Math 21B
Disussion Sheet 6 - Key
Answers by Doug
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Note: These answers are not endorsed by Dr. Gravner and may be incorrect!

1. Compute

a) $\int \frac{\mathbf{-2x+8}}{\mathbf{x^3+4x}}\ \mathbf{dx}$

By partial fraction decomposition

$$\frac{-2x+8}{x^3+4x} = \frac{-2x+8}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4} = \frac{A(x^2+4)+x(Bx+C)}{x(x^2+4)} = \frac{(A+B)x^2+Cx+4A}{x^3+4x}$$

Thus we get that $C=-2$, $A+B=0$, and $4A=8$. Thus this gives us

$$\int \frac{-2x+8}{x^3+4x}\,dx = \int \frac{2}{x} + \frac{-2x+-2}{x^2+4}\,dx = 2\ln|x| - \ln\left|x^2+4\right| - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$$

b) $\int \frac{\mathbf{x}}{\mathbf{x^2-x-6}}\ \mathbf{dx}$

c) $\int \frac{\mathbf{x^2}}{\mathbf{x^2-x-1}}\ \mathbf{dx}$

d) $\int \frac{\mathbf{x^3}}{\mathbf{(1+x^2)^4}}\ \mathbf{dx}$

e) $\int \frac{\mathbf{1}}{\mathbf{x^4+1}}\ \mathbf{dx}$

2. Compute

a) $\int \frac{\mathbf{1}}{\mathbf{x^{1/3}-4x}}\ \mathbf{dx}$

b) $\int \frac{\mathbf{1}}{\mathbf{\sqrt{x}(x+4)\left(\sqrt{x}-1\right)^2}}\ \mathbf{dx}$

3. Compute

a) $\int_{\mathbf{2}}^{\mathbf{3}} \frac{\mathbf{1}}{\mathbf{\sqrt{x^2-1}}}\ \mathbf{dx}$

b) $\int \frac{\cos \mathbf{x}}{\mathbf{(2+\sin x)(1+\sin x)}}\ \mathbf{dx}$

4. Compute

$$\int \frac{\mathbf{2x+1}}{\mathbf{2x^2+x+2}}\ \mathbf{dx}$$

We are going to cheat here by first recognizing

$$2x^2+x+2 = 2\left((x+1/4)^2+15/16\right)$$

Then we get

$$\int \frac{2x+1}{2x^2+x+2}\ dx = \int \frac{2x+1}{2\left((x+\frac{1}{4})^2+\frac{15}{16}\right)}\ dx$$

Then let $u = x+1/4$, then $du = dx$ and

$$\begin{aligned}
\int \frac{2x+1}{2\left((x+\frac{1}{4})^2+\frac{15}{16}\right)}\ dx &= \int \frac{2u+\frac{1}{2}}{2\left(u^2+\frac{15}{16}\right)}\ du \\
&= \int \frac{u+\frac{1}{4}}{u^2+\frac{15}{16}}\ du \\
&= \int \frac{16}{15}\frac{u}{\frac{16}{15}u^2+1}\ du + \frac{1}{4}\int \frac{16}{15}\frac{1}{\left(\sqrt{\frac{16}{15}}u\right)^2+1}\ du \\
&= \frac{1}{2}\ln\left|\frac{16}{15}u^2+1\right| + \frac{4}{15}\arctan\sqrt{\frac{16}{15}}u + C \\
&= \frac{1}{2}\ln\left|\frac{16}{15}(x+1/4)^2+1\right| + \frac{4}{15}\arctan\left(\sqrt{\frac{16}{15}}(x+1/4)\right) + C
\end{aligned}$$