

|      |                        |
|------|------------------------|
| Step | Algorithm:             |
| 1a   |                        |
| 4    | where                  |
| 2    |                        |
| 3    | while do               |
| 2,3  | $\wedge$               |
| 5a   | where                  |
| 6    |                        |
| 8    |                        |
| 5b   |                        |
| 7    |                        |
| 2    |                        |
|      | endwhile               |
| 2,3  | $\wedge \neg( \quad )$ |
| 1b   |                        |

|      |   |
|------|---|
| Step | <b>Algorithm:</b> $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$  |
| 1a   | $C = \widehat{C}$   |
| 4    | $A \rightarrow \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$<br><b>where</b> $A_{TL}$ is $0 \times 0$ , $B_T$ has 0 rows, $C_T$ has 0 rows   |
| 2    | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right)$   |
| 3    | <b>while</b> $m(A_{TL}) < m(A)$ <b>do</b>   |
| 2,3  | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \wedge m(A_{TL}) < m(A)$   |
| 5a   | $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$<br><b>where</b> $\alpha_{11}$ is $1 \times 1$ , $b_1$ has 1 row, $c_1$ has 1 row |
| 6    | $\left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right) = \left( \begin{array}{c} A_{00}B_0 + \widehat{C}_0 \\ \hline a_{01}^TB_0 + \widehat{c}_1 \\ \hline A_{02}B_0 + \widehat{C}_2 \end{array} \right)$   |
| 8    | $C_0 := a_{01}b_1^T + C_0$<br>$c_1^T := \alpha_{11}b_1^T + c_1^T$<br>$C_2 := a_{12}^Tb_1^T + C_2$   |
| 5b   | $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$   |
| 7    | $\left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right) = \left( \begin{array}{c} A_{00}B_0 + a_{01}b_1^T + \widehat{C}_0 \\ \hline a_{01}^TB_0 + \alpha_{11}b_1^T + \widehat{c}_1 \\ \hline A_{02}B_0 + a_{12}^Tb_1^T + \widehat{C}_2 \end{array} \right)$  |
| 2    | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right)$   |
|      | <b>endwhile</b>   |
| 2,3  | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \wedge \neg(m(A_{TL}) < m(A))$   |
| 1b   | $[C] = \text{symm\_lu}(A, B, \widehat{C})$  |

**Algorithm:**  $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$

$$A \rightarrow \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$$

**where**  $A_{TL}$  is  $0 \times 0$ ,  $B_T$  has 0 rows,  $C_T$  has 0 rows

**while**  $m(A_{TL}) < m(A)$  **do**

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

**where**  $\alpha_{11}$  is  $1 \times 1$ ,  $b_1$  has 1 row,  $c_1$  has 1 row

$$C_0 := a_{01}b_1^T + C_0$$

$$c_1^T := \alpha_{11}b_1^T + c_1^T$$

$$C_2 := a_{12}^Tb_1^T + C_2$$

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

**endwhile**

|      |   |
|------|---|
| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$ |
| 1a   | $C = \hat{C}$   |
| 4    | where   |
| 2    |   |
| 3    | while do  |
| 2,3  | $\wedge$  |
| 5a   | where   |
| 6    |   |
| 8    |   |
| 5b   |   |
| 7    |   |
| 2    |   |
|      | endwhile  |
| 2,3  | $\wedge \neg( \quad )$                                  |
| 1b   | $[C] = \text{symm\_lu}(A, B, \hat{C})$                  |

|      |  |
|------|--|
| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$                                |
| 1a   | $C = \hat{C}$  |
| 4    |  |
|      | where  |
| 2    | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\hat{C}_T}{\hat{C}_B}\right)$              |
| 3    | while do   |
| 2,3  | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\hat{C}_T}{\hat{C}_B}\right) \wedge$       |
| 5a   |  |
|      | where  |
| 6    |  |
| 8    |  |
| 5b   |  |
| 7    |  |
| 2    | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\hat{C}_T}{\hat{C}_B}\right)$              |
|      | endwhile   |
| 2    | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\hat{C}_T}{\hat{C}_B}\right) \wedge \neg($ |
| 1b   | $[C] = \text{symm\_lu}(A, B, \hat{C})$   |

|      |   |
|------|---|
| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$   |
| 1a   | $C = \hat{C}$   |
| 4    | where   |
| 2    | $\left( \frac{C_T}{C_B} \right) = \left( \frac{\hat{C}_T}{\hat{C}_B} \right)$                               |
| 3    | while $m(A_{TL}) < m(A)$ do   |
| 2,3  | $\left( \frac{C_T}{C_B} \right) = \left( \frac{\hat{C}_T}{\hat{C}_B} \right) \wedge m(A_{TL}) < m(A)$       |
| 5a   | where   |
| 6    |   |
| 8    |   |
| 5b   |   |
| 7    |   |
| 2    | $\left( \frac{C_T}{C_B} \right) = \left( \frac{\hat{C}_T}{\hat{C}_B} \right)$                               |
|      | endwhile  |
| 2,3  | $\left( \frac{C_T}{C_B} \right) = \left( \frac{\hat{C}_T}{\hat{C}_B} \right) \wedge \neg(m(A_{TL}) < m(A))$ |
| 1b   | $[C] = \text{symm\_lu}(A, B, \hat{C})$  |

|      |   |
|------|---|
| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$   |
| 1a   | $C = \hat{C}$   |
| 4    | $A \rightarrow \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$ <p>where <math>A_{TL}</math> is <math>0 \times 0</math>, <math>B_T</math> has 0 rows, <math>C_T</math> has 0 rows</p> |
| 2    | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \hat{C}_T \\ \hline \hat{C}_B \end{array} \right)$   |
| 3    | while $m(A_{TL}) < m(A)$ do   |
| 2,3  | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \hat{C}_T \\ \hline \hat{C}_B \end{array} \right) \wedge m(A_{TL}) < m(A)$   |
| 5a   | where   |
| 6    |   |
| 8    |   |
| 5b   |   |
| 7    |   |
| 2    | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \hat{C}_T \\ \hline \hat{C}_B \end{array} \right)$   |
|      | endwhile  |
| 2,3  | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \hat{C}_T \\ \hline \hat{C}_B \end{array} \right) \wedge \neg(m(A_{TL}) < m(A))$   |
| 1b   | $[C] = \text{symm\_lu}(A, B, \hat{C})$  |

|      |   |
|------|---|
| Step | <b>Algorithm:</b> $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$  |
| 1a   | $C = \widehat{C}$   |
| 4    | $A \rightarrow \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$ <p>where <math>A_{TL}</math> is <math>0 \times 0</math>, <math>B_T</math> has 0 rows, <math>C_T</math> has 0 rows</p>   |
| 2    | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right)$   |
| 3    | <b>while</b> $m(A_{TL}) < m(A)$ <b>do</b>   |
| 2,3  | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \wedge m(A_{TL}) < m(A)$   |
| 5a   | $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$ <p>where <math>\alpha_{11}</math> is <math>1 \times 1</math>, <math>b_1</math> has 1 row, <math>c_1</math> has 1 row</p> |
| 6    |   |
| 8    |   |
| 5b   | $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$   |
| 7    |   |
| 2    | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right)$   |
|      | <b>endwhile</b>   |
| 2,3  | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \wedge \neg(m(A_{TL}) < m(A))$   |
| 1b   | $[C] = \text{symm\_lu}(A, B, \widehat{C})$  |



|      |   |
|------|---|
| Step | <b>Algorithm:</b> $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$  |
| 1a   | $C = \widehat{C}$   |
| 4    | $A \rightarrow \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$<br><b>where</b> $A_{TL}$ is $0 \times 0$ , $B_T$ has 0 rows, $C_T$ has 0 rows   |
| 2    | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right)$   |
| 3    | <b>while</b> $m(A_{TL}) < m(A)$ <b>do</b>   |
| 2,3  | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \wedge m(A_{TL}) < m(A)$   |
| 5a   | $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$<br><b>where</b> $\alpha_{11}$ is $1 \times 1$ , $b_1$ has 1 row, $c_1$ has 1 row |
| 6    | $\left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right) = \left( \begin{array}{c} A_{00}B_0 + \widehat{C}_0 \\ \hline a_{01}^T B_0 + \widehat{c}_1 \\ \hline A_{02}B_0 + \widehat{C}_2 \end{array} \right)$  |
| 8    |   |
| 5b   | $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$   |
| 7    |   |
| 2    | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right)$   |
|      | <b>endwhile</b>   |
| 2,3  | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \wedge \neg(m(A_{TL}) < m(A))$   |
| 1b   | $[C] = \text{symm\_lu}(A, B, \widehat{C})$  |

|      |   |
|------|---|
| Step | <b>Algorithm:</b> $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$  |
| 1a   | $C = \widehat{C}$   |
| 4    | $A \rightarrow \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$<br><b>where</b> $A_{TL}$ is $0 \times 0$ , $B_T$ has 0 rows, $C_T$ has 0 rows   |
| 2    | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right)$   |
| 3    | <b>while</b> $m(A_{TL}) < m(A)$ <b>do</b>   |
| 2,3  | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \wedge m(A_{TL}) < m(A)$   |
| 5a   | $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$<br><b>where</b> $\alpha_{11}$ is $1 \times 1$ , $b_1$ has 1 row, $c_1$ has 1 row |
| 6    | $\left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right) = \left( \begin{array}{c} A_{00}B_0 + \widehat{C}_0 \\ \hline a_{01}^TB_0 + \widehat{c}_1 \\ \hline A_{02}B_0 + \widehat{C}_2 \end{array} \right)$   |
| 8    |   |
| 5b   | $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$   |
| 7    | $\left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right) = \left( \begin{array}{c} A_{00}B_0 + a_{01}b_1^T + \widehat{C}_0 \\ \hline a_{01}^TB_0 + \alpha_{11}b_1^T + \widehat{c}_1 \\ \hline A_{02}B_0 + a_{12}^Tb_1^T + \widehat{C}_2 \end{array} \right)$  |
| 2    | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right)$   |
|      | <b>endwhile</b>   |
| 2    | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \wedge \neg(m(A_{TL}) < m(A))$   |
| 1b   | $[C] = \text{symm\_lu}(A, B, \widehat{C})$  |

|      |   |
|------|---|
| Step | <b>Algorithm:</b> $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$  |
| 1a   | $C = \widehat{C}$   |
| 4    | $A \rightarrow \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$<br><b>where</b> $A_{TL}$ is $0 \times 0$ , $B_T$ has 0 rows, $C_T$ has 0 rows   |
| 2    | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right)$   |
| 3    | <b>while</b> $m(A_{TL}) < m(A)$ <b>do</b>   |
| 2,3  | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \wedge m(A_{TL}) < m(A)$   |
| 5a   | $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$<br><b>where</b> $\alpha_{11}$ is $1 \times 1$ , $b_1$ has 1 row, $c_1$ has 1 row |
| 6    | $\left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right) = \left( \begin{array}{c} A_{00}B_0 + \widehat{C}_0 \\ \hline a_{01}^T B_0 + \widehat{c}_1 \\ \hline A_{02}B_0 + \widehat{C}_2 \end{array} \right)$  |
| 8    | $C_0 := a_{01}b_1^T + C_0$<br>$c_1^T := \alpha_{11}b_1^T + c_1^T$<br>$C_2 := a_{12}^T b_1^T + C_2$  |
| 5b   | $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$   |
| 7    | $\left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right) = \left( \begin{array}{c} A_{00}B_0 + a_{01}b_1^T + \widehat{C}_0 \\ \hline a_{01}^T B_0 + \alpha_{11}b_1^T + \widehat{c}_1 \\ \hline A_{02}B_0 + a_{12}^T b_1^T + \widehat{C}_2 \end{array} \right)$  |
| 2    | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right)$   |
|      | <b>endwhile</b>   |
| 2,3  | $\left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} \widehat{C}_T \\ \hline \widehat{C}_B \end{array} \right) \wedge \neg(m(A_{TL}) < m(A))$   |
| 1b   | $[C] = \text{symm\_lu}(A, B, \widehat{C})$  |

|      |   |
|------|---|
| Step | <b>Algorithm:</b> $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$  |
|      |   |
|      | $A \rightarrow \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$ <p>where <math>A_{TL}</math> is <math>0 \times 0</math>, <math>B_T</math> has 0 rows, <math>C_T</math> has 0 rows</p>   |
|      |   |
|      | <b>while</b> $m(A_{TL}) < m(A)$ <b>do</b>   |
|      |   |
|      | $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$ <p>where <math>\alpha_{11}</math> is <math>1 \times 1</math>, <math>b_1</math> has 1 row, <math>c_1</math> has 1 row</p> |
|      |   |
|      | $C_0 := a_{01}b_1^T + C_0$ $c_1^T := \alpha_{11}b_1^T + c_1^T$ $C_2 := a_{12}^Tb_1^T + C_2$   |
|      | $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$   |
|      |   |
|      |   |
|      | <b>endwhile</b>   |
|      |   |
|      |   |

**Algorithm:**  $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$

$$A \rightarrow \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$$

where  $A_{TL}$  is  $0 \times 0$ ,  $B_T$  has 0 rows,  $C_T$  has 0 rows

while  $m(A_{TL}) < m(A)$  do

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

where  $\alpha_{11}$  is  $1 \times 1$ ,  $b_1$  has 1 row,  $c_1$  has 1 row

$$C_0 := a_{01}b_1^T + C_0$$

$$c_1^T := \alpha_{11}b_1^T + c_1^T$$

$$C_2 := a_{12}^Tb_1^T + C_2$$

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array} \right)$$

endwhile