Truncated Tooplitz Operators

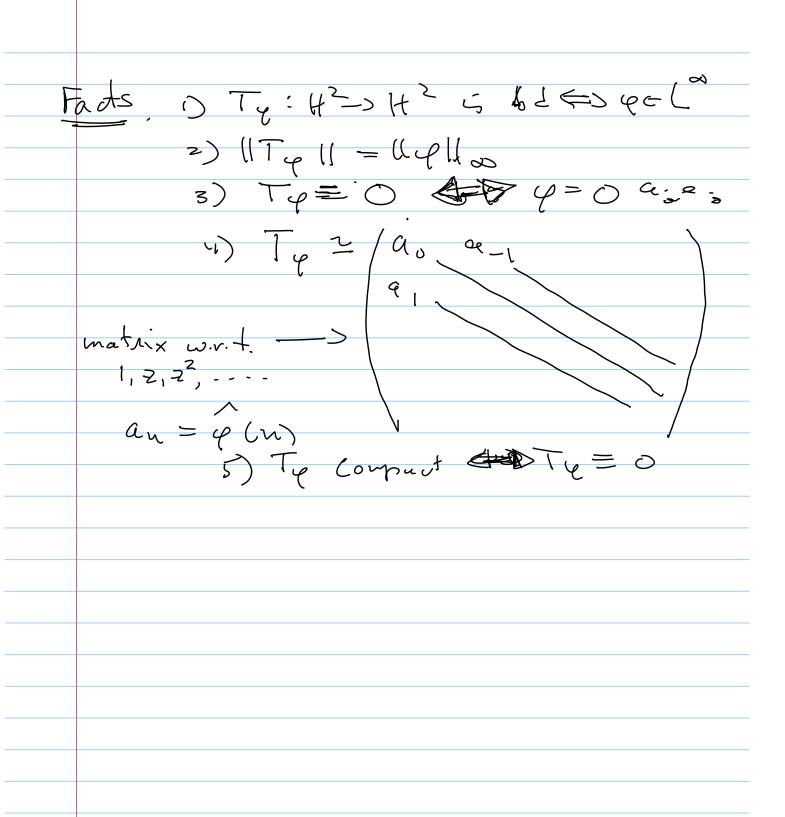
ote Title 1/16/2008

Topplitz Operators

H2, 12, P: 12 > H2 (Ruesz Pray)

$$(Pf)(2) = \int \frac{f(\xi)}{1 - \sqrt{\xi}} dm(\xi)$$

For $\psi \in \mathbb{Z}$, define $T\psi : H \rightarrow Hol(N)$ $T\psi f)(z) = \int \frac{\psi(\xi) f(\xi)}{1 - \xi z} du(\xi)$



Q when in T = B(H²) a Toep. op?

Let S = unclateral shelt on H².

(S*TySf, Sg)

= (MySf, Sg)

= (Myf, g)

= (Tpf,g)

So S*TyS = Ty

Thus (Brown-Halmes) TEB(H2) us a Toop of S*TS=T Truncated Toeplitz Opeatous

0 = inver function

Ko = H² (o H²)

model speace

Note S* Ko = Ko

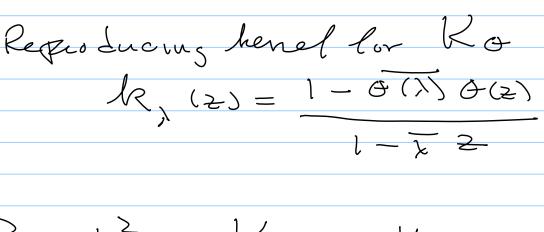
Thus (Douglas - Shapiro - Shields)

For feH2, TFAE.

D Je Ko

2) ff = H2(T) \(O H_0(T) \)

3) for hear a P.C. For H3 (De)



 $P_{\alpha}: L^{2} \rightarrow K_{\alpha} \text{ or this. Perj.}$ $(P_{\alpha}f)(\lambda) = \langle f, k_{\lambda} \rangle$

For Geld
Ag: Kon DHollD

 $(4gf)(x) = \int \varphi(\xi) f(\xi) le_{\chi}(\xi) du(\xi)$

Ay to a truncated Toppletz

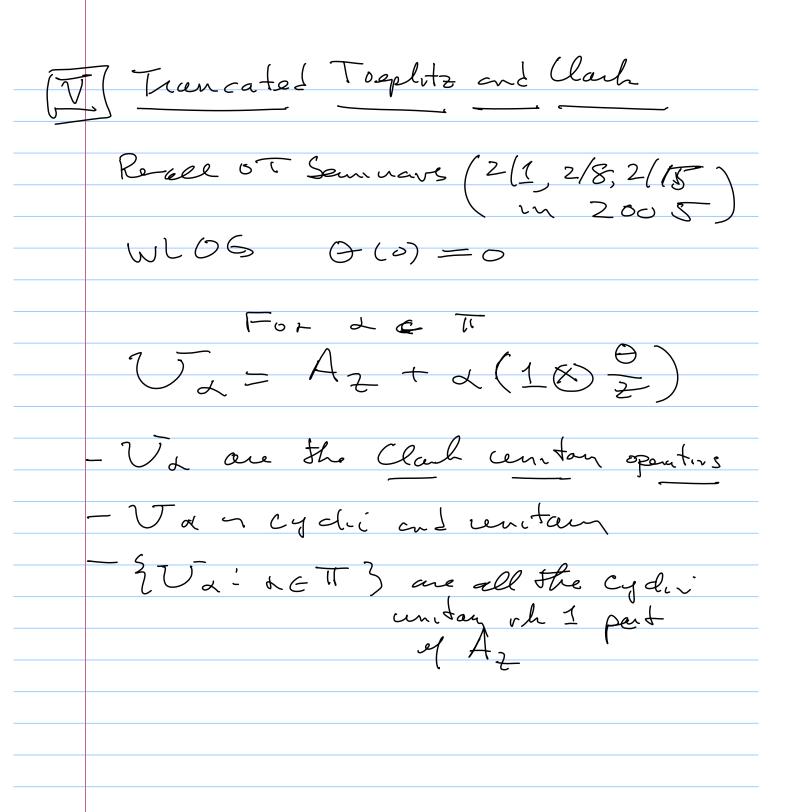
Rewals: 1; Wen y c Lo Aq = Po Tylko is bounded and in the compenser of the Toppletz specutar top 2. Unlike Te where Tq=0=0=0 it is possible to have Aq=0 but & \$0 Sall Aq is be con we lind a fle La so shat Aq = Aq1. Ex: If ye H and A q y bd the NF delation saws that A==Ay for some 4 = Hx. Moreover 11 AGIL = 11 yllas

Varque vess when if Ap = 0? QCOH2+ OH2 (thenh on

Suppose q = Ohy + (h,, hz & t (2) ff=Ohif+Ohzf T; Po (ohyf) = 0 (since Ohyfe OH2 f e 0 H 00 Of etto hz (6f) = H2 Po (h, 0 f) = 0 > Kind of technical

Wembership.

Zerael: TEB(H2) ma Toep. op S*TS = T To = { Aq: QEL , Aq: Ko-sko} (vector space of bl T.T. 0) The (Sanason) $A \in B(K_0)$ belongs to To J $g_{1},g_{2} \in K_{0}$ A = A= A= AA= +91 & S+6 + S+0 &92 (Nice condition but difficult to apply) Cov. Po is wor closed.



Thun (Clark) There Exists per & M+(t)
and a centrary Vz : Ko -> L'(rx) Such that $\nabla^{*}_{\alpha}\left(M_{q_{1}}L^{2}(q_{1})\right)\sqrt{\alpha}=\nabla_{\alpha}$ Con For say a be-Boul fewerten of $\nabla_{x}^{+}(M_{\varphi},L^{2}(\gamma_{x}))\nabla_{x}=\varphi(\nabla_{x})$ The (Alebertan) m = { Mx 2m (x) J fdm = [((fdmx) dm(x) lu (Peltaratshi) For fo Ko lun f(z) = (4)(2)\$2->€

For a bl. Bord y and fige Ko < Ay 1, g > = \ 9 + \ Ju (Hele) $\Rightarrow = \int \left(\int \varphi + \overline{g} \, d\mu_z \right) \, dm (a)$ (Polt) = S (S q VI Vx : q CMa) Jun(2) Thus Aq = { Q(Va) &m (a) (Coof familia;)

