

Final Winter 2015

1. (a) Capitalist's problem

$$\left\{ \begin{array}{l} V^c(k^c, z) \quad \text{(This can be } K \text{ instead of } k^c) \\ = \max_{c^c, h^c, i^c} \left[u(c^c, h^c) + \beta E V^c(k^c, z) \right] \\ \text{s.t. } c^c + i^c \leq w h^c + r k^c \\ k^c = (1-\delta) k^c + i^c \end{array} \right.$$

Workers' problem

$$\left\{ \begin{array}{l} V^w(k^c, z) \\ = \max_{c^w, h^w} \left[u(c^w, h^w) + \beta E V^w(k^c, z) \right] \\ \text{s.t. } c^w = w h^w \end{array} \right.$$

RCIE is (i) HH's value func,

$V^c(\text{---})$, $V^w(\text{---})$, policy func.

$c^c(\text{---})$, $h^c(\text{---})$, $i^c(\text{---})$

$c^w(\text{---})$, $h^w(\text{---})$

(ii) firm's policy $l^d(-), k^d(-)$

(iii) prices ---

(iv) LOM of capital $K^c = g(K^c, z)$
such that

- L/H optimization

- Firm optimization

- Market clearing

$$\begin{cases} l = l^d \\ k = k^d \\ C + I = z F(K, H) \end{cases}$$

$$\text{where } K = s K^c, H = s H^c + (1-s) H^w, \\ C = s C^c + (1-s) C^w, I = s I^w$$

- Consistency

$$g(K^c, z) = (1-s) K^c + \alpha(K^c, K^c, z)$$

1. (b) Sequential social planner's problem

$$\max_{\left\{ \begin{array}{l} C_t^c, H_t^c \\ C_t^w, H_t^w, I_t \end{array} \right\}} \sum \beta^t \left[s u(C_t^c, H_t^c) + (1-s) u(C_t^w, H_t^w) \right]$$

$$\left\{ \begin{array}{l} \text{s.t. } C_t + I_t \leq Z_t = (1 + r_{t-1}) K_{t-1} + (1 + h_t) L_t \\ K_t = (1 - \delta) K_{t-1} + I_t \\ s C_t^c + (1-s) C_t^w = C_t \\ s H_t^c + (1-s) H_t^w = H_t \end{array} \right.$$

Note: Competitive eqn. is different from the solution to planner's problem!

{ Social planner (with equal weights) will allocate same level of C and H to capitalists and workers.

This wouldn't happen in competitive eq.

(c) Capitalist's FONC are the standard conditions.

(Enter eq. and labor supply condition)

Workers' FONC is only one.

$$\frac{1}{c_{\pi}^w} w_{\pi} = (h_{\pi}^w)^{\gamma} \quad \text{--- } \textcircled{*}$$

Also note capitalists and workers face the same wage, which is equal to the marginal product of labor.

(d) Substitute workers' b.c. into $\textcircled{*}$

$$\frac{1}{w_{\pi} h_{\pi}^w} \cdot w_{\pi} = (h_{\pi}^w)^{\gamma}$$

$$\rightarrow 1 = (h_{\pi}^w)^{1+\gamma}$$

$$\rightarrow h_{\pi}^w \text{ const. over business cycle.}$$

This is because the util. function

implies that income and substitution effects cancel out.

2(a) See lecture note.

(s) See lecture note.

$$\text{Note } \left\{ \begin{array}{l} U_c = [C + m\alpha]^{-\sigma} \\ U_h = 1 + \alpha \\ U_m = [C + m\alpha]^{-\sigma} \end{array} \right.$$

(c) No, money is non-neutral in this economy.
Since U_c affected by m , real side affected by change in money level.

3. (a) Forward-looking iteration process
with real marginal cost as a
driving force.

(b) $\lambda_t = d_t C_t^{-\sigma}$

$\rightarrow \hat{\lambda}_t = \hat{d}_t - \sigma \hat{C}_t$

$E_t \hat{d}_{t+1} = \rho \hat{d}_t$

Also since

$\hat{\lambda}_t = \hat{\lambda}_t - E_t \pi_{t+1} + E_t \hat{\lambda}_{t+1}$ \checkmark

$\hat{d}_t - \sigma \hat{C}_t = \hat{\lambda}_t - E_t \pi_{t+1} + \rho \hat{d}_t - \sigma C_{t+1}$

Also use $\hat{C}_t = \hat{Y}_t$. Divide both
sides by $-\sigma$

$-\frac{(1-\rho)}{\sigma} \hat{d}_t + \hat{Y}_t = E_t \hat{Y}_{t+1} - \left(\frac{1}{\sigma}\right) (\hat{\lambda}_t - E_t \pi_{t+1})$

$\Rightarrow -\frac{(1-\rho)}{\sigma} \hat{d}_t + \hat{X}_t = E_t \hat{X}_{t+1} - \left(\frac{1}{\sigma}\right) (\hat{\lambda}_t - E_t \pi_{t+1})$

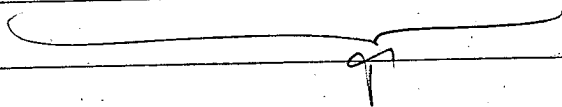
$u_t \equiv E_t \hat{Y}_{t+1} - \hat{Y}_t + u_t$

(c) RHS const. Assume $\sigma > 0$.

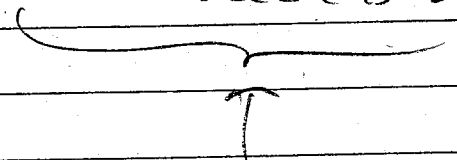
$\Rightarrow \hat{y}_\sigma$ increases, when \hat{d}_r increases,

(d) Since price sticky, real interest rate \uparrow

Output decreases and inflation
(gap) decreases.



through
Euler eq.



Through
NKPC