| Step | Algorithm: |
|---|---|
| 1a | |
| 4 | where |
| 2 | |
| 3 | while do |
| 2,3 | $\wedge$ |
| 5a | where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | |
| | endwhile |
| 2,3 | $\wedge\ \neg($ ) |
| 1b | |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_BLK\_VAR1}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | $B \rightarrow \left(\begin{array}{c\|c} B_L & B_R \end{array}\right)$ , $C \rightarrow \left(\begin{array}{c\|c} C_L & C_R \end{array}\right)$<br>where $B_L$ has 0 columns, $C_L$ has 0 columns |
| 2 | $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_L & \widehat{C}_R \end{array}\right)$ |
| 3 | while $n(B_L) < n(B)$ do |
| 2,3 | $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_L & \widehat{C}_R \end{array}\right) \wedge n(B_L) < n(B)$ |
| 5a | **Determine block size** $b$<br>$\left(\begin{array}{c\|c} B_L & B_R \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} B_0 & B_1 & B_2 \end{array}\right)$ , $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} C_0 & C_1 & C_2 \end{array}\right)$<br>where $B_1$ has $b$ columns, $C_1$ has $b$ columns |
| 6 | $\left(\begin{array}{c\|c\|c} C_0 & C_1 & C_2 \end{array}\right) = \left(\begin{array}{c\|c\|c} AB_0 + \widehat{C}_0 & \widehat{C}_1 & \widehat{C}_2 \end{array}\right)$ |
| 8 | $C_1 := AB_1 + C_1$ |
| 5b | $\left(\begin{array}{c\|c} B_L & B_R \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} B_0 & B_1 & B_2 \end{array}\right)$ , $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} C_0 & C_1 & C_2 \end{array}\right)$ |
| 7 | $\left(\begin{array}{c\|c\|c} C_0 & C_1 & C_2 \end{array}\right) = \left(\begin{array}{c\|c\|c} AB_0 + \widehat{C}_0 & AB_1 + \widehat{C}_1 & \widehat{C}_2 \end{array}\right)$ |
| 2 | $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_L & \widehat{C}_R \end{array}\right)$ |
| | endwhile |
| 2,3 | $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_L & \widehat{C}_R \end{array}\right) \wedge \neg(n(B_L) < n(B))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Algorithm: $[C] := \text{SYMM\_LU\_BLK\_VAR1}(A, B, C)$ |
|---|
| $B \rightarrow \left(\begin{array}{c\|c} B_L & B_R \end{array}\right)$ , $C \rightarrow \left(\begin{array}{c\|c} C_L & C_R \end{array}\right)$<br>where $B_L$ has 0 columns, $C_L$ has 0 columns |
| while $n(B_L) < n(B)$ do |
| **Determine block size** $b$<br>$\left(\begin{array}{c\|c} B_L & B_R \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} B_0 & B_1 & B_2 \end{array}\right)$ , $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} C_0 & C_1 & C_2 \end{array}\right)$<br>where $B_1$ has $b$ columns, $C_1$ has $b$ columns |
| $C_1 := AB_1 + C_1$ |
| $\left(\begin{array}{c\|c} B_L & B_R \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} B_0 & B_1 & B_2 \end{array}\right)$ , $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} C_0 & C_1 & C_2 \end{array}\right)$ |
| endwhile |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_BLK\_VAR1}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | where |
| 2 | |
| 3 | while do |
| 2,3 | $\wedge$ |
| 5a | **Determine block size**<br>where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | |
| | endwhile |
| 2,3 | $\wedge \neg($ $)$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_BLK\_VAR1}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | where |
| 2 | $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_L & \widehat{C}_R \end{array}\right)$ |
| 3 | while do |
| 2,3 | $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_L & \widehat{C}_R \end{array}\right) \wedge$ |
| 5a | **Determine block size**<br>where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_L & \widehat{C}_R \end{array}\right)$ |
| | endwhile |
| 2 | $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_L & \widehat{C}_R \end{array}\right) \wedge \neg($ $)$ |
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| 3 | while $n(B_L) < n(B)$ do |
| 2,3 | $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_L & \widehat{C}_R \end{array}\right) \wedge n(B_L) < n(B)$ |
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| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_L & \widehat{C}_R \end{array}\right)$ |
| | endwhile |
| 2,3 | $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_L & \widehat{C}_R \end{array}\right) \wedge \neg(n(B_L) < n(B))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_BLK\_VAR1}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | $B \rightarrow \left(B_L \middle\vert B_R\right)$ , $C \rightarrow \left(C_L \middle\vert C_R\right)$<br>where $B_L$ has 0 columns, $C_L$ has 0 columns |
| 2 | $\left(C_L \middle\vert C_R\right) = \left(\widehat{C}_L \middle\vert \widehat{C}_R\right)$ |
| 3 | while $n(B_L) < n(B)$ do |
| 2,3 | $\left(C_L \middle\vert C_R\right) = \left(\widehat{C}_L \middle\vert \widehat{C}_R\right) \wedge n(B_L) < n(B)$ |
| 5a | **Determine block size**<br><br>where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | $\left(C_L \middle\vert C_R\right) = \left(\widehat{C}_L \middle\vert \widehat{C}_R\right)$ |
| | endwhile |
| 2,3 | $\left(C_L \middle\vert C_R\right) = \left(\widehat{C}_L \middle\vert \widehat{C}_R\right) \wedge \neg(n(B_L) < n(B))$ |
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| 6 | |
| 8 | |
| 5b | $\left(\begin{array}{c\|c} B_L & B_R \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} B_0 & B_1 & B_2 \end{array}\right)$ , $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} C_0 & C_1 & C_2 \end{array}\right)$ |
| 7 | |
| 2 | $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_L & \widehat{C}_R \end{array}\right)$ |
| | endwhile |
| 2,3 | $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_L & \widehat{C}_R \end{array}\right) \wedge \neg(n(B_L) < n(B))$ |
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| 6 | $\left(\begin{array}{c\|c\|c} C_0 & C_1 & C_2 \end{array}\right) = \left(\begin{array}{c\|c\|c} AB_0 + \widehat{C}_0 & \widehat{C}_1 & \widehat{C}_2 \end{array}\right)$ |
| 8 | |
| 5b | $\left(\begin{array}{c\|c} B_L & B_R \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} B_0 & B_1 & B_2 \end{array}\right)$ , $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} C_0 & C_1 & C_2 \end{array}\right)$ |
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| 6 | $\left(\begin{array}{c\|c\|c} C_0 & C_1 & C_2 \end{array}\right) = \left(\begin{array}{c\|c\|c} AB_0 + \widehat{C}_0 & \widehat{C}_1 & \widehat{C}_2 \end{array}\right)$ |
| 8 | |
| 5b | $\left(\begin{array}{c\|c} B_L & B_R \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} B_0 & B_1 & B_2 \end{array}\right)$ , $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} C_0 & C_1 & C_2 \end{array}\right)$ |
| 7 | $\left(\begin{array}{c\|c\|c} C_0 & C_1 & C_2 \end{array}\right) = \left(\begin{array}{c\|c\|c} AB_0 + \widehat{C}_0 & AB_1 + \widehat{C}_1 & \widehat{C}_2 \end{array}\right)$ |
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| 2 | $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_L & \widehat{C}_R \end{array}\right)$ |
| 3 | while $n(B_L) < n(B)$ do |
| 2,3 | $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) = \left(\begin{array}{c\|c} \widehat{C}_L & \widehat{C}_R \end{array}\right) \wedge n(B_L) < n(B)$ |
| 5a | **Determine block size** $b$<br>$\left(\begin{array}{c\|c} B_L & B_R \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} B_0 & B_1 & B_2 \end{array}\right)$ , $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} C_0 & C_1 & C_2 \end{array}\right)$<br>where $B_1$ has $b$ columns, $C_1$ has $b$ columns |
| 6 | $\left(\begin{array}{c\|c\|c} C_0 & C_1 & C_2 \end{array}\right) = \left(\begin{array}{c\|c\|c} AB_0 + \widehat{C}_0 & \widehat{C}_1 & \widehat{C}_2 \end{array}\right)$ |
| 8 | $C_1 := AB_1 + C_1$ |
| 5b | $\left(\begin{array}{c\|c} B_L & B_R \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} B_0 & B_1 & B_2 \end{array}\right)$ , $\left(\begin{array}{c\|c} C_L & C_R \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} C_0 & C_1 & C_2 \end{array}\right)$ |
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|---|---|
| | |
| | $B \rightarrow \left(B_L \middle\vert B_R\right)$ , $C \rightarrow \left(C_L \middle\vert C_R\right)$<br>where $B_L$ has 0 columns, $C_L$ has 0 columns |
| | |
| | while $n(B_L) < n(B)$ do |
| | |
| | **Determine block size** $b$<br>$\left(B_L \middle\vert B_R\right) \rightarrow \left(B_0 \middle\vert B_1 \middle\vert B_2\right)$ , $\left(C_L \middle\vert C_R\right) \rightarrow \left(C_0 \middle\vert C_1 \middle\vert C_2\right)$<br>where $B_1$ has $b$ columns, $C_1$ has $b$ columns |
| | |
| | $C_1 := AB_1 + C_1$ |
| | $\left(B_L \middle\vert B_R\right) \leftarrow \left(B_0 \middle\vert B_1 \middle\vert B_2\right)$ , $\left(C_L \middle\vert C_R\right) \leftarrow \left(C_0 \middle\vert C_1 \middle\vert C_2\right)$ |
| | |
| | |
| | endwhile |
| | |
| | |

| Algorithm: $[C] := \text{SYMM\_LU\_BLK\_VAR1}(A, B, C)$ |
|---|
| $B \rightarrow \left(B_L \middle\vert B_R\right)$ , $C \rightarrow \left(C_L \middle\vert C_R\right)$<br>where $B_L$ has 0 columns, $C_L$ has 0 columns<br>while $n(B_L) < n(B)$ do<br>**Determine block size** $b$<br>$\left(B_L \middle\vert B_R\right) \rightarrow \left(B_0 \middle\vert B_1 \middle\vert B_2\right)$ , $\left(C_L \middle\vert C_R\right) \rightarrow \left(C_0 \middle\vert C_1 \middle\vert C_2\right)$<br>where $B_1$ has $b$ columns, $C_1$ has $b$ columns |
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| $\left(B_L \middle\vert B_R\right) \leftarrow \left(B_0 \middle\vert B_1 \middle\vert B_2\right)$ , $\left(C_L \middle\vert C_R\right) \leftarrow \left(C_0 \middle\vert C_1 \middle\vert C_2\right)$<br>endwhile |