

Banach spaces of functions of bounded generalized variation

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(joint work with D. Apatsidis)

The spaces V_2^0, V_2 of functions of bounded quadratic variation have been thoroughly studied. One of the characteristic properties of the space V_2^0 , is that it is separable with non separable dual, not containing ℓ_1 . Moreover, the second dual of V_2^0 naturally coincides with the space V_2 . The subspace structure of the spaces V_2^0, V_2 is quite rich, as c_0 and $\ell_p, p \geq 2$ all embed into V_2^0 and ℓ_∞ embeds into $V_2 \cap C[0, 1]$.

In the present lecture we will generalize the notion of variation, in order to obtain spaces D_X^0, D_X of functions of bounded generalized variation, where X is a reflexive space with an unconditional basis. These spaces share similar general properties with the spaces V_2^0, V_2 , as they are separable with non separable dual, not containing ℓ_1 and $(D_X^0)^{**}$ naturally coincides with D_X .

However, the subspace structure of these spaces is more homogeneous, as both D_X^0 and D_X are saturated with subspaces of X . In particular, the spaces D_2^0, D_2 are ℓ_2 saturated and the spaces D_T^0, D_T are saturated with subspaces of Tsirelson space, hence they do not contain c_0 or ℓ_p .