

# Locally Robust Estimation and Machine Learning in Dynamic Structural Econometrics

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# INTRODUCTION

Many interesting objects depend on a regression or other first step.

- Discrete dynamic structural economic models depend on conditional choice probabilities.
- Average equivalent variation bounds depend on average demand.
- Average treatment effect (ATE) depends on average outcome given covariates and treatment and/or propensity score.

Estimators of objects of interest, like structural parameters, can be constructed using modern machine learners of the first step such as Lasso, neural nets (NN), and random forests, but just plugging them into formulas or moment functions leads to large bias that invalidates inference.

A very common source of this bias is model selection, which also occurs with older nonparametric methods like series or kernel estimation, see Leeb and Potscher (2005).

A seemingly attractive model selection property, that the right model is chosen with probability approaching one, invalidates standard confidence intervals for parameters of interest.

Large bias also comes from the regularization used for machine learning, like penalization for Lasso or early stopping of gradient descent for NN.

Such regularization helps reduce variance of the first step, but passes through to estimator of object of interest, so that its bias shrinks slower than standard deviation, making the estimator not even root-n consistent.

A solution to regularization and model selection biases is Neyman orthogonal moment functions for GMM.

Orthogonality means first step has no effect, to first order, on average moment function.

Orthogonal moment functions can be constructed by adding to identifying moment functions the nonparametric influence function of the expected moment functions evaluated at the plim of the first step when data distribution is unrestricted (i.e. under general misspecification).

This orthogonality is model free in only depending on the plim of the first step.

Model free orthogonality leads to standard errors that are robust to misspecification (not true with other debiasing, e.g. Lasso propensity scores used in augmented inverse propensity score weighting for ATE).

We also use cross-fitting, where first steps are estimated from different observations than used to construct sample moments.

Cross-fitting removes some other biases, only requires mean-square convergence rates for the machine learners, which is all we know about NN, and leads to remainders of smaller order in some settings.

Exposition from "Locally Robust Semiparametric Estimation," (2016, 2022 *Econometrica*, Chernozhukov, Escanciano, Ichimura, Newey, Robins; LR henceforth).

Orthogonal moment function depends on another unknown function  $\alpha_0$  in addition to the first step.

Can use orthogonality property to estimate  $\alpha_0$ .

Leads to "automatic" methods of estimating  $\alpha_0$  that only use formulas for object of interest and do not require knowing a formula for  $\alpha_0$ .

These automatic methods often give estimator of parameter of interest with better properties than estimator based on plugging into a formula for  $\alpha_0$ , perhaps because they do not use inverses of high dimensional estimators.

See V. Chernozhukov, W.K. Newey, V. Quintas-Martinez, V. Syrgkanis (2022) "RieszNet and ForestRiesz: Automatic Debiased Machine Learning with Neural Nets and Random Forests," *Proceedings of the 39th International Conference on Machine Learning* 162;

<https://arxiv.org/abs/2110.03031>;

AutoDML improves on state of the art, Dragonnet NN estimator of ATE using inverse of NN estimator of propensity score in Monte Carlo example.

See also V. Chernozhukov, W.K. Newey, V. Quintas-Martinez, V. Syrgkanis (2021) "Automatic Debiased Machine Learning via Neural Nets for Generalized Linear Regression,"

<https://arxiv.org/abs/2104.14737>;

AutoDML developed here; previous paper is an application of this one.

Large sample theory only requires the first step converges faster than  $n^{-1/4}$  in mean square and the product of mean-square convergence rates for the learner of  $\alpha_0$  and the regression learner is faster than  $n^{-1/2}$ .

LR gives regularity conditions for dynamic discrete choice with Lasso regression but not automatic estimator of  $\alpha_0$ .

# ORTHOGONAL MOMENT FUNCTIONS

$\theta$  : finite dimensional parameter vector of interest;

$\gamma$  : unknown first step function, represent possible machine learner of the function;

$W$  : data observation with unknown cumulative distribution function (CDF)  $F_0$ ;

$g(w, \gamma, \theta)$  : vector moment functions,  $w$  a possible realization  $w$  of  $W$ ;

Moment condition

$$E[g(W, \gamma_0, \theta)] = 0,$$

$\theta_0$  is assumed to be the unique solution this equation, i.e.  $\theta_0$  is identified by the moment condition, where  $E[\cdot]$  is the expectation under the true distribution, and  $\gamma_0$  is the probability limit of a first step estimator (machine learner) of  $\gamma$ .

Example 1: Simple and illustrative;

$$\theta_0 = E[Z\gamma_0(X)], \quad \gamma_0(X) = E[Y|X].$$

Here the identifying moment function is

$$g(W, \gamma, \theta) = Z\gamma(X) - \theta.$$

In general the  $\gamma$  in  $g(W, \gamma, \theta)$  allows the moment conditions to depend on the whole function  $\gamma$ .

In this example the moment function depends on  $\gamma$  through  $\gamma(X)$ .

Here the limit  $\gamma_0$  of the machine learner is assumed to be the conditional expectation  $E[Y|X]$  of  $Y$  given  $X$ .

Construct orthogonal moment functions using the plim  $\gamma(F)$  of  $\hat{\gamma}$  when  $F$  is distribution of a single observation  $W$ ;

$\gamma(F)$  is plim of  $\hat{\gamma}$  under general misspecification where  $F$  is unrestricted except for regularity conditions;

$$\gamma_0 = \gamma(F_0).$$

Example 1 is for nonparametric regression  $\hat{\gamma}$  where  $\gamma(F)(x) = E_F[Y|X = x]$ .

Let  $H$  be some alternative distribution that is unrestricted except for regularity conditions, and  $F_\tau = (1 - \tau)F_0 + \tau H$  for  $\tau \in [0, 1]$ .

Assume  $\gamma(F_\tau)$  exists for  $\tau$  small enough and possibly other regularity conditions are satisfied.

Key assumption is existence of unknown functions  $\alpha$  and  $\phi(w, \gamma, \alpha, \theta)$  such that for all  $H$  and  $\theta$ ,

$$\begin{aligned} \frac{d}{d\tau} E[g(W, \gamma(F_\tau), \theta)] &= \int \phi(w, \gamma_0, \alpha_0, \theta) H(dw), \\ E[\phi(W, \gamma_0, \alpha_0, \theta)] &= 0, \quad E[\phi(W, \gamma_0, \alpha_0, \theta)^2] < \infty, \end{aligned}$$

$$\frac{d}{d\tau} E[g(W, \gamma(F_\tau), \theta)] = \int \phi(w, \gamma_0, \alpha_0, \theta) H(dw),$$

$$E[\phi(W, \gamma_0, \alpha_0, \theta)] = 0, \quad E[\phi(W, \gamma_0, \alpha_0, \theta)^2] < \infty,$$

$\alpha_0$  is the  $\alpha$  such that these equations hold; and  $d/d\tau$  is the derivative from the right (i.e. for nonnegative values of  $\tau$ ) at  $\tau = 0$ .

The equations define  $\phi(w, \gamma_0, \alpha_0, \theta)$  to be the *influence function* of  $\mu(F) = E[g(W, \gamma(F), \theta)]$ , as in Von Mises (1947), Hampel (1974), and Huber (1981).

$\phi(w, \gamma_0, \alpha_0, \theta)$  is unique because  $H$  is unrestricted.

Existence of  $\phi(w, \gamma_0, \alpha_0, \theta)$  is equivalent to finite semiparametric variance bound for  $\mu(F)$ .

Here  $\gamma_0$  and  $\alpha_0$  can depend on  $\theta$ ; equations hold for each  $\theta \in \Theta$ ;  $F_0$  and  $H$  do not depend on  $\theta$ .

We refer to  $\phi(w, \gamma, \alpha, \theta)$  as the *first step influence function* (FSIF); is "adjustment term" of Newey (1994).

Example 1: Here  $g(W, \gamma, \theta) = Z\gamma(X) - \theta$  and we assume that  $\gamma(F)$  is  $E_F[Y|X]$ , so that

$$E[g(W, \gamma(F), \theta)] = E[ZE_F[Y|X]] - \theta.$$

In this example

$$\phi(W, \gamma, \alpha, \theta) = \alpha(X)\{Y - \gamma(X)\}, \quad \alpha_0(X) = E[Z|X].$$

Follows from Newey (1994, "Asymptotic Variance of Semiparametric Estimators," EMA).

Construct orthogonal moment functions by adding FSIF to identifying moment functions

$$\psi(W, \gamma, \alpha, \theta) = g(W, \gamma, \theta) + \phi(W, \gamma, \alpha, \theta).$$

LR shows two key orthogonality properties:

Property I : For a linear set  $\Gamma \supseteq \{\gamma(F)\}$  of possible first steps,

$$\frac{d}{d\delta} E[\psi(W, \gamma_0 + \delta(\gamma - \gamma_0), \alpha_0, \theta)] = 0 \text{ for all } \gamma \in \Gamma \text{ and } \theta \in \Theta$$

where  $\delta \in \mathbb{R}$  is a scalar and the derivative is evaluated at  $\delta = 0$ .

Varying  $\gamma$  has no effect (locally to  $\gamma_0$ ) on  $E[\psi(W, \gamma, \alpha_0, \theta)]$  for  $\gamma \in \Gamma$ .

Property II : For the set  $\mathcal{A}$  of  $\alpha_0$  such that influence function equation is satisfied

$$E[\phi(W, \gamma_0, \alpha, \theta)] = 0 \text{ for all } \theta \in \Theta \text{ and } \alpha \in \mathcal{A}.$$

Implies  $E[\psi(W, \gamma_0, \alpha, \theta_0)] = 0$  for all  $\alpha \in \mathcal{A}$ , so orthogonal moment function  $\psi(W, \gamma_0, \alpha, \theta_0)$  is globally robust in  $\alpha$ .

In general there are distinct  $\phi(W, \gamma, \alpha, \theta)$  and  $\alpha$  for each element of  $g(W, \gamma, \theta)$ .

Example 1: Orthogonal moment function is

$$\psi(W, \gamma, \alpha, \theta) = Z\gamma(X) - \theta + \alpha(X)\{Y - \gamma(X)\}.$$

Property II follows by iterated expectations, which gives

$$E[\phi(W, \gamma_0, \alpha, \theta)] = E[\alpha(X)\{Y - E[Y|X]\}] = 0.$$

To show Property I, note that by iterated expectations,

$$\begin{aligned} E[\psi(W, \gamma, \alpha_0, \theta)] &= E[Z\gamma(X)] - \theta + E[\alpha_0(X)\{Y - \gamma(X)\}] \\ &= E[\alpha_0(X)\gamma(X)] - \theta + E[\alpha_0(X)Y] - E[\alpha_0(X)\gamma(X)] \\ &= E[\alpha_0(X)Y] - \theta. \end{aligned}$$

Here  $E[\psi(W, \gamma, \alpha_0, \theta)]$  does not depend on  $\gamma$ , i.e. it is globally robust to  $\gamma$ , and hence directional derivative is zero and have property I.

Here  $\psi(W, \gamma, \alpha, \theta)$  is an example of *doubly robust* moment function, meaning global robustness in  $\gamma$  as well as  $\alpha$ .

Necessary and sufficient condition for double robustness is  $E[\psi(W, \gamma, \alpha_0, \theta)]$  is linear (affine) in  $\gamma$ ; see LR for more discussion and many examples.

Orthogonal moment functions are

$$\psi(W, \gamma, \alpha, \theta) = g(W, \gamma, \theta) + \phi(W, \gamma, \alpha, \theta).$$

Orthogonality sometimes equated with semiparametric efficient influence functions.

$\psi(W, \gamma, \alpha, \theta)$  is nonparametric efficient influence function for  $E_F[g(W, \gamma(F), \theta)]$ ;

This moment function accounts fully for behavior of first step through  $\gamma(F)$ ;

Orthogonality characterization here useful because  $g(W, \gamma, \theta)$  and  $\phi(W, \gamma, \alpha, \theta)$  have distinct roles as identifying moment functions and bias correction, leading to Property II:  $E[\phi(W, \gamma_0, \alpha, \theta)] = 0$  for all  $\alpha, \theta$ .

Orthogonality property here is model free and robust to misspecification.

Standard errors robust to misspecification for  $g(W, \gamma, \theta)$  with same dimension as  $\theta$ .

$$\psi(W, \gamma, \alpha, \theta) = g(W, \gamma, \theta) + \phi(W, \gamma, \alpha, \theta).$$

Many formulae for  $\phi(W, \gamma, \alpha, \theta)$  are available.

Newey (1994) gives formulae when  $\gamma$  is

- conditional means and other nonparametric least squares projections.
- conditional pdfs.

Ichimura and Newey (2022, "The Influence Function of Semiparametric Estimators", Quantitative Economics) gives formulae when  $\gamma$  is

- conditional quantile;
- conditional expectile;
- other conditional location and scale measures;
- projection versions of conditional location and scale;
- solutions to conditional moment restrictions, i.e. for nonparametric IV.

# CROSS-FITTING

Combine orthogonal moment functions with cross-fitting, a form of sample splitting, to construct debiased sample moment functions; e.g. see Bickel (1982), Schick (1986), Klaassen (1987), and Chernozhukov et al. (2018).

Partition the observation indices ( $i = 1, \dots, n$ ) into  $L$  groups  $I_\ell$ , ( $\ell = 1, \dots, L$ ).

Let  $\hat{\gamma}_\ell$ ,  $\hat{\alpha}_\ell$ , and  $\tilde{\theta}_\ell$  be estimators that are constructed using all observations *not* in  $I_\ell$ . Debiased sample moment functions are  $\hat{\psi}(\theta) = \hat{g}(\theta) + \hat{\phi}$ ,

$$\hat{g}(\theta) := \frac{1}{n} \sum_{\ell=1}^L \sum_{i \in I_\ell} g(W_i, \hat{\gamma}_\ell, \theta), \quad \hat{\phi} := \frac{1}{n} \sum_{\ell=1}^L \sum_{i \in I_\ell} \phi(W_i, \hat{\gamma}_\ell, \hat{\alpha}_\ell, \tilde{\theta}_\ell).$$

$L = 5$  works well based on a variety of empirical examples and in simulations, for medium sized data sets; see Chernozhukov et al. (2018);

$L = 10$  works well for small data sets; larger  $L$  is better but  $L = 10$  get most of gain; Velez (2024).

This cross-fitting a) eliminates "own observation" bias, like jackknife instrumental variables; b) helps remainders converge faster to zero, e.g. Newey and Robins (2017); c) eliminates need for Donsker conditions not satisfied by many machine learners

Debiased GMM is

$$\hat{\theta} = \arg \min_{\theta \in \Theta} \hat{\psi}(\theta)' \hat{\Upsilon} \hat{\psi}(\theta),$$

where  $\hat{\Upsilon}$  is a positive semi-definite weighting

As usual  $\hat{\Upsilon}$  that minimizes the asymptotic variance of  $\hat{\theta}$  is  $\hat{\Upsilon} = \hat{\Psi}^{-1}$ , for

$$\hat{\Psi} = \frac{1}{n} \sum_{\ell=1}^L \sum_{i \in I_\ell} \hat{\psi}_{i\ell} \hat{\psi}'_{i\ell}, \quad \hat{\psi}_{i\ell} = g(W_i, \hat{\gamma}_\ell, \tilde{\theta}_\ell) + \phi(W_i, \hat{\gamma}_\ell, \hat{\alpha}_\ell, \tilde{\theta}_\ell).$$

No need to account for the presence of  $\hat{\gamma}_\ell$  and  $\hat{\alpha}_\ell$  in  $\hat{\psi}_{i\ell}$  because of orthogonality.  
See paper for initial estimator  $\tilde{\theta}_\ell$ .

An estimator  $\hat{V}$  of the asymptotic variance of  $\sqrt{n}(\hat{\theta} - \theta_0)$  is

$$\hat{V} = (\hat{G}' \hat{\Upsilon} \hat{G})^{-1} \hat{G}' \hat{\Upsilon} \hat{\Psi} \hat{\Upsilon} \hat{G} (\hat{G}' \hat{\Upsilon} \hat{G})^{-1}, \quad \hat{G} = \frac{\partial \hat{g}(\hat{\theta})}{\partial \theta}.$$

# AUTOMATIC ESTIMATION of $\alpha_0$

Need  $\hat{\alpha}_\ell$  with  $\text{plim } \alpha_0$  for debiased GMM.

Can use Property I, orthogonality of  $\psi(w, \gamma, \alpha_0, \theta)$  with respect to  $\gamma$ , to construct estimators of  $\alpha_0$  without knowing the form of  $\alpha_0$ .

Chernozhukov, Newey, and Singh (2022) give AutoDML for Lasso regression.

Chernozhukov, Newey, Quintas-Martinez, Syrgkanis (2021, "Automatic Debiased Machine Learning via Neural Nets for Generalized Linear Regression," developed AutoDML for NN and other first steps.

Based on the observation that the orthogonality in Property I can sometimes be interpreted as first order conditions for minimizing an objective function.

An important case of this is where  $\gamma_F(X) = E[Y|X]$ .

In this case

$$\phi(W, \gamma, \alpha, \theta) = \alpha(X, \theta)\{Y - \gamma(X)\}.$$

Can use orthogonality to get objective function to which NN can be applied for estimating  $\alpha_0$ .

Example 1: Orthogonal moment function is

$$\psi(W, \gamma, \alpha, \theta) = Z\gamma(X) - \theta + \alpha(X)\{Y - \gamma(X)\}.$$

Property I is that for every function  $\gamma(X)$  and  $\Delta(X) = \gamma(X) - \gamma_0(X)$  of  $X$ ,

$$0 = \frac{dE[\psi(W, \gamma_0 + \delta\Delta, \alpha_0, \theta)]}{d\delta} = E[\{Z - \alpha_0(X)\}\Delta(X)] = 0.$$

This is the first order condition for  $\alpha_0$  given by

$$\begin{aligned}\alpha_0 &= \arg \min_{\alpha} E[-2Z\alpha(X) + \alpha(X)^2] = \arg \min_{\alpha} E[-2g(W, \alpha, \theta) + \alpha(X)^2] \\ &= \arg \min_{\alpha} E[\{Z - \alpha(X)\}^2] = E[Z|X].\end{aligned}$$

Here Neyman orthogonality implies that  $\alpha_0(X)$  minimizes the objective function  $E[-2g(W, \alpha, \theta) + \alpha(X)^2]$  that only depends on the identifying moment function.

Can then use NN to estimate  $\alpha_0(X)$  by minimizing a sample version of this objective function over a neural net choice of  $\alpha$ ; here this is neural net regression of  $Z$  on space of neural nets.

Can do this in general when identifying moment  $g(W, \gamma, \theta)$  is smooth in  $\gamma$  and first step is a conditional mean.

Suppose that there is  $D(W, \Delta; \gamma, \theta)$  that is linear in  $\Delta$  such that for any  $\Delta(X)$ ,

$$\frac{dg(W, \gamma + \delta\Delta, \theta)}{d\delta} = D(W, \Delta; \gamma, \theta).$$

Then Property I of Neyman orthogonality is that for all  $\Delta$ ,

$$\begin{aligned} 0 &= \frac{dE[\psi(W, \gamma_0 + \delta\Delta, \alpha_0, \theta_0)]}{d\delta} = \frac{E[dg(W, \gamma_0 + \delta\Delta, \theta_0)]}{d\delta} \\ &\quad + \frac{E[d\alpha_0(X)\{Y - \gamma_0(X) - \delta\Delta(X)\}]}{d\delta} \\ &= E[D(W, \Delta; \gamma_0, \theta_0) - \alpha_0(X)\Delta(X)]. \end{aligned}$$

This is the first order condition for

$$\alpha_0 = \arg \min_{\alpha} E[-2D(W, \alpha; \gamma_0, \theta_0) + \alpha(X)^2].$$

An NN estimator  $\hat{\alpha}(X)$  of  $\alpha_0(X)$  can then be formed from minimizing a sample analog of this objective function

$$\hat{\alpha} = \arg \min_{\alpha \in \mathcal{A}} \frac{1}{n} \sum_{i=1}^n \{-2D(W_i, \alpha; \hat{\gamma}, \hat{\theta}) + \alpha(X_i)^2\},$$

where  $\mathcal{A}$  is set of NN.

## EXAMPLE 2: DYNAMIC BINARY CHOICE

Estimate structural parameters via learners of conditional choice probabilities.

Use Hotz and Miller approach (1993) that replaces computation of expected value functions with nonparametric estimation of choice probabilities.

Individuals choose between two alternatives  $j = 1$  and  $j = 2$  to maximize the expected present discounted value of per period utility  $U_{tj} = D_j(X_t)' \theta_0 + \varepsilon_{tj}$ ,  $(j = 1, 2; t = 1, \dots, T)$ , where  $\varepsilon_{jt}$  is i.i.d. with known CDF independent of the entire history  $\{X_s\}_{s=1}^{\infty}$  of a state variable vector  $X$ , and  $X_t$  is Markov of order 1 and stationary.

The parameter vector of interest is  $\theta_0$ .

Assume that choice 1 is a renewal choice where the conditional distribution of  $X_{t+1}$  given  $X_t$  and choice 1 does not depend on  $X_t$  and  $D_1(X_t) = (-1, 0')'$  and first element of  $D_2(X_t) = 0$ .

Three first steps:

$$\begin{aligned}\gamma_{10}(X_t) &= \Pr(Y_{2t} = 1|X_t), \quad \gamma_{20}(X_t) = E[H(\gamma_{10}(X_{t+1}))|X_t, Y_{2t} = 1], \\ \gamma_{30} &= E[H(\gamma_{10}(X_{t+1}))|Y_{1t} = 1]\end{aligned}$$

Here  $H(\gamma_1)$  is the known function from Hotz and Miller (1993) such that for the expected value function

$$E[V(X_{t+1})|X_t, Y_{2t} = 1] - E[V(X_{t+1})|Y_{1t} = 1] = \gamma_{20}(X_t) - \gamma_{30}.$$

Then for the CDF  $\Lambda(a)$  of  $\varepsilon_{t1} - \varepsilon_{t2}$ ,  $D(X_t) = D_2(X_t) - D_1(X_t)$ , and  $\delta$  the discount factor (assumed known) the conditional choice probability for  $j = 2$  is

$$\begin{aligned}\Pr(Y_{2t} = 1|X_t) &= \Lambda(a(X_t, \theta_0, \gamma_{20}, \gamma_{30})), \\ a(x, \theta, \gamma_2, \gamma_3) &= D(x)' \theta + \delta\{\gamma_2(x) - \gamma_3\}.\end{aligned}$$

Data are i.i.d. observations on individuals each followed for  $T$  time periods, that also includes the  $T + 1$  observation  $X_{T+1}$  of the state variables, where  $W = (X'_1, Y_{21}, \dots, X'_T, Y_{2T}, X'_{T+1})'$ .

First step  $\hat{\gamma}_1(x)$  choice probability can be NN or anything with fast enough mean square convergence rate.

First step  $\hat{\gamma}_2(x)$  obtained from NN or other regression of  $H(\hat{\gamma}_1(X_{it+1}))$  on functions of  $X_{it}$ , for  $Y_{i2t} = 1$ .

For relative simplicity take identifying moment functions to be IV first order conditions for quasi maximum likelihood, where

$$g(W, \gamma, \theta) = \frac{1}{T} \sum_{t=1}^T Z(X_t)[Y_{2t} - \Lambda(a(X_t, \theta, \gamma_2, \gamma_3))].$$

For each element  $g_j(W, \gamma, \theta)$  of  $g(W, \gamma, \theta)$  there are three bias correction terms  $\phi_{1j}, \phi_{2j}, \phi_{3j}$  corresponding to  $\hat{\gamma}_1, \hat{\gamma}_2$ , and  $\hat{\gamma}_3$ .

Each term is obtained treating the other  $\gamma$  functions as equal to the true ones.

This works by chain rule argument; bias correction term is sum of terms for each first step in the nested sequence; terms are derived in LR.

Have for each  $g_j(W, \gamma, \theta)$ ,

$$\phi_{1j}(W, \gamma_1, \gamma_2, \gamma_3, \alpha_{1j}, \theta) = \frac{1}{T} \sum_{t=1}^T \alpha_{1j}(X_t) \{Y_{2t} - \gamma_1(X_t)\},$$

$$\phi_{2j}(W, \gamma_1, \gamma_2, \gamma_3, \alpha_{2j}, \theta) = \frac{1}{T} \sum_{t=1}^T \alpha_{2j}(X_t) Y_{2t} \{H(\gamma_1(X_{t+1})) - \gamma_2(X_t)\}$$

$\phi_3$  given in LR paper.

Describe here NN estimator of  $\alpha_{1j0}(X_t)$  from H. Nguyen, student of John Rust.

$$\begin{aligned} D_{1j}(W, \alpha; \hat{\gamma}, \hat{\theta}) &= \frac{dg_j(W, \hat{\gamma}_1 + \delta\alpha; \hat{\gamma}_2, \hat{\gamma}_3, \hat{\theta})}{d\delta} \\ &= \frac{-\delta}{T} \sum_{t=1}^T Z_j(X_t) \Lambda_a(a(X_t, \hat{\theta}, \hat{\gamma}_2, \hat{\gamma}_3)) [\hat{\gamma}_{21}(\alpha, X_t) - \hat{\gamma}_{31}(\alpha)], \\ \hat{\gamma}_{21}(\alpha, X_t) &= \hat{E}[H'(\hat{\gamma}_1(X_{t+1})) \alpha(X_{t+1}) | X_t, Y_{2t} = 1], \\ \hat{\gamma}_{31}(\alpha, X_t) &= \hat{E}[H'(\hat{\gamma}_1(X_{t+1})) \alpha(X_{t+1}) | Y_{2t} = 1]. \end{aligned}$$

A NN estimator of  $\alpha_{10}$  is given by

$$\hat{\alpha}_{1j} = \arg \min_{\alpha \in \mathcal{A}} \frac{1}{n} \sum_{i=1}^n \left\{ -2D_{1j}(W_i, \alpha; \hat{\gamma}, \hat{\theta}) + \frac{1}{T} \sum_{t=1}^T \alpha(X_{it})^2 \right\}$$

# MONTE CARLO

Update to LR Monte Carlo that corrects computer code and design mistakes; code mistake found by H. Nguyen.

Also add covariates here.

Here give results for bias correction using explicit formulas in LR.

Those using automatic NN estimator are similar, perhaps because choice probabilities are not near zero or one very often in the design.

To highlight the bias correction we report results for Lasso penalty equal five times the cross-validated value.

There is not much bias in the plug-in estimates when cross-validated penalty is used.

	PI Bias	DB Bias	PI SE	DB SE	PI SD	DB SD	PI Cvg	DB Cvg
θ_1 (n=100)	0.143	0.028	0.244	0.191	0.224	0.230	0.928	0.894
θ_2 (n=100)	0.105	0.010	0.129	0.090	0.103	0.106	0.936	0.892
θ_1 (n=300)	0.039	0.008	0.142	0.120	0.132	0.136	0.958	0.900
θ_2 (n=300)	0.029	0.004	0.075	0.055	0.059	0.063	0.980	0.908
θ_1 (n=1000)	0.005	0.006	0.077	0.070	0.072	0.074	0.956	0.938
θ_2 (n=1000)	0.006	0.002	0.041	0.032	0.033	0.034	0.984	0.942
θ_1 (n=10000)	0.004	0.003	0.024	0.023	0.022	0.023	0.966	0.940
θ_2 (n=10000)	0.002	0.001	0.013	0.011	0.010	0.010	0.994	0.960

## SUMMARY

Described here how to do automatic debiased machine learning for estimators with nonlinear moment functions that depend on NN and other estimators of conditional expectations.

Regularity conditions in Chernozhukov, Newey, Quintas-Martinez, and Syrgkanis (2021) cited above.

These methods can be used for automatic debiased machine learning of a variety of structural dynamic models.