Dynamic Incentives for Screening and Monitoring in Venture Capital

Chunlan Xu
Coauthored with Xin Huang and Zhaojun Yang

Department of Finance
Southern University of Science and Technology (SUSTech)

DSE2025HKU Conference

December 10, 2025

Overview

Contents

Motivation

Venture Capital Markets

- **Venture Capital (VC) Role:** VCs select and nurture high-potential startups.
- **Two Key Activities:**
  - **Screening:** Selecting high-quality projects (ex-ante, **one-shot**)
  - **Monitoring:** Managing projects post-investment (ex-post, **ongoing**)
- **Agency Conflict:**
  - Both efforts are costly and **unobservable** to oustside investors
  - Screen carelessly and monitor poorly without proper incentives
  - Misalignment leads to project failures and investor losses
- how do we design optimal incentive mechanisms for these two very different types of effort?

Research Gap in the Literature

- **Existing Continuous-Time Models:**
  - Focus on **ongoing monitoring** efforts (Sannikov, 2008)
  - Standard approach: ongoing actions with transient effects
- **Missing:** Models for **one-shot screening** with **persistent effects**
- **Recent Related Work:**
  - Gryglewicz et al. (2024): screening/monitoring for **bank loans**
  - Uncertainty driven by **exponential distribution** (default risk)
  - Our contribution: **high-risk VC projects** with **Brownian motion** uncertainty

Our Contributions

1. **Novel Methodology:**
   - Generalized Lagrange multipliers + measure transformation
   - First continuous-time model for **screening** high-risk projects
2. **Screening-Only Model:**
   - Optimal contract: **Lump-sum payment** when output hits time-dependent threshold
   - Threshold: linear + exponential growth in time
   - **Special Case ($\gamma = r$): First-best optimality** achieved!
3. **Extension:** Screening + Monitoring model
   - VC keeps equity fraction with time-varying dividend threshold
4. **Practice:** Theoretical foundation for hurdle rates

Contents

The Players

Principal-Agent Framework

- **Principal (P):** Risk-neutral investor
  - Discount rate: r
  - Deep pockets (unlimited capital)
- **Agent (A):** Risk-neutral VC/Manager
  - Discount rate: $\gamma > r$ (more impatient)
  - Limited liability: $dC_t \geq 0$
  - Outside option value: R

Key Assumption: $\gamma > r$ captures **A**'s borrowing constraints or higher opportunity cost

The Project

Cumulative Output Process

$$dX_t = \mu dt + \sigma dZ_t \tag{1}$$

- $\mu \geq 0$: Project's intrinsic potential (drift)
- Determined by **A**'s one-shot screening effort at $t = 0$
- Cost of screening: $\frac{1}{2}\psi\mu^2$ (quadratic)
- μ is **hidden** (moral hazard); X_t is **observable**
- $\sigma > 0$: Volatility (project risk, exogenous)

Key Feature: Screening effort is **one-shot** but affects output **forever**

Timeline of Actions

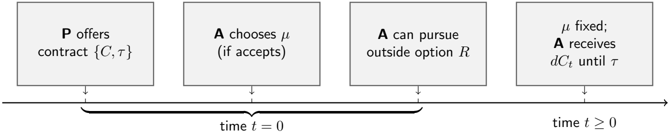

Figure: Sequential actions: **P** offers contract $\Rightarrow$ **A** screens $\Rightarrow$ Project runs

The Optimization Problem

P solves:

$$\max_{\mathcal{C}=\{C,\tau\},\mu} \mathbb{E}\left[\int_0^\infty e^{-rt} dX_t - \int_0^\tau e^{-rt} dC_t\right] \tag{2}$$

subject to:

1. **Incentive Compatibility (IC):**

$$\mu^* \in \arg\max_\mu W^C(\mu) \equiv \mathbb{E}\left[\int_0^\tau e^{-\gamma t} dC_t\right] - \frac{1}{2}\psi\mu^2 + R$$

2. **Participation Constraint (PC):** $W^C(\mu) \geq R$ (Outside Option)
3. **Limited Liability (LL):** $dC_t \geq 0$

First-Best case (full information)

First Best case

- The first-best (FB) benchmark model: all decisions are **contractible** (full information).
- **A** should be paid as early as possible and any delayed payment to **A** would undermine **P**'s earnings or contract efficiency.

The Solution

- **P** must pay **A** the value of $M^{FB} = w + \frac{1}{2}\psi\mu^2 - R$ at the very beginning.
- Optimal recommended effort: $\mu^{\mathbf{FB}} = \frac{\mathbf{1}}{\psi\mathbf{r}}$.
- P's value: $b^{FB}(w) = \frac{1}{2\psi r^2} - (w - R)$, $w \in [R, \frac{1}{2\psi r^2} + R]$.

Second-best case (moral hazard)

Second-best case

- **A**'s **screening effort** (or drift μ) is not contractible.
- If **P** paid **A** the lump sum $w + \frac{1}{2\psi r^2} - R$ as what she does in the FB situation, what would be the best choice of **A** with hidden screening? Doing nothing, i.e., $\mu = 0$.
- Therefore, **P** must **postpone the payment** to incentivize **A**.

A's problem:

$$\max_{\mu} \mathbb{E}\left[\int_0^{\tau} e^{-\gamma t} dC_t\right] - \frac{1}{2}\psi\mu^2 + R.$$

Challenge: $dC_t \leftarrow dX_t \leftarrow \mu$ (in an unknown form).

Measure Transformation Method

Our Solution: Measure Transformation

- **Key insight**: μ changes **probability distribution** rather than drift of X_t.
- Under $\mathbb{P}$: $dX_t = \sigma dZ_t$ (no effort)
- Under $\mathbb{P}^\mu$: $dX_t = \mu dt + \sigma dZ_t^\mu$ (with effort μ)
- The Radon-Nikodym derivative between $\mathbb{P}^\mu$ and $\mathbb{P}$ is given by

$$\mathbf{\Gamma_t} \equiv \left.\frac{d\mathbb{P}^\mu}{d\mathbb{P}}\right|_{\mathcal{F}_t} = \exp\left(\frac{\mu}{\sigma} Z_t - \frac{\boldsymbol{\mu^2}}{2\sigma^2} t\right),$$

where Z_t and $Z_t^\mu = Z_t - (\mu/\sigma)t$ are standard Brownian motions under $\mathbb{P}$ and $\mathbb{P}^\mu$, respectively.

Reformulated Problem

- **A's problem**:

$$\max_{\mu} \mathbb{E}^{\mu}\left[\int_0^{\tau} e^{-\gamma t} dC_t\right] - \frac{1}{2}\psi\mu^2 + R \tag{3}$$
$$= \max_{\mu} \mathbb{E}\left[\int_0^{\tau} e^{-\gamma t} \boldsymbol{\Gamma_t} dC_t\right] - \frac{1}{2}\psi\mu^2 + R$$

- By FOC, we get the following agent's IC:

$$\mathbb{E}\left[\int_0^{\tau} e^{-\gamma t} \Gamma_t \left(\frac{Z_t}{\sigma} - \frac{\mu}{\sigma^2} t\right) dC_t\right] = \psi\mu\sigma,$$

or equivalently,

$$\mathbb{E}^{\mu}\left[\int_0^{\tau} e^{-\gamma t} Z_t^{\mu} dC_t\right] = \psi\sigma\mu. \tag{4}$$

Reformulated Problem

- **P's problem:**

$$\max_{\mathcal{C}=\{C,\tau\},\mu} \quad \frac{\mu}{r} - \mathbb{E}^{\mu}\left[\int_0^{\tau} e^{-rt} dC_t\right]$$

$$\text{s.t.} \quad \begin{cases} \mathbb{E}^{\mu}\left[\int_0^{\tau} e^{-\gamma t} Z_t^{\mu} dC_t\right] = \psi\sigma\mu, \ \text{(IC)} \\ \mathbb{E}^{\mu}\left[\int_0^{\tau} e^{-\gamma t} dC_t\right] - \psi\mu^2/2 + R = w \geq R. \ \text{(PC)}. \end{cases}$$

- By the Lagrangian method, **P** solves

$$\max_{\mathcal{C}=\{C,\tau\},\mu,\lambda_1,\lambda_2} \quad G(C,\tau,\mu,\lambda)$$
$$\equiv \mathbb{E}^{\mu}\left[\int_0^{\tau} e^{-rt}\left(\boldsymbol{\lambda_1 e^{-(\gamma-r)t} Z_t^{\mu} + \lambda_2 e^{-(\gamma-r)t} - 1}\right) dC_t\right]$$
$$+ \left(\frac{1}{r} - \lambda_1\psi\sigma\right)\mu - \frac{1}{2}\lambda_2\psi\mu^2 + \lambda_2 R - \lambda_2 w. \tag{5}$$

for some Lagrange multipliers λ_1 and λ_2.

Optimal Contract, general form

Theorem 1

Suppose **A** *is more impatient* ($\gamma > r$). *In the optimal contract:*

1. **P**'*s payment to* **A** *is made* **at most once** *(Lump-sum* $\mathbf{M}(\tau^*)$*).*
2. *The payment occurs at stopping time* τ^**: the first time the standard BM* $\{\mathbf{Z_t^\mu}\}$ *hits a threshold* $[\mathbf{e^{(\gamma-r)t}} - \lambda_2^*]/\lambda_1^*$. *Equivalently, payment triggers when cumulative output* X_t *hits a* **time-dependent curve** $(X_0 + \mu^* t + \sigma Z_t^\mu)$.
3. *The optimal contract is determined by* $(\tau^*, \mu^*, \lambda_1^*, \lambda_2^*, \mathbf{M}(\tau^*))$ *, satisfying the following system of equations*

$$\begin{cases} \left(\frac{1}{r} - \lambda_1^*\psi\sigma\right) - \lambda_2^*\psi\mu^* - \frac{1}{\sigma}\lambda_1^*\mathbb{E}^{\mu^*}\left[M(\tau^*)e^{-\gamma\tau^*}\tau^*\right] = 0, \text{ (FOC)} \\ \mathbb{E}^{\mu^*}\left(M(\tau^*)e^{-r\tau^*} - \lambda_2^* M(\tau^*)e^{-\gamma\tau^*}\right) = \lambda_1^*\psi\sigma\mu^*, \text{ (IC)} \\ \mathbb{E}^{\mu^*}\left[M(\tau^*)e^{-\gamma\tau^*}\right] = \frac{1}{2}\psi(\mu^*)^2 + w - R. \text{ (PC)} \end{cases} \tag{6}$$

Optimal Contract, simplified Form

Proposition 1

- *The payment threshold is a time-dependent curve given by* $\xi \mathbf{e}^{(\gamma-\mathbf{r})\mathbf{t}} + \eta$ *for some constants* ξ *and* η *with the lump-sum payment* M *being a* **constant** *independent of the payment time.*
- **P***'s problem can be reduced to*

$$\max_{\mathbf{M},\xi,\eta,\mu} \quad \frac{\mu}{r} - M\mathbb{E}^{\mu}(\mathbf{e}^{-\mathbf{r}\tau^*})$$
$$\text{s.t.} \quad \begin{cases} M\mathbb{E}^{\mu}\left[e^{-\gamma\tau^*}(\xi e^{(\gamma-r)\tau^*} + \eta)\right] = \psi\sigma\mu, \ \textit{(IC)} \\ M\mathbb{E}^{\mu}\left[e^{-\gamma\tau^*}\right] = \psi\mu^2/2 + w - R, \ \textit{(PC)} \end{cases} \tag{7}$$

where τ^* *is the stopping time when the standard BM* $\{Z_t^{\mu}\}_{t\geq 0}$ *first hits the deterministic time curve (threshold)* $\xi e^{(\gamma-r)t} + \eta$*.*

Optimal Contract, simplified Form

- Denote $\mathbf{D}(\xi, \eta; \delta) = \mathbb{E}^{\mu}(\mathbf{e}^{-\delta\tau^*})$, where $\delta = \gamma$ or r.
- **P**'s problem can be changed into the following unconstrained optimization:

$$\begin{aligned} \max_{M,\xi,\eta,\mu;\alpha_1,\alpha_2} \quad & \mathcal{V}(M, \xi, \eta, \mu; \alpha_1, \alpha_2) \\ & \equiv \tfrac{\mu}{r} - MD(\xi, \eta; r) + \alpha_1(\xi MD(\xi, \eta; r) \\ & \quad +\eta MD(\xi, \eta; \gamma) - \psi\sigma\mu) \\ & \quad +\alpha_2(MD(\xi, \eta; \gamma) - \tfrac{1}{2}\psi\mu^2 - w + R). \end{aligned} \tag{8}$$

- When $\gamma = r$, $D(\xi, \eta; \delta) = \mathbb{E}^{\mu}(e^{-\delta\tau^*})$ has explicit solution.
- When $\gamma > r$, $D(\xi, \eta; \delta)$ is computed by Monte Carlo method and we have numerical solution.

First Best Optimality

Main Finding

When **P** and **A** are equally patient ($\gamma = r$), the optimal contract achieves **First-Best Efficiency**! Moral hazard is **completely eliminated**.

Theorem 2 (Explicit Solution)

If $\gamma = r$, constant threshold, $\mathbb{E}^{\mu}[\mathbf{e}^{-\mathbf{r}\tau^*}] = e^{-\sqrt{2r}\bar{Z}^*}$.

- *Payment threshold: $\bar{Z}^* = \frac{\sigma\psi r}{0.5+(w-R)\psi r^2}$ (**constant**)*
- *Lump-sum amount: $M^* = \frac{\sigma}{r\bar{Z}^*}e^{\sqrt{2r}\bar{Z}^*}$*
- *Optimal effort: $\mu^* = \frac{\mathbf{1}}{\psi\mathbf{r}}$ (**same as first-best**)*
- ***P**'s value: $b^*(w) = \frac{1}{2\psi r^2} - w + R$ (first-best)*

Why First-Best is Achieved

- **Intuition:** No cost from delaying payment when $\gamma = r$
- **P** can set threshold **high enough** to perfectly infer μ
- **Zero incentive cost** $\Rightarrow$ 100% contract efficiency

Comparison

- Generally: First-best impossible in principal-agent models
- Our result: Possible when discount rates align
- If $\gamma > r$: First-best unattainable (deadweight loss from delayed payment)

Optimal Contract ($\gamma > r$)

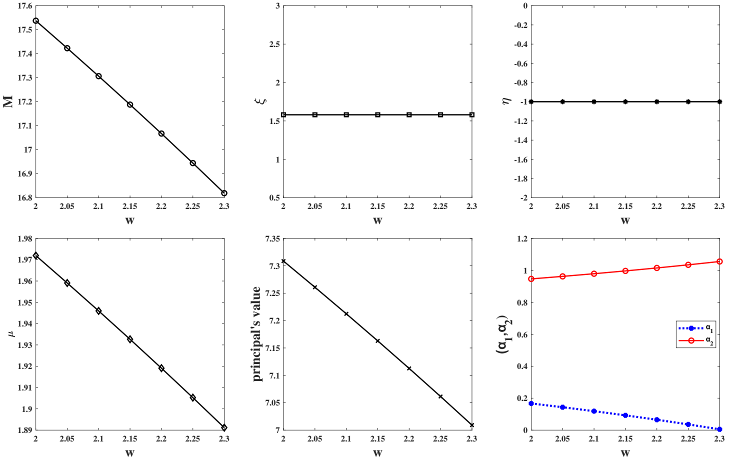

Figure: The effect of **A**'s promised value on the lump-sum payoff M, payment threshold $\bar{Z}$, afforded effort μ, and **P**'s value $b(w)$ with parameter values $\gamma = 0.1, r = 0.05, R = 0, \psi = 2, \sigma = 5$.

Optimal Contract $(\gamma > r)$

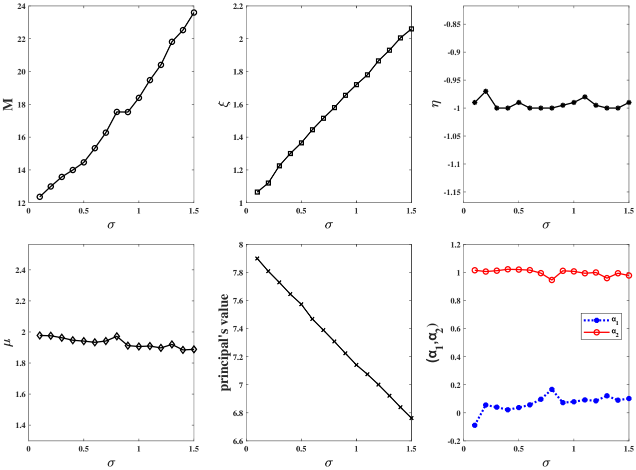

Figure: The effect of project risk on the lump-sum payoff M, payment threshold $\bar{Z}$, afforded effort μ, and **P**'s value $b(w)$ with parameter values $r = 0.05, R = 0, \psi = 2, w = 5$.

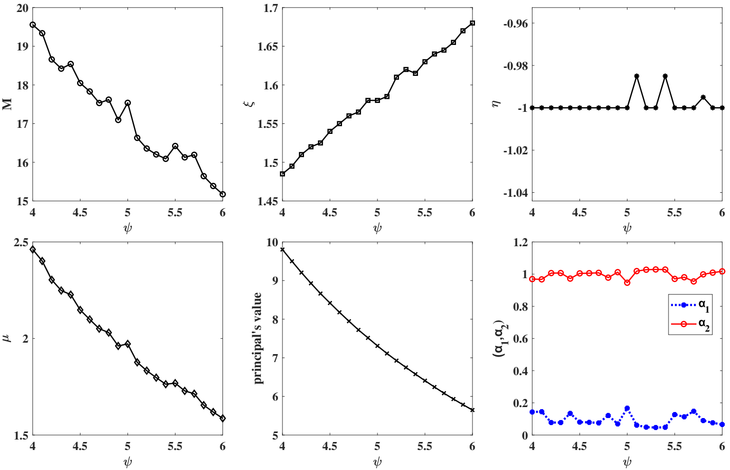

Figure: The effect of screening cost coefficient on the lump-sum payoff M, payment threshold $\bar{Z}$, afforded effort μ, and **P**'s value $b(w)$ with parameter values $r = 0.05, R = 0, \sigma = 5, w = 5$.

Optimal Contract $(\gamma > r)$

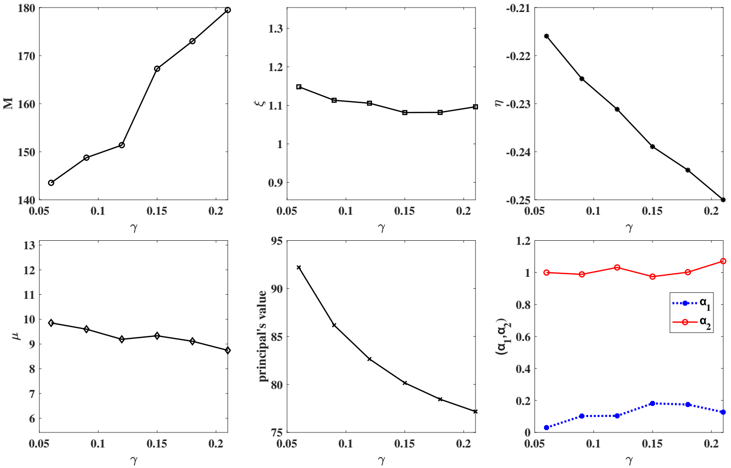

Figure: The effect of screening cost coefficient on the lump-sum payoff M, payment threshold $\bar{Z}$, afforded effort μ, and **P**'s value $b(w)$ with parameter values $r = 0.05, R = 0, \psi = 2, \sigma = 5, w = 5$.

Contents

Incorporating Monitoring

Extended Model

$$dX_t = (\mu + a_t)dt + \sigma dZ_t \tag{9}$$

- μ: One-shot screening (initial, persistent effect)
- $a_t \in [0, \bar{a}]$: Ongoing monitoring (continuous, hidden)
- Monitoring cost: θa_t per unit time

Double Moral Hazard:

1. Monitoring incentive: **A** must bear continuous risk
2. Screening incentive: **A** needs skin in the game from start

Optimal Contract with Both

Theorem 3 (Screening + Monitoring)

If $\gamma > r$, the optimal contract:

- **A** *maintains promised value account W_t*
- **Payment:** *Dividends paid if and only if $W_t \geq w_t^1$ (time-varying threshold,* **endogenously determined***)*
- **Termination:** *Project stops if W_t hits R*
- **Monitoring:** $a_t^* = \bar{a}$ *(maximum effort) with $\beta_t = \theta\sigma$*
- **Structure:** *Resembles equity with dividend cap*

Contrast:

- Screening-only: Single lump-sum (knock-in option)
- Screening + Monitoring: Continuous equity stake with threshold

Contents

Practical Implications

- **Hurdle Rates in VC/PE/Hedge Funds:**
  - Common practice: Managers paid only after returns exceed threshold
  - Our model: Provides **theoretical foundation**
  - Threshold is **endogenous**, determined by risk σ and cost ψ
  - Typical range: 6-10% (Metrick & Yasuda, 2010)

Contents

Conclusion

Summary

- We modeled one-shot screening incentives in continuous time.
- **Screening Only:** Optimal contract is a **Perpetual Knock-In Option**.
  - The threshold is linear + exponential in time.
  - First-Best: Achievable if P and A share discount rates.
- **Screening + Monitoring:** Optimal contract is **Equity-like with a Dividend Threshold**.

References

- Sannikov, Y. (2008). A continuous-time version of the principal-agent problem. *Review of Economic Studies*, 75(3), 957-984.
- Gryglewicz, S., Hartman-Glaser, B., & Zheng, H. (2024). Dynamics of screening and loan sales. *Review of Financial Studies*, 37(4), 1142-1186.
- DeMarzo, P. M., & Sannikov, Y. (2006). Optimal security design and dynamic capital structure in a continuous-time agency model. *Journal of Finance*, 61(6), 2681-2724.
- Shearer, B. (2004). Piece rates, fixed wages and incentives: Evidence from a field experiment. *Review of Economic Studies*, 71(2), 513-534.
- Metrick, A., & Yasuda, A. (2010). The economics of private equity funds. *Review of Financial Studies*, 23(6), 2303-2341.

Thank You!

Questions & Discussion

Contact: 12331469@mail.sustech.edu.cn
Paper available at SSRN (5200035)