

A Dynamic Analysis of Parental Beliefs and Investments

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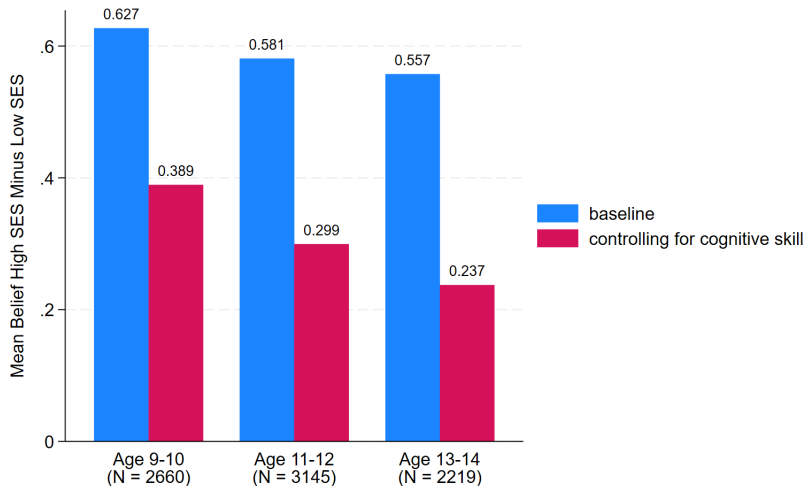
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Introduction

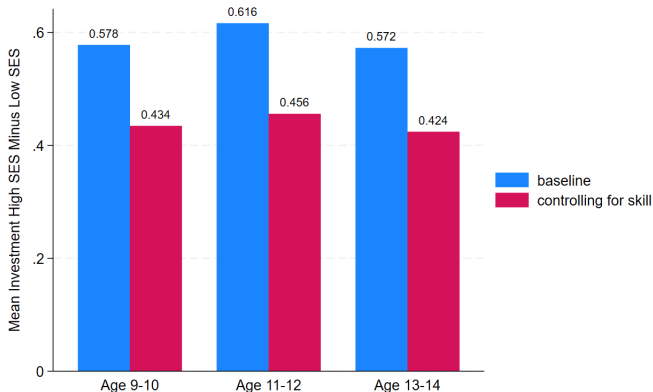
- SES investment gap: Rich parents invest more than poor parents in their children
(Bolt et al., 2021; Caucutt & Lochner, 2020; Carneiro, Reis & Toppeta 2024)
- Differences in investments \Rightarrow differences in children's human capital & life outcomes
(Attanasio, Cattan & Meghir, 2022)
- What contributes to the SES investment/skill gap?
 - Resources, skill of child & preferences differ across families
 - Parental beliefs (Attanasio & Kaufmann, 2009; Dizon-Ross, 2019; List, Pernaudet & Suskind, 2021)
 - Self-fulfilling prophecy of parental beliefs?

High SES hold higher beliefs, after accounting for child skills



correlation no anchoring

High SES invest more, after accounting for child skills



High SES: Above median family income

Low SES: Up to median family income

Investment factor score is standardised

This Paper

- **Goal:** Quantify the role of parental beliefs about child skill in explaining the SES skill gap
- **Data:** Longitudinal data on beliefs, investments and skills (NLSY)
- **Strategy:** Dynamic model of parent investments with parental beliefs
 - Parents do not observe skill, hold a belief
 - Invest and learn about skill over time
 - Feedback between beliefs and investments: channel for self-fulfilling prophecy

Preview of Findings

Q: Is there a mechanism consistent with the self-fulfilling prophecy?

- Yes, higher beliefs \Rightarrow invest more
- Because investments are more productive in children with higher skills

Q: What is the relative contribution of parental beliefs about child skill towards the SES skill gap?

- Equalising initial beliefs reduces SES skill gap by 2.59%
- This is around 4% of the impact of equalising the initial skills or initial income of parents

Outline

- Data
- Patterns in the Data
- Model
- Relative Contribution of Parental Beliefs to SES skill gap

Data

Data

- National Longitudinal Survey of Youth 1979 (NLSY 1979):
 - U.S. survey follows individuals who were 14-22 yr in 1979
- NLSY Child and Young Adult (CYA) starting 1986:
 - Longitudinal study of children of females in the NLSY 1979
 - Children enter when born
- Repeated measures of:
 - Parental beliefs
 - Parent investments
 - Child skills
 - Household demographics

summary statistics

literature

Parental Belief

1. Rating of child's academic standing in class:
 - One of the best students in the class, above the middle, in the middle, below the middle, near the bottom
2. Expectations of child educational attainment:
 - Up to high school, some college, college, more than college
3. Rating of child's future prospects:
 - Poor/fair, good, excellent

Estimated with a factor model: adjust for measurement error (Cunha, Heckman and Schennach 2010, Attanasio, Meghir & Nix 2020, Attanasio et al. 2020)

expectations

academic standing

future prospects

variation

factor model: belief

Parent Investment & Child Skills

Parent Investment

- Measures of goods and time investments
- Example components:
 - How many books the child has
 - Does child get special lessons or do extracurricular activities
 - How often bring child to museum/theatre
- Estimated with a factor model: adjust for measurement error
(Cunha, Heckman and Schennach 2010, Attanasio, Meghir & Nix 2020)

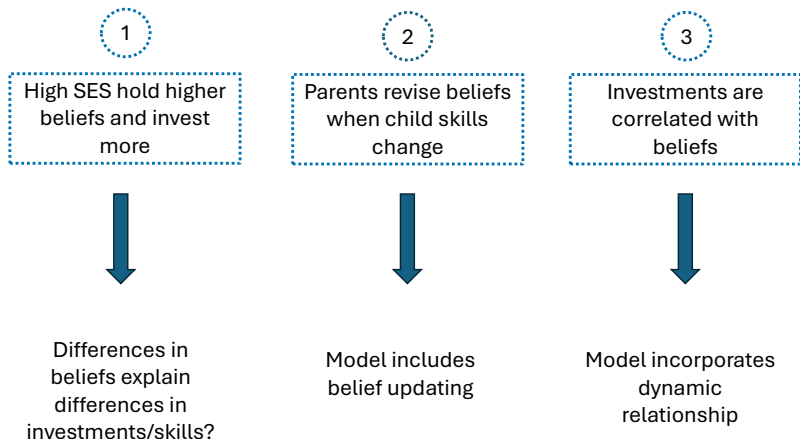
Child Cognitive Skill

- Achievement test scores in mathematics, reading recognition and reading comprehension

factor model: investment

factor model: skills

Patterns in the Data and Implications



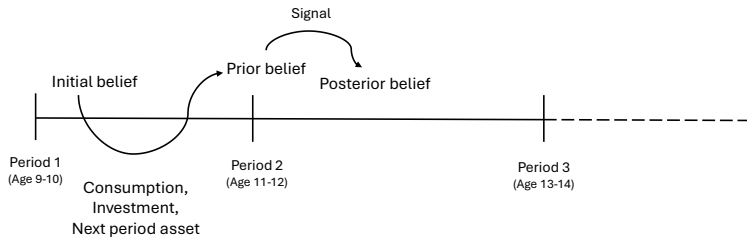
Next: Model (belief updating + how beliefs affect investments + how investments affect skills)

pattern 2: revise beliefs

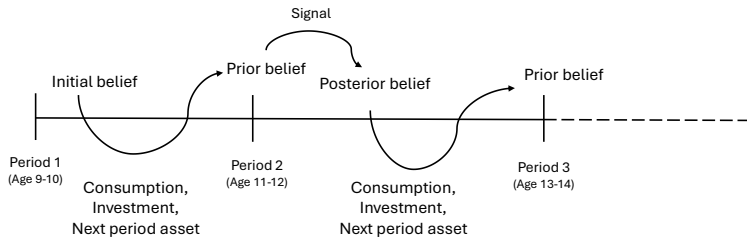
pattern 3: revise beliefs

Model

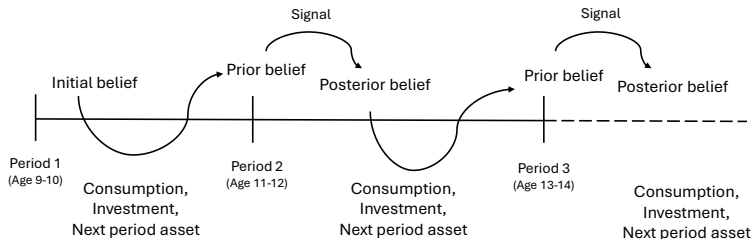
Dynamic Model of Parent Investments + Parental Beliefs About Skill



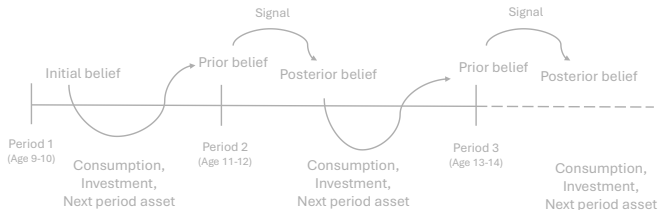
Dynamic Model of Parent Investments + Parental Beliefs About Skill



Dynamic Model of Parent Investments + Parental Beliefs About Skill



Dynamic Model of Parent Investments + Parental Beliefs About Skill



Initial Conditions

Parent belief, income, assets, skill of child

SES affects
dynamic
processes

Skill production function, belief evolution, income process

Unobserved
heterogeneity

Discrete preference types

value function

belief evolution 1

belief evolution 2

child skill production

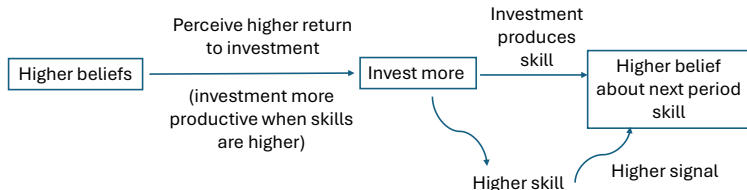
utility

budget constraint + income process

unobserved type

model overview text

Mechanism Consistent with Self-fulfilling Prophecy



estimation + model fit

identification

Relative Contribution of Beliefs to SES Skill Gap

SES Skill gap = Mean (high SES terminal skill) - Mean (low SES terminal skill)

Factors contributing to skill gap: (1) Beliefs; (2) Resources (income + assets); (3) Skills; (4) Preferences

Table: Percentage Change in SES Skill Gap

| Scenario | Percentage |
|------------------------------|------------|
| Equalise initial belief | -2.59 |
| Equalise initial child skill | -63.15 |
| Equalise initial income | -66.86 |
| Equalise initial assets | -167.38 |
| Equalise preferences | -0.53 |

Notes: This table presents the percentage change from the baseline in the SES skill gap when initial beliefs are equalised, initial skills are equalised, initial income is equalised, initial assets are equalised or preferences are equalised. All families have the same belief updating process, income process and skill production function.

Conclusion & Future Research

- Introduce dynamic model of parent investments with parental beliefs about child skill level
 - Two-way interaction between beliefs and investments
 - Self-fulfilling prophecy
 - Higher beliefs \Rightarrow invest more because perceive higher return to investment \Rightarrow higher beliefs
 - Invest more \Rightarrow higher beliefs
- Relative contribution of beliefs to the SES skill gap: equalising beliefs will lower SES skill gap by 2.59%
- Extend: Developing countries? Younger children?

Thank you! Comments/suggestions are much
appreciated

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Appendix

Literature

- **Economics of Education: Subjective expectations/beliefs**

Cunha, Elo & Culhane, 2022; Conti, Giannola & Toppeta, 2022; Dizon-Ross 2019; Attanasio, Cunha & Jervis 2019; List, Pernaudet & Suskind, 2021; Kinsler & Pavan 2021; Attanasio, Boneva & Rauh 2022; Tincani, Kosse & Miglino 2023; Greaves, Hussain, Rabe & Rasul 2023; Chikhale 2025; Lee 2025

This paper: Parental beliefs about child skill, dynamics between beliefs and investments (feedback)

- **Belief formation, learning and belief dynamics**

Akerberg, 2003; Erdem & Keane, 1996; Crawford & Shum, 2005; Zafar, 2011; Sanders, 2012; Stinebrickner & Stinebrickner 2014; Coffman, Ugalde Araya & Zafar 2021; Arcidiacono, Aucejo, Ransom & Maurel, 2024

This paper: Parent belief updating. Endogenous updating of evolving unobserved state (child skill)

Pattern 2: Parents Revise Beliefs

- Belief at time t depends linearly on belief at time $t - 1$ (captures previous information parents had) and cognitive skill of child at time t
- Estimated with Blundell-Bond dynamic panel data estimator: system GMM

$$\mu_{jt} = \alpha + \beta_1 \mu_{j,t-1} + \beta_2 \text{cognitive}_{jt} + \gamma X_{jt} + \underbrace{\delta_j}_{\text{child FE}} + \epsilon_{jt} \quad (1)$$

X_{jt} : age of child, region of residence, urban/rural, mother employed, biological father lives with child, education of spouse, marital status, log(family income)

[back: patterns in the data](#)

Pattern 2: Parents Revise Beliefs

| | Belief |
|---------------------------------------|---------------------|
| Lag Belief Factor Score | 0.198*** (0.030) |
| Cognitive Skill | 0.871*** (0.062) |
| Observations | 5,829 |
| Child FE | Yes |
| Cognitive Skill Treated as Endogenous | Yes |

- Parents use previous information: lag belief positive and significant
- Within-child changes in beliefs correlated with within-child changes in cognitive skill

Pattern 3: Investments are Correlated with Beliefs

- Relationship between investment and belief: Regress investment on belief
- Unobserved factors affecting both beliefs and investments:
 - Child FE, lag investment (persistent preferences)
- Specification with lag investment and child FE estimated with Blundell-Bond dynamic panel data estimator

$$\underbrace{l_{jt}}_{\text{investment}} = \alpha + \beta_1 \underbrace{\mu_{jt}}_{\text{belief}} + \gamma \underbrace{l_{j,t-1}}_{\text{lag investment}} + \underbrace{\delta_j}_{\text{child FE}} + \eta_{jt} \quad (2)$$

X_{jt} : log (family income), age of child and more

Assumptions: X_{jt} weakly exogenous, limited time dependence in η_{jt}

[back: patterns in the data](#)

Pattern 3: Investments are Correlated with Beliefs

Table: Dependent variable is Investment Factor Score

| | (1) | (2) | (3) |
|---------------------|---------------------|---------------------|---------------------|
| Belief Factor Score | 0.202*** (0.009) | 0.049*** (0.011) | 0.058*** (0.012) |
| Observations | 9557 | 9557 | 9557 |
| Child FE | No | Yes | Yes |
| Lag Investment | No | No | Yes |

Child FE model: 1 unit \uparrow in belief factor score is associated with 0.049 SD \uparrow in investment factor score

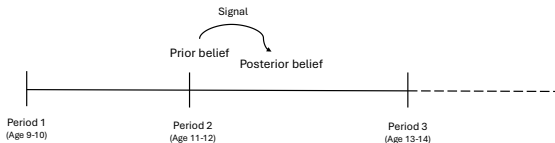
[lag belief](#)

[back: patterns in the data](#)

Model Overview

- Parent-child investment model with belief updating
- 3 periods: age 9-10, age 11-12, age 13-14
- Choices: consumption, investment, savings (next period assets)
 - Utility: consumption, skill of child
 - Trade-off: consumption, investment
- Features of the model:
 - Parents do not observe the skill of the child, hold a belief
 - Receive signal (noisy measure of child skill) and update belief
 - Feedback between beliefs and investments:
 - Beliefs affect investments
 - Parents invest more \Rightarrow higher next period belief
- Parent-child pairs heterogeneous: beliefs, income, assets, skill of child, mother's education, unobserved discrete type

Model: Timing & Evolution of Beliefs



Stage 1: Update belief about log skill

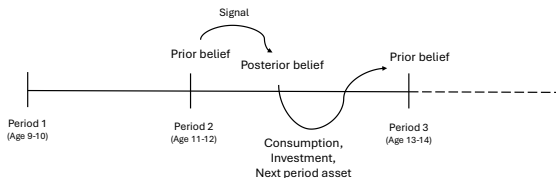
$$\underbrace{\ln \theta_t \sim \mathcal{N}(\mu_t, .)}_{\text{prior}} \xrightarrow{\text{signal } g_t} \underbrace{\ln \theta_t \sim \mathcal{N}(\mu_t^{\text{int}}, .)}_{\text{posterior}}$$

$$\text{Signal } g_t = \ln \theta_t + \epsilon_t, \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

$$\mu_t^{\text{int}} = \underbrace{\psi_s \mu_t + (1 - \psi_s) g_t}_{\text{weighted average of prior \& signal}}$$

[back: model overview](#)

Model: Timing & Evolution of Beliefs



Stage 2: Form belief about next period log skill $\ln \theta_{t+1} \sim \mathcal{N}(\mu_{t+1}, \cdot)$

$$\mu_{t+1} \equiv \tilde{E} [\ln \theta_{t+1}] = \underbrace{\tilde{E} [f(\theta_t, i_t, \eta_{t+1})]}_{\text{expected value given posterior belief, skill production function \& investment}}$$

Invest more \Rightarrow higher beliefs about next period skill

[back: model overview](#)

Model: Child Skill Production Technology

Translog Production Function (by SES s)

$$\ln \theta_{t+1} = \underbrace{A_t}_{\text{time-varying external factors}} + \gamma_{s,1} \ln \theta_t + \gamma_{s,2} \ln i_t + \gamma_{s,3} \ln \theta_t \times \ln i_t + \eta_{t+1}, \eta_{t+1} \sim \mathcal{N}(0, \sigma_{s,\eta}^2)$$

$\gamma_{s,3}$: determines whether parents invest more/less when beliefs are higher

Initial skill: estimated from factor model on achievement test scores

[back: model overview](#)

Model: Flow Utility

$$u \left(\underbrace{c_t}_{\text{consumption}}, \underbrace{\tilde{\theta}_t}_{\text{perceived skill}} \right)$$
$$= \ln c_t + \underbrace{\kappa_h}_{\text{relative preference for child skill over consumption}} \times \underbrace{\tilde{E}_t \ln \theta_t}_{\text{utility from perceived skill}}$$

Parents invest because they value child skill: higher investment \Rightarrow higher skill

κ depends on unobserved discrete type h [back: model overview](#)

Model: Value Function

$$V_t(\underbrace{\chi_t}_{\text{state}}) = \max_{c_t, i_t, a_{t+1}} \underbrace{u\left(\underbrace{c_t}_{\text{consumption}}, \underbrace{\tilde{\theta}_t}_{\text{perceived skill}}\right)}_{\text{Utility}} + \beta \tilde{E}_t V_{t+1}\left(\underbrace{\chi_{t+1}}_{\text{next period state}}\right)$$

s.t. Child Skill Production Function & **Belief Updating Process** & Budget Constraint + Income Process

State Variables:

1. Belief about skill
2. Income
3. Assets
4. SES \Rightarrow belief updating, skill production function, income process
5. Unobserved discrete type $h \Rightarrow$ relative preference for skill over consumption

Model: Resource Constraint and Income Process

Family Income Process (by SES s)

$$\ln(\underbrace{y_{t+1}}_{\text{family income}}) = \tau_{s,0} + \tau_{s,1} \text{age}_{t+1} + \tau_{s,2} \text{age}_{t+1}^2 + \tau_{s,3} \text{age}_{t+1}^3 + \nu_{t+1}$$

$$\nu_{t+1} = \rho_s \nu_t + u_{y,t}, u_{y,t} \sim \mathcal{N}(0, \sigma_{s,u}^2)$$

Budget Constraint

$$\underbrace{c_t}_{\text{consumption}} + \underbrace{i_t}_{\text{investment}} + \underbrace{a_{t+1}}_{\text{next period asset}} = \underbrace{(1+r)a_t + y_t}_{\text{resources}}$$

[back: model overview](#)

Pattern 2: Parents Revise Beliefs

| | All | High SES | Low SES |
|----------------------------|---------------------|---------------------|---------------------|
| Lag Belief Factor Score | 0.198*** (0.030) | 0.184*** (0.038) | 0.278*** (0.053) |
| Cognitive Skill | 0.871*** (0.062) | 0.925*** (0.081) | 0.724*** (0.101) |
| Observations | 5,829 | 3,477 | 2,352 |
| Child FE | Yes | Yes | Yes |
| Cognitive Skill Endogenous | Yes | Yes | Yes |

[back: parents revise beliefs](#)

Unobserved Heterogeneity: Type Probability

- Type probability determined by logistic function
- Depends on first period log skills of the child
- Parents who have higher preferences would have invested more in earlier periods \Rightarrow child has higher skills

$$Pr(\text{Type 2} | \ln \theta_1) = \frac{\exp(\pi_0 \ln \theta_1)}{1 + \exp(\pi_0 \ln \theta_1)} \quad (3)$$

back: unobserved discrete type

Estimated Parameters: Family Income

[back: self-fulfilling prophecy](#)

Table: Family Income Process Parameters

| Parameter | Description | Estimate | S.E. |
|--------------------------------------|--------------------------------------|-------------------------|---------|
| Low SES ($s = 1$) | | | |
| $\tau_{s=1,0}$ | Intercept term | 10.747 | 0.44918 |
| $\tau_{s=1,1}$ | Coefficient on age | 0.088 | 0.15816 |
| $\tau_{s=1,2}$ | Coefficient on age squared | -0.009 | 0.01749 |
| $\tau_{s=1,3}$ | Coefficient on age cubed | 3.605×10^{-4} | 0.00061 |
| $\rho_{s=1}$ | AR(1) residual persistence parameter | 0.633 | 0.03862 |
| $\sigma_{s=1,u}^2$ | AR(1) residual variance | 0.230 | 0.03265 |
| High SES ($s = 2$) | | | |
| $\tau_{s=2,0}$ | Intercept term | 11.725 | 0.44912 |
| $\tau_{s=2,1}$ | Coefficient on age | -0.129 | 0.15652 |
| $\tau_{s=2,2}$ | Coefficient on age squared | 0.018 | 0.01719 |
| $\tau_{s=2,3}$ | Coefficient on age cubed | -6.781×10^{-4} | 0.00060 |
| $\rho_{s=2}$ | AR(1) residual persistence parameter | 0.525 | 0.01326 |
| $\sigma_{s=2,u}^2$ | AR(1) residual variance | 0.372 | 0.03942 |

Notes: Robust standard errors reported for the log family income regression. Standard errors for the AR(1) process computed via bootstrap with 100 replications.

Estimation & Model Fit

Two-Step Estimation

1. Discount factor taken from literature, estimate family income process directly from the data
2. Estimate remaining parameters via method of simulated moments

Model Fit

Table: Selected Moments

| Moment | Data | Model |
|---|--------|--------|
| Mean skill age 13-14: Family income Q1 | 13.816 | 13.850 |
| Mean skill age 13-14: Family income Q2 | 13.944 | 13.946 |
| Mean skill age 13-14: Family income Q3 | 14.104 | 14.036 |
| Mean skill age 13-14: Family income Q4 | 14.318 | 14.193 |
| Mean belief age 13-14: Family income Q1 | 13.775 | 13.777 |
| Mean belief age 13-14: Family income Q2 | 13.761 | 13.861 |
| Mean belief age 13-14: Family income Q3 | 14.187 | 14.086 |
| Mean belief age 13-14: Family income Q4 | 14.454 | 14.202 |

Key Parameter Estimates

back: self-fulfilling prophecy

| Parameter | Description | Estimate | S.E. |
|---|---|----------|---------|
| Preference Parameters | | | |
| $\kappa_{h=1}$ | Relative preference for skill over consumption : Type 1 | 1.914 | 0.11843 |
| $\kappa_{h=2}$ | Relative preference for skill over consumption : Type 2 | 3.017 | 0.01016 |
| π_0 | Type probability logistic: coefficient on log skill | 1.765 | 0.06207 |
| Skill Production Function Parameters | | | |
| A_1 | Age 9-10 total factor productivity | 0.354 | 0.05702 |
| $A_{2,3}$ | Age 11-12, Age 13-14 total factor productivity | 0.234 | 0.06732 |
| $\gamma_{s=1,1}$ | Weight on log skill (Low SES) | 0.689 | 0.00217 |
| $\gamma_{s=1,2}$ | Weight on log investment (Low SES) | 0.039 | 0.00475 |
| $\gamma_{s=1,3}$ | Weight on interaction between log skill and log investment (Low SES) | 0.026 | 0.00051 |
| $\sigma_{s=1,\eta}^2$ | Variance of idiosyncratic shock (Low SES) | 0.132 | 0.01647 |
| $\gamma_{s=2,1}$ | Weight on log skill (High SES) | 0.778 | 0.00490 |
| $\gamma_{s=2,2}$ | Weight on log investment (High SES) | 0.048 | 0.00203 |
| $\gamma_{s=2,3}$ | Weight on interaction between log skill and log investment (High SES) | 0.016 | 0.00012 |
| $\sigma_{s=2,\eta}^2$ | Variance of idiosyncratic shock (High SES) | 0.098 | 0.02130 |
| Belief Parameters | | | |
| σ_ϵ^2 | Variance of error in signal | 6.051 | 1.29386 |
| $\psi_{s=1}$ | Weight on prior (Low SES) | 0.671 | 0.03809 |
| $\psi_{s=2}$ | Weight on prior (High SES) | 0.649 | 0.04217 |

Parent Belief about Child Skill: Variation

- Sub-sample of children in cross-section who had measure reported 3 times between 9 and 14 years
- Evaluation of child academic standing in class: 61% made change
- Expectations of child educational attainment: around 57% made change
- Rating of child's future prospects: around 45% made change

[back: belief measures](#)

Transcript Scores and PIAT Tests

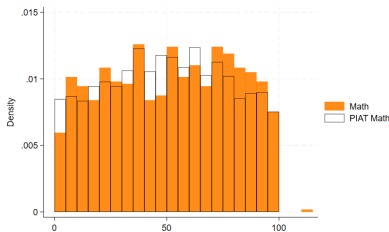


Figure: PIAT Mathematics

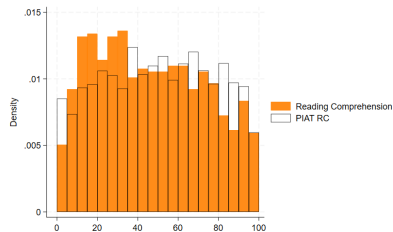
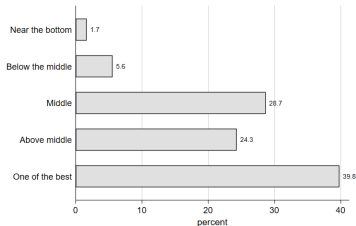


Figure: PIAT Reading Comprehension

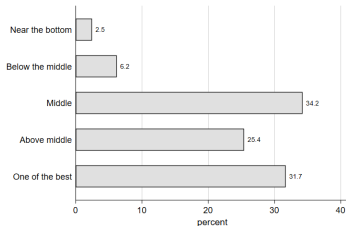
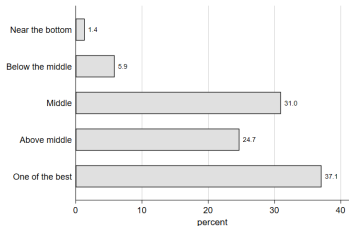
[back: investment and child skill measures](#)

Measures of Parent Beliefs: Academic Standing



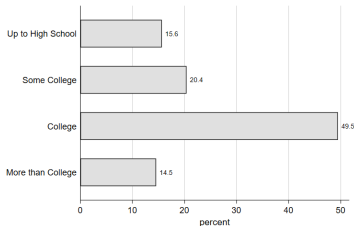
In anticlockwise direction

1. Age 9-10
2. Age 11-12
3. Age 13-14



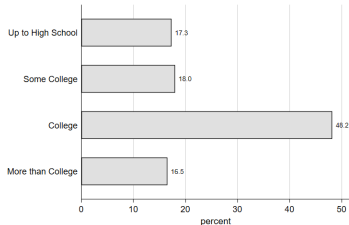
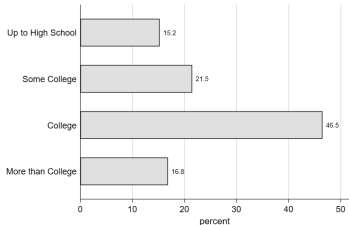
[back: belief measures overview](#)

Measures of Parent Beliefs: Expected Education



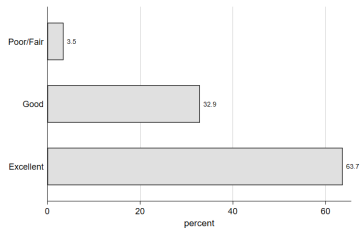
In anticlockwise direction

1. Age 9-10
2. Age 11-12
3. Age 13-14



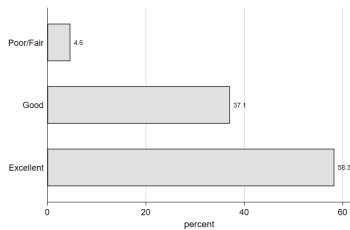
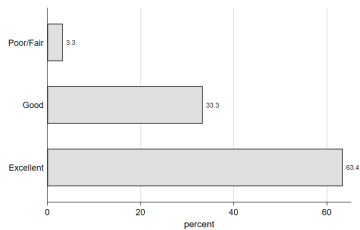
[back: belief measures overview](#)

Measures of Parent Beliefs: Child Future Prospects



In anticlockwise direction

1. Age 9-10
2. Age 11-12
3. Age 13-14



[back: belief measures overview](#)

Behaviour Problems Index

Measures the frequency, range, and type of childhood behavior problems for children age four and over

- Cheats or tells lies
- Does not seem to feel sorry after misbehaving
- Has sudden changes in mood or feeling
- Feels/complains no one loves him/her
- Is too dependent on others
- Demands a lot of attention
- Is rather high strung, tense, and nervous
- Has difficulty concentrating/paying attention
- Is not liked by other children
- Is withdrawn, does not get involved with others

Options: Often true, sometimes true, not true

[back: investment and child skill measures](#)

Factor Model: Estimation

- Estimated using STATA gsem command
- Maximum likelihood estimation
- Assumptions:
 - Each measurement is independent and identically distributed across the sample, conditional on latent variable
 - Measurements are independent, conditional on latent variable
- Likelihood is computed by integrating out the latent variables using mean variance adaptive Gauss-Hermite quadrature

Factor Model for Log Skills

Log skills assumed to be normally distributed

Measurements of skills are continuous

Equations for the latent factor model of skills for $t = 1, 2, 3$

$$Q = \alpha + \ln \theta_T + \epsilon_Q \quad (4)$$

$$m_{1t} = \alpha_{m1,t} + \lambda_{m1,t} \ln \theta_t + \epsilon_{m1,t} \quad (5)$$

$$m_{2t} = \alpha_{m2,t} + \lambda_{m2,t} \ln \theta_t + \epsilon_{m2,t} \quad (6)$$

$$m_{3t} = \alpha_{m3,t} + \lambda_{m3,t} \ln \theta_t + \epsilon_{m3,t} \quad (7)$$

- Latent log skills anchored to adult outcome Q : years of education of the child at age 24 and above
- Loading on the $\ln \theta_T$ is fixed at 1: an 1 unit \uparrow in $\ln \theta_T$ corresponds to a 1 unit \uparrow in the conditional expectation of the adult outcome Q
- Linking measure: intercept and factor loading of raw score on PIAT mathematics is constrained to be the same at all ages

Factor Model for Latent Belief

Latent belief assumed to be normally distributed

Measurements of belief are ordered discrete

Equations for the latent factor model of belief for $t = 1, 2, 3$

$$Q = \alpha + \mu_T^{obs} + \nu_Q \quad (8)$$

$$m_{1t}^* = \alpha_{m1*} + \lambda_{m1*} \mu_t^{obs} + \epsilon_{m1*,t} \quad (9)$$

$$m_{2t}^* = \alpha_{m2*} + \lambda_{m2*} \mu_t^{obs} + \epsilon_{m2*,t} \quad (10)$$

$$m_{3t}^* = \alpha_{m3*} + \lambda_{m3*} \mu_t^{obs} + \epsilon_{m3*,t} \quad (11)$$

- Latent belief anchored to adult outcome Q : years of education of the child at age 24 and above
- Loading on the μ_T^{obs} is fixed at 1: a 1 unit \uparrow in μ_T^{obs} corresponds to a 1 unit \uparrow in the conditional expectation of the adult outcome Q

back: parental belief

Factor Model for Latent Belief

The unobserved m^* are mapped to the discrete ordered values in the data according to a threshold model

For example, if the measurement m_{ijt} takes 4 values, the threshold model is like this:

$$m_{ijt} = \begin{cases} 0 & \text{if } m_{ijt}^* \leq \tau_{1,jt} \\ 1 & \text{if } m_{ijt}^* \in [\tau_{1,jt}, \tau_{2,jt}] \\ 2 & \text{if } m_{ijt}^* \in [\tau_{2,jt}, \tau_{3,jt}] \\ 3 & \text{if } m_{ijt}^* > \tau_{3,jt} \end{cases}$$

[back: parental belief](#)

Factor Model for Latent Investment

Latent log (investment) assumed to be normally distributed

Measurements of investment are binary or ordered discrete

Sample equations for the latent factor model of investment for

$t = 1, 2, 3$

$$m_{1t}^* = \alpha_{m1^*} + \lambda_{m1^*} \ln I_t + \epsilon_{m1^*,t} \quad (12)$$

$$m_{2t}^* = \alpha_{m2^*} + \lambda_{m2^*} \ln I_t + \epsilon_{m2^*,t} \quad (13)$$

$$m_{3t}^* = \alpha_{m3^*} + \lambda_{m3^*} \ln I_t + \epsilon_{m3^*,t} \quad (14)$$

- Linking measure: how often child is brought to the museum

[back: parent investment](#)

Factor Model for Latent Investment

The unobserved m^* are mapped to the discrete ordered values in the data according to a threshold model

For example, if the measurement m_{ijt} takes 4 values, the threshold model is like this:

$$m_{ijt} = \begin{cases} 0 & \text{if } m_{ijt}^* \leq \tau_{1,jt} \\ 1 & \text{if } m_{ijt}^* \in [\tau_{1,jt}, \tau_{2,jt}] \\ 2 & \text{if } m_{ijt}^* \in [\tau_{2,jt}, \tau_{3,jt}] \\ 3 & \text{if } m_{ijt}^* > \tau_{3,jt} \end{cases}$$

[back: parent investment](#)

3. Investments are Correlated with Beliefs (Lag Belief)

Table: Dependent variable is Investment Factor Score

| | (1) | (2) | (3) |
|----------------|---------------------|--------------------|---------------------|
| Lag Belief | 0.178*** (0.013) | -0.025* (0.015) | -0.038** (0.017) |
| Observations | 6619 | 6619 | 6619 |
| Child FE | No | Yes | Yes |
| Lag Investment | No | No | Yes |

[back: regression investment on belief](#)

Summary Statistics

Table: Summary Statistics of Model Estimation Sample

| | Mean | Std. Dev. | Min | Max. |
|-----------------------------|-----------|-----------|----------|----------|
| Family Income Age 9-10 | 105940.91 | 98,973.33 | 5,906.35 | 1.52e+06 |
| Assets Age 9-10 | 304110.50 | 527918.06 | 0.00 | 3.37e+06 |
| Belief Age 9-10 | 13.80 | 1.11 | 9.90 | 15.68 |
| Log Skill Age 9-10 | 13.27 | 0.74 | 9.59 | 15.37 |
| Mother's Years of Education | 14.04 | 2.36 | 7.00 | 20.00 |
| Mother's AFQT Percentile | 56.58 | 27.28 | 0.82 | 100.00 |
| Mother's Age at Birth | 27.03 | 3.98 | 17.00 | 37.00 |
| White | 0.88 | 0.33 | 0.00 | 1.00 |
| Black | 0.06 | 0.23 | 0.00 | 1.00 |
| Hispanic | 0.07 | 0.25 | 0.00 | 1.00 |
| Male | 0.50 | 0.50 | 0.00 | 1.00 |

Notes: This table presents the sample statistics of the children were used in the model estimation. The Armed Forces Qualification Test (AFQT) percentile is a measure of cognitive skill.

Summary of Identification

| Parameter | Identifying Data Features |
|--|--|
| $\kappa_{h=1}, \kappa_{h=2}$ | Ratio of next period assets to investment factor score, moments of investment factor score conditional on observables |
| $A_1, A_2, \gamma_{s=1,1}, \gamma_{s=1,2}, \gamma_{s=1,3}, \gamma_{s=2,1}, \gamma_{s=2,2}, \gamma_{s=2,3}, \sigma_{s=1,\eta}^2, \sigma_{s=2,\eta}^2$ | Coefficients and residuals of regression of log skill factor score on lag log skill factor score, lag investment factor score and interaction between lag log skill factor score and lag investment factor score |
| $\psi_{s=1}, \psi_{s=2}, \sigma_{\epsilon}^2$ | Coefficients and residuals of regression of the posterior belief on lag log skills, lag investment and the current log skills |
| π_0 | Residuals of regression of investment factor score on belief factor score and the correlation of investment factor score conditional on quartile of skill factor score |

[back: self-fulfilling prophecy](#)

Model: Unobserved Discrete Type

Concern 1: Different preferences are driving differences in beliefs and investments

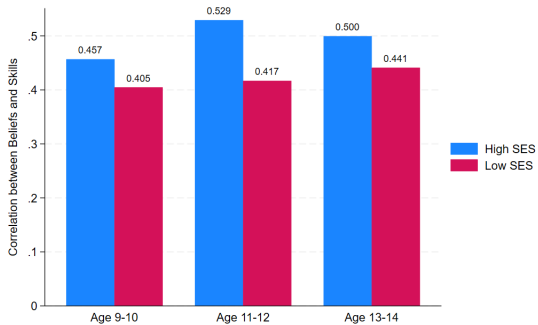
Concern 2: Initial conditions (at age 9-10) are endogenous

- Parents are one of two discrete types (Heckman & Singer, 1983; Keane & Wolpin, 1997; French & Jones, 2011)
- One type: Higher relative value for child skill over consumption κ
- Type probability depends on first period child skills

back: model value function

type probability

Pattern 1: SES Differences in Beliefs



back: SES differences in beliefs