Math 21C
Kouba
Disussion Sheet 2 - Key

1.) Determine convergence or divergence of each series using the test indicated. I suggest that your read all of the assumptions and conclusions for each test as you do each problem.

a.) $\sum_{\mathbf{n=3}}^{\infty} \mathbf{\frac{2n+3}{3n+2}}$ **(Use the nth term test.)**

Let $a_n := \frac{2n+3}{3n+2}$ then we obtain

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2n+3}{3n+2} = \lim_{n\to\infty} \frac{\frac{2n}{n}+\frac{3}{n}}{\frac{3n}{n}+\frac{2}{n}} = \frac{2+0}{3+0} = \frac{2}{3} \neq 0$$

Therefore, since the terms in ther series $\sum_{n=3}^{\infty} a_n$ do not approach 0, we conclude this series diverges by the nth term test.

b.) $\sum_{\mathbf{n=4}}^{\infty} \mathbf{7\frac{(-2)^{n+1}}{3^{n-1}}}$ **(Use the geometric series test.)**

With the geometric series test we must get our series into the form of $\sum (r)^n$ for $|r| < 1$. So, for the given series we obtain

$$\sum_{n=4}^{\infty} 7\frac{(-2)^{n+1}}{3^{n-1}} = \sum_{n=4}^{\infty} 7\frac{(-2)^{n-1}(-2)^2}{3^{n-1}} = \sum_{n=4}^{\infty} 7(-2)^2 \left(\frac{-2}{3}\right)^{n-1} = 28\sum_{n=4}^{\infty} \left(-\frac{2}{3}\right)^{n-1}$$

Thus since $|-2/3| = 2/3 < 1$ we conclude that this series converges by the geometric series test.

c.) $\sum_{\mathbf{n=1}}^{\infty} \mathbf{\frac{1}{n^{\sqrt{2}}}}$ **(Use the p-series test.)**

By the p-series test we have the series $\sum \frac{1}{n^p}$ converge if $p > 1$. For our series we obtain

$$\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{2}}} \approx \sum_{n=1}^{\infty} \frac{1}{n^{1.4\cdots}}$$

Thus, since $\sqrt{2} \approx 1.4... > 1$, we conclude that our series converges by the p-series test with $p = \sqrt{2} > 1$.

d.) $\sum_{\mathbf{n=2}}^{\infty} \mathbf{\frac{n}{n^2+4}}$ **(Use the integral test.)**

e.) $\sum_{\mathbf{n=1}}^{\infty} \mathbf{\frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n+2}}}$ **(Use the sequence of partial sums test.)**

f.) $\sum_{\mathbf{n=2}}^{\infty} \mathbf{\frac{n-1}{n^3+2}}$ **(Use the comparison test.)**

g.) $\sum_{\mathbf{n=1}}^{\infty} \mathbf{\frac{n^3+7n^2-3}{n^4-4n+9}}$ **(Use the limit comparison test.)**

h.) $\sum_{\mathbf{n=0}}^{\infty} \mathbf{\frac{3^{n-1}}{(n+1)!}}$ **(Use the ratio test.)**

i.) $\sum_{\mathbf{n=1}}^{\infty} \mathbf{\left(1.01 - \frac{5}{n^3}\right)^n}$ **(Use the root test.)**