → higher price → lower quantity

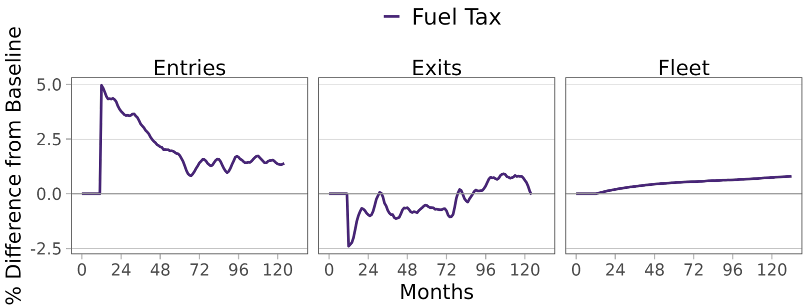
Speed Eqn Profit Eqn

What is the *mid-run* response to a **fuel tax**?



- Emissions drop is persistent

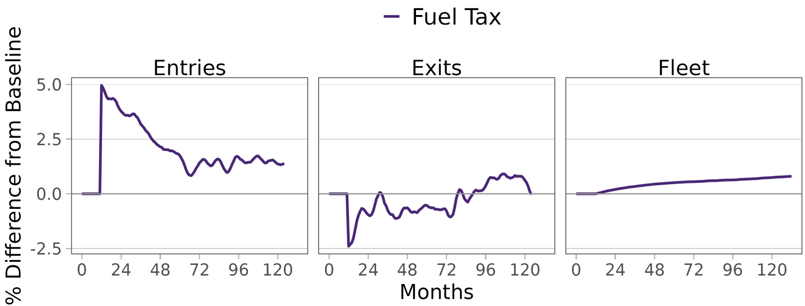
How does the fleet respond?



- Fuel tax initially **suppresses exits**!

Speed Effect

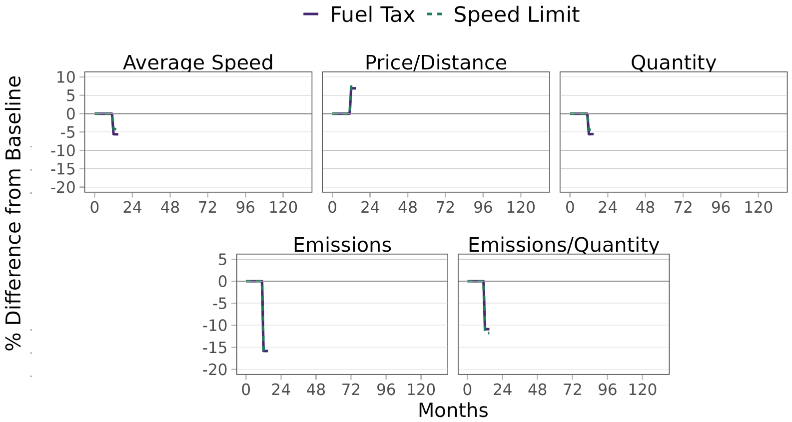
How does the fleet respond?



- Fuel tax initially **suppresses exits**!
- Entry response fades, but has a **persistent** impact

Speed Effect

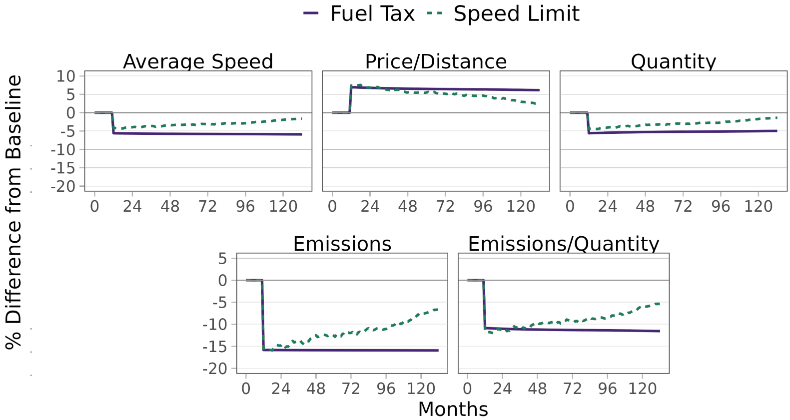
Immediate response to a **speed limit** is similar



Slower ships $\rightarrow$ higher price $\rightarrow$ lower quantity

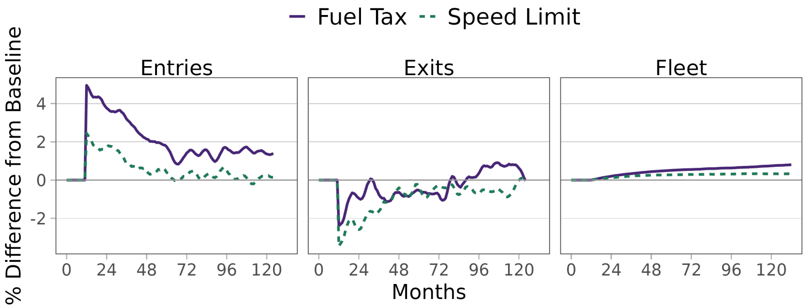
Speed Eqn Profit Eqn

Impact of a **speed limit** diminishes over time!



- Over half of initial emissions reduction disappears

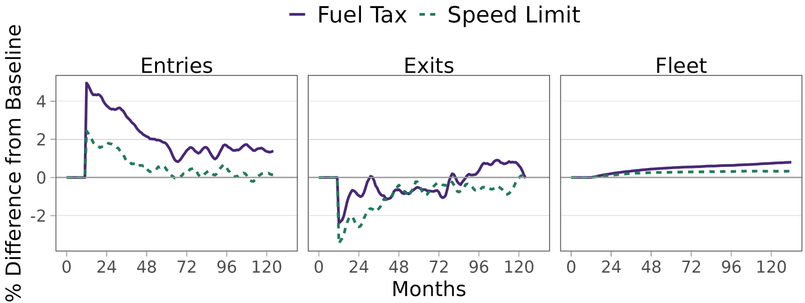
What drives the dynamic response?



$$s_{j,q} = \max\left\{\left(\frac{pq}{2c\eta(j,q)}\right)^{1/3}, \bar{s}\right\}$$

Speed Effect

What drives the dynamic response?

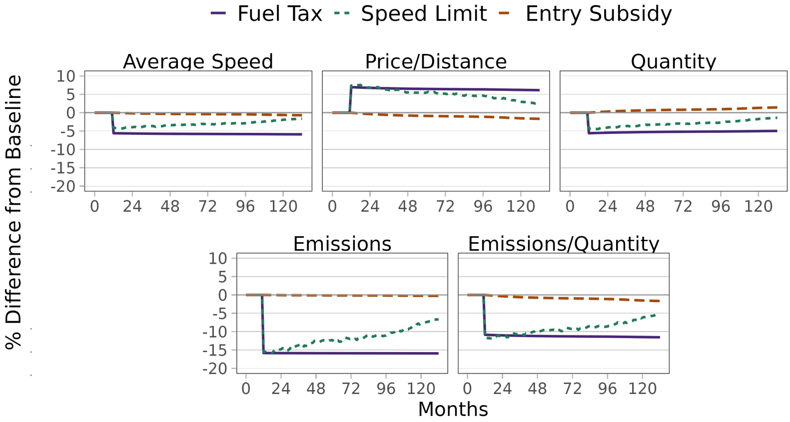


- **Both** policies initially **suppress exits**

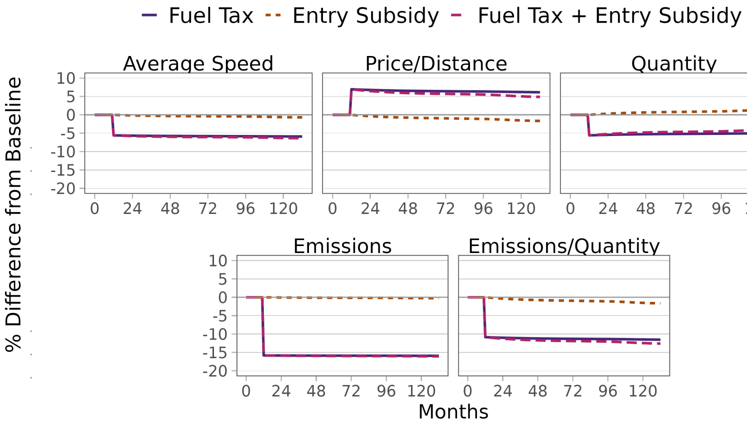
Speed Effect

Entry subsidy induces exit. . .

...But has a small effect on emissions



The best of both worlds?



Entry/Exit Speed Limit Combo

Policy Implications

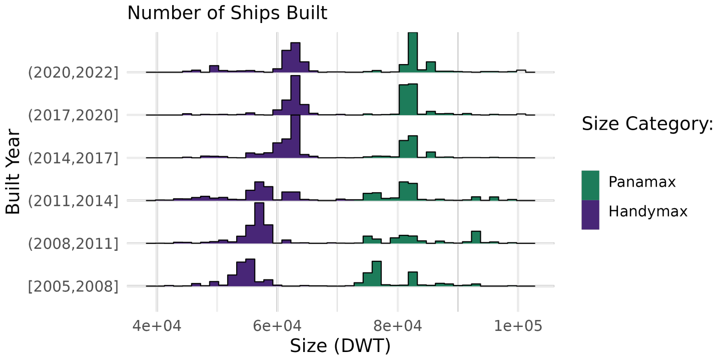
- Reducing speeds decreases emissions in the short-run
- Accelerating turnover will require policy directed at the extensive margin
- Large emissions reductions will require a technological improvement
- New ship efficiency standards may inhibit turnover

Conclusion

- Structural model of emissions from dry bulk shipping
  - Endogenous **speed**, **exit**, and **entry**
  - Important heterogeneity over age and **size**
  - Estimated from rich data on fleet and its operation
- Both static and dynamic equilibrium adjustments are essential to understand policy effects

Appendix

Ship Size vs. Vintage



- Distinct size categories
- Sizes have increased

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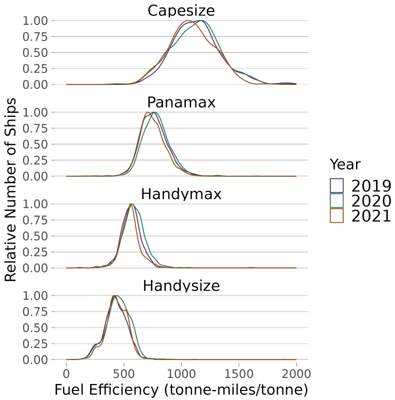
Fuel Efficiency Variation



Ship:

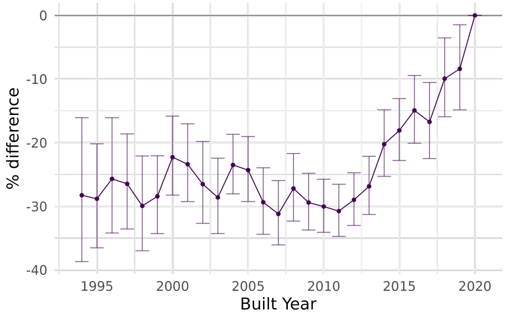
- Size ✓
- Age (deterioration) ✓
- Vintage (technology) ✗ Details

Operation:

- Speed of travel ✓
- Loading factor ✗
- Route/season (weather) ✗

Size vs. Vintage

$$\log\left(fuel\ \ efficiency\right)_{it} = built\ \ year_i + \beta \cdot log(size_i) + \varepsilon_{it},$$



- Mostly flat, except recently
- If linear, one year newer $\approx$ 2% larger

$\Rightarrow$ Focus on impact of size

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Fuel Consumption

$$\eta(j, q) = \alpha_0 q^{\alpha} \exp(\delta j)$$

where

- $\delta > 0$ represents ship ageing
- $\alpha \approx 0.4$, related to wetted surface area to volume ratio

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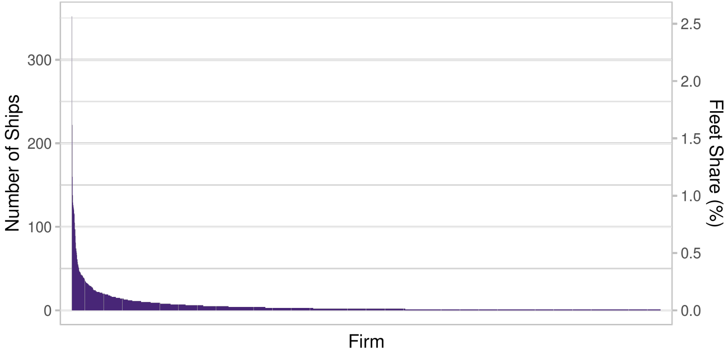
Firm Size



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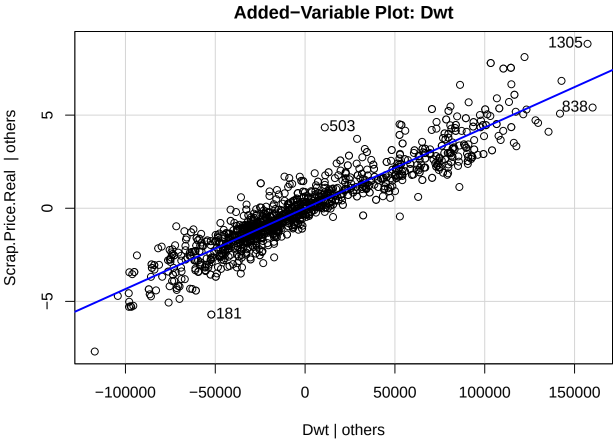
Scrap Price Linearity



$$Scrap.Price.Real \sim Dwt + c|Month$$

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Exit Probability

- Given stochastic scrap values, exit occurs with probability:

$$\begin{aligned}\zeta(j, q; d_t, \boldsymbol{n}_t) &\equiv Pr(q\phi_{it} > VC(j, q; d_t, \boldsymbol{n}_t)) \\ &= 1 - F_\phi\left(\frac{VC(j, q; d_t, \boldsymbol{n}_t)}{q}\right)\end{aligned}$$

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Equilibrium Profit

$$Q = \psi \sum_{j,q} n_{j,q} q s^*_{j,q}$$
$$= \left(\frac{p}{2c\alpha_0}\right)^{1/3} \Sigma_Q(\boldsymbol{n})$$

where $\Sigma_Q(\boldsymbol{n}) \equiv \psi \sum_{j',q'} n_{j',q'} \left(\frac{q'^{4-\alpha}}{e^{\delta j'}}\right)^{1/3}$

Equilibrium owner profit:

$$\pi^*(j, q; d, \boldsymbol{n}) = \underbrace{e^{\frac{3d}{2-\gamma}}}_{\text{demand term}} (2c)^{\frac{\gamma+1}{\gamma-2}} \underbrace{\left(\frac{1}{\Sigma_Q(\boldsymbol{n})^{\gamma}\Sigma_s(\boldsymbol{n})}\right)^{\frac{3}{\gamma-2}}}_{\text{supply term}} q - F_{j,q}$$

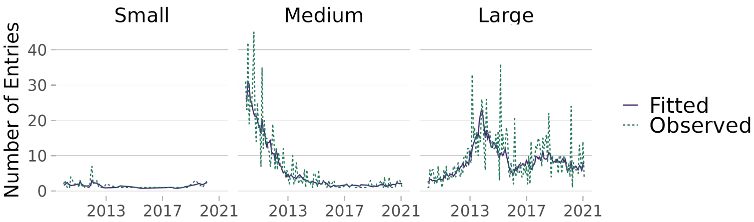
where $\Sigma_s(\boldsymbol{n}) \equiv \frac{1}{n} \sum_{j',q'} n_{j',q'} \left(\frac{q'^{1-\alpha}}{e^{\delta j'}}\right)^{1/3}$

Profit Results

First-Step Entry

$$\underbrace{\lambda(q, d_t, \boldsymbol{n}_t)}_{\text{Entries}} = \sinh\left(\gamma_d d_t + \underbrace{\gamma_q}_{\text{Size FE}} + \underbrace{\sum_{j',q'} \gamma_{q,n}^{j',q'} n_t^{j',q'}}_{\text{Size-specific supply}} \right)$$

- Regression fit:



- Similarly for exit Offline Exit

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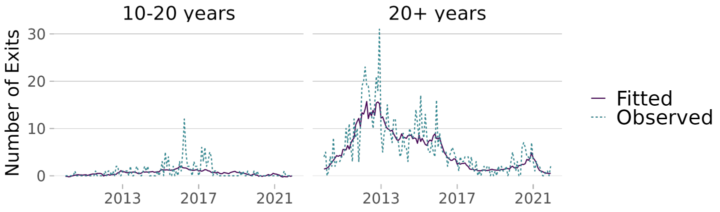
First Step Exit

- Number of exits of **each age**:

$$\mu_j(d_t, \boldsymbol{n}_t) = \sinh\left(\delta_j + \delta_{j,d} d_t + \underbrace{\sum_{j',q'} \delta_{j,n}^{j',q'} n_t^{j',q'}}_{\text{supply bins}} \right)$$

- Regression fit:



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First Step Entry/Exit Estimation

	asinh(Entries)
	(1)
d	0.52574* (0.23439)
$\sum n^{1,q}$	-0.00226*** (0.00051)
$\sum n^{2,q}$	-0.00044 (0.00051)
$\sum n^{3,q}$	-0.00366* (0.00159)
Constant	-3.46680 (5.84681)
Observations	144
Adjusted R^2	0.23

	asinh(Exits)	
	(1)	(2)
	Age Bin [120,240)	Age Bin [240,Inf]
d	-0.2158 (0.2422)	-0.0985 (0.2216)
$n^{1,1}$	-0.0224*** (0.0000)	-0.0143*** (0.0000)
$n^{1,2}$	0.0003*** (0.0000)	0.0036*** (0.0000)
$n^{1,3}$	0.0050*** (0.0000)	0.0091*** (0.0000)
$n^{2,1}$	0.0033*** (0.0000)	0.0198*** (0.0000)
$n^{2,2}$	-0.0061*** (0.0000)	0.0033*** (0.0000)
$n^{2,3}$	0.1403*** (0.0000)	0.0710*** (0.0000)
$n^{3,1}$	-0.0024*** (0.0000)	0.0106*** (0.0000)
$n^{3,2}$	-0.0356*** (0.0000)	-0.0505*** (0.0000)
$n^{3,3}$	-0.0103*** (0.0000)	0.4257*** (0.0000)
Constant	10.0933 (9.7512)	-11.0437 (8.6610)
Observations	144	144
R^2	0.30	0.70

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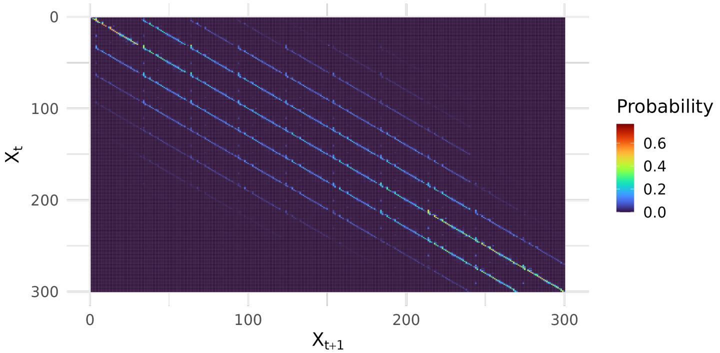
Transition Matrix



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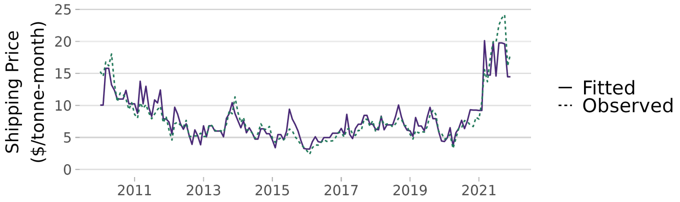
Variable Cost

- Model implied equilibrium price:

$$p = \underbrace{e^{\frac{3d}{2-\gamma}}}_{\text{demand term}} (2c)^{\frac{\gamma+1}{\gamma-2}} \underbrace{\left(\frac{1}{\Sigma_Q(\boldsymbol{n})^{\gamma}\Sigma_s(\boldsymbol{n})}\right)^{\frac{3}{\gamma-2}}}_{\text{supply term}}$$

- Recover c by fitting to observed prices via NLS
- Regression fit:



Speed fit Estimation Transition

Fixed Costs

- **Non-parametric profit** from Bellman equations:

$$\boldsymbol{\pi}^*_{j,q} = \boldsymbol{V}_{j,q} - \beta(1-\tilde{\zeta}_{j,q})\boldsymbol{VC}_{j,q} - \beta\tilde{\zeta}_{j,q} q E\left[\phi | \phi > \frac{\boldsymbol{VC}_{j,q}}{q}\right],$$

Fixed Costs

- **Non-parametric profit** from Bellman equations:

$$\boldsymbol{\pi}^*_{j,q} = \boldsymbol{V}_{j,q} - \beta(1-\tilde{\zeta}_{j,q})\boldsymbol{VC}_{j,q} - \beta\tilde{\zeta}_{j,q} q E\left[\phi | \phi > \frac{\boldsymbol{VC}_{j,q}}{q}\right],$$

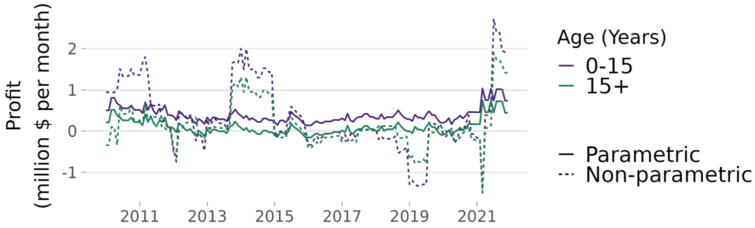
- **Parametric profit**: $\pi^*_{j,q} = pq - F_{j,q}$

Fixed Costs

- **Non-parametric profit** from Bellman equations:

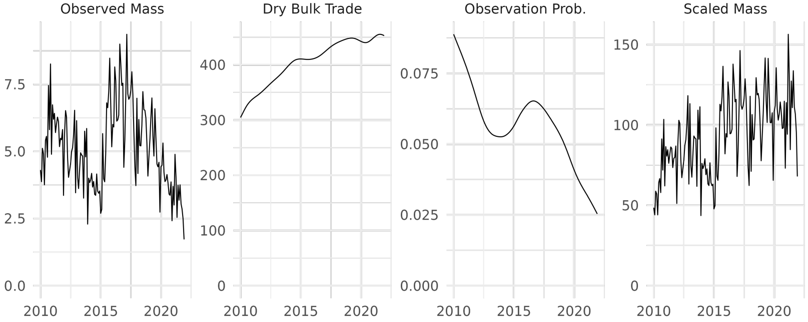
$$\pi^*_{j,q} = \boldsymbol{V}_{j,q} - \beta(1-\tilde{\zeta}_{j,q})\boldsymbol{VC}_{j,q} - \beta\tilde{\zeta}_{j,q} q E\left[\phi|\phi > \frac{\boldsymbol{VC}_{j,q}}{q}\right],$$

- Recover $\{F_{j,q}\}$ via NLS
- Regression fit:



Estimation Transition

Quantity Scaling



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Demand Elasticity

	(1) 1st Stage	(2) 2nd Stage
	log(Quantity)	*log(Rate)*
log(SD Age)	0.551 (0.545)	
log(Fleet Capacity)	1.974*** (0.393)	
log(WIP)	-0.057 (0.572)	2.269*** (0.581)
log(Food Prices)	-0.865* (0.432)	-0.504 (0.532)
log(Ag. Raw Materials Prices)	1.451*** (0.374)	-0.492· (0.296)
log(Minerals, Ores, Metals Prices)	-0.355· (0.202)	1.591*** (0.265)
log(Ship Fuel Price)	-0.202· (0.121)	-0.132 (0.161)
log(Previous Size Utilization)	0.131** (0.046)	0.255** (0.097)
log(Next Size Utilization)	0.606*** (0.110)	0.555** (0.212)
log(Quantity)		-1.172*** (0.227)
Constant	-17.145· (9.495)	-4.421 (4.599)
Observations	144	144
F-test (IV only)	30.2	64.5

- Instruments: fleet capacity, standard deviation of age

Back

Scrap Value Distribution

- Model implies relation between continuation values ν and observed exit frequencies:

$$\frac{Z_{j,q}}{n_{j,q}} = 1 - F_{j,\phi}\left(\frac{\nu}{q}\right) \tag{1}$$

- Assume scrap value distributions by age bin j, with mean $\mu_{j,\phi}$, sd $\sigma_{j,\phi}$
- Exits follow Binomial distribution so ML is:

$$\max_{\mu_{j,\phi},\sigma_{j,\phi}} \sum_t \log\left[F_{j,\phi}\left(\frac{VC_j(\boldsymbol{X}_t)}{q};\mu_{j,\phi},\sigma_{j,\phi}\right)^{n_t^j - Z_t^j}\left(1 - F_{j,\phi}\left(\frac{VC_j(\boldsymbol{X}_t)}{q};\mu_{j,\phi},\sigma_{j,\phi}\right)\right)^{Z_t^j}\right]$$

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Entry costs

From the free entry condition, we can forecast the value function one period ahead to obtain the entry cost:

$$\begin{aligned}\kappa_q(\boldsymbol{X}_t) &= \beta E_{X_{t+1},\lambda_{-q,t}}[V(0,q,\boldsymbol{X}_{t+1})|\boldsymbol{X}_t] \\ &= \beta \boldsymbol{P}_X \boldsymbol{V}_{0,q}\end{aligned}$$

where

- $\boldsymbol{V}_{0,q} \in \mathbb{R}^{L\times K}$ is the matrix of values of an age 0, size q ship for each aggregate state and size
- $\boldsymbol{P}_X$ is the row of $\boldsymbol{P}$ corresponding to state $\boldsymbol{X}$

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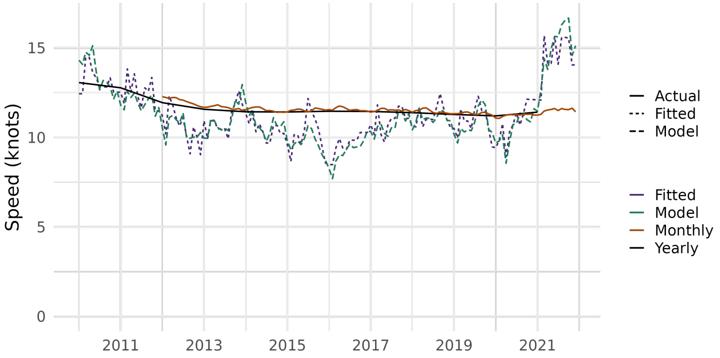
Speed Calibration Fit



- 'Actual' speeds are averages for Handymax vessels from independent data

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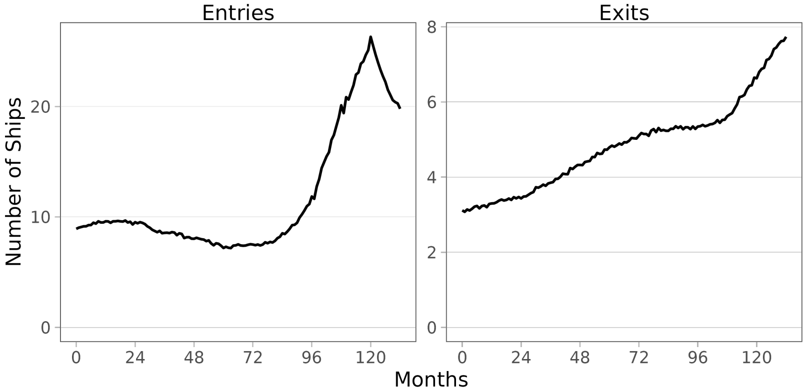
Baseline Evolution: Entry/Exit



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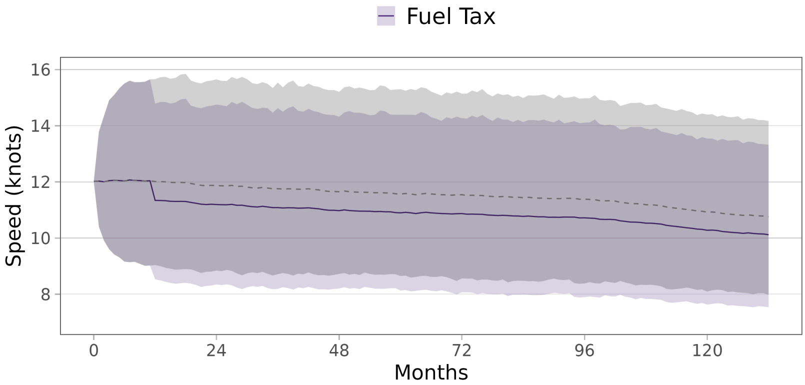
Speed Effect - Tax



- Tax maintains downward pressure

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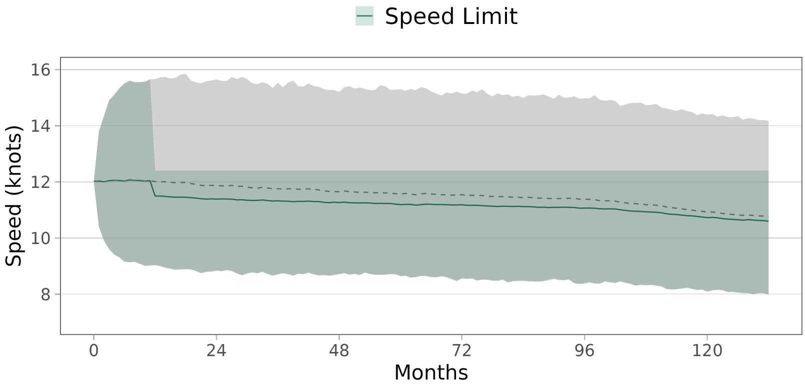
Speed Effect - Speed Limit



- Speed limit becomes less binding

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Demand

- Elasticity:

$$\log\left(\frac{p_t}{\bar{x}_t}\right) = \gamma \log(Q_t) + \omega_0 + \omega_1 \log(\boldsymbol{H}_t) + \varepsilon_t$$

  where
  - $\boldsymbol{H}$ is demand shifters
  - Q_t is scaled number of contracts observed $\times$ capacity $\times$ distance
  - Instrument for Q with supply shifters: fleet capacity, sd of age
- Define demand state variable as intercept:

$$d_t = \hat{\omega_0} + \hat{\omega_1} \log(\boldsymbol{H}_t) + \hat{\varepsilon}_t$$

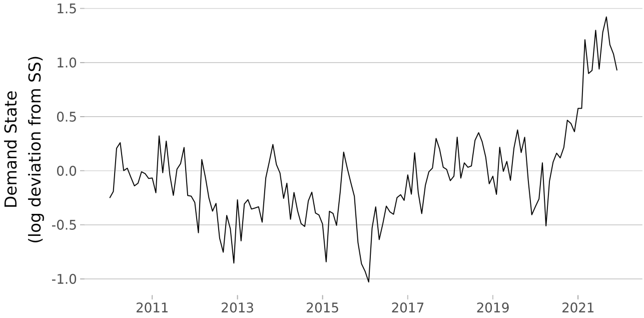
- Demand transitions:

$$d_t = \rho d_{t-1} + \rho_0 + \epsilon_t$$

Quantity Scaling

Demand Results

- Demand state:



- Elasticity: $\frac{1}{\gamma} = -0.85$
- Transition: $\rho = 0.863$

Regression Back

Value Function

Interpret second-hand ship price as value function:

$$V(j, q; d, \boldsymbol{n}) = E[p^{SH} | j, q, d, \boldsymbol{n}]$$

Local linear regression on gridpoints $[j, q, d, \boldsymbol{n}]$:

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Value Function

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Transition Probabilities

- Deterministic ageing + three stochastic processes:
  - Demand: $d_t = \rho d_{t-1} + k + \epsilon_t$
  - Entry: $M_t^q \sim Poisson(\lambda(q, d_t, \boldsymbol{n}_t))$
  - Exit: $Z_t^{j,q} \sim Poisson(\mu_j(d_t, \boldsymbol{n}_t))$
  - Ageing and exit $\Rightarrow n_{t+1}^{j,q} = n_t^{j-1,q} - Z_t^{j-1,q}$
- **Simulate** many transitions

- **Count** transitions

Transition Matrix | First Stage | Costs Est. | Back

Transition Probabilities

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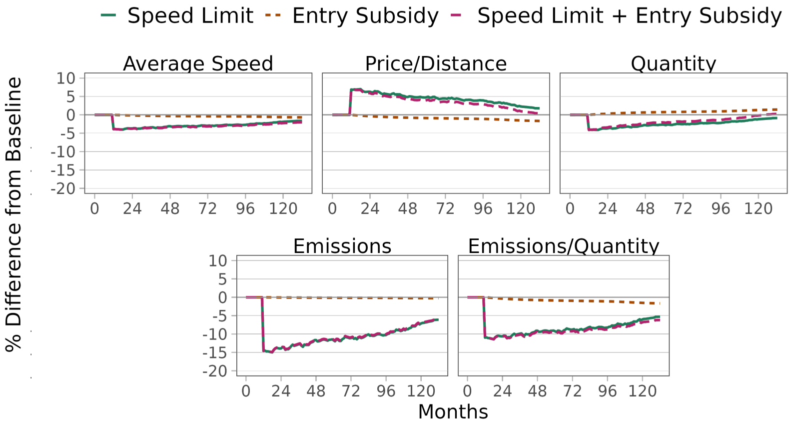
- **Simulate** many transitions
- **State-space aggregation**:
  - Nine age-size bins
  - Define levels of aggregated states using k-means clustering
  - Assign nearest aggregated state
- **Count** transitions

Transition Matrix | First Stage | Costs Est. | Back

Speed Limit + Entry Subsidy Combination



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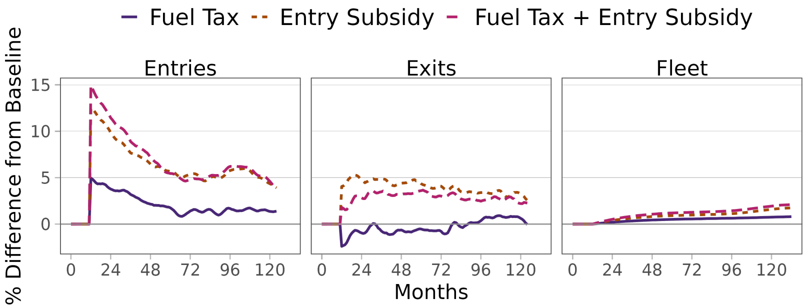
Entry/Exit with Tax/Subsidy Combo



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