Ch 4 - Démonstration 13 non faite en classe.

Proposition :

Soit f une fonction de $\mathbb{R}$ dans $\mathbb{R}$.

$$\begin{aligned} & \exists\, a,b \in \mathbb{R} \text{ tels que } (a,b) \neq (0,0), \quad \forall\, t \in \mathbb{R}, \quad f(t) = a\cos(t) + b\sin(t) \\ \iff \quad & \exists\, A \in \mathbb{R}_+^*, \quad \exists\, \varphi \in \mathbb{R}, \quad \forall\, t \in \mathbb{R}, \quad f(t) = A\cos(t-\varphi) \end{aligned}$$

Démo 13 :

$$\begin{aligned} & \exists\, a,b \in \mathbb{R} \text{ tels que } (a,b) \neq (0,0), \quad \forall\, t \in \mathbb{R}, \quad f(t) = a\cos(t) + b\sin(t) \\ \iff \quad & \exists\, a,b \in \mathbb{R} \text{ tels que } (a,b) \neq (0,0), \quad \forall\, t \in \mathbb{R}, \quad f(t) = \operatorname{Re}\left((a-ib)e^{it}\right) \end{aligned}$$

(en effet : pour a, b, t réels,

$$(a-ib)e^{it} = (a-ib)\left(\cos t + i\sin t\right)$$

$$(a-ib)e^{it} = \underbrace{a\cos t + b\sin t}_{\in\mathbb{R}} + i\left(\underbrace{a\sin t - b\cos t}_{\in\mathbb{R}}\right)$$

d'où le résultat.)

$$\begin{aligned} \iff \quad & \exists\, a,b \in \mathbb{R} \text{ tels que } (a,b) \neq (0,0), \quad \forall\, t \in \mathbb{R}, \quad f(t) = \operatorname{Re}\left(\overline{(a+ib)}e^{it}\right) \\ \iff \quad & \exists\, z \in \mathbb{C}^*, \quad \forall\, t \in \mathbb{R}, \quad f(t) = \operatorname{Re}\left(\overline{z}e^{it}\right) \\ \iff \quad & \exists\, A \in \mathbb{R}_+^*, \quad \exists\, \varphi \in \mathbb{R}, \quad \forall\, t \in \mathbb{R}, \quad f(t) = \operatorname{Re}\left(\overline{Ae^{i\varphi}}e^{it}\right) \\ \iff \quad & \exists\, A \in \mathbb{R}_+^*, \quad \exists\, \varphi \in \mathbb{R}, \quad \forall\, t \in \mathbb{R}, \quad f(t) = \operatorname{Re}\left(Ae^{-i\varphi}e^{it}\right) \\ \iff \quad & \exists\, A \in \mathbb{R}_+^*, \quad \exists\, \varphi \in \mathbb{R}, \quad \forall\, t \in \mathbb{R}, \quad f(t) = \operatorname{Re}\left(Ae^{i(t-\varphi)}\right) \\ \iff \quad & \exists\, A \in \mathbb{R}_+^*, \quad \exists\, \varphi \in \mathbb{R}, \quad \forall\, t \in \mathbb{R}, \quad f(t) = A\operatorname{Re}\left(e^{i(t-\varphi)}\right) \text{ car } A \text{ réel} \\ \iff \quad & \exists\, A \in \mathbb{R}_+^*, \quad \exists\, \varphi \in \mathbb{R}, \quad \forall\, t \in \mathbb{R}, \quad f(t) = A\cos(t-\varphi) \end{aligned}$$