Unified Phase-Geometric Theory (UPGT): Foundations

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October 10, 2025

DOI (concept): 10.5281/zenodo.17334971
DOI (version 2): 10.5281/zenodo.17401263

This is version 2, published on October 20, 2025. Minor corrections applied.

Abstract

A unified geometric framework is formulated in which classical mechanics, electrodynamics, relativity, and quantum mechanics emerge as limiting cases of a single SU(2) phase dynamics on the compact three-sphere S^3. The internal phase field $U(x) \in \mathrm{SU}(2)$ defines curvature, spin, and temporal structure within a globally finite, locally continuous manifold. All physical entities — matter, radiation, and gravitation — are interpreted as manifestations of curvature and evolution of this phase geometry. The model introduces a dual notion of time, with local operational time $t(x)$ determined by the phase rate $\omega(x)$ relative to a global evolution parameter T, thereby unifying gravitational and kinematic time dilation. Newtonian dynamics, Maxwell's equations, and the Einstein field equations are recovered as successive limits of the same Lagrangian structure, while quantum discreteness arises from the compactness of S^3 and the SU(2) spin topology. The framework offers geometric interpretations of relativistic and quantum phenomena, including light deflection, entanglement, tunneling, and cosmological redshift, and predicts testable deviations from the ΛCDM relation between redshift and luminosity. The SU(2) phase geometry thus provides a coherent and self-contained foundation linking classical and quantum domains through a single compact phase manifold.

Contents

1 Introduction and Motivation

The purpose of this work is to establish a unified geometric framework in which all known physical interactions — gravitational, electromagnetic, and quantum — emerge from a single phase structure defined on the compact three-sphere S^3. The internal geometry of this phase space is described by the Lie group SU(2), which is topologically equivalent to S^3. Within this setting, matter, radiation, and spacetime curvature appear as different manifestations of a single underlying field — the *phase field* $U(x) \in \mathrm{SU}(2)$.

The choice of SU(2) and S^3[1] is not arbitrary. The compactness of the three-sphere naturally enforces discretization of eigenmodes, providing quantization of physical states without additional postulates. The group SU(2) represents the minimal non-Abelian structure that admits both rotational and spinor behavior, allowing a direct connection between geometric curvature and internal spin degrees of freedom. In this sense, the SU(2) phase manifold constitutes the simplest closed and self-consistent arena that accommodates the observed coexistence of wave-like and particle-like phenomena.

Geometrically, the stereographic projection of S^3 onto $\mathbb{R}^3$ illustrates how parallel and meridional families form an orthogonal grid, preserving local Euclidean structure while maintaining global curvature. This property allows local inertial frames to exist on a globally closed manifold, providing a natural geometric basis for relativistic and quantum effects.

The motivation for the present approach originates from the requirement that the physical universe be both globally finite and locally continuous. A compact phase space ensures the existence of normalizable eigenmodes and a finite total phase energy, avoiding divergences typical of non-compact formulations. Furthermore, the SU(2) structure permits the unification of gauge, spin, and gravitational properties within a single mathematical object, thereby linking the geometry of spacetime with the geometry of matter.

Earlier expositions introduced the qualitative idea of a phase-based physical world in a semi-popular form. The present paper reformulates it in a strict mathematical language and establishes the theoretical foundation required for further developments. Subsequent works will extend this framework to describe atomic, nuclear, and cosmological systems, each as specific realizations of the same SU(2)-phase geometry.

2 Ontological Background and Relation to the Global S^3 Model

Before introducing the local SU(2) phase-field formalism, it is useful to recall the ontological picture from which this formulation originates. The earlier version of the theory, as

[1] On cosmological scales, this framework remains consistent with current observations of spatial flatness. For a three-sphere radius $R_{S^3} \gtrsim 10^{28}\,\mathrm{m}$, the intrinsic curvature becomes observationally indistinguishable from Euclidean space, so the model does not conflict with large-scale measurements of the cosmic geometry (Planck Collaboration 2020, reporting $|\Omega_k| < 10^{-3}$). While spatial compactness of S^3 is essential for the emergence of quantization—ensuring a discrete spectrum of phase eigenmodes—it does not require the Universe to be globally spherical. Topological variants such as the Poincaré dodecahedral space or the three-torus T^3 can serve as equivalent compact manifolds, affecting only the cosmological aspect of the theory without altering its local quantum or dynamical predictions. In the present formulation, the three-sphere S^3 is adopted as the *minimal sufficient configuration* that allows the emergence of all observable physical phenomena within the proposed SU(2) phase geometry, while keeping the global structure mathematically closed and self-consistent.

developed in Ref. [3], described the Universe itself as a compact three-sphere S^3 embedded in four-dimensional Euclidean space $\mathbb{R}^4$. In that representation, all physical entities arise from the intrinsic phase dynamics of the hypersphere, and spacetime, matter, and energy are different aspects of one self-contained geometric system.

The Compact Hyperspherical Universe. In this ontological picture, the Universe is identified with a closed hypersurface $S^3 \subset \mathbb{R}^4$ of constant radius R, defined by $x_1^2 + x_2^2 + x_3^2 + x_4^2 = R^2$. Each point on this manifold corresponds to a physical state of the world, while its evolution in the embedding space represents a global phase rotation. Physical quantities such as mass, momentum, and spin are interpreted as manifestations of curvature and rotation within this global phase geometry. Time itself emerges as the cyclic evolution of the hyperspherical phase.

Compactness and Quantization. The compactness of the global hypersphere ensures that only discrete standing-wave configurations can exist on S^3. This property provides a direct geometric origin for quantization, conservation, and stability of physical modes, without introducing external constraints or boundary conditions. Finite total energy and self-consistency arise naturally from the closed topology of the manifold.

Limitations of the Global Formulation. While the hyperspherical model offers a philosophically cleaner and conceptually unified picture of reality, its purely geometric structure is difficult to express within the standard Lagrangian and gauge-field language of modern physics. A natural mathematical framework for this ontology would be geometric algebra, for example the Euclidean Clifford algebra Cl(4, 0), which treats rotations, spinors, and bivectors within a single coherent formalism. Such an approach would eliminate the need for matrix representations, clarify the dual $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ structure of $SO(4)$, and express quantization, curvature, and field interactions directly as properties of the underlying geometry 10. However, this reformulation is highly radical from the standpoint of current theoretical practice and is unlikely to be immediately accepted by the mainstream scientific community. For compatibility with established field-theoretic methods, the present work therefore introduces a locally differentiable reformulation in terms of SU(2) phase fields.

Transition to Local SU(2) Phase Fields. To express the same geometry in a local differential form, each spacetime point can be assigned a unitary orientation $U(x) \in SU(2) \simeq S^3$, representing the local phase state of the global hypersphere. The differential of this orientation, described by the Maurer–Cartan current $J_\mu = U^{-1}\nabla_\mu U$, encodes the curvature and phase variation responsible for physical fields. In this way, the SU(2) phase field represents a localized version of the same intrinsic geometry that defined the global S^3 universe.

Preservation of Compactness and Quantization. Although the local formulation no longer requires spacetime itself to be globally closed, the essential compactness of the theory is preserved internally through the SU(2) group manifold. Each local phase orientation $U(x)$ belongs to a compact three-sphere, ensuring that quantization and conservation remain inherent consequences of the geometry, not external assumptions. The discrete, self-contained character of the original model therefore persists at the local level.

Recovery of the Global Picture. When all local SU(2) phases are globally coherent, $U(x) = U_0(\xi)$, the field reconstructs the original hyperspherical geometry. In this limit, the local SU(2) formulation collapses back into a single compact S^3 universe, and cosmological closure reappears as a condition of phase coherence rather than as an imposed boundary. Hence, the global and local descriptions are not alternative models but complementary limits of the same unified structure.

Transition to the Local Field Formalism. The following sections develop this local SU(2) phase-field framework in detail. Its Lagrangian structure, stress–energy tensor, and curvature properties provide a direct differential realization of the intrinsic dynamics of the compact hypersphere. Thus, the ontological model of a closed S^3 universe and the operational SU(2) field theory represent two perspectives of the same unified phase geometry.

3 Geometric Arena: The SU(2) Phase Field on S^3

The fundamental arena of the present formulation is a four-dimensional Lorentzian spacetime manifold $(\mathcal{M}, g_{\mu\nu})$, endowed with an internal compact phase space isomorphic to the group manifold $\mathrm{SU}(2) \simeq S^3_{\text{phase}}$. Each spacetime point $x \in \mathcal{M}$ carries an element $U(x) \in \mathrm{SU}(2)$, representing the local orientation of the phase field. Physical quantities are constructed from the left-invariant Maurer–Cartan current

$$J_\mu \equiv U^{-1}\nabla_\mu U \ \in \ \mathfrak{su}(2), \tag{1}$$

where ∇_μ denotes the Levi–Civita covariant derivative associated with the spacetime metric $g_{\mu\nu}$.

Geometric and dimensional conventions

The four-dimensional Lorentzian manifold $(\mathcal{M}, g_{\mu\nu})$ represents the observable spacetime geometry with local operational time $t(x)$. At each spacetime point we attach a compact internal phase space topologically equivalent to S^3, described by a group element $U(x) \in \mathrm{SU}(2)$. It can be parametrized by $(a_0, a_1, a_2, a_3) \in \mathbb{R}^4$ with $a_0^2 + a_1^2 + a_2^2 + a_3^2 = 1$, i.e. an embedded hypersphere in $\mathbb{R}^4$. The fourth embedding coordinate does not extend external spacetime; it encodes the internal phase orientation of $U(x)$ and evolves with respect to a global phase parameter T. Thus, global evolution in T reflects the progression of the compact SU(2) phase rather than motion along an external spatial direction. This internal degree of freedom is geometric rather than spatial; it manifests through observables such as spin and phase curvature, not as an extra spacetime coordinate.

We use the Lorentzian signature $(-, +, +, +)$. Greek indices $\mu, \nu, \ldots$ denote spacetime components; Latin indices $a, b, \ldots$ label $\mathfrak{su}(2)$ components. The generators T_a are normalized as

$$\mathrm{Tr}(T_a T_b) = \tfrac{1}{2}\,\delta_{ab}.$$

With this normalization the phase current

$$J_\mu := U^{-1}\nabla_\mu U \in \mathfrak{su}(2)$$

is an algebra-valued one-form with mass dimension 1 (i.e. $[J_\mu] = L^{-1}$). Projections such as a fixed electromagnetic direction $T_{\rm em}$ satisfy $\mathrm{Tr}(T_{\rm em}^2) = \frac{1}{2}$, which ensures the standard normalization of the topological charge and of the emergent electromagnetic field strength used later.

We adopt natural units ($\hbar = c = 1$) unless stated otherwise. In these units the action is dimensionless (with constants restored, $[S] = \hbar$), energies and masses have dimension $[E] = [M] = L^{-1}$, while $[T] = [L]$. Electromagnetic quantities are expressed in the *Heaviside–Lorentz* system with $c = 1$, so that the vacuum permittivity and permeability satisfy $\varepsilon_0 = \mu_0 = 1$ and

$$u_{\rm em} = \tfrac{1}{2}(E^2 + B^2), \qquad \mathbf{S} = \mathbf{E} \times \mathbf{B}.$$

For later reference, once the phase–field Lagrangian density $\mathcal{L}_{\rm phase}$ is introduced, its coupling constants κ and α are fixed *in natural units* by the requirement that $\mathcal{L}_{\rm phase}$ has the dimension of an energy density:

$$[\kappa] = L^{-2}, \qquad [\alpha] = L^0,$$

so that the characteristic scales read

$$L_* \sim \sqrt{\frac{\alpha}{\kappa}}, \qquad E_* \sim \sqrt{\kappa\,\alpha}.$$

(Conversion note.) When converting to SI one may equivalently read $[\kappa] = E/L$ and $[\alpha] = E \cdot L$, which is consistent with the above after restoring $\hbar$ and c.

The normalization of coefficients follows the same dimensional structure as in the standard Skyrme and nonlinear–sigma models, where the Lagrangian is often written as

$$\mathcal{L}_{\rm Skyrme} = -\frac{f_\pi^2}{4}\,\mathrm{Tr}(J_\mu J^\mu) + \frac{1}{32e^2}\,\mathrm{Tr}\big([J_\mu, J_\nu][J^\mu, J^\nu]\big).$$

Our constants κ and α correspond to $f_\pi^2/4$ and $1/(32e^2)$ respectively, hence $[\kappa] = L^{-2}$ and $[\alpha] = L^0$ in natural units. Physically, κ quantifies the phase stiffness of the SU(2) field and sets the intrinsic energy and length scales $E_* = \sqrt{\kappa\alpha}$ and $L_* = \sqrt{\alpha/\kappa}$.

Phase Lagrangian and Total Action

The intrinsic dynamics of the phase field are defined by the nonlinear sigma and Skyrme-type Lagrangian density

$$\mathcal{L}_{\rm phase} = \frac{\kappa}{2}\,\mathrm{Tr}(J_\mu J^\mu) + \frac{\alpha}{4}\,\mathrm{Tr}\big([J_\mu, J_\nu][J^\mu, J^\nu]\big) - V(U). \tag{2}$$

Here κ and α are positive coupling constants, and $V(U)$ is a gauge-invariant potential term. The first term governs smooth phase variations (phase stiffness), while the second provides a stabilizing correction against excessive local curvature, analogous to the Skyrme term in chiral field theory [4]. The competition between these two contributions defines a characteristic length scale

$$L_* \sim \sqrt{\frac{\alpha}{\kappa}},$$

which sets the typical size of localized phase excitations such as solitons or vortices.

In natural units ($\hbar = c = 1$) the Lagrangian density has the dimension of an energy density, so that

$$[\kappa] = L^{-2}, \qquad [\alpha] = L^0.$$

The corresponding characteristic energy (or mass) scale is

$$E_* \sim \sqrt{\kappa\,\alpha}.$$

(Conversion note.) In SI units this corresponds to $[\kappa] = E/L$ and $[\alpha] = E\cdot L$, consistent with the same dimensional ratios after restoring $\hbar$ and c. These parameters can be calibrated so that L_* matches empirical microscopic scales (for instance, nuclear or atomic radii) once the fundamental constants are reinstated.

The total action functional combines the gravitational, phase, and matter sectors as

$$S = \int d^4x\,\sqrt{-g}\left[\frac{c^3}{16\pi G}\,R(g) + \mathcal{L}_{\text{phase}}(U,g) + \mathcal{L}_{\text{matter}}\right], \tag{3}$$

where $R(g)$ is the Ricci scalar of the metric $g_{\mu\nu}$.
All terms in Eq. (3) share the same dimensional normalization as energy densities in curved spacetime. Fundamental constants are restored here explicitly to make this correspondence manifest.

Units and parameter scaling. In natural units ($\hbar = c = 1$),

$$[\kappa] = L^{-2}, \qquad [\alpha] = L^0,$$

so that the characteristic solitonic scales are

$$M_{\text{sol}} \sim E_* = \sqrt{\kappa\,\alpha}, \qquad L_* \sim \sqrt{\frac{\alpha}{\kappa}}.$$

This normalization is used consistently throughout the subsequent *Atomic* and *Nuclear* extensions of the model.

Connection to effective electroweak dynamics. The potential term $V(U)$ in the phase Lagrangian can incorporate effective symmetry–breaking contributions analogous to the Higgs potential. In a low–energy expansion, the local phase amplitude $\phi(x)$ may be parameterized by coefficients (μ^2, λ) as

$$V(U) \;\simeq\; -\tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda\phi^4,$$

where the parameters emerge from the underlying SU(2) couplings and global curvature,

$$\mu^2 = \zeta_2\,\kappa + \zeta_R/R^2, \qquad \lambda = \zeta_4\,\alpha, \qquad v = \mu^2/\lambda.$$

This effective representation, used in the *Atomic* and *Nuclear* extensions, provides the electroweak mass scale and vacuum expectation value as derived quantities rather than postulated constants, while the present foundational formulation keeps $V(U)$ general.
Since $U(x)$ is a dimensionless SU(2) phase variable, the potential $V(U)$ carries the same dimensional normalization as the kinetic term in Eq. (2). Consequently, the coupling λ is dimensionless in natural units ($\hbar = c = 1$), consistent with the standard normalization adopted in nonlinear–sigma and Skyrme–type field theories. This convention ensures that the parameters (μ^2, λ) retain the correct physical dimensions when expressed through κ and α as above.

Field Equations

Variation of the total action with respect to the metric yields the Einstein tensor together with the phase–induced stress contribution, extending the classical formulation of general relativity [1]:

$$G_{\mu\nu}(g) + \Phi_{\mu\nu}(U, g) = \frac{8\pi G}{c^4}\, T^{(\text{matter})}_{\mu\nu}, \tag{4}$$

where

$$\Phi_{\mu\nu} \equiv -\frac{8\pi G}{c^4}\, T^{(\text{phase})}_{\mu\nu}, \qquad T^{(\text{phase})}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta\big(\sqrt{-g}\,\mathcal{L}_{\text{phase}}\big)}{\delta g^{\mu\nu}}. \tag{5}$$

This definition follows directly from the variational principle applied to the total action, so that $\Phi_{\mu\nu}$ is not an ad hoc term but a genuine geometric contribution arising from the SU(2) phase dynamics itself.

The contracted Bianchi identity, $\nabla_\mu G^{\mu\nu} = 0$, together with the above definition, implies the covariant conservation law

$$\nabla_\mu\big(T^{(\text{matter})\,\mu\nu} + T^{(\text{phase})\,\mu\nu}\big) = 0,$$

ensuring that the total energy–momentum tensor of matter and phase fields is conserved consistently within the curved spacetime background.

Explicitly, the symmetric (Hilbert) energy–momentum tensor of the phase field, obtained by varying the action with respect to the metric, decomposes into three contributions:

$$T^{(\sigma)}_{\mu\nu} = \kappa\,\mathrm{Tr}(J_\mu J_\nu) - \frac{\kappa}{2}\, g_{\mu\nu}\,\mathrm{Tr}(J_\alpha J^\alpha), \tag{6}$$

$$T^{(\text{Sk})}_{\mu\nu} = \alpha\,\mathrm{Tr}([J_\mu, J_\alpha][J_\nu, J^\alpha]) - \frac{\alpha}{4}\, g_{\mu\nu}\,\mathrm{Tr}([J_\alpha, J_\beta][J^\alpha, J^\beta]), \tag{7}$$

$$T^{(V)}_{\mu\nu} = -\, g_{\mu\nu}\, V(U). \tag{8}$$

Here κ controls the phase stiffness, while α quantifies the curvature (Skyrme) rigidity of the SU(2) field.

This expression represents the Hilbert (symmetric) form of the phase stress tensor. It is equivalent, up to the standard Belinfante symmetrization, to the canonical tensor that follows from translational invariance of the Lagrangian, and will be used throughout to define the total energy and angular momentum of the phase configuration.

The phase stress tensor can be placed either on the geometric (left) or energy (right) side of Eq. (4); in the present interpretation it is treated as a geometric modification of curvature, in analogy with Kaluza–Klein theories. In the limit of a homogeneous phase field ($J_\mu \to 0$, $V(U) \to V_0 = \text{const}$), one obtains

$$T^{(V)}_{\mu\nu} = -\, g_{\mu\nu} V_0, \qquad \Phi_{\mu\nu} = -\frac{8\pi G}{c^4} T^{(V)}_{\mu\nu} = \Lambda_{\text{phase}}\, g_{\mu\nu},$$

with $\Lambda_{\text{phase}} = \frac{8\pi G}{c^4} V_0$. Thus, a constant phase potential acts as an effective cosmological term. Setting $V_0 = 0$ (vacuum normalization) reduces Eq. (4) exactly to Einstein's equations of general relativity.

Variation with respect to the phase field. To obtain the field equation for $U(x)$, consider infinitesimal variations on the SU(2) group manifold,

$$\delta U = U\,\varepsilon, \qquad \varepsilon(x) \in \mathfrak{su}(2).$$

Then $\delta J_\mu = [J_\mu, \varepsilon] + \nabla_\mu \varepsilon$, and the variations of the sigma and Skyrme terms yield, respectively,

$$\delta\mathcal{L}_\sigma = -\,\kappa\,\mathrm{Tr}(\varepsilon\,\nabla_\mu J^\mu)\,, \qquad \delta\mathcal{L}_{\mathrm{Sk}} = -\,\alpha\,\mathrm{Tr}(\varepsilon\,\nabla_\mu([J_\nu,[J^\mu,J^\nu]]))\,,$$

while $\delta V = \mathrm{Tr}\big(\frac{\partial V}{\partial U}\,\varepsilon\big)$. Since $\varepsilon(x)$ is arbitrary, the Euler–Lagrange equation follows as

$$\nabla_\mu(\kappa\,J^\mu) + \alpha\,\nabla_\mu([J_\nu,[J^\mu,J^\nu]]) - \frac{\partial V}{\partial U}\,U^{-1} = 0, \tag{9}$$

representing the nonlinear dynamics of the SU(2) phase field on curved spacetime. Thus, the complete system of field equations—both for gravity and for the internal SU(2) phase—follows from the extremization of a single action functional, ensuring theoretical self-consistency and unification of geometry and internal phase dynamics within a single variational framework.

Dual Time and Phase Rate

To connect the internal phase evolution with the operational notion of time, a global evolution parameter T is introduced. Let $n^\mu(x)$ denote a future-directed unit timelike vector field satisfying $g_{\mu\nu}n^\mu n^\nu = -1$. Defining a scalar phase angle $\theta(x)$ along a fixed generator of $\mathfrak{su}(2)$, the local phase rate is

$$\omega(x) = n^\mu \partial_\mu \theta(x) = \frac{d\theta}{d\tau}, \tag{10}$$

where $d\tau$ is the proper time along the integral curves of n^μ. The observable or operational time $t(x)$ is related to the global phase parameter T by the local ratio of phase rates,

$$\boxed{\frac{dt(x)}{dT} = \frac{\omega(x)}{\omega_*}}, \tag{11}$$

where ω_* is a universal reference frequency that defines the rate of uniform phase evolution.

Hence, when the local phase rate $\omega(x)$ decreases due to curvature or phase energy, the ratio dt/dT becomes smaller, meaning that the local operational time flows more slowly relative to the global phase parameter T. This provides a geometric origin of time dilation and redshift: regions of lower phase frequency correspond to slower physical clocks. In the uniform phase limit, $\omega(x) \equiv \omega_*$, the correspondence $t \equiv T$ is recovered, and standard relativistic time emerges as a limiting case. This interpretation agrees with the conventional gravitational and kinematic time–dilation laws, where $\omega/\omega_* \simeq 1 + \phi/c^2$ in the weak–field approximation.

In practical applications such as the *Atomic* and *Nuclear* extensions, this local limit is adopted: the phase rate $\omega(x)$ varies negligibly within atomic scales, so the operational time $t(x)$ coincides with the global evolution parameter T. Only on cosmological scales, where $\omega(x)$ slowly drifts due to global curvature R, does the distinction between t and T become observable, manifesting as frequency shifts or cosmological redshifts.

4 Elementary Phase Excitations: Electron and Photon

The SU(2) phase field defined above admits two fundamental classes of excitations: localized topological vortices corresponding to material particles, and delocalized oscillatory modes corresponding to radiation. Both arise as self-consistent solutions of the phase equation (9), with the difference determined by their topological and dynamical properties.

Electron as a Localized SU(2) Vortex

A stationary localized configuration with nontrivial winding number

$$B = \frac{1}{24\pi^2} \int_{S^3_{\text{space}}} \epsilon^{ijk} \,\mathrm{Tr}(J_i J_j J_k)\, d^3x, \tag{12}$$

represents a quantized SU(2) vortex. The orientation and normalization are chosen such that a spherically symmetric hedgehog configuration $U(\mathbf{x}) = \exp[i\, F(r)\, \hat{\mathbf{x}}\cdot\boldsymbol{\sigma}]$ carries topological charge $B = +1$, fixing the overall sign convention and ensuring consistency with the standard Skyrme-model definition of the winding number.

The integer B measures the topological mapping degree $S^3_{\text{space}} \to S^3_{\text{phase}}$, ensuring that such configurations are globally stable. The lowest nontrivial configuration $B = 1$ corresponds to a localized SU(2) vortex, identified with the electron. Here S^3_{space} denotes a compactified spatial section of the physical three-space, while S^3_{phase} refers to the internal SU(2) phase manifold.

Topological remark. In the static approximation used in the atomic model, the electron can be represented by a localized SU(2) phase configuration characterized by a local winding number $B = 1$. Within the full dynamical framework, however, the electron is better described as a delocalized SU(2) phase excitation whose instantaneous field structure may locally exhibit $B = 1$, while the total topological charge of the time–dependent configuration remains $B = 0$. This resolves the apparent conflict between topological quantization and the wave nature of the electron, allowing for continuous propagation and pair annihilation without violating topological consistency.

In this interpretation, the electromagnetic field components arise as effective projections of the SU(2) curvature tensor. Defining

$$F_{\mu\nu} = +\,i\,\mathrm{Tr}(T_{\text{em}}[J_\mu, J_\nu])\,, \tag{13}$$

one recovers the standard identifications

$$E_i = F_{0i}, \qquad B_i = \tfrac{1}{2}\,\epsilon_{ijk} F_{jk}.$$

Hence, at the local level,

$$E_i \propto +\,i\,\mathrm{Tr}(T_{\text{em}}[J_0, J_i])\,, \qquad B_i \propto +\,\tfrac{i}{2}\epsilon_{ijk}\,\mathrm{Tr}(T_{\text{em}}[J_j, J_k])\,. \tag{14}$$

Here T_{em} is the fixed generator specifying the electromagnetic orientation in the internal SU(2) space. The local circulation of the phase field gives rise to an intrinsic magnetic moment, while the double-connected topology of SU(2) naturally accounts for the spin-$\frac{1}{2}$ property.

In bound systems, the localized vortex becomes distributed along a resonant eigenmode of the atomic phase field. The resulting extended configuration retains the same local topological structure but exhibits spatially distributed energy density and angular momentum, corresponding to the familiar orbital structure of atomic states.

These relations are consistent with the tensor identity

$$F_{\mu\nu} = \partial_\mu J_\nu - \partial_\nu J_\mu + [J_\mu, J_\nu],$$

which follows from the Maurer–Cartan structure equation and from the phase equation (9). Its projection onto the fixed generator T_{em} recovers the electromagnetic field tensor (29), demonstrating that standard electrodynamics arises naturally as the Abelian limit of the SU(2) phase geometry.

Photon as a Phase Wave

A second class of excitations arises from small-amplitude oscillations of the phase field with vanishing topological charge,

$$B = 0, \qquad U(x) \simeq \exp[i\theta(x)T_{\mathrm{em}}], \tag{15}$$

which lead to a linearized wave equation for the phase component. The corresponding field satisfies the homogeneous and inhomogeneous Maxwell equations in the weak-field limit, matching the curvature-based definition used in Eq. (13).

The polarization state corresponds to the internal SU(2) orientation of the oscillation, while the frequency ω and wavevector k^μ are associated with the temporal and spatial gradients of the phase angle $\theta(x)$. The photon therefore represents a *propagating curvature of the SU(2) phase field*, transmitting energy and momentum through variations of the internal orientation.

In regions where localized vortices and delocalized waves coexist, the interaction between these two types of excitations yields the familiar phenomena of absorption, emission, and radiation pressure. No external gauge field is required; electromagnetic interactions arise as intrinsic deformations of the phase geometry itself.

Matter and Radiation as Unified Phase States

Both particle-like and wave-like excitations thus emerge from the same fundamental object $U(x)$. Localized vortices correspond to finite-energy topological defects, while photons are smooth periodic modulations of the same field. The transition between these regimes is continuous and governed by the relative magnitude of the local curvature term in Eq. (2). This provides a unified physical interpretation of matter and radiation as different expressions of the same SU(2) phase dynamics.

5 Classical Mechanics as Phase Kinematics

Classical mechanics arises as the macroscopic limit of the SU(2) phase dynamics when the phase variations are smooth on the scale of the compact manifold and the nonlinear contributions in Eq. (9) are small. In this regime, localized phase vortices behave as quasiparticles, and their collective motion obeys the familiar laws of Newtonian mechanics.

Phase Momentum and Energy

The canonical energy–momentum tensor associated with the phase field follows from the Noether theorem for infinitesimal coordinate translations. It is equivalent, up to the usual Belinfante symmetrization, to the symmetric (Hilbert) tensor derived in Sec. 3, and provides a convenient form for computing conserved quantities.

$$T^{\mu\nu}_{(\text{phase})} = \kappa\,\mathrm{Tr}(J^\mu J^\nu) + \alpha\,\mathrm{Tr}([J^\mu, J_\alpha][J^\nu, J^\alpha]) - g^{\mu\nu}\mathcal{L}_{\text{phase}}. \tag{16}$$

For a localized configuration, the total four-momentum is

$$P^\mu = \int T^{0\mu}_{(\text{phase})}\, d^3x, \tag{17}$$

and the invariant rest mass follows as

$$mc^2 = \int T^{00}_{(\text{phase})}\, d^3x. \tag{18}$$

The integrand corresponds to the phase energy density stored in the curvature of the field $U(x)$.

In the nonrelativistic limit, where $J_0 \gg J_i$, the temporal part of Eq. (9) reduces to an effective equation of motion for the vortex center,

$$m\,\frac{d^2x^i}{dt^2} = F^i, \tag{19}$$

where the effective force F^i originates from spatial gradients of the surrounding phase energy. Equation (19) reproduces the second law of Newton as the translational dynamics of a localized SU(2) vortex.

Angular Momentum and Rotational Dynamics

Rotational motion corresponds to internal reorientation of the phase field. The conserved angular momentum follows from the global internal SU(2) invariance of the Lagrangian, obtained by varying the action under infinitesimal internal rotations $U \to e^{\epsilon^a T_a} U$. It receives contributions from both the σ-term and the Skyrme term:

$$L^a = \int \epsilon^{ijk} x_i\,\mathrm{Tr}\Big(T^a\big[\kappa\,J_j J_k + \alpha\,[J_j, J_\ell][J_k, J^\ell]\big]\Big)\, d^3x. \tag{20}$$

where T^a are the generators of the algebra $\mathfrak{su}(2)$. For slowly rotating configurations, one can introduce an effective moment of inertia tensor I_{ab} defined by

$$L^a = I_{ab}\,\Omega^b, \tag{21}$$

where Ω^b are generalized angular velocities describing the internal rotation of the phase orientation. The kinetic energy of rotation then takes the standard quadratic form

$$E_{\text{rot}} = \frac{1}{2} I_{ab}\,\Omega^a \Omega^b. \tag{22}$$

Thus, the conventional relations between torque, angular acceleration, and angular momentum arise as macroscopic manifestations of SU(2) phase reorientation.

Potential Energy and Phase Deformation

External fields and interactions correspond to spatial variations of the background phase configuration. A gradient of the scalar phase angle $\theta(x)$ produces an effective potential

$$V_{\text{eff}}(\mathbf{x}) \propto \frac{\kappa}{2}\,\mathrm{Tr}\big(J_i J^i\big)\,, \tag{23}$$

which acts as a restoring or attractive force depending on the curvature of the surrounding phase. A small deviation δU from equilibrium satisfies a linearized equation of motion analogous to a harmonic oscillator,

$$\kappa\,\partial_t^2 \delta U = -\frac{\partial^2 V}{\partial U^2}\,\delta U, \tag{24}$$

demonstrating that inertial and potential effects share a common origin in the local curvature of the phase manifold.

Hamiltonian and Lagrangian Structure

The local Lagrangian density (2) defines conjugate momenta

$$\Pi^\mu = \frac{\partial \mathcal{L}_{\text{phase}}}{\partial(\partial_\mu U)} = \kappa\, U^{-1}\partial^\mu U + \alpha\,[J_\nu, [J^\mu, J^\nu]], \tag{25}$$

leading to a Hamiltonian density

$$\mathcal{H}_{\text{phase}} = \mathrm{Tr}(\Pi_\mu\, \partial^\mu U) - \mathcal{L}_{\text{phase}}. \tag{26}$$

In the smooth-field limit, $\mathcal{H}_{\text{phase}}$ reduces to the standard kinetic plus potential form,

$$\mathcal{H}_{\text{phase}} \simeq \frac{\kappa}{2}\,\mathrm{Tr}(J_0 J^0) + \frac{\kappa}{2}\,\mathrm{Tr}(J_i J^i) + V(U), \tag{27}$$

which corresponds to the conventional decomposition of mechanical energy into motion and deformation terms.

Emergence of Classical Laws

Equations (18)–(19) demonstrate that mass, momentum, and force can be interpreted as measures of curvature and energy flow within the phase field. The classical laws of motion, including the conservation of energy and angular momentum, follow as direct consequences of the underlying SU(2) symmetry and spacetime invariance of the action. Therefore, Newtonian mechanics is recovered not as an independent axiom but as the low-frequency limit of the general SU(2) phase dynamics.

6 Electrodynamics from Phase Geometry

Electrodynamics emerges as the projection of SU(2) phase dynamics onto a fixed internal direction associated with electromagnetic orientation. In this view, the electromagnetic field represents the Abelian sector of the non-Abelian phase geometry, obtained by projecting the Maurer–Cartan current J_μ onto a single generator of $\mathfrak{su}(2)$.

Projection and Effective Four-Potential

Let $T_{\rm em}$ be a fixed normalized generator of $\mathfrak{su}(2)$ satisfying $\mathrm{Tr}(T_{\rm em}^2) = \frac{1}{2}$. The projection of the SU(2) current onto this direction defines an effective electromagnetic four-potential:

$$a_\mu = -\,i\,\mathrm{Tr}\big(T_{\rm em}\,U^{-1}\nabla_\mu U\big)\,. \tag{28}$$

The corresponding field tensor, representing the curvature of the projected phase connection, is

$$\mathcal{F}_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu = +\,i\,\mathrm{Tr}(T_{\rm em}[J_\mu, J_\nu])\,, \tag{29}$$

where the *plus* sign follows from the Maurer–Cartan identity $\partial_\mu J_\nu - \partial_\nu J_\mu + [J_\mu, J_\nu] = 0$ for a left-invariant SU(2) connection. In curved spacetime one replaces partial derivatives by covariant ones; for slowly varying phase fields the Abelian projection remains valid up to small non-Abelian corrections of order $[J_\mu, J_\nu]^2$.

In the weak-field limit, where nonlinear commutator terms are small, the Bianchi identity for the SU(2) curvature implies

$$\partial_{[\lambda}\mathcal{F}_{\mu\nu]} = 0, \tag{30}$$

corresponding to the homogeneous Maxwell equations. Variation of the action with respect to the phase components aligned with $T_{\rm em}$ yields

$$\partial_\mu \mathcal{F}^{\mu\nu} = j^\nu, \tag{31}$$

where j^ν denotes the effective current generated by motion of charged phase vortices.

Equations (29)–(31) therefore reproduce Maxwell's equations as a limiting case of the SU(2) field equation (9). The projection selects an Abelian subsector of the full SU(2) dynamics and is accurate whenever higher-order commutator corrections are negligible, ensuring that standard electrodynamics emerges naturally from the underlying phase geometry.

Electric and Magnetic Fields

In the local rest frame, the electric and magnetic fields are recovered as

$$E_i = \mathcal{F}_{0i}, \qquad B_i = \frac{1}{2}\epsilon_{ijk}\,\mathcal{F}^{jk}. \tag{32}$$

The energy density and Poynting vector follow directly from the stress tensor of the phase field:

$$u_{\rm em} = \frac{1}{2}\big(E^2 + B^2\big)\,, \qquad \mathbf{S} = \mathbf{E}\times\mathbf{B}. \tag{33}$$

Hence, electromagnetic energy flow corresponds to transport of phase curvature across spacetime.

Gauge symmetry remark. The projection onto a fixed internal generator $T_{\rm em}$ explicitly breaks the full local SU(2) gauge symmetry down to its U(1) subgroup. This reduction identifies a preferred internal direction corresponding to the electromagnetic phase, so that the projected potential $a_\mu \equiv -i\,\mathrm{Tr}(T_{\rm em}\,J_\mu)$ behaves as an Abelian connection. Consequently, the resulting field tensor $F_{\mu\nu}$ satisfies the Maxwell equations as the curvature of this residual U(1) sector within the unified SU(2) phase geometry.[2]

[2]The calligraphic notation $\mathcal{F}_{\mu\nu}$ used earlier denotes the SU(2) curvature projected onto the electromagnetic direction prior to the explicit SU(2)$\to$U(1) reduction. In this Abelian limit, $\mathcal{F}_{\mu\nu}$ reduces to the conventional electromagnetic tensor $F_{\mu\nu}$, consistent with Eq. (29).

Current, Voltage, and Resistance

The effective electric current j^μ in Eq. (31) arises from gradients of the SU(2) phase angle carried by localized vortices:

$$j^\mu \propto \nabla^\mu \theta(x), \tag{34}$$

where $\theta(x)$ is the local projection of the internal phase on T_{em}.

The scalar potential difference between two points A and B along a path Γ is

$$V_{AB} = -\int_\Gamma \partial_i a_0 \, dx^i, \tag{35}$$

where the scalar potential

$$a_0 = -\, i \,\mathrm{Tr}\big(T_{\mathrm{em}}\, U^{-1} \partial_0 U\big)$$

is the temporal component of the projected phase connection. In the quasistatic limit, where the temporal variation of the field is slow and $\partial_t a_i \approx 0$, one may set $a_0 \propto \dot{\theta}$, so Eq. (35) reduces to $V_{AB} = -\int_\Gamma \partial_i \theta \, dx^i$, representing the purely potential contribution to the electromotive difference.

In the general case, the electric field is given by the full relation

$$E_i = F_{0i} = -\,\partial_t a_i - \partial_i a_0,$$

so inductive and radiative effects appear whenever the quasistatic approximation is not valid.

The local resistive behavior is determined by the rate of phase decoherence. For an ensemble of vortices with phase coherence time τ_c, the conductivity may be written as

$$\sigma \propto \tau_c, \tag{36}$$

indicating that perfect coherence ($\tau_c \to \infty$) leads to infinite conductivity. Thus, ohmic resistance is not a fundamental property but a measure of phase disorder.

Superconductivity as Phase Coherence

In a perfectly coherent phase domain, the covariant derivative of the phase angle vanishes,

$$\nabla_\mu \theta(x) = 0, \tag{37}$$

and the projected electric field is zero within the material, while surface currents maintain constant phase. This condition corresponds to the Meissner state, in which magnetic flux is expelled and resistance disappears. Therefore, superconductivity emerges naturally as a macroscopic manifestation of coherent SU(2) phase alignment.

At the microscopic level, the supercurrent density is proportional to the gradient of the global phase,

$$\mathbf{j}_s \propto \nabla \theta, \tag{38}$$

and satisfies the London-type equation,

$$\nabla^2 \mathbf{B} = \frac{1}{\lambda_L^2}\,\mathbf{B}, \qquad \lambda_L^{-2} \propto \kappa\,|\Psi|^2. \tag{39}$$

Here λ_L is the London penetration depth and $|\Psi|$ denotes the magnitude of the coherent phase amplitude. The proportionality $\lambda_L^{-2} \propto \kappa\,|\Psi|^2$ is understood phenomenologically:

the exact coefficient depends on microscopic properties of the condensate and can be calibrated by comparison with the Ginzburg–Landau or BCS frameworks. This ensures that the macroscopic limit of the SU(2) phase theory reproduces the standard London behavior without assuming any specific microscopic pairing mechanism. In this view, superconductivity arises as an emergent consequence of global phase coherence on the compact SU(2) manifold rather than from an external potential or interaction term.

Summary

Electrodynamics is thus identified as the Abelian projection of the SU(2) phase geometry. Charge corresponds to topological winding of the phase, electric and magnetic fields represent its gradients and curls, and current flow reflects collective transport of the internal phase orientation. Maxwell's equations, electromagnetic energy transport, and superconductivity all appear as limiting expressions of the underlying SU(2) phase dynamics.

7 Relativity and the Phase Stress Tensor

The geometric structure of the SU(2) phase field provides a natural foundation for both special and general relativity. The compactness of S^3 ensures local Euclidean neighborhoods that support inertial frames, while variations of the phase rate induce effective curvature and time dilation. Relativistic phenomena thus arise as manifestations of the spacetime response to internal phase energy.

Special Relativity as a Limit of Uniform Phase Flow

Consider a homogeneous phase configuration in which the phase rate $\omega(x)$ is constant, $\omega(x) = \omega_*$. In this case, the local operational time t defined by Eq. (11) is identical to the global parameter T, and the spacetime metric is locally Minkowskian:

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2. \tag{40}$$

Small perturbations of the phase field correspond to local boosts of the internal orientation, giving rise to the Lorentz transformations between moving observers. The invariance of the phase action under such transformations ensures that the speed of light c remains constant, since it represents the propagation velocity of infinitesimal phase disturbances on S^3.

Hence, special relativity is recovered as the symmetry of uniform phase evolution, where all regions share the same phase rate ω_* and the internal geometry is isotropic.

Gravitational Effects as Phase-Rate Variations

When the phase rate $\omega(x)$ varies spatially due to local curvature of the SU(2) field, the relation between operational time $t(x)$ and the global phase parameter T becomes nontrivial, as given by Eq. (11). The ratio $\omega(x)/\omega_*$ acts as an effective gravitational redshift factor, and time dilation follows directly:

$$\frac{dt}{dT} = \frac{\omega(x)}{\omega_*} \quad \Rightarrow \quad \frac{d\tau_1}{d\tau_2} = \frac{\omega(x_1)}{\omega(x_2)}. \tag{41}$$

Regions of increased phase curvature correspond to a lower local frequency $\omega(x)$, so clocks located there run more slowly relative to regions of weaker curvature. In this picture, gravitational potential wells arise as regions of reduced phase rate, and redshift emerges as a direct manifestation of SU(2) phase geometry.

The Newtonian potential ϕ is recovered in the weak–field limit through

$$\frac{\omega(x)}{\omega_*} \simeq 1 + \frac{\phi}{c^2}, \qquad g_{00} \simeq -\left(1 + \frac{2\phi}{c^2}\right), \tag{42}$$

which reproduces the standard post–Newtonian approximation. Thus, gravitational effects correspond to spatial variations of the intrinsic phase rate, and general–relativistic time dilation arises naturally from the local SU(2) phase dynamics.

Phase Stress and the Einstein Tensor

The geometric coupling between phase energy and curvature is expressed by Eq. (4),

$$G_{\mu\nu} + \Phi_{\mu\nu} = \frac{8\pi G}{c^4} T^{(\text{matter})}_{\mu\nu}, \tag{43}$$

where the phase stress tensor $\Phi_{\mu\nu}$ represents curvature induced by internal SU(2) dynamics. Explicitly,

$$\Phi_{\mu\nu} = -\frac{8\pi G}{c^4}\, T^{(\text{phase})}_{\mu\nu}, \tag{44}$$

with $T^{(\text{phase})}_{\mu\nu}$ given in Eqs. (16)–(20). This tensor encodes the full influence of the phase field on spacetime geometry and may be viewed as a geometric analogue of the stress–energy of the gravitational field itself.

In regions where $\mathcal{L}_{\text{phase}} \to 0$, the tensor $\Phi_{\mu\nu}$ vanishes, and the conventional Einstein field equations are recovered:

$$G_{\mu\nu} = \frac{8\pi G}{c^4}\, T^{(\text{matter})}_{\mu\nu}. \tag{45}$$

Thus, general relativity appears as the limit of vanishing internal phase curvature, while nonzero $\Phi_{\mu\nu}$ introduces higher-order geometric corrections that can account for additional curvature effects typically attributed to dark energy or vacuum polarization.

Geodesics and Phase Motion

The motion of a test vortex in curved spacetime follows from the conservation law $\nabla_\mu T^{\mu\nu}_{(\text{phase})} = 0$, which yields the standard geodesic equation in the limit of small phase stress gradients:

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\, \frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0. \tag{46}$$

Hence, free motion corresponds to transport along extremal paths of the phase manifold. Deviations from geodesic behavior arise only when internal phase interactions contribute non-negligible stress, leading to gravitational self-coupling or radiation reaction.

Unification of Gravitational and Electromagnetic Geometry

Equations (4) and (29) show that both gravitation and electromagnetism originate from different aspects of the same SU(2) phase geometry. The curvature associated with the diagonal generator $T_{\rm em}$ produces the electromagnetic field, while the overall SU(2) curvature enters the spacetime metric through the tensor $\Phi_{\mu\nu}$. This correspondence mirrors the structure of Kaluza–Klein and gauge–gravity unifications, but without introducing extra spacetime dimensions: the internal compact space itself generates both gauge and gravitational phenomena.

Summary

Special relativity arises from the uniform evolution of the phase field, and general relativity follows when local variations of the phase rate $\omega(x)$ modify the metric through the stress tensor $\Phi_{\mu\nu}$. Both inertial and gravitational effects thus have a common geometric origin in the curvature of the SU(2) phase field, providing a single coherent foundation for relativistic physics.

8 Quantum Mechanics as Compact Phase Dynamics

Quantum mechanics emerges as the natural description of compact phase evolution on the SU(2) manifold. Because the internal space S^3 is finite and closed, all eigenmodes of the Laplacian operator on this manifold form a discrete spectrum. Quantization, therefore, follows as a geometric necessity rather than an additional postulate.

Wave Equation on the Compact Manifold

Consider small harmonic perturbations of the phase field around a stationary configuration,

$$U(x) = U_0 \exp\big[i\,\psi(x)\big], \qquad |\psi| \ll 1. \tag{47}$$

Linearizing Eq. (9) yields

$$\nabla_\mu \nabla^\mu \psi + M^2 \psi = 0, \tag{48}$$

where M^2 is an effective mass term derived from the curvature of the potential $V(U)$. On the compact three-sphere of radius R, eigenfunctions of the Laplacian satisfy

$$\nabla^2_{S^3} Y_n = -\frac{n(n+2)}{R^2}\, Y_n, \qquad n = 0, 1, 2, \ldots, \tag{49}$$

so that energy levels of stationary modes are quantized as

$$E_n = \hbar\omega_n = \hbar c\sqrt{k^2 + \frac{n(n+2)}{R^2}}, \tag{50}$$

providing a geometric explanation for discrete spectra in bound systems.

Emergence of the Schrödinger Equation

In the nonrelativistic limit where spatial gradients are small compared to temporal oscillations, the field $\psi(x)$ may be factorized as

$$\psi(x,t) = \Psi(\mathbf{x},t)\, e^{-iMc^2 t/\hbar}, \tag{51}$$

and Eq. (48) reduces to

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2M} \nabla^2 \Psi + V_{\text{eff}} \Psi, \tag{52}$$

which is the Schrödinger equation for the effective wave function $\Psi(\mathbf{x},t)$. Thus, the fundamental quantum evolution arises as the slow modulation of the internal SU(2) phase oscillation, with Planck's constant $\hbar$ serving as a proportionality between internal angular momentum and phase rate.

Commutation Relations and Uncertainty

The canonical structure of the phase field reproduces the standard quantum operator algebra in the local (infinitesimal) limit of the compact SU(2) manifold. Translations of the phase coordinate along spatial directions are generated by the momentum operator

$$\hat{p}_i = -\, i\hbar\, \partial_i. \tag{53}$$

The basic commutation relation

$$[x_i, \hat{p}_j] = i\hbar\, \delta_{ij}$$

emerges as the flat–space (tangent–space) limit of the SU(2) Lie algebra, in which the noncommuting group parameters reduce locally to linear coordinates on S^3. Thus, the canonical commutator reflects the intrinsic curvature of the compact phase space, and ordinary quantum mechanics appears as its linearized approximation.

The uncertainty relation

$$\Delta x_i\, \Delta p_i \geq \frac{\hbar}{2}$$

therefore expresses the geometric noncommutativity of conjugate phase variables on the compact SU(2) manifold. Finite curvature introduces higher–order corrections to this relation at very small scales, while in the macroscopic limit ($R_{\text{phase}} \to \infty$) the standard Heisenberg uncertainty principle is exactly recovered.

Probability and Normalization on S^3

The probability density associated with a quantum state is interpreted as the norm of the phase amplitude on S^3:

$$\int_{S^3} |\Psi(\xi)|^2\, dV_{S^3} = 1. \tag{54}$$

For physical observables in three-dimensional space, the density is obtained by integrating over the hidden coordinate of the compact manifold,

$$\rho(\mathbf{x}) = \int_{\pi^{-1}(\mathbf{x})} |\Psi(\xi)|^2\, d\mu_{\text{fiber}}(\xi), \tag{55}$$

where $\pi : S^3 \to \mathbb{R}^3$ denotes the stereographic projection and $d\mu_{\text{fiber}}$ is the measure along the fiber direction. Hence, the usual probabilistic interpretation of quantum mechanics corresponds to a projection of the normalized SU(2) phase amplitude.

Spin and Intrinsic Angular Momentum

The double-connected topology of SU(2) implies that a 2π rotation in physical space corresponds to a sign reversal of the internal phase orientation. This property naturally yields the spin-$\frac{1}{2}$ behavior of fermions without additional assumptions. The Pauli matrices represent the infinitesimal generators of SU(2), and spin operators follow directly from the internal algebra:

$$\hat{S}_i = \frac{\hbar}{2}\sigma_i, \qquad [\hat{S}_i, \hat{S}_j] = i\hbar\,\epsilon_{ijk}\hat{S}_k. \tag{56}$$

The intrinsic magnetic moment of the electron corresponds to the coupling between spin orientation and the electromagnetic projection T_{em}.

Coherence and Quantum Superposition

Because the SU(2) manifold is compact and unitary, linear superposition of phase eigenmodes corresponds to constructive or destructive interference of internal orientations. Coherent states occupy minimal volumes on the group manifold and exhibit stable phase relationships, while decoherence corresponds to diffusion of the phase orientation and loss of off-diagonal coherence in the density matrix. Hence, the statistical nature of quantum mechanics originates from the coarse-graining of microscopic SU(2) phase configurations.

Summary

Quantum mechanics arises as the compact-mode limit of SU(2) phase dynamics. The discreteness of energy levels results from the finiteness of S^3; the Schrödinger equation represents the nonrelativistic limit of the phase-wave equation; and the uncertainty principle reflects the inherent curvature of the internal phase manifold. Thus, the quantum behavior of matter follows directly from the same geometric principles that give rise to classical mechanics, electrodynamics, and relativity within the unified SU(2) phase framework.

9 Geometric Interpretation of Relativistic and Quantum Effects

The unified SU(2) phase geometry not only reproduces the formal structure of known physical laws, but also provides intuitive geometric interpretations of a wide range of experimentally verified phenomena. Several key effects traditionally regarded as paradoxical or counterintuitive in relativity and quantum theory are here understood as direct consequences of the curvature and evolution of the compact phase manifold.

Light Deflection and Gravitational Redshift

The deflection of light in a gravitational field follows from spatial variations of the local phase rate $\omega(x)$. Photons propagate along null geodesics of the effective metric

$$g_{\mu\nu}^{\text{eff}} = g_{\mu\nu} + \delta g_{\mu\nu}(\omega),$$

where the perturbation $\delta g_{\mu\nu}$ depends on the gradient of $\omega(x)$. Expanding to first order in the gravitational potential ϕ,

$$\frac{\omega(x)}{\omega_*} \simeq 1 + \frac{\phi}{c^2},$$

one recovers the standard Einstein deflection angle for light grazing a massive body,

$$\Delta\theta = \frac{4GM}{c^2 b},$$

where b is the impact parameter. Thus, gravitational lensing and the Einstein redshift arise directly from the geometric modulation of the phase rate, without additional postulates.

Time Dilation and Phase Geometry

Equation (11) implies that local clocks measure time according to

$$dt = \frac{\omega(x)}{\omega_*}\, dT,$$

so that slower phase evolution corresponds to slower passage of time. In moving reference frames, the effective phase rate decreases by the Lorentz factor,

$$\omega(x, v) = \omega_* \sqrt{1 - \frac{v^2}{c^2}},$$

reproducing kinematic time dilation of special relativity. Hence, both gravitational and kinematic time dilation are unified as manifestations of local variations of the SU(2) phase rate.

Observer Effect as Phase Synchronization

Measurement or observation corresponds to synchronization between the local phase of the measuring apparatus and that of the observed system. During this interaction, the system's internal orientation $U(x)$ becomes aligned with the phase frame of the observer, reducing its accessible state space. The apparent "collapse" of the wave function therefore represents a geometric projection of the distributed phase configuration onto a synchronized submanifold of the total SU(2) manifold. Probabilistic outcomes reflect the relative volumes of these submanifolds under the invariant group measure. *The relative volumes of these submanifolds define the probabilities of the corresponding outcomes, thus providing a direct geometric foundation for the statistical interpretation of quantum mechanics.*

Quantum Entanglement as Correlated Phase Orientation

For two spatially separated excitations described by correlated SU(2) phase fields $U_1(x_1)$ and $U_2(x_2)$, a single global constraint on the composite phase orientation ensures that their internal states remain correlated even when spatially distant. The correlations arise from the shared global parameter T of the phase evolution, while each observer experiences local time $t(x)$. Since no causal signal is exchanged in $t(x)$, relativistic causality remains intact. Entanglement thus represents a nonlocal constraint on the joint SU(2) phase configuration, not a physical superluminal interaction.

Quantum Tunneling as Continuous Phase Transition

Barrier penetration is interpreted as continuous rotation of the SU(2) phase through a region of higher curvature energy. The exponential suppression of transmission probability,

$$P \propto e^{-2\int \sqrt{2m(V-E)}\, dx/\hbar},$$

corresponds to the geometric attenuation of the phase amplitude across the curved segment of the manifold. This mechanism agrees with experimental observations of macroscopic quantum tunneling, such as those reported by John Clarke, Michel Devoret, and John Martinis in superconducting systems [2], for which the tunneling rate is governed by the same phase-curvature relation.

Cosmological Redshift as Phase Aging

The cosmological redshift need not imply metric expansion of the universe. In the SU(2) phase framework, light emitted at global time T_{emit} and observed at T_{obs} experiences a frequency change due to slow evolution of the global phase rate:

$$1 + z = \frac{\omega_{\text{emit}}}{\omega_{\text{obs}}} = \frac{\omega_*(T_{\text{emit}})}{\omega_*(T_{\text{obs}})}.$$

If $\omega_*(T)$ decreases monotonically with global time, the same redshift pattern observed in distant galaxies arises naturally as a consequence of *phase aging*, without requiring expansion of spatial distances. The photon thus "ages" in phase while propagating through the global SU(2) field, its frequency decreasing as the universe evolves toward lower global phase energy. *This interpretation predicts redshift–luminosity relations for standard candles that differ from the ΛCDM model, offering a potential observational test in future astronomical surveys.*

Summary

Relativistic and quantum phenomena acquire transparent geometric meaning within the SU(2) phase model. Light deflection, time dilation, entanglement, tunneling, and cosmological redshift are unified as different expressions of curvature and evolution on the compact phase manifold. No paradoxes arise when both matter and radiation are viewed as manifestations of a single SU(2) phase geometry evolving in global time T.

10 Concluding Remarks and Outlook

The formulation presented above establishes a single geometric framework in which classical mechanics, electrodynamics, relativity, and quantum mechanics arise as limiting cases of the same underlying SU(2) phase dynamics on the compact three-sphere S^3. Matter, radiation, and gravitation are not treated as separate entities but as different manifestations of curvature and motion within the same internal phase geometry.

The compactness of the SU(2) manifold ensures both global finiteness and local continuity, providing natural quantization, intrinsic spin structure, and the absence of singularities. In this setting, the conventional physical quantities — mass, charge, field strength, and energy — are identified with geometric invariants of the phase field: mass with localized curvature energy, charge with topological winding, field strength with phase gradients, and radiation with propagating curvature waves. Time itself acquires a dual interpretation as both a global evolution parameter T and a locally measured rate determined by the phase frequency $\omega(x)$.

The analysis demonstrates that:

- Newtonian dynamics emerge as the low-frequency limit of localized phase motion;
- Electromagnetism corresponds to the Abelian projection of the SU(2) curvature;
- Relativistic effects arise from spatial variations of the phase rate and curvature coupling through the tensor $\Phi_{\mu\nu}$;
- Quantum mechanics represents the compact eigenmode dynamics on S^3, yielding discrete spectra and intrinsic spin structure without external quantization rules.

This unified geometric interpretation preserves the full content of established physics while removing the artificial division between particles, fields, and spacetime. All observable phenomena are described as aspects of a single differentiable object $U(x) \in \mathrm{SU}(2)$, whose local orientation encodes internal degrees of freedom and whose curvature defines both forces and dynamics.

The present work focuses on the general formalism and its direct consequences for fundamental interactions. Further developments will elaborate specific domains of application:

1. **Atomic systems:** bound configurations of delocalized SU(2) vortices along resonant modes, explaining magnetic moments, spectral structure, and stability conditions. This topic deserves a separate dedicated study.
2. **Nuclear structure:** collective phase configurations on the global S^3 shell, describing magic numbers and binding energies as stable multi-vortex modes.
3. **Cosmology:** large-scale curvature and expansion of the global SU(2) phase manifold, relating gravitational constants and cosmic evolution to the geometry of the compact phase space.

The SU(2) phase model thus provides a continuous pathway from microscopic to cosmological scales within a single mathematical principle. Future work will refine quantitative predictions, explore the stability of composite configurations, and compare measurable consequences with experimental data. Ultimately, the goal is to establish a fully geometric representation of physical reality in which the apparent diversity of natural phenomena is unified by the topology and curvature of the compact SU(2) phase manifold.

Emergent hierarchy of physical scales. The SU(2) phase field describes the internal geometry at each spacetime point rather than an external spatial structure. The vast difference between hadronic, atomic, and cosmological scales emerges dynamically from the balance between the gradient energy (with coupling constant κ) and the stabilizing Skyrme term (with coefficient α) in the unified Lagrangian. This competition defines a characteristic length scale,

$$L_* \sim \sqrt{\frac{\alpha}{\kappa}},$$

which sets the order of magnitude for equilibrium configurations of the field.

In the topological sector with winding number $B = 1$, the minimization of the static energy

$$E_{\rm stat}(r_0) \;=\; A\,\kappa\,r_0 \;+\; B\,\frac{\alpha}{r_0}$$

yields a compact soliton of size $r_0^* \sim \sqrt{\alpha/\kappa}$, naturally identified with the nucleon core — the high–energy–density branch of the theory. For the leptonic branch ($B = 0$), the dominant gradient and electromagnetic self–energy terms favor an extended configuration, with characteristic size on the order of the Compton wavelength $\lambda_C = \hbar/(m_e c)$, corresponding to atomic or subatomic structures. At the opposite extreme, the background curvature radius R of the global S^3 defines the cosmological scale of the model. The physical sizes of nuclei and atoms thus appear as small ratios $r_{\rm nuc}/R$ and a_0/R, naturally explaining the observed hierarchy of scales.

Hence, a single SU(2) phase field admits multiple classes of stable solutions — localized solitons, extended leptonic modes, and global background curvature — each with its characteristic length determined by the energy minimization of the same unified Lagrangian. The group structure remains universal, while the observed scales are emergent consequences of its internal dynamics.

Appendix: Geometric Algebra Formulation of the S^3 Ontology

The global hyperspherical model can be expressed most naturally in the language of geometric algebra. Within the Euclidean Clifford algebra Cl$(4, 0)$, points of the three-sphere $S^3 \subset \mathbb{R}^4$ correspond to unit rotors R satisfying $R\tilde{R} = 1$, where $\tilde{R}$ denotes reversion. These rotors generate rotations of vectors through the geometric product $x' = R\,x\,\tilde{R}$, and collectively form the group Spin$(4) \simeq SU(2)_L \times SU(2)_R$.

The local phase structure of the hypersphere may then be represented by a rotor field $R(X)$, whose differential defines a bivector-valued current

$$J_\mu = R^{-1}\partial_\mu R.$$

This object is directly analogous to the Maurer–Cartan form $U^{-1}\nabla_\mu U$ used in the SU(2) formalism, but now appears as a geometric quantity rather than a matrix operator. The scalar part of $J_\mu J^\mu$ provides the kinetic term of the Lagrangian, while its wedge products describe the intrinsic curvature of the phase field.

A coordinate-free expression for the Lagrangian can therefore be written as

$$\mathcal{L} = \frac{\kappa}{2}\,\langle J_\mu J^\mu\rangle_0 + \frac{\alpha}{4}\,\langle (J_\mu \wedge J_\nu)(J^\mu \wedge J^\nu)\rangle_0 - V(R),$$

where $\langle\cdot\rangle_0$ denotes the scalar (grade-0) part of a multivector. This formulation eliminates the need for explicit trace operations and matrix bases, treating rotations, spinors, and bivectors as elements of a single unified structure.

Electromagnetic and gauge-like projections can be defined geometrically by choosing a unit simple bivector B_e that specifies an oriented two-plane within the self-dual subspace of Cl$(4, 0)$. The effective electromagnetic potential and field strength then follow as

$$a_\mu = -\langle B_e J_\mu\rangle_0, \qquad F_{\mu\nu} = -\langle B_e(J_\mu \wedge J_\nu)\rangle_0.$$

In this way, what appears in the SU(2) formulation as a privileged generator T_{em} is replaced by a purely geometric selection of a two-plane within the phase space.

This geometric-algebra representation clarifies the dual SU$(2)_L\times$SU$(2)_R$ structure of the hyperspherical symmetry and expresses quantization, curvature, and field interactions directly through the algebra of multivectors. Although such an approach is mathematically elegant and philosophically consistent with the original S^3 ontology, it departs significantly from conventional field-theoretic practice and is therefore presented here only as a formal equivalence to the SU(2) matrix representation developed in the main text.

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