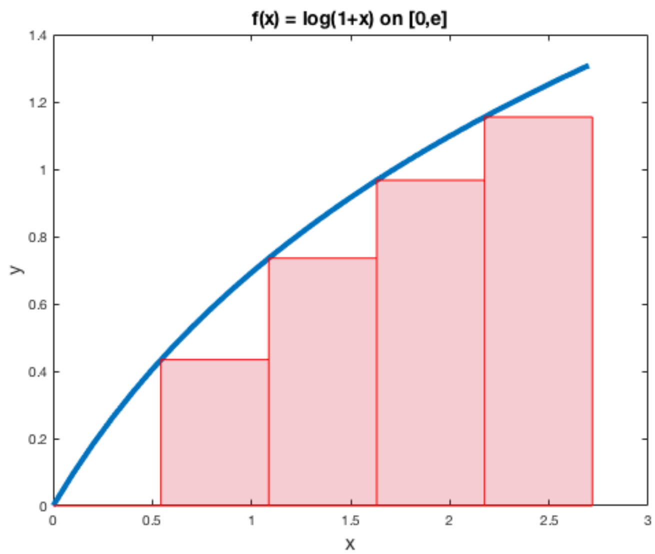
Math 21B
Disussion Sheet 2 - Key
Answers by Doug
DESherman@UCDavis.edu
Note: These answers are not endorsed by Dr. Gravner and may be incorrect!

1. Partition the interval $[0, e]$ into 5 subintervals of equal length and let c_i be the *left* endpoints of the subintervals. Form the approximating sum (i.e., the Riemann sum) for $\int_0^e \log(x+1)\, dx$. Does the sum underestimate or overestimate the integral?

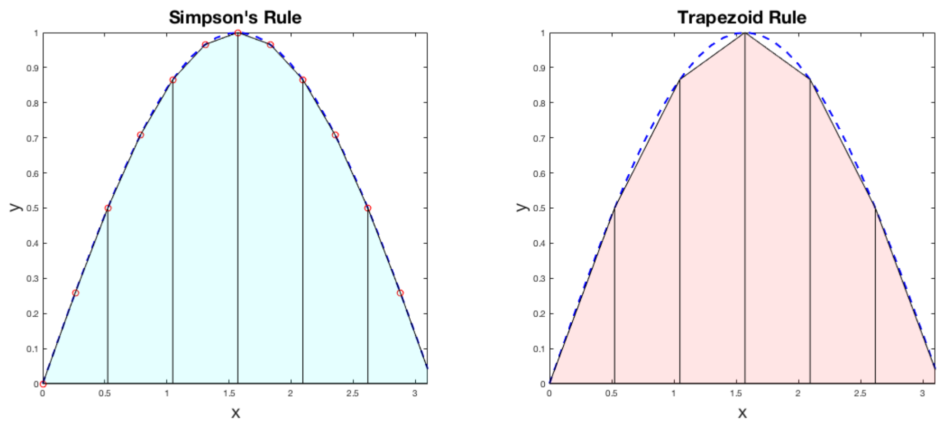


1. Left hand rule approximation of $f(x) = \log(1+x)$ over the interval $[0, e]$.

Let $f(x) = \log(1+x)$, then using the left endpoint rule we get 5 rectangles given by

$$\begin{aligned} f(x) &\approx (x_1 - x_0)f(x_0) + (x_2 - x_1)f(x_1) + (x_3 - x_2)f(x_2) + (x_4 - x_3)f(x_3) + (x_5 - x_4)f(x_4) \\ &= \frac{e}{5}\left(\log(1) + \log(1 + e/5) + \log(1 + 2e/5) + \cdots + \log(4e/5)\right) = 1.79 \end{aligned}$$

2. Use Simpson's rule with $n = 6$ to approximate $\int_0^\pi \sin x\, dx$. Simplify your answer to the point where a calculator would be useful. Then do the same with the trapezoidal method. Does the trapezoidal method underestimate or overestimate the integral?



2. Comparison of Simpson's Rule and the Trapezoid Method for estimating the definite integral of $f(x) = \sin x$ for $x \in [0, \pi]$.

3. Find a definite integral that is approximated by

$$\boldsymbol{\sum_{i=1}^{100} \frac{1}{200+i}}$$

Is the sum larger or smaller than the integral?

The definition of an approximate sum for a function $f(x)$ is given by

$$f(x) \approx \frac{b-a}{n} \sum_{i=1}^{n} f\left(a + \frac{b-a}{n} i\right)$$

For the given function we see that n must be 100. So, this gives us

$$\frac{b-a}{n} = \frac{b-a}{100} = 1 \iff b = a + 100$$

Moreover, there is no constant on i, so we can't factor anything out of i in the sum above, so we conclude that $a = 200$, thus $b = 200 + 100 = 300$. Thus we have that this approximates the integral

$$\sum_{i=1}^{100} \frac{1}{200+i} \approx \int_{200}^{300} \frac{1}{x} \, dx$$