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Investment practitioners and many academics are understandably preoccupied with identifying stock characteristics and strategies that offer the prospect of high risk-adjusted returns. Many sensible investment beliefs, when translated into portfolio weights, result in historical outperformance relative to the cap-weighted benchmark.

The naive expectation is that, when we invert the weighting algorithms of these sensible investment heuristics, effectively turning them upside down, these inverted strategies should underperform by roughly as much as the original algorithm outperformed. Instead, we find that these upside-down strategies also beat the cap-weighted benchmark, often by more than the upright originals. Indeed, even a portfolio generated by Malkiel's blindfolded monkey[1] throwing darts outperforms the market.[2]

How can it be that a monkey—who may have great skill with darts, but presumably has no skill in evaluating investments—adds value? Our findings suggest that the investment beliefs upon which many investment strategies are ostensibly based play little or no role in their outperformance.[3]

This does not mean that these strategies' outperformance is suspect. Rather, as it turns out, these investment beliefs work because they introduce, often unintentionally, value and small cap tilts into the portfolio. Counter-intuitively, when we invert these strategies, the resulting portfolios continue to display value and small cap bias. We demonstrate this paradoxical effect mathematically in Appendix A.

The results we present are puzzling until one grasps the derivations in the literature, starting with Berk [1997]. Berk [1997] and Arnott et al. [2011] argue that low prices create size and value effects. Berk ascribes this to hidden risk factors; Arnott et al. [2011] ascribe this to mean-reverting errors in price. Either way, falling prices lead to low book-to-market ratios and low market capitalization; whenever prices mean-revert, value and small stocks outperform. Hsu [2006] and Arnott and Hsu [2008] offer a conceptual framework where non-price-weighted portfolios, which contra-trade against price changes at each rebalancing, necessarily result in value and size tilts, *regardless of the weighting method chosen*.

In summary, value and small-cap exposures are naturally occurring portfolio characteristics, unless an investor constructs a portfolio to have a positive relationship between price and portfolio weights. In this article, we illustrate these theoretical results with simple, easily replicable portfolio back-tests. We do not attempt to comment on the interesting debate regarding the nature of value and small-cap premiums.

RESEARCH DESIGN

This research is motivated by the proliferation of quasi-passive equity index strategies and their noteworthy long-term outperformance against traditional cap-weighted benchmarks in back-tests, despite sometimes diametrically opposed investment beliefs. This leads to a natural skepticism. It's hard to believe that they could all work, in light of the longstanding literature on the mean-variance efficiency of the cap-weighed benchmark and the underperformance of active management. However, the empirical evidence from domestic and global market data, which extend back as far as data are available, suggests a robust outperformance.[4]

Our examination of this puzzle starts with portfolios formulated from an array of arguably sensible investment beliefs. We then invert these beliefs to create less intuitive strategies. In inverting the strategies, we tacitly examine whether these strategies outperform because they are predicated on meaningful investment theses and deep insights on capital markets, or for reasons unrelated to the investment theses. If the investment beliefs are the source of outperformance, then contradicting those beliefs should lead to underperformance.

For each of the investment beliefs, we create long-only equity portfolios using simple weighting heuristics. We then turn them upside down. For each quasi-index strategy, we form two inverse portfolios: 1) an inverse ratio portfolio, formed by normalizing the inverse weight 1/w and 2) an inverse complement portfolio, formed by normalizing the original portfolio's complementary weight (max(w)-w).[5] Except for some special situations, the two inverse portfolios generally have comparable characteristics. We also compute portfolios based on Malkiel's blindfolded dart-throwing monkey.[6]

To ensure that we invest in sufficiently liquid stocks, we restrict our universe to the largest 1,000 U.S. stocks by market capitalization.[7] We extend the analysis to global markets at both an individual country level and a global portfolio level. For the global country portfolios, we use the largest stocks by market capitalization, matching the number of stocks to the most popular local cap-weighted benchmark indices.[8] The global country results are generally qualitatively similar to the U.S. results, but often with a considerably larger magnitude of CAPM and Fama–French Four Factor model (FF4) alpha.[9]

We rebalance all portfolios annually, on the last trading day of the year. We back-test the portfolio schemes using as much historical data as are available in the CRSP/CompuStat merged database for the United States, and the Worldscope and Datastream databases for other developed countries. When necessary for portfolio construction, we estimate the risk parameters, such as variances and covariances, using the previous five years of monthly data. For example, for a covariance-based strategy portfolio for 2003, we will use the sample covariance matrix from 1998 to 2002.[10] Appendix B contains a strategy summary.

We break our analysis into five categories.

Reference Portfolios

We establish two reference portfolios. Our first reference portfolio is the cap-weighted portfolio, which most people consider a reasonable representation of the market. Exhibit 1 summarizes key attributes of the U.S. cap-weighted portfolio, using data from 1964 to 2012.

The second line of Exhibit 1 displays the equal-weighted (EW) portfolio, which represents perhaps the strongest level of investor naiveté, tacitly believing that all stocks have identical expected returns and risk attributes. This makes EW an interesting and sensible secondary reference portfolio. One might also interpret EW as an effective approach for capturing stock-price mean reversion where, at each rebalancing, the portfolio mechanically buys stocks that have fallen in price relative to others—unless they've fallen so far that they no longer make the size cut for the country—and sells stocks that have risen in price relative to others.

The EW portfolio produces 180 basis points (bps) per year of incremental performance over the cap-weighted reference benchmark. This incremental performance is almost entirely due to substantial size and value factor loading; EW delivers a 0.15 percent annualized FF4 alpha, with no statistical or economic significance. Throughout this article, it will become increasingly clear why the EW portfolio is a sensible reference benchmark for other non-price-weighted strategy indices.

EXHIBIT 1

Performance Summary, Strategies, Inverse Strategies, and Random Portfolios: United States (1964–2012)

	Strategy	Return	Standard Deviation	Sharpe Ratio	Value Added	Tracking Error	Information Ratio	CAPM Alpha	CAPM Beta	Alpha *t*-stat	Annual FF4 Alpha	Alpha *t*-stat	Market Exposure	Size Exposure	Value Exposure	Momentum Exposure
	U.S. Cap Weighted	9.66%	15.29%	0.29	0.00%	0.00%	0.00	0.00%	1.00	0.00	0.00%	0.00	1.00	0.00	0.00	0.00
	Equal Weight	11.46%	17.37%	0.36	1.80%	5.00%	0.36	1.63%	1.09	2.21	0.15%	0.38	1.05	0.38	0.12	−0.02
High Risk = High Reward	Volatility Weighted	12.15%	19.13%	0.36	2.49%	7.24%	0.34	1.98%	1.17	1.91	0.23%	0.46	1.10	0.55	0.16	−0.04
	Market Beta Weighted	11.89%	19.76%	0.34	2.23%	7.60%	0.29	1.55%	1.21	1.47	0.56%	1.01	1.13	0.54	0.13	−0.09
	Downside Semi-Deviation Weighted	12.13%	18.92%	0.37	2.47%	6.90%	0.36	1.99%	1.16	2.02	0.26%	0.52	1.10	0.52	0.17	−0.04
	Inverse-Ratio of Volatility Weighted	12.53%	15.64%	0.47	2.86%	5.36%	0.53	3.24%	0.96	3.97	0.58%	1.13	0.97	0.28	0.33	−0.03
	Inverse-Complement of Volatility Weighted	12.59%	16.40%	0.45	2.92%	5.30%	0.55	3.08%	1.02	3.79	0.64%	1.37	1.01	0.35	0.29	−0.03
	Inverse-Ratio of Market Beta Weighted	13.48%	15.02%	0.55	3.81%	7.22%	0.53	4.58%	0.87	4.30	0.86%	1.07	0.91	0.25	0.43	0.03
	Inverse-Complement of Market Beta Weighted	12.63%	16.16%	0.46	2.97%	5.35%	0.55	3.20%	1.00	3.89	0.48%	1.01	0.99	0.34	0.31	−0.01
	Inverse-Ratio of Downside Semi-Deviation Weighted	12.45%	15.62%	0.46	2.78%	5.30%	0.53	3.16%	0.96	3.91	0.48%	0.95	0.97	0.28	0.33	−0.02
	Inverse-Complement of Downside Semi-Deviation Weighted	12.51%	16.04%	0.45	2.84%	5.25%	0.54	3.09%	0.99	3.85	0.51%	1.04	0.99	0.31	0.31	−0.02
Optimization-Based	Minimum Variance	11.75%	11.69%	0.56	2.09%	8.04%	0.26	3.77%	0.65	4.06	1.05%	1.39	0.70	0.13	0.34	0.00
	Maximum Diversification	11.99%	13.96%	0.48	2.32%	6.58%	0.35	3.28%	0.82	3.57	0.40%	0.54	0.83	0.26	0.26	0.04
	Risk-Efficient (λ=2)	12.50%	16.81%	0.43	2.83%	5.35%	0.53	2.87%	1.04	3.52	0.63%	1.32	1.03	0.36	0.26	−0.03
	Risk Cluster Equal Weight	11.18%	14.61%	0.41	1.51%	4.92%	0.31	2.13%	0.91	2.95	0.31%	0.49	0.94	0.03	0.21	0.03
	Inverse-Ratio of Minimum Variance	12.66%	18.14%	0.41	2.99%	6.29%	0.48	2.70%	1.12	2.93	0.54%	1.07	1.08	0.45	0.25	−0.04
	Inverse-Complement of Minimum Variance	12.51%	17.41%	0.42	2.85%	5.83%	0.49	2.74%	1.08	3.13	0.47%	0.98	1.05	0.41	0.26	−0.04
	Inverse-Ratio of Maximum Diversification	12.48%	17.58%	0.41	2.82%	6.01%	0.47	2.68%	1.08	2.97	0.52%	0.94	1.07	0.38	0.28	−0.05
	Inverse-Complement of Maximum Diversification	12.37%	17.30%	0.41	2.71%	5.70%	0.48	2.63%	1.07	3.06	0.36%	0.76	1.05	0.40	0.26	−0.03
	Inverse-Ratio of Risk−Efficient (λ=2)	12.35%	17.32%	0.41	2.68%	5.81%	0.46	2.61%	1.07	2.97	0.25%	0.51	1.04	0.41	0.27	-0.03
	Inverse-Complement of Risk-Efficient (λ=2)	12.34%	17.53%	0.41	2.67%	5.96%	0.45	2.55%	1.08	2.85	0.21%	0.41	1.05	0.42	0.26	−0.03
	Inverse−Ratio of RCEW	13.23%	18.96%	0.42	3.57%	8.98%	0.40	3.37%	1.10	2.48	−0.16%	−0.19	1.06	0.62	0.41	−0.02
	Inverse-Complement of RCEW	12.43%	17.21%	0.42	2.76%	5.68%	0.49	2.71%	1.06	3.15	0.41%	0.85	1.04	0.40	0.26	−0.03
Fundamentals-Based	Book Value Weighted	11.23%	15.66%	0.38	1.57%	4.51%	0.35	1.87%	0.98	2.71	0.54%	1.56	1.03	0.03	0.34	−0.10
	5yr avg Earnings Weighted	11.18%	15.08%	0.40	1.52%	4.16%	0.36	1.95%	0.95	3.11	0.64%	1.92	1.00	0.00	0.31	−0.08
	Fundamental Weighted	11.60%	15.45%	0.41	1.93%	4.64%	0.42	2.30%	0.96	3.26	0.64%	1.83	1.01	0.05	0.37	−0.09
	Earnings Growth Weighted	12.42%	19.03%	0.38	2.76%	7.26%	0.38	2.29%	1.16	2.19	0.96%	1.34	1.09	0.47	0.04	0.00
	Inverse-Ratio of Book Value Weighted	13.86%	18.52%	0.47	4.19%	8.22%	0.51	4.03%	1.09	3.24	1.39%	2.14	1.05	0.56	0.39	−0.11
	Inverse-Complement of Book Value Weighted	13.04%	17.49%	0.45	3.38%	6.55%	0.52	3.33%	1.06	3.35	1.09%	2.05	1.05	0.39	0.37	−0.11
	Inverse-Ratio of 5yr avg Earnings Weighted	14.38%	18.34%	0.50	4.71%	8.58%	0.55	4.66%	1.06	3.56	1.65%	2.19	1.03	0.57	0.41	−0.09
	Inverse-Complement of 5yr avg Earnings Weighted	13.16%	17.08%	0.47	3.50%	6.44%	0.54	3.56%	1.04	3.62	1.12%	2.00	1.03	0.37	0.38	−0.09
	Inverse-Ratio of Fundamental Weighted	14.06%	18.77%	0.47	4.39%	8.63%	0.51	4.21%	1.10	3.22	1.40%	2.06	1.05	0.60	0.41	−0.11
	Inverse-Complement of Fundamental Weighted	13.34%	17.60%	0.46	3.67%	6.89%	0.53	3.63%	1.06	3.47	1.19%	2.11	1.05	0.41	0.40	−0.11
	Inverse-Ratio of Earnings Growth Weighted	10.26%	18.05%	0.28	0.59%	5.64%	0.10	0.26%	1.13	0.33	−0.95%	−2.17	1.07	0.42	0.10	−0.02
	Inverse-Complement of Earnings Growth Weighted	11.37%	17.27%	0.36	1.70%	4.90%	0.35	1.55%	1.09	2.14	0.08%	0.20	1.04	0.37	0.13	−0.02
	Average of 100 Malkiel's Monkey Portfolios	11.26%	18.34%	0.33	1.60%	7.76%	0.21	1.43%	1.09	1.22	−0.29%	−0.31	1.05	0.37	0.13	−0.02
	Average for Non-Cap-Weight Strategies, excl. Inverses	**11.75%**	**16.60%**	**0.40**	**2.09%**	**6.15%**	**0.35**	**2.23%**	**1.02**	**2.63**	**0.47%**	**0.96**	**1.00**	**0.28**	**0.22**	**−0.03**
	Average for All Inverse−Ratio Strategies	**12.88%**	**17.45%**	**0.44**	**3.22%**	**6.91%**	**0.46**	**3.23%**	**1.05**	**3.08**	**0.60%**	**0.88**	**1.03**	**0.44**	**0.33**	**−0.05**
	Average for All Inverse-Complement Strategies	**12.57%**	**17.04%**	**0.43**	**2.91%**	**5.80%**	**0.50**	**2.91%**	**1.05**	**3.30**	**0.60%**	**1.16**	**1.03**	**0.38**	**0.29**	**−0.05**

Source: Research Affiliates, based on CRSP/Compustat data.

Favoring High-Risk Stocks in our Portfolios

Given the theoretical and empirical links between risk and return, one might expect a link between higher returns and higher-risk stocks. A naive way to act on this belief, for investors willing to accept higher risk in the quest for higher returns, would be to build a portfolio that tilts toward more volatile stocks, or higher-beta stocks, or stocks with higher downside semi-deviation. We might expect these strategies to earn higher portfolio returns, rewarding us for our willingness to bear one of these types of incremental risk. This investment belief anchors our second set of strategies: weighting a portfolio proportional to conventional risk measures, such as market beta, volatility, or downside semi-variance of the constituent stocks. The second block of Exhibit 1—labeled "High Risk = High Reward"—explores these three strategies and their inverted forms. These strategies all work splendidly, beating the reference cap-weighted benchmark by between 2.23 percent and 2.49 percent per year.

When we flip the algorithm to favor companies with low volatility, low beta, or low downside semi-deviation, we get the expected drop in risk, relative to the risk-seeking strategies. Nonetheless, for all three risk-seeking strategies, our returns are even higher when we flip them and shun risk. The inverted portfolios add between 2.78 percent and 3.81 percent per year. These low-risk portfolios, as a result, have higher Sharpe ratios and higher CAPM alphas.

How can overweighting high-risk stocks and overweighting low risk stocks both lead to higher returns versus the cap-weighted benchmark? An examination of the FF4 factor decomposition in Table 1 reveals the key differences between the risk-seeking and risk-averse strategies: the latter have roughly two to three times as large a loading on the value factor and lower loading on the market factor. Net of the value effect and other factor tilts, we are left with annualized FF4 alphas that are statistically similar to zero.

Popular Covariance-Based Strategy Indices versus their Inverted Counterparts

The recent surge in interest in non-price-weighted market indices is a noteworthy development in the evolution of the indexing business. The revival of minimum variance, with roots dating back to the late 1960s, is the first among many.[11] The CAPM Capital Market Line is empirically flatter than theory would predict. Indeed, empirically, it often is downward sloping: in many markets, we find that low-volatility stocks produce higher returns than do high-volatility stocks. The minimum-variance (MinVar) portfolio represents a simple strategy for capturing this anomaly.

Well-respected quantitative index providers have introduced two other new strategies, which lean heavily on the Markowitz mean-variance optimization framework. The risk-efficient index assumes, among other things, that stock returns are related to downside semi-variances. The maximum-diversification index portfolio, on the other hand, assumes a linear relationship between stock returns and volatility. These differ from our earlier exploration of weighting in proportion to volatility or downside semi-variance in using an explicit mean-variance optimizer in portfolio construction. Another covariance-based index strategy is the "risk-cluster equal weight" portfolio, also known as the diversification-based index. The RCEW approach uses equally weighted industry-country clusters, selected on the basis of covariance, to form a portfolio that is less concentrated in individual countries and sectors, relative to cap-weighting.[12]

Empirically, they all work. In the United States, MinVar outperforms the cap-weighted market by 209 bps annually. Because of its very low beta and low volatility, the Sharpe ratio is the highest of any strategies that we tested, with the highest statistical significance on the CAPM alpha. However, the excess return is almost fully explained by exposure to FF4 factors, leaving no statistically meaningful FF4 alpha. The other covariance-based strategy indices also offer historical returns that outperform the cap-weighted market benchmark. As with MinVar, their CAPM alphas are economically large and statistically significant. As with MinVar, the FF4 four-factor model largely explains the excess returns.

In this section, when we invert the strategies, we focus on companies with high rather than low covariance. Again, our inverse strategies deliver outperformance over the cap-weighted benchmark, and we observe meaningfully positive CAPM alphas. And again, positive exposure to value and size explain most of the excess returns, leaving insignificant FF4 alphas.

Favoring Stocks with Large Fundamental Scale or Earnings Growth

Traditional analysts believe that fundamentals matter for stock price valuation: low prices relative to fundamentals suggest undervaluation and better subsequent returns. This fundamental approach anchors the value investing style popularized by Ben Graham in the 1930s and 1940s, which remains influential today. In this section, we test three portfolios weighted by the following fundamental measures: 1) book value, which tacitly creates a higher book-to-price ratio relative to the cap-weighted benchmark, 2) five-year average of reported earnings, leading to a higher earnings-to-price ratio than the cap-weighted benchmark, and 3) the four-metric composite method described by Arnott et al. [2005]. All three methods weight stocks drawn from a universe of the 1,000 largest companies in proportion to their financial fundamentals, using the method described in Arnott et al. [2005].[13] These portfolios are expected to have a value tilt, relative to the cap-weighted market, as the weighting metrics are value oriented.

The fourth portfolio in this category is explicitly constructed with a growth emphasis: it weights stocks proportional to their recent earnings growth, a strategy that emphasizes companies with the strongest recent earnings growth.[14] The Gordon Growth Model suggests that earnings growth drives stock returns. This has motivated the belief that fast-growing companies deliver high returns.

Consistent with the previous sections, Exhibit 1 shows that all of these strategies produce economically meaningful excess returns with no statistically significant FF4 alpha. The first three fundamental-weighted portfolios earn their excess returns from a value tilt, while the earnings growth-weighted strategy outperforms because of its small-cap tilt. We have constructed a growth-oriented portfolio that outperforms the cap-weighted benchmark, unlike most growth strategies, albeit without using cap weights to allocate to growth stocks. Note that, using FF4 metrics, our growth portfolio actually has a value tilt, not a growth tilt, in the U.S. data.

The inverse portfolios should intuitively result in the opposite characteristics and symmetrical results. They do not. Similar to what we previously observed, the upside-down strategies all win, often by substantial margins, because of positive exposures to value and small cap. We do observe that a few of these inverted strategies also deliver statistically significant alpha, net of the FF4 factor attributes. The positive FF4 alpha is somewhat surprising, because these strategies are mechanistic, with no special insights into the subtleties that drive the markets, and thus do not have skill.

Accordingly, we see two possible interpretations of these significant FF4 alphas. First, they could simply be statistical outliers. After all, 1 in 20 completely random time series will appear to have statistical significance at the five-percent level. Alternatively, the outliers could reflect a significant risk factor that is missing from the FF4 model. We leave the exploration of these observations for the future, and welcome others' investigations into this interesting topic.

Malkiel's Blindfolded Monkey

In the last section of Exhibit 1, we examine the performance of random portfolios. For those doubting the benefits of active management, the go-to portfolio strategy has been cap-weighted indexing, ever since the dawn of the capital asset pricing model (CAPM). The conventional wisdom generally assumes that the cap-weighted portfolio is the mean-variance efficient, neutral portfolio for investors without stock-picking skills. We challenge this premise by simulating random portfolios managed by Malkiel's dart-throwing monkey for comparison against the cap-weighted benchmark.

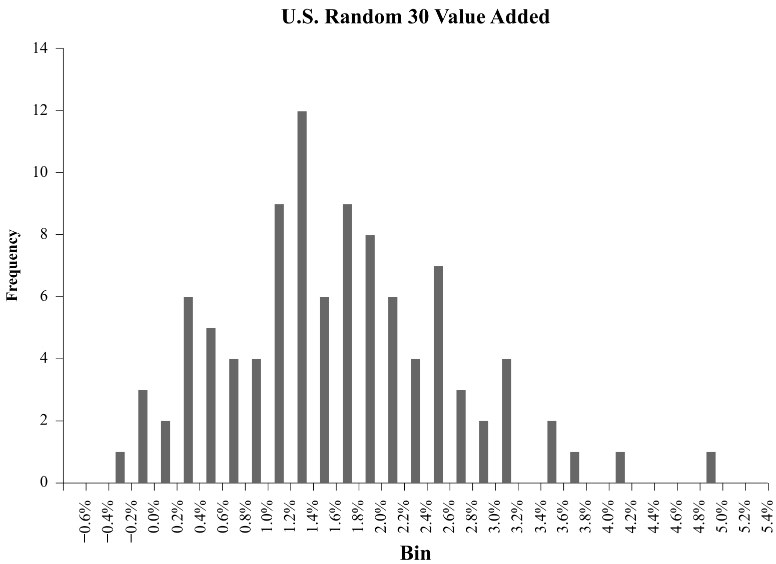
It would be time-consuming and costly to arrange for a monkey to throw darts at the *Wall Street Journal*'s stock pages, not to mention tracking down 50 years of their archived stock lists. We simulate a dart-throwing monkey by annually picking a random 30-stock portfolio from the top 1,000 largest stocks, by market capitalization. We then equally weight the random stock selections to form the portfolio. We repeat the exercise 100 times and examine both the individual year trials and the trials' average.

Malkiel surmised that his monkey would perform as well as the market; he was too modest. Our simulated monkey appears to be proficient in security selection, adding an average of 160 bps per year. True, the risk (volatility and beta) and tracking error are large, but we still have a respectable Sharpe ratio and an information ratio that looks like skill. Exhibit 2, panel A shows that the dartboard portfolio matches or beats the cap-weighted portfolio in 96 of the 100 trials. Better still, our

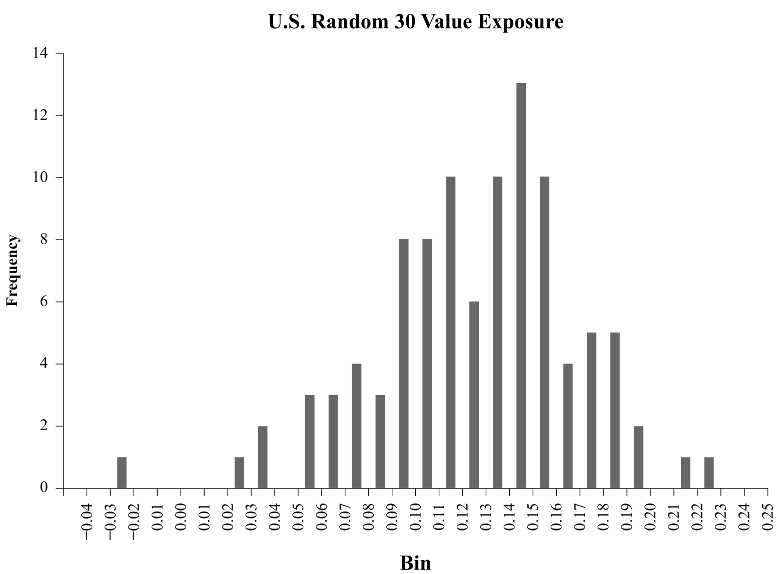
EXHIBIT 2

Random Strategies: 100 Simulations, United States (1964–2012)

Panel A: Histogram of Outperformance Frequencies



Panel B: Histogram of Value Loading—100 Simulations U.S. (1964–2012)



Source: Research Affiliates, based on CRSP/Compustat data.

monkey has an average CAPM alpha that is economically large and verges on statistical significance.[15]

Once again, the FF4 model explains essentially all the CAPM alpha. As in the other strategy indices we have examined so far, the monkey is introducing a size and value tilt. In Exhibit 2, panel B we can see that the monkey has a value tilt, on average over the 49 years, in 99 of the 100 trials. The astute observer will note that the average of our 100 monkey-managed portfolios has FF4 factor loadings identical to the equal-weight portfolio; this is, of course, a trivial convergence result associated with the law of large numbers.

WHY DO THESE STRATEGIES ALL WORK?

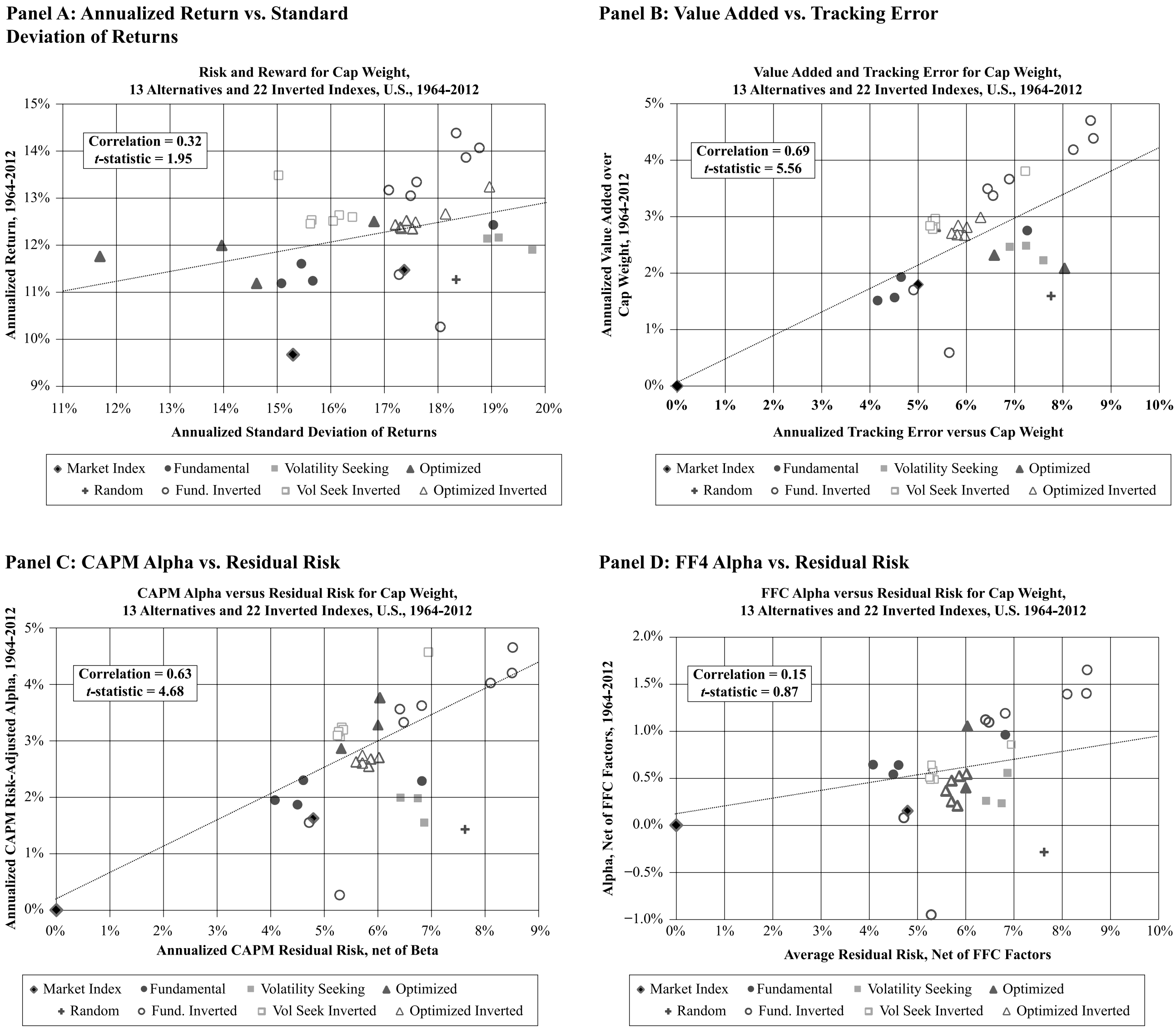
The well-reasoned and carefully crafted strategies tested in this article, which have spawned countless journal articles and white papers, all appear to work remarkably well, as shown in the summary statistics at the bottom of Exhibit 1. They only differ by their exposures to market, value, and size, which contributes to their differences in risk and return over time.

When we turn these strategies upside-down, inverting the resulting portfolio weights, we again find a near-perfect pattern of outperformance. Paradoxically, these upside-down strategies generally performed better than the right-side-up strategies that inspired them, with higher returns, Sharpe ratios, information ratios, and CAPM alphas. This clearly implies that the thesis for these alternative non-cap-weight index strategies is not the reason for their outperformance.

The graphs in Exhibit 3 provide us with a visual description of the excess-return driver. Panel A shows the conventional link between volatility and average returns. Because portfolio volatility is largely determined by its market beta, panel A would seem to suggest a classic CAPM relationship between beta and return. However, market beta is clearly not the only return driver, given the empirical evidence on value, size, and the low-volatility effect. Panel B shows the link between tracking error and value added, while panel C shows a similar link between CAPM residual risk and CAPM alpha, which is conventionally attributed to skill, if it's statistically significant. These two graphs suggest that the entirety of the value-added return shown in panel A is driven by non-market exposure(s).

Exhibit 3

Performance Characteristics of Market Cap, 13 Strategy Indexes, and 22 Inverses of Same (1964–2012)



Source: Research Affiliates, based on CRSP/Compustat data.

Panel D shows that, adjusting for the FF4 factor loadings, we are left with a small, unexplained alpha and a weak relationship between FF4 factor model residual and returns. This demonstrates that the FF4 factors are the key drivers of returns. The small, unexplained alpha and the weakly positive slope point to a path for future research, which is outside the scope of this article. There appear to be other priced risk factors (if it's not skill it presumably must be a risk factor), capable of producing economically meaningful and statistically significant sources of equity returns, which the FF4 factor model does not fully capture.

Although many of these strategies' performance—and their FF4 style attributes—seem markedly similar, especially in their divergence from the less-profitable cap-weighted strategies, their differences are noteworthy. This is best observed in Exhibit 4, which shows the top 10 holdings of a selected roster of these strategies. A casual examination of this table reveals the main problem for the inverse strategies: the top-10 roster is often populated by an array of relatively obscure companies, generally more thinly traded and less liquid than the cap-weight market leaders. The exceptions are self-evident, and appear only in the original strategies, never their inverse variants.

We draw two important lessons from this research. First, the investment thesis behind each of these strategies—no matter how thoughtful, intuitive, or compelling—is not the source of the incremental return, alpha, or information ratio. The thesis matters little; the resulting value and size tilts are the dominant reason behind these strategies' success.

Second, a size bias and, more significantly, a value bias exist in almost all of these strategy indices, whether we engineer for it or not. By comparison, a growth bias seems nearly impossible to find. That's a good thing, given the historical evidence of growth-biased portfolios' weak performance. Indeed, even a portfolio weighted toward stocks with strong historical fundamental growth in earnings exhibits a modest value tilt, instead of a growth tilt.

In Appendix A, we provide the theoretical explanation for these perplexing empirical observations. Intuitively, any strategy that implicitly weights by a valuation metric that is not price-based would tend to have a lower price-to-value ratio, relative to the cap-weighted index. We shouldn't attribute much, if any, of a strategy's success to the investment thesis that was the basis of its development.

Further, the inverse portfolios demonstrate that cap weighting appears to be surprisingly easy to beat, at least historically. Random portfolios selected by dart-throwing monkeys, and other inane or bizarre portfolios, would evidently do the job.

INTERNATIONAL EVIDENCE

We extend our analysis to global markets and find that the U.S. results are by no means an aberration. Exhibit 5 shows the results for the Global Developed World Markets (using the current MSCI definition for our country roster), from 1991 to 2012.[16] With only one exception, all these global strategies historically added value. And, with only one exception, the inverted strategies also add value. The CAPM alphas for the strategies are almost all positive, many showing statistical significance. For 18 of the 22 inverted strategies, results are better than the underlying strategy. The FF4 alphas in the global arena are generally stronger, both in economic terms and in statistical significance, than for the United States, despite a shorter history. Let the quest for the missing risk factor(s) begin!

SUMMARY

Many sensible investment beliefs, when translated into portfolio-weighting strategies, result in outperformance against the cap-weighted benchmark index. But so do the arguably nonsensical inverses of those weighting strategies. This paradoxical empirical result, which is observed in a large array of long-only strategies globally, is a consequence of the fact that seemingly unrelated strategies that are not based on value or small cap size often have unintended and almost unavoidable value and small-cap tilts, as do their inverse strategies.

The resulting factor tilts are the primary sources of outperformance, rather than the underlying investment beliefs. Even Malkiel's blindfolded monkey throwing darts at the *Wall Street Journal* would produce a portfolio strategy with a value and size bias that would have outperformed historically. Our empirical results support an assertion that value and size arise naturally in non-price-weighted strategies and constitute the main source of their return advantage.

What are we to make of the result that popular strategy indexes, when inverted, produce even better outperformance? It may behoove investors to emphasize more the FF4 factor-based analysis when analyzing investment philosophies. When random portfolios and irrational investment strategies all lead to outperformance, a simple outperformance measure becomes an unreliable gauge of skill.

For simplicity's sake, we omit the discussion of transaction costs and investment capacity. At the same time, costs and capacity differences between strategies can make a significant difference for investors who are interested in assessing these strategies' true investment benefits. Given that both sensible and senseless strategies outperform for the same reasons (value and small-cap tilts), potential investors would do well to base much of their decisions on a comparison of implementation costs associated with turnover and market-price impact.[17]

EXHIBIT 4

Top 10 Holdings and Weights, Selected Strategies,* and Inverse Strategies: United States (January 1, 2012)

Market		High Risk = High Reward		Optimization-Based				Fundamentals-Based			
Capitalization Weighted		Volatility Weighted		Minimum Variance		Risk Cluster Equal Weight		Fundamental Weighted		EPS Growth	
Exxon Mobil Corp	3.1%	Human Genome Sciences Inc	0.5%	Illumina Inc	3.3%	Altria Group Inc	5.0%	Exxon Mobil Corp	3.0%	H S N Inc New	1.5%
Apple Inc	2.9%	Questcor Pharmaceuticals Inc	0.4%	Kimberly Clark Corp	2.9%	Constellation Brands Inc	5.0%	General Electric Co	2.5%	Wendys Arbys Group Inc	1.5%
Microsoft Corp	1.7%	Pier 1 Imports Inc De	0.4%	Harleysville Group Inc	2.8%	Reynolds American Inc	3.4%	Bank Of America Corp	2.2%	American Water Works Co Inc	1.2%
International Business Machs Cor	1.7%	American International Group Inc	0.4%	General Mills Inc	2.8%	Coca Cola Co	2.4%	A T & T Inc	2.1%	Solera Holdings Inc	1.0%
Chevron Corp New	1.6%	Dollar Thrifty Automotive Grp In	0.4%	Wal Mart Stores Inc	2.8%	Lorillard Inc	2.2%	Wal Mart Stores Inc	1.9%	Biomarin Pharmaceutical Inc	1.0%
Wal Mart Stores Inc	1.6%	M G M Mirage	0.3%	Campbell Soup Co	2.6%	Loews Corp	2.1%	Citigroup Inc	1.9%	Healthsouth Corp	1.0%
General Electric Co	1.4%	M B I A Inc	0.3%	Newmont Mining Corp	2.5%	Mohawk Industries Inc	1.9%	Chevron Corp New	1.9%	Ariba Inc	1.0%
Procter & Gamble Co	1.4%	Genworth Financial Inc	0.3%	P G & E Corp	2.4%	Nike Inc	1.6%	Jpmorgan Chase & Co	1.7%	Dr Pepper Snapple Group Inc	1.0%
A T & T Inc	1.4%	Las Vegas Sands Corp	0.3%	S A I C Inc	2.4%	Pepsico Inc	1.6%	Berkshire Hathaway Inc Del	1.7%	T W Telecom Inc	0.9%
Johnson & Johnson	1.4%	Medivation Inc	0.3%	Flowers Foods Inc	2.2%	Mcdonalds Corp	1.5%	Pfizer Inc	1.5%	Domtar Corp	0.9%

Inverse-Ratio of Volatility Weighted		Inverse-Ratio of Minimum Variance	Inverse-Ratio of Risk Cluster Equal Weight		Inverse-Ratio of Fundamental Weighted		Inverse-Ratio of EPS Growth
Progress Energy Inc	0.3%	Not Applicable	P M C Sierra Inc	0.7%	C B R L Group Inc	0.3%	Not Applicable
Duke Energy Corp New	0.3%	Hundreds of Companies	Netgear Inc	0.7%	Arris Group Inc	0.3%	Hundreds of Companies
Southern Co	0.3%	at the Top of the List	Vishay Intertechnology Inc	0.6%	John Bean Technologies Corp	0.2%	at the Top of the List
Kimberly Clark Corp	0.2%	Have Identical Weight,	Intersil Corp	0.6%	Geo Group Inc	0.2%	Have Identical Weight,
General Mills Inc	0.2%	Because of Zero Weight	International Rectifier Corp	0.6%	W & T Offshore Inc	0.2%	Because of Zero Weight
Wisconsin Energy Corp	0.2%	in the Original Index	First American Corp Calif	0.6%	Old Dominion Freight Line Inc	0.2%	in the Original Index
Nstar	0.2%		Coinstar Inc	0.6%	Comstock Resources Inc	0.2%	
Consolidated Edison Inc	0.2%		Itron Inc	0.6%	Mcmoran Exploration Co	0.2%	
U G I Corp New	0.2%		Microsemi Corp	0.6%	Macquarie Infrastructure Co Llc	0.2%	
X C E L Energy Inc	0.2%		M K S Instruments Inc	0.6%	Bon Ton Stores Inc	0.2%	

Inverse-Complement of Volatility Weighted		Inverse-Complement of Minimum Variance	Inverse-Complement of Risk Cluster Equal Weight		Inverse-Complement of Fundamental Weighted		Inverse-Complement of EPS Growth
Progress Energy Inc	0.1%	Not Applicable	P M C Sierra Inc	0.1%	C B R L Group Inc	0.1%	Not Applicable
Duke Energy Corp New	0.1%	Hundreds of Companies	Netgear Inc	0.1%	Arris Group Inc	0.1%	Hundreds of Companies
Southern Co	0.1%	at the Top of the List	Vishay Intertechnology Inc	0.1%	John Bean Technologies Corp	0.1%	at the Top of the List
Kimberly Clark Corp	0.1%	Have Identical Weight,	Intersil Corp	0.1%	Geo Group Inc	0.1%	Have Identical Weight,
General Mills Inc	0.1%	Because of Zero Weight	International Rectifier Corp	0.1%	W & T Offshore Inc	0.1%	Because of Zero Weight
Wisconsin Energy Corp	0.1%	in the Original Index	First American Corp Calif	0.1%	Old Dominion Freight Line Inc	0.1%	in the Original Index
Nstar	0.1%		Coinstar Inc	0.1%	Comstock Resources Inc	0.1%	
Consolidated Edison Inc	0.1%		Itron Inc	0.1%	Mcmoran Exploration Co	0.1%	
U G I Corp New	0.1%		Microsemi Corp	0.1%	Macquarie Infrastructure Co Llc	0.1%	
X C E L Energy Inc	0.1%		M K S Instruments Inc	0.1%	Bon Ton Stores Inc	0.1%	

*We exclude equal-weight and random portfolios, neither of which has a well-defined top 10 list. Nor do we include portfolios with large rosters of companies with identical weights at the top of their respective lists.

Source: Research Affiliates based on CRSP/Compustat data.

Exhibit 5

Performance Summary, Strategies, Inverse Strategies, and Random Portfolios: Global Developed Markets (1991–2012)

	Strategy	Return	Standard Deviation	Sharpe Ratio	Value Added	Tracking Error	Information Ratio	CAPM Alpha	CAPM Beta	Alpha t-stat	Annual FF4 Alpha	Alpha t-stat	Market Exposure	Size Exposure	Value Exposure	Momentum Exposure	FFC Residual	CAPM Residual
	Global Cap Weighted	7.15%	15.15%	0.26	0.00%	0.00%	0.00	0.00%	1.00	0.00	0.00%	0.00	1.00	0.00	0.00	0.00	0.00%	0.00%
	Equal Weight	8.36%	15.45%	0.34	1.21%	2.71%	0.45	1.32%	1.00	2.04	0.28%	0.65	1.02	0.25	0.15	–0.02	1.86%	2.80%
High Risk = High Reward	Volatility Weighted	7.86%	16.89%	0.28	0.71%	3.84%	0.19	0.51%	1.08	0.55	0.12%	0.20	1.10	0.31	0.13	–0.06	2.71%	3.67%
	Market Beta Weighted	6.58%	18.81%	0.18	–0.57%	5.81%	–0.10	–1.24%	1.20	–0.89	–0.13%	–0.13	1.19	0.37	0.03	–0.15	4.00%	5.13%
	Downside Semi-Deviation Weighted	8.29%	16.78%	0.31	1.14%	3.88%	0.29	0.97%	1.07	1.04	0.55%	0.83	1.09	0.29	0.15	–0.07	2.82%	3.76%
	Inverse-Ratio of Volatility Weighted	9.32%	13.94%	0.44	2.17%	4.11%	0.53	2.73%	0.89	2.77	0.77%	1.28	0.92	0.13	0.34	–0.04	2.55%	3.91%
	Inverse-Complement of Volatility Weighted	8.99%	14.81%	0.40	1.84%	3.42%	0.54	2.16%	0.95	2.63	0.69%	1.28	0.98	0.19	0.27	–0.05	2.28%	3.49%
	Inverse-Ratio of Market Beta Weighted	9.44%	12.34%	0.51	2.29%	6.85%	0.33	3.49%	0.72	2.12	0.66%	0.64	0.77	0.01	0.44	0.01	4.35%	5.65%
	Inverse-Complement of Market Beta Weighted	9.31%	14.31%	0.43	2.16%	3.84%	0.56	2.65%	0.91	2.87	0.71%	1.12	0.94	0.17	0.30	–0.01	2.69%	3.74%
	Inverse-Ratio of Downside Semi-Deviation Weighted	9.11%	13.89%	0.43	1.96%	4.08%	0.48	2.53%	0.88	2.58	0.54%	0.90	0.92	0.14	0.33	–0.03	2.54%	3.86%
	Inverse-Complement of Downside Semi-Deviation Weighted	8.83%	14.41%	0.40	1.68%	3.71%	0.45	2.11%	0.92	2.37	0.36%	0.62	0.95	0.17	0.30	–0.04	2.42%	3.68%
Optimization-Based	Minimum Variance	8.40%	9.89%	0.53	1.25%	9.65%	0.13	3.20%	0.53	1.38	1.73%	1.33	0.55	0.02	0.30	–0.06	5.49%	6.15%
	Maximum Diversification	7.14%	11.33%	0.35	0.00%	9.09%	0.00	1.59%	0.62	0.73	0.12%	0.08	0.65	0.11	0.24	0.01	6.52%	6.77%
	Risk–Efficient (λ=2)	9.00%	14.82%	0.40	1.85%	3.47%	0.53	2.17%	0.95	2.61	0.53%	0.93	0.98	0.19	0.28	–0.03	2.40%	3.53%
	Risk Cluster Equal Weight	9.48%	15.90%	0.40	2.33%	6.54%	0.36	2.63%	0.95	1.68	0.97%	0.66	1.00	0.25	0.21	0.08	6.25%	6.67%
	Inverse-Ratio of Minimum Variance	8.70%	16.22%	0.34	1.55%	3.46%	0.45	1.50%	1.04	1.80	0.42%	0.76	1.07	0.24	0.23	–0.05	2.36%	3.50%
	Inverse-Complement of Minimum Variance	8.77%	15.50%	0.36	1.62%	3.32%	0.49	1.75%	0.99	2.20	0.47%	0.88	1.02	0.22	0.25	–0.05	2.28%	3.44%
	Inverse-Ratio of Maximum Diversification	8.90%	15.86%	0.36	1.75%	3.67%	0.48	1.81%	1.01	2.06	0.50%	0.88	1.04	0.21	0.29	–0.07	2.41%	3.80%
	Inverse-Complement of Maximum Diversification	8.80%	15.35%	0.37	1.65%	3.38%	0.49	1.83%	0.98	2.26	0.49%	0.91	1.01	0.21	0.26	–0.05	2.30%	3.50%
	Inverse-Ratio of Risk-Efficient (λ=2)	8.55%	15.46%	0.35	1.40%	3.52%	0.40	1.56%	0.99	1.85	0.44%	0.75	1.01	0.22	0.25	–0.06	2.48%	3.65%
	Inverse-Complement of Risk-Efficient (λ=2)	8.51%	15.68%	0.34	1.37%	3.59%	0.38	1.47%	1.00	1.71	0.45%	0.75	1.02	0.23	0.24	–0.07	2.57%	3.72%
	Inverse-Ratio of RCEW	9.44%	16.70%	0.38	2.29%	6.53%	0.35	2.34%	1.02	1.50	0.63%	0.42	1.05	0.14	0.28	0.02	6.41%	6.78%
	Inverse-Complement of RCEW	8.74%	15.22%	0.37	1.59%	3.41%	0.47	1.80%	0.97	2.21	0.47%	0.86	1.00	0.21	0.26	–0.05	2.32%	3.52%
Fundamentals-Based	Book Value Weighted	9.50%	16.09%	0.40	2.35%	4.78%	0.49	2.47%	1.00	2.15	1.31%	2.22	1.02	0.09	0.40	–0.12	2.50%	4.79%
	5yr avg Earnings Weighted	11.20%	15.28%	0.51	3.83%	5.01%	0.76	3.65%	0.95	3.04	2.36%	3.28	0.97	–0.01	0.39	–0.09	3.05%	4.97%
	Fundamental Weighted	11.00%	15.33%	0.49	3.63%	5.06%	0.72	3.43%	0.96	2.82	1.93%	2.98	0.98	0.09	0.43	–0.11	2.74%	5.03%
	Earnings Growth Weighted	8.83%	17.06%	0.33	1.68%	4.19%	0.40	1.37%	1.11	1.36	1.55%	1.91	1.11	0.27	–0.02	–0.04	3.44%	3.99%
	Inverse-Ratio of Book Value Weighted	10.60%	15.51%	0.48	3.45%	5.65%	0.61	3.76%	0.95	2.78	1.94%	2.60	0.98	0.33	0.46	–0.13	3.16%	5.76%
	Inverse-Complement of Book Value Weighted	10.51%	15.60%	0.47	3.37%	5.30%	0.64	3.64%	0.96	2.86	1.95%	2.90	0.99	0.26	0.45	–0.13	2.84%	5.41%
	Inverse-Ratio of 5yr avg Earnings Weighted	12.45%	15.40%	0.58	5.08%	6.12%	0.83	4.82%	0.94	3.29	2.70%	3.28	0.98	0.29	0.50	–0.12	3.49%	6.06%
	Inverse-Complement of 5yr avg Earnings Weighted	12.40%	15.35%	0.58	5.03%	5.70%	0.88	4.77%	0.94	3.49	2.79%	3.63	0.98	0.21	0.48	–0.11	3.25%	5.64%
	Inverse-Ratio of Fundamental Weighted	12.53%	15.67%	0.58	5.16%	6.41%	0.80	4.73%	0.95	3.08	2.81%	3.44	0.99	0.35	0.51	–0.15	3.45%	6.37%
	Inverse-Complement of Fundamental Weighted	12.32%	15.50%	0.57	4.95%	5.91%	0.84	4.56%	0.95	3.22	2.74%	3.70	0.98	0.28	0.49	–0.13	3.14%	5.87%
	Inverse-Ratio of Earnings Growth Weighted	6.60%	15.92%	0.22	–0.55%	4.51%	–0.12	–0.41%	0.99	–0.38	–1.20%	–1.57	1.02	0.43	0.06	0.02	3.25%	4.57%
	Inverse-Complement of Earnings Growth Weighted	8.36%	15.24%	0.34	1.22%	2.73%	0.45	1.39%	0.99	2.12	0.23%	0.54	1.01	0.25	0.17	–0.02	1.81%	2.81%
	Average of 100 Malkiel's Monkey Portfolios	8.12%	16.36%	0.31	0.97%	6.35%	0.16	1.10%	1.00	0.72	0.15%	0.10	1.02	0.23	0.18	–0.03	5.92%	6.34%
	Average for Non-Cap-Weight Strategies, excl. Inverses	**8.75%**	**15.38%**	**0.37**	**1.57%**	**5.41%**	**0.34**	**1.78%**	**0.96**	**1.48**	**0.88%**	**1.15**	**0.98**	**0.19**	**0.22**	**–0.05**	**3.82%**	**4.89%**
	Average for All Inverse-Ratio Strategies	**9.60%**	**15.17%**	**0.43**	**2.41%**	**4.99%**	**0.47**	**2.62%**	**0.94**	**2.13**	**0.93%**	**1.22**	**0.98**	**0.23**	**0.34**	**–0.06**	**3.31%**	**4.90%**
	Average for All Inverse-Complement Strategies	**9.60%**	**15.18%**	**0.42**	**2.41%**	**4.03%**	**0.56**	**2.56%**	**0.96**	**2.54**	**1.03%**	**1.56**	**0.99**	**0.22**	**0.32**	**–0.06**	**2.54%**	**4.07%**

Source: Research Affiliates, based on Worldscope/Datastream data.

Exhibit 6

Global Strategies Performance Summary (1991–2012)*

Country	Strategy	Cap Weighted	Volatility Weighted	Market Beta Weighted	Inverse-Ratio of Volatility Weighted	Inverse-Complement of Volatility Weighted	Inverse-Ratio of Market Beta Weighted	Inverse-Complement of Market Beta Weighted	Minimum Variance	Maximum Diversification	Inverse-Ratio of Minimum Variance	Inverse-Complement of Minimum Variance	Inverse-Ratio of Maximum Diversification	Inverse-Complement of Maximum Diversification	Book Weighted	Inverse-Ratio of Book Weighted	Inverse-Complement of Book Weighted
Australia	Return	12.4%	16.7%	16.4%	15.3%	16.6%	17.6%	16.6%	17.4%	23.8%	15.5%	16.2%	14.4%	15.5%	14.3%	14.2%	13.9%
	Volatility	20.9%	27.4%	26.4%	20.8%	22.2%	22.1%	22.3%	19.2%	25.9%	28.8%	24.3%	22.3%	22.8%	21.2%	23.4%	22.2%
	Value Added		4.3%	4.0%	2.9%	4.2%	5.1%	4.2%	5.0%	11.3%	3.0%	3.7%	1.9%	3.1%	1.9%	1.8%	1.5%
	4-factor Alpha		5.7%	4.8%	3.8%	4.9%	6.4%	5.0%	5.6%	12.1%	5.4%	4.7%	3.0%	4.0%	2.0%	3.7%	2.7%
	4-factor Alpha t-stat		1.76	2.03	3.04	3.14	2.84	2.76	3.34	3.36	1.48	2.24	2.17	2.52	2.46	2.09	1.93
	Size Exposure		0.48	0.32	0.20	0.26	0.31	0.29	0.12	0.41	0.48	0.35	0.25	0.29	–0.02	0.40	0.25
	Value Exposure		–0.05	–0.05	–0.01	–0.01	–0.02	–0.02	0.02	–0.04	–0.04	–0.02	–0.03	–0.03	0.01	–0.02	–0.01
Canada	Return	10.5%	10.5%	10.3%	12.6%	12.3%	10.9%	12.1%	11.7%	10.5%	10.8%	11.6%	12.6%	12.1%	11.8%	11.8%	12.0%
	Volatility	19.3%	20.7%	20.6%	17.1%	17.7%	19.2%	18.0%	15.1%	17.7%	23.5%	20.2%	18.9%	18.9%	18.2%	18.4%	18.2%
	Value Added		–0.1%	–0.2%	2.0%	1.8%	0.3%	1.5%	1.1%	0.0%	0.2%	1.1%	2.1%	1.6%	1.3%	1.3%	1.4%
	4-factor Alpha		–0.5%	0.5%	1.6%	1.3%	–0.6%	0.8%	1.6%	0.0%	–0.8%	0.3%	1.4%	0.9%	1.5%	1.2%	1.4%
	4-factor Alpha t-stat		–0.32	0.36	1.40	1.08	–0.30	0.59	1.10	0.01	–0.39	0.21	0.98	0.75	1.77	0.80	1.02
	Size Exposure		0.20	0.18	0.07	0.08	0.04	0.10	0.08	0.18	0.14	0.14	0.04	0.11	–0.01	0.16	0.11
	Value Exposure		0.15	0.04	0.31	0.30	0.43	0.33	0.33	0.20	0.22	0.22	0.35	0.25	0.25	0.35	0.35
France	Return	8.7%	10.2%	9.8%	11.3%	11.3%	11.9%	11.1%	12.3%	10.8%	10.9%	10.3%	10.6%	10.6%	9.5%	12.8%	11.7%
	Volatility	20.0%	21.2%	22.2%	19.5%	19.6%	18.8%	19.2%	16.8%	18.6%	23.3%	21.9%	21.7%	21.6%	21.5%	20.4%	20.6%
	Value Added		1.5%	1.1%	2.7%	2.7%	3.2%	2.5%	3.7%	2.2%	2.2%	1.6%	1.9%	1.9%	0.8%	4.1%	3.1%
	4-factor Alpha		1.8%	2.2%	2.2%	2.3%	2.2%	1.6%	3.0%	1.7%	3.0%	1.8%	2.2%	1.8%	0.9%	3.9%	3.0%
	4-factor Alpha t-stat		1.62	1.96	2.35	2.40	1.99	1.57	2.24	1.34	2.15	1.65	1.89	1.69	1.26	2.90	2.72
	Size Exposure		0.42	0.40	0.35	0.35	0.38	0.38	0.31	0.44	0.44	0.41	0.32	0.38	0.09	0.60	0.47
	Value Exposure		0.16	0.11	0.22	0.20	0.27	0.25	0.23	0.16	0.16	0.18	0.24	0.23	0.29	0.30	0.29
Japan	Return	0.3%	0.9%	0.6%	2.2%	1.9%	1.5%	1.7%	1.2%	–0.6%	1.5%	1.7%	1.9%	1.8%	3.4%	3.9%	3.8%
	Volatility	20.1%	21.0%	21.9%	18.8%	19.1%	19.1%	19.3%	15.4%	17.8%	20.8%	20.2%	20.3%	20.0%	20.0%	20.5%	20.3%
	Value Added		0.6%	0.3%	1.9%	1.7%	1.3%	1.4%	0.9%	–0.9%	1.2%	1.4%	1.6%	1.5%	3.2%	3.6%	3.5%
	4-factor Alpha		–0.2%	0.4%	0.1%	0.0%	–0.6%	–0.2%	–1.8%	–2.5%	0.1%	0.1%	0.3%	0.2%	1.3%	1.5%	1.5%
	4-factor Alpha t-stat		–0.23	0.34	0.08	0.04	–0.39	–0.22	–1.15	–1.67	0.18	0.12	0.33	0.20	2.52	1.71	1.88
	Size Exposure		0.27	0.21	0.23	0.24	0.31	0.26	0.15	0.20	0.27	0.26	0.25	0.25	0.05	0.39	0.30
	Value Exposure		0.20	0.10	0.29	0.26	0.29	0.26	0.29	0.20	0.24	0.25	0.26	0.25	0.33	0.39	0.38
U.K.	Return	8.0%	8.7%	7.5%	9.2%	9.1%	10.1%	9.4%	9.5%	10.1%	9.0%	8.8%	8.5%	9.0%	9.5%	10.0%	10.4%
	Volatility	16.4%	18.3%	20.0%	16.3%	16.2%	15.5%	16.0%	14.9%	15.4%	20.4%	18.1%	18.9%	18.1%	18.9%	19.1%	18.6%
	Value Added		0.7%	–0.5%	1.2%	1.1%	2.1%	1.4%	1.5%	2.1%	1.0%	0.8%	0.5%	1.0%	1.5%	2.0%	2.4%
	4-factor Alpha		1.9%	2.6%	1.4%	1.2%	1.1%	1.1%	1.4%	1.7%	2.5%	1.7%	2.0%	1.9%	1.6%	2.2%	2.4%
	4-factor Alpha t-stat		1.65	1.78	1.41	1.24	0.97	1.20	0.96	1.49	1.55	1.56	1.58	1.75	1.69	1.56	2.03
	Size Exposure		0.30	0.25	0.21	0.21	0.24	0.24	0.20	0.21	0.23	0.26	0.25	0.26	0.08	0.41	0.31
	Value Exposure		–0.03	–0.13	0.09	0.10	0.15	0.12	0.10	0.10	–0.03	0.03	0.02	0.04	0.31	0.35	0.34
Global	Return	7.1%	7.9%	6.6%	9.3%	9.0%	9.4%	9.3%	8.4%	7.1%	8.7%	8.8%	8.9%	8.8%	9.5%	10.6%	10.5%
	Volatility	15.1%	16.9%	18.8%	13.9%	14.8%	12.3%	14.3%	9.9%	11.3%	16.2%	15.5%	15.9%	15.4%	16.1%	15.5%	15.6%
	Value Added		0.7%	–0.6%	2.2%	1.8%	2.3%	2.2%	1.2%	0.0%	1.5%	1.6%	1.7%	1.7%	2.4%	3.4%	3.4%
	4-factor Alpha		0.1%	–0.1%	0.8%	0.7%	0.7%	0.7%	1.7%	0.1%	0.4%	0.5%	0.5%	0.5%	1.3%	1.9%	2.0%
	4-factor Alpha t-stat		0.20	–0.13	1.28	1.28	0.64	1.12	1.33	0.08	0.76	0.88	0.88	0.91	2.22	2.60	2.90
	Size Exposure		0.31	0.37	0.13	0.19	0.01	0.17	0.02	0.11	0.24	0.22	0.21	0.21	0.09	0.33	0.26
	Value Exposure		0.13	0.03	0.34	0.27	0.44	0.30	0.30	0.24	0.23	0.25	0.29	0.26	0.40	0.46	0.45

**Due to space limitations, we report only a fraction of the inverse strategies for the international markets. Omitted simulations display similar results and are available by request.*

Source: Research Affiliates, based on Worldscope/Datastream data.

EXHIBIT 7

Global Random Strategies Performance Summary (1991–2012)

Country	Strategy	Return	Volatility	Sharpe Ratio	Value Added	Tracking Error	Information Ratio	Outperformed out of 100
	Cap Weighted	12.41%	20.90%	0.44				
Australia	100 Portfolio Avg	12.83%	22.62%	0.43	0.42%	7.84%	0.05	68
	100 Portfolio Std Dev	1.36%	0.52%	0.06	1.36%	0.48%	0.18	
	Cap Weighted	10.55%	19.32%	0.38				
Canada	100 Portfolio Avg	11.82%	19.05%	0.46	1.27%	7.31%	0.17	88
	100 Portfolio Std Dev	1.11%	0.51%	0.06	1.11%	0.47%	0.15	
	Cap Weighted	8.67%	19.98%	0.28				
France	100 Portfolio Avg	10.72%	20.81%	0.37	2.05%	6.00%	0.34	100
	100 Portfolio Std Dev	0.85%	0.38%	0.04	0.85%	0.32%	0.14	
	Cap Weighted	0.29%	20.10%	–0.14				
Japan	100 Portfolio Avg	1.71%	20.63%	–0.07	1.43%	7.37%	0.20	88
	100 Portfolio Std Dev	1.51%	0.56%	0.07	1.51%	0.49%	0.21	
	Cap Weighted	7.99%	16.42%	0.30				
U.K.	100 Portfolio Avg	9.12%	17.66%	0.34	1.12%	5.91%	0.19	92
	100 Portfolio Std Dev	0.82%	0.40%	0.05	0.82%	0.36%	0.14	
	Cap Weighted	7.15%	15.15%	0.27				
Global	100 Portfolio Avg	8.12%	16.36%	0.31	0.97%	6.35%	0.16	76
	100 Portfolio Std Dev	1.26%	0.62%	0.08	1.26%	0.79%	0.20	

Source: Research Affiliates, based on Worldscope/Datastream data.

APPENDIX A

The Mathematics Behind Our Consistent Pattern of FF4 Factor Loadings

Let us examine the expected return characteristics of an arbitrary strategy that invests in n stocks where each stock has weight w_i. The return for this portfolio R_p can be shown to be a sum of two components: the average return of all stocks and the sum of covariance terms between a stock's return and its weight.

$$Rp = E[r_i] + n \cdot cov\ [r_i, w_i] \qquad \text{(A–1)}$$

Equation (1) can be derived trivially by noting the definition of covariance: cov [a,b] = E[ab] – E[a]E[b].

$$\begin{aligned} R_p &= n \cdot E[r_i w_i] = n \cdot E\ [r_i]E[w_i] + n \cdot cov\ [r_i, w_i] \\ &= E[r_i] + n \cdot cov\ [r_i, w_i] \end{aligned}$$

If the strategy weights are unrelated to the future company returns, then the strategy's return is equal to the average stock's return. For example, an equally weighted index or a randomly weighted portfolio will, on average, have returns equal to the average return of all stocks.

The returns of the various non-price-weighted investment strategies are similar in magnitude to those of the random portfolios. This is surprising. It implies that the portfolio weights associated with the various investment beliefs are only very weakly related to future returns, if at all. This, however, perfectly explains why the inverse portfolios generally outperform by a comparable level. If the original weights are nearly uncorrelated with future returns, then the inverse of these weights would generally also be uncorrelated.

The remaining puzzle is why cap weighting stands out as the unique portfolio strategy that underperforms everything else. That is, why is the covariance term, $n \cdot cov\ [r_i, w_i]$, negative for cap weighting? The answer is now obvious. By design, a cap-weighted portfolio has larger allocations to the higher-price stocks, which have lower returns. A negative correlation between price (and therefore stock weights) and subsequent returns would explain the unique underperformance of a market-cap portfolio compared to almost any other strategy where little or no correlation exists.

Why then does a fundamentally weighted portfolio, which also assigns large weights to large stocks, not suffer from the same effect? The answer is quite simple: price. Most practitioners agree that prices at times can include errors, although the extent of the error is not visible. Berk [1997] supports this empirical observation, arguing that there is no ex ante relationship between a company's accounting size and its expected return. Because valuation ratios, expressed by capitalization divided by an accounting fundamental (e.g., price to book), predict returns, then it must be that price (capitalization) predicts returns. That is, because book does not predict returns, but low price-to-book predicts high returns, then low price (capitalization) must predict high returns.[18] From this perspective it becomes clear why cap weighting appears sub-optimal and suffers a return deficit against all other non-price-weighted strategies in our examination. This is exactly consistent with Hsu's [2006] prediction.

How do we explain the ubiquitous value and small-cap effect measured for all of the strategies, whether sensible, random, wacky, or upside-down, examined in this article? Again, this is no puzzle. Arnott and Hsu [2008] predict that any non-price-weighted portfolio will naturally register a value and small-cap bias without explicitly screening for valuation ratio or capitalization. We must work very hard to build a growth-tilted portfolio, in an FF4 context, without deliberately focusing on high-price or high-multiple companies.

APPENDIX B

DESCRIPTION OF STRATEGY DEFINITIONS

The number of stocks by country: Australia—200; Canada—100; France—80; Japan—400; United Kingdom—100; United States—1,000; Global—1,000.

Strategy name	Portfolio Construction Method
Cap Weighted	Weighted based on market capitalization. We compute market capitalization using the December close of the year prior to index construction.
Volatility weighted	Weighted based on the standard deviation of monthly returns over the five-year window prior to index construction.
Market-Beta Weighted	Weighted based on CAPM betas using market factor kindly provided by Kenneth French on his website. We estimate market-beta loading using monthly returns data over a five-year window prior to index construction.
Downside Semi-Deviation Weighted	Weighted based on downside semi-deviation of the monthly returns over a five-year period prior to index construction.
Minimum Variance	We use Clarke et al. method [2006] to construct the minimum variance strategy.
Book Weighted	Weighted based on equity book value. We use the book value from the fiscal year ending two years prior to index construction. We introduce delay to avoid forward-looking bias.
Five-Year Average Earnings Weighted	Weighted based on the five-year earnings average. The averaging period covers the five fiscal years ending with the fiscal year two years prior to index construction. We introduce delay to avoid forward-looking bias.
EPS Growth	Weighted based on the five-year average dollar change in earnings, divided by the average absolute dollar value of earnings over the five-year period. The last fiscal years of the measuring window is taken two years prior to index construction. We introduce delay to avoid forward-looking bias.
RCEW	We apply statistical methods to identify major market-risk factors, assumed to be driven by industries and geographies, and then equally weight these uncorrelated risk clusters.
Fundamental Weighted	Weighted based on the five-year averages of cash flows, dividends, sales, and the most recent equity book value. We introduce a two-year delay to avoid forward-looking bias. Following the original method, we select top stocks with the largest fundamental weight. For details see Arnott et al. [2005].
Risk-Efficient (λ=2)	Mean-variance optimized portfolio assumes that expected excess returns are proportional to the stocks' downside semi-deviation, and with stringent constraint to limit portfolio concentration. For details see Amenc et al. [2010].
Maximum Diversification	Portfolio optimized to maximize expected diversification ratio, defined as the ratio of weighted average risk to the expected portfolio risk. For details see Choueifaty and Coignard [2008].

Source: Research Affiliates.

ENDNOTES

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[1]In his bestselling book *A Random Walk Down Wall Street*, Burton Malkiel claimed that "a blindfolded monkey throwing darts at a newspaper's financial pages could select a portfolio that would do just as well as one carefully selected by experts." The experts, he believed, would on average produce results that were no better than the cap-weighted benchmark. The implicit assumption is that both monkeys and equity portfolio managers have no skills when prices are random walks and therefore would perform no better than the cap-weighted benchmark. As it turns out, Malkiel's assessment of his monkey was too modest; in empirical testing, the monkey reliably outperforms, at least before transaction costs.

[2]This result is unsurprising. A sufficiently large random portfolio converges on equal weight, which has well documented, well-understood value added over corresponding cap weighting of the same names.

[3]The aim of this article is neither to recommend for or against any particular strategy index. Essentially all of the strategies examined in this article, and their inverses, have provided highly profitable factor tilts; some of them also have attractively low turnover, vast capacity, and appealing core-like portfolio composition, making them interesting investment options. There is value in strategies that give well-constructed access to value and small-cap exposure.

[4]Our research draws on the work of Chow et al. [2011], who find that popular alternative equity indexing strategies outperform, due largely to their value and size exposures.

[5]In the inverse ratio strategies, for stocks with a weight of 0 in the original portfolio, the inverted 1/w weight is set to the inverse of the lowest non-zero weight, to avoid singularity. Note that when a strategy sets most of the 1,000 stocks to zero weight, the inverse portfolio becomes similar to equal weighting.

[6]We do not invert these portfolios because the inverse of a monkey-managed portfolio is the equally weighted portfolio of 970 stocks, which is virtually indistinguishable from the equal weighted portfolio that is also present in our study.

[7]For the accounting of fundamentally weighted portfolios, we instead follow the original universe selection criteria (select the top 1,000 largest stocks by accounting fundamentals) proposed by Arnott et al. [2005], which are also designed to ensure liquidity. Using the largest 1,000 stocks by market cap has similar but less dramatic results.

[8]The number of stocks by country: Australia—200; Canada—100; France—80; Germany—60; Japan—400; United Kingdom—100; United States—1,000; Global—1,000.

[9]The Fama–French four-factor model is an extension of the original Fama–French model, which attributes return to market beta, size (SMB, or small minus big), value (HML, or high minus low), and momentum (UMD, or up minus down). This last component was added based on the work of Asness [1994] and Carhart [1997].

[10]Similar to Chow et al. [2011], we find that varying the methods and data frequency for the risk estimates has no meaningful impact on the results.

[11]Bob Haugen championed minimum variance in the 1980s, during his tenure at UC Irvine. In the late 1960s to early 1970s, Haugen and his co-authors empirically documented that portfolios with low-volatility stocks outperform the cap-weighted market (see, for example, Haugen and Heins [1975]).

[12]The details of maximum-diversification and risk-efficient index strategies can be found in articles by Choueifaty and Coignard [2008] and Amenc et al. [2010], respectively. RCEW is based on QS Investors' Diversity-Based Index methodology. See Chow et al. [2011] for a review of the portfolio construction strategies associated with the three quantitative strategy indices described in this section.

[13]Following Arnott et al. [2005], the strategies weighted by book, five-year average earnings, or composite four metrics select top 1,000 stocks using fundamental measures to capture the fundamental economic footprint of the companies' businesses, rather than selecting the top 1,000 based on market capitalization.

[14]To measure the earnings growth, we use five-year average dollar change in reported earnings, divided by the average absolute dollar value of earnings over the five-year period. The last fiscal year of the measuring window is two years prior to index construction.

[15]Surprisingly, Graham [2012] found no alpha for his random portfolio. In fact, he found that a randomly generated EW portfolio asymptotically converged on the cap-weighted portfolio in simulation. After reviewing his work, we have concluded that it is a mistake. A more comprehensive study by Clare et al. [2013] of the Cass School of Business, City University London, found alpha for the random portfolio, which is consistent with our result for random portfolios.

[16]In Exhibits 6 and 7, we show selected results for a few individual developed countries. The individual countries demonstrate the same general pattern we observe in the U.S. or global developed markets.

[17]Readers can find a detailed comparison of implementation costs and investability of the popular alternative beta strategies in the article by Chow et al. [2011].

[18]See Arnott et al. [2011] for an explicit derivation of the value and size effect using the noise-in-price framework.

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