A (Brief) Introduction to SEM

DATA 695 Research Capstone Project

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The University of Calgary, located in the heart of Southern Alberta, both acknowledges and pays tribute to the traditional territories of the peoples of Treaty 7, which include the Blackfoot Confederacy (comprised of the Siksika, the Piikani, and the Kainai First Nations), the Tsuut'ina First Nation, and the Stoney Nakoda (including Chiniki, Bearspaw, and Goodstoney First Nations). The City of Calgary is also home to the Métis Nation of Alberta (Districts 5 and 6).

Chapter 1: Introduction, Background, & Review

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What is Structural Equation Modelling?

*"Structural equation modeling is a **linear model framework** that models both simultaneous regression equations with latent variables."*

— Johnny Lin, PhD (2024)

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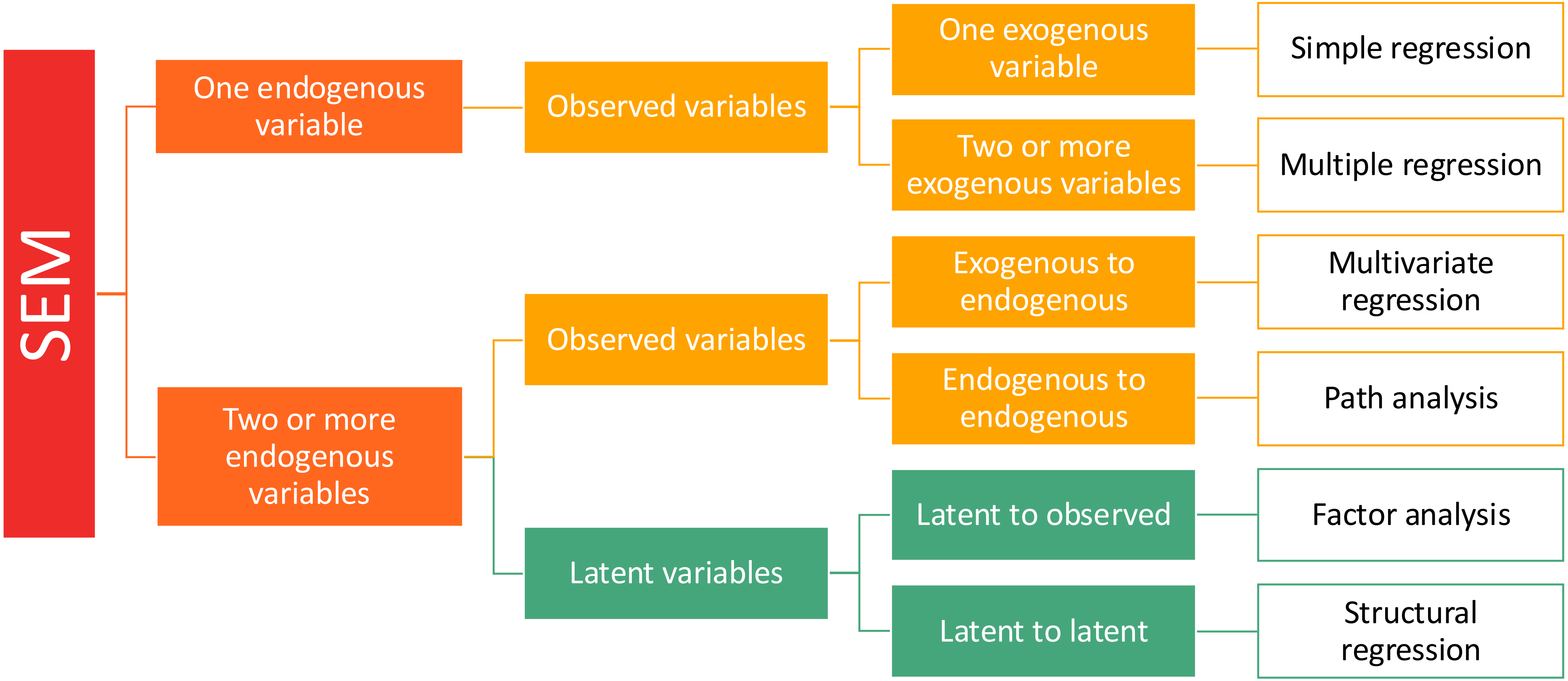
What is Structural Equation Modelling?



What is Structural Equation Modelling?

Definitions / Terminology

- ***Exogenous*** (a.k.a. ***independent***) *variables*:
  - Variables that are **not** expressed as a function of other variables; they exist "**outside**" the system of variables under study.
  - Notation:
    - x (single); or
    - x_i (multiple)
- ***Endogenous*** (a.k.a. ***dependent***) *variables*:
  - Variables that are expressed as a function of one or more other variables; they exist "**inside**" the system of variables under study.
  - Notation:
    - y (single); or
    - y_i (multiple)

Definitions / Terminology

- ***Observed*** *variable(s)*:
  - Variables that can be directly measured or “observed”.
  - Examples: Height, weight, age, etc.

- ***Latent*** *variable(s)*:
  - Variables that (usually) cannot be directly measured; often “inferred”.
  - Example: Intelligence.
  - Notation:
    - η “eta” (single); or
    - η_i (multiple)

Linear Regression as a Structural Equation Model

- Step 1: Start with linear regression equation.

$$y = \beta_0 + \beta_1 x_1 + \varepsilon$$

Linear Regression as a Structural Equation Model

- Step 2: Add subscript i to each variable.

$$y_i = \beta_0 + \beta_1 x_{1i} + \varepsilon$$

Linear Regression as a Structural Equation Model

- Step 3: Take the intercept…

$$y_i = \boxed{\beta_0} + \beta_1 x_{1i} + \varepsilon$$

Intercept

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Linear Regression as a Structural Equation Model

- Step 3: And replace it with α.

$$y_i = \boxed{\alpha} + \beta_1 x_{1i} + \varepsilon$$

Intercept

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Linear Regression as a Structural Equation Model

- Step 4: Take the regression coefficient(s)...

$$y_i = \underbrace{\boxed{\alpha}}_{\text{Intercept}} + \underbrace{\boxed{\beta_1}}_{\text{Coefficient}} x_{1i} + \varepsilon$$

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Linear Regression as a Structural Equation Model

- Step 4: And replace it(them) with γ.

$$y_i = \boxed{\alpha} + \boxed{\gamma_1} x_{1i} + \varepsilon$$

Intercept

Coefficient

Linear Regression as a Structural Equation Model

- Step 5: Take the residual error term...

$$y_i = \underbrace{\boxed{\alpha}}_{\text{Intercept}} + \underbrace{\boxed{\gamma_1}}_{\text{Coefficient}} x_{1i} + \overbrace{\boxed{\varepsilon}}^{\text{Error}}$$

Linear Regression as a Structural Equation Model

- Step 5: And replace it with ζ_i.

$$y_i = \underbrace{\boxed{\alpha}}_{\text{Intercept}} + \underbrace{\boxed{\gamma_1}}_{\text{Coefficient}} x_{1i} + \overbrace{\boxed{\zeta_i}}^{\text{Error}}$$

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References

- Lin, J. (2024). *Introduction to Structural Equation Modeling (SEM) in R with lavaan.* OARC Stats – Statistical Consulting Web Resources | UCLA. https://stats.oarc.ucla.edu/r/seminars/rsem/