| Step | Algorithm: |
|------|------------|
| 1a | |
| 4 | where |
| 2 | |
| 3 | while do |
| 2,3 | ∧ |
| 5a | where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | |
| | endwhile |
| 2,3 | ∧ ¬( ) |
| 1b | |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$ |
|------|----------------------------------------------------------|
| 1a | $C = \widehat{C}$ |
| 4 | $B \to \left( B_L \middle\| B_R \right) , C \to \left( C_L \middle\| C_R \right)$ <br> where $B_R$ has 0 columns, $C_R$ has 0 columns |
| 2 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right)$ |
| 3 | while $n(B_R) < n(B)$ do |
| 2,3 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right) \wedge n(B_R) < n(B)$ |
| 5a | $\left( B_L \middle\| B_R \right) \to \left( B_0 \middle\| b_1 \middle\| B_2 \right) , \left( C_L \middle\| C_R \right) \to \left( C_0 \middle\| c_1 \middle\| C_2 \right)$ <br> where $b_1$ has 1 column, $c_1$ has 1 column |
| 6 | $\left( C_0 \middle\| c_1 \middle\| C_2 \right) = \left( \widehat{C}_0 \middle\| \widehat{c}_1 \middle\| AB_2 + \widehat{C}_2 \right)$ |
| 8 | $c_1 := Ab_1 + c_1$ |
| 5b | $\left( B_L \middle\| B_R \right) \leftarrow \left( B_0 \middle\| b_1 \middle\| B_2 \right) , \left( C_L \middle\| C_R \right) \leftarrow \left( C_0 \middle\| c_1 \middle\| C_2 \right)$ |
| 7 | $\left( C_0 \middle\| c_1 \middle\| C_2 \right) = \left( \widehat{C}_0 \middle\| Ab_1 + \widehat{c}_1 \middle\| AB_2 + \widehat{C}_2 \right)$ |
| 2 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right)$ |
| | endwhile |
| 2,3 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right) \wedge \neg(n(B_R) < n(B))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

---

**Algorithm:** $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$

$B \to \left( B_L \middle\| B_R \right) , C \to \left( C_L \middle\| C_R \right)$

    where $B_R$ has 0 columns, $C_R$ has 0 columns

while $n(B_R) < n(B)$ do

    $\left( B_L \middle\| B_R \right) \to \left( B_0 \middle\| b_1 \middle\| B_2 \right) , \left( C_L \middle\| C_R \right) \to \left( C_0 \middle\| c_1 \middle\| C_2 \right)$

    where $b_1$ has 1 column, $c_1$ has 1 column

    $c_1 := Ab_1 + c_1$

    $\left( B_L \middle\| B_R \right) \leftarrow \left( B_0 \middle\| b_1 \middle\| B_2 \right) , \left( C_L \middle\| C_R \right) \leftarrow \left( C_0 \middle\| c_1 \middle\| C_2 \right)$

endwhile

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR}2(A, B, C)$ |
|------|---------------------------------------------------------|
| 1a | $C = \widehat{C}$ |
| 4 | where |
| 2 | |
| 3 | while do |
| 2,3 | $\wedge$ |
| 5a | where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | |
| | endwhile |
| 2,3 | $\wedge \neg ($ $)$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$ |
|------|------------------------------------------------------------|
| 1a | $C = \widehat{C}$ |
| 4 | where |
| 2 | $\left( C_L \middle| C_R \right) = \left( \widehat{C}_L \middle| \widehat{C}_R \right)$ |
| 3 | while                do |
| 2,3 | $\left( C_L \middle| C_R \right) = \left( \widehat{C}_L \middle| \widehat{C}_R \right) \wedge$ |
| 5a | where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | $\left( C_L \middle| C_R \right) = \left( \widehat{C}_L \middle| \widehat{C}_R \right)$ |
|  | endwhile |
| 2 | $\left( C_L \middle| C_R \right) = \left( \widehat{C}_L \middle| \widehat{C}_R \right) \wedge \neg(\qquad\qquad\quad)$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] :=$ SYMM_LU_UNB_VAR2$(A, B, C)$ |
|------|--------------------------------------------------|
| 1a | $C = \widehat{C}$ |
| 4 | where |
| 2 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right)$ |
| 3 | while $n(B_R) < n(B)$ do |
| 2,3 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right) \wedge n(B_R) < n(B)$ |
| 5a | where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right)$ |
| | endwhile |
| 2,3 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right) \wedge \neg(n(B_R) < n(B))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$ |
|------|--------------------------------------------------------|
| 1a | $C = \widehat{C}$ |
| 4 | $B \to \left( B_L \middle\| B_R \right) , C \to \left( C_L \middle\| C_R \right)$ <br> where $B_R$ has 0 columns, $C_R$ has 0 columns |
| 2 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right)$ |
| 3 | while $n(B_R) < n(B)$ do |
| 2,3 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right) \wedge n(B_R) < n(B)$ |
| 5a | where |
| 6 | |
| 8 | |
| 5b | |
| 7 | |
| 2 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right)$ |
| | endwhile |
| 2,3 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right) \wedge \neg(n(B_R) < n(B))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | $B \to \left( B_L \middle\| B_R \right), C \to \left( C_L \middle\| C_R \right)$ <br> where $\quad B_R$ has 0 columns, $C_R$ has 0 columns |
| 2 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right)$ |
| 3 | while $n(B_R) < n(B)$ do |
| 2,3 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right) \wedge n(B_R) < n(B)$ |
| 5a | $\left( B_L \middle\| B_R \right) \to \left( B_0 \middle\| b_1 \middle\| B_2 \right), \left( C_L \middle\| C_R \right) \to \left( C_0 \middle\| c_1 \middle\| C_2 \right)$ <br> where $\quad b_1$ has 1 column, $c_1$ has 1 column |
| 6 | |
| 8 | |
| 5b | $\left( B_L \middle\| B_R \right) \leftarrow \left( B_0 \middle\| b_1 \middle\| B_2 \right), \left( C_L \middle\| C_R \right) \leftarrow \left( C_0 \middle\| c_1 \middle\| C_2 \right)$ |
| 7 | |
| 2 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right)$ |
| | endwhile |
| 2,3 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right) \wedge \neg(n(B_R) < n(B))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | $B \to \left( B_L \middle\| B_R \right), C \to \left( C_L \middle\| C_R \right)$ <br> where $\quad B_R$ has 0 columns, $C_R$ has 0 columns |
| 2 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right)$ |
| 3 | while $n(B_R) < n(B)$ do |
| 2,3 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right) \wedge n(B_R) < n(B)$ |
| 5a | $\left( B_L \middle\| B_R \right) \to \left( B_0 \middle\| b_1 \middle\| B_2 \right), \left( C_L \middle\| C_R \right) \to \left( C_0 \middle\| c_1 \middle\| C_2 \right)$ <br> where $\quad b_1$ has 1 column, $c_1$ has 1 column |
| 6 | $\left( C_0 \middle\| c_1 \middle\| C_2 \right) = \left( \widehat{C}_0 \middle\| \widehat{c}_1 \middle\| AB_2 + \widehat{C}_2 \right)$ |
| 8 | |
| 5b | $\left( B_L \middle\| B_R \right) \leftarrow \left( B_0 \middle\| b_1 \middle\| B_2 \right), \left( C_L \middle\| C_R \right) \leftarrow \left( C_0 \middle\| c_1 \middle\| C_2 \right)$ |
| 7 | |
| 2 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right)$ |
| | endwhile |
| 2,3 | $\left( C_L \middle\| C_R \right) = \left( \widehat{C}_L \middle\| \widehat{C}_R \right) \wedge \neg(n(B_R) < n(B))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | $B \rightarrow \left( B_L \mid B_R \right), C \rightarrow \left( C_L \mid C_R \right)$<br>where $B_R$ has 0 columns, $C_R$ has 0 columns |
| 2 | $\left( C_L \mid C_R \right) = \left( \widehat{C}_L \mid \widehat{C}_R \right)$ |
| 3 | while $n(B_R) < n(B)$ do |
| 2,3 | $\left( C_L \mid C_R \right) = \left( \widehat{C}_L \mid \widehat{C}_R \right) \wedge n(B_R) < n(B)$ |
| 5a | $\left( B_L \mid B_R \right) \rightarrow \left( B_0 \mid b_1 \mid B_2 \right), \left( C_L \mid C_R \right) \rightarrow \left( C_0 \mid c_1 \mid C_2 \right)$<br>where $b_1$ has 1 column, $c_1$ has 1 column |
| 6 | $\left( C_0 \mid c_1 \mid C_2 \right) = \left( \widehat{C}_0 \mid \widehat{c}_1 \mid AB_2 + \widehat{C}_2 \right)$ |
| 8 | |
| 5b | $\left( B_L \mid B_R \right) \leftarrow \left( B_0 \mid b_1 \mid B_2 \right), \left( C_L \mid C_R \right) \leftarrow \left( C_0 \mid c_1 \mid C_2 \right)$ |
| 7 | $\left( C_0 \mid c_1 \mid C_2 \right) = \left( \widehat{C}_0 \mid Ab_1 + \widehat{c}_1 \mid AB_2 + \widehat{C}_2 \right)$ |
| 2 | $\left( C_L \mid C_R \right) = \left( \widehat{C}_L \mid \widehat{C}_R \right)$ |
| | endwhile |
| 2 | $\left( C_L \mid C_R \right) = \left( \widehat{C}_L \mid \widehat{C}_R \right) \wedge \neg(n(B_R) < n(B))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

<br>

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$ |
|---|---|
| 1a | $C = \widehat{C}$ |
| 4 | $B \rightarrow \left( B_L \mid B_R \right), C \rightarrow \left( C_L \mid C_R \right)$<br>where $B_R$ has 0 columns, $C_R$ has 0 columns |
| 2 | $\left( C_L \mid C_R \right) = \left( \widehat{C}_L \mid \widehat{C}_R \right)$ |
| 3 | while $n(B_R) < n(B)$ do |
| 2,3 | $\left( C_L \mid C_R \right) = \left( \widehat{C}_L \mid \widehat{C}_R \right) \wedge n(B_R) < n(B)$ |
| 5a | $\left( B_L \mid B_R \right) \rightarrow \left( B_0 \mid b_1 \mid B_2 \right), \left( C_L \mid C_R \right) \rightarrow \left( C_0 \mid c_1 \mid C_2 \right)$<br>where $b_1$ has 1 column, $c_1$ has 1 column |
| 6 | $\left( C_0 \mid c_1 \mid C_2 \right) = \left( \widehat{C}_0 \mid \widehat{c}_1 \mid AB_2 + \widehat{C}_2 \right)$ |
| 8 | $c_1 := Ab_1 + c_1$ |
| 5b | $\left( B_L \mid B_R \right) \leftarrow \left( B_0 \mid b_1 \mid B_2 \right), \left( C_L \mid C_R \right) \leftarrow \left( C_0 \mid c_1 \mid C_2 \right)$ |
| 7 | $\left( C_0 \mid c_1 \mid C_2 \right) = \left( \widehat{C}_0 \mid Ab_1 + \widehat{c}_1 \mid AB_2 + \widehat{C}_2 \right)$ |
| 2 | $\left( C_L \mid C_R \right) = \left( \widehat{C}_L \mid \widehat{C}_R \right)$ |
| | endwhile |
| 2,3 | $\left( C_L \mid C_R \right) = \left( \widehat{C}_L \mid \widehat{C}_R \right) \wedge \neg(n(B_R) < n(B))$ |
| 1b | $[C] = \text{symm\_lu}(A, B, \widehat{C})$ |

| Step | Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$ |
|------|--------------------------------------------------------|
| | |
| | $B \to \left( B_L \middle\| B_R \right), C \to \left( C_L \middle\| C_R \right)$ <br> where $\quad B_R$ has 0 columns, $C_R$ has 0 columns |
| | |
| | while $n(B_R) < n(B)$ do |
| | |
| | $\left( B_L \middle\| B_R \right) \to \left( B_0 \middle\| b_1 \middle\| B_2 \right), \left( C_L \middle\| C_R \right) \to \left( C_0 \middle\| c_1 \middle\| C_2 \right)$ <br> where $\quad b_1$ has 1 column, $c_1$ has 1 column |
| | |
| | $c_1 := Ab_1 + c_1$ |
| | $\left( B_L \middle\| B_R \right) \leftarrow \left( B_0 \middle\| b_1 \middle\| B_2 \right), \left( C_L \middle\| C_R \right) \leftarrow \left( C_0 \middle\| c_1 \middle\| C_2 \right)$ |
| | |
| | |
| | endwhile |
| | |
| | |

| Algorithm: $[C] := \text{SYMM\_LU\_UNB\_VAR2}(A, B, C)$ |
|--------------------------------------------------------|
| $B \to \left( B_L \middle\| B_R \right), C \to \left( C_L \middle\| C_R \right)$ <br> where $\quad B_R$ has 0 columns, $C_R$ has 0 columns <br> while $n(B_R) < n(B)$ do <br> $\quad\left( B_L \middle\| B_R \right) \to \left( B_0 \middle\| b_1 \middle\| B_2 \right), \left( C_L \middle\| C_R \right) \to \left( C_0 \middle\| c_1 \middle\| C_2 \right)$ <br> where $\quad b_1$ has 1 column, $c_1$ has 1 column |
| $\quad c_1 := Ab_1 + c_1$ |
| $\quad\left( B_L \middle\| B_R \right) \leftarrow \left( B_0 \middle\| b_1 \middle\| B_2 \right), \left( C_L \middle\| C_R \right) \leftarrow \left( C_0 \middle\| c_1 \middle\| C_2 \right)$ <br> endwhile |