| Step | Algorithm: |
|---|---|
| 1a | |
| 4 | where |
| 2 | |
| 3 | while do |
| 2,3 | $\wedge$ |
| 5a | where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | |
| | endwhile |
| 2,3 | $\wedge \neg($ $)$ |
| 1b | |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR1}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$, $C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right)$ <br> where $A_{TL}$ is $0 \times 0$, $B_T$ has 0 rows, $C_T$ has 0 rows |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C_T} \\ \hline A_{TR}^T B_T + \widehat{C_B} \end{array}\right)$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C_T} \\ \hline A_{TR}^T B_T + \widehat{C_B} \end{array}\right) \wedge m(A_{TL}) < m(A)$ |
| 5a | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$ <br> where $\alpha_{11}$ is $1 \times 1$, $b_1$ has 1 row, $c_1$ has 1 row |
| 6 | $\left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right) = \left(\begin{array}{c} A_{00}B_0 + a_{01}b_1^T + A_{02}B_2 + \widehat{C_0} \\ \hline a_{01}^T B_0 + \widehat{c^T} \\ \hline A_{02}^T B_0 + \widehat{C_2} \end{array}\right)$ |
| 8 | $c_1^T := a_{11}b_1 + a_{12}^T B_2 + c_1^T$ <br> $c_2 := a_{12}^{TT} b_1^T + C_2$ |
| 5b | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$ |
| 7 | $\left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right) = \left(\begin{array}{c} A_{00}B_0 + a_{01}b_1^T + A_{02}B_2 + \widehat{C_0} \\ \hline a_{01}^T B_0 + \alpha_{11}b_1 + \alpha_{12}^T B_2 + \widehat{c^T} \\ \hline A_{02}^T B_0 + a_{12}^{TT} b_1^T + \widehat{C_2} \end{array}\right)$ |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C_T} \\ \hline A_{TR}^T B_T + \widehat{C_B} \end{array}\right)$ |
| | endwhile |
| 2,3 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C_T} \\ \hline A_{TR}^T B_T + \widehat{C_B} \end{array}\right) \wedge \neg(m(A_{TL}) < m(A))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

**Algorithm:** $[C] := \text{SYMM\_LU\_UNB\_VAR1}(A, B, C)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right), B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right), C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right)$$

where $A_{TL}$ is $0 \times 0$, $B_T$ has 0 rows, $C_T$ has 0 rows

**while** $m(A_{TL}) < m(A)$ **do**

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$$

where $\alpha_{11}$ is $1 \times 1$, $b_1$ has 1 row, $c_1$ has 1 row

$c_1^T := a_{11} b_1 + a_{12}^T B_2 + c_1^T$

$c_2 := a_{12}^{TT} b_1^T + C_2$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$$

**endwhile**

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR1}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | where |
| 2 | |
| 3 | while do |
| 2,3 | $\wedge$ |
| 5a | where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | |
| | endwhile |
| 2,3 | $\wedge \neg( \quad )$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \textsc{symm\_lu\_unb\_var1}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | where |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right)$ |
| 3 | while do |
| 2,3 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right) \wedge$ |
| 5a | where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right)$ |
| | endwhile |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right) \wedge \neg( \qquad )$ |
| 1b | $[C] = \mathrm{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR1}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | where |
| 2 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{A_{TL}B_T + A_{TR}B_B + \widehat{C}_T}{A_{TR}^T B_T + \widehat{C}_B}\right)$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{A_{TL}B_T + A_{TR}B_B + \widehat{C}_T}{A_{TR}^T B_T + \widehat{C}_B}\right) \wedge m(A_{TL}) < m(A)$ |
| 5a | where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{A_{TL}B_T + A_{TR}B_B + \widehat{C}_T}{A_{TR}^T B_T + \widehat{C}_B}\right)$ |
| | endwhile |
| 2,3 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{A_{TL}B_T + A_{TR}B_B + \widehat{C}_T}{A_{TR}^T B_T + \widehat{C}_B}\right) \wedge \neg(m(A_{TL}) < m(A))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR1}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right), B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right), C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right)$ <br> where $A_{TL}$ is $0 \times 0$, $B_T$ has 0 rows, $C_T$ has 0 rows |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right)$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right) \wedge m(A_{TL}) < m(A)$ |
| 5a | where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right)$ |
| | endwhile |
| 2,3 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right) \wedge \neg(m(A_{TL}) < m(A))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR1}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$, $C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right)$<br>where $A_{TL}$ is $0 \times 0$, $B_T$ has 0 rows, $C_T$ has 0 rows |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right)$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right) \wedge m(A_{TL}) < m(A)$ |
| 5a | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$<br>where $\alpha_{11}$ is $1 \times 1$, $b_1$ has 1 row, $c_1$ has 1 row |
| 6 | |
| 8 | |
| 5b | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$, $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$ |
| 7 | |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right)$ |
| | endwhile |
| 2,3 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right) \wedge \neg(m(A_{TL}) < m(A))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR1}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right)$ , $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$ , $C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right)$ <br> where $A_{TL}$ is $0 \times 0$, $B_T$ has 0 rows, $C_T$ has 0 rows |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right)$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right) \wedge m(A_{TL}) < m(A)$ |
| 5a | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$ , $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$ <br> where $\alpha_{11}$ is $1 \times 1$, $b_1$ has 1 row, $c_1$ has 1 row |
| 6 | $\left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right) = \left(\begin{array}{c} A_{00}B_0 + a_{01}b_1^T + A_{02}B_2 + \widehat{C}_0 \\ \hline a_{01}^T B_0 + \widehat{c}^T \\ \hline A_{02}^T B_0 + \widehat{C}_2 \end{array}\right)$ |
| 8 | |
| 5b | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$ , $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$ |
| 7 | |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right)$ |
| | endwhile |
| 2,3 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right) \wedge \neg(m(A_{TL}) < m(A))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR1}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right)$ , $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$ , $C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right)$ where $A_{TL}$ is $0 \times 0$, $B_T$ has 0 rows, $C_T$ has 0 rows |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C_T} \\ \hline A_{TR}^T B_T + \widehat{C_B} \end{array}\right)$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C_T} \\ \hline A_{TR}^T B_T + \widehat{C_B} \end{array}\right) \wedge m(A_{TL}) < m(A)$ |
| 5a | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$ , $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$ where $\alpha_{11}$ is $1 \times 1$, $b_1$ has 1 row, $c_1$ has 1 row |
| 6 | $\left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right) = \left(\begin{array}{c} A_{00}B_0 + a_{01}b_1^T + A_{02}B_2 + \widehat{C_0} \\ \hline a_{01}^T B_0 + \widehat{c^T} \\ \hline A_{02}^T B_0 + \widehat{C_2} \end{array}\right)$ |
| 8 | |
| 5b | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$ , $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$ |
| 7 | $\left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right) = \left(\begin{array}{c} A_{00}B_0 + a_{01}b_1^T + A_{02}B_2 + \widehat{C_0} \\ \hline a_{01}^T B_0 + \alpha_{11}b_1 + \alpha_{12}^T B_2 + \widehat{c^T} \\ \hline A_{02}^T B_0 + a_{12}^{TT} b_1^T + \widehat{C_2} \end{array}\right)$ |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C_T} \\ \hline A_{TR}^T B_T + \widehat{C_B} \end{array}\right)$ |
| | endwhile |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C_T} \\ \hline A_{TR}^T B_T + \widehat{C_B} \end{array}\right) \wedge \neg(m(A_{TL}) < m(A))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR1}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right), B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right), C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right)$ where $A_{TL}$ is $0 \times 0$, $B_T$ has 0 rows, $C_T$ has 0 rows |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right)$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right) \wedge m(A_{TL}) < m(A)$ |
| 5a | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$ where $\alpha_{11}$ is $1 \times 1$, $b_1$ has 1 row, $c_1$ has 1 row |
| 6 | $\left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right) = \left(\begin{array}{c} A_{00}B_0 + a_{01}b_1^T + A_{02}B_2 + \widehat{C}_0 \\ \hline a_{01}^T B_0 + \widehat{c}^T \\ \hline A_{02}^T B_0 + \widehat{C}_2 \end{array}\right)$ |
| 8 | $c_1^T := a_{11}b_1 + a_{12}^T B_2 + c_1^T$<br>$c_2 := a_{12}^{TT} b_1^T + C_2$ |
| 5b | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$ |
| 7 | $\left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right) = \left(\begin{array}{c} A_{00}B_0 + a_{01}b_1^T + A_{02}B_2 + \widehat{C}_0 \\ \hline a_{01}^T B_0 + \alpha_{11}b_1 + \alpha_{12}^T B_2 + \widehat{c}^T \\ \hline A_{02}^T B_0 + a_{12}^{TT} b_1^T + \widehat{C}_2 \end{array}\right)$ |
| 2 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right)$ |
| | endwhile |
| 2,3 | $\left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + A_{TR}B_B + \widehat{C}_T \\ \hline A_{TR}^T B_T + \widehat{C}_B \end{array}\right) \wedge \neg(m(A_{TL}) < m(A))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR1}(A, B, C)$ |
|---|---|
| | |
| | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right), B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right), C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right)$ where $A_{TL}$ is $0 \times 0$, $B_T$ has 0 rows, $C_T$ has 0 rows |
| | |
| | while $m(A_{TL}) < m(A)$ do |
| | |
| | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$ where $\alpha_{11}$ is $1 \times 1$, $b_1$ has 1 row, $c_1$ has 1 row |
| | |
| | $c_1^T := a_{11} b_1 + a_{12}^T B_2 + c_1^T$<br>$c_2 := a_{12}^{TT} b_1^T + C_2$ |
| | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$ |
| | |
| | |
| | endwhile |
| | |
| | |

**Algorithm:** $[C] := \textsc{symm\_lu\_unb\_var1}(A, B, C)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right), B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right), C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right)$$

where $A_{TL}$ is $0 \times 0$, $B_T$ has 0 rows, $C_T$ has 0 rows

**while** $m(A_{TL}) < m(A)$ **do**

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$$

where $\alpha_{11}$ is $1 \times 1$, $b_1$ has 1 row, $c_1$ has 1 row

$c_1^T := a_{11} b_1 + a_{12}^T B_2 + c_1^T$

$c_2 := a_{12}^{TT} b_1^T + C_2$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline 0 & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$$

**endwhile**