Math 21B
Disussion Sheet 6 - Key
Answers by Doug
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Note: These answers are not endorsed by Dr. Gravner and may be incorrect!

1. Compute

a) $\int \frac{-2x+8}{x^3+4x}\,dx$

By partial fraction decomposition

$$\frac{-2x+8}{x^3+4x} = \frac{-2x+8}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4} = \frac{A(x^2+4)+x(Bx+C)}{x(x^2+4)} = \frac{(A+B)x^2+Cx+A}{x^3+4x}$$

Thus we get that $C=-2$, $A+B=0$, and $A=8$. Thus this gives us

$$\int \frac{-2x+8}{x^3+4x}\,dx = \int \frac{8}{x} + \frac{-8x+-2}{x^2+4}\,dx = 8\ln|x| - 4\ln\left|x^2+4\right| - \arctan\left(\frac{x}{2}\right) + C$$

b) $\int \frac{x}{x^2-x-6}\,dx$

c) $\int \frac{x^2}{x^2-x-1}\,dx$

d) $\int \frac{x^3}{(1+x^2)^4}\,dx$

e) $\int \frac{1}{x^4+1}\,dx$

2. Compute

a) $\int \frac{1}{x^{1/3}-4x}\,dx$

b) $\int \frac{1}{\sqrt{x}(x+4)\left(\sqrt{x}-1\right)^2}\,dx$

3. Compute

a) $\int_2^3 \frac{1}{\sqrt{x^2-1}}\,dx$

b) $\int \frac{\cos x}{(2+\sin x)(1+\sin x)}\,dx$

4. Compute

$$\int \frac{\mathbf{2x+1}}{\mathbf{2x^2+x+2}}\,\mathbf{dx}$$

We are going to cheat here by first recognizing

$$2x^2+x+2 = 2\left((x+1/4)^2+15/16\right)$$

Then we get

$$\int \frac{2x+1}{2x^2+x+2}\,dx = \int \frac{2x+1}{2\left((x+\frac{1}{4})^2+\frac{15}{16}\right)}\,dx$$

Then let $u = x+1/4$, then $du = dx$ and

$$\begin{aligned}
\int \frac{2x+1}{2\left((x+\frac{1}{4})^2+\frac{15}{16}\right)}\,dx &= \int \frac{2u+\frac{1}{2}}{2\left(u^2+\frac{15}{16}\right)}\,du \\
&= \int \frac{u+\frac{1}{4}}{u^2+\frac{15}{16}}\,du \\
&= \int \frac{16}{15}\frac{u}{\frac{16}{15}u^2+1}\,du + \frac{1}{4}\int \frac{16}{15}\frac{1}{\left(\sqrt{\frac{16}{15}}u\right)^2+1}\,du \\
&= \frac{1}{2}\ln\left|\frac{16}{15}u^2+1\right| + \frac{4}{15}\arctan\sqrt{\frac{16}{15}}u + C \\
&= \frac{1}{2}\ln\left|\frac{16}{15}(x+1/4)^2+1\right| + \frac{4}{15}\arctan\left(\sqrt{\frac{16}{15}}(x+1/4)\right) + C
\end{aligned}$$