

**Alternating Series Test**  $\sum (-1)^n a_n$  converges if and only if  $a_n$  is a positive decreasing sequence and  $\lim_{n \rightarrow \infty} a_n = 0$ .

**Absolute Convergence Test** If the series  $\sum |a_n|$  converges, then  $\sum a_n$  converges as well. In this case we say that  $\sum a_n$  converges absolutely.

**1. Determine convergence or divergence of each series using the test indicated**

a.)  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{7}{n^2+3}$  (Use the alternating series test)

a.)  $1 + \frac{1}{3} - \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} - \frac{1}{3^5} + \frac{1}{3^6} + \frac{1}{3^7} - \frac{1}{3^8}$  (Absolute convergence test)

**2. Determine if the following series, which contain both positive and negative terms, converge conditionally, converge absolutely, or diverge. Remember what the absolute convernence test says.**

a.)  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n+2}$

b.)  $\sum_{n=4}^{\infty} (-1)^{n+1} \frac{1}{n^2+4}$

c.)  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n+1}{n^2+99}$  (Use a derivative to show that  $f(x) = (x+1)/(x^2+99)$  decreases after some point)

d.)  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2+50}{n^5+50}$

e.)  $\frac{5}{1^4} + \frac{5}{2^4} - \frac{5}{3^4} - \frac{5}{4^4} + \frac{5}{5^4} + \frac{5}{6^4} - \frac{5}{7^4} - \frac{5}{8^4} + \dots$

f.)  $\sum_{n=1}^{\infty} (-1)^n \frac{(\ln n)^3}{n}$  (Use a derivative to show that  $f(x) = (\ln x)^3/x$  decreases after some point)

**3. Use the absolute ratio test to determine the interval of convergence for each series.**

a.)  $\sum_{n=1}^{\infty} \frac{x^n}{n+1}$

b.)  $\sum_{n=1}^{\infty} \frac{x^n}{n^2+1}$

c.)  $\sum_{n=1}^{\infty} (-1)^n \frac{(x-1)^{n+1}}{n2^n}$

d.)  $\sum_{n=1}^{\infty} \frac{x^{n-1}}{(n+1)!}$

e.)  $\sum_{n=1}^{\infty} (n!)^2 x^n$

f.)  $\sum_{n=1}^{\infty} \frac{(n!)x^n}{n^{2n}}$

g.)  $\sum_{n=0}^{\infty} \left(\frac{n+1}{n+3}\right)^n x^n$  (Hint: Use the absolute root test.)

4. a.) Consider the series  $\sum_{n=1}^{\infty} (\cos(2n\pi) + \cos((2n+1)\pi))$

i.) Write out the first 4 terms of the series.

The first 4 terms are given by

$$(1 + -1) + (1 + -1) + (1 + -1) + (1 + -1)$$

ii.) Use a particular test to determine if the series converges or diverges.

Diverges by  $n$ th term test

b.) Consider the series  $\sum_{n=1}^{\infty} (-1)^{n+1}$

i.) Write out the first 8 terms of the series.

The first 8 terms are given by

$$1 + -1 + 1 + -1 + 1 + -1 + 1 + -1$$

ii.) Use a particular test to determine if the series converges or diverges.

Divergent by  $n$ th term test.

c.) The series in a.) and b.) appear to be the same, but the answers in a.) ii.) and b.) ii.); show otherwise

Although both of these series appear to be the same, we can use the identity  $\cos(a+b) = \cos(a)\cos(b) + \sin(a)\sin(b)$  to change the sum into

$$\sum_{n=1}^{\infty} (\cos(2n\pi) + \cos((2n+1)\pi)) = \sum_{n=1}^{\infty} (\cos(2n\pi) + \cos(2n\pi)\cos(n\pi) + \sin(2n\pi)\sin(n\pi))$$

Note that  $\sin(2n\pi) = 0$  for any  $n$ , so this series becomes

$$\sum_{n=1}^{\infty} \cos(2n\pi)(1 + \cos(n\pi))$$

and the first 4 terms are now

$$1 \cdot (1 + -1) + 1 \cdot (1 + 1) + 1 \cdot (1 + -1) + 1 \cdot (1 + 1) + 1 \cdot (1 + -1) = 0 + 1 + 0 + 1$$

Thus the two series are not the same.

5. a.) Consider the series  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$

i.) Write out the first 8 terms of the series.

The first 8 terms of this series are

$$\frac{1}{2} - \frac{2}{3} + \frac{3}{4} - \frac{4}{5} + \frac{5}{6} - \frac{6}{7} + \frac{7}{8} - \frac{8}{9}$$

ii.) Use a particular test to determine if the series converges or diverges.

Diverges because  $\lim a_n = 1$ .

b.) Consider the series  $\sum_{n=1}^{\infty} \left( \frac{2n-1}{2n} - \frac{2n}{2n+1} \right)$

i.) Write out the first 4 terms of the series.

The first 4 terms are given by

$$\left( \frac{1}{2} - \frac{2}{3} \right) + \left( \frac{3}{4} - \frac{4}{5} \right) + \left( \frac{5}{6} - \frac{6}{7} \right) + \left( \frac{7}{8} - \frac{8}{9} \right)$$

ii.) Use a particular test to determine if the series converges or diverges.

Combining the two fractions gives us

$$\sum_{n=1}^{\infty} \left( \frac{2n-1}{2n} - \frac{2n}{2n+1} \right) = \sum_{n=1}^{\infty} \left( \frac{1}{2n(2n+1)} \right)$$

Note that

$$\frac{1}{4n^2 + 2n} < \frac{1}{n^2}$$

for any  $n > 0$ , so by the comparison test, we conclude that this series converges.

c.) The series in a.) and b.) appear to be the same, but the answers in a.) ii.) and b.) ii.); show otherwise

One converges and the other diverges, so they are not equal; wow.

6. a.) Consider the series  $\sum_{n=0}^{\infty} (-1/2)^n$

i.) Write out the first 8 terms of the series.

The first 8 terms are

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64} - \frac{1}{128}$$

ii.) Use a particular test to determine if the series converges or diverges.

Converges absolutely by geometric series test.

b.) Consider the series  $\sum_{n=0}^{\infty} \left( \frac{1}{2^{2n}} - \frac{1}{2^{2n+1}} \right)$

i.) Write out the first 4 terms of the series.

The first 4 terms are

$$\left( 1 - \frac{1}{2} \right) + \left( \frac{1}{4} - \frac{1}{8} \right) + \left( \frac{1}{16} - \frac{1}{32} \right) + \left( \frac{1}{64} - \frac{1}{128} \right)$$

ii.) Use a particular test to determine if the series converges or diverges.

Some algebra gives us

$$\sum_{n=0}^{\infty} \left( \frac{1}{2^{2n}} - \frac{1}{2^{2n+1}} \right) = \sum_{n=0}^{\infty} \frac{2^{2n+1} - 2^{2n}}{2^{2n} 2^{2n+1}} = \sum_{n=0}^{\infty} \frac{2^{2n}(2 - 1)}{2^{2n} 2^{2n+1}} = \sum_{n=0}^{\infty} \frac{1}{2^{2n+1}}$$

Which converges by geometric series test

c.) The series in a.) and b.) appear to be the same, but the answers in a.) ii.) and b.) ii.); bear out that they are. What characteristic in 6.) have that those in 4.) and 5.) do not? The series (All are the same.) in 6.) is an *absolutely convergent* series, which converges to the same number no matter how the terms in the series are grouped or in what order they are added. This is not always true of *conditionally convergent* or *divergent* series.

Both series converge so these are the same. In Divergent series, terms can be arbitrarily grouped and thus they can look similar without being the same.