

*Note: These answers are not endorsed by Dr. Gravner and may be incorrect!*

## 1. Compute

a)  $\int x^3 e^{x^2} dx$

Let  $u = x^2$ , then  $du = 2x dx$ , so we obtain

$$\int x^3 e^{x^2} dx = \int \frac{1}{2} u e^u du$$

Then by integration by parts, let  $w = \frac{1}{2}u$ , and  $dv = e^u$ , then we get

$$\int \frac{1}{2} u e^u du = \frac{1}{2} u e^u - \int \frac{1}{2} e^u du = \frac{1}{2} (x^2 - 1) e^{x^2}$$

b)  $\int \arcsin x dx$

By integration by parts, let  $u = \arcsin x$  and  $dv = dx$ , then

$$\int \arcsin x dx = x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx = x \arcsin x + \frac{1}{2} \ln |1-x^2| + C$$

c)  $\int \arccos x dx$

We would solve this the same way, so this would be

$$x \arccos x + \frac{1}{2} \ln |1-x^2| + C$$

## 2. Compute

a)  $\int \sin 3x \cos 4x dx$

b)  $\int \sin^4 x \cos^4 x dx$

Recall that we have

$$\sin^2 x = \frac{1}{2} (1 + \cos 2x) \quad \text{and} \quad \cos^2 x = \frac{1}{2} (1 + \cos 2x)$$

Then we obtain

$$\begin{aligned}
\int \sin^4 x \cos^4 x \, dx &= \int (\sin^2 x)^2 (\cos^2 x)^2 \, dx \\
&= \int \left( \frac{1}{2} (1 + \cos 2x) \right)^2 \left( \frac{1}{2} (1 - \cos 2x) \right)^2 \, dx \\
&= \int \left( \frac{1}{2} \right)^2 (1 + 2 \cos 2x + \cos^2 2x) \left( \frac{1}{2} \right)^2 (1 - 2 \cos 2x + \cos^2 2x) \, dx \\
&= \frac{1}{16} \int 1 - 2 \cos^2 2x + \cos^4 2x \, dx \\
&= \frac{1}{16} \int 1 - 2 \left( \frac{1}{2} (1 + \cos 4x) \right) + \left( \frac{1}{2} (1 + \cos 4x) \right)^2 \, dx \\
&= \frac{1}{16} \int \frac{1}{2} \cos 4x + \frac{1}{2} (1 - 2 \cos 4x + \cos^2 4x) \, dx \\
&= \frac{1}{16} \int \frac{1}{2} \cos 4x + \frac{1}{2} - \cos 4x + \frac{1}{2} \left( \frac{1}{2} (1 + \cos 8x) \right) \, dx \\
&= \frac{1}{16} \int \frac{1}{4} - \frac{1}{2} \cos 4x - \frac{1}{4} \cos 8x \, dx \\
&= \frac{1}{16} \left( \frac{1}{4} x - \frac{1}{2} \frac{\sin 4x}{4} - \frac{1}{4} \frac{\sin 8x}{8} \right) + C \\
&= \frac{24x - 8 \sin 4x + \sin 8x}{1024} + C
\end{aligned}$$

c)  $\int (\sin 2x)^2 \sin^2 x \, dx$

d)  $\int \frac{\tan x}{\cos^2 x} \, dx$

### 3. Compute

a)  $\int_{\pi}^{\pi} (\sin 2x)^3 \sin^2 x \, dx$

b)  $\int_{\pi}^{17\pi} (\sin 2x)^3 \sin^2 x \, dx$

c)  $\int_{\pi/2}^{\pi/2} \cos^2(2x) \, dx$

4. Compute  $\int \frac{x^2}{\sqrt{x^2+9}} \, dx$