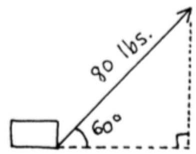
1. A force of 80 pounds is applied in the given diagram. Find the horizontal and vertical components of this force.



The y component of the force is given by

$$F\sin\theta = 80\sin 60^\circ = 40\text{lb}$$

The x component of the force is given by

$$F\cos\theta = 80\cos 60^\circ = 40\sqrt{3}\text{lb}$$

2. Let vector $\tilde{\mathbf{A}} = (\mathbf{3}, \tilde{\mathbf{4}})$. Find a vector of length 4 which is

a.) parallel to $\tilde{\mathbf{A}}$

Any vector pointing in the same direction works. So,

$$2\vec{A} = \langle 6, 8\rangle$$

b.) perpendicular to $\tilde{\mathbf{A}}$

To be perpendicular we need the angle to be $\pi/2$, or the dot product equal to zero. Let $\vec{B} = \langle b_1, b_2\rangle$ then

$$\vec{A}\cdot\vec{B} = 3b_1 + 4b_2 = 0 \iff b_1 = -\frac{4}{3}b_2$$

Thus let $b_2 = 1$ and we get

$$\vec{B} = \left\langle -\frac{4}{3}, 1\right\rangle$$

3. Find two unit vectors each of which is perpendicular to both $\tilde{\mathbf{A}} = (\mathbf{1}, \mathbf{0}, \tilde{-}\mathbf{2})$ and $\tilde{\mathbf{B}} = (\mathbf{0}, \tilde{\mathbf{3}}, \mathbf{4})$

To be perpendicular to both of these vectors we need to be perpendicular to the plane spanned by these vectors. Thus we need to find a normal vector of this plane. The easiest way to do this is by computing the cross-product of both vectors. Thus

$$\vec{A}\times\vec{B} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -2 \\ 0 & 3 & 4 \end{bmatrix} = \left\langle \left|\begin{bmatrix} 0 & -2 \\ 3 & 4 \end{bmatrix}\right|, \left|\begin{bmatrix} 1 & -2 \\ 0 & 4 \end{bmatrix}\right|, \left|\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}\right| \right\rangle = \langle 6, 4, 0\rangle$$

Thus the two vectors are

$$\langle 6, 4, 0\rangle \quad \text{and} \quad \langle -6, -4, 0\rangle$$

4. Construct the projection vector $\mathbf{proj}_{\tilde{\mathbf{B}}}\tilde{\mathbf{A}}$ for each pair of vectors.

a.) $\tilde{\mathbf{A}} = (\mathbf{3}, \tilde{\mathbf{4}}), \tilde{\mathbf{B}} = (-\tilde{\mathbf{1}}, \mathbf{2})$

The projection vector can be found by

$$\text{proj}_{\vec{B}}\vec{A} = \left(\vec{A} \cdot \hat{B}\right)\hat{B}$$

where $\hat{B}$ is the unit vector in the same direction of $\vec{B}$ given as $\hat{B} = \vec{B}/\left|\vec{B}\right|$. Then we get

$$\hat{B} = \frac{\vec{B}}{\left|\vec{B}\right|} = \frac{\langle -1, 2\rangle}{|\langle\langle -1, 2\rangle|} = \left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\rangle$$

then we get

$$\text{proj}_{\vec{B}}\vec{A} = \left(\vec{A} \cdot \hat{B}\right)\hat{B} = \left(-\frac{3}{\sqrt{5}} + \frac{8}{\sqrt{5}}\right)\left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\rangle = \langle -1, 2\rangle$$

Note, the formula that is used in class is equivalent, but I like this one because it's a little more intuitive. The projection of $\vec{A}$ onto $\vec{B}$ has magnitude of $\vec{A}$ dotted with the unit vector $\hat{B}$ and is in the direction of $\vec{B}$.

b.) $\tilde{\mathbf{A}} = (\mathbf{1}, \tilde{\mathbf{2}}, \mathbf{3}), \tilde{\mathbf{B}} = (\mathbf{3}, \tilde{\mathbf{2}}, \mathbf{1})$

$$\hat{B} = \frac{\vec{B}}{\left|\vec{B}\right|} = \frac{\langle 3, 2, 1\rangle}{|\langle 3, 2, 1\rangle|} = \left\langle \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right\rangle$$

then we get

$$\text{proj}_{\vec{B}}\vec{A} = \left(\vec{A} \cdot \hat{B}\right)\hat{B} = \left(\frac{3}{\sqrt{14}} + \frac{4}{\sqrt{14}} + \frac{3}{\sqrt{14}}\right)\left\langle \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right\rangle = \frac{10}{14}\langle 3, 2, 1\rangle = \frac{5}{7}\langle 3, 2, 1\rangle$$

5. Determine the area of the triangle formed by the points

a.) (1,1),(4,2), and (-3,3)

We could go through the process of trying to compute the area with geometry, but we can also use the fact that the magnitude of the parallelogram from two vectors is the magnitude of their cross product. Thus the triangle would be half this area. Thus we get

$$\text{Area of Triangle} = \frac{1}{2}\left|\vec{A} \times \vec{B}\right|$$

where $\vec{A} = \langle 3, 1\rangle$ and $\vec{B} = \langle 4, 2\rangle$. Then we get

$$\text{Area of Triangle} = \frac{1}{2}\left|\vec{A} \times \vec{B}\right| = \frac{1}{2}|\langle 0, 0, 2\rangle| = \frac{1}{2}2 = 1$$

b.) (0,0,0),(3,-2,1), and (1,0,2)

$$\text{Area of Triangle} = \frac{1}{2}\left|\vec{A} \times \vec{B}\right| = \frac{1}{2}|\langle 3, -2, 1\rangle \times \langle 1, 0, 2\rangle| = \frac{1}{2}|\langle -4, 5, 2\rangle| = \frac{1}{2}\sqrt{45} = \frac{3}{2}\sqrt{5}$$

6. Compute the area of the parallelogram formed by the vectors $\tilde{\mathbf{A}} = (\mathbf{4}, \tilde{-\mathbf{1}}, \mathbf{2})$ and $\tilde{\mathbf{B}} = (\mathbf{2}, \tilde{\mathbf{3}}, \mathbf{0})$

As stated in 5 we can just compute the magnitude of the cross product, so we get

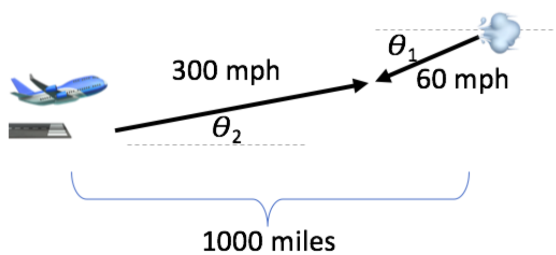
$$\left|\vec{A} \times \vec{B}\right| = |\langle 4, -1, 2\rangle \times \langle 2, 3, 0\rangle| = |\langle 6, 4, 14\rangle| = \sqrt{248} = 2\sqrt{62}$$

7. Compute the volume of the parllelpiped formed by the vectors $\tilde{\mathbf{u}} = \tilde{\mathbf{i}} + 2\tilde{\mathbf{j}} - \tilde{\mathbf{k}}, \tilde{\mathbf{v}} = 2\tilde{\mathbf{i}} + \tilde{\mathbf{j}} + 3\tilde{\mathbf{k}}$, and $\tilde{\mathbf{w}} = \tilde{\mathbf{i}} - \tilde{\mathbf{j}} + 2\tilde{\mathbf{k}}$

Here we can compute the magnitude of the cross product to get the area of the parallelogram spanned by two of the vectors, then multiply that by the magnitude of the third vector. I'm just going to do this in Matlab.

$$|\vec{u} \times \vec{v}|\,|\vec{w}| \approx 22.32$$

8. A jet airplane wants to fly in a straight line from airport A directly East to airport B, which is 1000 miles away. The jet is pushed by a tailwind from 30° South of West at 60 mph. If the jet flies at a constant speed of 300 mph (relative to the surrounding air space),



1. Plane travelling east with a SE headwind

a.) in what direction should the jet fly?

We need the north component of the plane's trajectory to balance the south component of the wind. Thus we get

$$300 \sin\theta_2 = 60 \sin\theta_1 = 60 \sin 30^\circ = 30$$

solving for θ_2 gives us

$$\theta_2 = \arcsin\left(\frac{1}{10}\right)$$

b.) what is the jet's actual flying speed (relative to the ground)?

This is the plane's x-component, but we need to also subtract the wind's x component as well. Thus

$$300\cos\theta_2 - 60\cos\theta_1 = \sqrt{10^2 - 1} - 30\sqrt{3} \approx 268.5$$

c.) how long will the flight take?

This is just 1000 miles divided by the speed or

$$\frac{1000}{268.4962} = 3.7244 \text{ hours}$$

9. Determine parametric equations for the line L passing through the points (1,-1,2) and (3,0,-4).

The line passing through this equation must solve the system of equations given by

$$\begin{cases} x - y + 2z = 0 \\ 3x - 4z = 0 \end{cases}$$

Solving this we get $x = \frac{4}{3}z$, $y = \frac{10}{3}z$, and let $z = t$. Then we have

$$L : \begin{cases} x = \frac{4}{3}t \\ y = \frac{10}{3}t \\ z = y \end{cases}$$

10. Determine parametric equation for the line L passing through the point (2,1,-3) and which is parallel to the line M given by.

$$M : \begin{cases} x &= & 1 + t \\ y &= & 2 - t \\ z &= & 3t \end{cases}$$

11. Determine if the following lines intersect. If they do, find the point of intersection and the angle between the lines.

$$L : \begin{cases} x &= & 3 - t \\ y &= & 2 + t \\ z &= & t \end{cases}$$

$$M : \begin{cases} x &= & 3 + s \\ y &= & 3 - s \\ z &= & 2s + 7 \end{cases}$$

For the points to intersect we need

$$3 - t = 3 + s \iff t = -s$$

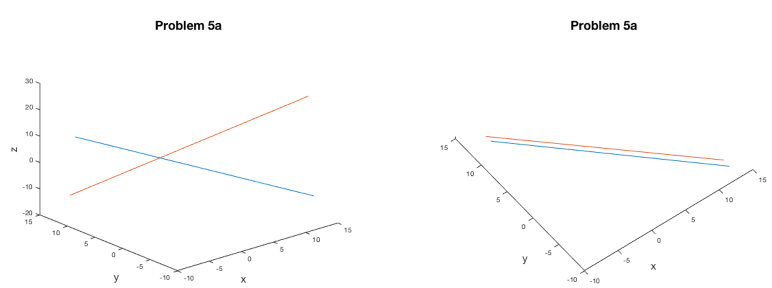
and

$$2 + t = 3 - s \iff t = 1 - s$$

and

$$t = 2s + 7$$

and since each of these equations has different solutions for t and s, we can't find a value for t or s that satisfies each of them. Thus they don't intersect.



2. Plotting two different angles of the Lines M and L

12. Determine parametric equations for the line L representing the intersection of the planes $\mathbf{x + y - 2z = 10}$ and $\mathbf{3x + 2y + z = 5}$.

13. Determine the angle between the intersecting planes $\mathbf{x + y - 2z = 10}$ and $\mathbf{3x + 2y + z = 5}$

14. Consider the plane given by $\mathbf{2x - 4y + 5z = 20}$.

a.) Find 3 points lying on this plane.

We can just plug in anything for x, y, and z that satisfies this equation. Chooosing points with as many zeros as possible is always my go to. Thus

$$2(0) - 4(0) + 5z = 20 \iff z = 4$$

thus $(0, 0, 4)$ is on the plane. The next two points can be found similarly as $(10, 0, 0)$ and $(0, -5, 0)$. Thus we have

$$(0, 0, 4),\ (10, 0, 0),\ \text{and}\ (0, -5, 0)$$

b.) Find 2 vectors perpendicular (normal) to the plane.

The easiest way to find two vectors normal to the plane is to cheat and recognize that any plane given by $ax + by + cz = d$ has normal vector $\langle a, b, c\rangle$. Thus the two vectors are

$$\langle 2, -4, 5\rangle\ \text{and}\ \langle -2, 4, -5\rangle$$

Note, I think the point of this problem was to construct two vectors from the 3 points in **a)** and then compute the cross product of those vectors. Should give you the same answer (scaled by some value).

c.) Find a vector parallel to the plane's surface.

Can just use two of the 3 points we computed. Thus a vector is

$$\langle 10 - 0, 0 - 0, 0 - 4\rangle = \langle 10, 0, -4\rangle$$

15. Determine an equation of the plane passing through the points (0,0,0),(1,0,-2), and (0,3,4).

Can just produce a system of equation by plugging each point into a general plane $ax + by + cz = d$. Thus we have

$$\begin{cases} 0 = d \\ a - 2c = d \\ 3b + 4c = d \end{cases}$$

Thus we have $d = 0$, $a = 2c$, and $b = \frac{4}{3}c$. Since 3 equations can't solve for 4 variables, we have to settle for a parametrized solution. Let $c = 1$, then the equation for this plane is

$$2x + \frac{4}{3}y + z = 0$$

16. Compute the distance from the origin to the plane given by $\mathbf{x + 2y + 3z = 6}$ and $\mathbf{x + 2y + 3z = 0}$

17. Compute the distance between the parllel planes given by $\mathbf{x + 2y + 3z = 6}$ and $\mathbf{x + 2y + 3z = 0}$

It's just 6

18. Find the point of intersection of the plane given by $\mathbf{x + 2y + 3z = 6}$ and the line given by

$$L : \begin{cases} x &= 3 - t \\ y &= 2 + t \\ z &= t \end{cases}$$

19. Consider the vectors $\tilde{\mathbf{A}} = (\mathbf{a_1}, \tilde{\mathbf{a_2}}, \mathbf{a_3})$,$\tilde{\mathbf{B}} = (\mathbf{b_1}, \tilde{\mathbf{b_2}}, \mathbf{b_3})$, and $\tilde{\mathbf{C}} = (\mathbf{c_1}, \tilde{\mathbf{c_2}}, \mathbf{c_3})$. Prove that $\tilde{\mathbf{A}} \cdot (\tilde{\mathbf{B}} + \tilde{\mathbf{C}}) = \tilde{\mathbf{A}} \cdot \tilde{\mathbf{B}} + \tilde{\mathbf{A}} \cdot \tilde{\mathbf{C}}$

Let $\vec{A} = (a_1, \vec{a_2}, a_3)$,$\vec{B} = (b_1, \vec{b_2}, b_3)$, and $\vec{C} = (c_1, \vec{c_2}, c_3)$. The we obtain

$$\begin{aligned}
\vec{A} \cdot (\vec{B} + \vec{C}) &= vec(a_1, a_2, a_3) \cdot \Big((b_1, \vec{b_2}, b_3) + (c_1, \vec{c_2}, c_3) \Big) \\
&= (a_1, \vec{a_2}, a_3) \cdot \Big((b_1 + c_1, b_2 \vec{+} c_2, b_3 + c_3) \Big) \\
&= a_1(b_1 + c_1) + a_2(b_2 + c_2) + a_3(b_3 + c_3) \\
&= a_1 b_1 + a_1 c_1 + a_2 b_2 + a_2 c_2 + a_3 b_3 + a_3 b_3 \\
&= (a_1 b_1 + a_2 b_2 + a_3 b_3) + (a_1 c_1 + a_2 c_2 + a_3 b_3) \\
&= \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}
\end{aligned}$$

20. Consider the vectors $\tilde{\mathbf{A}} = (\mathbf{a_1}, \tilde{\mathbf{a_2}}, \mathbf{a_3})$,$\tilde{\mathbf{B}} = (\mathbf{b_1}, \tilde{\mathbf{b_2}}, \mathbf{b_3})$, and $\tilde{\mathbf{C}} = (\mathbf{c_1}, \tilde{\mathbf{c_2}}, \mathbf{c_3})$. Prove that $\tilde{\mathbf{A}} \times (\tilde{\mathbf{B}} + \tilde{\mathbf{C}}) = \tilde{\mathbf{A}} \times \tilde{\mathbf{B}} + \tilde{\mathbf{A}} \times \tilde{\mathbf{C}}$

Gross. Do the same thing as 19.

21. Consider the vectors $\tilde{\mathbf{A}} = (\mathbf{a_1}, \tilde{\mathbf{a_2}}, \mathbf{a_3})$ and $\tilde{\mathbf{B}} = (\mathbf{b_1}, \tilde{\mathbf{b_2}}, \mathbf{b_3})$. Prove that $\left|\tilde{\mathbf{A}} \times \tilde{\mathbf{B}}\right|^{\mathbf{2}} = \left|\tilde{\mathbf{A}}\right|^{\mathbf{2}} \left|\tilde{\mathbf{B}}\right|^{\mathbf{2}} - (\tilde{\mathbf{A}} \cdot \tilde{\mathbf{B}})^{\mathbf{2}}$

22. Consider the vectors $\tilde{\mathbf{A}} = (\mathbf{a_1}, \tilde{\mathbf{a_2}}, \mathbf{a_3})$ and $\tilde{\mathbf{B}} = (\mathbf{b_1}, \tilde{\mathbf{b_2}}, \mathbf{b_3})$. Prove that $\tilde{\mathbf{A}} \perp \tilde{\mathbf{A}} \times \tilde{\mathbf{B}}$

23. Consider the vector $\tilde{\mathbf{A}} = (\mathbf{a_1}, \tilde{\mathbf{a_2}}, \mathbf{a_3})$. Prove that $\tilde{\mathbf{A}} \times \tilde{\mathbf{A}} = \tilde{\mathbf{O}}$

24. A 12ft. by 30 ft. room has a 12 ft. ceiling. In the middle of one end wall, one foot above the floor, is a spider. The spider wants to capture a fly in the middle of the opposite end wall, one foot below the ceiling. What is the length of the shortest path the spider can walk (no spider webs allowed) in order to reach the fly?

Code

```

% Problem 6
u = [1,2,-1];
v = [2,1,3];
w = [1,-1,2];
area = norm(cross(u,v))*norm(w);
sprintf('Area of Parallelpiped is %f',area)


% Problem 11
t = -10:.1:10;
s = -10:.1:10;
L = [3-t;2+t;t];
M = [3+s;3-s;2*s+7];
figure;
plot3(L(1,:),L(2,:),L(3,:)); hold on;
plot3(M(1,:),M(2,:),M(3,:));
title('Problem 5a','Fontsize',18);xlabel('x','Fontsize',15);
ylabel('y','Fontsize',15);zlabel('z','Fontsize',15);
```