| Step | Algorithm: | | |
|------|------------|------|---|
| 1a | | | |
| 4 | | | |
| | where | | |
| 2 | | | |
| 3 | while do | | |
| 2,3 | | ٨ | |
| 5a | | | |
| | where | | |
| 6 | | | |
| 8 | | | |
| 5b | | | |
| 7 | | | |
| 2 | | | |
| | endwhile | | |
| 2,3 | | ^ ¬(|) |
| 1b | | | |

| Step | Algorithm: $[C] := \text{SYMM_LU_UNB_VAR2}(A, B, C)$ |
|------|--|
| 1a | $C = \widehat{C}$ |
| 4 | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $ \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) $ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \wedge m(A_{TL}) < m(A)$ |
| 5a | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \to \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \to \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \to \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ C_2 \end{array}\right) $ where $a_1 = a_1 + a_2 + a_3 + a_4 + a_4 + a_4 + a_4 + a_5 $ |
| 6 | where α_{11} is 1×1 , b_1 has 1 row, c_1 has 1 row $ \left(\frac{C_0}{c_1^T}\right) = \left(\frac{A_{00}B_0 + \widehat{C}_0}{a_{01}^T B_0 + \widehat{c}_1}\right) \\ \frac{a_{01}^T B_0 + \widehat{c}_1}{A_{02}B_0 + \widehat{C}_2} $ |
| 8 | $C_0 := a_{01}b_1^T + C_0$ $c_1^T := \alpha_{11}b_1^T + c_1^T$ $C_2 := a_{12}^Tb_1^T + C_2$ |
| 5b | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right) $ |
| 7 | $ \left(\frac{C_0}{c_1^T}\right) = \left(\frac{A_{00}B_0 + a_{01}b_1^T + \widehat{C}_0}{a_{01}^TB_0 + \alpha_{11}b_1^T + \widehat{c}_1} - \frac{a_{01}^TB_0 + \alpha_{11}b_1^T + \widehat{C}_2}{A_{02}B_0 + a_{12}^Tb_1^T + \widehat{C}_2}\right) $ |
| 2 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right)$ |
| | endwhile |
| 2,3 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \land \neg (m(A_{TL}) < m(A))$ |
| 1b | $[C] = \operatorname{symm} \operatorname{lu}(A, B, \widehat{C})$ |

Algorithm: $[C] := \text{SYMM_LU_UNB_VAR2}(A, B, C)$

$$A \to \left(\frac{A_{TL} | A_{TR}}{A_{BL} | A_{BR}}\right), B \to \left(\frac{B_T}{B_B}\right), C \to \left(\frac{C_T}{C_B}\right)$$

where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows

while $m(A_{TL}) < m(A)$ do

$$\left(\frac{A_{TL} | A_{TR}}{A_{BL} | A_{BR}}\right) \to \left(\frac{A_{00} | a_{01} | A_{02}}{0 | a_{11} | a_{12}^T}\right), \left(\frac{B_T}{B_B}\right) \to \left(\frac{B_0}{b_1^T}\right), \left(\frac{C_T}{C_B}\right) \to \left(\frac{C_0}{c_1^T}\right)$$

where α_{11} is 1×1 , b_1 has 1 row, c_1 has 1 row

$$C_0 := a_{01}b_1^T + C_0$$

$$c_1^T := \alpha_{11} b_1^T + c_1^T$$

$$C_2 := a_{12}^T b_1^T + C_2$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$$

endwhile

| Step | Algorithm: $[C] := \text{SYMM_LU_UNB_VAR2}(A, B, C)$ |
|------|---|
| 1a | $C = \widehat{C}$ |
| 4 | where |
| 2 | |
| 3 | while do |
| 2,3 | \wedge |
| 5a | |
| | where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | |
| | endwhile |
| 2,3 | $\wedge \neg ($ |
| 1b | $[C] = \operatorname{symm_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM_LU_UNB_VAR2}(A, B, C)$ |
|------|--|
| 1a | $C = \hat{C}$ |
| 4 | |
| _ | 1 |
| | where $\langle C \rangle \langle \hat{C} \rangle$ |
| 2 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right)$ |
| 3 | while do |
| 2,3 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \land$ |
| | |
| 5a | |
| | where |
| | |
| 6 | |
| | |
| | |
| 8 | |
| | |
| 5b | |
| | |
| | |
| 7 | |
| | |
| 2 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right)$ |
| | ondwhile |
| 2 | $ \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \land \neg () $ $ [C] = \operatorname{argmin} \ln(A \cdot B \cdot \widehat{C}) $ |
| 1b | $[C] = \operatorname{symm} \operatorname{lu}(A, B, \widehat{C})$ |
| | |

| Step | Algorithm: $[C] := \text{SYMM_LU_UNB_VAR2}(A, B, C)$ |
|------|---|
| 1a | $C = \widehat{C}$ |
| 4 | |
| 2 | $ \frac{C_T}{C_B} = \frac{\widehat{C}_T}{\widehat{C}_B} $ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \wedge m(A_{TL}) < m(A)$ |
| 5a | |
| | where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right)$ |
| | endwhile |
| 2,3 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \land \neg (m(A_{TL}) < m(A))$ |
| 1b | $[C] = \operatorname{symm_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM_LU_UNB_VAR2}(A, B, C)$ |
|------|--|
| 1a | $C = \widehat{C}$ |
| 4 | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows $ \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) $ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \wedge m(A_{TL}) < m(A)$ |
| 5a | |
| | |
| | where |
| 6 | |
| | |
| 8 | |
| 5b | |
| 7 | |
| 2 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right)$ |
| | endwhile |
| 2,3 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \wedge \neg (m(A_{TL}) < m(A))$ |
| 1b | $[C] = \operatorname{symm} \operatorname{lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM_LU_UNB_VAR2}(A, B, C)$ |
|------|---|
| 1a | $C = \hat{C}$ |
| 4 | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $ \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) $ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $ \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \wedge m(A_{TL}) < m(A) $ |
| 5a | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right) $ where α_{11} is 1×1 , b_1 has 1 row, c_1 has 1 row |
| 6 | |
| 8 | |
| 5b | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right) $ |
| 7 | |
| 2 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right)$ |
| | endwhile |
| 2,3 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \land \neg (m(A_{TL}) < m(A))$ |
| 1b | $[C] = \operatorname{symm_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM_LU_UNB_VAR2}(A, B, C)$ |
|------|---|
| 1a | $C = \widehat{C}$ |
| 4 | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $ \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) $ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \wedge m(A_{TL}) < m(A)$ |
| 5a | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ C_2 \end{array}\right) $ where α_{11} is 1×1 , b_1 has 1 row, c_1 has 1 row |
| 6 | $ \begin{pmatrix} \frac{C_0}{c_1^T} \\ C_2 \end{pmatrix} = \begin{pmatrix} \frac{A_{00}B_0 + \hat{C}_0}{a_{01}^T B_0 + \hat{c}_1} \\ \frac{a_{02}B_0 + \hat{C}_2}{A_{02}B_0 + \hat{C}_2} \end{pmatrix} $ |
| 8 | |
| 5b | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ C_2 \end{array}\right) $ |
| 7 | |
| 2 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right)$ |
| | endwhile |
| 2,3 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \land \neg (m(A_{TL}) < m(A))$ |
| 1b | $[C] = \operatorname{symm} \operatorname{lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM_LU_UNB_VAR2}(A, B, C)$ |
|------|--|
| 1a | $C = \widehat{C}$ |
| 4 | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $ \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) $ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \wedge m(A_{TL}) < m(A)$ |
| 5a | $ \left(\frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BR} \end{vmatrix} \rightarrow \left(\frac{A_{00}}{0} \begin{vmatrix} a_{01} & A_{02} \\ 0 & \alpha_{11} & a_{12}^T \\ 0 & 0 & A_{22} \end{vmatrix}, \left(\frac{B_T}{B_B}\right) \rightarrow \left(\frac{B_0}{b_1^T} \\ B_2\right), \left(\frac{C_T}{C_B}\right) \rightarrow \left(\frac{C_0}{c_1^T} \\ C_2\right) $ where c_0 is 1×1 by loss 1 years c_1 by c_2 by c_3 by c_4 b |
| 6 | where α_{11} is 1×1 , b_1 has 1 row, c_1 has 1 row $ \left(\frac{C_0}{c_1^T}\right) = \left(\frac{A_{00}B_0 + \widehat{C}_0}{a_{01}^TB_0 + \widehat{c}_1}\right) \\ \frac{a_{01}^TB_0 + \widehat{c}_1}{A_{02}B_0 + \widehat{C}_2} $ |
| 8 | |
| 5b | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1^T \\ C_2 \end{array}\right) $ |
| 7 | $ \begin{pmatrix} \frac{C_0}{c_1^T} \\ \overline{C_2} \end{pmatrix} = \begin{pmatrix} \frac{A_{00}B_0 + a_{01}b_1^T + \widehat{C}_0}{a_{01}^TB_0 + \alpha_{11}b_1^T + \widehat{c}_1} \\ \overline{A_{02}B_0 + a_{12}^Tb_1^T + \widehat{C}_2} \end{pmatrix} $ |
| 2 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right)$ |
| | endwhile |
| 2 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \land \neg (m(A_{TL}) < m(A))$ |
| 1b | $[C] = \operatorname{symm} \operatorname{lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM_LU_UNB_VAR2}(A, B, C)$ |
|------|--|
| 1a | $C = \widehat{C}$ |
| 4 | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $ \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) $ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \wedge m(A_{TL}) < m(A)$ |
| 5a | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \to \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \to \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \to \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right) $ where a_1 is 1 × 1, b_2 has 1 row, a_1 has 1 row. |
| 6 | where α_{11} is 1×1 , b_1 has 1 row, c_1 has 1 row $ \left(\frac{C_0}{c_1^T}\right) = \left(\frac{A_{00}B_0 + \widehat{C}_0}{a_{01}^TB_0 + \widehat{C}_1}\right) \\ \frac{a_{01}^TB_0 + \widehat{C}_1}{A_{02}B_0 + \widehat{C}_2} $ |
| 8 | $C_0 := a_{01}b_1^T + C_0$ $c_1^T := \alpha_{11}b_1^T + c_1^T$ $C_2 := a_{12}^Tb_1^T + C_2$ |
| 5b | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right) $ |
| 7 | $ \left(\frac{C_0}{c_1^T}\right) = \left(\frac{A_{00}B_0 + a_{01}b_1^T + \widehat{C}_0}{a_{01}^TB_0 + \alpha_{11}b_1^T + \widehat{c}_1} - A_{02}B_0 + a_{12}^Tb_1^T + \widehat{C}_2\right) $ |
| 2 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right)$ |
| | endwhile |
| 2,3 | $\left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{\widehat{C}_B}\right) \land \neg (m(A_{TL}) < m(A))$ |
| 1b | $[C] = \operatorname{symm_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM_LU_UNB_VAR2}(A, B, C)$ |
|------|---|
| | |
| | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| | |
| | while $m(A_{TL}) < m(A)$ do |
| | |
| | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline 0 & \alpha_{11} & a_{12}^T \\ \hline 0 & 0 & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ C_2 \end{array}\right) $ where α_{11} is 1×1 , b_1 has 1 row, c_1 has 1 row |
| | |
| | $C_0 := a_{01}b_1^T + C_0$ $c_1^T := \alpha_{11}b_1^T + c_1^T$ $C_2 := a_{12}^Tb_1^T + C_2$ |
| | $ \left(\frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BR} \end{vmatrix} \leftarrow \left(\frac{A_{00}}{0} \begin{vmatrix} a_{01} & A_{02} \\ 0 & \alpha_{11} & a_{12}^T \\ 0 & 0 & A_{22} \end{vmatrix}, \left(\frac{B_T}{B_B}\right) \leftarrow \left(\frac{B_0}{b_1^T} \\ B_2\right), \left(\frac{C_T}{C_B}\right) \leftarrow \left(\frac{C_0}{c_1^T} \\ C_2\right) $ |
| | |
| | |
| | endwhile |
| | |
| | |
| | |
| | |

Algorithm: $[C] := \text{SYMM_LU_UNB_VAR2}(A, B, C)$

$$A \to \left(\frac{A_{TL} | A_{TR}}{A_{BL} | A_{BR}}\right), B \to \left(\frac{B_T}{B_B}\right), C \to \left(\frac{C_T}{C_B}\right)$$

where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows

while $m(A_{TL}) < m(A)$ do

$$\left(\frac{A_{TL} | A_{TR}}{A_{BL} | A_{BR}}\right) \to \left(\frac{A_{00} | a_{01} | A_{02}}{0 | a_{11} | a_{12}^T}\right), \left(\frac{B_T}{B_B}\right) \to \left(\frac{B_0}{\frac{b_1^T}{B_2}}\right), \left(\frac{C_T}{C_B}\right) \to \left(\frac{C_0}{\frac{c_1^T}{C_2}}\right)$$

where α_{11} is 1×1 , b_1 has 1 row, c_1 has 1 row

$$C_0 := a_{01}b_1^T + C_0$$

$$c_1^T := \alpha_{11} b_1^T + c_1^T$$

$$C_2 := a_{12}^T b_1^T + C_2$$

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
A_{00} & a_{01} & A_{02} \\
\hline
0 & \alpha_{11} & a_{12}^T \\
\hline
0 & 0 & A_{22}
\end{array}\right), \left(\begin{array}{c}
B_T \\
\hline
B_B
\end{array}\right) \leftarrow \left(\begin{array}{c}
B_0 \\
\hline
b_1^T \\
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
C_B
\end{array}\right) \leftarrow \left(\begin{array}{c}
C_0 \\
\hline
c_1^T \\
C_2
\end{array}\right)$$

endwhile