College Admission Policies and Pre-College Human Capital Investment: Evidence from China

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September 2025

Abstract

We construct a model of pre-college human capital investment in which students with heterogeneous abilities face college admissions competition based on exam scores. Human capital investment is driven by both productive and competitive channel, and the latter is the source of overinvestment and efficiency loss. We solve for a competitive equilibrium in which students take the admission cutoff score of colleges as given. Using data from Chinese college admissions, we estimate that the current system generates an efficient loss equal to 1.5 percent of post-college wage level, with competitive channel dominating productive channel at the margin. Policies of reducing college quality gap or expanding high-quality college quota can reduce excess effort and efficiency loss.

Keywords: college admissions, pre-college education, education production function, contest

JEL Code: C78, I26, I28, J24, L11

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I. Introduction

In each year, over 10 million Chinese students participate in the national college entrance examination (*Gaokao*). Nowadays, young people indeed have higher opportunity of entering colleges: The college enrollment rate of population aged 18-22 has increased dramatically from around 30% to 60% in the recent decade. But opportunities for high-quality colleges are still scarce: the enrollment rate for four-year colleges have stayed around 50%, and for top 100 universities is kept as low as 6%, among all the applicants. In addition, the quality gap among colleges becomes even larger, reflected in their selectivity, government fundings as well as job earnings.

Competitions for college admissions have a huge impact on student pre-college human capital investment, which has not been thoroughly studied in previous studies. Unlike competing for commodity goods, where only the "winners" pay the monetary price, college admission competitions are an all-pay auction (or, *tournament* or *contest*) costing both time and money for all the students and their families, and potentially involve student overinvestment in human capital and efficiency loss for the society. In China, the daily study hours of a typical high school student have increased rapidly from 6.3 hours in 2008 to 9.6 hours in 2018; family education expenditure is one of the highest in the world.[1]

In this paper, we ask the following questions in the context of China's college admissions: (1) How does a college admissions system based on exam scores affect students' pre-college human capital investment? (2) How can we measure the possibly excess human capital accumulation (i.e., overinvestment or "involution") and its welfare consequences? (3) What policies from supply-side can reduce excess human capital accumulation and welfare loss?[2]

We construct a theoretical model to study the impact of college admission competitions on student pre-college human capital investment. Students are endowed with heterogeneous abilities only observed to themselves and make effort in accumulating human capital and strive for high college entrance exam score. Colleges, different in their qualities, admit students

[1] For various documents of huge pressures of China's college admission imposed on daily life of children, parents, and schools, so called "Gaokao grind", see The Economist (2018), Howlett (2021), Lin (2023), among others.

[2] There can be either policies from demand-side or supply-side to alleviate excess efforts and social welfare loss in pre-college human capital investment. An example of demand-side policy is that government in China has put a limit on school study hours, and regulated (or even shut down) extracurricular training institutions. In this paper we focus on supply-side policy, i.e., policies changing the quality-quantity composition of colleges in the market.

according to their exam scores up to admission quotas.[3] Wages after college graduation are jointly determined by student pre-college human capital level and the college quality. We solve for a competitive equilibrium in which students take the admission cutoff scores as given, and decide their pre-college human capital level. The equilibrium is achieved under cutoff scores in which the admission quota of each college (i.e., supply) is equal to the number of students who choose to enroll in that college by meeting its cutoff (i.e., demand).

The incentive for a student to invest in pre-college human capital is twofold: First, human capital is a productive asset. It enables students to have a higher wage after college graduation. Imagine a perfect information world in which students are admitted into colleges according to their intrinsic abilities. In other words, students are “born to” a college with certain quality. They would still invest in pre-college human capital because it is productive in their future careers. Furthermore, students “born” to higher-quality colleges may indeed invest more, since human capital and college quality are complements. This is an incentive working through a “productive channel”. In fact, the first best is achieved under this hypothetical situation, i.e., when student abilities are observed and colleges admit students according to their abilities.

Second, pre-college human capital generates academic scores which are used as signals and provide access to high-quality college education, even if it does not add to labor productivity. It incentivizes students to work hard just to outperform others. This forms the "competitive" channel of pre-college human capital investment. The allocation of students to colleges would be (roughly) the same as in the first best: Students with higher ability still enter the higher-quality colleges. However, since study effort generates negative externality on others in the tournament, study efforts are always excess and efficiency loss is unavoidable in the equilibrium.

We calibrate our model using data from China’s college admissions, by estimating two Cobb-Douglas form education production functions. The first is the pre-college human capital production function which captures the effects of student abilities and study efforts on the pre-college human capital level. The second is the wage determination equation which quantifies how the pre-college human capital and college quality jointly determine the wage level after college graduation. Based on parameter estimations, we simulate equilibrium outcomes under the current system and find a sizable excess effort and efficient loss. High-school students spend

[3] We focus on the matching between college quality and student ability in the college admissions system, and ignore the issue of matching between heterogeneous student preference and horizontally differentiable college attributes. These issues are largely separable and can be addressed, e.g., by reforming the matching algorithm.

8.2 hours per day in study, 2.2 hours more than under the first best scenario (i.e., 6.0 hours). The efficiency loss is equivalent to 1.5% reduction in the post-college wage level, which is about one fourth or fifth of the rate of return of one-year schooling.[4] In addition, the competitive channel dominates the productive channel, implying the study effort at the margin is largely wasteful. Noticeably, there are also large distributive effects among different students. Higher-ability students suffer much more from the system.

By counterfactual policy analysis, we explore the possibility of changing the quality-quantity composition (i.e., the "industrial organization") of colleges in the market to alleviate the excess efforts and efficiency loss. Both reducing the quality-gap between colleges and expanding the high-quality college quotas can reduce excess effort and deadweight loss, with the highest magnitude as 0.5 hour for effort and 0.5 percentage point (or one third) for wage level.

Our paper is one of the first structural models studying the effect of college admission policies on pre-human capital investment, with a focus in China, a large country with a centralized college admission system. The model combines the core features of the two backbone models of economics of education originated from Becker (1962) and Spence (1973): the one-dimensional study effort plays both the role of human capital accumulation and signaling, connected through the key role of college quality. Our results shed lights on various aspects of policy making in college admissions, especially on the supply-side of the market.

The paper is organized as the following: Section II is about institutional background and literature review. Section III is on model setup and theoretical analysis. Section IV is on empirical calibrations. Section V is counterfactual policy analysis. Section VI concludes.

II. Institutional Background and Literature Review

II.1 Institutional Background

China's higher education has stepped into popularization stage. Today, nearly 60% of young people (with age 18-22) goes to colleges. In each year, over 10 million students (most of them are regular high school graduates) participate the nation-wide college entrance examination (*Gaokao*) as the only access to enter colleges. Among them, admission rate for any

[4] The rate of return of one-year schooling is around 6-12% in the U.S., see, for example, Lovenheim and Turner (2018). A recent annual report on China's human capital suggests the rate of return of one-year schooling in China can be as low as 5% (Li, 2023). A World Bank report (Psacharopoulos & Patrinos, 2018) indicates that the world average rate of return to education is 9%.

higher education institution (i.e., college) has reached 90 percent. Only 40 percent of college applicants can join four-year universities providing undergraduate programs. 50 percent of them would go to non-university tertiary colleges, mostly vocational/technical colleges, which provide three-year programs.

Admissions into high-quality four-year universities are very competitive. In China, there are 39 colleges that have received the Project 985 government designation and are acknowledged as the very top colleges, and 116 colleges (including all project-985 colleges) have received the Project 211 designation and represent the broadly defined top universities. This is a very small number, compared with the total number of four-year universities in China, i.e., 1,200 in year 2022. The admissions rates to Project 985 and Project 211 universities among applicants are 1.6% and 5.0%, which have been very low and unchanged in more than 15 years. Not only the very top universities are competitive. Almost all the higher education institutions have been stratified by the "college admission batches", in which colleges assigned into higher batches have priorities in recruiting students and widely recognized as higher-quality colleges (Wu and Zhong, 2020).[5]

The quality-gap of colleges is large. Return to 4-year college education vs vocational college in China is 20%, i.e., the wage from attending a 4-year college is 20% higher than a vocational college; return to top 100 colleges can be as high as 50% (Jia and Li, 2021; Li. et al., 2012; and our estimation in Table 3). The quality gap of colleges has also been reflected in the huge gap of their revenues. According to a recent news report (Huang, 2024), among all the surveyed 789 public universities, the top 10 universities have a total revenue of 16 percent of the total, and the 134 top universities (roughly equal to Project-211 universities) have a total revenue of over 50 percent of all the universities.[6]

The gap of college selectivity in China has also been rising in recent years. Cutoff scores are widely used as the measure of selectivity of universities (Hoxby, 2009). Li (2019) collected cutoff scores of 637 colleges covering all quality levels from year 2000 to year 2017. She found that during this period, the cutoff scores of colleges have shown a clearly divergent time trend.

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[5] Admission batches are decided by provincial governments, but commonly acknowledged top colleges are always listed in higher batches. Therefore, project 985 and 211 colleges are always in batch-1. Other high-quality colleges, which may be provincially administered and well-known within its province, are also listed in batch-1. Universities issuing bachelor's degrees of all quality levels are usually listed within batch 1-3, and non-university colleges (i.e., vocational colleges) are in batch 4.

[6] Increasing stratification in higher education has also been found in other countries such as the U.S., with the increasing concentration of peer and finical resources at more selective colleges (Hoxby, 2009; Bound, Hershibein &Long, 2009; Bound, Lovenheim & Turner, 2010).

That is, universities with high original cutoffs (in year 2000) would usually have a higher "growth rate" of their cutoffs, after controlling for year-province fixed effect. The divergence of cutoffs would imply a widened vertical differentiation in college quality.

To compete for high-quality colleges, students in regular high schools[7] and their families investment heavily in pre-college education. Children's study time is extremely long and increases rapidly. As our own calculation (see Section III.3) using data from National Time Use Survey, the average daily study hours (including weekdays and weekends) of a high school student (age 15-18) increased from 6.3 to 9.6 (by 67%) from 2008 to 2018. A recent time use survey (Du, et al. 2018, 2024) found that from 2017 to 2021, the average daily study hours of high school students increased from 8.8 to 9.7 hours, less rapidly than early years but remaining high.

Heavy investment in children's education is also reflected in family education expenditures. In year 2018, education expenditure per household has reached 11,000 RMB yuan, accounting for 15% of household total expenditure, and the total value accounts for 2.4% of China's total GDP (Wei, 2023), much higher than the world average.[8] Ever-increasing education investment is also reflected in the total cost (including all the opportunity costs) of raising a child until entering college. According to a report by Liang et al. (2024), the total cost of raising a child till age 18 for a family is 538,000 RMB-yuan (or 77,000 U.S. dollar), accounting for 6.3 times of GDP per capita in China - the highest in the world. The high burden of raising children has been reported as the most important factor for low fertility rate of Chinese women.

The household demand for children education also stimulated the unprecedent expansion of the extra-curriculum training business in the past decade.[9] The government has made continuous efforts to regulate the industry since year 2018. On July 23th, 2021, the central

[7] Students enrolling in regular high schools constitute the dominant group who intends to pursue college education. In 2022, there are 16 million students graduating from junior secondary school, and over half of them (9 million) join the regular high schools.

[8] According to a recent report by the World Bank (2022), from a dataset of 100 low- and middle-income countries between 2009 and 2020, the average household education expenditure accounts for 3.2 percent of their total expenditure, and from a dataset of 140 countries including all income levels, total household education expenditure in 2020 accounts for 1.9 percent of total GDP (averaged among countries).

[9] From a report by a consulting firm (Qianzhan Industry Research Institute 2021), from 2014 to 2021, the number of newly established educational training firms in each year increased from 23,000 to 68,000. The annual investment in the whole industry increased from 0.58 billion yuan in 2010 to 16.5 billion yuan in 2021, in which 73 percent (in 2021) is poured into K-12 (i.e., pre-college) education. The total sales reached 2.37 trillion yuan in 2019 at its peak, of which one third is earned from K-12 education.

government made the "final" solution plan (usually called "double-reduction" policy) and almost completely shut down the whole K-12 education training business.

A final note on institutional background is that college tuitions play a very limited role for selecting students into colleges, as compared with other countries (e.g., the U.S.). Chinese colleges are almost entirely public, with very low and uniform college tuitions.[10] Full-time students are also provided with highly subsidized room and board in the campus.

II.2 Literature Review

The effect of admission policy on pre-college human capital investment. There is a growing (yet still small) literature on the effect of college admission policies on pre-college human capital investment. Many studies focus on the effect of the "Affirmative Actions (AA)" policy in the United States and other countries, which covers a broad type of policy helping minority students to go to college. Some research found AA policy has positive effects on the minority group: it induced more pre-college study effort and higher academic performance, and higher college entrance (Caldwell, 2010; Akhtari & Laliberté, 2020; Cotton, Hickman &Price, 2022; Cassan, 2019; Khanna 2020). Other research found negligible or even negative effect (Antonovics & Backes, 2014; Ferman & Assunçâo, 2011; Estevan et al., 2019; Tincani et al., 2020).[11]

Recent literature has begun to use structural model to look at various aspects of the policy and its counterfactuals. Bodoh-Creed & Hickman (2017), based on the theory framework from their early work (Bodoh-Creed & Hickman, 2018), estimated a structural model analyzing pre-college human capital investment, college admissions, and post-college income of minority and non-minority students under the AA policy. It shows that the counterfactual abolition of the AA system would increase the pre-college human capital investment of high-ability minorities, while decrease human capital investment of low-ability minorities. The policy would also drive minority students out of the best colleges with a corresponding reduction in household

[10] In our CCSS data set (described in Section IV.1), the average tuition in year 2013 is 5,760 RMB yuan (account for 13% of per capita GDP), with a standard deviation of 4 yuan. From a recent data source with possibly more representative sample, the average college tuition in China is 4,200 yuan in 2020 (National Bureau of Statistics of China, 2021).

[11] A non-race-based affirmative action policy is the percentile plans. A leading example is the "Top 10 Percent Plan" implemented in Texas since 1997, guaranteeing that the top 10 percent of students in the graduating class (12th-grade) of any Texas high school be admitted to a public university. The plan substantially improves the probability of admissions to state flagship public universities for students from low-performing Texas high schools. Literature also finds ambiguous effects of such a policy on student study efforts and academic performance in pre-college education. See Cortes & Zhang (2012), Leeds et al. (2017).

income.[12]

Our model has the similar conceptual framework as theirs, but makes unique contributions. First, we solve for a competitive equilibrium instead of a Nash equilibrium. This would simplify the model solving while not affecting the equilibrium outcome given the large number of students. Second, we go a step further back and introduce student study effort (measured by daily study time) as the input of pre-college human capital, while in theirs the pre-college human capital is a variable directly chosen by students. Thirdly, literature often focuses on specific polies such as AA policy, while our paper examines a more fundamental institutional feature, i.e., the market structure of higher education.

China's college admission policies. There are also literature studying the impact of policy changes in China's college admissions on students' pre-college human capital investment. Luo & Meng (2016) found that rural students have lower high school enrollment than urban students, just because they have lower opportunities of attaining colleges. Xing (2013) supported this view by finding that the college enrollment expansion policy since 1999 has incentivized pre-college human capital investment of rural students and increased their enrollment in high school. Xue & Fang (2020) found that the increase in admission rate of universities as a whole encourages extracurricular tutoring, while the increase in admission rate of top universities (of project 985 or 211) discourages students' extracurricular tutoring. None of them uses structural model to evaluate the welfare consequences of counterfactual policies.

Stratification in higher education. Our study also speaks to the debated issue of stratification in higher education not fully studied in literature. Stratification intensified competition for college admissions, which is no doubt a fundamental force driving pre-college human capital investment. Bound, Hershibein and Long (2009) documented that high school students in the U.S. have been driven by increased college competition to invest more in "signals of ability", e.g., taking calculus course, AP exams, and SAT or ACT exams, resulting in increased homework and private tutoring time. The authors argued that those activities are largely non-productive by measuring students' college academic performance or future earnings. In a theoretical paper, however, Sallee et al. (2008) argue that stratification, i.e., "a

[12] Grau (2018) used administrative data from the 2009 university admissions in Chile to estimate a rank-order tournament model, in which high school students decide their level of effort and whether to take college entrance exams, taking into account how these decisions affect their future chances of college admission. It evaluated the impact of two affirmative action policies, and found that increasing the weight of high school GPA in admissions is more effective in promoting high school students' academic efforts, while the quota system imposing the population's socioeconomic group distribution for each university is more effective in assigning high-quality students to high-quality colleges.

tiered system of higher education that sorts students by ability", may be optimal since student ability and school resource are complements, and establishing new schools incur nonnegligible fixed cost.

Matching in a large market. In this paper we consider a large matching market, in which there are many students and a few colleges. Azevedo & Leshno (2016) is one of the first taking this approach to study matching market. An important feature of this approach is to connect the stable matching to a cutoff system which plays a role similar to market prices. Our paper focus on student behavior especially their optimal efforts when taking admission cutoffs as given; in their paper they mainly focus on the university (firm) behavior.

Matching with efforts. As we mentioned, college admissions competition can be regarded as an all-pay auction or a contest with effort levels as the payment. In a canonical paper, Moldovanu and Sela (2001) studied how to allocate prizes according to performance rankings among participants with privately known abilities to maximize expected effort. Under the same spirit but the specific context of college admissions competition, Bodoh-Creed & Hickman (2018) compared two affirmative action mechanisms, i.e., admissions preferences and quotas, while Hafalir et al. (2018) compared centralized vs decentralized admissions mechanisms of their effect on student efforts and matching outcomes.

III. Model and Solution

We start out analysis by building a model in which students invest in pre-college human capital, competing for high-quality colleges and high wages in the labor market afterwards. The model highlights two incentive channels for human capital investment: the productive channel in which students accumulate human capital directly for higher labor productivity, and the competitive channel in which students only concern about entering high-quality colleges.

III.1 Model Setup

Consider a colleges admissions market in which each student i makes efforts to build on their pre-college human capital. The human capital production function is characterized as:

$$S_i = L_i = \alpha_0 A_i^{\alpha_1} E_i^{\alpha_2}, \tag{1}$$

where L_i is the student i's accumulated pre-college human capital. A_i is her inner ability, and E_i is her effort level. Three parameters are involved: $\alpha_0, \alpha_1 > 0,\ \alpha_2 \in (0,1)$. The human capital production function indicates that a student's pre-college human capital level is determined jointly by her ability and effort. For simplicity, we also assume that the pre-college human capital determines student gaokao performance deterministically, that is, $S_i = L_i$, where S_i is gaokao score of student i.

Efforts incur cost from the student and are described as:

$$C(E_i, A_i) = \frac{{E_i}^{\mu}}{A_i}, \tag{2}$$

where parameter $\mu > 1$, implying a convex function. The study cost increases with the effort level (E_i) while decreases with student ability (A_i).

For each student i, wage after graduation from colleges is characterized by the wage determination equation as:

$$\mathbb{E}[w_i|P_i, L_i] = \alpha_w {P_i}^{\alpha_P} {L_i}^{\alpha_L}, \tag{3}$$

where w_i is wage after college graduation, and P_i is the quality of college attended by student i, with larger value indicating higher quality. The expectation ($\mathbb{E}[\cdot]$) is taken to capture any uncertainty of wage level after college graduation when students make pre-college human capital investment decision. Three parameters are involved: $\alpha_w, \alpha_P > 0$ and $\alpha_L \in (0,1)$.

The utility maximization problem of student i is:

$$\max U = \mathbb{E}[w_i|P_i, L_i] - C(E_i, A_i). \tag{4}$$

Note that the only choice variable to maximize utility is her effort level E_i.

We assume there are many students but a few colleges in the market. In particular, we assume there are a continuum of students with a unit mass and two colleges.[13] The student ability in the market follows a continuous distribution with a CDF as $F(\cdot)$, with $A_i \epsilon (\underline{A}, \overline{A})$. In our model, $F(\cdot)$ can be any differentiable CDF.

The two colleges, H and L, are high-quality and low-quality respectively, i.e., $P_H > P_L$. The admission quotas $q_{H,L}$ are real numbers with $q_H + q_L = 1$. The colleges admit students according to their gaokao score (S_i) and higher score is preferred.

III.2 Competitive Equilibrium

III.2.1 First best outcome and competitive equilibrium

We first derive the first best outcome. Consider in the market student ability is public

[13] In our numerical simulations and counterfactual analysis, we will randomly draw a large number of students from a given distribution of student ability, and consider any finite (but small) number of colleges.

information. Colleges admit students according to their intrinsic ability from high to low. Let $A^{\#}$ denote the lowest student ability level eligible for the high-quality college, i.e., $1 - F(A^{\#}) = q_H$ or $F(A^{\#}) = q_L$. Therefore, any student i with $A_i \geq A^{\#}$ would be admitted by the high-quality college for certain and their optimal effort level is:

$$E^o(A_i, P_H) \equiv \underset{E_i}{argmax}\, \{\beta_0 A_i^{\beta_1} E_i^{\beta_2} P_H^{\alpha_P} - \frac{E_i^{\mu}}{A_i}\}, \tag{5}$$

by using eq. (1) to (4), where $\beta_0 \equiv \alpha_w {\alpha_0}^{\alpha_L}, \beta_1 \equiv \alpha_1 \alpha_L, \beta_2 \equiv \alpha_2 \alpha_L \in (0,1)$. Any student i with $A_i \leq A^{\#}$ would be admitted by the low-quality college for certain and their optimal effort level is:

$$E^o(A_i, P_L) \equiv \underset{E_i}{argmax}\{\beta_0 A_i^{\beta_1} E_i^{\beta_2} P_L^{\alpha_P} - \frac{E_i^{\mu}}{A_i}\}. \tag{6}$$

We call such an outcome as the *first best outcome*. It is the socially optimal (i.e., maximizing total student utilities) outcome which a social planner would like to achieve.

Now suppose student abilities are their private information and unobserved to any other agents. Colleges can only rely on observed student gaokao score (S_i) to admit students. Suppose the cutoff score of college H is $\bar{S}$, which is publicly known and taken as given for any single student. Any student with pre-college human capital $S_i \geq \bar{S}$ would be admitted by college H, otherwise she would be admitted by college L. Now each student, given the cutoff $\bar{S}$, would have two choices. She may choose an effort level at least to meet the cutoff and go to the high-quality college as:

$$E(A_i, \bar{S}),\ \text{s.t.}\ \bar{S} = \alpha_0 A_i^{\alpha_1} E_i^{\alpha_2}, \tag{7}$$

or she chooses to go to the low-quality college without any prerequisite (i.e., "lie flat"). In this case, she will choose the corresponding optimal effort as $E^o(A_i, P_L)$ in eq. (6).

A "single-crossing property" is clearly satisfied. That is, if a student i chooses to meet the cutoff $\bar{S}$ instead of "lying flat", then any student i' with $A_{i'} > A_i$ would also do that. Intuitively, students with higher ability have both lower cost and higher return of making efforts on accumulating human capital (as indicated in eq. (1) -(3)). Let $A(\bar{S})$ be the "cutoff" ability such that student with ability $A(\bar{S})$ is indifferent between making efforts for meeting the cutoff $(E(A, \bar{S}))$ and "lying flat" $(E^o(A, P_L))$.[14]

[14] That is, $\beta A(\bar{S})^{\beta_1} E\big(A(\bar{S}), \bar{S}\big)^{\beta_2} P_H^{\alpha_P} - \frac{E(A(\bar{S}),\bar{S})^{\mu}}{A(\bar{S})} = \beta A(\bar{S})^{\beta_1} E^o(A(\bar{S}), P_L)^{\beta_2} P_L^{\alpha_P} - \frac{E^o(A(\bar{S}),P_L)^{\mu}}{A(\bar{S})}$.

We define the demand for high-quality college under any given cutoff $\bar{S}$ as the number of students who choose to meet the cutoff, i.e.:

$$D(\bar{S}) = 1 - F(A(\bar{S})). \tag{8}$$

It is immediately clear that $D'(\bar{S}) < 0$, since $F' > 0$ and $A'(\bar{S}) > 0$. That is, the demand curve for high-quality college is strictly downward sloping.

The *competitive equilibrium* is reached under the cutoff $\bar{S}^*$ such that:

$$D(\bar{S}^*) = q_H. \tag{9}$$

It is immediately clear that $A(\bar{S}^*) = A^\#$, by eq. (8), (9) and the definition of $A^\#$ (i.e., $1 - F(A^\#) = q_H$). That is, under the competitive equilibrium, the same group of students would attend the high-quality college (or the low-quality college) as under the first best.

The competitive equilibrium can be illustrated in Figure 1, in which the equilibrium effort level for each student ability level is shown by the bolded lines. As we shown, under competitive equilibrium, student with ability lower than $A^\#$ would choose $E^o(A, P_L)$, and students with ability higher than $A^\#$ would choose efforts weakly higher than $E(A, \bar{S}^*)$. As shown in the figure, some of high-ability students may choose an effort level $E^o(A, P_H)$ which is *strictly* higher than $E(A, \bar{S}^*)$. Intuitively, those students can "lie flat but get into" the high-quality college. We denote $\tilde{A}$ as the student ability at which two curves intersect, i.e., $E^o(\tilde{A}, P_H) = E(\tilde{A}, \bar{S}^*)$.[15] Although at the equilibrium, the allocation of student abilities to college qualities are the same as under the first best, students with ability $A \in (A^\#, \tilde{A})$ make higher efforts than under the first best (i.e., $E(A, \bar{S}^*) > E^o(A, P_H)$). We call such students as *marginal students*. The excess effort of marginal students is the source of social welfare loss.

We summary our findings as the follow proposition:

Proposition 1. Suppose there is a continuum of students and two colleges in the market. Then: (1) there is a unique competitive equilibrium. Under the equilibrium, (2) students make efforts with a level higher than under the first best, and (3) their total welfare is strictly lower than under the first best.

[15] However, whether there is a positive mass of students "lucky" enough to "lie flat but get into" high-quality colleges depends on the quota distribution. It is possible that q_H is small and $\bar{S}^*$ is high enough such that $E(A, \bar{S}^*)$ shift upward until it cannot intersect with $E^o(A, P_H)$ for any $A \leq \bar{A}$. In this case, all students who choose to go to high-quality college have to choose $E(A, \bar{S}^*)$, and cannot "lie flat".

The first conclusion is due to $D'(\bar{S}) < 0$, and $D(0) = 1, D(\infty) = 0$.[16] All the other conclusions are clear from our analysis above. The following comparative static results are immediately obtained (the proof is in Appendix A):

Proposition 2 (comparative statics). Under the competitive equilibrium:(1) As the college quality of the low-quality college (P_L) increases, the cutoff of the high-quality college ($\bar{S}^*$) would decrease, the mass of marginal students ($F(\tilde{A}) - F(A^{\#})$) decrease, and the welfare loss (i.e., difference in the total student utilities between the first best and the competitive equilibrium) decreases. (2) As the quota of the high-quality college (q_H) decreases, the cutoff of the high-quality college ($\bar{S}^*$) would increase.

A numerical example. We give a numerical example to illustrate our theoretical model. For this, we simulate 1,000 students with the quota of high-quality college as 600 (i.e., $q_H = 0.6$). Student abilities (A) are assumed to follow a uniform distribution between 0 and 10. The quality ratio between high-quality college and low-quality college is $P_H/P_L = 4$. The model parameters are given in Table 1, and the numerical solving procedure is given in Appendix C.1.

The simulated equilibrium outcomes are shown in Figure 2. Figure 2(a) illustrates the equilibrium effort, which replicates Figure 1 numerically. Figure 2(b) shows the equilibrium pre-college human capitals (HCs, or scores). Note that all the marginal students have equal scores. Figure 2(c) shows the equilibrium utilities. Dotted lines show student utilities under the first best, in which students with ability lower than $A^{\#}$ have the utility indicated by the lower dotted line, while those with abilities higher than $A^{\#}$ have the utility indicated by the upper dotted line. The solid line corresponds to the equilibrium utility. Only marginal students (with $A \in (A^{\#}, \tilde{A})$) have lower utility levels than under the first best outcome. The shaded triangle indicates the total welfare reduction (i.e., "deadweight loss") of the system.

III.2.2 Competitive equilibrium with random perturbations

Our previous benchmark model does not incorporate uncertainty. We now incorporate one important uncertainty: the random perturbation imposed on pre-college human capitals. In particular, the gaokao score is determined as:

[16] $D(0) = 1$ implies that if high-quality college has no any human capital prerequisites, all students would be happy to join since high-quality college would always be preferred by any student. $D(\infty) = 0$ implies that if high-quality college requires a high enough human capital level, all the students would not choose it. Note that as $\overline{S}^* \to \infty, E \to \infty$. This is nonoptimal for students since the cost function (eq. (2)) is convex while the wage determination equation (eq. (3)) and human capital production function (eq. (1)) are both concave.

$$S_i = L(A_i, E_i)e^{\varepsilon_i} = \alpha_0 A_i^{\alpha_1} E_i^{\alpha_2} e^{\varepsilon_i}, \tag{10}$$

where $L_i = L(A_i, E_i) = \alpha_0 A_i^{\alpha_1} E_i^{\alpha_2}$ is the pre-college human capital, and $\varepsilon_i \sim i.i.d. N(0, \sigma_\varepsilon^2)$ is the random perturbation. Now college entrance exam scores do not perfectly reflect pre-college human capital because random shocks (e.g., student mentality or health at the exam day, randomness in exam contents, etc.) affect the exam performance in addition to human capital.[17]

Now the student utility maximization problem becomes:

$$max\ U = \mathbb{E}_\varepsilon \left\{ \alpha_w P_i(S_i)^{\alpha_P} L_i{}^{\alpha_L} - \frac{E_i^\mu}{A_i} \right\}. \tag{11}$$

The expectation is taken over ε_i for each student i. Note that the randomness is contained only in $P_i(S_i)$.

Consider the competitive equilibrium with the random perturbation. For any fixed number of students in the market, given student efforts ($\{E_i\}$), the resulted cutoff ($\bar{S}$) is itself a random variable. However, as the number of students become larger and larger, it shrinks to a point value. Alternatively, we might think students would take the expectation value of the cutoff ($E_{\{\varepsilon_i\}}(\bar{S})$) in a college admissions system as her expected equilibrium cutoff. By a bit of abuse of notation, we denote the expected cutoff still as $\bar{S}$.

Given the single-valued (expected) cutoff score, the expected utility of any student is easy to compute. Note that in eq. (11), the expectation is only taken on $P_i(S_i)^{\alpha_P}$, therefore:

$$\begin{aligned}
U &= \alpha_w \mathbb{E}_\varepsilon \{P_i(S_i)^{\alpha_P}\} L_i{}^{\alpha_L} - \frac{E_i^\mu}{A_i} \\
&= \alpha_w * \left[Prob(S_i \geq \bar{S}) * P_H^{\alpha_P} + Prob(S_i \leq \bar{S}) * P_L^{\alpha_P}\right] * L_i{}^{\alpha_L} - \frac{E_i^\mu}{A_i} \\
&= Prob\big(Ln(S_i) \geq Ln(\bar{S})\big) * W_H(E_i) + Prob\big(Ln(S_i) \leq Ln(\bar{S})\big) * W_L(E_i) - \frac{E_i^\mu}{A_i} \\
&= \left[1 - F_{N(0,\sigma_\varepsilon^2)}\big(Ln(\bar{S}) - Ln(L_i)\big)\right] * W_H(E_i) \\
&\qquad + F_{N(0,\sigma_\varepsilon^2)}\big(Ln(\bar{S}) - Ln(L_i)\big) * W_L(E_i) - \frac{E_i^\mu}{A_i},
\end{aligned} \tag{12}$$

where $W_j(E_i) = \alpha_w P_j^{\alpha_P} L_i{}^{\alpha_L} = \beta_0 A_i^{\beta_1} E_i^{\beta_2} P_j^{\alpha_P}, j = H, L$ are wages of being admitted by high-

[17] Another interpretation would be that students are not perfectly assortative matched (PAM) with colleges, even if pre-college human capitals are perfectly reflected in the exam scores. That is, students with higher scores may not exactly matched with higher-quality colleges, due to admission mechanisms or even student application behavior.

or low-quality college for certain with effort E_i, $F_{N(0,\sigma_\varepsilon^2)}(\cdot)$ is the CDF of $N(0,\sigma_\varepsilon^2)$. We denote the optimal effort level chosen to maximize eq. (12) as $E_i(\bar{S})$.

The *competitive equilibrium cutoff* $(\bar{S}^*)$ *under random perturbation* is solved recursively as the following: (1) it is the cutoff when all the students take it as given and make efforts $E_i(\bar{S}^*)$ to maximize its utility, and (2) the resulted empirical score distribution $F(\cdot\,|\{E_i(\bar{S}^*)\})$ satisfies $1-F(\bar{S}^*|\{E_i(\bar{S}^*)\})=q_H$.

Competitive vs Productive Channel. We now introduce the competitive channel and productive channel of pre-college human capital investment, which are first introduced in Bodoh-Cree & Hickman (2017). Recall that the "revenue" part of a student's utility function, i.e., the expected wage, is:

$$R_i \equiv \mathbb{E}[w_i|P_i,L_i] = \mathbb{E}_\varepsilon\{\alpha_w P_i^{\alpha_P} L_i^{\alpha_L}\} = \alpha_w {L_i}^{\alpha_L}\mathbb{E}_\varepsilon\{P_i^{\alpha_P}\}. \quad (13)$$

The marginal revenue explains the effect of one unit increase in human capital on wage increase:

$$MR_i(L_i) \equiv \frac{\partial\mathbb{E}[w_i|P_i,L_i]}{\partial L_i} = \alpha_L\alpha_w L_i^{\alpha_L-1}\mathbb{E}_\varepsilon\{P_i^{\alpha_P}\} + \alpha_w L_i^{\alpha_L}\frac{\partial\mathbb{E}_\varepsilon\{P_i^{\alpha_P}\}}{\partial L_i}. \quad (14)$$

The first (positive) item on the right of eq. (14) explains that, given the (expected) college quality, how human capital investment (ΔL_i) would change the student's wage level $(\Delta\mathbb{E}[w_i])$, through its direct effect on "producing" wages. We call it the *productive channel*, denoted as:

$$MR_{p,i}(L_i) \equiv \alpha_L\alpha_w L_i^{\alpha_L-1}\mathbb{E}_\varepsilon\{P_i^{\alpha_P}\}. \quad (15)$$

Under the first best, this is the only channel through which human capital investment affects wage level.[18] The second item on the right explains how human capital investment would affect the expected college quality. By eq. (12), this item is still positive. We call it the *competitive channel* through which human capital investment affects wage level, denoted as:[19]

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[18] Under the first best with no random perturbations, $\mathbb{E}_\varepsilon\{P_i^{\alpha_P}\}$ is replaced by either $P_L^{\alpha_P}$ or $P_H^{\alpha_P}$, depending on whether $A_i < A^\#$ or not.

[19] Note that under the equilibrium with no random perturbations, competitive channel $MR_{c,i}(L_i)$ is not well-defined for student i with $A_i \in [A^\#, \tilde{A}]$: their effort levels exactly make human capital equal to the cutoff score, therefore an infinitely small decrease in E_i or L_i would make the student "jump" from the high-quality college to the low-quality college. Note also that for any student i with $A_i < A^\#$ or $A_i > \tilde{A}$, $\frac{\partial\mathbb{E}_\varepsilon\{P_i^{\alpha_P}\}}{\partial L_i} = 0$, because those students are admitted by either high-quality or low-quality college for certain.

$$MR_{c,i}(L_i) \equiv \alpha_w L_i^{\alpha_L} \frac{\partial \mathbb{E}_\varepsilon\{P_i^{\alpha_P}\}}{\partial L_i}. \tag{16}$$

We measure the relative force of competitive channel (vs. productive channel) by the ratio of marginal revenue from the competitive channel to the total marginal revenue, i.e.:

$$Ratio_{c,i} = \frac{MR_{c,i}}{MR_i} \in [0,1]. \tag{17}$$

A larger $Ratio_{c,i}$ implies a higher relative force of competitive channel. If $Ratio_{c,i} > 0.5$, we say that competitive channel dominates the productive channel.

A brief note on calculating $MR_{c,i}(L_i)$. It might be difficult to calculate it directly by eq. (12) and (16). However, as $MR_i(E_i^*) = MC_i(E_i^*) = \mu \frac{{E_i^*}^{\mu-1}}{A_i}$ under the equilibrium, then:

$$MR_i(L_i^*) = \frac{MC_i(E_i^*)}{\partial L_i^*/\partial E_i^*} = \frac{\mu {E_i^*}^{\mu-\alpha_2}}{\alpha_0\alpha_2 A_i^{1+\alpha_1}} = \frac{\mu {L_i^*}^{(\frac{\mu}{\alpha_2}-1)}}{\alpha_0^{\frac{\mu}{\alpha_2}}\alpha_2 A_i^{1+\alpha_1*\frac{\mu}{\alpha_2}}}. \tag{18}$$

We can calculate $MR_{c,i}(L_i^*)$ as $MR_{c,i}(L_i^*) = MR_i(L_i^*) - MR_{p,i}(L_i^*)$ by eq. (18) and (15).

III.3 Comparative statics

We use the same numerical example as in Table 1 but adding random perturbations to illustrate the equilibrium with random perturbations. The numerical solving procedure is in Appendix C.2. We also provide several comparative statics under alternative policies which change quantity-quality compositions of colleges.

Measuring the equilibrium outcomes. We evaluate all equilibrium outcomes using the following measurements: (1) the average effort of students; $\overline{E} = \frac{1}{N}\sum_{i=1}^N E_i^*$; (2) total student welfare; $\sum_{i=1}^N U_i^*$; (3) (total) deadweight loss, $\frac{\sum_{i=1}^N (U_i^o - U_i^*)}{\sum_{i=1}^N U_i^o}$, i.e., the percentage change of total welfare compared with the first best; (4) the cutoff score of the high-quality college ($\bar{S}^*$); (5) the number of marginal students, i.e., the number of students with equilibrium effort level higher than the first best effort level, $\sum_i 1(E_i^* > E_i^o)$,; and (6) the average competitive channel ratio, $\frac{1}{N}\sum_{i=1}^N Ratio_{c,i}$.

III.3.1 Random perturbations

We first show how the random perturbations change the equilibrium. Figure 3 illustrates the results. We list the results with three magnitudes of perturbations: $\sigma_\varepsilon = 0.16, 0.32, 0.64$; more specifications are in Table A1. Figure 3(a) shows that, as the random perturbations increases, the group of marginal students spreads to both lower- and higher-ability students, as more students want to catch the chance of high-quality college. However, the magnitude of excess effort around the cutoff ability gets smaller, since an increase in effort has a smaller effect on the chance of the high-quality college. As a result, the "jump" on the equilibrium effort gradually disappears and the curve become flatter. The total effort level of all students still increases (as shown in Table A1).

Since the effort level of the students *at the intensive margin* decreases as the perturbation becomes larger, the expected cutoff score (measured by the true pre-college human capital) also decreases, shown in Figure 3(b). In Figure 3(c), the welfare of low-ability students in general increases with the perturbation, because now they have (larger) chance to get into high-quality college, while the welfare of high-quality students decreases due to less chance for high-quality college. The total welfare changes non-monotonically (as shown in Table A1). In fact, there are two forces offsetting each other. On one hand, when the perturbation becomes larger, students are less likely to be positively assortative matched (i.e., high-ability students being matched to high-quality college), which tends to lower the total welfare. On the other hand, the random perturbation attenuates the signaling effect of human capital investment thus reduces the excess effort and welfare loss.

III.3.2 Other comparative statics

We now conduct several comparative statics studies, focusing on quality-quantity composition of colleges. We fix the random perturbation as $\sigma_\varepsilon = 0.32$.[20]

Quality gap. We first consider changing the quantity-gap of colleges in the market. We do this by fixing the quality of the high-quality college ($P_H = 40$) while increasing the quality of the low-quality college ($P_L = 10$ to 20).[21]

Figure 4 shows the results: reducing college quality gap decreases the cutoff score, reduce the mass of marginal students, and improve student welfare. Table A2 shows more specifications and the results are the same: As the quality gap *increases*, the average effort level of all students generally increases, even if the average quality of colleges decreases. Student welfare decreases and deadweight loss increases, with a higher proportion of marginal students

[20] Comparative statics under equilibrium without random perturbations (i.e., $\sigma_\varepsilon = 0$) are shown in Figure A1 and Table A2-A6 (panel A) in Appendix.

[21] In comparative static studies, to make the ceteris paribus effect clear, we will not stick to the resource neutrality assumption. In our counterfactual analysis later, we will stick to the resource neutrality assumption.

and higher competitive channel ratio.

Admission quota. We then consider changing the college quotas. We decrease the quota of the high-quality college (i.e., $q_H = 0.6$ to 0.5), while increasing the quota of the low-quality college accordingly (i.e., $q_L = 0.4$ to 0.5). Figure 5 shows the results. As the high-quality college becomes scarcer, students intending for it (i.e., those with abilities above the new cutoff) make larger efforts, and the cutoff score increases. Total student welfare decreases due to lower total college quality as well as higher competitiveness as the source of excess effort.

Table A3 shows more results. Average student effort can increase or decrease when we reduce the quota of the high-quality college. The averaged competitive channel ratio, as well as the deadweight loss also changes non-monotonically. The nonmonotonicity comes from two offsetting effects: reducing the quota of the high-quality college finally decreases student effort at the extensive margin, by lowering the proportion of marginal students, but increases student effort at the intensive margin, for those still targeting for the high-quality colleges.

Number of Colleges. Thirdly, we consider adding one median-quality college into the market. The quota of the newly-added college is taken entirely from its two neighboring colleges by one third of each of them, with its quality equal to the quota-weighted average quality of the neighboring colleges. Note that by this specification, the total quality of all colleges, i.e., $\sum_j q_j * P_j$, is kept constant. In particular, we change from $P_H = 40, P_L = 10$ and $q_H = 0.3, q_L = 0.7$ to a situation of three colleges with $P_H = 40, P_M = 19, P_L = 10; q_H = 0.2, q_M = 0.333, q_L = 0.467$.

The results are shown in Figure 6. Since now there are two cutoffs instead of one, the proportion of marginal students increase. However, adding one median-quality college reduces the quality-gap between neighboring colleges, resulting in lower excess efforts and competitive channel ratios around the two cutoffs, compared with the case of single cutoff. The cutoff score of the high-quality college also decreases accordingly.

In Table A4, we show results of adding more colleges. In general, when more colleges are added, both the proportion of marginal students and the average competitive channel ratio increase. However, student effort and deadweight loss not always increase or decrease. As more colleges added, student efforts always increase in the equilibrium without randomness and decrease in the equilibrium with randomness, while for both cases, the deadweight loss decreases first and then increases. The non-monotonicity also comes from two offsetting effects. On one hand, adding more colleges reduces college-quality gaps and brings in finer matching between student ability and college quality, which is conducive to reduce incentive distortion and improve student welfare. On the other hand, adding more colleges introduce a higher number of college quality gaps, which may introduce more excess effort and deadweight loss. Deadweight loss can even explode (as when the number of colleges becoming 9 in Panel A in Table A4), since the jump-ups of student efforts at the lower cutoff can induce further jump-ups at the higher cutoff, thus forming a reaction chain.

Importance of pre-college human capital vs. college quality. Finally, we consider changing the power of human capital and college quality respectively in the wage determination equation. Increasing the power of human capital can be interpreted as reforming the pre-college education curriculum such that the accumulated human capital would be more useful in long-term for increasing labor productivity. Increasing the power of college quality can be interpreted as the college quality becomes more valuable in the labor market. We change $\alpha^L = 0.4$ to 0.6, and $\alpha^P = 0.5$ to 0.6.

The results are shown in Figure 7 and 8. Both changes increase average effort level, cutoff scores, and total welfare, as both being favorable productivity shocks. However, they are different in other aspects. Increasing the power of human capital reduces the proportion of marginal students. Students are now less likely to be forced to investment in pre-college human capital than to do it voluntarily. Both the deadweight loss and the competitive channel ratio are reduced. On the contrary, increasing the power of college quality increases the proportion of marginal students, resulting in a larger deadweight loss and a higher competitive channel ratio, because getting into a high-quality college now becomes more important. Table A5 and A6 provide more specifications for either case, and the results are essentially the same.

III.4 General Model

In our model until now, students only differ in one-dimensional intrinsic abilities (A), and we focus on two-college case. The model is useful for highlighting our basic theoretical insights and empirical exercises. The model can be extended to multi-dimensional student attributes (but not multidimensional student effort or choice variable) with multi-college, which will be used in our model estimation and counterfactual analysis. In Appendix B, we analyze a more general theoretical model with multi-dimensional student attributes and multiple colleges.

We first define the first best in this general framework. First best is not solely determined by student ability but also other student attributes. The first best can be solved numerically as an integer linear programing problem (Appendix B.1).

For the equilibrium model without random perturbations, we show that there still exists a unique competitive equilibrium. In particular, the law of demand still holds, i.e., the demand for any college would decrease if its cutoff increases (Appendix B.2). The theoretical results thus are similar with the simple model of one-dimensional student attribute and two colleges, as well as the numerical solving procedure. The numerical solving procedure for equilibrium without perturbations is in Appendix C.1.

However, under the model with random perturbations, the equilibrium exists (due to the fixed-point theorem) but may not be unique. Moreover, the law of demand may not hold, therefore the uniqueness of the equilibrium cannot be guaranteed. We then have to rely on numerical solving procedure to check whether the equilibrium is unique.[22] The numerical solving procedure (in Appendix C.2) tries to find the equilibrium cutoff as the fixed-point, by directly using its definition: the equilibrium cutoff (vector), which students maximized their utilities by taking as given, would generate a score distribution which is consistent with those cutoffs, given the quotas of all colleges.

IV. Empirical Calibration

In this section we put our model into the real world by applying it in China's college admissions. We will have a first glance at the system by exploring several empirical patterns. Then we estimation key parameters in our model for counterfactual analysis.

IV.1 Data and Variables

We collect data containing individual student's information on pre-college (at least high school) human capital investment, college admission outcomes and wage after college graduation from several sources.

IV.1.1 China Family Panel Survey (CFPS)

China Family Panel Studies (CFPS) is a nationwide comprehensive panel survey that tracks and collects data at the individual, family, and community levels. The survey is conducted every two years since 2010. The baseline survey contained near 15,000 households and over 40,000 individuals in 25 provinces, and all family members were then followed by subsequent surveys. We use dataset of six rounds from 2010 to 2020. CFPS dataset contains

[22] Since we prove the uniqueness of equilibrium without perturbations, it is natural to believe that if the perturbation is small relative to the variation of human capital (or exam score) in the student population, the uniqueness is likely to hold. In Appendix B.3, we also give a sufficient condition for the law of demand to hold. Under such conditions, the uniqueness of the equilibrium can be guaranteed.

individual information on their pre-college education, which is useful to estimate parameters in our human capital production function. We focus on the sample of individuals in regular high school, which is the most relevant stage for college admission. We do not track them in earlier education since that would greatly reduce sample size. There are finally around 1,200 to 5,000 students in our sample, depending on the variables we include in the regressions.

Unfortunately, CFPS does not contain college entrance exam scores of students. We use the cognitive ability test (CAT) score after graduating from high school to proxy for their pre-college human capital (L). In addition, we use students' earliest CAT score in the sample to proxy for their abilities (A). Cognitive ability test includes both math and word/memory test items; we use the total score of two tests, normalized to 0-100. For study time (E), CFPS reports student daily study hours including both in class and out of class. It is reported separately for weekdays and weekends, so we average it with weights 5/7 and 2/7 to get average daily study time across a whole week. Wage level (w) is measured by the salary in the first year after graduating from colleges. Due to lengthy time span of the survey, wage levels are adjusted for inflation based on the price level of year 2013, the same year as another data source, using national Consumer Price Index (CPI). Finally, college quality (P) is measured by categorical variables and divided into 4 tiers from high- to low-quality: 211 university (College211), the first-batch universities (Tier1), the second- and third-batch universities (Tier23), and non-university tertiary colleges (default group).

The descriptive statistics is shown in Table A7. Column (1) includes all surveyed students studying in regular high school. As more variables are included in column (2) & (3), sample size shrinks significantly (from 4,675 to 2,668, and then 1,248). Students have an average ability and scores as around 70-80 out of 1000. They study around 8.5 hours per day and their annual wage level after graduation (with college degree or beyond) is 32.03 thousand RMB yuan. The proportions of students admitted in different tiers are close to the aggregate measures as we mentioned in Section II.1.

IV.1.2 China College Student Survey (CCSS)

China College Student Survey is a survey conducted by the China Economic and Social Data Research Center of Tsinghua University. Colleges are selected by stratified random sampling based on the geographical location of universities (Northeast, East, Central and West) and quality tier (Project 985 University, Project 211 University, and other colleges). In each of the college surveyed, college students in their final college year were randomly selected. The survey sampled 65 colleges in China, with more than 10,000 individuals. It was conducted in each year of 2009-2013. We use its 2013 survey.[23] By dropping samples with missing variables and non-targeted groups (e.g., students not admitted from normal college entrance exam), the final sample size in our study is 4,160.

CCSS dataset contains student information of their colleges attended, wage after college education, and college entrance exam scores, which is useful for estimating parameters in the wage determination equation. Note that college entrance exam scores cannot be directly used as students' true human capital (L) for two reasons: First, it is a noisy measure of true human capital as we modeled. Second, college entrance exams are independently conducted by each province in China, thus not directly comparable across provinces. In the CCSS data, we observe both student college entrance exam scores and their college GPA. The former is comparable and consistent with a student's true human capital within a province, while the latter is comparable and consistent with a student' true human capital within a college. We explore those data patterns and use a measurement error model to identify students' true human capital following Diao et al. (2024). The details are omitted in this paper and can be found in Appendix B of Diao et al. (2024).

The descriptive statistics for the CCSS sample are in Table A8. Comparing with CFPS (column (3), Table A7), CCSS oversampled high-quality colleges. The mean value of wage level is 3 thousand RMB yuan per month, or 36(=3*12) thousand RMB yuan per year, slightly higher than the value in CFPS. The constructed human capital (L) is around 66 (normalized to 0-100), which is significantly lower than the average CAT score (around 76) after high school graduation in CFPS. CCSS also sampled more urban students compared with the CFPS data (43% vs 17%).

IV.1.3 National Time Use Survey (NTUS)

National Time Use Survey (NTUS) records time use in various activities (such as work, study, housework, etc.) of the respondents in an on-time daily base, therefore, should have the smallest possible measurement errors on time use. Unfortunately, it contains limited information on other variables (e.g., academic performance) for students, so we only use it for statistical summary and simple regressions to explore time use patterns. NTUS is carried out by the National Bureau of Statistics and has two rounds (year 2008 and 2018), surveying over

[23] College entrance exam scores are noncomparable across years due to variance in exam difficulty. To avoid measure errors on human capital derived from these scores, we focus on the latest survey of CCSS and explore variations only in the cross-sectional data.

16,000 households in 2008 and another 20,000 households in 2018, in 10 provinces.

The survey does not ask about students' education status, so we select the individuals aged 15-18 with non-zero formal education hours, who are assumed to be high school students (either regular or vocational). The sample size was 1,457 for 2008 and 1,332 for 2018. The average daily study time is obtained through weighting average of the student's study time in weekdays and weekends by 5/7 and 2/7.[24]

As shown in Table A9, from 2008 to 2018, the average study time of Chinese high school students increased from 6.2 hours to 9.6 hours per day (by 55%). The standard errors are getting smaller, indicating a converging trend of study time across individuals, both within and across provinces.[25]

IV.2 Some Empirical Patterns

IV.2.1 Student ability and effort

We now explore some data patterns using the datasets we just introduce. It is an interesting question whether a smarter student would spend more or less time on study. In our model, since student ability and effort are partially complements in producing human capital (as expressed by Cobb-Douglas function), we might expect a positive relationship. Yet, in equilibrium, student effort and ability may be negatively related for marginal students (see Figure 1 and 2).

We draw a scatter plot to examine the relationship between student ability and study time. The sample is from CFPS (corresponding to column (2), Table A7). Date is censored by eliminating samples with the highest and lowest 1% study time, those with extreme values such as over 15 or close to 0 hours per day. The result is shown in Figure 9. Study hours concentrates between 5 to 10 hours, and student ability concentrates between 50 and 100. Student effort and ability show a strong positive relationship, with a correlation coefficient of 0.31, significant at 1% level. Students with higher abilities reveal a much stronger positive relationship, especially those eligible for high-quality (i.e., project- 211) colleges, as shown in panel (b) of Figure 9. The findings are consistent with our theory, since the opportunity of enrolling into 211-colleges

[24] We add up two components of high school study time as the total study time: (1) formal educational activities, including school educational activities, recess or waiting for classes, and distance education learning activities; (2) homework, after-school revision, and other activities related to formal education.

[25] Since we cannot distinguish between students from regular and vocational high school, study hours may be underestimated for regular high school students, because the common sense is that they study more hours than vocational high school students.

is scarce and students at the competitive edge must work harder. In particular, the pattern is consistent with our equilibrium with moderate or large random perturbations (Figure 3, middle and right column).

IV.2.2 Admission rate and student effort

Our theoretical model indicates that the effect of admission rate on student efforts are non-monotonic (see Table A3). We now examine the relationship between college admission rate and student study effort by exploring admission rate variations across provinces and years.

The regression equation is the following:

$$E_{ipt} = \beta_0 + \beta_1 AdmRate_{p,t-1} + \beta_2 AdmRate^2_{p,t-1} + \beta_3 Q_{ipt} + \mu_p + \lambda_t + \varepsilon_{ipt}, \tag{19}$$

where E_{ipt} is the average daily study hours of a high school student i in province p in the survey year t, $AdmRate_{p,t-1}$ is the admission rate of batch-1 (i.e., high-quality) colleges in this province one-year before, Q_{ipt} are students' socioeconomic characteristics, including gender, hukou (urban vs rural), family income, parental education, μ_p is provincial fixed effect and λ_t is year fixed effect. The sample is high school students in CFPS (column (1), Table A7). We also include student grades (i.e., grade 1, 2 or 3 in high school) as control variables.

Table 2 report the results. Results with or without year fixed effects are both reported, in column (1)-(2) and (3)-(4) respectively. When we include only linear term of the admission rate as explanatory variable, in column (1) and (3), the study time has no significant relation with the admission rate. When we include both the linear and quadratic term of the admission rate, in column (2) and (4), the study time and the admission rate reveal a significantly inverted U-shape relationship: with the increase of the admission rate, the study time first increases and then decreases. The turning point is at the admission rate around 15%, which is close to the sample mean (13.1%) or median (11.8%). Table A10 shows the results for students categorized by grade. The inverted U-shaped relationship between study time and admission rate exists for all grades but only significant for grade 2 students.

Table A11 shows the results for the same regression equation but using the National Time Use Survey data. The results are similar. The turning points of the inverted U-shape curve, however, is around admission rate of 25%.

IV.3 Model Calibration

Now we use our datasets to estimate parameters in the model. There are three sets of parameters: First, parameters in the pre-college human capital production function with perturbations (eq. (10)); second, parameters in the wage determination equation (eq. (3)); third, parameters in cost function (eq. (2)).

IV.3.1 Estimating human capital production function

The pre-college human capital production to be estimated is a variant of equation (10):

$$\tilde{S}_i = \alpha_0 A_i^{\alpha_1} E_i^{\alpha_2} Q_{ki}^{\alpha_{3k}} e^{\varepsilon_i}, \tag{20}$$

where $\tilde{S}_i$ is a student's pre-college human capital, proxied by cognitive ability test (CAT) score after graduation from a high school. Student ability A_i is measured (or proxied) by the student's earliest CAT score in CFPS.[26] Student effort E_i is measured by the average daily study time (in hours) in all the survey years between the two cognitive tests, restricted to their high school period.[27] For student attributes other than the ability, we include four student socioeconomic characteristics including family income, parental education, gender and hukou: $Q_{ki} = \{Finc_i, ParentEdu_i, Male_i, Urban_i\}, k = 1, \ldots 4.$ $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$ is the error term.

After taking the logarithm of eq. (20), we get the following equation:

$$ln\,(\tilde{S}_i) = ln(\alpha_0) + \alpha_1\, ln(A_i) + \alpha_2\, ln(E_i) + \alpha_{3k}\, ln(Q_{ki}) + \varepsilon_i. \tag{21}$$

where dummies in Q_{ki} remain to be dummies (not taking the logarithm). The data source is CFPS (Column (2), Table A7). For getting a nationally representative estimation, we do not add provincial or year fixed effect. Adding these two fixed effects will almost not change other estimated coefficients.

All the coefficients ($\{\alpha_{0,1,2,3k}\}$) can be estimated by linear regressions. The results are shown in Table 3. Column (1), with all regular high school students having at least two CAT scores in CFPS, shows that student ability plays the most important role, while the effect of student effort is significant but plays a minor role. In our regression, the earliest cognitive ability test, used as the proxy for student ability, may still contain information of a student's human capital, when it is surveyed shortly before the cognitive ability test after high school graduation, resulting in an underestimation of the effect of student effort. Because the effort variable (E) only measures the study time of students in high school, it is natural to define study ability (A)

[26] For example, if a student first appears in the CFPS sample in 2010 and his or her high school graduation year is 2016, then the 2010 cognitive ability test score is the student's ability, and the 2016 cognitive ability test score is the student human capital ($\tilde{S}_i$).

[27] Because CFPS is conducted biennially, students are surveyed either in Grade-1 and 3, or Grade-2 in their high school period. Daily study hours then are measured either by study hours in Grade-2 only, or a simple average of study hours in both Grade-1 and 3.

as its human capital right before entering high school, i.e., the last cognitive test in middle school. The result is shown in column (2). Now student ability plays an even larger rule, while student effort become even less important. It may be because students already accumulate a lot of human capital in the middle school. To measure student ability more precisely as their true intrinsic ability, in Column (3), we restrict our sample to students with an interval of more than 6 years between the two cognitive tests, removing at least the effect of human capital accumulation during the whole secondary school period. The sample size shrinks to 1,300. The coefficient of students' cognitive ability decreases from 0.48-0.56 to 0.13, and student effort (measured by study time) has roughly the same effect on human capital, with a coefficient of 0.039.

CFPS dataset may contain measurement errors on two key variables, i.e., study time (E) and CAT scores (A). To correct for this, we use two instrumental variables. We use the average daily wake time to correct for the measurement error in study time, and CAT scores reported in the closest subsequent survey to our instrumented CAT scores to correct for its measurement error. The results are in Column (4). Although the first stage results are significant (not reported in the table), the overall IV regressions do not reveal significancy for both variables. The coefficients are close to the OLS results, except for the effort variable, which almost doubles the OLS value.[28]

Our model ignores other inputs in pre-college human capital production, in particular, the family education expenditure. In Column (5) we further add this variable and find small (yet significant) effects of it on human capital and negligible influence on the coefficient of ability and effort. Note that our model introduces a single choice variable, i.e., student effort (or study time). Including multidimensional inputs or choice variables would complicate our model both theoretically and numerically. Since student efforts and other inputs (e.g., parental time inputs and family education expenditure) should be highly complementary, ignoring them in the model may not cause a serious omitted variable problem: their effects on human capital accumulation and incurred cost can be largely captured by student effort.

In our future analysis, the parameters in the human capital production function, i.e.,

[28] There is very rare literature studying the elasticity of student effort on human capital. Stinebrickner and Stinebrickner (2007) study the causal effect of study time on GPA using a survey data from Berea College. Their OLS regression result leads to an elasticity of 0.04 (by translating their estimated coefficients as slope into elasticity), while IV estimation leads to an elasticity of 0.41. In another paper of them (Stinebrickner & Stinebrickner, 2004), the elasticity estimated by OLS while correcting for measure error in study time is 0.26. However, those estimations are for colleges instead of pre-college education.

$\{\alpha_0, \alpha_1, \alpha_2, \alpha_{3k}\}$, take the value of the OLS estimates reported in column (3) of Table 3. Robust checks will be reported for some other specifications.

As a final remark, we acknowledge that it is hard to measure a student's true ability. In our specification, we measure it by the CAT score well ahead of time of high school graduation. It can be interpreted as that we restrict student human capital investment within the period between the year in which this early CAT scores are measured and the high school graduation year, regarding the human capital level at the beginning as their abilities. Such a specification tends to underestimate the effect of effort but overestimate the effect of true ability. This in turn may cause to an underestimation of excess effort and welfare loss.

IV.3.2 Estimating wage determination equation

We use CCSS dataset to estimate wage determination equation. Regression equation is a variant of eq. (3) in our model:

$$ln(w_i) = ln(\alpha_w) + \alpha_{P1} College211_i + \alpha_{P2} Tier1_i + \alpha_{P3} Tier23_i + \alpha_L\, ln(L_i) + \alpha_{Qk} Q_{ki} + \eta_i. \quad (22)$$

Here we take the logarithm of eq. (3), while keeping college quality (P) as categorical variables. As we mentioned before, we divide colleges into 4 tiers from high- to low-quality: 211 university ($College211$), the first-batch universities ($Tier1$), the second- and third-batch universities (Tier2,3), and non-university tertiary colleges (default group).[29] As in the human capital production function, we include in the model student socioeconomic characteristics including family income, parental education, gender and hukou, i.e., $Q_{ki} = \{Finc_i, ParentEdu_i, Male_i, Urban_i\}$, $k = 1,2,3,4$. To get a nationally average coefficients, we do not add provincial and track (i.e., humanity or science track of college entrance exam) fixed effect in the model. Adding those fixed effects have negligible influence on other coefficients.

Table 4 shows the OLS regression results. Column (1) uses the unweighted sample of CCSS, while Column (2) uses the weighted sample. The sample weights are provided by CCSS dataset mainly to correct the non-representativeness on quality and location of surveyed colleges. The two results are similar. College quality has a significant positive effect on students' post-graduation wages. The starting salary of students attending Project-211, the first batch, and lower batch university is around 45%, 28% and 17%, respectively, higher than that of non-

[29] We do not divide tier-1 (i.e., 211-college) students further into 985-college and other 211-college students, because the number of tier-1 students in our simulation sample is very small (#=59). The sample size for each tier of students is in the last row of Table A13.

university college students (the default group).[30] Students' pre-college human capital also has a significantly positive effect on post-graduation income. Every 1% increase in the level of pre-college human capital would increase post-graduation salary by 0.45~0.50%. We will use the estimated parameter values $\{\alpha_w, \alpha_P, \alpha_L, \alpha_{Qk}\}$ in column (2) in the counterfactual analysis.[31]

Connecting $\tilde{S}_i$ and L_i. As we mentioned in Section IV.1.2., the constructed human capital (L_i) in CCSS is significantly lower than the average cognitive ability test score ($\tilde{S}_i$) in CFPS (66 vs 76), even if CCSS contains students with stronger socioeconomic status. This suggests that $\tilde{S}_i$ measured in CFPS may be systematically different from L_i estimated from CCSS dataset, although both $\widetilde{S}_i$ and L_i are normalized to 0-100. We estimate the following statistic relation between the two:

$$ln\,(\widetilde{S}_i/100) = b\,ln(L_i/100) + \delta_i$$

with $\delta_i \sim N(0,\, \sigma_\delta^2)$. The equation guarantees, at the mean level, both $\widetilde{S}_i$ and L_i are still normalized to 0-100, while their measuring sensitivity may vary among students. We estimate b through the following procedure: (1) We predict $ln\,(L_i)$ in CCSS by a linear regression of $ln\,(L_i)$ on covariates including college qualities dummies and individual characteristics (gender, hukou, income, parental education)[32]; (2) Replace all the covariates with its mean value in CFPS (as shown in Column (3) in Table A7) in the estimated linear regression to get a predicted mean value of true human capital $\mu(ln(L_i))$ in the sample of CFPS, (3) Estimate b

[30] These estimation results are largely consistent with those in literature on education returns to college quality. For the U.S., Brewer et al. (1999) find that students attend a top-ranked private or public college have earnings 20-25% than those of students who attend a bottom-ranked public college. Hoekstra (2009) finds that individuals attending the flagship state university have approximately 24% higher earnings compared with those who narrowly missed the admission cutoff of those colleges. For China, Li et al. (2012) and Jia & Li (2021) find the return to top 100 elite colleges vs. non-elite colleges are 10% and 28-45% respectively. For a literature reviews on the education returns to college quality, see Lovenheim & Smith (2023) and Lovenheim &Turner (2018).

[31] The estimation of college quality parameters $\{\alpha_{P1}, \alpha_{P2}, \alpha_{P3}\}$ may be upward biased because we do not control for student ability due to data availability in CCSS. We re-estimate those parameters by using regression discontinuity (RD) model as in Jia &Li (2021). For each college quality type, we calculate the college admission cutoff score of universities belonging to this type in each province and (humanities vs science) track in CCSS, and select students above the cutoff by points [0, 20] and below the cutoff by points [-20, 0) to construct the sample, and estimate the corresponding parameters. The results are shown in Table A12. The probability of admission into colleges with different qualities does "jump" at the discontinuous point (in Column (2)) while the wage level does not (Column (1)). The estimated coefficients are in Column (3), which are unfortunately insignificant. The corresponding coefficients are $\alpha_{P3} = 0.112$, $\alpha_{P2} = 0.112 + 0.185 = 0.297, \alpha_{P1} = 0.112 + 0.185 + 0.312 = 0.609$, which are not much different from the OLS estimation in Table 4.

[32] The estimated linear regression function is:
$\widehat{\mathrm{Ln}(L_i)} = -0.737 + 0.342 * College211_i + 0.33 * Tier1_i + 0.218 * Tier23_i + 0.012 * ln(Finc_i) + 0.011 * ln(ParentEdu_i) - 0.005 * Male_i - 0.012 * Urban_i$, with $R^2 = 0.3577$.

as:

$$\hat{b} = \frac{\mu(\ln(\widetilde{S_i}/100))}{\mu(ln(L_i/100))}$$

where $\mu(\ln(\widetilde{S_i}))$ is the sample mean of $\ln(\widetilde{S_i})$ in CFPS. The estimate result is: $\hat{b}$ =0.536. We then convert all the estimated coefficients in the human capital function through $\hat{b}$: $\widehat{ln(\alpha_0)}' = \frac{1}{\hat{b}} * \widehat{Ln(\alpha_0)} + \left(1 - \frac{1}{\hat{b}}\right) * Ln(100), \widehat{\alpha_i}' = \frac{1}{\hat{b}} * \widehat{\alpha_i}, i = 1,2,3,k$. The predicted value from the human capital function after this correction is then used as the input of $ln(L_i)$ in the wage determination equation when we simulate the equilibrium.

Estimating $\boldsymbol{\sigma_\varepsilon^2}$. Remember that σ_ε^2 is the parameter measuring the standard deviation of the random perturbations between the "true" pre-college human capital and college entrance exam score. We estimate σ_ε^2, by exploring information on a student's college entrance exam scores as well as her college grades in CCSS. The estimated result is: $\sigma_\varepsilon = 0.075$. See Appendix D for details.[33]

IV.3.3. Estimating cost function

There is only one parameter μ in the cost function to be estimated (eq (2)). We estimate its value by minimizing the distance between the model's predicted equilibrium and the actual equilibrium, based on other estimated parameters we just described. The solving procedure is as follows: (1) set a (initial) value μ; (2) solve the utility maximization problem (eq. (11) s.t. eq. (10) and (12)) to get the equilibrium outcome of effort, human capital and wage, i.e. $\{\hat{E}(\mu), \hat{L}(\mu), \hat{w}(\mu)\}$; (3) repeat procedure (1) and (2) for different values of μ, to get the numerical functions of $\{\hat{E}(\mu), \hat{L}(\mu), \hat{w}(\mu)\}$; (4) the final estimated value of μ is solved by minimizing the true value of the effort level and the predicted effort level $\hat{E}(\mu)$ as follows:

$$\mu = argmin \sum_{i=1}^{N} \left(\hat{E}_i(\mu) - E_i\right)^2 . \tag{23}$$

The baseline data is CFPS (column (3) Table A7). The estimated value is $\hat{\mu} = 4.052$.[34]

[33] It might be considered estimating σ_ε^2 by calculating regression residuals from eq. (21). However, the independent variable in eq. (21), i.e., $\tilde{S}$, is the CAT score after the high school graduation. Therefore, the regression residuals only contain measurement error for CAT scores, not σ_ε^2, the measurement error contained in CEE scores. Furthermore, it may also contain unobserved variables for researchers but observable for the agents, resulting in an overestimation of measurement errors even for CAT score. We do not use this method.

[34] A more general form of the cost function would be $C(E_i, A_i) = c * \frac{{E_i}^\mu}{A_i}$, with two parameters c, μ. We try several alternative specifications by fixing $\mu = 2, 3$ etc., and solving for c. The best solution with the minimum distance between the predicted and actual equilibrium is still the one with c=1 and $\mu = 4.052$.

The estimated values of all the parameters in the model are summarized in Table 5.

V. Counterfactual Analysis

Based on the model estimation, we now simulate the equilibrium under the current system and alternative policies which may improve the welfare of the current system.

V.1 Simulated equilibrium under the current system

Based on our theoretical model and parameter estimation, we can simulate the equilibrium and evaluate the welfare consequences of the current college admissions system in China. Our baseline sample is sample 2 of CFPS (column (3), Table A7). The distribution of student attributes (A_i, Q_i) for simulation is therefore drawn from this sample. The simulation procedure is the same as for our numerical example with random perturbations (see Appendix C.2). The number of colleges is set as 4, i.e., the number of college tiers.

The simulated equilibrium fits our data well. As shown in Table A13, the model predicts that the average study time of all students is 8.23, which is the same as the value observed in the data (8.23). The predicted wage after graduation is 28.66 (thousand yuan), which is 10.5% lower than the observed value (32.03). The underestimation may be partly due to the conversion of monthly wage (in CCSS) into annual wage (in CFPS) by using a multiplier of 12, which may ignore any bonus or retained wage issued annually. The average pre-college human capital of all students predicted is 66.59, 11.8% lower than the observed value (75.51). The model fits the study time most closely, probably because we estimate the cost function coefficient (μ) by minimizing the MSE of study effort instead of human capital or wage level. The model also predicts well the demographic distribution of students admitted in each college tier.

We now calculate the equilibrium outcome under the current system and compare it with the first-best or socially optimal outcome. The results are shown in panel (a)-(d) Figure 10[35] and row (0) and (1) of Table 7.

As we can see, first, simulated equilibrium student efforts are much higher than the first best. The first-best effort is around 6.0 study hours per day, while the equilibrium is 8.2 hours and 37% higher than the optimal. Second, the difference in pre-college human capital is smaller:

[35] For clarity, we draw the plots with various equilibrium outcomes (in Y-axis) against student ability (in X-axis) but not other student attributes. Ability is the single most important attribute affecting human capital accumulation (as shown in Table 3).

the gap is only 3 percentage (64.61 vs 66.59). Excess effort does not convert into higher human capital but be wasted in rat-racing. Third, student welfares are on average lower and more concentrated than under the first best[36]. The deadweight loss (i.e., the percentage change of total student welfare from the equilibrium to the first best) is equal to 1.5% reduction in monthly post-college wage level (2,315 vs 2,351 RMB yuan). This is a sizable efficient loss: it is one fourth or fifth of the rate of return of one-year schooling. Fourth, under the equilibrium, for most of the students especially the high-ability ones, the competitive channel ratio is well above 0.5 (out of 1), and the average for all students is 0.56. The competitive channel dominates the productive channel.

The incentive distortion and welfare loss are not evenly distributed among students. As shown in panel (c) Figure 10, a significant proportion of students may gain from the tournament, while high-ability students are more likely to be loser. In Table A14, we run a regression of various individual equilibrium outcomes (efforts, welfares, etc., all translated into percentage changes or percentage points) on student attributes. Higher-ability students suffer more from welfare loss and incentive distortion. As student ability moves from 50 to 100, excess effort increase by 20 = (50*0.406) percentage points (with the average value as 38 percentage points), while deadweight loss increases by 5 (=50*0.090) percentage points (with the average value as -1.2 percentage points)[37] and competitive channel increases by 35 (=50*0.690) percentage points (with the average value as 56 percentage points). The standard deviation of all equilibrium outcome measures is above 9%.

Robust check for benchmark. We do four robust checks for the benchmark simulation. The results are summarized in Table A15. First, we replace the human capital parameters using the estimated parameter in column (2) instead of column (3) of Table 3. The coefficient for ability (lnA) is 0.559 instead of 0.132, and the coefficient for effort (lnE) is now 0.034 instead of 0.039. The simulated human capital is now lower than under the benchmark, as well as the competitive channel ratio and the deadweight loss, but with a tolerant difference.

[36] The first-best student utilities in Figure 10(c) show a layered pattern. This is because the college quality is at discrete value and students with various attributes are grouped into different layers of colleges under the first best.

[37] The average value of deadweight loss in Table A14 is negative and smaller than the corresponding value in Table 7 (i.e., 1.53%). This is because in Table A14 we calculate the deadweight loss measure by percentage points for each student first, and then average across them. In Table 7, we sum up all the student utilities in the equilibrium and in the first best, and then calculate the "total" deadweight loss, which arguably is the better measure of total welfare loss.

Second, we replace the measure of random perturbation (σ_ε) by 0.046 (instead of 0.075), from an independent estimation by Zhao et al. (2022).[38] The results are close to our benchmark, except for a bit higher excess effort, deadweight loss and competitive channel.

Thirdly, we use sample weight provided by CFPS to correct for potential sampling bias of the data. We use the "individual panel weight" attached to each high-school student in the dataset, and normalize the total weights of the sample in each surveyed year so that each year would have the equal weight. The results are very similar to the benchmark.

Finally, we use coefficients estimated by IV regression in the human capital production function in Column (4), Table 3. The major change is the higher coefficient for effort. This results in a higher deadweight loss; other results are quite similar with the benchmark.

Equilibrium without perturbations. In addition to robust checks, we also calculate equilibrium outcomes without perturbations. The motivations are two: First, our first best result is calculated without considering random perturbations. To elicit a "net" effect of the current college admissions system, we may want to compare an equilibrium outcome with the first best when both are without randomness. Second, we may regard it as another extreme counterfactual analysis when the random perturbations are dramatically decreased, due to, e.g., an improved college entrance exam system closely reflecting the true human capital of student.

We use the same parameters as in equilibrium with perturbations, except that σ_ε=0. The algorithm is also different (Appendix C.1). The results are shown in Row (1') in Table A16. Compared with the equilibrium with perturbations (Row (1) in Table 7), the deadweight loss is much higher (4.7% vs. 1.5%), while with a lower effort level (7.85 vs. 8.22). The results indicate a potentially larger efficient loss in the system when the college entrance exam itself become more precise.

V.2 Counterfactual policy analysis

We now evaluate various policy portfolios by counterfactual analysis. We consider four types of policies: (1) reducing college quality gap, (2) increasing high-quality college quota, (3)

[38] Zhao et al. (2022) regresses students' total college entrance exam scores to their two mock college entrance exam scores and Grade 1&2 annual total scores (which can be regarded as their true human capital), using data of students graduating in year 2018 and 2019 from 8 high schools in 4 provinces (Shandong, Sichuan, Guangdong, and Hainan), with a sample size of 2,699. The regression results find a root of mean squared error (RMSE) as 25.864 (in their Table 10). With the mean value of college entrance exam score as 563.8611 (calculated from their Table 1), the RMSE translates into a percentage point as 25.864/563.8611=4.5869%, which can be regarded as an estimation of σ_ε.

adding more median-quality colleges, and (4) changing student numbers. Except for the fourth type, we restrict to a "resource-neutral" policy by keeping the total quality of colleges (i.e., $\sum_j P_j q_j$) unchanged, to mimic a balanced-budget policy. The fourth type policy would instead consider the case when the college market as a whole become tight or loose. Reducing student number is equal to a policy expanding resource per student, and vice versa. All the policies we experiment are listed in Table 6. Table 7 shows the results of simulated equilibrium outcomes under counterfactuals. In addition, Figure A2 shows the results for competitive channel ratio for several typical scenarios.

Reducing college quality gap. The first counterfactual policy is to narrow the gap in quality between universities. Quality gap is reduced through reallocation of educational resources from higher-quality colleges to lower-quality colleges. We consider a radical policy which totally eliminates quality gap of neighboring college tiers. Three scenarios are included: (1) eliminating quality gap between tier 1 and 2 colleges; (2) eliminating quality gap between tier 2 and 3; (3) eliminating quality gap between tier 3 and 4.

Policies under all scenarios can reduce the average student effort, deadweight loss and competitive channel ratio. Among them, a bit surprisingly, the policy of eliminating quality gap between tier 3 and 4 colleges has the largest effect (Row (4) of Table 7). It reduces daily study time from 8.2 hours to 7.7 hours, the deadweight loss from 1.5% to 1.1% (i.e., one third), and the competitive channel ratio from 0.56 to 0.43. In China, the gap between tier 3 and 4, i.e., universities vs non-university colleges, can be taken very seriously by students and parents, because the former issue a bachelor's degree and access to postgraduate education, while the latter do not issue any degree but only graduation certificate. Another reason for its relatively large effect is the large proportion of students covered by these two college tiers (70% of total, shown in the last row of Table A13).

Increasing quotas of high-quality colleges. The second counterfactual policy is to increase the admission quota of high-quality colleges. Every 1% increase in the quota of any college tier will result in a 1% quality decrease of that tier, due to our "resource neutrality" assumption. If there is economy of scale, the quality decrease may be less than 1%, then our results tend to overestimate the welfare loss of the policy. We include three scenarios: (1) Tier 1 colleges increase their quotas by 10%; (2) Both tier 1 and 2 colleges increase their quota by 10%, and (3) All tier 1-3 colleges increase their quota by 10%. That is, we gradually include more lower-quality college tiers into "quota expansion movement".

Again, all the policies can reduce the average student effort, deadweight loss and competitive channel ratio. The largest effect happens when we increase quota of all tier 1-3 colleges. It reduces student effort by 0.2 study hours, deadweight loss by over 0.3 percentage point (i.e., one fifth), and competitive channel ratio by 0.04.

Adding more colleges. The third counterfactual policy is to add one median-quality college tier to the existing four tiers of colleges. We consider three scenarios: (1) adding one median-quality college between tier 1&2; (2) adding one between tier 2&3; and (3) adding one between tier 3&4.

All three polices *increase* the competitive channel ratio, although not much. The excess effort and the deadweight loss decrease but with a negligible magnitude. Increasing quality differentiation in the market have small positive or even negative effect on reducing excess efforts and deadweight loss.

As a final note, all the policies mentioned above have negligible effects on average human capital and student total welfare, probably due to the "resource-neutrality" assumption we made. Furthermore, as we indicated in our comparative statics (Section III.3.2), many policies themselves generate offsetting effects with or without resource constraints.[39]

Changing total student number. Another important policy change would consider changing the total number of students. China now faces a long-term demographic trend toward a lower birthrate, while in recent 15 years, college applicants are predicted to increase, due to the baby boom in earlier years when the government removed birth control. The demand shock may be mild, since the supply also increased steadily in recent years. Our final policy considers changing total student number by increasing or decreasing it by 10%. For the increase of the student number, quotas would be added to the college tier with the lowest quality by the same amount. For the decrease of student number, we first reduce quotas of the 4^{th} tier, and if there is still excess supply, quota of the 3^{rd} tier will be reduced, and so on. By this method, when the number of student increases (or decreases), the higher-quality colleges would become relatively scarcer (or more abundant), and the market becomes tighter (or looser).

Reducing the number of students decreases excess effort, deadweight loss and competitive

[39] The resource neutrality itself can generate offsetting allocative effects. For example, eliminating quality gap between tier 1&2 inevitably enlarges the gap between tier 2&3, as the quality of tier 2 colleges increases. Many policies have offsetting effects even without resource constraints. For example, increasing the quota of high-quality colleges may encourage more students to compete for them, although it reduces the scarcity of the high-quality colleges. Policies by adding more median-quality colleges reduces the competition at the intensive margin, while increases competition at the extensive margin.

ratio, but the effect is small. Increasing student number has the opposite yet still small effect.[40] Note also that these two policies have a larger effect on total student welfare, compared with resource-neutral policies. Therefore, students still benefit from a decrease in total demand or an increase in total supply, and hurt in the opposite changes.

In Table A16, we consider several counterfactuals under the equilibrium without random perturbations, which may highlight the "pure" effects of institutional changes. Almost every policy has a larger effect, compared with their effects under equilibrium with random perturbations. In addition, adding one median-quality college now help to reduce deadweight loss quite a bit (by 1.5 percentage points).

VI. Conclusions

College admissions are not only a market for allocating education resource, but also a rat racing game. Students make pre-college human capital investment not only for their long-term labor productivity, but also for winning the one-shot game of high-quality colleges. By using data from China's college admissions, we find that the current system generates a noticeable excess study effort and welfare loss – students study 2 more hours per day just for competing others, in which one fourth of their education returns are wasted in the game.

Supply-side policies, which change the quality and quantity composition of colleges (i.e., the "industrial organization") in the market, can alleviate excess effort and welfare loss to some extent. Probably surprisingly, the largest effect comes from policy changes positioning at the low-end of college market, either by reducing quality gap between lower-quality college groups or increasing quota including them. This may reflect the high population of college applicants covered by lower-quality colleges. Although the policy implications are preliminary and suggestive, our research can be valuable for supply-side reform in college admissions from the perspective of overinvestment in pre-college education.

Our research can be extended in several way. First, our model is still stylized. Student efforts are one-dimensional. Incorporating multi-dimensional efforts into the model would greatly increase the model-solving difficulty. When multi-dimensional efforts and multi-dimensional admission rules are considered, the incentive distortions may become smaller since

[40] The small effect may come from two offsetting effects. Consider decreasing the student number. Students would have a higher chance of entering the high-quality colleges. On one hand, the policy may lead to a higher *proportion* of students competing for higher-quality colleges, thus expanding the extensive margin; on the other hand, it eases the competitiveness for students *previously* targeting the high-quality college, i.e., at the intensive margin.

students may choose different competition tracks in which they have comparative advantage.

Second, some features of the real-world system have not been included in our model. To mention just a few: (1) Repetitive college exam takers increase rapidly in recent years, which may exacerbate the incentive distortions. (2) College admission policies such as affirmative action plans (e.g., policies favoring students from disadvantaged provinces or studying majors of national need), and reforms on examinations (e.g., from fixed exam subjects to freely chosen subjects) may also influence pre-college human capital investment. (3) High school graduates seeking for overseas college education become common, which may lessen the demand pressure in domestic market. (4) Reforms in high school education may also matter. For example, government has advocated policies of more active tracking system in high school and even imposed a minimum required ratio (e.g., 50%) of students flowing into vocational schools. All those issues are worth further empirical and theoretical research.

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