Math 21C
Kouba
Discussion Sheet 1

1.) Compute the first five terms (starting with n=1) of each sequence. Determine whether each sequence converges or diverges.

a.) $\{3\}$ b.) $\{3^n\}$ c.) $\left\{\frac{3}{n}\right\}$ d.) $\left\{\left(\frac{1}{3}\right)^n\right\}$

e.) $\{3^{1/n}\}$ f.) $\left\{\frac{n+5}{n+2}\right\}$ g.) $\{n(3-n)\}$ h.) $\left\{\frac{n^3+n^2-n+7}{4n^3+5n^2-2}\right\}$

i.) $\{(0.9999)^n\}$ j.) $\{(1.00001)^n\}$ k.) $\left\{\left(\frac{-2}{3}\right)^n\right\}$ l.) $14/3, 15/5, 16/7, 17/9, \ldots$

m.) $\left\{\frac{\sin 3n}{n}\right\}$ n.) $\left\{\left(\frac{\sqrt{7}}{\ln 14}\right)^n\right\}$ o.) $\{\cos(2n\pi)\}$ p.) $\{(1+1/n)^n\}$

q.) $\left\{\frac{3^n}{n!}\right\}$ r.) $\{\sin(\pi/2+n\pi)\}$ s.) $\left\{\frac{1000^n}{n!}\right\}$ t.) $\left\{\frac{n^2}{e^n}\right\}$

u.) $\left\{(n-1)(n-2)(n-3)(n-4)(n-5)\right\}$ v.) $\{3+(-1)^n\}$ w.) $\left\{\sum_{i=1}^{n} i^2\right\}$

x.) $\left\{\sum_{i=1}^{n}(2/3)^{i-1}\right\}$ y.) $\left\{\sum_{i=1}^{n}(1+i/n)^3(1/n)\right\}$

2.) Find a formula (starting with $n=1$) for each of the following sequences.

a.) $4, 8, 12, 16, 20, 24, \ldots$ b.) $4, 6, 8, 10, 12, 14, \ldots$

c.) $4, -9, 25, -36, 49, 64, \ldots$ d.) $2, 12, 30, 56, 90, 132, \ldots$

e.) $2, 12, 2, 12, 2, 12, \ldots$ f.) $\frac{1}{2}, \frac{0}{6}, \frac{1}{12}, \frac{4}{20}, \frac{9}{30}, \frac{16}{42}, \ldots$

3.) Determine whether the following series converge or diverge.

a.) $\sum_{n=0}^{\infty} 2^n$ b.) $\sum_{n=1}^{\infty} 2$ c.) $\sum_{n=2}^{\infty} \frac{1}{2^n}$ d.) $\sum_{n=1}^{\infty} 0.000001$

e.) $\sum_{n=1}^{\infty}(0.98)^{n+3}$ f.) $\sum_{n=1}^{\infty} 5(-2/3)^{n-1}$ g.) $\sum_{n=0}^{\infty}(-1)^n$ h.) $\sum_{n=0}^{\infty}(1/4)(3/2)^{n+3}$

i.) $\sum_{n=1}^{\infty} \frac{n+2}{n+1000}$ j.) $\sum_{n=2}^{\infty}(1-1/n)^n$ k.) $\sum_{n=1}^{\infty} \cos n\pi$

l.) $\sum_{n=7}^{\infty}\left(\frac{1}{\ln n} - \frac{1}{\ln(n+1)}\right)$ m.) $\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6}$ HINT: Use partial fractions first.

4.) Compute the exact value of the following convergent series :

$$6 - 2 + \frac{2}{3} - \frac{2}{9} + \frac{2}{27} - \frac{2}{81} + \ldots$$

5.) Find the 25th term in the following sequence : $1, 1, 2, 3, 5, 8, 13, 21, 34, ...$

6.) Find the sum of the first 200 terms for each of the following sequences.
a.) $3, 7, 11, 15, 19, 23, ...$ a.) $9, 25, 49, 81, 121, 169, ...$

7.) A ball bearing is dropped from a building 200 feet high. Each time the ball bearing rebounds to 40% of its falling distance. What is the total distance the ball bearing will travel ?

8.) Determine the limit of the following sequence :

$$2, \quad 2-\frac{1}{2}, \quad 2-\frac{1}{2-\frac{1}{2}}, \quad 2-\frac{1}{2-\frac{1}{2-\frac{1}{2}}}, \quad 2-\frac{1}{2-\frac{1}{2-\frac{1}{2-\frac{1}{2}}}} \quad ...$$

9.) Start at the origin and move 10 units along the positive y-axis. Turn 90 degrees to the right and move 70% of that distance. Turn 90 degrees to the right and move 70% of that distance. Turn 90 degrees to the right and move 70% of that distance. Turn 90 degrees to the right and move 70% of that distance. Continue this process. At what point (x,y) will you "end" ?

"The mind is not a vessel to be filled, but a fire to be ignited." – Plutarch