

# Policy Choice in Time Series by Empirical Welfare Maximization

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# Introduction

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- How about in macro/finance? **Statistical treatment (policy) choice with time-series data?**
- E.g.,
  - 1 FOMC's monetary policy choice?
  - 2 Central bank's intervention to exchange markets?
  - 3 Policy choice of relaxing or tightening the Covid containment policy?

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- Assuming treatments are randomized conditional on observables in the data, we propose the *time series empirical welfare maximization* (**T-EWM**): estimate welfare impact of the today's policy choice using the past data and maximize it
- We formulate the decision problem using potential outcome time-series and study theoretical guarantee of the proposal in terms of welfare convergence

## Review: (Static) Statistical Treatment Choice

Assume randomized controlled trial (RCT) or observational data for  $i = 1, \dots, n$  iid individuals

- $X_i \in \mathcal{X}$  - pre-treatment observed covariates (e.g., education, previous earnings)
- $W_i \in \{0, 1\}$  - randomized treatment (possibly conditional on  $X_i$ ) (e.g., job training program)
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- $Y_i \in \mathbb{R}$  - post-treatment observed outcome (e.g., employment, income)
- An assignment rule  $g : \mathcal{X} \rightarrow \{0, 1\}$
- Let  $(Y(1), Y(0))$  be the potential outcomes. Additive (utilitarian) welfare:

$$\mathcal{W}(g) = E[Y(1) \cdot g(X) + Y(0) \cdot (1 - g(X))].$$

## Manski's Empirical Success Rule

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$$\hat{g}_{CES}(x) \equiv 1\{\hat{\tau}(x) \geq 0\},$$

where  $\hat{\tau}(x)$  is the difference-in-means estimator for CATE,

$$\hat{\tau}(x) = \widehat{E}[Y|W=1, X=x] - \widehat{E}[Y|W=0, X=x] \quad (1)$$

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- Manski assesses the performance of the CES rule by bounding the **expected welfare regret**,

$$E_{P^n} \left[ \max_g \mathcal{W}(g) - \mathcal{W}(\hat{g}_{CES}) \right] \leq \frac{C}{\sqrt{n}}.$$

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## Questions and challenges:

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- ② Observations are no longer iid.
- ③ What could be a reasonable welfare objective? How to obtain its empirical analogue?
- ④ Can we obtain welfare regret convergence? Under what condition?  
What rate?

## Baseline Setting

- Suppose a social planner is at the beginning of period  $T$ , wants to perform the policy choice at  $T$ ,  $W_T$ , based on the information available at the beginning of  $T$ .
- The planner has access to time-series observations: for  $t = 0, 1, 2, \dots, T - 1$ .
- $Y_t \in \mathbb{R}$ : outcome, e.g., output, unemployment, stock price, Covid deaths, etc.
- $W_t \in \{0, 1\}$ : (manipulatable) treatment, e.g., high/low target rate, lockdown or not, structural policy shock, etc
- $Z_t \in \mathbb{R}$ : contextual information, e.g., macroeconomic indices other than  $Y_t$ ,

- Denote the sample by  $X_{0:T-1} = (Y_{0:T-1}, W_{0:T-1}, Z_{0:T-1})$ , given initial value  $X_0$ .
- A **treatment path**  $w_{0:T} \in \{0, 1\}^{T+1}$ .
- Potential outcomes indexed by the treatment paths: for each  $t = 1, \dots, T$ ,  $Y_t(w_{0:T})$  denotes the counterfactual outcome at period  $t$  when the treatment path were exogenously set to  $w_{0:T}$ .

## Definition: Potential Outcome Time-Series (Bojinov & Shephard 19 JASA, and Rambachan & Shephard 21)

- ① **Non-anticipating potential outcomes:** For each  $t = 1, \dots, T$ ,

$$Y_t(w_{0:t}, w_{t+1:T}) = Y_t(w_{0:t}, w'_{t+1:T})$$

holds a.s. for any  $w_{0:t}$ , and  $w_{t+1:T} \neq w'_{t+1:T}$ , i.e., the potential outcomes are indexed only by the past and current treatments

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- ② **Non-anticipating treatment paths (sequential unconfoundedness):** For every  $t = 1, \dots, T - 1$  and  $s = t, t + 1, \dots, T$ , and any  $w_{t:T}$ ,

$$W_t \perp Y_s(W_{0:t-1}, w_{t:s}) | X_{0:t-1}$$

In the data,  $W_t$  is randomized once conditioned on the history of observables

# Policies and Welfare

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- **One-period (nonrandomized) policy:**  $g : \mathcal{X}_{0:T-1} \rightarrow \{0, 1\}$
- In this talk, we focus on the **one-period conditional welfare**:

$$\begin{aligned}\mathcal{W}_T(g|X_{0:T-1}) \\ \equiv E [Y_T(W_{0:T-1}, 1)g(X_{0:T-1}) + Y_T(W_{0:T-1}, 0)(1 - g(X_{0:T-1}))|X_{0:T-1}]\end{aligned}$$

- In the paper, we also discuss **the unconditional welfare** in the time-series setting, which differs from that in the cross-sectional setting

# Let's simplify (and generalize later)

## Toy model: one-period Markovian model

Bivariate time-series,  $X_t = (Y_t, W_t) \in \mathbb{R} \times \{0, 1\}$ ,  $t = 0, 1, 2, \dots$ , given  $X_0$ .

- ① **Markovian Exclusion:** For each  $t = 2, \dots, T$ ,

$$Y_t(w_{0:t-2}, w_{t-1}, w_t) = Y_t(w'_{0:t-2}, w_{t-1}, w_t) \equiv Y_t(w_{t-1}, w_t)$$

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- ② **Markovian Exogeneity:** For each  $t = 1, \dots, T - 1$  and  $w_{0:t}$ ,

$$(Y_t(w_{0:t}), W_t) \perp X_{0:t-1}|W_{t-1}, \text{ and } Y_T(w_{T-1}, w_T) \perp X_{0:T-1}|W_{T-1}$$

- Under this simplification, the planner's objective (one-period social welfare) satisfies

$$\mathcal{W}_T(g|X_{0:T-1})$$

$$= E[Y_T(W_{T-1}, 1)g(X_{0:T-1}) + Y_T(W_{T-1}, 0)(1 - g(X_{0:T-1}))|X_{0:T-1}]$$

(∴ Markovian exclusion )

$$= E[Y_T(W_{T-1}, 1)|W_{T-1}]g(X_{0:T-1}) + E[Y_T(W_{T-1}, 0)|W_{T-1}](1 - g(X_{0:T-1}))$$

(∴ Markovian exogeneity )

$$= (E[Y_T(W_{T-1}, 1) - Y_T(W_{T-1}, 0)|W_{T-1}]) \cdot g(X_{0:T-1}) + E[Y_T(W_{T-1}, 0)|W_{T-1}]$$

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$$\begin{aligned}
 & \mathcal{W}_T(g|X_{0:T-1}) \\
 &= E[Y_T(W_{T-1}, 1)g(X_{0:T-1}) + Y_T(W_{T-1}, 0)(1 - g(X_{0:T-1}))|X_{0:T-1}] \\
 &\quad (\because \text{Markovian exclusion}) \\
 &= E[Y_T(W_{T-1}, 1)|W_{T-1}]g(X_{0:T-1}) + E[Y_T(W_{T-1}, 0)|W_{T-1}](1 - g(X_{0:T-1})) \\
 &\quad (\because \text{Markovian exogeneity}) \\
 &= (E[Y_T(W_{T-1}, 1) - Y_T(W_{T-1}, 0)|W_{T-1}]) \cdot g(X_{0:T-1}) + E[Y_T(W_{T-1}, 0)|W_{T-1}]
 \end{aligned}$$

- We can reduce  $g(\cdot)$  to a binary map of  $W_{T-1}$  without loss of the welfare and focus on maximizing

$$\mathcal{W}_T(g|W_{T-1}) \equiv E[Y_T(W_{T-1}, 1)g + Y_T(W_{T-1}, 0)(1 - g)|W_{T-1}]$$

# Empirical Welfare

- Empirical analogue of  $\mathcal{W}_T(g|W_{T-1} = w)$ ? Try a historical simple average.

## Policy rule: Time-Series CES

We obtain  $\hat{g}_{CES}(w)$  by maximizing

$$\widehat{\mathcal{W}}(g|W_{t-1} = w) = \frac{1}{T(w)} \sum_{t: W_{t-1} = w} \left[ \frac{Y_t W_t}{e_t(W_{t-1})} g + \frac{Y_t(1 - W_t)}{1 - e_t(W_{t-1})} (1 - g) \right]$$

in  $g \in \{0, 1\}$ , where  $T(w) = |\{t : W_{t-1} = w\}|$  and  
 $e_t(W_{t-1}) = \Pr(W_t = 1|W_{t-1})$  assumed to satisfy a strict overlap condition

Any concentration regarding  $\widehat{W}(g|W_{t-1} = w)$ ?

- Let  $\{\mathcal{F}_t = \sigma(X_{1:t-1}) : t = 1, \dots, T-1\}$  be the filtration and consider decomposing  $\widehat{W}(g|W_{t-1} = w)$  into sums of innovations and conditional expectations w.r.t  $\{\mathcal{F}_t\}$  (Bojinov & Shephard 19).

$$\widehat{W}(g|W_{t-1} = w) = \frac{1}{T(w)} \sum_{t=1}^T \xi_{t,w}(g) + \bar{W}(g|w)$$

where

$$\bar{W}(g|w) \equiv \frac{1}{T(w)} \sum_{t: W_{t-1}=w} E [Y_t(W_{t-1}, 1)g + Y_t(W_{t-1}, 0)(1-g)|\mathcal{F}_{t-1}],$$

$$\xi_{t,w}(g) \equiv 1\{W_{t-1} = w\} \cdot \left[ \begin{matrix} \frac{Y_t W_t}{e_t(W_{t-1})} g + \frac{Y_t(1-W_t)}{1-e_t(W_{t-1})} (1-g) \\ -E [Y_t(W_{t-1}, 1)g + Y_t(W_{t-1}, 0)(1-g)|\mathcal{F}_{t-1}] \end{matrix} \right]$$

- Under Markovian exogeneity and sequential unconfoundedness:

$$\bar{\mathcal{W}}(g|w) \equiv \frac{1}{T(w)} \sum_{t: W_{t-1}=w} E[Y_t(W_{t-1}, 1)g + Y_t(W_{t-1}, 0)(1-g)|W_{t-1}=w],$$

$$E[\xi_{t,w}(g)|\mathcal{F}_{t-1}] \equiv 1\{W_{t-1} = w\} \cdot \begin{bmatrix} E\left[\frac{Y_t W_t}{e_t(W_{t-1})} g + \frac{Y_t(1-W_t)}{1-e_t(W_{t-1})} (1-g)|W_{t-1}\right] \\ -E[Y_t(W_{t-1}, 1)g + Y_t(W_{t-1}, 0)(1-g)|W_{t-1}] \end{bmatrix} = 0$$

- I.e.,  $\xi_{t,w}(g)$  is a martingale difference sequence (MDS), and the empirical welfare centered at  $\bar{\mathcal{W}}(g|w)$  is an average of MDS w.r.t  $\{\mathcal{F}_t\}$ ,

$$\widehat{\mathcal{W}}(g|w) - \bar{\mathcal{W}}(g|w) = \frac{1}{T(w)} \sum_{t=1}^T \xi_{t,w}(g),$$

## Key assumption: Invariance of welfare ordering

Bridge the period- $T$  welfare regret and the past average welfare regret

### Assumption: Invariance of welfare ordering

Let  $g^* = \arg \max_g \mathcal{W}_T(g|W_{T-1} = w)$ . There exists  $c > 0$  such that for any  $w$  and  $g \in \{0, 1\}$ ,

$$\mathcal{W}_T(g^*|w) - \mathcal{W}_T(g|w) \leq c(\bar{\mathcal{W}}(g^*|w) - \bar{\mathcal{W}}(g|w))$$

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- The welfare ordering of the policies today agrees with the historical average over the periods sharing the conditioning events similar to today's
- Weaker than stationarity of welfare(i.e.,  $\mathcal{W}_t(g|w)$  independent of  $t$ )
- In structural MA(2):  $Y_t = \alpha_t + \beta_t W_t + \gamma_t W_{t-1} + \epsilon_t$ , this assumption holds if

$$sign(\beta_T) = sign\left(\frac{1}{T(w)} \sum_{t: W_{t-1}=w} \beta_t\right)$$

- Putting everything together,

$$\begin{aligned} \mathcal{W}_T(g^*|w) - \mathcal{W}_T(\hat{g}_{CES}|w) &\leq c(\bar{\mathcal{W}}(g^*|w) - \bar{\mathcal{W}}(\hat{g}_{CES}|w)) \\ &\leq 2c \sup_g |\widehat{\mathcal{W}}(g|w) - \bar{\mathcal{W}}(g|w)| \end{aligned}$$

- Hence, the average conditional welfare regret can be bounded as follows:

$$\begin{aligned} E_P[\mathcal{W}_T(g^*|w) - \mathcal{W}_T(\hat{g}_{CES}|w)] \\ \leq 2cE_P \left[ \sup_g \left| \frac{1}{T(w)} \sum_{t=1}^{T-1} \xi_{t,w}(g) \right| \right] \end{aligned}$$

- Imposing additional regularity ( $Y_t$  bounded for all  $t$ ), we can apply Freedman's large deviation inequality for MDS to obtain a finite-sample regret bound uniform in  $w$  and  $P$

$$E_P[\mathcal{W}_T(g^*|w) - \mathcal{W}_T(\hat{g}_{CES}|w)] \leq \frac{C}{\sqrt{T-1}}$$

- **Result:** Under the imposed assumptions, Manski's CES-rule can be generalized to time-series and attain  $\sqrt{T}$ -convergence

## Extension: more covariates?

- Maintaining the one-period Markovian model, consider adding continuous variables to  $X_{t-1}$ :

$$X_{t-1} = (W_{t-1}, Y_{t-1}, Z_{t-1}) \in \{0, 1\} \times \mathbb{R} \times \mathbb{R}^d$$

- The conditional welfare  $\mathcal{W}_T(g|x_{T-1})$  as planner's objective:

$$\begin{aligned}\mathcal{W}_T(g|x_{T-1}) \\ \equiv E [Y_T(W_{T-1}, 1)g + Y_T(W_{T-1}, 0)(1 - g)|X_{T-1} = x_{T-1}]\end{aligned}$$

- Conditioning on the continuous variables, a simple sample analogue is not available

- **Our approach:** analogous to EWM, put a class of policies  $\mathcal{G} \equiv \{g\}$ , and optimize a sample analogue of the **unconditional** welfare over  $\mathcal{G}$
- Define the **unconditional welfare** at period  $T$ :

$$\mathcal{W}_T(g) \equiv E [Y_T(W_{T-1}, 1)g(X_{T-1}) + Y_T(W_{T-1}, 0)(1 - g(X_{T-1}))]$$

## Key assumptions for T-EWM:

- ① **Correct specification** at  $x_{T-1} \in \mathcal{X}_{T-1}$ ,

$$\arg \sup_{g \in \mathcal{G}} \mathcal{W}_T(g) \subset \arg \sup_{g \in \mathcal{G}} \mathcal{W}_T(g|x_{T-1}),$$

A sufficient condition:

$$\{x_{T-1} : E[Y_T(W_{T-1}, 1) - Y_T(W_{T-1}, 0)|x_{T-1}] \geq 0\} \in \mathcal{G}$$

- With additional assumption on the marginal distribution of  $X_{T-1}$ , we can bound the conditional welfare regret by the unconditional welfare regret, i.e.,  $\exists c > 0$  such that  $\forall G \in \mathcal{G}$ ,

$$\sup_{g \in \mathcal{G}} \mathcal{W}_T(g|x_{T-1}) - \mathcal{W}_T(g|x_{T-1}) \leq c(\sup_{g \in \mathcal{G}} \mathcal{W}_T(g) - \mathcal{W}_T(g))$$

## EWM policy and regret bounds

$$\hat{g}^{EWM} \in \arg \max_{g \in \mathcal{G}} \frac{1}{T-1} \sum_{t=1}^{T-1} \left[ \frac{Y_t W_t}{e_t(X_{t-1})} g(X_{t-1}) + \frac{Y_t(1-W_t)}{1-e_t(X_{t-1})} (1-g(X_{t-1})) \right]$$

### Theorem: Time-series EWM regret bounds

Assume the one-period Markovian model with bounded outcome, correct specification of  $\mathcal{G}$ , and a complexity restriction of  $\mathcal{G}$  (i.e.,  $\{\xi_{t,x_{t-1}}(G) : G \in \mathcal{G}\}$  has finite VC dimension  $v$ ), there exists a constant  $C > 0$  such that

$$E_P[\sup_{g \in \mathcal{G}} \mathcal{W}_T(g|x_{T-1}) - \mathcal{W}_T(\hat{g}_{EWM}|x_{T-1})] \leq C \sqrt{\frac{v \log T}{T-1}}$$

holds.

## Extension: longer dependence?

- Introducing  $q$ th-order Markovian structure,  $2 \leq q < \infty$ , is feasible
- $q = \infty$  is more challenging, though common SVAR modeling implies so, e.g.,

$$Y_t = \sum_{h=0}^{\infty} (\theta_h W_{t-h} + \phi_h \epsilon_{t-h})$$

- Approximate  $\infty$ -Markov by finite order  $m$ -th Markov and run T-EWM. The regret bounds depend also on the approximation bias,

$$\begin{aligned} & \mathcal{W}_T(g^* | \mathcal{F}_{T-1}) - \mathcal{W}_T(\hat{g}_{CES} | \mathcal{F}_{T-1}) \\ & \leq 2c \sup_g |\widehat{\mathcal{W}}(g | w_{T-m:T-1}) - \bar{\mathcal{W}}(g | \mathcal{F}_{T-1})| + bias(m), \end{aligned}$$

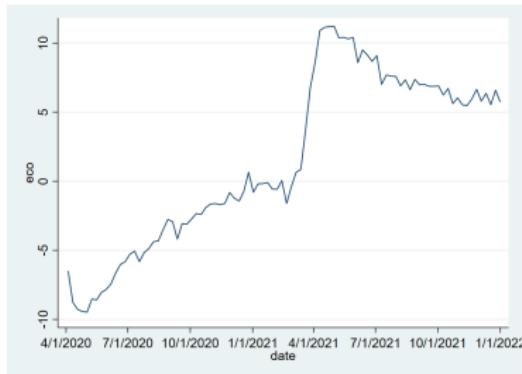
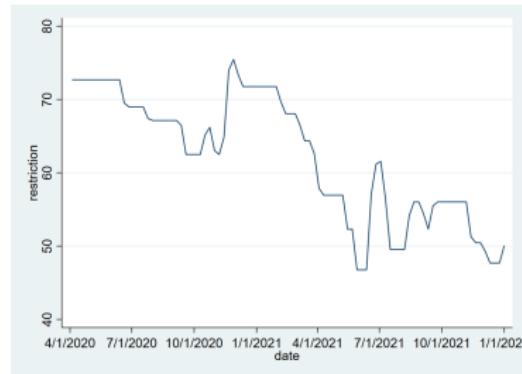
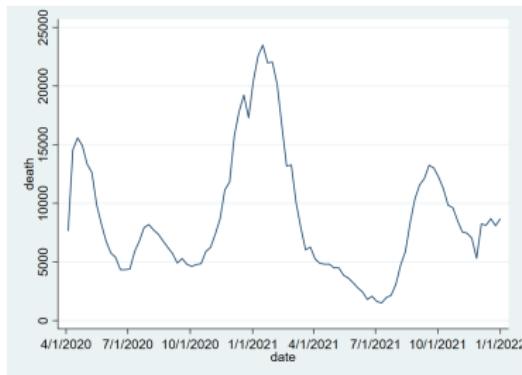
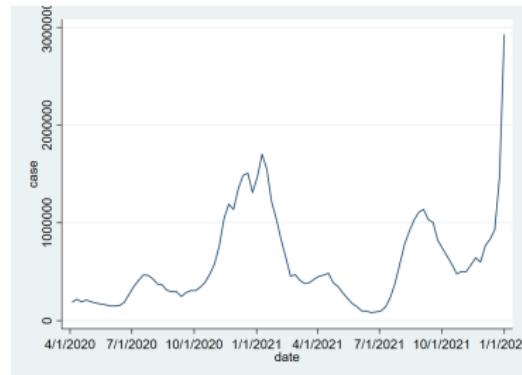
where  $bias(m) = 2 \sup_g |\mathcal{W}_T(g | w_{-\infty:T-1}) - \mathcal{W}_T(g | w_{T-m:T-1})|$ .

## Empirical Example

- Binary treatment: relax the covid restriction level ( $W = 0$ ) or not ( $W = 1$ )
- Outcome (welfare):  $-1 \times$  two-week ahead deaths in US:
- Contextual information for the model of propensity score:  
 $X_t = (\text{cases}_t, \text{deaths}_t, \text{change of cases}_t, \text{change of deaths}_t, \text{restriction level}_t, \text{vaccine coverage}_t, \text{economic condition}_t)$
- We maximize the expected welfare,  $E(-1 \cdot \text{deaths}_{T+1})$ , over the set of quadrant (threshold) policies based on  
( $\text{change of deaths}_{T-1}, \text{restriction level}_{T-1}$ )
- Data source:
  - ▶ Cases, deaths, and vaccinations: The CDC website
  - ▶ Restriction level: The Oxford Stringency Index
  - ▶ Economic condition: The Lewis-Mertens-Stock business conditions index

# Empirical Example

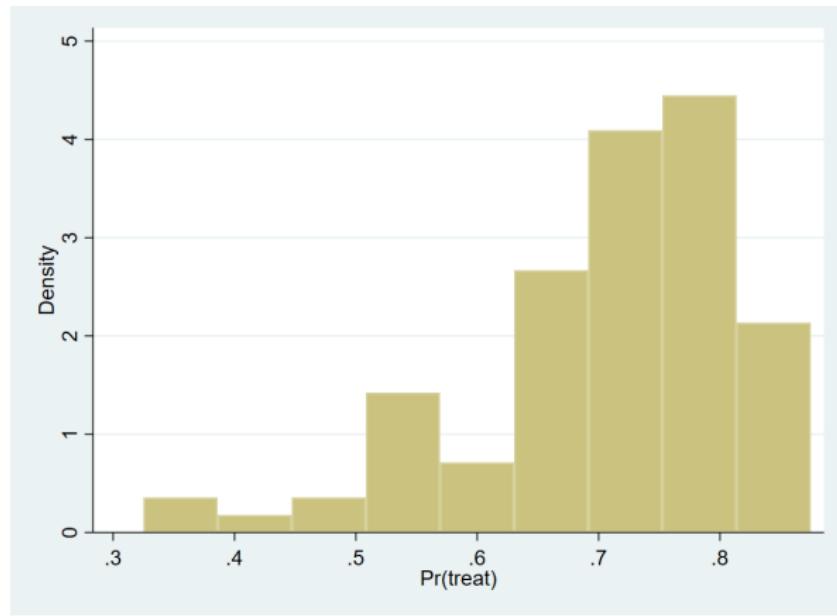
Data overview (from the top left to the bottom right: cases, deaths, restriction level, and economic condition)



## Empirical Example (preliminary)

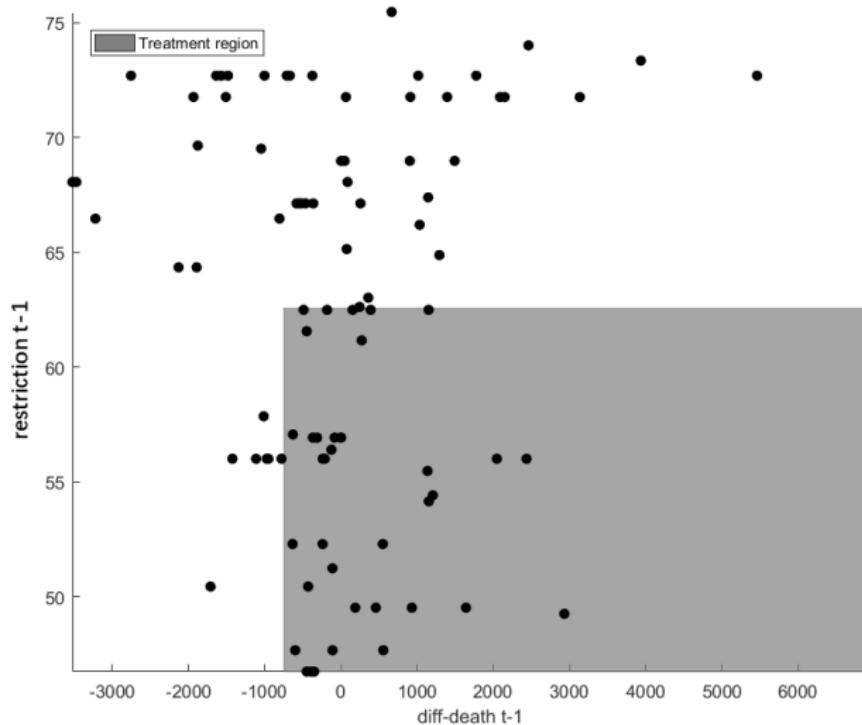
The propensity score model and the estimated propensity score

$$\log \left( \frac{\Pr(\text{keeping or increasing restriction at week } t)}{\Pr(\text{decreasing restriction at week } t)} \right) = \alpha + \beta X_{t-1}$$



# Empirical Example (preliminary)

## Policy choice



# Literature

- **Potential outcome time-series:** Angrist, Jorda, Kuersteiner (18 JBES), Bojinov & Shephard (19 JASA), Rambachan & Shephard (21)
- **Treatment choice:** Manski(04 Ecta), Dehejia (05 JoE), Hirano & Porter (09 Ecta), Stoye (09, 12 JoE), Tetenov (12 JoE), Chamberlain (11 Handbook Chap), Kitagawa & Tetenov (18 Ecta), Mbakop & Tabord-Meehan (21 Ecta), Athey & Wager (21 Ecta), and more
- **(Individualized) Dynamic treatment regimes** (Large  $N$  - short  $T$  panel): Murphy (03 JRSSB), Zhao, Zeng, Laber, Kosorok (15 JASA), Sakaguchi (21), among many others
- **Empirical risk minimization in time-series:** Jiang & Tanner (10 ET), Brownlees & Guðmundsson (21), Brownlees & Llorens-Terrazas (21)

## Concluding Remarks

- Propose a framework for data-driven policy choice in the time-series setting
- Non-trivial challenges distinguishing the time-series policy choice from the static one
- Under sequential unconfoundedness, finite-order Markovian, and invariance of welfare ordering, Manski's CES rule can be extended to time-series
- With the correct specification assumption added, EWM can be extended to time-series as well
- Our approach relies only on the causal and conditional independence structure among the variables, and is free from functional form, stationarity (in the outcome generating process), or prior distribution thereof