Introduction to SEM

DATA 695 Research Capstone Project

David Errington
Master of Data Science & Analytics Student
Faculty of Graduate Studies

July 6, 2025

UNIVERSITY OF CALGARY

Table of Contents

The University of Calgary, located in the heart of Southern Alberta, both acknowledges and pays tribute to the traditional territories of the peoples of Treaty 7, which include the Blackfoot Confederacy (comprised of the Siksika, the Piikani, and the Kainai First Nations), the Tsuut'ina First Nation, and the Stoney Nakoda (including Chiniki, Bearspaw, and Goodstoney First Nations). The City of Calgary is also home to the Métis Nation of Alberta (Districts 5 and 6).

Chapter 1: Introduction, Background, & Review

UNIVERSITY OF CALGARY

What is Structural Equation Modelling?

What is Structural Equation Modelling?

"…a linear model framework that models both *simultaneous regression equations* with *latent variables*."

— Johnny Lin, PhD (2024)

What is Structural Equation Modelling?

- SEM
  - One endogenous variable
    - Observed variables
      - One exogenous variable → Simple regression
      - Two or more exogenous variables → Multiple regression
  - Two or more endogenous variables
    - Observed variables
      - Exogenous to endogenous → Multivariate regression
      - Endogenous to endogenous → Path analysis
    - Latent variables
      - Latent to observed → Factor analysis
      - Latent to latent → Structural regression

Terminology: Exogenous vs. Endogenous Variables

- *Exogenous variables*:
  - Variables that are not expressed as a function of other variables; they exist "outside" the system of variables under study.
  - Often referred to as "independent" variables (denoted as x or x_i).

- *Endogenous variables*:
  - Variables that are expressed as a function of one or more other variables; they exist "inside" the system of variables under study.
  - Often referred to as "dependent" variables (denoted as y or y_i).

Terminology: Observed vs. Latent Variables

- *Observed variable(s)*:
  - Variables that can be directly measured or “observed”.
  - Example: Height, weight, age, etc.

- *Latent variable(s)*:
  - Variables that (usually) cannot be directly measured; instead, they are often “inferred” (denoted as lowercase “eta” η or η_i)
  - Example: Intelligence.

Brief Review of Linear Regression

- Given a data set $\{y_i, x_{i1}, \ldots, x_{ip}\}$ of n samples,

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} + \varepsilon_i$$

for $i = 1, 2, \ldots, n$

where $p =$ number of “independent” variables/predictors.

Brief Review of Linear Regression

- Given a data set $\{y_i, x_{i1}, \dots, x_{ip}\}$ of n samples,

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2} + \cdots + \hat{\beta}_p x_{ip}$$

for $i = 1, 2, \dots, n$

where $p =$ number of "independent" variables/predictors.

Note: the residual term is difference between the *observed* and *predicted* values of the outcome (i.e., $\varepsilon_i = y_i - \hat{y}_i$).

Brief Review of Linear Regression

- In R (or RStudio):

```
> model <- lm(y ~ x1 + ... + xp, data = ...)
```

Operator	Description
<- or =	Assign a value to a variable.
~ (tilde)	Define formula/relationship between two or more variables.

Linear Regression as a Structural Equation Model

- Step 1: Start with linear regression equation.

$$y_i = \beta_0 + \beta_1 x_{i1} + \varepsilon_i$$

Linear Regression as a Structural Equation Model

- Step 2: Take the intercept…

$$y_i = \boxed{\beta_0} + \beta_1 x_{i1} + \varepsilon_i$$

Intercept

Linear Regression as a Structural Equation Model

- Step 2: And replace it with "alpha".

$$y_i = \boxed{\alpha} + \beta_1 x_{i1} + \varepsilon_i$$

Intercept

UNIVERSITY OF CALGARY

Linear Regression as a Structural Equation Model

- Step 3: Take the regression coefficient(s)...

$$y_i = \underbrace{\boxed{\alpha}}_{\text{Intercept}} + \underbrace{\boxed{\beta_1}}_{\text{Coefficient}} x_{i1} + \varepsilon_i$$

UNIVERSITY OF CALGARY

Linear Regression as a Structural Equation Model

- Step 3: And replace it/them with “gamma”.

$$y_i = \boxed{\alpha} + \boxed{\gamma_1} x_{i1} + \varepsilon_i$$

Intercept

Coefficient

Linear Regression as a Structural Equation Model

- Step 4: Take the residual error term...

Error

$$y_i = \boxed{\alpha} + \boxed{\gamma_1} x_{i1} + \boxed{\varepsilon_i}$$

Intercept

Coefficient

UNIVERSITY OF CALGARY

Linear Regression as a Structural Equation Model

- Step 4: And replace it with “zeta”.

Error

$$y_i = \boxed{\alpha} + \boxed{\gamma_1} x_{i1} + \boxed{\zeta_i}$$

Intercept

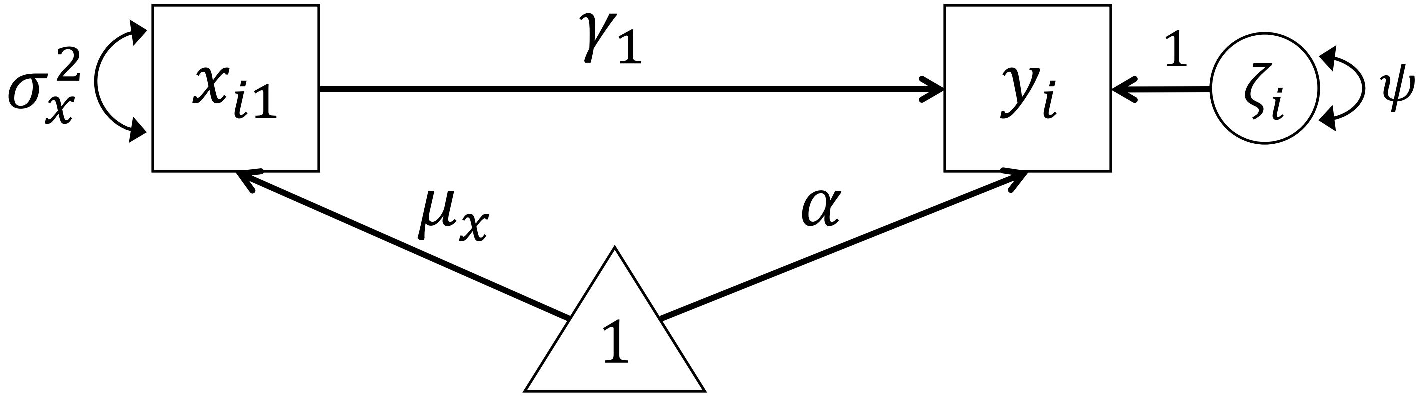
Coefficient

UNIVERSITY OF CALGARY

Chapter 2: Path Analysis

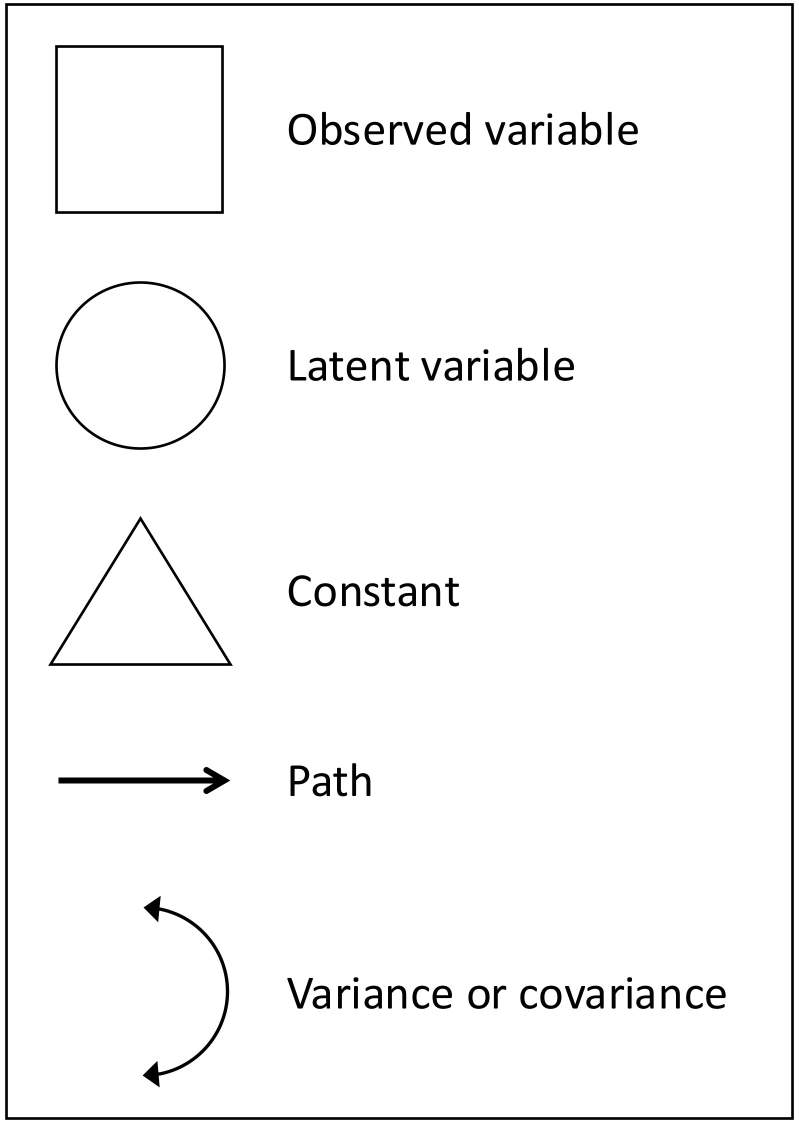
UNIVERSITY OF CALGARY

Example: Path Diagram

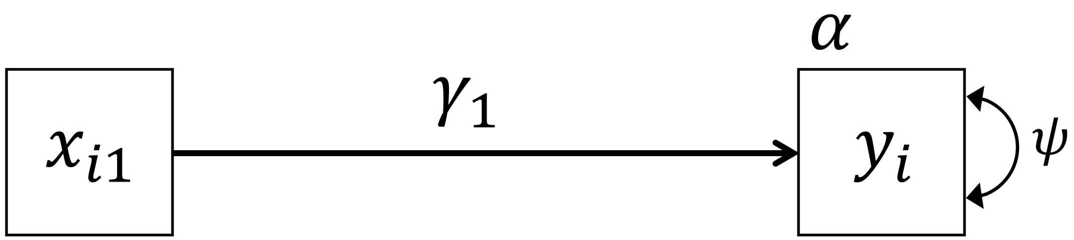


UNIVERSITY OF CALGARY

Path Diagram Legend



Example: (Simplified) Path Diagram



UNIVERSITY OF CALGARY

References

1. Bauer, D. J., & Curran, P. J. (2024). *Introduction to Statistical Equation Modeling*. CenterStat. https://centerstat.org/workshop/introduction-to-structural-equation-modeling/
2. Bollen, K. A. (1989). *Structural Equations with Latent Variables*. John Wiley & Sons, Incorporated. https://doi.org/10.1002/9781118619179
3. Lin, J. (2024). *Introduction to Structural Equation Modeling (SEM) in R with lavaan.* OARC Stats – Statistical Consulting Web Resources | UCLA. https://stats.oarc.ucla.edu/r/seminars/rsem/

UNIVERSITY OF CALGARY