

Introduction to SEM

DATA 695 Research Capstone Project

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The University of Calgary, located in the heart of Southern Alberta, both acknowledges and pays tribute to the traditional territories of the peoples of Treaty 7, which include the Blackfoot Confederacy (comprised of the Siksika, the Piikani, and the Kainai First Nations), the Tsuut'ina First Nation, and the Stoney Nakoda (including Chiniki, Bearspaw, and Goodstoney First Nations). The City of Calgary is also home to the Métis Nation of Alberta (Districts 5 and 6).



Chapter 1: Introduction, Background, & Review

What is Structural Equation Modelling?

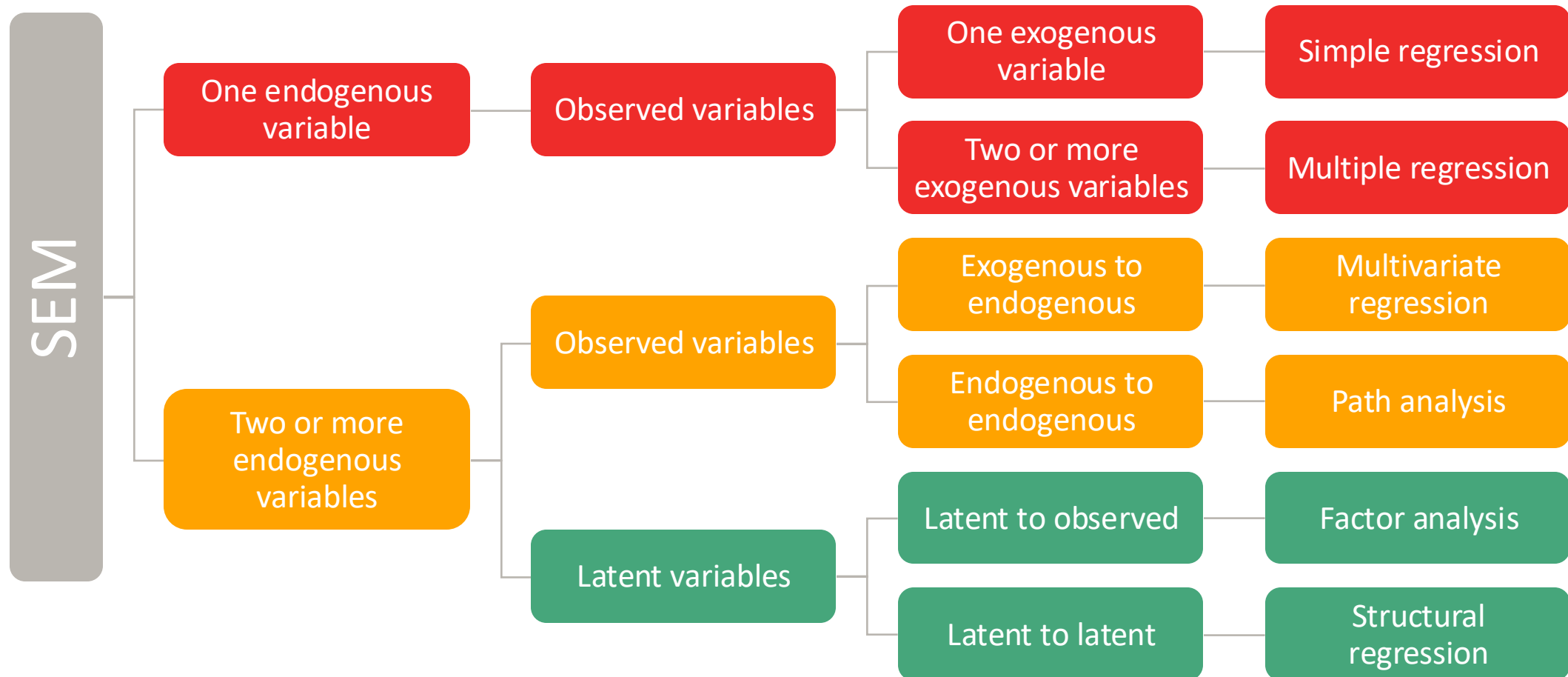


What is Structural Equation Modelling?

“...a linear model framework that models both *simultaneous regression equations* with *latent variables*.”

— Johnny Lin, PhD (2024)

What is Structural Equation Modelling?



Terminology: Exogenous vs. Endogenous Variables

- *Exogenous variables:*
 - Variables that are not expressed as a function of other variables; they exist “outside” the system of variables under study.
 - Often referred to as “independent” variables (denoted as x or x_i).
- *Endogenous variables:*
 - Variables that are expressed as a function of one or more other variables; they exist “inside” the system of variables under study.
 - Often referred to as “dependent” variables (denoted as y or y_i).

Terminology: Observed vs. Latent Variables

- *Observed variable(s)*:
 - Variables that can be directly measured or “observed”.
 - Example: Height, weight, age, etc.
- *Latent variable(s)*:
 - Variables that (usually) cannot be directly measured; instead, they are often “inferred” (denoted as lowercase “eta” η or η_i)
 - Example: Intelligence.

Brief Review of Linear Regression

- Given a data set $\{y_i, x_{i1}, \dots, x_{ip}\}$ of n samples,

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip} + \varepsilon_i$$

for $i = 1, 2, \dots, n$

where p = number of “independent” variables/predictors.

Brief Review of Linear Regression

- Given a data set $\{y_i, x_{i1}, \dots, x_{ip}\}$ of n samples,

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2} + \dots + \hat{\beta}_p x_{ip}$$

for $i = 1, 2, \dots, n$

where p = number of “independent” variables/predictors.

Note: the residual term is difference between the *observed* and *predicted* values of the outcome (i.e., $\varepsilon_i = y_i - \hat{y}_i$).

Brief Review of Linear Regression

- In R (or RStudio):

```
> model <- lm(y ~ x1 + ... + xp, data = ...)
```

Operator	Description
<- or =	Assign a value to a variable.
~ (tilde)	Define formula/relationship between two or more variables.

Linear Regression as a Structural Equation Model


- Step 1: Start with linear regression equation.

$$y_i = \beta_0 + \beta_1 x_{i1} + \varepsilon_i$$

Linear Regression as a Structural Equation Model

- Step 2: Take the **intercept**...

$$y_i = \boxed{\beta_0} + \beta_1 x_{i1} + \varepsilon_i$$


Intercept 

Linear Regression as a Structural Equation Model

- Step 2: And replace it with “alpha”.

$$y_i = \boxed{\alpha} + \beta_1 x_{i1} + \varepsilon_i$$



Intercept



Linear Regression as a Structural Equation Model

- Step 3: Take the regression **coefficient**(s)...



$$y_i = \boxed{\alpha} + \boxed{\beta_1} x_{i1} + \varepsilon_i$$

Intercept  Coefficient 

Linear Regression as a Structural Equation Model

- Step 3: And replace it/them with “gamma”.

$$y_i = \boxed{\alpha} + \boxed{\gamma_1} x_{i1} + \varepsilon_i$$

Intercept  Coefficient 

Linear Regression as a Structural Equation Model

- Step 4: Take the residual **error** term...

$$y_i = \boxed{\alpha} + \boxed{\gamma_1} x_{i1} + \boxed{\varepsilon_i}$$

Intercept

Coefficient

Error

The diagram shows the linear regression equation $y_i = \alpha + \gamma_1 x_{i1} + \varepsilon_i$. The term α is enclosed in a blue box, with a blue arrow pointing from the label 'Intercept' below it to the box. The term γ_1 is enclosed in a red box, with a red arrow pointing from the label 'Coefficient' below it to the box. The term ε_i is enclosed in a green box, with a green arrow pointing from the label 'Error' above it to the box.

Linear Regression as a Structural Equation Model

- Step 4: And replace it with “zeta”.

$$y_i = \boxed{\alpha} + \boxed{\gamma_1} x_{i1} + \boxed{\zeta_i}$$

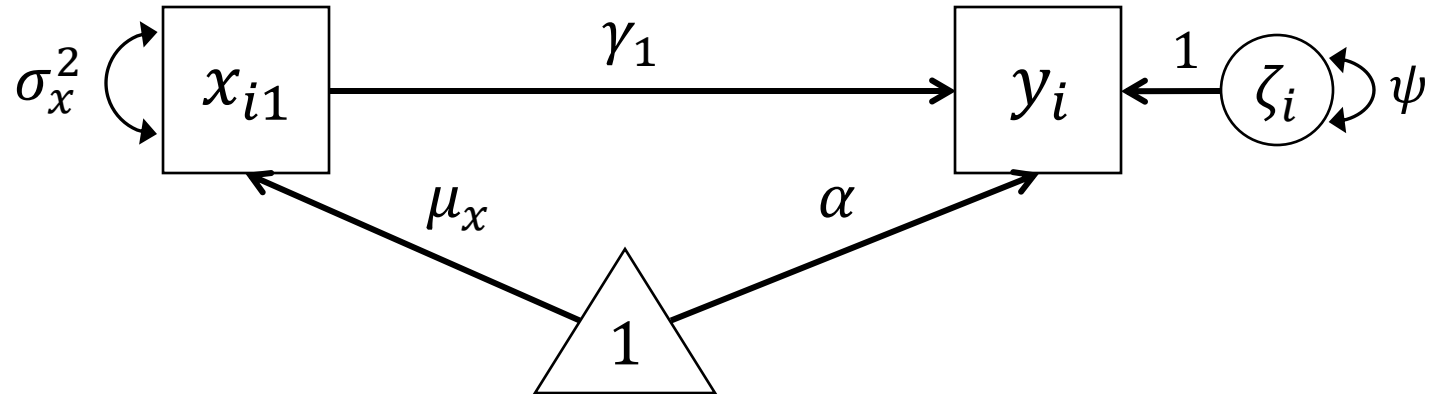
Intercept

Coefficient

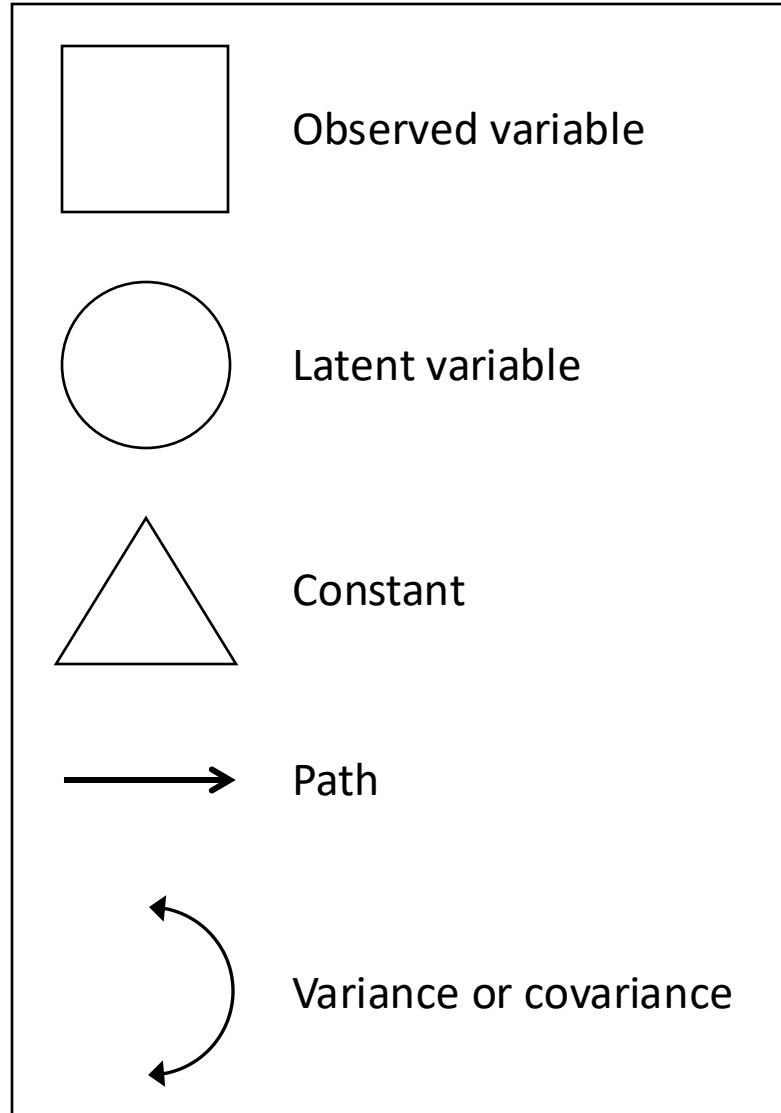
Error

The diagram shows the linear regression equation $y_i = \alpha + \gamma_1 x_{i1} + \zeta_i$. The term α is enclosed in a blue box, and a blue arrow points from the label 'Intercept' below to it. The term γ_1 is enclosed in a red box, and a red arrow points from the label 'Coefficient' below to it. The term ζ_i is enclosed in a green box, and a green arrow points from the label 'Error' above to it.

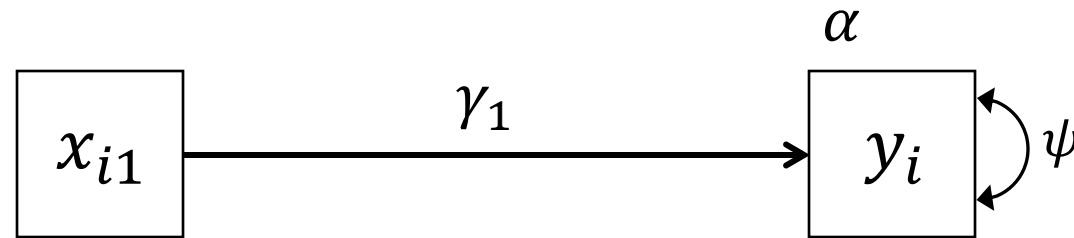
The Path Diagram



Path Diagram Legend

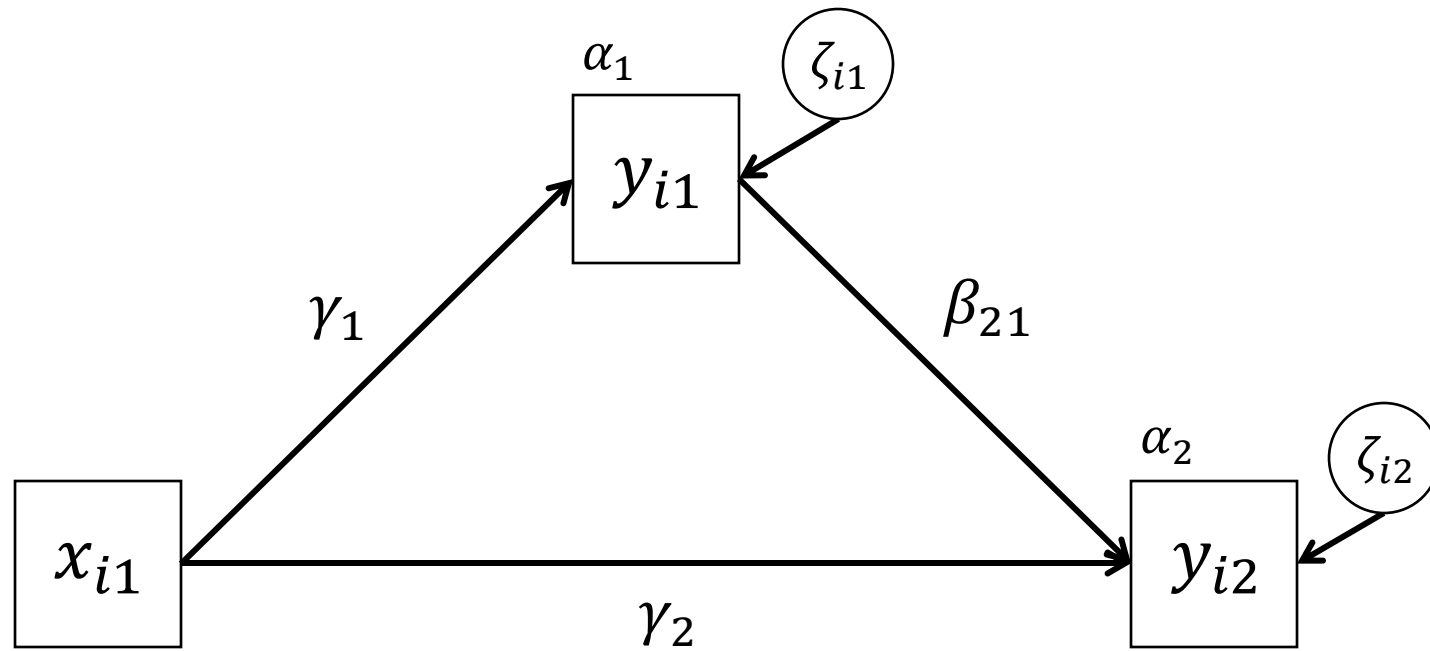


(Simplified) Path Diagram



Chapter 2: Path Analysis

Example: Recursive Model (Mediation)



References

1. Bauer, D. J., & Curran, P. J. (2024). *Introduction to Statistical Equation Modeling*. CenterStat.
<https://centerstat.org/workshop/introduction-to-structural-equation-modeling/>
2. Bollen, K. A. (1989). *Structural Equations with Latent Variables*. John Wiley & Sons, Incorporated.
<https://doi.org/10.1002/9781118619179>
3. Lin, J. (2024). *Introduction to Structural Equation Modeling (SEM) in R with lavaan*. OARC Stats – Statistical Consulting Web Resources | UCLA. <https://stats.oarc.ucla.edu/r/seminars/rsem/>