

NEARLY INVARIANT SUBSPACES AND SYMMETRIC OPERATORS

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A subspace S of the Hardy space of upper half-plane H^2 is said to be nearly invariant for the backwards shift if given any $f \in S$ which vanishes at i , the function $f(z)/(z - i)$ belongs to S . Any nearly invariant subspace S has the form $S = h(\Theta H^2)^\perp$ where Θ is an inner function vanishing at i , and h is an isometric multiplier of $(\Theta H^2)^\perp$ onto S . We call a subspace S of L^2 of the real line nearly invariant if $S = uS'$ where u is a unimodular function and S' is a nearly invariant subspace of H^2 .

In this talk we show that a subspace $S = uh(\Theta H^2)^\perp$ is nearly invariant with meromorphic inner function Θ if and only if the operator M of multiplication by the independent variable has a regular simple symmetric restriction to S with deficiency indices $(1, 1)$. To achieve this result we will use the dilation theory of completely positive maps.

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