Math 21C
Kouba
Discussion Sheet 10

1.) Consider the function given by $f(x,y) = xy^2 - x^2y$ and the point $P = (1,-1)$. Compute

a.) the exact change of f and
b.) use a differential to estimate the exact change of f

if point P moves in a straight line to point $Q = (1.5,-0.7)$.

2.) Consider the function given by $f(x,y) = \ln(3x + 4y^2)$ and the point $P = (5,2)$. Compute

a.) the exact change of f and
b.) use a differential to estimate the exact change of f

if point P moves a distance of $ds = 1.4$ in the direction of vector $\overrightarrow{A} = 5\overrightarrow{i} + 12\overrightarrow{j}$.

3.) Find the point on the plane $x + 2y + 3z = 6$ nearest the origin.

4.) Determine the dimensions and minimum surface area of a closed rectangular box with volume 8 ft.^{3}

5.) Determine the dimensions and minimum surface area of the closed right circular cylinder with volume 16 π ft.^{3}

6.) Material for the top and bottom of a rectangular box costs \$4/ft.^{2} and that for the sides costs \$2/ft.^{2} Determine the dimensions of the least expensive box of volume \$4/ft.^{2}

7.) Among all open (no top) rectangular boxes with surface area 300 in.^{2}, determine the dimensions of the box of maximum volume.

8.) Determine the absolute extrema for each function on the indicated region.

a.) $f(x,y) = 2x + 4y + 12$ on

i.) the triangle with vertices $(0,0), (0,3)$, and $(3,0)$ and its interior.
ii.) the circle $x^2 + y^2 = 4$ and its interior.

b.) $f(x,y) = xy - x - 3y$ on the triangle with vertices $(0,0), (0,4)$, and $(5,0)$ and its interior.

c.) $f(x,y) = x^2 - 3y^2 - 2x + 6y$ on the square with vertices $(0,0), (0,2), (2,0)$ and $(2,2)$ and its interior.

9.) Use Lagrange multipliers to determine the extreme values for each of the following.

a.) Minimize $f(x,y) = x^2 + y^2$ subject to $2x + 4y = 5$.

b.) Maximize $f(x, y) = x^2 - y^2$ subject to $y = x^2$.

c.) Maximize and minimize $f(x, y) = 3x + 4y + 2$ subject to $x^2 + y^2 = 9$.

d.) Minimize $f(x, y, z) = x^2 + y^2 + z^2$ subject to $x + 2z = 4$ and $x + y = 8$.

"Do just once what others say you can't do, and you will never pay attention to their limitations again." – James R. Cook