

## Entraînement au calcul algébrique : corrigé.

**Correction de la question 1.** 1°)  $\frac{1}{(-1)^n} = (-1)^n$

2°)  $(-1)^{n+2} = (-1)^n(-1)^2 = (-1)^n$

3°)  $(-1)^{2n} = ((-1)^2)^n = 1^n = 1$

4°)  $(-1)^{2n+1} = (-1)(-1)^{2n} = -1$  grâce à la question précédente

**Correction de la question 2.** 1°)

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$(a+b)(a-b) = a^2 - b^2$$

2°)

$$\begin{aligned} (a+b+c)^2 &= (a+b+c)(a+b+c) \\ &= a^2 + ab + ac + ba + b^2 + bc + ca + cb + c^2 \\ &= \boxed{a^2 + b^2 + c^2 + 2(ab + ac + bc)} \end{aligned}$$

**Correction de la question 3.** 1°)  $(a-b)(a^2+ab+b^2) = a^3 + a^2b + ab^2 - ba^2 - ab^2 - b^3 = \boxed{a^3 - b^3}$ .

2°)  $a^3 + b^3 = a^3 - (-b^3) = a^3 - (-b)^3 = (a - (-b))(a^2 + a(-b) + (-b)^2) = \boxed{(a+b)(a^2 - ab + b^2)}$ .

3°)  $27x^3 + 8 = 3^3x^3 + 2^3 = (3x)^3 + 2^3 = (3x+2)((3x)^2 - 3x \times 2 + 2^2) = \boxed{(3x+2)(9x^2 - 6x + 4)}$ . Le discriminant du trinôme qui apparaît est strictement négatif, on ne factorise pas plus dans  $\mathbb{R}$ .

**Correction de la question 4.** 1°)  $A = x^4 - x^2 = x^2x^2 - x^2 = x^2(x^2 - 1) = \boxed{x^2(x-1)(x+1)}$

2°)

$$\begin{aligned} B &= x^2 - 2x + 1 - (x-1)(2x+3) \\ &= (x-1)^2 - (x-1)(2x+3) \\ &= (x-1)(x-1-2x-3) \\ &= (x-1)(-x-4) = \boxed{(1-x)(x+4)} \end{aligned}$$

3°)

$$\begin{aligned} C &= (3x+2+x-1)(3x+2-(x-1)) \\ &= \boxed{(4x+1)(2x+3)} \end{aligned}$$

4°)

$$\begin{aligned} D &= x^2(x+1) + x + 1 \\ &= \boxed{(x^2+1)(x+1)} \end{aligned}$$

5°)

$$\begin{aligned} E &= (3x)^2 - 7^2 + (3x+7)(2x+3) \\ &= (3x+7)(3x-7) + (3x+7)(2x+3) \\ &= (3x+7)(3x-7+2x+3) \\ &= \boxed{(3x+7)(5x-4)} \end{aligned}$$

**Correction de la question 5.** 1°)  $\frac{7}{8} + \frac{5}{12} - \frac{4}{3} = \frac{7 \times 3 + 5 \times 2 - 4 \times 8}{24} = -\frac{1}{24}$

2°)  $\frac{1}{(n+1)^2} + \frac{1}{n+1} - \frac{1}{n} = \frac{n + n(n+1) - (n+1)^2}{n(n+1)^2} = -\frac{1}{n(n+1)^2}$

**Correction de la question 6.**

$$x = \frac{\frac{a}{\frac{b}{\frac{c}{d}}}}{\frac{c}{d}} = \frac{a}{b} \frac{1}{\frac{c}{d}} = \frac{a}{b} \frac{d}{c} = \boxed{\frac{ad}{bc}}.$$

$$y = \frac{\frac{a}{\frac{b}{\frac{c}{d}}}}{\frac{c}{d}} = \frac{a}{bd} = \boxed{\frac{ac}{bd}}.$$

$$z = \frac{\frac{a}{\frac{b}{\frac{c}{d}}}}{\frac{c}{d}} = \frac{ac}{bd} = \boxed{\frac{ac}{bd}}.$$

$$t = \frac{\frac{a}{\frac{b}{\frac{c}{d}}}}{\frac{c}{d}} = \frac{a}{bcd} = \boxed{\frac{a}{bcd}}.$$

$$A = \frac{\frac{a^2}{\frac{3b}{ac}}}{\frac{6b}{6b}} = \frac{6ba^2}{3bac} = \boxed{\frac{2a}{c}}.$$

$$B = \frac{\frac{\frac{3}{10} \times \frac{15}{9}}{\frac{2}{9} \times \frac{5}{15}}}{\frac{5 \times 2}{3 \times 3} \times \frac{5 \times 3}{3 \times 3} \times 2} = \frac{1}{\frac{3}{2}} = \boxed{\frac{2}{3}}.$$

$$C = \frac{6(3-\frac{1}{2})(4+\frac{1}{3})}{12(5+\frac{1}{4})(7-\frac{1}{3})} = \frac{\frac{5}{2} \frac{13}{3}}{\frac{21}{4} \frac{20}{3}} = \frac{\frac{5 \times 13}{2 \times 3}}{\frac{21 \times 4 \times 5}{2 \times 3}} = \frac{13}{21 \times 4} = \boxed{\frac{13}{84}}.$$

**Correction de la question 7.**  $A = \frac{x+1}{(x-1)(x+1)} - \frac{x-1}{(x+1)(x-1)} + \frac{2x}{1-x^2}$   
 $= \frac{x+1-x+1}{x^2-1} - \frac{2x}{x^2-1} = \frac{2-2x}{x^2-1} = \frac{-2(x-1)}{(x-1)(x+1)} = \frac{-2}{x+1}.$

$B = \frac{b(x-a)}{abx} + \frac{x(a-b)}{abx} + \frac{a(b-x)}{axb} = \frac{bx-ba+xa-xb+ab-ax}{abx} = 0$

**Correction de la question 8.**  $A = \frac{4}{3-\sqrt{5}} = \frac{4(3+\sqrt{5})}{9-5} = \boxed{3+\sqrt{5}}$

$B = \frac{1}{\sqrt{\sqrt{2}}} \sqrt{\frac{1+\sqrt{2}}{2}} = \sqrt{\frac{1+\sqrt{2}}{2\sqrt{2}}} = \sqrt{\frac{\sqrt{2}+2}{4}} = \boxed{\frac{\sqrt{\sqrt{2}+2}}{2}}$

$C = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})^2}{3-2} - \frac{(\sqrt{3}-\sqrt{2})^2}{3-2}$   
 $= 3+2+2\sqrt{6} - (3+2-2\sqrt{6}) = \boxed{4\sqrt{6}}$

**Correction de la question 9.**

$A = \frac{\frac{1}{1+x} + \frac{1-x}{(1+x)^2}}{\sqrt{\frac{1-x}{1+x} \left(1 + \frac{1-x}{1+x}\right)}} = \frac{\frac{1+x+1-x}{(1+x)^2}}{\frac{\sqrt{1-x}}{\sqrt{1+x}} \frac{1+x+1-x}{1+x}} = \frac{2}{(1+x)^2} \frac{1}{\frac{2\sqrt{1-x}}{(1+x)^{\frac{3}{2}}}} = \frac{1}{(1+x)^{\frac{1}{2}}} \frac{1}{\sqrt{1-x}}$   
 $A = \frac{1}{\sqrt{(1+x)(1-x)}} = \boxed{\frac{1}{\sqrt{1-x^2}}}$

**Correction de la question 10.**  $\frac{14^2 \times 9^2}{3^5 \times 7} = \frac{7^2 \times 2^2 \times (3^2)^2}{3^5 \times 7} = \frac{7 \times 4 \times 3^4}{3^5} = \frac{7 \times 4}{3} = \boxed{\frac{28}{3}}$

**Correction de la question 11.**  $x = \frac{2^3 3^2}{3^{-2} 2^{-2} 3^4 2^8} = \frac{2^3 3^2}{3^2 2^6} = \boxed{2^{-3}}$

$y = 2^{100} + 2^{101} = 2^{100}(1+2) = \boxed{3 \cdot 2^{100}}$

$z = 2^{101} - 2^{100} = 2^{100}(2-1) = \boxed{2^{100}}$

$t = \boxed{2 \cdot 3^{15}}$

$u = \frac{(3^2(-2)^4)^8}{((-3)^5 2^3)^{-2}} = 3^{16} 2^{32} ((-3)^5 2^3)^2 = 3^{16} 2^{32} 3^{10} 2^6 = \boxed{2^{38} 3^{26}}$

**Correction de la question 12.**

$A = 3x^2y^3 - y(xy)^2 = 3x^2y^3 - yx^2y^2 = 3x^2y^3 - x^2y^3 = \boxed{2x^2y^3}.$

$B = \frac{4x^2y^3 - (xy)^2y}{x^2y^2 \times (-x)^3} = \frac{4x^2y^3 - x^2y^3}{-x^2y^2x^3} = -\frac{3x^2y^3}{x^5y^2} = -\frac{3x^2y^2 \times y}{x^2y^2 \times x^3} = \boxed{-\frac{3y}{x^3}}.$

$C = \frac{(-a)^7 \times (-b^2c^3)^3}{-b^3c \times (-a)^5} = \frac{(-a^7) \times (-b^6c^9)}{-b^3c \times (-a^5)} = \frac{a^7b^6c^9}{a^5b^3c} = \boxed{a^2b^3c^8}.$

**Correction de la question 13.** Pour  $A = \frac{\frac{1}{1 + \frac{1}{1 + \frac{1}{\frac{1}{1 + \frac{1}{1 - \frac{1}{2}}}}}}}{\frac{1}{1 + \frac{1}{1 - \frac{1}{2}}}}$  :

$$1 + \frac{1}{1 + \frac{1}{2}} = 1 + \frac{1}{\frac{3}{2}} = 1 + \frac{2}{3} = \frac{5}{3} \quad \text{et} \quad 1 + \frac{1}{1 - \frac{1}{2}} = 1 + \frac{1}{\frac{1}{2}} = 1 + 2 = 3.$$

Donc  $A = \frac{\frac{5}{3}}{\frac{1}{3}} = \boxed{\frac{9}{5}}$ .

$$B = \frac{2 + \frac{2+a}{2-a}}{2 - \frac{2+a}{2-a}} = \frac{\frac{2(2-a) + 2+a}{2-a}}{\frac{2(2-a) - 2-a}{2-a}} = \frac{4-2a+2+a}{4-2a-2-a} = \boxed{\frac{6-a}{2-3a}}$$

**Correction de la question 14.**

$$\begin{aligned} A &= \frac{\frac{b+a-x}{ab}(x+a+b)}{\frac{b^2+a^2+2ab-x^2}{a^2b^2}} \\ &= \frac{(b+a-x)(x+a+b)}{b^2+a^2+2ab-x^2} ab \\ &= ab \end{aligned}$$

$$\begin{aligned} B &= \frac{(2x+3)^2}{2(2x-3)(2x+3)} - \frac{24x}{2(2x-3)(2x+3)} + \frac{(3-2x)(2x-3)}{2(2x+3)(2x-3)} \\ &= \frac{(2x+3)^2 - 24x - (2x-3)^2}{2(2x-3)(2x+3)} \\ &= \frac{(2x+3 - (2x-3))(2x+3 + 2x-3) - 24x}{2(2x-3)(2x+3)} \\ &= \frac{6.4x - 24x}{2(2x-3)(2x+3)} \\ &= 0 \end{aligned}$$

**Correction de la question 15.**  $7840 = 10 \times 784 = 2 \times 5 \times 2 \times (350 + 42) = 2^2 \times 5 \times 2 \times (175 + 21) = 2^3 \times 5 \times 196 = 2^3 \times 5 \times 2 \times 98 = 2^4 \times 5 \times 2 \times 49 = \boxed{2^5 \times 5 \times 7^2}$

**Correction de la question 16.**  $A = \frac{-3\left(\frac{2}{3}\right)^2 + 8\left(\frac{7}{2}\right)^2}{5\left(\frac{2}{5}\right)^2 - 6\left(\frac{4}{3}\right)^2} = \frac{-\frac{4}{3} + 2 \times 7^2}{\frac{4}{5} - 2\frac{16}{3}} = \frac{\frac{2}{3}(-2 + 3 \times 49)}{\frac{4}{15}(3 - 5 \times 8)} = \frac{\frac{2}{3} \times 145}{\frac{4}{15} \times (-37)} =$   
 $-\frac{2 \times 145 \times 15}{4 \times 3 \times 37} = -\frac{145 \times 5}{2 \times 37} = \boxed{-\frac{725}{74}}$

**Correction de la question 17.**

$$12^3 \times 3^3 = (12 \times 3)^3 = \boxed{36^3}$$

$$125^2 \times 3^6 = (5^3)^2 \times 3^6 = 5^6 \times 3^6 = \boxed{15^6}$$

$$3^3 \times 5^6 = 3^3 \times (5^2)^3 = (3 \times 25)^3 = \boxed{75^3}.$$

$7^2 \times 2^3$  ne peut pas s'écrire sous la forme voulue.

**Correction de la question 19.** 1°)

$$\begin{aligned} A &= 16(2x+7)^2 - 25(3x-7)^2 = (4(2x+7))^2 - (5(3x-7))^2 \\ &= (4(2x+7) - 5(3x-7))(4(2x+7) + 5(3x-7)) \\ &= (-7x+63)(23x-7) = \boxed{7(9-x)(23x-7)} \end{aligned}$$

2°)

$$\begin{aligned} B &= 9x^2(2x+1) - (2x+1) \\ &= (9x^2-1)(2x+1) \\ &= ((3x)^2-1)(2x+1) \\ &= \boxed{(3x-1)(3x+1)(2x+1)} \end{aligned}$$

3°)

$$\begin{aligned} C &= (4x^2-25)(x+2) - (x^2-4)(2x+5) + (5x+10)(2x+5) \\ &= (2x-5)(2x+5)(x+2) - (x-2)(x+2)(2x+5) + 5(x+2)(2x+5) \\ &= (2x+5)(x+2)(2x-5-(x-2)+5) \\ &= (2x+5)(x+2)(x+2) = \boxed{(2x+5)(x+2)^2} \end{aligned}$$

**Correction de la question 20.** Comme 4 et 5 sont strictement positifs,  $\frac{7}{4} < \frac{9}{5} \iff 7 \times 5 < 9 \times 4$  ie  $35 < 36$ , ce qui est bien le cas donc on a bien :  $\frac{7}{4} < \frac{9}{5}$ .

**Correction de la question 21.** Comme  $1 + \sqrt{2}$  et  $\sqrt{3}$  sont positifs, il suffit de comparer leurs carrés.  $(1 + \sqrt{2})^2 = 1 + 2 + 2\sqrt{2} = 3 + 2\sqrt{2} > \sqrt{3}^2$ . Donc,  $1 + \sqrt{2} > \sqrt{3}$ .

**Correction de la question 22.** Calculons  $\left(\frac{1+\sqrt{5}}{4}\right)^2 = \frac{1+5+2\sqrt{5}}{16} = \frac{3+\sqrt{5}}{8}$ .

Comme  $\frac{1+\sqrt{5}}{4}$  est un nombre positif, on a bien  $\sqrt{\frac{3+\sqrt{5}}{8}} = \frac{1+\sqrt{5}}{4}$ .

**Correction de la question 23.** Justifions d'abord que  $A$  existe.

$(4\sqrt{3})^2 = 48 < 7^2 = 49$  donc  $4\sqrt{3} < 7$  donc  $7 - 4\sqrt{3} > 0$ .

Calculons  $A^2$  pour commencer.

$$\begin{aligned} A^2 &= \left( \sqrt{7-4\sqrt{3}} - \sqrt{7+4\sqrt{3}} \right)^2 \\ &= 7 - 4\sqrt{3} + 7 + 4\sqrt{3} - 2\sqrt{7-4\sqrt{3}}\sqrt{7+4\sqrt{3}} \\ &= 14 - 2\sqrt{(7-4\sqrt{3})(7+4\sqrt{3})} = 14 - 2\sqrt{49-48} = \boxed{12} \end{aligned}$$

$0 \leq 7 - 4\sqrt{3} < 7 + 4\sqrt{3}$  donc  $\sqrt{7-4\sqrt{3}} < \sqrt{7+4\sqrt{3}}$ . Ainsi,  $A < 0$ .

On en déduit que :  $\boxed{A = -\sqrt{12} = -2\sqrt{3}}$ .

**Correction de la question 24.**  $e^{3\ln 2} = 2^3 = 8$ .

**Correction de la question 25.** Cette expression existe sur  $\mathbb{R}^*$ .

$\forall x \in \mathbb{R}^*, \ln(x^2) = \ln(|x|^2) = 2\ln(|x|)$  car  $|x| > 0$ .

**Correction de la question 26.**

$$\begin{aligned} 2\ln\left(\frac{3}{4}\right) - 3\ln\left(\frac{3}{8}\right) &= 2\ln(3) - 2\ln(4) - 3\ln(3) + 3\ln(8) \\ &= -\ln(3) - 2\ln(2^2) + 3\ln(2^3) \\ &= -\ln(3) - 2.2\ln(2) + 3.3\ln(2) \\ &= 5\ln(2) - \ln(3) \\ \ln\left(\frac{1}{\sqrt{2}}\right) &= -\ln(\sqrt{2}) \\ &= -\frac{1}{2}\ln(2) \end{aligned}$$

**Correction de la question 27.** L'expression existe ssi  $x > -1$ .

$$e^{x-\ln(x+1)} = e^x e^{-\ln(x+1)} = e^x \frac{1}{e^{\ln(x+1)}} = \frac{e^x}{x+1}.$$