Robust Structural Estimation under Misspecified Latent-State Dynamics

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This paper deals with

- Dynamic models contain **serially correlated latent variables**:
  e.g., unobserved product characteristics, supply shocks, utility shocks.

- Estimation and counterfactual analysis rely on assumptions about the **dynamic process** of latent variables.

- We propose a **computationally tractable** framework to **quantify** the **sensitivity** of **counterfactual parameters** to distributional assumptions.
- We apply it to:
  1. **infinite-horizon** dynamic demand for new cars in Europe.
  2. **finite-horizon** dynamic labor supply for New York City taxi drivers.

- We focus on **time-homogeneous** models in this talk.

Toy Example: Optimal Stopping

- At period t, an agent decides whether to stop or wait for the next period.
- The period utility is given by:

$$u(a_t, \xi_t, \varepsilon_t) = \begin{cases} \xi_t + \varepsilon_t(1), & \text{if } a_t = 1 \text{ (stop)} \\ \varepsilon_t(0), & \text{if } a_t = 0 \text{ (wait)} \end{cases}$$

  - $\xi_t \in \Xi$: latent variable, e.g, product characteristics, labor supply shock.
  - ε_t: i.i.d. extreme value type I utility shocks.
- The value function $V(\xi)$ is the solution to the Bellman equation:

$$V(\xi) = \log\left(\exp\left(v_1(\xi)\right) + \exp\left(v_0(\xi)\right)\right)$$

  - $v_1(\xi) = \xi$ is the value of stopping.
  - $v_0(\xi) = \beta \mathbb{E}_{\xi'|\xi}[V(\xi')|\xi]$ is the value of waiting.
  - $\beta \in (0, 1)$: discount factor.

- Misspecified latent-state dynamics biases welfare, i.e., $V(\xi)$.

Examples of Latent Variables

- Search costs in consumer search (Koulayev 2014)
- Patent profitability in optimal stopping (Pakes 1986)
- Quality in technology adoption (De Groote and Verboven 2019)
- Product characteristics in demand estimation (Nair 2007; Schiraldi 2011; Gowrisankaran and Rysman 2012)
- Firm productivity in trade (Piveteau 2021)
- Health shocks in insurance (Fang and Kung 2021)
- Beliefs about ability in labor (Miller 1984; Arcidiacono et al. 2025)

Related Literature

- **Identification and estimation:**
  - **Unobserved heterogeneity:** Kasahara and Shimotsu 2009; Higgins and Jochmans 2023; Luo et al. 2022; Hu and Shum 2012
  - **Finite-dimensional parameters:** Schennach 2014; Fan et al. 2023; Fan et al. 2025; Kalouptsidi et al. 2021a

  This paper: perturbation set around a reference dynamic process.

- **Sensitivity analysis and robustness:**
  - **Local misspecification:** Andrews et al. 2017; Armstrong and Kolesár 2021; Bonhomme and Weidner 2022
  - **Dynamic discrete choice:** Norets and Tang 2014; Bugni and Ura 2019; Kalouptsidi et al. 2021b; Christensen and Connault 2023

  This paper: misspecified latent-state dynamics.

- **Distributionally robust optimization:** Kuhn et al. 2019; Rahimian and Mehrotra 2019; Blanchet et al. 2022; Gao and Kleywegt 2023

Roadmap

1. Framework
2. Practical Implementation
3. Statistical Properties
4. Interpreting the Results
5. Empirical Application

Framework

Framework Overview

- We will define a **perturbation set** around a **reference distribution**.
- The perturbed dynamic process still has a **stationary** distribution.
- We are interested in a **counterfactual parameter**, e.g. elasticity.

- The **lower bound** on the **elasticity**:

$$\begin{aligned} \inf_{\substack{\text{other model} \\ \text{parameters}}} \ \inf_{\text{perturbation set}} \ & \text{elasticity} \\ \text{s.t.} \quad & \text{Moment condition} \\ & \text{Structural constraint, e.g., Bellman equation} \end{aligned}$$

- The main challenge is the optimization over the **perturbation set**.
- The value function in the Bellman equation is **infinite-dimensional**.

Implementation Overview [1/3]

- The framework is a constrained optimization problem.

$$\begin{aligned} \inf_{\substack{\text{other model}\\\text{parameters}}} \quad & \color{red}{\inf_{\text{perturbation set}} \text{elasticity}} \\ \text{s.t.} \quad & \text{Moment condition} \\ & \text{Structural constraint, e.g., Bellman equation} \end{aligned}$$

- The red part is the Distributionally Robust Optimization (DRO):

$$\inf_{\text{perturbation set}} \text{elasticity}$$

whose implementation depends on the choice of the perturbation set.

- In most cases, we will derive the **dual** of the DRO problem because:
  1. It is often **much easier to solve**.
  2. It provides **closed-form** and **smoothness** of the **worst-case distribution**.

Implementation Overview [2/3]

- The framework is a constrained optimization problem.

$$\inf_{\substack{\text{other model}\\\text{parameters}}} \inf_{\text{perturbation set}} \text{elasticity}$$

$$\text{s.t.} \quad \text{Moment condition}$$

$$\text{Structural constraint, e.g., Bellman equation}$$

whose Lagrangian is:

$$\inf_{\substack{\text{other model}\\\text{parameters}}} \color{red}{\inf_{\text{perturbation set}}} \color{blue}{\sup_{\substack{\text{Lagrange}\\\text{multipliers}}}} \color{black}{\text{Lagrangian}}$$

- We will use a minimax theorem to swap $\color{red}{\inf}$ and $\color{blue}{\sup}$.

$$\inf_{\substack{\text{other model}\\\text{parameters}}} \color{blue}{\sup_{\substack{\text{Lagrange}\\\text{multipliers}}}} \color{red}{\inf_{\text{perturbation set}}} \color{black}{\text{Lagrangian}}$$

where the inner problem is a distributionally robust optimization problem.

Implementation Overview [3/3]

- The framework is a constrained optimization problem.

$$\inf_{\substack{\text{other model}\\\text{parameters}}} \; \inf_{\text{perturbation set}} \; \text{elasticity}$$

$$\text{s.t.} \quad \text{Moment condition}$$

$$\qquad \text{Structural constraint, e.g., Bellman equation}$$

- By a minimax theorem, we swap $\color{red}{\inf}$ and $\color{blue}{\sup}$:

$$\inf_{\substack{\text{other model}\\\text{parameters}}} \; \color{blue}{\sup_{\substack{\text{Lagrange}\\\text{multipliers}}}} \; \color{red}{\inf_{\text{perturbation set}}} \; \text{Lagrangian}$$

- Proposed algorithm overview:
  - The **dual** of the red part returns the **worst-case distribution**.
  - The worst-case distribution is used in the **Structural constraint** to update the **value function** in the Bellman equation.
  - Update the **Lagrange multipliers** and **other model parameters** (other than the value function).

Recap [1/2]: Stationary Dynamic Discrete Choice (DDC)

- Consider an infinite horizon stationary single agent DDC.
- At period t, an agent chooses $j_t \in \mathcal{J}$ to maximize the lifetime utility:

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left[u_{j_t}(x_t, \xi_t; \theta) + \varepsilon_t(j_t)\right] \middle| x_0, \xi_0, \varepsilon_0\right]$$

  - $\mathcal{J} := \{1, \cdots, J\}$ is the action space.
  - $\beta \in (0, 1)$ is the discount factor.
  - $x_t \in \mathcal{X}$ is discrete observable (to researcher) state variable, e.g., mileage.
  - $\xi_t \in \Xi$ is the exogenously evolving latent variable, e.g., technology shock.
  - $\varepsilon_t(j)$ is i.i.d extreme value type I utility shock.
  - $u_j(x, \xi; \theta)$ is the period utility function parameterized by $\theta \in \Theta$.

Recap [2/2]: Stationary Dynamic Discrete Choice (DDC)

- Conditional value function, v, is the solution to the Bellman equation:

$$v_j(x,\xi) = \underbrace{u_j(x,\xi;\theta)}_{\textbf{period utility}} + \underbrace{\beta\mathbb{E}_{\xi'|\xi}\mathbb{E}_{x'|x,j}\left[\log\left(\sum_{j'\in\mathcal{J}}\exp(v_{j'}(x',\xi'))\right)\right] + \beta\gamma}_{\textbf{continuation value}}$$

  - γ is the Euler constant due to i.i.d extreme value type I utility shocks.
  - $\xi \in \Xi$ is the exogenously evolving latent variable, e.g., technology shock.
  - Future states are denoted by (x',ξ').
- To solve the Bellman equation, we need to compute $\mathbb{E}_{\xi'|\xi}$.

Example: Perturbation around a reference distribution

- The conditional distribution $F_0(d\xi'|\xi)$ has stationary distribution $\nu_0(d\xi)$
- Stationarity requires that $\int F_0(d\xi'|\xi)\nu_0(d\xi) = \nu_0(d\xi')$.
- The **reference distribution** is:

$$dF_0(\xi, \xi') := \nu_0(d\xi) \cdot F_0(d\xi'|\xi)$$

- Stationarity is equivalent to $F_0 \in \Pi(\nu_0, \nu_0)$ where $\Pi(\nu_0, \nu_0)$ is the set of distributions of (ξ, ξ') with marginals (ν_0, ν_0).
- For $\forall\ F \in \Pi(\nu_0, \nu_0)$, $F(d\xi'|\xi)$ has a stationary distribution $\nu_0(d\xi)$.
- Our **perturbation set** is:

$$\mathcal{F} := \left\{ F \in \mathcal{P}(\Xi^2) \mid \underbrace{F \in \Pi(\nu_0, \nu_0)}_{\text{Stationarity}}, \underbrace{D_{KL}(F \| F_0) \leq \delta}_{\text{Perturbation}} \right\}$$

  where D_{KL} is the Kullback-Leibler divergence ball of radius $\delta \geq 0$.
- In the paper, we also perturb the stationary distribution.

Example: Structural Constraint

- The Bellman equation is a continuum of conditional moment restrictions:

$$v_j(x,\xi) = u_j(x,\xi;\theta) + \beta \mathbb{E}_{\xi'|\xi} \mathbb{E}_{x'|x,j} \left[\log \left(\sum_{j' \in \mathcal{J}} \exp(v_{j'}(x',\xi')) \right) \right] + \beta\gamma$$

- We assume v is the solution to the Bellman equation if and only if:

$$\sup_{g \in \mathcal{G}} \mathbb{E}_F \mathbb{E}_{x,j,x'} \Big[g_j(x,\xi) \Big(v_j(x,\xi) - u_j(x,\xi;\theta) - \beta V(x',\xi') - \beta\gamma \Big) \Big] = 0$$

  where $(\xi,\xi') \sim F$, $V(x',\xi') := \log \left(\sum_{j' \in \mathcal{J}} \exp(v_{j'}(x',\xi')) \right)$, and $g \in \mathcal{G}$ is the test function.

- Let the **structural constraint** (Bellman equation) be:

$$\sup_{g \in \mathcal{G}} \mathbb{E}_F \big[\psi(\xi,\xi';\theta,v,g) \big] = 0$$

Example: Moment Condition and Counterfactual Parameter

- The finite-dimensional **moment condition** is:

$$\mathbb{E}_F\left[m(\xi,\xi';\theta,v)\right] = P_0$$

e.g., conditional choice probability $p(j|x,\xi) = \frac{\exp(v_j(x,\xi))}{\exp(V(x,\xi))}$.

- The **counterfactual parameter** is:

$$\mathbb{E}_F\left[s(\xi,\xi';\theta,v)\right]$$

e.g., average welfare $\mathbb{E}_{\nu_0}\mathbb{E}_x \log\left(\sum_{j\in\mathcal{J}}\exp\left(v_j(x,\xi)\right)\right)$.

Example: Stationary DDC

- The lower bound on the counterfactual parameter is:

$$\kappa(\delta, P_0) := \inf_{(\theta,v,F)\in\Theta\times\mathcal{V}\times\mathcal{F}} \mathbb{E}_F\left[s(\xi,\xi';\theta,v)\right]$$
$$\text{s.t.} \quad \mathbb{E}_F\left[m(\xi,\xi';\theta,v)\right] = P_0$$
$$\sup_{g\in\mathcal{G}} \mathbb{E}_F\left[\psi(\xi,\xi';\theta,v,g)\right] = 0$$

where $\mathcal{F} := \left\{F \in \mathcal{P}(\Xi^2) \mid F \in \Pi(\nu_0,\nu_0), D_{KL}(F\|F_0) \leq \delta\right\}$

- **Question 1:** the worst-case distribution has closed-form? Smoothness?
- **Question 2:** the expectations are taken with respect to F.

- It works for multi-marginal cases, e.g., cross-sectional dependence.
- It also works for panel discrete choice models.

Practical Implementation

Duality [1/2]: Minimax part

- The **Lagrangian** is:

$$\inf_{\substack{(\theta,v)\in\Theta\times\mathcal{V}\\ F\in\Pi(\nu_0,\nu_0)}} \sup_{\substack{\lambda\in\mathbb{R}^{d_P}\\ \lambda_{KL}\geq 0, g\in\mathcal{G}}} \mathbb{E}_F\left[c(\xi,\xi';\theta,v,g,\lambda)\right]+\lambda_{KL}(D_{KL}(F\|F_0)-\delta)-\lambda^T P$$

where $c(\xi,\xi';\theta,v,g,\lambda) = s(\xi,\xi';\theta,v) + \lambda^T m(\xi,\xi';\theta,v) + \psi(\xi,\xi';\theta,v,g)$

- For given (θ,v), we **swap** the $\inf$ and $\sup$ by a minimax theorem:

$$\sup_{\substack{\lambda\in\mathbb{R}^{d_P}\\ \lambda_{KL}\geq 0, g\in\mathcal{G}}} \inf_{F\in\Pi(\nu_0,\nu_0)} \mathbb{E}_F\left[c(\xi,\xi';\theta,v,g,\lambda)\right]+\lambda_{KL}(D_{KL}(F\|F_0)-\delta)-\lambda^T P$$

- The inner problem is **entropic optimal transport** (EOT) ($\lambda_{KL} > 0$):

$$\mathcal{C}(\theta,v,g,\lambda,\lambda_{KL}) := \inf_{F\in\Pi(\nu_0,\nu_0)} \mathbb{E}_F\left[c(\xi,\xi';\theta,v,g,\lambda)\right] + \lambda_{KL}D_{KL}(F\|F_0)$$

Duality [2/2]: Entropic Optimal Transport (EOT)

- The EOT problem ($\lambda_{KL} > 0$):

$$\mathcal{C}(\theta, v, g, \lambda, \lambda_{KL}) := \inf_{F \in \Pi(\nu_0, \nu_0)} \mathbb{E}_F \left[c(\xi, \xi'; \theta, v, g, \lambda) \right] + \lambda_{KL} D_{KL}(F \| F_0)$$

- The EOT problem has a unique worst-case distribution.

- Based on the EOT duality,
  - The worst-case distribution has closed form + smoothness.
  - The **Sinkhorn algorithm** is a fast numerical method to find it.
  - Expectation is taken with respect to F_0.

Duality Theorem [1/2]: Minimax

Theorem: Minimax Duality

Under mild regularity conditions for minimax theorem and EOT duality,

$$\kappa(\delta, P) = \inf_{(\theta,v)\in\Theta\times\mathcal{V}} \sup_{\substack{\lambda\in\mathbb{R}^{d_P} \\ \lambda_{KL}\geq 0, g\in\mathcal{G}}} \mathcal{C}(\theta, v, g, \lambda, \lambda_{KL}) - \lambda_{KL}\delta - \lambda^T P \qquad \text{(Dual)}$$

where $\mathcal{C}(\theta, v, g, \lambda, \lambda_{KL})$ is the EOT problem:

$$\mathcal{C}(\theta, v, g, \lambda, \lambda_{KL}) := \inf_{F\in\Pi(\nu_0,\nu_0)} \mathbb{E}_F\left[c(\xi, \xi'; \theta, v, g, \lambda)\right] + \lambda_{KL} D_{KL}(F\|F_0)$$

Duality Theorem [2/2]: EOT duality

Theorem: EOT duality

For $\lambda_{KL} > 0$, it holds that:

$$\mathcal{C}(\theta, v, g, \lambda, \lambda_{KL}) = \lambda_{KL} +$$

$$\sup_{\{\phi_i\}_{i=1}^2 \in L^1(\nu_0)} \mathbb{E}_{\nu_0}\left[\phi_1(\xi) + \phi_2(\xi')\right] - \lambda_{KL}\mathbb{E}_{F_0} \exp\left(\frac{\phi_1(\xi) + \phi_2(\xi') - c(\xi, \xi')}{\lambda_{KL}}\right)$$

Moreover, the unique worst-case distribution F^* has the density:

$$\frac{dF^*(\xi, \xi')}{dF_0(\xi, \xi')} = \exp(\frac{\phi_1^*(\xi) + \phi_2^*(\xi') - c(\xi, \xi'; \theta, v, g, \lambda)}{\lambda_{KL}}) \quad F_0\text{-a.s.}$$

In addition, if c is continuous in (ξ, ξ'), then optimizing over $\lambda_{KL} > 0$ is equivalent to optimizing over $\lambda_{KL} \geq 0$.

Statistical Properties

Statistical Properties: Consistency and Convergence Rate

- **Key Assumption:** for $\forall\ (\theta, F) \in \Theta \times \mathcal{F}$, the structural constraint F-a.s. has a unique solution $v_{\theta,F} \in \mathcal{V}$.
- Our estimator is $\kappa(\delta, P_n, \epsilon_n)$ where P_n is an estimator of P_0, $\epsilon_n \to 0$ is the tolerance for the moment condition and n is the sample size.
- The estimator of the identified set $\mathcal{A}_I$ is:

$$\hat{\mathcal{A}}_I := \left\{(\theta, F) \in \Theta \times \mathcal{F} \mid \left\|\mathbb{E}_F\left[m(\xi, \xi'; \theta, v_{\theta,F})\right] - P_n\right\|_\infty \leq \epsilon_n\right\}$$

- Consistency and convergence rate follow from Chernozhukov et al. 2007.
- Under continuity condition on $\mathbb{E}_F\left[s(\xi, \xi'; \theta, v_{\theta,F})\right]$, the consistency and convergence rate of $\kappa(\delta, P_n, \epsilon_n)$ follow.

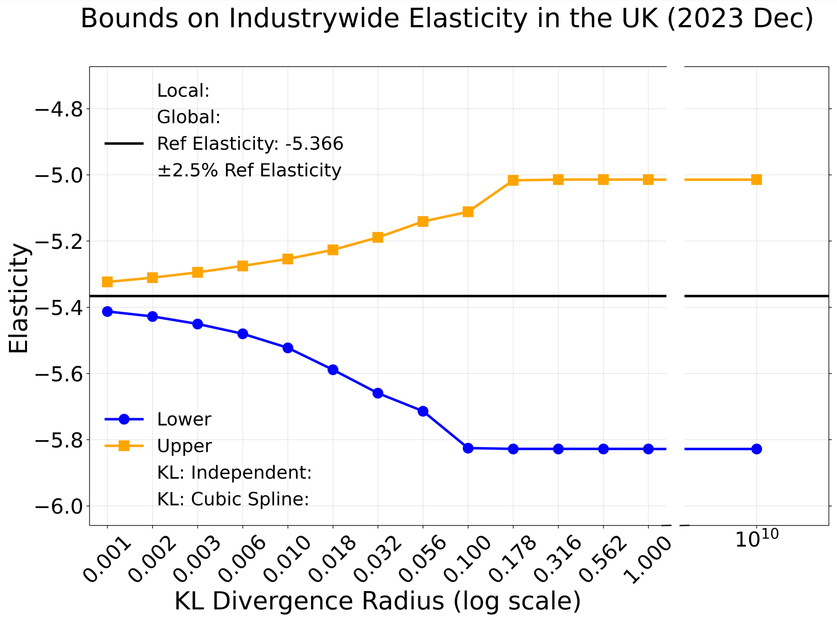
Statistical Properties: Asymptotic Distribution

- We show $\kappa(\delta, P)$ is Hadamard directionally differentiable at P_0.

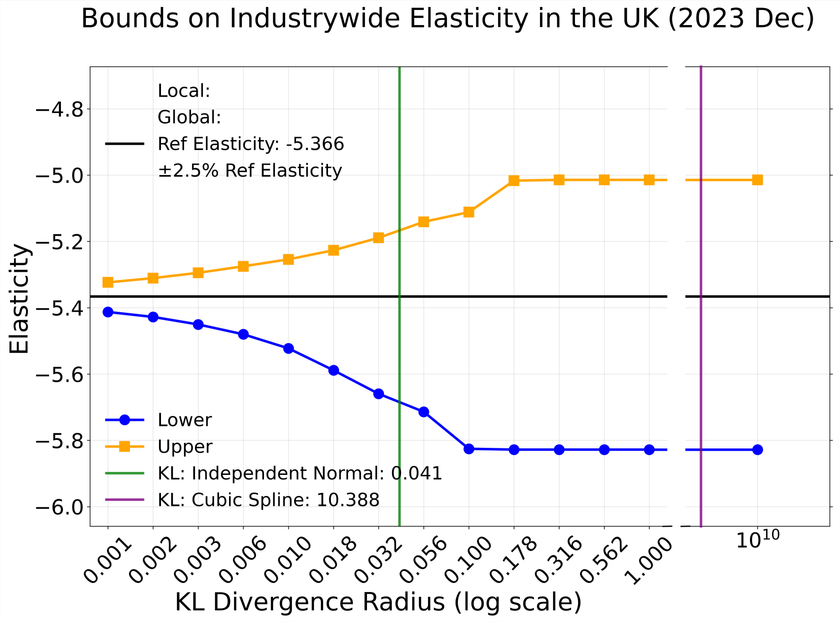
- If $\sqrt{n}(P_n - P_0) \xrightarrow{d} Z \sim \mathcal{N}(0, \Sigma)$, $\sqrt{n}(\kappa(\delta, P_n) - \kappa(\delta, P_0)) \xrightarrow{d} \kappa'(\delta, P_0; Z)$ where $\kappa'(\delta, P_0; Z)$ is the directional derivative.

Interpreting the Results

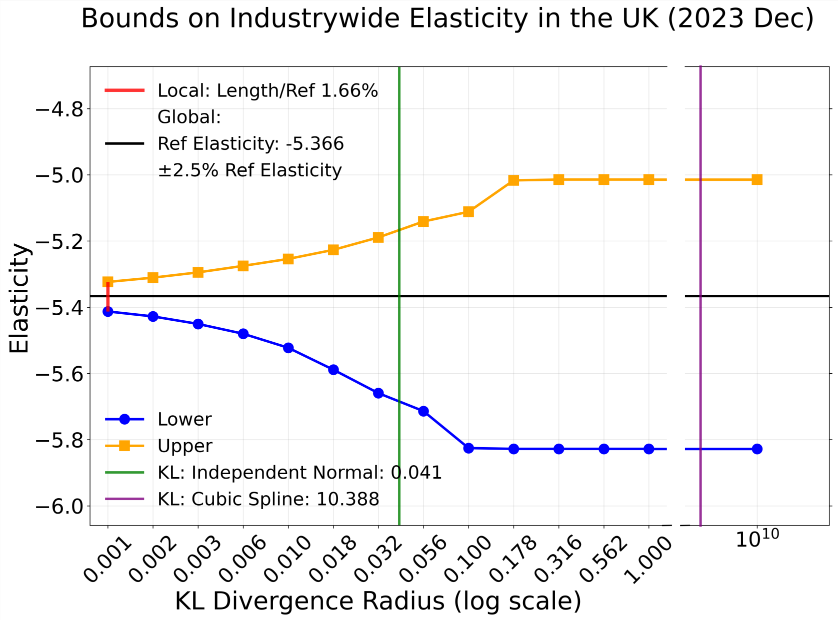
Plot the bounds against δ



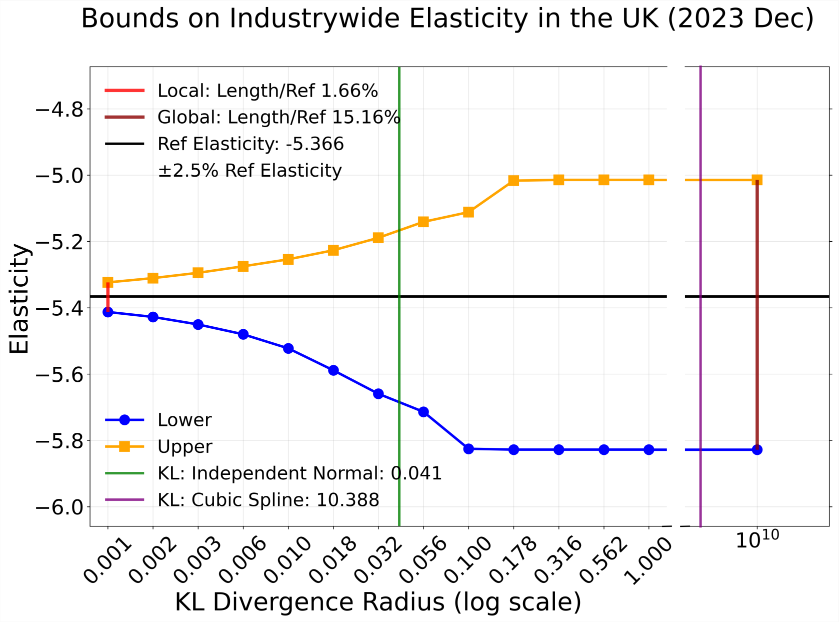
Estimate alternative models to choose δ



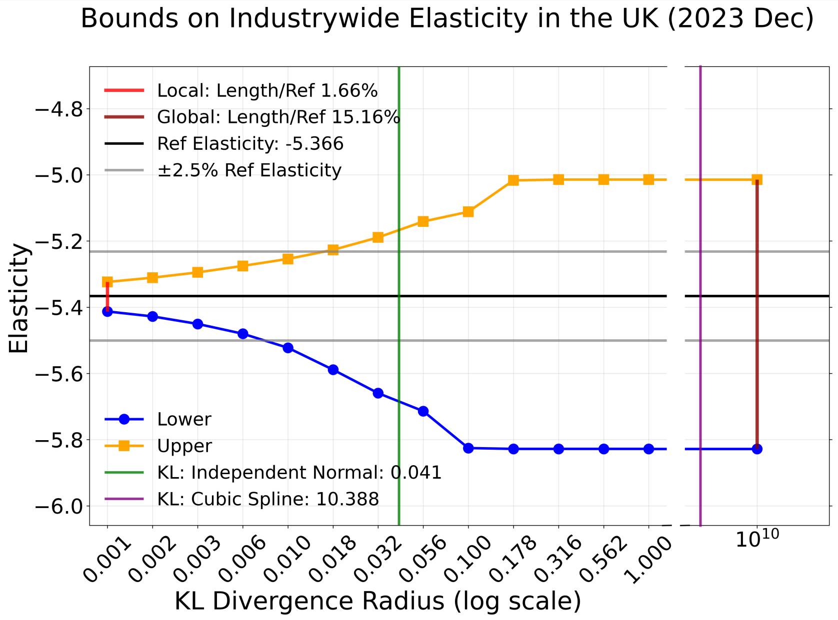
Three complementary sensitivity measures: Local Sensitivity



Three complementary sensitivity measures: Global Sensitivity



Three complementary sensitivity measures: Robustness metric



Three complementary sensitivity measures

- We can estimate an alternative model (e.g., independent normal distribution) and compute its KL divergence to the reference distribution.

- **Local sensitivity**: compute the right-derivative $\frac{\partial_+ \kappa(0, P_0)}{\partial \delta}$.

- **Global sensitivity**: plot bounds $(\kappa(\delta, P_0))$ against δ until, e.g., 10^{10}.
  - It provides an explicit upper bound on the approximation error to the global bounds.

- **Robustness metric**: the smallest deviation from the reference distribution that can generate sensitive results (Spini 2024).
  - s_{Ref} is the reference elasticity, $\bar{s} = 0.9 \cdot s_{Ref}$ is the threshold.
  - If the elasticity is less than $\bar{s}$, then we worry about the robustness.
  - $\delta(\bar{s})$ is the smallest deviation from the reference distribution.

Empirical Application

Empirical Application: Dynamic Demand for New Cars

- The IHS Markit new car registration data from 2014 to 2023 in the UK, France and Germany.
- The period utility of individual i in month t is:

$$u(j, x_t, p_t, \xi_t, \varepsilon_{it}) = \begin{cases} \alpha p_{jt} + x_{jt}^T \theta + \xi_{jt} + \varepsilon_{ijt} & \text{if } j \in \mathcal{J}_t \\ \varepsilon_{i0t} & \text{if } j = 0 \end{cases}$$

  where x_{jt} is observable car characteristics, ξ_{jt} is the unobserved car characteristics, and ε_{ijt} is the Extreme Value Type I utility shock.

- We assume the purchase is a terminating action (optimal stopping).
- **Step 1:** Identify (α, θ) without distributional assumptions for ξ.
- **Step 2:** Sensitivity analysis of the following:
  - Price elasticities of demand
  - Consumer surplus from electric vehicle subsidy

Step 1: Identify Utility Parameters

- The conditional value function of purchasing car j is:

$$v_j(x_t, p_t, \xi_t) = \frac{x_{jt}^T\theta + \xi_{jt}}{1-\beta} + \alpha p_{jt}$$

- Let r be the reference car, $\Delta\xi_{jt} = \xi_{jt} - \xi_{rt}$, and s_{jt} be the market share:

$$\log\left(\frac{s_{jt}}{s_{rt}}\right) = \frac{\Delta x_{jt}^T\theta + \Delta\xi_{jt}}{1-\beta} + \alpha\Delta p_{jt}$$

  where (α, θ) are identified by BLP instruments.
- **However,** we still need to compute value of waiting (outside option) to solve the Bellman equation.

Step 2.1: The Reference Distribution

- The indirect utility (inclusive value) of purchasing is:

$$\begin{aligned}\omega_t &= \log \sum_{j \in \mathcal{J}_t} \exp\left(\frac{x_{jt}^T\theta + \xi_{jt}}{1-\beta} + \alpha p_{jt}\right) \\ &= \underbrace{\log \sum_{j \in \mathcal{J}_t} \exp\left(\frac{x_{jt}^T\theta + \Delta\xi_{jt}}{1-\beta} + \alpha p_{jt}\right)}_{\text{Known}} + \underbrace{\frac{\xi_{rt}}{1-\beta}}_{\text{Unknown}}\end{aligned}$$

- We assume the inclusive value sufficiency (IVS) assumption.
- The reference distribution for ω_t is the AR(1) process:

$$\omega_t = \gamma_0 + \gamma_1\omega_{t-1} + \eta_t$$

- We set the estimated AR(1) as the reference distribution.

Step 2.2: The Structural Constraint

- By the IVS assumption and Hotz and Miller Inversion Lemma,

$$v_1(\omega) = \omega, \quad v_0(\omega) = \beta \mathbb{E}\Big[\underbrace{\omega' - \log(s_1(\omega'))}_{=V(\omega')}|\omega\Big]$$

  where 1 is purchase and 0 is not purchase, and s_1 is the market share.

- The fixed point problem on the market share space:

$$\log\left(\frac{1 - s_0(\omega)}{s_0(\omega)}\right) = \omega - \beta \mathbb{E}\left[\omega' - \log(1 - s_0(\omega'))|\omega\right]$$

- Convert the above to an unconditional moment condition:

$$\sup_{g \in C(\Omega)} \mathbb{E}_F\left[g(\omega)\left(\log(\frac{1 - s_0(\omega)}{s_0(\omega)}) - \omega + \beta\omega' - \beta \log(1 - s_0(\omega'))\right)\right] = 0$$

Step 2.3: Sensitivity Analysis

- The industrywide elasticity at $T =$ December 2023 is:

$$\frac{(1 - s_0(\omega_T')) - (1 - s_{0T})}{1 - s_{0T}} \times 100$$

where ω_T' is ω_T plus 1% increase in price of all cars.

- The lower bound is given by:

$$\inf_{s_0 \in C(\Omega), F \in \mathcal{F}} \frac{s_{0T} - s_0(\omega_T')}{1 - s_{0T}} \times 100$$

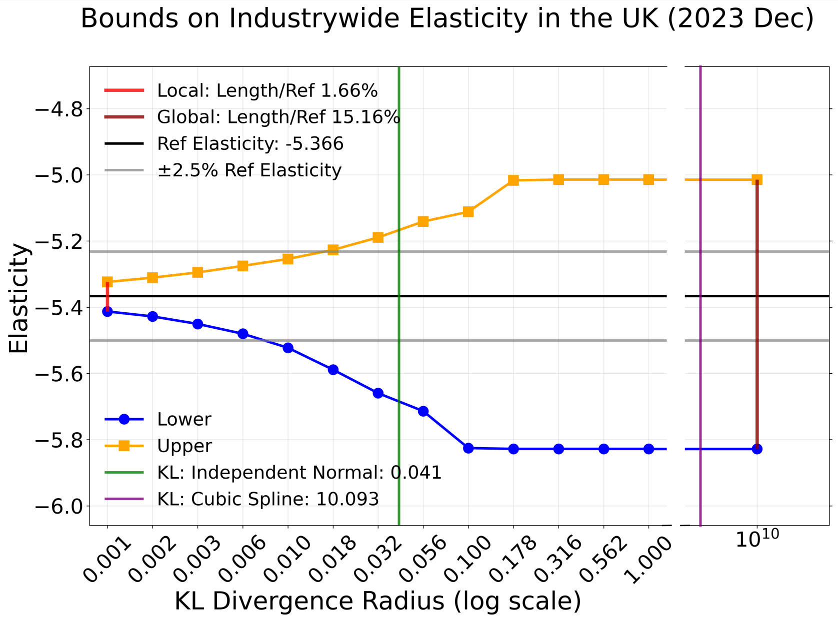
$$\text{s.t.} \quad s_0(\omega_t) = s_{0t} \text{ for } t = 1, \ldots, T$$

$$\sup_{g \in C(\Omega)} \mathbb{E}_F \left[g(\omega) \left(\log(\frac{1 - s_0(\omega)}{s_0(\omega)}) - \omega + \beta\omega' - \beta \log(1 - s_0(\omega')) \right) \right] = 0$$

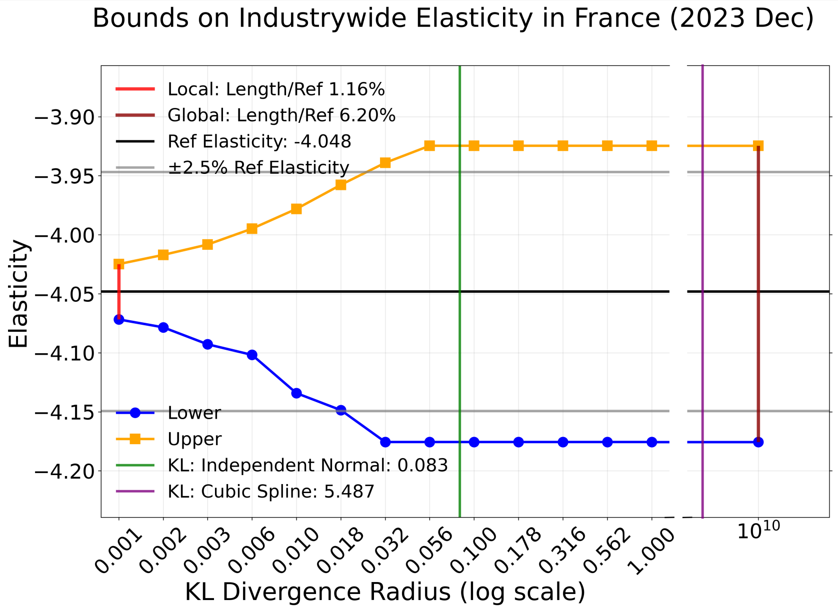
$$D_{KL}(F \| \hat{F}) \leq \epsilon_T$$

where $\hat{F}$ is the estimator for the distribution of $\{\omega, \omega'\}$ using the recovered $\{\omega_t, \omega_{t+1}\}_{t=1}^{T-1}$.

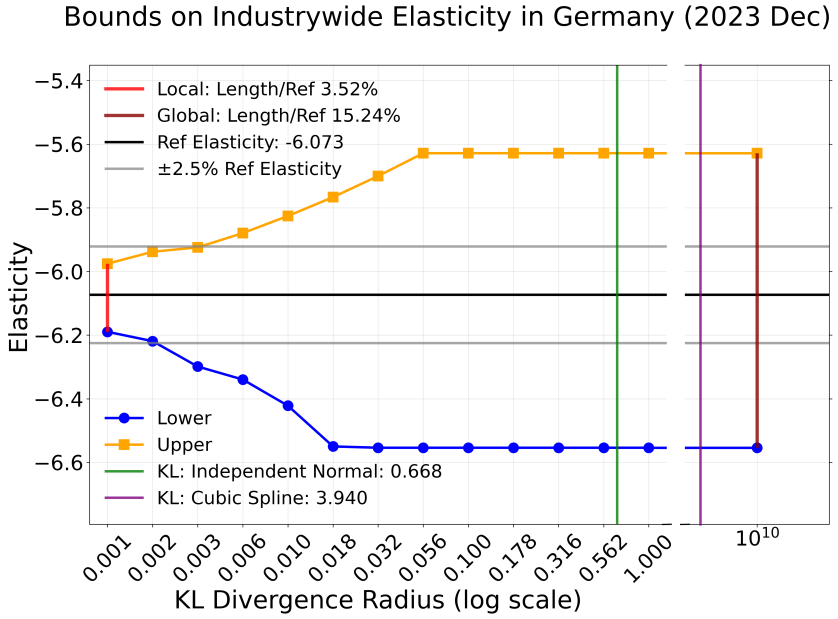
Elasticity Bounds for the UK December 2023



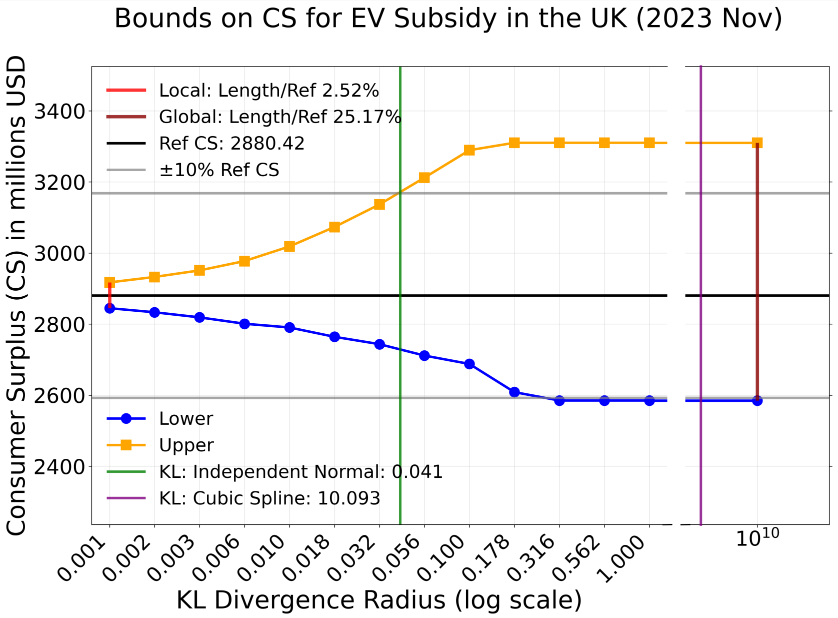
Elasticity Bounds for France December 2023



Elasticity Bounds for Germany December 2023



Consumer Surplus Bounds for additional 3000 USD EV Subsidy



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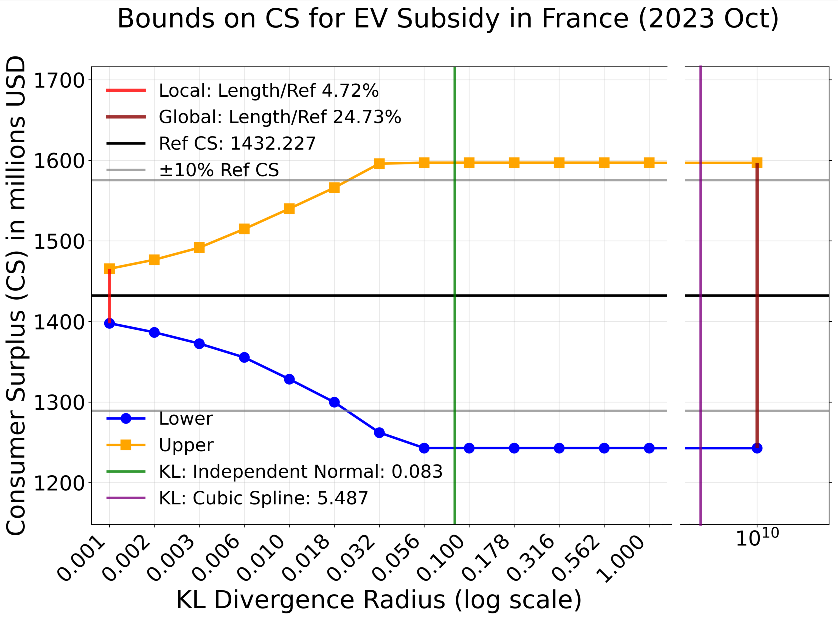

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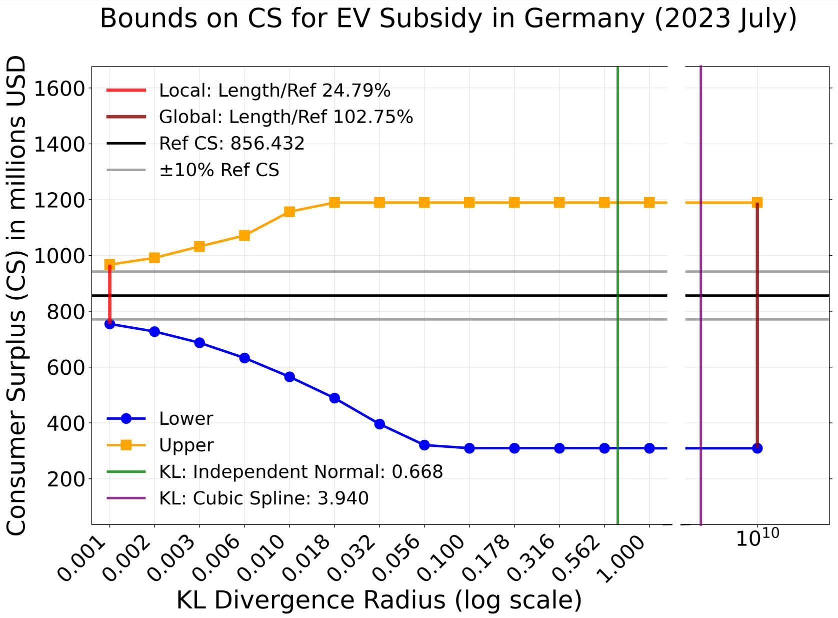

Conclusion

- We propose a computationally tractable framework to quantify the sensitivity of counterfactual parameters to assumptions about the dynamic process of latent variables.

- We apply it to:
  1. infinite-horizon dynamic demand for new cars in Europe.
  2. finite-horizon dynamic labor supply for New York City taxi drivers.

Thank you!

The Sinkhorn Algorithm for Entropic Optimal Transport (EOT)

Let $F_\otimes := \nu_0 \otimes \nu_0$ be the product measure. The EOT and its dual are:

$$\inf_{F \in \Pi(\nu_0, \nu_0)} \mathbb{E}_F \left[c(\xi, \xi') \right] + \lambda_{KL} D_{KL}(F \| F_\otimes) \tag{EOT}$$

$$\sup_{\{\phi_i\}_{i=1}^2 \in L^1(\nu_0)} \mathbb{E}_{\nu_0} \left[\phi_1(\xi) + \phi_2(\xi') \right] - \lambda_{KL} \mathbb{E}_{F_0} \exp\left(\frac{\phi_1(\xi) + \phi_2(\xi') - c(\xi, \xi')}{\lambda_{KL}} \right) \tag{EOT Dual}$$

- The optimal $\{\phi_i^*\}_{i=1}^2$ are the solutions to the Schrödinger equations:

$$\phi_1(\xi) = -\lambda_{KL} \log \left(\mathbb{E}_{\nu_0} \exp\left(\frac{\phi_2(\xi') - c(\xi, \xi')}{\lambda_{KL}} \right) \right) \quad \nu_0\text{-a.s.}$$

$$\phi_2(\xi') = -\lambda_{KL} \log \left(\mathbb{E}_{\nu_0} \exp\left(\frac{\phi_1(\xi) - c(\xi, \xi')}{\lambda_{KL}} \right) \right) \quad \nu_0\text{-a.s.}$$

- The Sinkhorn algorithm iterates the above equations until convergence.

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