Experimental Design for Policy Choice

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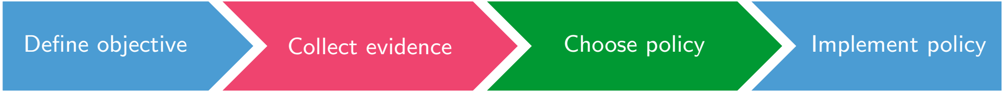
Linking data to objective

Experiments are a valuable tool for **data-driven policymaking**

- Prominent examples: negative income tax, Progresa, Moving to Opportunity, Oregon health insurance experiment, UBI experiments

Linking data to objective



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Linking data to objective

Define objective | Collect evidence | Choose policy | Implement policy

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What makes an **experiment** optimal for choosing a **policy**?

- Experiments often designed to maximize power/precision
- **Detecting effects** neither necessary nor sufficient for **choosing good policy**
- Should tax/subsidy be higher/lower? How should it vary across population?

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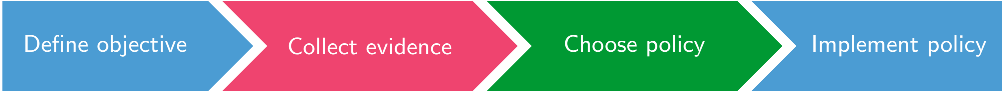

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Experimental design and **policy choice** as a **dynamic decision problem**

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Motivating example: Progresa

Cash offered to families conditional on children attending school

- **Our objective:** design cash transfer to $\uparrow$ graduation rates, $\downarrow$ gender gap
- **Experiment:** randomly offer transfer, amount can vary by grade and gender
- **Policy:** money offered by grade/gender (can differ from experiment)
- **Model:** dynamic school choice (Todd and Wolpin 2006, Attanasio, Meghir, and Santiago 2012)

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Experimental design:

- Larger subsidy or larger treatment group?
- Should we focus on particular subgroups?
- What are the constraints on the design?

Policy choice:

- How large should the subsidy be?
- Should the subsidy vary by observables?
- What are the constraints on the policy?

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Key quantity: **marginal effect** of extra peso

- Should an extra peso go to boys vs girls?
- Primary vs secondary school?
- Sufficient to characterize optimal policy
- Optimal experiment focuses on these parameters

Overview

This paper provides a **general** method for designing experiments for policy choice

- Tailored to objective and constraints of policy
- Asymptotically optimal — resulting policy has highest expected welfare

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- **Model:** link experimental variation to structural parameters
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Two results make the problem tractable

1. Information environment characterized by Gaussian **limit experiment**
2. Policy choice problem approximated by simple **quadratic program**

Together, imply that experiment need only estimate **marginal effect**

A quadratic and Gaussian example

General setup and decision problem

Asymptotic equivalence

Application to Progresa

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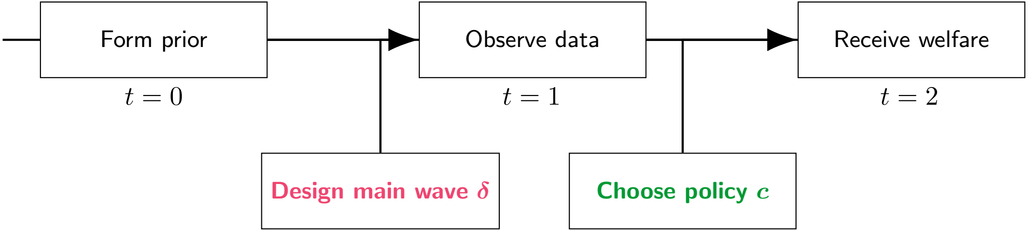
Timing


Form prior
$t = 0$
Observe data
$t = 1$
Receive welfare
$t = 2$
Design main wave δ
Choose policy c

Ingredients

Set of feasible **designs** given by $\boldsymbol{\delta} \in \Delta$

Model for data $\hat{h} \sim N(h, J(\boldsymbol{\delta})^{-1})$

Set of feasible **policies** given by $G_1 \boldsymbol{c} = 0,\ G_2 \boldsymbol{c} \leq 0$

Welfare $W(\boldsymbol{c}, h) = \boldsymbol{c}' B_{c,h} h + \boldsymbol{c}' B_{c,c} \boldsymbol{c}$

Dynamic formulation

Period 2: Policy choice

- Objective: maximize expected welfare
- Subject to budget, other constraints
- Given posterior $h \sim N(\mu_1, \Sigma_1) \mid \hat{h}$

Defines **value function** of posterior

Dynamic formulation

Period 1: Experimental design

- Objective: maximize expected value
- Subject to budget, other constraints
- Given prior $h \sim N(0, \Sigma_0)$

Governs **law of motion** for posterior

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Defines **value function** of posterior

Observe $\hat{h} \sim N(h, J(\boldsymbol{\delta})^{-1})$, form posterior

Value function

Period 2: Policy choice

$$\max_{\boldsymbol{c}} \quad \mathbb{E}[W(\boldsymbol{c}, h) \mid \hat{h}] \quad \text{s.t.} \quad G_1 \boldsymbol{c} = 0,\ G_2 \boldsymbol{c} \leq 0$$

given posterior

$$h \sim N(\mu_1, \Sigma_1) \mid \hat{h}$$

details

Value function

Period 2: Policy choice

$$\max_{\boldsymbol{c}} \quad \boldsymbol{c}'\bar{\gamma} + \boldsymbol{c}' B_{c,c} \boldsymbol{c} \quad \text{s.t.} \quad G_1 \boldsymbol{c} = 0, \; G_2 \boldsymbol{c} \leq 0$$

where

$$\gamma = B_{c,h} h \quad \bar{\gamma} = B_{c,h} \mu_1$$

Define value function $V(\bar{\gamma})$

details

Value function

Period 1: Experimental design

$$\max_{\boldsymbol{\delta}} \quad \mathbb{E}_{\boldsymbol{\delta}}\left[V\left(\bar{\gamma}\right)\right] \quad \text{s.t.} \quad \boldsymbol{\delta} \in \Delta$$

given prior

$$h \sim N(0, \Sigma_0)$$

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Law of motion is

$$\boxed{\text{Update beliefs given } \hat{h} \sim N(h, J(\boldsymbol{\delta})^{-1})}$$

Period 2: Policy choice

$$\max_{\boldsymbol{c}} \quad \boldsymbol{c}'\bar{\gamma} + \boldsymbol{c}' B_{c,c}\boldsymbol{c} \quad \text{s.t.} \quad G_1\boldsymbol{c} = 0,\ G_2\boldsymbol{c} \leq 0$$

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Solution algorithm

Solve with standard value function approximation

- Sample $\{\bar{\gamma}_j\}_{j=1}^{M}$ from state space
- Compute $\{V(\bar{\gamma}_j)\}_{j=1}^{M}$ (quadratic program)
- Construct $\hat{V}(\cdot)$ by interpolating (use your favorite regression model)
- Choose δ to maximize $\mathbb{E}[\hat{V}(\bar{\gamma})]$ (quadrature or simulation for LOM)

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Functional form of $W(\boldsymbol{c}, h)$ key for tractability

- $\bar{\gamma}$ can be substantially lower dimensional than (μ_1, Σ_1)

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Gaussian/quadratic setting is easy– but too simple to reflect actual experiments?

- Progresa example had nonlinear dynamic structural model, non-quadratic welfare, many observations...

A quadratic and Gaussian example

General setup and decision problem

Asymptotic equivalence

Application to Progresa

Ingredients

Experimental design $z_i \sim p_{z|x}(z_i \mid x_i; \delta)$ for $i = 1, \ldots, n$

- Treatment may be binary, continuous, multidimensional, but parametric
- Progresa: current and future subsidy offered

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Model for data $y_i \sim p_{y|z,x}(y_i \mid z_i, x_i; \boldsymbol{\theta})$ for $i = 1, \ldots, n$

- Can be dynamic, equilibrium, but parametric
- Progresa: dynamic discrete choice

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- New taxes, subsidies, prices (not nec. same treatment as experiment)
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Welfare $w(\boldsymbol{\pi}, \boldsymbol{\theta})$

- Can be as rich as model: dynamic effects, equilibrium effects, utility of agents, profit
- Progresa: long-run completion rate

Value function

Period 2: Policy choice

$$\max_{\boldsymbol{\pi}} \quad \mathbb{E}[w(\boldsymbol{\pi}, \boldsymbol{\theta}) \mid \{y_i, z_i\}_{i=1}^n]$$

s.t.

$$g(\boldsymbol{\pi}) \leq 0$$

given posterior $\boldsymbol{\theta} \sim q(\boldsymbol{\theta} \mid \{y_i, z_i\}_{i=1}^n)$

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Law of motion is given by model

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$$\boxed{\begin{array}{c}(y_i, z_i) \sim p(y_i, z_i \mid x_i; \boldsymbol{\theta}, \boldsymbol{\delta}) \\ \text{for } i = 1, \ldots, n\end{array}}$$

Finite-sample problem is intractable

While structure is similar, this problem is **much more difficult**

Challenge: $v(\{y_i, z_i\}_{i=1}^n)$ is extremely high dimensional

- Well-known in adaptive experimentation literature
- Must compute value of policy choice for every possible dataset ▸ details
- Progresa: y_i binary, $z_i \in \mathbb{R}^3$, $n_1 = 1000 \implies$ state space is in $\mathbb{R}^{3000} \times \{0,1\}^{1000}$

Gaussianity & quadratic welfare helped **a lot**

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Gaussianity & quadratic welfare helped **a lot**

We show that this problem is well-approximated by the Gaussian problem, which is

- **Tractable:** limiting problem has simple structure
- **Interpretable:** relies on marginal effects of policy
- **Asymptotically optimal:** results in highest possible expected welfare

A quadratic and Gaussian example

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Two approximations

Data $\{y_i, z_i\}_{i=1}^n$ summarized by **efficient estimate** $\hat{\boldsymbol{\theta}}_n$

- Related to classical efficiency results
- Holds uniformly across designs
- Any policy $\hat{\boldsymbol{\pi}}_n$ converges in d to a policy which depends only on $\hat{h}$

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Value of policy choice v approximated by **quadratic program** V

- Related to extremum estimation, but complicated by constraints
- Quadratic approximation to welfare, linear approximation of constraints
- QP is directional derivative of value function

Two approximations

Suppose we have prior on $\boldsymbol{\theta}$

$$\sqrt{n}(\boldsymbol{\theta} - \boldsymbol{\theta}_0) \sim N(0, \Sigma_0)$$

which is **asyptotically informative**

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Posterior approximately normal as well

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Then define

$$\boldsymbol{c} := \sqrt{n}(\boldsymbol{\pi} - \boldsymbol{\pi}_0)$$

Then $W(\boldsymbol{c}, h)$ and G_1, G_0 constructed through Taylor approximation of Lagrangian ▸ details

Where does the prior come from?

Where does asymptotically informative prior come from?

Paper uses **pilot data** to construct prior: If $\hat{\boldsymbol{\theta}}_{n,\text{pilot}}$ is pilot estimate,

$$\boldsymbol{\theta}_0 = \hat{\boldsymbol{\theta}}_{n,\text{pilot}} \qquad\qquad \Sigma_0 = \frac{1}{n_{\text{pilot}}} J_0^{-1}$$

Prior and quadratic approximation in this case is **random**

- Guarantees in paper are fully ex-ante
- For any smooth pre-pilot prior, posterior after pilot is approximately Gaussian as above
- Asymptotically informative if pilot is nonnegligible vs main sample

Assumptions

Formal results stated in terms of regret

$$r(\boldsymbol{\pi}, \boldsymbol{\theta}) = w(\boldsymbol{\pi}^*, \boldsymbol{\theta}) - w(\boldsymbol{\pi}, \boldsymbol{\theta})$$
$$R(\boldsymbol{c}, h) = W(\boldsymbol{c}^*, h) - W(\boldsymbol{c}, h)$$

Maximizing welfare equivalent to minimizing regret under Bayes risk

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Maximizing welfare equivalent to minimizing regret under Bayes risk

Assumption

1. *Likelihood ratio of model is stochastically equicontinuous in* Δ
2. *Strict second-order condition is satisfied*
3. *Regret is uniformly integrable, other regularity conditions*

Main results

Quadratic/Gaussian problem is lower bound for any choice of design and policy

Theorem (Informal)

Let $(\boldsymbol{\delta}_n, \boldsymbol{\pi}_n)$ be any sequence of designs and policies in finite-sample experiment.

Let $(\boldsymbol{\delta}^*, \boldsymbol{c}^*)$ be optimal in limit experiment (i.e. maximize $E[V(\bar{\gamma})]$)

Under our assumptions,

$$\liminf_{n \to \infty} n \mathbb{E}_{\boldsymbol{\delta}_n} \left[r \left(\boldsymbol{\pi}_n, \boldsymbol{\theta} \right) \right] \geq \mathbb{E}_{\boldsymbol{\delta}^*} \left[R(\boldsymbol{c}^*, h) \right]$$

Empirical analog

Solving $V(\bar{\gamma})$ relies on unknown population quantities

- In particular, knowledge of which constraints bind

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Proposed estimation avoids pre-testing for set of binding constraints

- Use pilot estimate $\hat{\boldsymbol{\theta}}_{n,\mathrm{pilot}}$
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Resulting $\hat{V}(\bar{\gamma})$ can be solved similarly to $V(\bar{\gamma})$

- State space is the same– still low-dimensional
- Posterior mean on $\gamma = B_{c,h}h$ sufficient state variable as before
- Solving $\hat{V}(\bar{\gamma})$ becomes NLP rather than QP

details

Main results

Empirical analog attains lower bound

Theorem (Informal)

Let $(\hat{\boldsymbol{\delta}}_n, \hat{\boldsymbol{\pi}}_n)$ be optimal in empirical analog (i.e. maximize $\mathbb{E}[\hat{V}(\bar{\gamma})]$).

Under our assumptions,

$$\lim_{n \to \infty} n \mathbb{E}_{\hat{\boldsymbol{\delta}}_n} \left[r\left(\hat{\boldsymbol{\pi}}_n, \boldsymbol{\theta} \right) \right] = \mathbb{E}_{\boldsymbol{\delta}^*} \left[R(\boldsymbol{c}^*, h) \right]$$

Interpreting the state variable $\bar{\gamma}$

Policy choice only depends on $\bar{\gamma}$

- γ tells us **marginal effect** of changing policy from $\boldsymbol{\pi}_0$
- $\bar{\gamma}$ delivers posterior estimate of this marginal effect
- $\nabla_{\boldsymbol{\pi}} w(\boldsymbol{\pi}_0, \boldsymbol{\theta}) \approx \nabla_{\boldsymbol{\pi}} w(\boldsymbol{\pi}_0, \boldsymbol{\theta}_0) + \gamma$

Experimental designs can be compared by how informative they are about γ

- $W(\boldsymbol{c}, h)$ depends on h only **linearly** through γ
- Only **welfare effects** of $\boldsymbol{\pi}$ are policy-relevant

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Marginal effect of policy is **low-dimensional**

- γ is of the same dimension as $\boldsymbol{\pi}$
- Often low-dim for practical reasons (Kitagawa and Tetenov 2018, Athey and Wager 2021)
- Dimension of state depends only on complexity of **policy** rather than **model**

A quadratic and Gaussian example

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Setting

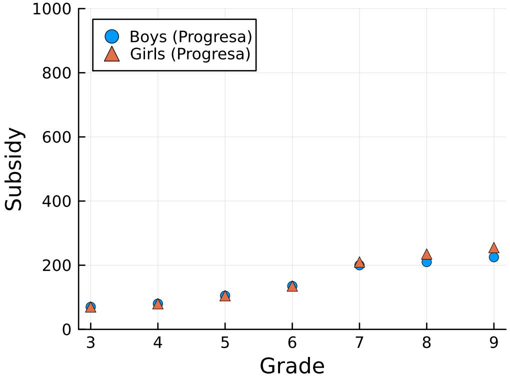
Cash transfer program in rural Mexico

- Receive transfer if children attend school
- Large frac ($\sim 20\%$) of household income

Original experiment:

- 62% randomly assigned to treatment
- Effects observed for two years
- Rolled out to broader population

Our goal: increase completion and reduce gender gap



62-38 treatment-control split

details

Model

Similar to Attanasio, Meghir, and Santiago (2012)

- At each age $\tau < 18$, households choose binary attendance y_τ
- If $y_\tau = 1$, gets transfer z_τ
- If $y_\tau = 0$, child works and earns wage w_τ
- If enrolled in grade s_τ, pass with probability $r(\tau, s_\tau)$

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- If enrolled in grade s_τ, pass with probability $r(\tau, s_\tau)$
- Utility of schooling also depends on covariates x_τ

$$
\begin{aligned}
u_{1\tau} &= \theta_0 + \theta_1 z_\tau + \theta_2'(\tau, s_\tau, x_\tau) + \theta_3'(\tau, s_\tau, x_\tau) z_\tau + \epsilon_{1\tau} \\
u_{0\tau} &= \theta_4 w_\tau + \epsilon_{0\tau}
\end{aligned}
$$

- At age $\tau = 18$, get terminal value

$$
v_{18} = \theta_5 s_{18}
$$

Model

Households solve

$$\max_{\{y_\tau\}_{\tau=6}^{18}} \sum_{\tau=6}^{18} \beta^\tau \mathbb{E}\left[y_\tau u_{1\tau} + (1 - y_\tau) u_{0\tau}\right]$$

$$s_{\tau+1} = s_\tau + y_\tau \text{Bernoulli}(r(\tau, s_\tau))$$

Under T1EV assumption on $\epsilon_{y\tau}$, we can compute the value function and obtain the likelihood of observed choices

Objective

Objective: Maximize school completion and reduce gender gap

$$W(\boldsymbol{\pi}, \boldsymbol{\theta}) = \kappa \underbrace{\mathbb{P}_{\boldsymbol{\theta}, \boldsymbol{\pi}}[s_{18} \geq 9]}_{\text{completion}} - (1-\kappa) \underbrace{\left(\mathbb{P}_{\boldsymbol{\theta}, \boldsymbol{\pi}}[s_{18} \geq 9 \mid \text{boy}] - \mathbb{P}_{\boldsymbol{\theta}, \boldsymbol{\pi}}[s_{18} \geq 9 \mid \text{girl}]\right)^2}_{\text{gender gap}}$$

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Period 2: Policy choice

- Subsidy is piecewise linear in grade, by gender
- Total amount of subsidy in policy must equal original policy

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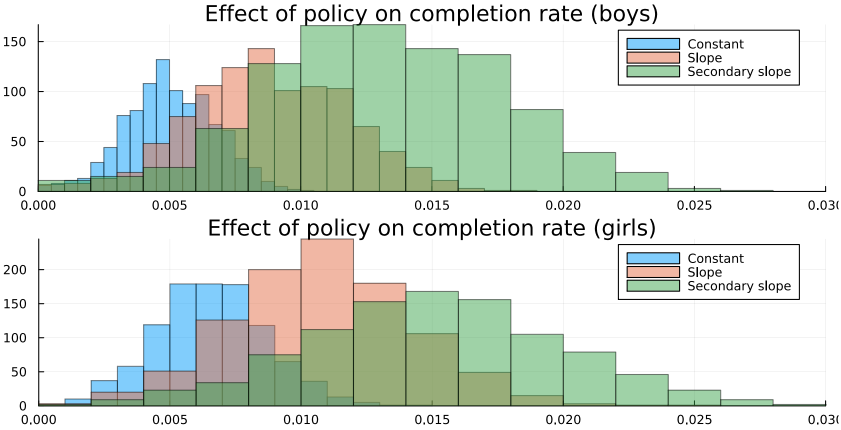
Period 1: Experimental design

- Subsidy is piecewise linear in grade, by gender
- Treatment prob depends on grade and gender
- Total amount of subsidy in experiment must equal original experiment
- $n_0 = 500$, consider $n_1 = 500, \ldots, 4000$

Period 2: Policy choice

- Subsidy is piecewise linear in grade, by gender
- Total amount of subsidy in policy must equal original policy

Uncertainty about effects of subsidy

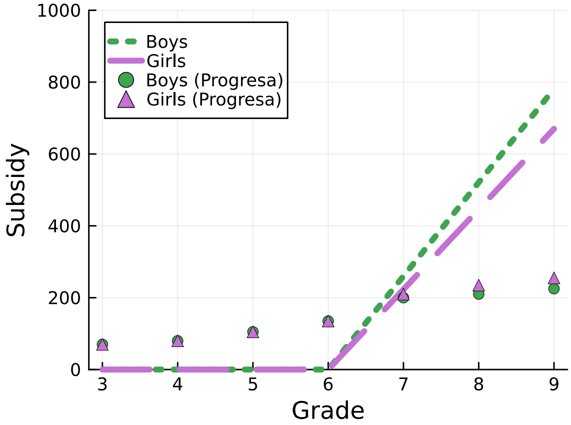


Best experiment for informing policy

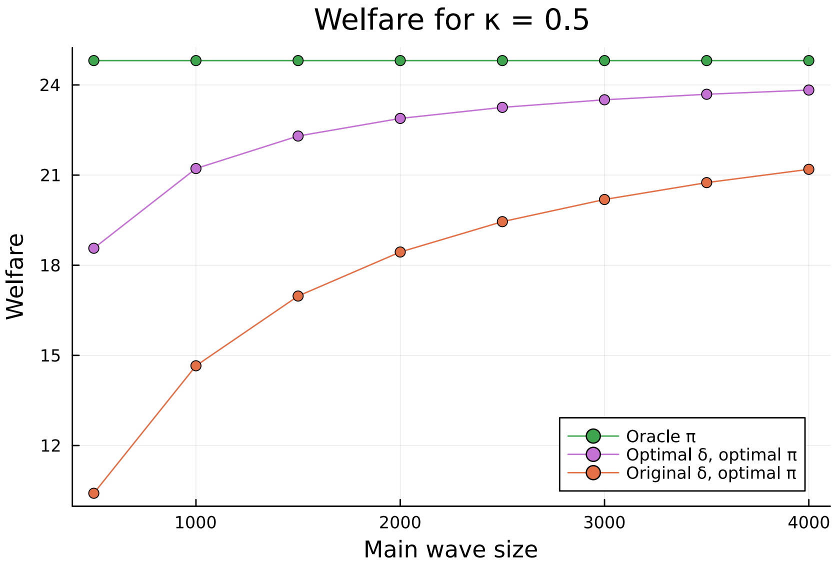
Treatment probabilities

- Boys in sec. school: 77%
- Girls in sec. school: 67%
- Boys in pri. school: 0%
- Girls in pri. school: 0%

Experimental subsidy



Expected welfare gain from policy-focused experiment



more

Constructing quadratic approximation

Let $(\boldsymbol{\pi}_0, \boldsymbol{\lambda}_0)$ be optimal policy and Lagrange multiplier under $\boldsymbol{\theta}_0$

$$L(\boldsymbol{\pi}, \boldsymbol{\theta}, \boldsymbol{\lambda}) = r(\boldsymbol{\pi}, \boldsymbol{\theta}) + \boldsymbol{\lambda}' g(\boldsymbol{\pi})$$

$$\begin{bmatrix} B_{c,c} & B_{c,h} \\ B_{h,c} & B_{h,h} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\nabla^2_{\boldsymbol{\pi}\boldsymbol{\pi}} L(\boldsymbol{\pi}_0, \boldsymbol{\theta}_0, \boldsymbol{\lambda}_0) & \nabla^2_{\boldsymbol{\pi}\boldsymbol{\theta}} L(\boldsymbol{\pi}_0, \boldsymbol{\theta}_0, \boldsymbol{\lambda}_0) \\ \nabla^2_{\boldsymbol{\theta}\boldsymbol{\pi}} L(\boldsymbol{\pi}_0, \boldsymbol{\theta}_0, \boldsymbol{\lambda}_0) & \frac{1}{2}\nabla^2_{\boldsymbol{\theta}\boldsymbol{\theta}} L(\boldsymbol{\pi}_0, \boldsymbol{\theta}_0, \boldsymbol{\lambda}_0) \end{bmatrix}$$

$$\begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = \begin{bmatrix} \nabla_{\boldsymbol{\pi}} g_j(\boldsymbol{\pi}_0), & j : \lambda_{0j} > 0 \\ \nabla_{\boldsymbol{\pi}} g_j(\boldsymbol{\pi}_0), & j : \lambda_{0j} = 0 \ \&\ g(\boldsymbol{\pi}_0) = 0 \end{bmatrix}$$

Recall

$$V(\bar{\gamma}) = \max_{\boldsymbol{c}} \quad \mathbb{E}[\boldsymbol{c}' B_{c,h} h] + \boldsymbol{c}' B_{c,c} \boldsymbol{c} \quad \text{s.t.} \quad G_1 \boldsymbol{c} = 0, \quad G_2 \boldsymbol{c} \leq 0$$

▸ back

Estimation

Construct welfare approximation

$$\begin{aligned}\hat{W}(\boldsymbol{\pi}, \boldsymbol{\theta}) = {} & \nabla_{\boldsymbol{\pi}} W(\boldsymbol{\pi}_0, \boldsymbol{\theta}_0)(\boldsymbol{\pi} - \boldsymbol{\pi}_0) \\ & + (\boldsymbol{\pi} - \boldsymbol{\pi}_0)' \nabla^2_{\boldsymbol{\pi}, \boldsymbol{\theta}} W(\boldsymbol{\pi}_0, \boldsymbol{\theta}_0)(\boldsymbol{\theta} - \boldsymbol{\theta}_0) \\ & + \frac{1}{2}(\boldsymbol{\pi} - \boldsymbol{\pi}_0)' \nabla^2_{\boldsymbol{\pi}, \boldsymbol{\pi}} W(\boldsymbol{\pi}_0, \boldsymbol{\theta}_0)(\boldsymbol{\pi} - \boldsymbol{\pi}_0)\end{aligned}$$

and define

$$\hat{V}(\bar{\gamma}) = \max_{\boldsymbol{\pi}} \quad \mathbb{E}[\hat{W}(\boldsymbol{\pi}, \boldsymbol{\theta}) \mid \hat{\boldsymbol{\theta}}_n] \quad \text{s.t.} \quad g(\boldsymbol{\pi}) \leq 0$$

back