Targeted Incentives for Charter Schools to Expand Capacity: a Dynamic Analysis

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School Choice and Charter School in the U.S.

- Goal of school choice: improve education quality and access
  - $\uparrow$ alternatives to the assigned public school
  - $\uparrow$ quality via competition in the education market
- A common form of school choice in the U.S. is charter schools
  - Publicly funded (tuition-free) and privately run
  - $3\% \rightarrow 6\%$ nationwide in a decade
  - Some are run by a for-profit management company

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Incentivize Capacity Expansion of Charter Schools

- Many charters, esp. high-quality ones, are capacity-constrained
  - Charters in lottery-based studies (Abdulkadiroglu al. 2011; Angrist et al. 2013; Angrist et al. 2016)
  - In Florida, 61% are oversubscribed in 2012
- Capacity constraints limit access and quality provision
  - Some charters might lack the incentive to improve performance
  - Traditional schools lack the incentive to compete with neighboring charters
- Since 2011, several states (e.g., FL, MA, LA, MO) started to **incentivize** “high-performing" charters with **eligibility to expand**

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Policy Context and Research Questions

New Scheme: *2011 Florida High-performing Charter school Statute*

- Target charters of high performance level

- Allow expansion without asking for permission

1. How do schools respond to the policy by adjusting capacity and performance?
   - High-performing charters' provision of seats
   - Traditional public schools' (TPS) enrollment and performance
2. What are the key mechanisms of the existing policy in influencing school performance, and how important are they?
3. What would happen to school performance and access to students if the policy targets charters with high **value-added** instead of **level**?

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This Paper

- **Descriptive and Causal Evidence**
  - Adjustment: HP charters $\uparrow$ classrooms
  - Reallocation: TPS $\downarrow$ enrollment, if more HP neighbors
  - Competition: TPSs test score $\uparrow$ 0.8%σ, if $\uparrow$ 1 HP neighbor
  - Target whom: charters in high SES regions
- **New Empirical Model**
  - Schools adjust capacity and performance dynamically
  - Schools react to dynamic competitive pressure
- **New Insights from Counterfactuals**
  - Existing v.s. Targeting value-added
    - Higher charter and TPS mean performance and charter seats
    - More equal access to high quality education

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Contribution and Literature Review

1. **School Choice and Charter Expansion in the U.S.** Friedman ('55), Hoxby ('03), Abdulkadiroğlu et al. ('11), Epple et al. ('17), Gilraine et al. ('21), Cohodes and Parham ('21), Campos and Kearn ('23)
   - **First to evaluate capacity regulation**
   - **Eligibility to expand as a new source of competitive pressure**
2. **Identification of Causal Effects of School Choice Expansion**: Hoxby ('03), Imberman ('11), Figlio and Hart ('14)
   - **Identifies causal effects of competitive spillover of new policy**
3. **Industrial Organization of Education Industry**: Neilson ('13), Ferreyra and Kosenok ('18), Singleton ('19), Allende ('19), Dinerstein and Smith ('21), Bodéré ('22), Sanchez ('23), Crema ('24)
   - **First to model the school dynamic adjustment and responses to competitive pressure in K–12 setting**
4. **Application of Oblivious-type of Equilibrium in Dynamic Game**: Weintraub et al. ('08), Ifrach and Weintraub('17), Jeon ('22), Caoui ('23)
   - **Application of Moment-based Markov Equilibrium**

Data Sources and Sample

1. NCES, and, FL-DOE and ACS 5-Year
   - Charter and TPS by school-year
   - Enrollment and operation status
   - Comprehensive performance score
   - Census tract demographics
   - Matched-cohort test scores (by grade-subject)
2. FL-DOE (Office of IEPC) and Audit General
   - Charter by school-year
   - Counts of the classroom for instruction
   - HP designation status
   - Expenditure by categories
3. FL-DOE (BAR)
   - Teacher by school-year
   - Teacher (matchable to schools) value-added score Details

Sample: regular charter and TPSs running any grade in K-8, 2007∼19

- School-year obs: TPS 29 k, charter 4.4 k

The Policy

- Designation criteria of “high-performing" (HP)
  - Better than “2A1B" in last 3 years
  - “A" : high students’ achievements in tests. Base more on levels
  - De-designation possible but rarely seen
- HP charter schools: expansion autonomy
  - Eligible to expand if enrollment capacity $\leq$ facility limit
  - Expansion requests can not be vetoed by district
- The HP charter schools Summary
  - account for 20% (2012) $\rightarrow$ 40% (2019) of all charters
  - higher performance score
  - higher capacity and enrollment
  - higher income, more educated neighborhood
  - enroll more proportion of less-disadvantaged students

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Adjustment Cost: HP Charters Changes Expansion Behaviors

- How does the charter sector react to the policy in terms of capacity?

$$Y_{it} = \beta HP_{it} + FE_i + FE_t + \epsilon_{it}.$$

  - Sample: charter schools, 2007-19

Table: Correlation of School Size and Designation

VARIABLES	(1) #Classrooms	(2) log(enroll)	(3) #Grades
HP	1.841***	0.090***	0.029
	(0.559)	(0.027)	(0.126)
Average of Dependent Var.	23.76	5.58	5.87

Heterogeneity

Reallocation: Less TPS Enrollment if More HP Charter Neighbors

- How does HP charters influence neighboring TPSs' enrollment?
  - Sample: TPS, 2007-19
  - $ExposureHP_i$: # HP charters in 5 miles of i in 2012

$$Y_{it} = \beta Post_t \times ExposureHP_i + \alpha ChartersNearby_{it} + FE_i + FE_t + \epsilon_{it}$$

Table: Effects on Log Enrollment of Exposure to HP Charter Schools

	Outcome: log(enroll)			
	(1)	(2)	(3)	(4)
#HP Charter in 5 miles X After 2011	-0.026***		-0.026***	
	(0.002)		(0.002)	
#Charters in 5 Miles		-0.003***	-0.000	-0.000
		(0.001)	(0.001)	(0.001)
#HP Charter in 3 miles X After 2011				-0.024***
				(0.003)
#HP Charter in 3-5 miles X After 2011				-0.027***
				(0.002)
Average of Dependent Var.	6.50			

Identify Competitive Effects: Challenge and Solution

- Given the enrollment loss pressure, what are TPSs' responses in test scores facing charters' increasing eligibility to expand?
- Hypothesis: Post-policy, TPSs with more # charter neighbors that became HP in 2012 faced more pressure
- Challenge: composition of students might change. No stud. level data.

Identify Competitive Effects: Challenge and Solution

- Solution: DiD and control for the **matched-cohort**'s lag scores

$$\underbrace{A_{igkt}}_{\substack{\text{Cohort}(i,g,t)\\ \text{test score}}} = \beta Post_t \times Treat_i + \alpha Post_t + \eta Treat_i + \rho \underbrace{A_{igkt}^{Last}}_{\substack{\text{Same}(i,g,t)\\ \text{t-1 test score}}} + \gamma \mathbf{Z}_{igkt} + \epsilon_{igkt}$$

- school i , grade g, year t , subject k
- Cohort is a tripple (i, g, t)

- Sample: all TPSs 2007-2014 with at least one charter in 2011, 4-8 grades
- Measurements
  - $Treat_i$: # of charter in 5 miles that become HP in 2012
  - $Post_t$: post-policy dummy
  - $\mathbf{Z}_{igkt}$: fixed effects (gt, ig, kt, kg), cohort match rate, charter presence, P-T ratio, etc.

Competition: TPS ↑ Scores When More HP Threats

- TPS ↑ 0.8-1.5% σ test scores if adding one HP charter in 5 mi. Robustness
- Comparable and larger magnitude
  - 0.21% σ (Figlio and Hart 2014)
  - 0.36%∼0.98% σ (Figlio et al. 2021)

Table: TPSs' Responses in Test Score to HP Threat

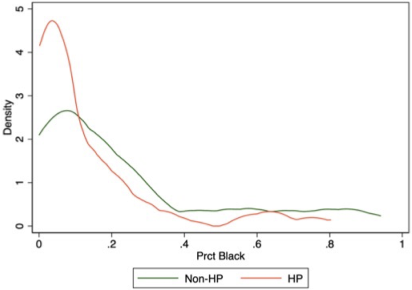
	Outcome: Normalized Average Test Score			
	(1)	(2)	(3)	(4)
#HP Charter in 5 miles X After 2011	0.015***	0.008***	0.008***	
	(0.002)	(0.002)	(0.002)	
#HP Charter in 3 miles X After 2011				0.014***
				(0.003)
#HP Charter in 3-5 miles X After 2011				0.004
				(0.003)
Charter Presence + School Demographics	N	Y	Y	Y
PT Ratio	N	N	Y	Y

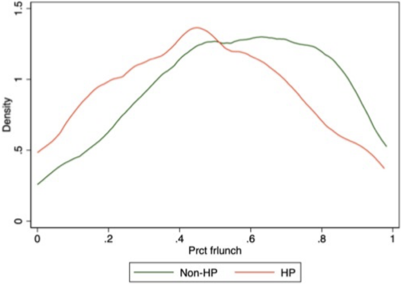
Target Whom? Charter Schools Serving High SES Regions

- What charters are “targeted"? i.e., which charters are easier to get the designation?
    - Education: targeted subsidies affect quality (Neilson 2021)
    - Other industry: “performance-based" criteria (e.g., Barwick et al.,’24)
- Answer using the distribution of the ratio of under-severed students among high value-added charter schools

Target Whom? Charter Schools Serving High SES Regions

Figure: Under-served Student Ratio Across Higher VA Charters in 2015





- L: % of black, R: % of free lunch
- Among high value-added (>2015 median) charters, non-HPs serve higher prct. underserved students
- Patterns robust across years and choices of value-added cutoff

Taking Stock

- Key takeaways
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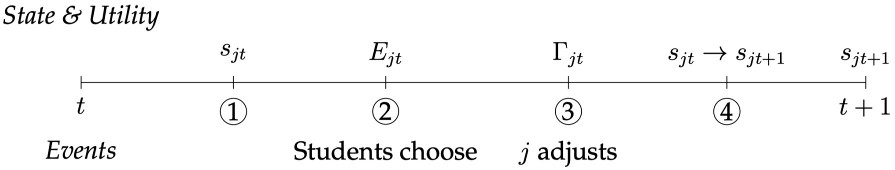
- Key mechanisms $\longrightarrow \longrightarrow \longrightarrow$ **Model Components**
  - Incentive for adjustments $\rightarrow$ **Forward-looking + adjustment**
  - Competitive responses $\rightarrow$ **Demand**

Market Environment and Event Flow

- A schooling market evolves in discrete time $t = 1, 2, 3, ...$
- A school : j, $j = 0, 1, 2, ..., J$
- No entry, exit, or change in ownership
- Schools are heterogeneous w.r.t. their own state x_{jt}
- Each j in time t, faces the state

$$s_{jt} = (x_{jt}, n_{jt}),$$

  where n_{jt}, the "market situation" j faces , is a function of j and other schools' states
- Event flow:



Model Overview

- Main school heterogeneity: type, performance, capacity, HP

$$x_{jt} = (o_j, q_{jt}, k_{jt}, hp_{jt}, \xi_{jt})$$

- Demand: school choice in BLP (1995),
  - The market is endowed with a set of locations l. $l \in L$
  - A representative student i lives in l. Travel $dist_{jl}$ to j, getting utility w_{ijlt}:

$$w_{ijlt} = \delta(x_{jt}; \alpha) + \lambda dist_{jl} + \zeta_{ijlt}.$$

  - The enrollment, E_{jt}, of j in t is : Details

$$E_{jt} = \exp(\delta(x_{jt}; \alpha)) \cdot n_{jt},$$

- Dynamic supply: investment

$$q_{jt} \xrightarrow{v} q_{jt+1} \qquad k_{jt} \xrightarrow{e} k_{jt+1}$$

$$hp_{jt} \xrightarrow{q_{jt}} hp_{jt+1}$$

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Dynamic Programming Problem of Both Types

- Flow utility of a charter j (assumed to be for-profit)

$$u_{jt} = \underbrace{rE(x_{jt}, n_{jt})}_{\text{Revenue}} - \underbrace{\Psi(E(x_{jt}, n_{jt}), x_{jt})}_{\text{Operating cost}} - \underbrace{\Gamma(v_{jt}, e_{jt}, hp_{jt})}_{\text{Adjustment costs}}.$$

- Charter school's DP

$$V(s_t) = \max_{v_t, e_t} rE(s_t) - \Psi(E(s_t), s_t) - \Gamma(v_t, e_t, s_t) + \beta \mathbb{E} V(s_{t+1}|s_t)$$

$$s.t. \quad q_{t+1} = \tau(v_t, q_t),\ k_{t+1} = k_t + e_t,\ \text{prob}(hp_{t+1}|q_t, hp_t) = \eta(q_t, hp_t),$$

n_{jt}'s transition is $AR(1)$ and it satisfies Consistent Belief

- Traditional public school's DP. They are assumed to have different different objectives , no expansion , and no designation

$$V(s_t) = \max_{v_t} r^E E(s_t) + r^q q_t - \Gamma(v_t, s_t) + \beta \mathbb{E} V(s_{t+1}|s_t)$$

$$s.t. \quad q_{t+1} = \tau(v_t, q_t), k_{t+1} = \bar{k}, hp_{t+1} = 0,$$

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Estimation Overview

- Demand specification: a form similar in Bayer and Timmins (2007)
- Estimation: Bajari, Benkard, and Levin (2007) Details OfflineSpec
- Estimate separately for charter and TPSs with post-policy data
  - Exclude K-2 charter and TPSs
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- Define market as a district (e.g., Miami-Dade School District)
  - Local market size: school-aged population from ACS
  - Outside option: private enrollment

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Demand Estimation: Implementation and Specification

- Utility specification (ξ_{jt} : quality shock for j in t)

$$\begin{aligned} w_{ijlt} &= \delta(s_{jt};\alpha) + \lambda dist_{jl} + \zeta_{ijlt} \\ &= \alpha_1 ClassSize_{jt} + \alpha_2 q_{jt} + \alpha_3 o_j + \xi_{jt} + \lambda dist_{jl} + \zeta_{ijlt}, \end{aligned}$$

  where $\text{ClassSize}_{jt} = (E_{jt}/k_{jt})$

- “Soft" cap. constraint: if $\alpha_1 < 0$, reducing ClassSize_{jt} attracts students
- Demand estimation
  - Instruments: $Z^{BayerTimmins2007}, Z^{BLP1995}, Z^{demo}$, Details
  - Nested fixed point (Berry et al. , 1995)
- With the estimated demand, back out n_{jt}

$$n_{jt} = \sum_{l \in L} m_{lt} \cdot \frac{\exp(\hat{\lambda} dist_{jl})}{1 + \sum_{j' \in J} \exp(\hat{\alpha} x^{\text{demand}}_{j't} + \hat{\lambda} dist_{j'l})},$$

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Demand Estimation: Results

Table: Mean Utility and Distance Taste Estimates

	Class Size (E/k)	Performance Score (q)	Charter (o)	Distance ($dist$)
Coefficient	-0.071	2.782	-0.321	-0.362
	(0.0109)	(0.313)	(0.0500)	(0.0386)

- For a medium-sized charter school having 20 classrooms and 400 students, with a performance score of 0.6 (B grade)
  - Enrollment elasticities : capacity: 0.58, performance: 1.20
  - + 2 classrooms $\rightarrow$ + 23.2 students
  - proceed to A grade $\rightarrow$ + 48 students
- The transition of n_{jt} shows large heterogeneity across markets Offlines

Structural Estimates of $\Gamma(.)$ and Public Finance Implications

$$\Gamma(v_{jt}, e_{jt}, hp_{jt}, \epsilon_{jt}) = \gamma_v v_{jt} + 1_{\{e_{jt}\geq 0\}} \cdot \left(\overbrace{\gamma_1}^{\text{Fixed Costs}} + \overbrace{\gamma_3 \cdot e_{jt} + \underset{\substack{\uparrow \\ \text{HP effect}}}{\gamma_4} \cdot e_{jt} \cdot hp_{jt}}^{\text{Variable Costs}} \right)$$

$$+ 1_{\{e_{jt}<0\}} \gamma_5 \cdot e_{jt}.$$

	Adjustment Cost	
	Charter	TPS
Value-added Cost	8.059	24.080
	0.312	0.214
Mean Fixed Cost of Expansion	-0.103	
	2.022	
Mean Variable Cost of Expansion	4.284	
	0.458	
HP's Effect in Variable Cost	0.817	
	0.227	
Variable Cost of Shrinkage	4.284	
	0.330	

Structural Estimates of $\Gamma(.)$ and Public Finance Implications

$$\Gamma(v_{jt}, e_{jt}, hp_{jt}, \epsilon_{jt}) = \gamma_v v_{jt} + 1_{\{e_{jt} \geq 0\}} \cdot \left(\overbrace{\gamma_1}^{\text{Fixed Costs}} + \overbrace{\gamma_3 \cdot e_{jt} + \underset{\substack{\uparrow \\ \text{HP effect}}}{\gamma_4} \cdot e_{jt} \cdot hp_{jt}}^{\text{Variable Costs}} \right) + 1_{\{e_{jt} < 0\}} \gamma_5 \cdot e_{jt}.$$

- Designation $\downarrow$ expansion cost by 18.8% for the HP
- Cheaper to add classrooms than entry
  - Singleton (2019): entry cost of an average charter of 250 stud. = 1.9 $\times$ (cost of adding 12 classrooms with my estimates for non-HP)
- Costs in perspectives Identification
  - Adding a classroom: \$734 per sqf. for a 900sqf. classroom
  - Adding a unit of value-added: \$0.81 million for charter and \$2.41 million for TPS (Grade C to almost A purely by $\uparrow$ value-added)

Policy Counterfactuals: Motivation

- No-HP v.s. Existing scheme
  - Q: What are the key mechanisms of the existing policy in influencing school performance, and how important are they?
- Existing scheme v.s. Targeting value-added
  - Q: What would happen to school performance and access to students if the policy targets charters with high value-added instead of high level?

Policy Counterfactuals: Implementation

Table: Changes of Primitives of Policy Counterfactuals

	Existing Scheme	"No-HP"	"Target-va"
Γ^{charter}	$\gamma_4 = \hat{\gamma}_4$	$\gamma_4 = 0$	
η	$\text{prob}(hp_{t+1} \mid hp_t, q_t) = \hat{\eta}(hp_t, q_t)$	$hp_t = 0, \forall t$	$hp_{t+1} = 1$ if $v_t \geq \tilde{v}$
ν	$n_{t+1} = \hat{\nu}(n_t)$	Change according to the Consistent Belief Assumption.	

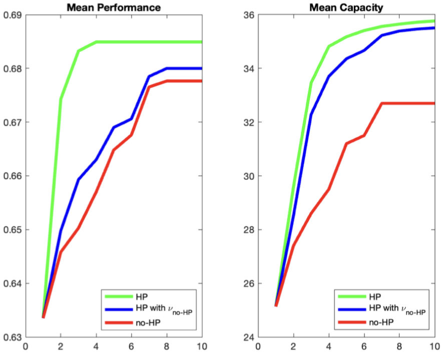
- Current version: Miami-Dade school district, shrinkage = 0, one draw
- Decomposition of channels (e.g., no-HP v.s. HP)

$$Y^{HP} - Y^{noHP} = \underbrace{Y^{HP} - Y^{HP}_{\nu_{\text{no-HP}}}}_{\text{Competition Effect}} + \underbrace{Y^{HP}_{\nu_{\text{no-HP}}} - Y^{noHP}}_{\text{Incentive Effect}}$$

- Consistent Belief: iterate $\nu(.)$ $\rightarrow$ re-solve value function for the $\nu(.)$ $\rightarrow$ forward-simulate $\rightarrow$ update $\nu(.)$ $\rightarrow$ repeat until converge Details

Charter Sector: No-HP v.s. HP

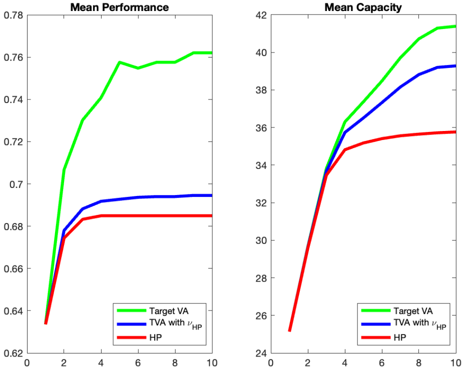
Figure: No-HP v.s. HP: Mean Charter Perf. and Cap. Flow in 10 Years



- $\Upsilon^{HP} - \Upsilon^{noHP}$ for charter
  - Performance: + 10.1%(0.073 points) , 67.8% via competition effect
  - Capacity: + 9.3%(3.0 classrooms), 91.6% via incentive effect

Charter Sector: HP v.s. Target Value-added

Figure: HP v.s. Target VA: Mean Charter Perf. and Cap. Flow in 10 Years



- $\Upsilon^{TVA} - \Upsilon^{HP}$ for charter
  - Performance: +11.2%(0.077 points) , 63.2% via competition effect
  - Capacity: +16% (5 classrooms) , 90.3% via incentive effect

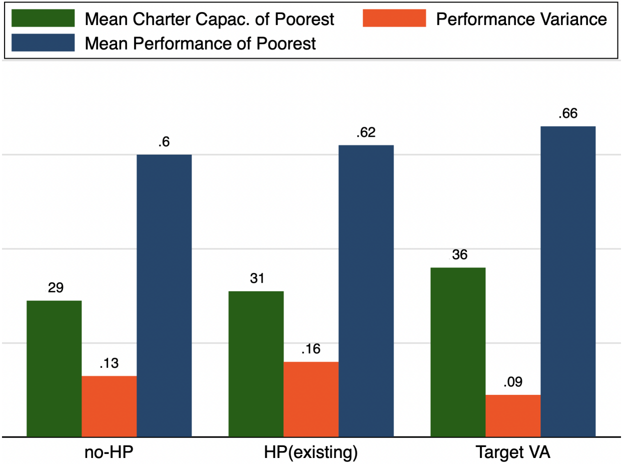
Traditional Sector: No-HP v.s. HP v.s. Target Value-added

Figure: Compare all Schemes: Mean Traditional Sector Perf. at 10^{th} Year

- Target VA has highest mean TPS performance at 10^{th} years

Equity of Access to High-quality Education

Figure: Compare all Schemes: Various Equity Measures at 10^{th} Year



- Target VA improves equity of access to high quality education

Concluding Remarks and Looking Ahead

- Evaluate a novel policy that incentivizes charters with expansion eligibility
  - $\uparrow$ charter capacity and reallocation of students across sectors
  - $\uparrow$ competitive spillover on TPS performance
  - The system targets the advantaged charters
- Estimate a tractable dynamic model
  - adjustments of capacity and performance
  - dynamic competitive responses
- Targeting value-added improves the mean performance of all schools, but also increases the equity of access to high-quality education
- Looking ahead
  - Revisit estimation and add more complete simulations
  - Evaluate allocative efficiency and sorting effects

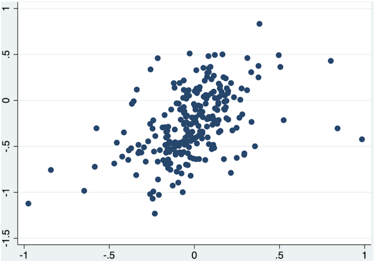
Appendix

Teacher Value-added Scores

- The FL-DOE uses a regression model:

$$y_{ti} = \mathbf{X}_i \boldsymbol{\beta} + y_{t-1,i}\gamma_1 + y_{t-2,i}\gamma_2 + \mathbf{Z}_{1i}\boldsymbol{\theta}_1 + \mathbf{Z}_{2i}\boldsymbol{\theta}_2 + \epsilon_{ti}$$

- i: student, t: year. Regressions are run using all students' scores separately for each year-subject cell
- $\mathbf{Z}_{1i}$: i's school;$\mathbf{Z}_{2i}$: i's teacher
- Raw teacher scores are from these fixed effect estimates, $\hat{\theta}_1, \hat{\theta}_2$
- I validate the mean teacher value-added of a school with the ones estimated using cohort-matched test scores (pooling years)



Difference of HP and Non-HP Charters

Table: Summary Statistics of 2015 Charter Schools by HP Status

	non-HP	HP		non-HP	HP
I. School Characteristics			**III. Location Characteristics**		
Total Performance Score (%)	0.50	0.72	Population Density (1000/square mile)	1.29	1.53
	(0.16)	(0.12)		(0.88)	(1.00)
Enrollment	357.25	560.24	Household Income	62755	68443
	(330.20)	(349.40)		(13625)	(19158)
Number of Classroom	21.88	33.04	Mean Reading Score of TPSs	-0.23	-0.04
	(16.90)	(19.41)		(0.51)	(0.53)
II. Student Composition			Mean Math Score of TPSs	-0.19	0.01
% of Free/Reduced Price Lunch	0.52	0.40		(0.49)	(0.53)
	(0.30)	(0.27)			
			Number of TPSs	24.40	24.60
% of Hispanic	0.32	0.43		(15.39)	(15.44)
	(0.28)	(0.32)			
			IV. Instructional Costs		
% of Black	0.31	0.13	Instructional Cost Per Pupil	4110	3838
	(0.31)	(0.19)		(2373)	(978)
% of Whilte	0.31	0.37			
	(0.28)	(0.30)	Number of Charter Schools	257	119

Return

HP Charters Changes Expansion Behaviors by Heterogeneity

- $LocalCond_{it}$: # TPS, local household income, ratio of black pop. Return

$$Y_{it} = \beta HP_{it} \times LocalCond_{it} + FE_i + FE_t + \epsilon_{it}.$$

Table: Correlation of School Size and Designation

	(1) #Classrooms	(2) log(enroll)	(3) #Grades	(4) #Classrooms	(5) #Classrooms	(6) #Classrooms
HP	1.841***	0.090***	0.029	1.828***	2.692***	1.362*
	(0.559)	(0.027)	(0.126)	(0.666)	(0.901)	(0.699)
#TPSs in 3 miles (normalized)				3.039		
				(3.570)		
HP X #TPSs in 3 miles				1.025**		
				(0.520)		
Locate in Higher Income Pop.					0.780	
					(0.995)	
HP X Locate in Higher Income Pop.					-1.245	
					(1.502)	
Locate in Higher Black Pop.						-0.440
						(2.258)
HP X Locate in Higher Black Pop.						1.240
						(1.266)
Average of Dependent Var.	23.76	5.58	5.87		23.76	

Other Variants of Reallocation Tests

- Regress log(enrollment) of TPS on post-policy×HP exposure, controlling for charter presence and fixed effects Return

$$Y_{it} = \beta Post_t \times ExposureHP_i + \alpha ChartesNearby_{it} + FE_i + FE_t + \epsilon_{it}$$

Table: Effects on Composition of Students of Exposure to HP Charter Schools

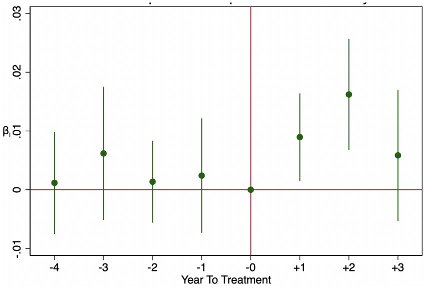
	log(enroll)			Ratio of Student in a School		
	(1) Black	(2) Hispanic	(3) FR Lunch	(4) Black	(5) Hispanic	(6) FR Lunch
#HP Charter in 5 miles X After 2011	-0.031***	-0.066***	-0.019**	0.001	-0.003***	0.006**
	(0.006)	(0.008)	(0.010)	(0.001)	(0.001)	(0.003)
#Charters in 5 Miles	-0.009**	-0.007***	-0.001	-0.000	0.000	0.001
	(0.004)	(0.002)	(0.003)	(0.000)	(0.001)	(0.001)
Average of Dependent Var.	4.60	4.80	5.93	.26	.28	.62

Event Study of TPS Competition Responses

$$A_{igkt} = \sum_{\ell=-4}^{3} \beta_\ell 1_{\ell=t-2011} \times Treat_i + \rho A_{igkt-1} + \sum_{\ell=-4}^{3} \alpha_\ell 1_{\ell=t-2011} + \eta Treat_i + \gamma \mathbf{Z}_{igkt} + \epsilon_{igkt}$$

- ℓ indicates years distant to treatment

Figure: Event Study of TPS Competition Responses



Other Tests on Competitive Spillover

Table: Other Variants of the TPS Competition Response Tests

Outcome: Test Score	By Subject		Alternative Treatment Measure					Sample Selection		
	Read	Math	#HP in 3	Exist in 3	Exist in 5	#A in 3	#A in 5	>80 Match	>90 Match	Full Sample
$Post_t \times Treat_i$	0.0090***	0.0076**	0.0132***	0.0189***	0.0176***	0.0055**	0.0031	0.0082***	0.0088***	0.0097***
	(0.0024)	(0.0033)	(0.0033)	(0.0066)	(0.0062)	(0.0028)	(0.0023)	(0.0024)	(0.0032)	(0.0023)
	(13.6531)	(18.7661)	(13.1629)	(13.1426)	(13.1283)	(13.1501)	(13.1679)	(13.6499)	(21.1103)	(10.1978)
Observations	27,593	27,593	55,304	55,304	55,304	55,304	55,304	52,286	27,599	83,004
R-squared	0.9504	0.9013	0.8973	0.8973	0.8973	0.8973	0.8972	0.8985	0.9097	0.8976
Fixed Effects	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Charter Entry + School Demo	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
PT Ratio	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Return

Spatial Demand: School Enrollment Derivation

- Choice probability of i living in l choosing j :

$$p_{lj} = \frac{\exp(\delta(x_{jt};\alpha) + \lambda dist_{jl})}{1 + \sum_{j' \in J} \exp(\delta(x_{j't};\alpha) + \lambda dist_{j'l})}.$$

- The enrollment of school j in period t, E_{jt}, is :

$$E_{jt} = \sum_{l \in L} m_{lt} \cdot p_{lj} = \sum_{l \in L} m_{lt} \cdot \frac{\exp(\delta(x_{jt};\alpha) + \lambda dist_{jl})}{1 + \sum_{j' \in J} \exp(\delta(x_{j't};\alpha) + \lambda dist_{j'l})}.$$

- Factor out $\exp(\delta(x_{jt};\alpha))$, one gets:

$$\begin{aligned} E_{jt} &= \exp(\delta(x_{jt};\alpha)) \cdot \sum_{l \in L} m_{lt} \cdot \frac{\exp(\lambda dist_{jl})}{1 + \sum_{j' \in J} \exp(\delta(x_{j't};\alpha) + \lambda dist_{j'l})}. \\ &= \exp(\delta(x_{jt};\alpha)) \cdot n_{jt}, \end{aligned}$$

where $n_{jt} = \sum_{l \in L} m_{lt} \cdot \frac{\exp(\lambda dist_{jl})}{1 + \sum_{j' \in J} \exp(\delta(x_{j't};\alpha) + \lambda dist_{j'l})}$.

Return

Literature Related to the Assumptions on n_{jt}

- Literature Return
    - Schools limited horizon (Sanchez 2023; Dinerstein et al. 2022)
    - Oblivious belief (Weintraub et al. 2008)
    - Inclusive value in dynamic demand (Hendel and Nevo 2006; Gowrisankaran and Rysman 2012)
    - Krusell and Smith (1998), Ifrach and Weintraub (2017)

Full Definition of Equilibrium

- An equilibrium of a market is characterized by a strategy $z = (v, e)$
  1. (Optimality) z satisfies the optimality condition. That is, for every state $s \in \mathcal{S}$, for every school, z is the optimal choice given others' choosing z
  $$\sup_{\tilde{z} \in Z} \bar{V}_{\tilde{z},z}(s) = \bar{V}_{z,z}(s).$$
  2. (Consistent Belief) Each school forms a rational expectation that $\nu(.)$ is an AR(1), and its belief is consistent with how the market would evolve when the school itself and its competitors make optimal dynamic decisions given their beliefs $\nu(.)$.
  $$\tilde{\nu}^z(.) = \nu(.),$$
  where $\tilde{\nu}^z(.)$ is the n's transition when all schools use z.

Computation Return

Estimation Procedure Details

- Structural estimation: Bajari, Benkard, and Levin (2007)
  - 1^{st} step: estimate "offline" functions
    - demand, operation cost, expansion policy, value-added policy, transition functions
  - 2^{nd} step: estimate $\hat{\Gamma}$ that satisfies
    - Get $\hat{v}(.), \hat{e}(.)$ from first step. Get $\tilde{v}(.), \tilde{e}(.)$ by perturbing $\hat{v}(.), \hat{e}(.)$
      i: an initial state, j: a perturbed policy

$$\min_{\hat{\Gamma}} \sum_j \sum_i \min\{0, \bar{V}(s_{i0}; \hat{v}(.), \hat{e}(.); \hat{\Gamma}) - \bar{V}(s_{i0}; \tilde{v}_j(.), \tilde{e}_j(.); \hat{\Gamma})\}^2$$

Return

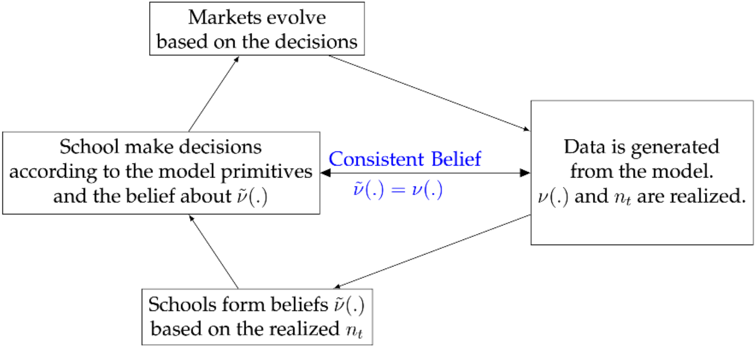
Other Offline functions: Estimation

- Expansion policy functions $e(.)$ of charters
  - Lumpiness: 83 % of observed e_t are zeros
  - (S, s) rule (Attanasio 2000) with flexible functional form
    - Target:
$$k^*_{jt} = h_1(s_{jt}) + u^*_{jt}$$
    - Lower and upper bands:
$$\underline{k}_{jt} = k^*_{jt} - \exp\left(h_2(s_{jt}) + \underline{u}^b_{jt}\right)$$
$$\bar{k}_{jt} = k^*_{jt} + \exp\left(h_2(s_{jt}) + \bar{u}^b_{jt}\right)$$
    - $e_{jt} > 0$ if $k^*_{jt} < \underline{k}_{jt}$
- Value-added $v(.)$ policy: flexible functional form
- Operating cost: flexible functional form
- Transitions: empirical distributions

Return

Estimation and Simulation under Consistent Belief

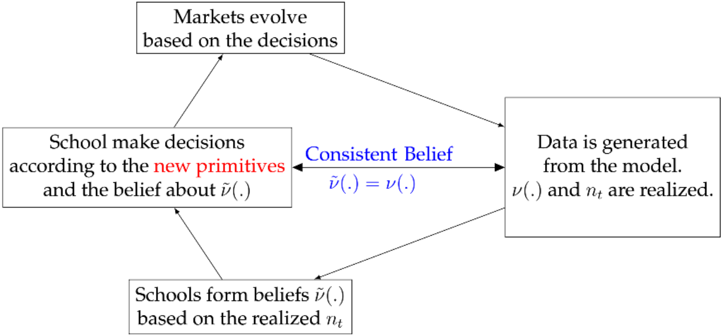
Figure: Process of the Updating of Beliefs about n_t in Estimation



- Data settle at where consistent belief holds $\rightarrow$ $\tilde{\nu}(.)$ approximated from the data is what schools use to make decisions $\rightarrow$ use data to back out the other school primitives under the approximated $\tilde{\nu}(.)$

Estimation and Simulation under Consistent Belief

Figure: Process of the Updating of Beliefs about n_t in Counterfactual



- In simulation, this implies an iterative algorithm forcing the implied $\nu(.)$ generated by schools' decision streams to be close enough to $\tilde{\nu}(.)$ (as similar in Krusell and Smith, 1998)

Computation Framework

- I use a perceived transition of n_t capturing the **short-term transitional dynamics** starting from picked initial states because
  - The goal is to characterize short term (< 15years) effects of a policy change
  - The existing data are perceived as not yet reaching a steady state
- Given this, the belief on the transition of n is made to be aligned with average paths from n_t to n_{t+1} over many short trajectories that start from the picked initial states
  - I modified Ifrach and Weintraub's (2017) algorithm designed for a perceived transition of long-term dynamics
- The initial states are from Miami School District in 2012
- Inspection period $T = 10$ years. Simulate $L = 0$ draws.

Simulation Procedures: Stationary Approach

1. Start from an initial guess $\nu^1(n)$. Solve the implied expected value function $\bar{V}^{(\nu^1)}(s)$. Pick a market whose state is

$$s_0 = \left((o_j, q_{j0}, k_{j0}, hp_{j0}, d_{j0}, \xi_{j0},)_{j=1}^{J}, m_0, n_0 \right)$$

2. Simulate paths for horizon T of interest, starting from s_0 for L times under the belief $\nu^i(n)$, i.e., the i's iterate of n's transition
   - Draw unobserved heterogeneity L times
   - Solve for $z^{(\nu_1)}(s)$ by value function iteration for each draw
   - Simulate using $z^{(\nu_1)}(s)$ for each j for each draw and get L paths of n using inclusive value formula: $\left\{ \hat{n}_t^l : t = 0, \ldots, T, l = 1, \ldots, L \right\}$
   - Get $\nu^{i+1}(n)$ by approximating an AR(1) using the average (across simulation) path of $\hat{n}$
3. Repeat until $\nu^{i+1}(n)$ is close enough to $\nu^i(n)$. Get a converged $\nu(n)$
4. Use the model under $\nu(n)$ with initial state s_0 to simulate outcomes. Repeat the above procedure for each picked market ReturnCF

Identification Assumptions for Demand

- x^{demand} : the inputs in δ specification except for class size and ξ.
  - The demand inputs x^{demand} is independent with ξ_{jt} because I assume ξ_{jt} exogenously evolve as an AR(1), as in Sweeting (2013)
- Z^{BT}: predicted class size using Bayer and Timmins (2007)
  - Z^{BT}: predicted enrollment $\hat{E}_{jt}$ divided by k where the construction of $\hat{E}_{jt}$ stems from this model

$$w_{ijlt} = \alpha_2 q_{jt} + \alpha_3 o_j + \lambda dist_{jl} + \zeta_{ijlt}.$$

  - Unique.& exist. of eq.: Bayer and Timmins (2006)
- $Z^{BLP1995}$: number of charter and TPSs within 5 miles and 5-10 miles, and the total capacity of those schools Return
- Z^{demo}: population density

Measurement

- An example: For the 2013-2014 school year ($t = 2014$)
  - q_t: 2013 accountability score
  - k_t: 2014 # of classrooms
    - For a TPS, it is the all-year largest enrollment / 22
  - hp_t:2014 designation
    - For a TPS, it is zero
  - d_t: 2014 demographics, including proportion of school-aged children and household income
  - n_t,ξ_t: Model implied
  - m: The sum of private and public enrollment in the market in 2014
  - v_t: 2014 mean teacher value-added
  - e_t: k_{2015} - k_{2014}
  - E_t: 2014 # of enrollment
  - Ψ_t: 2014 instructional expenditures

Return

Other Empirical Assumptions

- Same equilibrium is played in every market
- Schools believe the HP scheme (and any other counterfactual incentive schemes) persists forever

Return

Demand: Summary Stat. And Implied Transitions

Table: Summary Statistics of Demand Characteristics

Variables	Mean	Variance	Median
Class Size	18.56	11.20	18.39
Performance Score	0.60	0.12	0.61
Charter	0.15	0.36	0
ξ	0.07	2.63	-0.52
n	421.56	448.47	260.79

Table: Implied Transitions of ξ and n

Transition of ξ	0.923	0.0360	11493
	(0.0029)	(0.0051)	
Transition of n			
Miami-Dade	0.942	134.223	1525
	(.00314)	(9.213)	
Pinellas	0.942	49.274	461
		(7.890)	
Polk	0.942	2.042	256
		(8.096)	

Return

Identification of $\Gamma(.)$

- The HP-related effects
  - Identified by comparing the difference in expansion choices across charters or within that experience a change in their HP status
- γ_v and the HP-related effects (separately)
  - γ_v can be identified by the variation in a school's performance in the following year when its capacity remains
- γ_1 and γ_3 (separately)
  - In paper, assume fixed cost γ_1 and variable cost γ_3 :

$$\gamma_1 \sim N(\gamma_1^{\mu}, (\gamma_1^{\sigma})^2), \gamma_3 \sim N(\gamma_3^{\mu}, (\gamma_3^{\sigma})^2).$$

  - γ_1^{μ}, is identified by the (average) frequency of charters choosing not to expand
  - γ_3^{μ}, is identified by the (average) variation of expansion magnitude across or within schools.
  - The spread coefficients are identified by the unexplained variance in magnitude and frequency of expansion implied by the model

Return