

Policy Choice in Time Series by Empirical Welfare Maximization

Toru Kitagawa (Brown), Weining Wang (Bristol), Mengshan Xu (Mannheim)

Introduction

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- The treatment choice literature initiated by Manski (04, Ecta) uses the statistical decision theory to link causal inference and planner's policy decision in the microeconometrics context.
- How about in macro/finance? **Statistical treatment (policy) choice with time-series data?**
- E.g.,
 - ① FOMC's monetary policy choice?
 - ② Central bank's intervention to exchange markets?
 - ③ Policy choice of relaxing or tightening the Covid containment policy?

Motivating Example and Preview

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- Assuming treatments are randomized conditional on observables in the data, we propose the *time series empirical welfare maximization* (T-EWM): estimate welfare impact of the today's policy choice using the past data and maximize it
- We formulate the decision problem using potential outcome time-series and study theoretical guarantee of the proposal in terms of welfare convergence

Review: (Static) Statistical Treatment Choice

Assume **randomized controlled trial (RCT)** or observational data for $i = 1, \dots, n$ iid individuals

- $X_i \in \mathcal{X}$ - pre-treatment observed covariates (e.g., education, previous earnings)
- $W_i \in \{0, 1\}$ - randomized treatment (possibly conditional on X_i) (e.g., job training program)
- $Y_i \in \mathbb{R}$ - post-treatment observed outcome (e.g., employment, income)

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- $Y_i \in \mathbb{R}$ - post-treatment observed outcome (e.g., employment, income)
- An assignment rule $g : \mathcal{X} \rightarrow \{0, 1\}$
- Let $(Y(1), Y(0))$ be the potential outcomes. Additive (utilitarian) welfare:

$$\mathcal{W}(g) = E[Y(1) \cdot g(X) + Y(0) \cdot (1 - g(X))].$$

Manski's Empirical Success Rule

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$$\hat{g}_{CES}(x) \equiv 1\{\hat{\tau}(x) \geq 0\},$$

where $\hat{\tau}(x)$ is the difference-in-means estimator for CATE,

$$\hat{\tau}(x) = \widehat{E}[Y|W = 1, X = x] - \widehat{E}[Y|W = 0, X = x] \quad (1)$$

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- Manski assesses the performance of the CES rule by bounding the **expected welfare regret**,

$$E_{P^n} \left[\max_g \mathcal{W}(g) - \mathcal{W}(\hat{g}_{CES}) \right] \leq \frac{C}{\sqrt{n}}.$$

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- 1 Policy impact can have dynamic causal effects and can be heterogeneous (nonstationary). External validity?
- 2 Observations are no longer iid.
- 3 What could be a reasonable welfare objective? How to obtain its empirical analogue?
- 4 Can we obtain welfare regret convergence? Under what condition? What rate?

Baseline Setting

- Suppose a social planner is at the beginning of period T , wants to perform the policy choice at T , W_T , based on the information available at the beginning of T .
- The planner has access to time-series observations: for $t = 0, 1, 2, \dots, T - 1$.
- $Y_t \in \mathbb{R}$: outcome, e.g., output, unemployment, stock price, Covid deaths, etc.
- $W_t \in \{0, 1\}$: (manipulatable) treatment, e.g., high/low target rate, lockdown or not, structural policy shock, etc
- $Z_t \in \mathbb{R}$: contextual information, e.g., macroeconomic indices other than Y_t ,

- Denote the sample by $X_{0:T-1} = (Y_{0:T-1}, W_{0:T-1}, Z_{0:T-1})$, given initial value X_0 .
- A **treatment path** $w_{0:T} \in \{0, 1\}^{T+1}$.
- Potential outcomes indexed by the treatment paths: for each $t = 1, \dots, T$, $Y_t(w_{0:T})$ denotes the counterfactual outcome at period t when the treatment path were exogenously set to $w_{0:T}$.

Definition: Potential Outcome Time-Series (Bojinov & Shephard 19 JASA, and Rambachan & Shephard 21)

① **Non-anticipating potential outcomes:** For each $t = 1, \dots, T$,

$$Y_t(w_{0:t}, w_{t+1:T}) = Y_t(w_{0:t}, w'_{t+1:T})$$

holds a.s. for any $w_{0:t}$, and $w_{t+1:T} \neq w'_{t+1:T}$, i.e., the potential outcomes are indexed only by the past and current treatments

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- ② **Non-anticipating treatment paths (sequential unconfoundedness):** For every $t = 1, \dots, T - 1$ and $s = t, t + 1, \dots, T$, and any $w_{t:T}$,

$$W_t \perp Y_s(W_{0:t-1}, w_{t:s}) | X_{0:t-1}$$

In the data, W_t is randomized once conditioned on the history of observables

Policies and Welfare

- **One-period (nonrandomized) policy:** $g : \mathcal{X}_{0:T-1} \rightarrow \{0, 1\}$

Policies and Welfare

- **One-period (nonrandomized) policy:** $g : \mathcal{X}_{0:T-1} \rightarrow \{0, 1\}$
- In this talk, we focus on the **one-period conditional welfare**:

$$\begin{aligned} & \mathcal{W}_T(g|X_{0:T-1}) \\ & \equiv E[Y_T(W_{0:T-1}, 1)g(X_{0:T-1}) + Y_T(W_{0:T-1}, 0)(1 - g(X_{0:T-1}))|X_{0:T-1}] \end{aligned}$$

- In the paper, we also discuss **the unconditional welfare** in the time-series setting, which differs from that in the cross-sectional setting

Let's simplify (and generalize later)

Toy model: one-period Markovian model

Bivariate time-series, $X_t = (Y_t, W_t) \in \mathbb{R} \times \{0, 1\}$, $t = 0, 1, 2, \dots$, given X_0 .

① **Markovian Exclusion:** For each $t = 2, \dots, T$,

$$Y_t(w_{0:t-2}, w_{t-1}, w_t) = Y_t(w'_{0:t-2}, w_{t-1}, w_t) \equiv Y_t(w_{t-1}, w_t)$$

for all $w_{0:t-2} \neq w'_{0:t-2}$, and (w_{t-1}, w_t)

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② **Markovian Exogeneity:** For each $t = 1, \dots, T-1$ and $w_{0:t}$,

$$(Y_t(w_{0:t}), W_t) \perp X_{0:t-1} | W_{t-1}, \text{ and } Y_T(w_{T-1}, w_T) \perp X_{0:T-1} | W_{T-1}$$

- Under this simplification, the planner's objective (one-period social welfare) satisfies

$$\begin{aligned}
 & \mathcal{W}_T(g|X_{0:T-1}) \\
 &= E[Y_T(W_{T-1}, 1)g(X_{0:T-1}) + Y_T(W_{T-1}, 0)(1 - g(X_{0:T-1}))|X_{0:T-1}] \\
 &\quad (\because \text{Markovian exclusion}) \\
 &= E[Y_T(W_{T-1}, 1)|W_{T-1}]g(X_{0:T-1}) + E[Y_T(W_{T-1}, 0)|W_{T-1}](1 - g(X_{0:T-1})) \\
 &\quad (\because \text{Markovian exogeneity}) \\
 &= (E[Y_T(W_{T-1}, 1) - Y_T(W_{T-1}, 0)|W_{T-1}]) \cdot g(X_{0:T-1}) + E[Y_T(W_{T-1}, 0)|W_{T-1}]
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 \end{aligned}$$

- We can reduce $g(\cdot)$ to a binary map of W_{T-1} without loss of the welfare and focus on maximizing

$$\mathcal{W}_T(g|W_{T-1}) \equiv E[Y_T(W_{T-1}, 1)g + Y_T(W_{T-1}, 0)(1 - g)|W_{T-1}]$$

Empirical Welfare

- Empirical analogue of $\mathcal{W}_T(g|W_{T-1} = w)$? Try a historical simple average.

Policy rule: Time-Series CES

We obtain $\hat{g}_{CES}(w)$ by maximizing

$$\widehat{\mathcal{W}}(g|W_{t-1} = w) = \frac{1}{T(w)} \sum_{t: W_{t-1}=w} \left[\frac{Y_t W_t}{e_t(W_{t-1})} g + \frac{Y_t(1 - W_t)}{1 - e_t(W_{t-1})} (1 - g) \right]$$

in $g \in \{0, 1\}$, where $T(w) = |\{t : W_{t-1} = w\}|$ and $e_t(W_{t-1}) = \Pr(W_t = 1|W_{t-1})$ assumed to satisfy a strict overlap condition

Any concentration regarding $\widehat{\mathcal{W}}(g|W_{t-1} = w)$?

- Let $\{\mathcal{F}_t = \sigma(X_{1:t-1}) : t = 1, \dots, T-1\}$ be the filtration and consider decomposing $\widehat{\mathcal{W}}(g|W_{t-1} = w)$ into sums of innovations and conditional expectations w.r.t $\{\mathcal{F}_t\}$ (Bojinov & Shephard 19).

$$\widehat{\mathcal{W}}(g|W_{t-1} = w) = \frac{1}{T(w)} \sum_{t=1}^T \xi_{t,w}(g) + \bar{\mathcal{W}}(g|w)$$

where

$$\bar{\mathcal{W}}(g|w) \equiv \frac{1}{T(w)} \sum_{t: W_{t-1}=w} E[Y_t(W_{t-1}, 1)g + Y_t(W_{t-1}, 0)(1-g)|\mathcal{F}_{t-1}],$$
$$\xi_{t,w}(g) \equiv 1\{W_{t-1} = w\} \cdot \left[\frac{Y_t W_t}{e_t(W_{t-1})} g + \frac{Y_t(1-W_t)}{1-e_t(W_{t-1})} (1-g) \right. \\ \left. - E[Y_t(W_{t-1}, 1)g + Y_t(W_{t-1}, 0)(1-g)|\mathcal{F}_{t-1}] \right]$$

- Under Markovian exogeneity and sequential unconfoundedness:

$$\bar{\mathcal{W}}(g|w) \equiv \frac{1}{T(w)} \sum_{t: W_{t-1}=w} E[Y_t(W_{t-1}, 1)g + Y_t(W_{t-1}, 0)(1-g)|W_{t-1} = w],$$

$$\begin{aligned} E[\xi_{t,w}(g)|\mathcal{F}_{t-1}] &\equiv 1\{W_{t-1} = w\} \cdot \left[\frac{E\left[\frac{Y_t W_t}{e_t(W_{t-1})}g + \frac{Y_t(1-W_t)}{1-e_t(W_{t-1})}(1-g)|W_{t-1}\right]}{-E[Y_t(W_{t-1}, 1)g + Y_t(W_{t-1}, 0)(1-g)|W_{t-1}]} \right] \\ &= 0 \end{aligned}$$

- I.e., $\xi_{t,w}(g)$ is a martingale difference sequence (MDS), and the empirical welfare centered at $\bar{\mathcal{W}}(g|w)$ is an average of MDS w.r.t $\{\mathcal{F}_t\}$,

$$\widehat{\mathcal{W}}(g|w) - \bar{\mathcal{W}}(g|w) = \frac{1}{T(w)} \sum_{t=1}^T \xi_{t,w}(g),$$

Key assumption: Invariance of welfare ordering

Bridge the period- T welfare regret and the past average welfare regret

Assumption: Invariance of welfare ordering

Let $g^* = \arg \max_g \mathcal{W}_T(g|W_{T-1} = w)$. There exists $c > 0$ such that for any w and $g \in \{0, 1\}$,

$$\mathcal{W}_T(g^*|w) - \mathcal{W}_T(g|w) \leq c(\bar{\mathcal{W}}(g^*|w) - \bar{\mathcal{W}}(g|w))$$

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- The welfare ordering of the policies today agrees with the historical average over the periods sharing the conditioning events similar to today's
- Weaker than stationarity of welfare (i.e., $\mathcal{W}_t(g|w)$ independent of t)
- In structural MA(2): $Y_t = \alpha_t + \beta_t W_t + \gamma_t W_{t-1} + \epsilon_t$, this assumption holds if

$$\text{sign}(\beta_T) = \text{sign}\left(\frac{1}{T(w)} \sum_{t: W_{t-1}=w} \beta_t\right)$$

- Putting everything together,

$$\begin{aligned} \mathcal{W}_T(g^*|w) - \mathcal{W}_T(\hat{g}_{CES}|w) &\leq c(\bar{\mathcal{W}}(g^*|w) - \bar{\mathcal{W}}(\hat{g}_{CES}|w)) \\ &\leq 2c \sup_g |\widehat{\mathcal{W}}(g|w) - \bar{\mathcal{W}}(g|w)| \end{aligned}$$

- Hence, the average conditional welfare regret can be bounded as follows:

$$\begin{aligned} E_P[\mathcal{W}_T(g^*|w) - \mathcal{W}_T(\hat{g}_{CES}|w)] \\ \leq 2c E_P \left[\sup_g \left| \frac{1}{T(w)} \sum_{t=1}^{T-1} \xi_{t,w}(g) \right| \right] \end{aligned}$$

- Imposing additional regularity (Y_t bounded for all t), we can apply **Freedman's large deviation inequality for MDS** to obtain a finite-sample regret bound uniform in w and P

$$E_P[\mathcal{W}_T(g^*|w) - \mathcal{W}_T(\hat{g}_{CES}|w)] \leq \frac{C}{\sqrt{T-1}}$$

- **Result:** Under the imposed assumptions, Manski's CES-rule can be generalized to time-series and attain \sqrt{T} -convergence

Extension: more covariates?

- Maintaining the one-period Markovian model, consider adding continuous variables to X_{t-1} :

$$X_{t-1} = (W_{t-1}, Y_{t-1}, Z_{t-1}) \in \{0, 1\} \times \mathbb{R} \times \mathbb{R}^d$$

- The conditional welfare $\mathcal{W}_T(g|x_{T-1})$ as planner's objective:

$$\begin{aligned} &\mathcal{W}_T(g|x_{T-1}) \\ &\equiv E[Y_T(W_{T-1}, 1)g + Y_T(W_{T-1}, 0)(1 - g)|X_{T-1} = x_{T-1}] \end{aligned}$$

- Conditioning on the continuous variables, a simple sample analogue is not available

- **Our approach:** analogous to EWM, put a class of policies $\mathcal{G} \equiv \{g\}$, and optimize a sample analogue of the **unconditional** welfare over \mathcal{G}
- Define the **unconditional welfare** at period T :

$$\mathcal{W}_T(g) \equiv E[Y_T(W_{T-1}, 1)g(X_{T-1}) + Y_T(W_{T-1}, 0)(1 - g(X_{T-1}))]$$

Key assumptions for T-EWM:

- 1 **Correct specification** at $x_{T-1} \in \mathcal{X}_{T-1}$,

$$\arg \sup_{g \in \mathcal{G}} \mathcal{W}_T(g) \subset \arg \sup_{g \in \mathcal{G}} \mathcal{W}_T(g|x_{T-1}),$$

A sufficient condition:

$$\{x_{T-1} : E[Y_T(W_{T-1}, 1) - Y_T(W_{T-1}, 0)|x_{T-1}] \geq 0\} \in \mathcal{G}$$

- With additional assumption on the marginal distribution of X_{T-1} , we can bound the conditional welfare regret by the unconditional welfare regret, i.e., $\exists c > 0$ such that $\forall G \in \mathcal{G}$,

$$\sup_{g \in \mathcal{G}} \mathcal{W}_T(g|x_{T-1}) - \mathcal{W}_T(g|x_{T-1}) \leq c(\sup_{g \in \mathcal{G}} \mathcal{W}_T(g) - \mathcal{W}_T(g))$$

EWM policy and regret bounds

$$\hat{g}^{EWM} \in \arg \max_{g \in \mathcal{G}} \frac{1}{T-1} \sum_{t=1}^{T-1} \left[\frac{Y_t W_t}{e_t(X_{t-1})} g(X_{t-1}) + \frac{Y_t(1 - W_t)}{1 - e_t(X_{t-1})} (1 - g(X_{t-1})) \right]$$

Theorem: Time-series EWM regret bounds

Assume the one-period Markovian model with bounded outcome, correct specification of \mathcal{G} , and a complexity restriction of \mathcal{G} (i.e., $\{\xi_{t,X_{t-1}}(G) : G \in \mathcal{G}\}$ has finite VC dimension v), there exists a constant $C > 0$ such that

$$E_P[\sup_{g \in \mathcal{G}} \mathcal{W}_T(g|X_{T-1}) - \mathcal{W}_T(\hat{g}_{EWM}|X_{T-1})] \leq C \sqrt{\frac{v \log T}{T-1}}$$

holds.

Extension: longer dependence?

- Introducing q th-order Markovian structure, $2 \leq q < \infty$, is feasible
- $q = \infty$ is more challenging, though common SVAR modeling implies so, e.g.,

$$Y_t = \sum_{h=0}^{\infty} (\theta_h W_{t-h} + \phi_h \epsilon_{t-h})$$

- Approximate ∞ -Markov by finite order m -th Markov and run T-EWM. The regret bounds depend also on the approximation bias,

$$\begin{aligned} & \mathcal{W}_T(g^* | \mathcal{F}_{T-1}) - \mathcal{W}_T(\hat{g}_{CES} | \mathcal{F}_{T-1}) \\ & \leq 2c \sup_g |\widehat{\mathcal{W}}(g | w_{T-m:T-1}) - \bar{\mathcal{W}}(g | \mathcal{F}_{T-1})| + \text{bias}(m), \end{aligned}$$

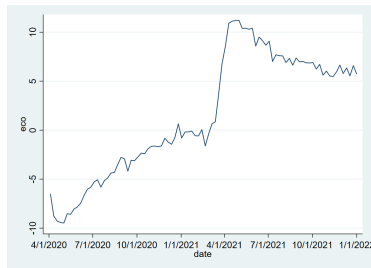
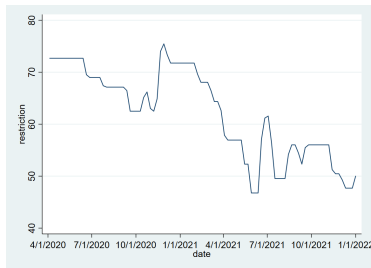
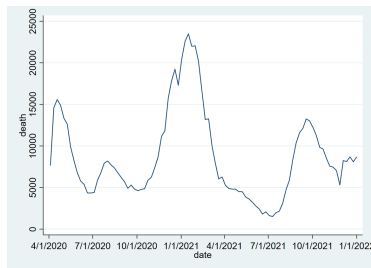
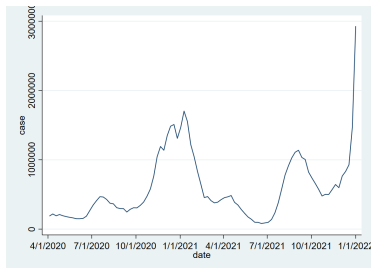
where $\text{bias}(m) = 2 \sup_g |\mathcal{W}_T(g | w_{-\infty:T-1}) - \mathcal{W}_T(g | w_{T-m:T-1})|$.

Empirical Example

- Binary treatment: relax the covid restriction level ($W = 0$) or not ($W = 1$)
- Outcome (welfare): $-1 \times$ two-week ahead deaths in US:
- Contextual information for the model of propensity score:
 $X_t = (\text{cases}_t, \text{deaths}_t, \text{change of cases}_t, \text{change of deaths}_t, \text{restriction level}_t, \text{vaccine coverage}_t, \text{economic condition}_t)$
- We maximize the expected welfare, $E(-1 \cdot \text{deaths}_{T+1})$, over the set of quadrant (threshold) policies based on
(change of deaths $_{T-1}$, restriction level $_{T-1}$)
- Data source:
 - ▶ Cases, deaths, and vaccinations: The CDC website
 - ▶ Restriction level: The Oxford Stringency Index
 - ▶ Economic condition: The Lewis-Mertens-Stock business conditions index

Empirical Example

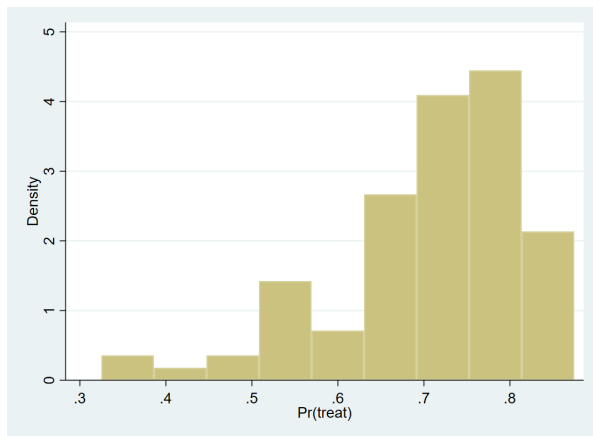
Data overview (from the top left to the bottom right: cases, deaths, restriction level, and economic condition)



Empirical Example (preliminary)

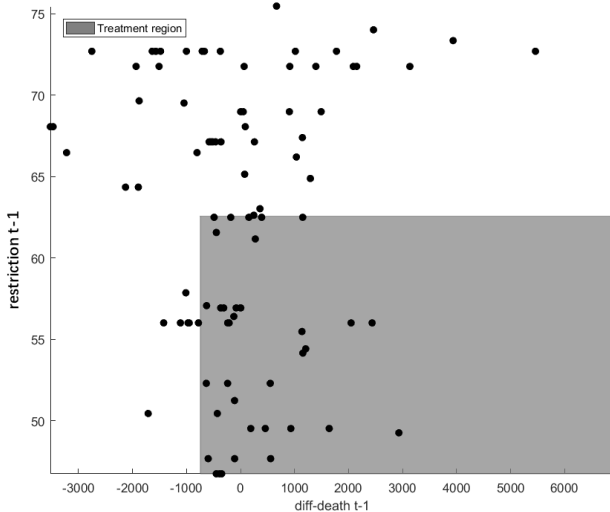
The propensity score model and the estimated propensity score

$$\log \left(\frac{\Pr(\text{keeping or increasing restriction at week } t)}{\Pr(\text{decreasing restriction at week } t)} \right) = \alpha + \beta X_{t-1}$$



Empirical Example (preliminary)

Policy choice



Literature

- **Potential outcome time-series**: Angrist, Jorda, Kuersteiner (18 JBES), Bojinov & Shephard (19 JASA), Rambachan & Shephard (21)
- **Treatment choice**: Manski(04 Ecta), Dehejia (05 JoE), Hirano & Porter (09 Ecta), Stoye (09, 12 JoE), Tetenov (12 JoE), Chamberlain (11 Handbook Chap), Kitagawa & Tetenov (18 Ecta), Mbakop & Tabord-Meehan (21 Ecta), Athey & Wager (21 Ecta), and more
- **(Individualized) Dynamic treatment regimes** (Large N - short T panel): Murphy (03 JRSSB), Zhao, Zeng, Laber, Kosorok (15 JASA), Sakaguchi (21), among many others
- **Empirical risk minimization in time-series**: Jiang & Tanner (10 ET), Brownlees & Guðmundsson (21), Brownlees & Llorens-Terrazas (21)

Concluding Remarks

- Propose a framework for data-driven policy choice in the time-series setting
- Non-trivial challenges distinguishing the time-series policy choice from the static one
- Under sequential unconfoundedness, finite-order Markovian, and invariance of welfare ordering, Manski's CES rule can be extended to time-series
- With the correct specification assumption added, EWM can be extended to time-series as well
- Our approach relies only on the causal and conditional independence structure among the variables, and is free from functional form, stationarity (in the outcome generating process), or prior distribution thereof