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Statistical Computing
Home Assignment (Summer Term 2018)

I. First, consider a univariate optimization problem. Given a sample from a Cauchy$(0, \theta)$ distribution of size $n = 20$ with a pdf

$$f(x, \theta) = \frac{1}{\pi(1 + (x - \theta)^2)}, \qquad x \in \mathbb{R}$$

and a log-likelihood function

$$l(\theta, \boldsymbol{x}) = -n \ln \pi - \sum_{i=1}^{n} \ln(1 + (x - \theta)^2),$$

find the value of θ that maximizes $l(\theta, \boldsymbol{x})$ (the data can be found in the R template online).

1. Write short procedures for the following univariate optimization algorithms:
   (a) bisection method;
   (b) Newton's method;
   (c) scaled fixed point iteration.

   Instead of using a fixed number of iterations and a for-loop (as in class), you need to use the stopping criteria and a while-loop. Also stop a non-converging algorithm by using a maximum number of iterations (100).

2. For all of the algorithms above perform a time analysis: track the system's time required to find a maximum.

3. Create an output matrix containing the value of the optimum, the number of required iterations and the running time as such:

	Max.value	Iterations	Time
Bisection	-	-	-
Newton	-	-	-
Fixed point	-	-	-

II. Now consider some Monte Carlo problems.

1. Suppose X has a standard Normal distribution and you want to estimate $\mu = \mathrm{E}[h(X)]$ with $h(x) = \frac{x^2}{e^x - 1}$.
   (a) Compute a standard Monte Carlo estimator based on $n = 100,000$ observations.
   (b) **Compute an antithetic estimator based on the original sample split in half. Here use the first half $X_i,\ i = 1, \ldots, n/2$ and its transformation $-X_i$ to create negatively correlated observations.**
2. Consider finding $\sigma^2 = \mathrm{E}[X^2]$ when X has the density that is proportional to $q(x) = \exp(-|x|^3/3)$. Estimate σ^2 using importance sampling with standard weights.

III. Implement a Metropolis-Hastings algorithm to simulate from the mixture distribution

$$\delta\mathcal{N}(7, 0.5^2) + (1-\delta)\mathcal{N}(10, 0.5^2)$$

with $\delta = 0.7$, using $\mathcal{N}(x^{(t)}, 0.01^2)$ as the proposal distribution.

1. For each of three starting values, $x^{(0)} = 0,\ 7$ and 15, run the chain for 10,000 iterations.
2. Plot the sample path of the output from each chain.
3. For each of the simulations, create a histogram of the realizations with the true density superimposed on the histogram.
4. Now, suggest a change for the proposal distribution to improve the convergence properties of the chain. Repeat steps (1)–(3) to confirm.

Remarks: Download the R template from OLAT. Write sufficient comments for your code. Codes that are not written using the template and/or that return error messages will not be evaluated. If you are working in groups, make sure to note down every participant's name,ID and e-mail address.
Submission: Submit your scripts via email to *atitova[at]stat-econ.uni-kiel.de* until the end of July 10th (until 00:00:00, July 11th)