Bimodules over Cartan Subalgebras, and Mercer's Extension Theorem

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Abstract

In a paper from 1990, Mercer asserts the following extension theorem:

> *For $i = 1, 2$, let $\mathcal{M}_i$ be a von Neumann algebra with separable predual, let $\mathcal{D}_i \subseteq \mathcal{M}_i$ be a Cartan subalgebra, and let $\mathcal{D}_i \subseteq \mathcal{A}_i \subseteq \mathcal{M}_i$ be a σ-weakly closed non-self-adjoint algebra which generates $\mathcal{M}_i$. If $\theta : \mathcal{A}_1 \to \mathcal{A}_2$ is an isometric algebra isomorphism such that $\theta(\mathcal{D}_1) = \mathcal{D}_2$, then there exists a unique $*$-isomorphism $\overline{\theta} : \mathcal{M}_1 \to \mathcal{M}_2$ such that $\overline{\theta}|_{\mathcal{M}_1} = \theta$.*

His proof relies on the *Spectral Theorem for Bimodules* of Muhly, Saito, and Solel (hereafter STB), which characterizes the σ-weakly closed $\mathcal{D}_i$-bimodules of $\mathcal{M}_i$ in terms of certain measure-theoretic data. Unfortunately, both proofs of STB in the literature contain gaps, and so the validity of STB, and therefore of Mercer's extension theorem, are in doubt. In this talk, based on joint work with Jan Cameron (Vassar) and David Pitts (Nebraska), we prove Mercer's extension theorem under the additional hypothesis that θ is σ-weakly continuous. Our argument makes use ideas from operator space theory (in particular an automatic complete boundedness theorem of Pitts), as well as a characterization of the Bures-closed $\mathcal{D}_i$-bimodules of $\mathcal{M}_i$ in terms of projections in an appropriate abelian von Neumann algebra, and does not require that $\mathcal{M}_i$ have separable predual. *(Remark: If this talk sounds familiar, it should be. We gave a similar talk last year at UVA, before discovering the issue with STB. This talk represents the aftermath of that realization.)*