

# The Unique Pseudo-Expectation Property, and a New Proof of Mercer's Extension Theorem

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UVA Seminar in Operator Theory and Operator Algebras  
17 April 2012

## Abstract

Let  $\mathcal{C}$  be a unital  $C^*$ -algebra and  $\mathcal{D} \subseteq \mathcal{C}$  be a unital  $C^*$ -subalgebra. We say that the pair  $(\mathcal{C}, \mathcal{D})$  has the **unique pseudo-expectation property** if there is a unique ucp map  $\Phi : \mathcal{C} \rightarrow I(\mathcal{D})$  such that  $\Phi|_{\mathcal{D}} = \text{id}$  (here  $I(\mathcal{D})$  is the injective envelope of  $\mathcal{D}$ ). For the first half of the talk, we explore the unique pseudo-expectation property, giving several examples and proving some key results. In the second half of the talk we show how the unique pseudo-expectation property combined with an automatic complete boundedness result of Pitts gives a new proof of Mercer's Extension Theorem, which states that every Cartan bimodule isomorphism extends to a  $\star$ -isomorphism of the containing von Neumann algebras (terms to be defined during the lecture). This talk is based on joint work with David Pitts (Nebraska) and Jan Cameron (Vassar), and should be accessible to graduate students in functional analysis.