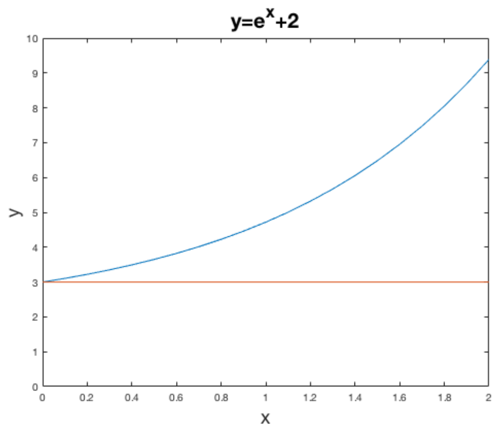
Math 21B
Disussion Sheet 4 - Key
Answers by Doug
DESherman@UCDavis.edu
Note: These answers are not endorsed by Dr. Gravner and may be incorrect!

1. Find the area of the planar region bounded by the curve $\mathbf{y = e^x + 2}$, and the lines $\mathbf{x = 0, x = 2}$, and $\mathbf{y = 0}$

Notice that $y(0) = 3$ and is increasing on $[0, 2]$. So, to compute this we are going find the integral of $y - 3$, and then add the rectangle spanned from $[0, 2] \times [0, 3]$.

Thus we have

$$\text{Area} = \int_0^2 (e^x + 2) - 3\ dx + (2)(3) = (e^x - x)|_{x=0}^2 + 6 = e^2 - e^0 + 2 + 6 = e^2 - 7$$



2. Compute the area.

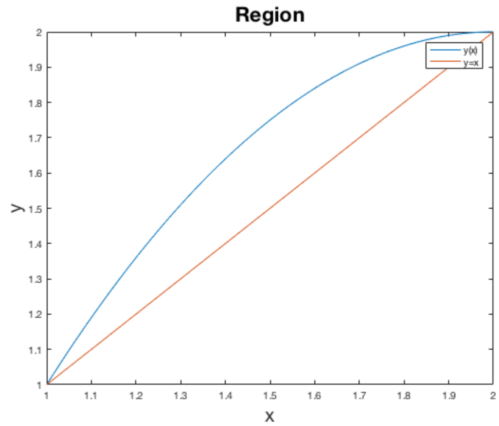
a) Region bounded by $\mathbf{y = -x^2 + 4x - 2}$ and $y = x$.

These two curves intersect at

$$-x^2 + 4x - 2 = x \iff 0 = x^2 - 3x + 2$$

Which has solutions $x = 1$ and $x = 2$. Moreover, plugging in $x = 1.5$ we see that the first plot is on top. Thus we need to solve the integral

$$\int_1^2 -x^2 + 4 - x\ dx = \int_1^2 -x^2 + 3x - 2\ dx = -\frac{1}{3}x^3 + \frac{3}{2}x^2 - 2x\Big|_{x=1}^2 = -\frac{8}{3} + 6 - 4 + \frac{1}{3} - \frac{3}{2} + 2 =$$



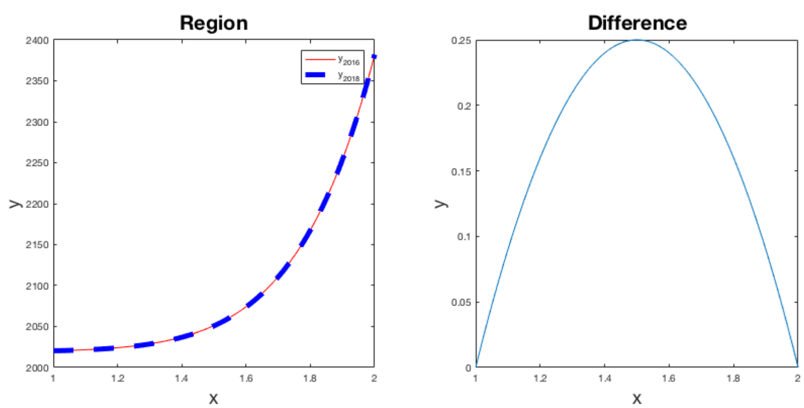
b1) The planar region R is bounded by the graph $\mathbf{y = -x^2 + 4x + \sqrt{x^{17}+1} + 2016}$ and the graph of $\mathbf{y = x + \sqrt{x^{17}+1} + 2018}$. Compute the area of R.

These two regions are equal when

$$-x^2 + 4x + \sqrt{x^{17}+1} + 2016 = x + \sqrt{x^{17}+1} + 2018 \iff 0 = x^2 - 3x + 2$$

which has solutions $x = 1$ and $x = 2$. Plugging in $x = 1.5$ we get that the first function is on top. Thus we need to solve the integral

$$\begin{aligned}\int_1^2 -x^2 + 4x + \sqrt{x^{17}+1} + 2016 - \left(x + \sqrt{x^{17}+1} + 2018\right)\, dx &= \int_1^2 -x^2 + 3x - 2\, dx \\ &= -\frac{1}{3}x^3 + \frac{3}{2}x^2 - 2x\bigg|_{x=1}^{2} \\ &= -\frac{1}{3}(8) + \frac{3}{2}(4) - 2(2) + \frac{1}{3} - \frac{3}{2} + 2 =\end{aligned}$$



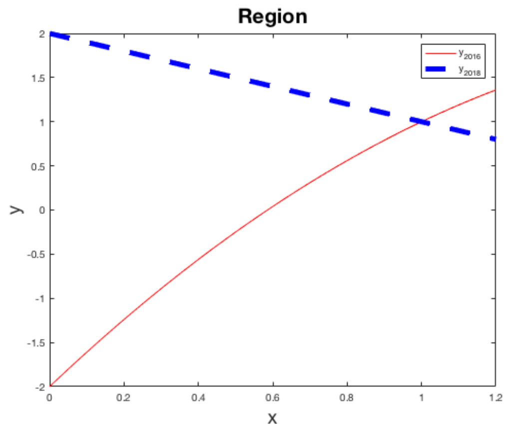
b2) Finally, R is bounded by the graph of $\mathbf{y = -x^2 + 4x - 2}$ and the lines $\mathbf{x = 0}$ and $\mathbf{x + y = 2}$

These plots intersect at

$$-x^2 + 4x - 2 = 2 - x \iff -x^2 + 5x - 4 = 0$$

which intersects at 1 and 4. Since we want to bound the region by $x = 0$. we are going to consider the interval $[0, 1]$ and on this interval, the 2nd curve is higher. Thus the area is

$$\int_0^1 -x^2 + 4x - 2 - (2 - x)\ dx = \int_0^1 -x^2 + 5x - 4\ dx = -\frac{1}{3}x^3 + \frac{5}{2}x^2 - 4x\Big|_{x=0}^{1} = -\frac{1}{3} + \frac{5}{2} - 4 =$$



3. The planar region R is bounded by curves $\mathbf{y = x + 2}$ and $\mathbf{x = y^3 - 2y^2}$. Compute its area.

4. Compute $\int_{-3}^{3} x\left(\sqrt{x+3} + \sin x^4 + \cos x^3\right)\ dx$.

5. If $\int_{-1}^{2} f(x)\ dx = 3$ and $\int_0^2 f(x)\ dx = -4$, what is $\int_0^{-1} f(x)\ dx$?

Note that we have

$$\int_{-1}^{2} f(x)\ dx = \int_{-1}^{0} f(x)\ dx + \int_0^2 f(x)\ dx$$

Then we can compute

$$\int_0^{-1} f(x)\ dx = -\int_{-1}^{0} f(x)\ dx = -\left(\int_{-1}^{2} f(x)\ dx - \int_0^2 f(x)\ dx\right) = -(3 - (-4)) = -7$$

Matlab Code

```matlab
% Problem 1
close all; clc;
x = 0:.1:2;
f = @(x) exp(x)+2;
plot(x,f(x)); hold on;
plot([0 2],[3 3]);
axis([0 2 0 10])
xlabel("x","Fontsize",18)
ylabel("y","Fontsize",18)
title("y=e^x+2","Fontsize",20)

% Problem 2
close all; clc
x = 1:.01:2
f = @(x) -x.^2+4*x-2;
plot(x,f(x)); hold on;
plot(x,x);
xlabel("x","Fontsize",18)
ylabel("y","Fontsize",18)
title("Region","Fontsize",20)
legend({"y(x)","y=x"})

clc; close all;
x = 1:.01:2;
f = @(x) -x.^2+4*x+sqrt(x.^(17)+1)+2016;
g = @(x) x+sqrt(x.^(17)+1)+2018;
subplot(1,2,1);
plot(x,f(x),'r-','Linewidth',1); hold on;
plot(x,g(x),'b--','Linewidth',5);
xlabel("x","Fontsize",18)
ylabel("y","Fontsize",18)
title("Region","Fontsize",20)
legend({"y_{2016}","y_{2018}"})

subplot(1,2,2);
h = @(x) f(x)-g(x);
plot(x,h(x))
xlabel("x","Fontsize",18)
ylabel("y","Fontsize",18)
title("Difference","Fontsize",20)


% Problem 2 b2
x = 0:.01:1.2;
f = @(x) -x.^2+4*x-2;
g = @(x) 2-x;
plot(x,f(x),'r-','Linewidth',1); hold on;
plot(x,g(x),'b--','Linewidth',5);
xlabel("x","Fontsize",18)
ylabel("y","Fontsize",18)
title("Region","Fontsize",20)
legend({"y_{2016}","y_{2018}"})
```