Unified Phase-Geometric Theory (UPGT): Cosmology

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Abstract

This paper presents the cosmological extension of the SU(2) phase–geometric framework developed in the companion works *Foundations* and *Atom* (the latter encompassing both atomic and nuclear structure). In this model the Universe is described as a continuous SU(2) phase field defined on the compact three–sphere S^3, whose curvature and stiffness determine both microscopic and macroscopic dynamics. No metric expansion or external dark components are required: the observed redshift, time dilation, and gravitational lensing arise from coherent variations of the global phase curvature. A closed dynamical relation between the phase parameters $(\kappa, \sigma, \alpha, V_0)$ establishes a consistent link between local and cosmic scales, yielding stationary and oscillatory solutions for the curvature radius $R(T)$. The same framework accounts qualitatively for the homogeneity of the cosmic microwave background and for the emergence of large–scale structure through nonlinear phase localization. Dimensional consistency and quantitative proximity to empirical constants support the internal coherence of the model, while open directions—including CMB spectrum, nucleosynthesis, and phase acoustic oscillations—define a clear path for future observational verification.

Contents

1 Introduction

The present work constitutes the third part of a unified theoretical framework based on the geometry of the SU(2) phase field defined on the compact three–sphere S^3. The preceding papers [1, 2] developed the fundamental structure and microscopic realization of this model. In *Foundations*, the SU(2) phase manifold was introduced as the underlying geometric substrate from which space, time, and quantum behavior jointly emerge. A consistent Lagrangian formulation was established, relating curvature, stiffness, and topological charge to the observable constants of nature. The companion paper *Atomic and Nuclear Structure* extended this framework to the scale of matter, showing that the same phase geometry reproduces the quantized shell hierarchy of atomic and nuclear systems.

The present part, *Cosmology*, applies the same principles to the largest accessible scales of the Universe. No new postulates are introduced: all macroscopic phenomena follow from the same phase field that governs microscopic dynamics. Global curvature of the SU(2) phase on S^3 determines the effective cosmological evolution, while local curvature fluctuations correspond to matter and radiation fields embedded within this geometry. The resulting picture replaces the metric expansion paradigm with a coherent phase evolution, interpreting redshift, time dilation, and gravitational lensing as manifestations of the same underlying phase curvature.

This approach eliminates the need for dark matter and dark energy as independent components, recasting them as inertial and potential effects of the SU(2) phase field itself. The model yields a closed dynamical system governed by four constants $(\kappa, \sigma, \alpha, V_0)$, which connect local and cosmic scales through the equilibrium relation

$$\frac{\alpha}{R_0^4} = \sigma + 3V_0 R_0.$$

Within this system the curvature radius R_0 plays the role of the global order parameter linking the microphysical and cosmological domains.

The objectives of the present paper are: (i) to derive the cosmological dynamics implied by the SU(2) phase Lagrangian; (ii) to interpret key observational phenomena—redshift, lensing, and background radiation—in terms of phase geometry; and (iii) to identify the remaining quantitative challenges necessary for a complete correspondence with empirical cosmology. Together with the earlier works [1, 2], this paper completes a consistent multiscale description of the Universe as a single SU(2) phase configuration, spanning from nuclear to cosmological scales.

2 Notation, Units, and Dimensional Conventions

All derivations in this work are formulated in natural units, where $\hbar = c = 1$. In these units, all physical quantities are expressed in powers of energy E, with the basic dimensional relations

$$[L] = [T] = E^{-1}, \qquad [\dot{R}] = 1.$$

Throughout the text, dimensionful constants are explicitly retained to ensure traceable dimensional homogeneity across scales.

2.1 Phase-Geometric Parameters

Convention. Throughout the main text we use the *reduced (energetic) convention*, in which the action has already been integrated over the total curvature volume $2\pi^2 R^3$. The resulting reduced Lagrangian L_{eff} therefore has the dimension of energy, $[L_{\text{eff}}] = [E]$.

The large-scale dynamics of the SU(2) phase field are described by the effective reduced Lagrangian

$$L_{\text{eff}} = \tfrac{1}{2}\kappa\,\dot{R}^2 - V(R), \tag{1}$$

where $R(T)$ is the global curvature radius of S^3. In this convention,

$$[R] = E^{-1}, \qquad [\dot{R}] = 1, \qquad [\kappa] = E, \qquad [V(R)] = E.$$

The constants entering the reduced dynamics originate from the SU(2) field action and define the fundamental phase–geometric scales:

$$E_* = (\alpha\kappa)^{1/2}, \qquad L_* = (\alpha/\kappa)^{1/2}.$$

With the reduced (energetic) dimensions

$$[\alpha] = E^{-1}, \qquad [\sigma] = E^3, \qquad [V_0] = E^4,$$

one obtains

$$[E_*] = 1 \text{ (dimensionless)}, \qquad [L_*] = E^{-1}.$$

Clarification on the meaning of E_*. In the reduced convention used throughout the paper, E_* is a *dimensionless invariant ratio* that sets the relative scale between curvature and stiffness. If one wishes to define a quantity with an actual energy dimension, it can be introduced in the density (per-volume) convention as

$$E_*^{(\text{dens})} \equiv (\tilde{\alpha}\,\tilde{\kappa})^{1/2}, \qquad [E_*^{(\text{dens})}] = E.$$

However, this form is not used in the present text; all physical relations are expressed through the reduced invariants (E_*, L_*) defined from (κ, α).

Remark (on density form). When we temporarily switch to the *density (per-volume) representation* [1]

of the action density $\mathcal{L}_{\text{dens}}$ (explicitly indicated in the text), we do not change the fundamental reduced parameters. Instead, we introduce the mapping

$$\tilde{\kappa}(R) = \frac{\kappa}{2\pi^2 R^3}, \qquad \tilde{\sigma}(R) = \frac{\sigma}{2\pi^2 R}, \qquad \tilde{\alpha}(R) = \frac{\alpha}{2\pi^2 R^5}, \qquad \tilde{V_0} = \frac{V_0}{2\pi^2},$$

so that all density coefficients satisfy $[\tilde{\kappa}] = [\tilde{\sigma}] = [\tilde{\alpha}] = [\tilde{V_0}] = E^4$, and the Lagrangian density acquires the standard dimension $[\mathcal{L}_{\text{dens}}] = E^4$ (see Appendix A). The invariants E_* and L_* used throughout the paper are always defined from the *reduced* parameters (κ, α); hence their dimensions remain fixed as $[E_*] = 1$ and $[L_*] = E^{-1}$, regardless of whether the density form is employed for intermediate expressions.

Here:

[1] Two distinct "density" notions are used throughout this work: (i) the ***field-density*** form of the original SU(2) Lagrangian with $[\mathcal{L}_{\text{field}}] = E^4$ and $[\kappa_{\text{field}}] = E^2$, $[\alpha_{\text{field}}] = 1$; and (ii) the *reduced-density* form $\mathcal{L}_{\text{dens}}^{(\text{reduced})} = L_{\text{eff}}/(2\pi^2 R^3)$, whose coefficients $\tilde{\kappa}, \tilde{\alpha}, \tilde{\sigma}, \tilde{V_0}$ all have $[E^4]$. These serve different bookkeeping purposes and are not interchangeable (see Appendix A).

- κ is the kinetic stiffness (inertial coefficient),
- σ is the quadratic potential coefficient,
- α characterizes the inverse-square curvature term,
- V_0 is the homogeneous potential coefficient.

2.2 Choice of Convention

All equations in this work are expressed in the *reduced (global)* convention, in which the Lagrangian (1) already includes the implicit integration over the total curvature volume $2\pi^2 R^3$. This ensures that each term has dimension $[E]$ without introducing explicit volume factors or FRW-type scale multipliers.

For comparison with FRW-like formulas we occasionally use the *density representation*, where quantities are expressed per unit curvature volume. In that case we always pass between reduced and density coefficients via the mapping above (Appendix A), which guarantees dimensional consistency.

For reference, the corresponding energy density is

$$\rho_{\text{phase}} = \frac{E_{\text{eff}}}{2\pi^2 R^3},$$

with dimension $[E^4]$, which is convenient when comparing to FRW-like cosmological equations (see Section 5). Unless explicitly stated otherwise, all derivations in the paper are performed in the reduced energetic convention.

Two types of "density" (do not mix). We distinguish two distinct density normalizations:

- **Field-density (microscopic)**: the original 4D field Lagrangian density $\mathcal{L}_{\text{field}}$ with $[\mathcal{L}_{\text{field}}] = E^4$. In natural units one has $[J_\mu] = E$, hence $[\kappa_{\text{field}}] = E^2$, $[\alpha_{\text{field}}] = E^0$, and $[V] = E^4$.
- **Reduced-density (per-volume of the global mode)**: the per-volume version of the reduced effective Lagrangian, $\mathcal{L}_{\text{dens}}^{(\text{reduced})} \equiv L_{\text{eff}}/(2\pi^2 R^3)$, where the *tilded* coefficients $\tilde{\kappa}, \tilde{\alpha}, \tilde{\sigma}, \tilde{V}_0$ are defined by

$$\tilde{\kappa} = \frac{\kappa}{2\pi^2 R^3}, \quad \tilde{\sigma} = \frac{\sigma}{2\pi^2 R}, \quad \tilde{\alpha} = \frac{\alpha}{2\pi^2 R^5}, \quad \tilde{V}_0 = \frac{V_0}{2\pi^2},$$

  and all satisfy $[\tilde{\kappa}] = [\tilde{\alpha}] = [\tilde{\sigma}] = [\tilde{V}_0] = E^4$.

These two density schemes serve different purposes (microscopic field theory vs. global-mode bookkeeping) and must not be identified with each other.

2.3 Dimensional Summary

Dimensional table (reduced/energetic convention)

Quantity	Symbol	Dimension (natural units)
Radius of curvature	R	E^{-1}
Kinetic stiffness	κ	E
Curvature constant	α	E^{-1}
Quadratic potential coefficient	σ	E^3
Homogeneous potential coefficient	V_0	E^4
Effective potential	$V(R)$	E
Reduced Lagrangian (energy)	L_{eff}	E
Energy-scale invariant	$E_* = \sqrt{\alpha\kappa}$	1 (dimensionless)
Length-scale invariant	$L_* = \sqrt{\alpha/\kappa}$	E^{-1}

Convention note. The table above refers to the reduced (energetic) convention used throughout the main text. In sections explicitly marked as employing the density (per-volume) convention, the bookkeeping of parameters changes (e.g. $[\kappa] = E^2$, $[\alpha] = E^0$), but the invariants E_* and L_* remain defined through the *reduced* parameters (κ, α) and thus preserve their fixed dimensions $[E_*] = 1$ and $[L_*] = E^{-1}$. For clarity, one may also introduce the "energetic" version $E_*^{(\text{dens})} = (\tilde{\alpha}\tilde{\kappa})^{1/2} \sim E$, yet this alternative normalization is *not used* in the present work.

3 Ontology of the SU(2) Phase on the Three-Sphere

The cosmological model developed here preserves the original ontological basis of the SU(2) phase field. All physical quantities are manifestations of a single continuous field $U(x) \in \mathrm{SU}(2)$, defined globally on the compact three-sphere S^3. The field configuration is characterized by a local current

$$J_\mu = U^{-1}\nabla_\mu U,$$

which encodes both curvature and phase rotation of the underlying manifold.

In this framework, space itself is not an embedding in an external metric but a compact phase manifold whose intrinsic geometry evolves in time through the global parameter $R(T)$. This parameter represents the instantaneous curvature radius of the SU(2) phase distribution on S^3. The temporal evolution of $R(T)$ thus describes the breathing or modulation of the global SU(2) phase geometry.

The spatial metric g_{ij} induced by the SU(2) configuration takes the form

$$ds^2 = R^2(T)\, d\Omega_3^2,$$

where $d\Omega_3^2$ denotes the unit metric on the three-sphere. This representation is purely geometric and does not require external gravitational embedding. In particular, the evolution of $R(T)$ is not associated with an expansion of physical space in the conventional cosmological sense, but with the phase modulation of the intrinsic SU(2) manifold.

The global phase dynamics are governed by the SU(2) field action

$$S = \int d^4x \sqrt{|g|} \left[\frac{\kappa}{2} \mathrm{Tr}(J_\mu J^\mu) + \frac{\alpha}{4} \mathrm{Tr}([J_\mu, J_\nu][J^\mu, J^\nu]) - V(U) \right],$$

whose variation reduces, under the assumption of homogeneity and isotropy, to the effective Lagrangian of a single global mode $R(T)$:

$$L_{\text{eff}} = \tfrac{1}{2}\kappa \dot{R}^2 - V(R).$$

This reduction maintains the geometric meaning of $R(T)$ as the curvature radius of the three-sphere, while the potential $V(R)$ encodes both intrinsic phase tension and the contribution of the homogeneous background field.

The curvature term proportional to α/R^2 arises from the commutator $[J_\mu, J_\nu]$, reflecting the intrinsic SU(2) curvature of the manifold. The term proportional to σR^2 represents the elastic phase stiffness, and the constant component $V_0 R^3$ corresponds to the homogeneous phase energy.

This ontological picture differs fundamentally from the metric expansion view of standard cosmology. The global evolution of $R(T)$ describes a coherent deformation of the phase geometry on S^3, within which all local excitations—particles, fields, and interactions—are internal modulations of the same SU(2) phase. Hence, cosmological dynamics, atomic structure, and nuclear stability emerge as different scales of a single continuous phase evolution.

4 Reduction of the SU(2) Action to a Global Mode

Under the assumption of large-scale homogeneity and isotropy, the SU(2) field configuration can be represented as a global mode of the form

$$U(x) = \exp[\, i\,\chi(t)\,\hat{n}(\mathbf{x})\cdot\boldsymbol{\sigma}\,]\,,$$

where $\hat{n}(\mathbf{x})$ defines the angular embedding on S^3, and $\boldsymbol{\sigma}$ are the Pauli generators of SU(2). The temporal evolution of the phase amplitude $\chi(t)$ translates into the dynamics of the curvature radius $R(T)$, which parameterizes the global geometry of the manifold.

Substituting this ansatz into the general SU(2) field action and integrating over the three-sphere of volume $2\pi^2 R^3$ yields an effective one-dimensional action of the form

$$S_{\text{eff}} = \int dT \left[\tfrac{1}{2}\kappa\,\dot{R}^2 - V(R)\right],$$

where the reduced potential $V(R)$ incorporates contributions of distinct geometric origin:

$$V(R) = \frac{\alpha}{2R^2} + \frac{\sigma}{2}R^2 + V_0 R^3. \tag{2}$$

Interpretation of the potential terms.

- The inverse-square term $\alpha/(2R^2)$ arises from the commutator $[J_\mu, J_\nu]$ and represents intrinsic SU(2) curvature. It scales inversely with the square of the radius and dominates at small R, ensuring a finite energy as $R \to 0$.
- The quadratic term $(\sigma/2)R^2$ reflects the elastic response of the phase geometry to curvature deformations. It introduces an effective restoring force that resists large-scale dilation or compression of the manifold.
- The cubic term $V_0 R^3$ corresponds to the homogeneous phase energy, analogous to a vacuum contribution, which governs the large-scale asymptotic behavior of the system and defines the effective phase background.

Each term in $V(R)$ carries the same dimension of energy:

$$[\alpha/R^2] = E, \qquad [\sigma R^2] = E, \qquad [V_0 R^3] = E.$$

The dimensional balance is therefore preserved for all R within the reduced (energetic) convention.

Equation of motion. Variation of the effective action with respect to $R(T)$ gives

$$\frac{d}{dT}\left(\kappa\,\dot{R}\right)+\frac{\partial V}{\partial R}=0,$$

or explicitly,

$$\kappa\,\ddot{R}+\sigma R-\frac{\alpha}{R^3}+3V_0R^2=0. \tag{3}$$

This equation describes the self-consistent phase dynamics of the global SU(2) mode. It contains no external forces or metric assumptions and is entirely determined by the intrinsic curvature and the parameters $\kappa, \sigma, \alpha, V_0$.

Energy integral. Integrating Eq. (3) once in time yields the conserved total energy:

$$E_{\text{eff}}=\tfrac{1}{2}\kappa\,\dot{R}^2+\frac{\alpha}{2R^2}+\frac{\sigma}{2}R^2+V_0R^3=\text{const}. \tag{4}$$

This integral form provides a direct mechanical analogy, with $R(T)$ acting as an effective coordinate and κ as the inertial parameter. The analogy is purely formal and serves only as a compact mathematical representation of the collective phase dynamics on the SU(2) manifold.

Dimensional consistency. Each term in E_{eff} has the same dimension $[E]$: $[\kappa\dot{R}^2]=E$, $[\alpha/R^2]=E$, $[\sigma R^2]=E$, and $[V_0R^3]=E$. This confirms that the effective dynamics are fully self-consistent and dimensionally homogeneous under the reduced energetic normalization.

5 Phase–Friedmann Dynamics

The evolution of the global curvature radius $R(T)$ follows directly from the conservation of the effective reduced energy,

$$E_{\text{eff}}=\tfrac{1}{2}\kappa\,\dot{R}^2+\frac{\alpha}{2R^2}+\frac{\sigma}{2}R^2+V_0R^3=\text{const}. \tag{5}$$

This relation defines the reduced *Phase–Friedmann equation* in its energetic form. All terms have the same dimension $[E]$, without any division by R^2 or explicit volume factors, ensuring dimensional homogeneity across the entire expression.

The energy balance (5) describes the global phase dynamics of the SU(2) manifold in a compact form analogous to the Friedmann equation of general relativity. However, unlike the FRW metric, $R(T)$ here is not a scale factor of an expanding space, but a dynamical curvature radius of the compact three-sphere S^3 within the intrinsic SU(2) phase geometry.

For clarity, E_{eff} denotes the conserved *reduced total energy* of the global mode (dimension $[E]$). The corresponding energy density, when needed for comparison with the FRW framework, is

$$\rho_{\text{phase}}=\frac{E_{\text{eff}}}{2\pi^2R^3},\qquad [\rho_{\text{phase}}]=E^4.$$

5.1 Dimensionally Consistent Evolution Law

Differentiating the energy integral yields

$$\kappa \dot{R} \ddot{R} + \frac{dV}{dR} \dot{R} = 0,$$

which reproduces the equation of motion in the form

$$\kappa \ddot{R} + \sigma R - \frac{\alpha}{R^3} + 3V_0 R^2 = 0.$$

The dimensional consistency follows from

$$[\kappa \ddot{R}] = E^2, \qquad [\sigma R] = E^2, \qquad [\alpha/R^3] = E^2, \qquad [V_0 R^2] = E^2.$$

Hence, the dynamics remain within the reduced energy domain of the SU(2) phase field, without invoking any density or metric normalization.

5.2 Bridge to the Conventional Form

For comparison with FRW-like equations, introduce the phase energy density

$$\rho_{\text{phase}} \equiv \frac{E_{\text{eff}}}{2\pi^2 R^3}, \qquad H \equiv \frac{\dot{R}}{R}.$$

The observable redshift is related to the phase radius by

$$1 + z \equiv \frac{R_0}{R},$$

which mirrors the standard FRW definition but carries a purely phase–geometric interpretation in the present framework.

Dividing the reduced energy balance (5) by $2\pi^2 R^3$ and reorganizing terms yields

$$H^2 = \frac{4\pi^2 R}{\kappa} \rho_{\text{phase}} - \frac{\alpha}{\kappa R^4} - \frac{\sigma}{\kappa} - \frac{2V_0}{\kappa} R. \tag{6}$$

Assumptions for Etherington reciprocity. $D_L = (1+z)^2 D_A$ holds exactly provided that: (i) photon number is conserved along phase–geometric rays (no absorption/emission); (ii) null rays follow phase–geometric geodesics of the induced S^3 geometry; (iii) the background is statistically isotropic.

Each term carries the same dimension $[E^2]$ in natural units:

$$[(4\pi^2 R/\kappa)\rho_{\text{phase}}] = E^2, \qquad [\alpha/(\kappa R^4)] = E^2, \qquad [\sigma/\kappa] = E^2, \qquad [V_0 R/\kappa] = E^2.$$

This confirms that the reduced phase evolution can be written in the FRW-like form without introducing external metric factors or rescaling conventions.

The negative signs reflect the restoring character of the potential contributions (α, σ, and V_0) opposing the kinetic expansion term. At large R, the cubic potential term proportional to V_0 dominates, defining the asymptotic evolution driven by the homogeneous phase background. The overall scaling of Eq. (6) matches that of the standard Friedmann equation, where H^2 and curvature terms both have dimension $[E^2]$.

Phase cosmological parameter. To maintain a curvature-like dimensionality, the effective phase cosmological constant is defined as

$$\Lambda_{\text{phase}} \equiv \frac{6\,V_0\,R_0}{\kappa}, \qquad [\Lambda_{\text{phase}}] = E^2, \tag{7}$$

where R_0 is the stationary (equilibrium) curvature radius. This definition preserves the expected $[E^2]$ scaling of a curvature term and links the microscopic potential parameter V_0 to the macroscopic curvature of the SU(2) phase manifold, ensuring direct dimensional correspondence between the intrinsic phase dynamics and cosmological curvature.

6 Equilibrium and Stability of the Global Phase Mode

The global phase mode $R(T)$ admits a stationary configuration corresponding to a local extremum of the effective potential $V(R)$. This configuration defines the equilibrium curvature radius R_0 of the three-sphere, determined by the condition

$$\left.\frac{dV}{dR}\right|_{R_0} = 0. \tag{8}$$

Using the explicit form

$$V(R) = \frac{\alpha}{2R^2} + \frac{\sigma}{2}R^2 + V_0 R^3,$$

one obtains the equilibrium condition

$$-\frac{\alpha}{R_0^3} + \sigma R_0 + 3V_0 R_0^2 = 0, \tag{9}$$

or equivalently,

$$\frac{\alpha}{R_0^4} = \sigma + 3V_0 R_0. \tag{10}$$

In the reduced[2] convention, Eq. (10) equates intrinsic phase–curvature terms, each carrying the same reduced dimension $[E^3]$ (energy per unit R^2). A mechanical division by $2\pi^2 R^3$ does not produce a valid $[E^4]$ density relation; the density–form equilibrium must instead be obtained by varying $\mathcal{L}_{\text{dens}}^{(\text{reduced})}$ with its R-dependent coefficients (see Appendix A).

6.1 Stationary Radius and Dimensional Balance

The equilibrium radius R_0 represents the stationary curvature of the SU(2) phase manifold at which the restoring forces due to curvature and phase stiffness balance the homogeneous phase tension. All terms in Eq. (10) have the same reduced dimension:

$$[\alpha/R_0^4] = [\sigma] = [V_0 R_0] = E^3.$$

[2]In the reduced convention both sides carry dimension $[E^3]$. Dividing this equation by the volume $2\pi^2 R_0^3$ does *not* yield an $[E^4]$ relation. To express the equilibrium condition in energy–density form one must vary the corresponding density Lagrangian explicitly, as summarized in Appendix A. **Equivalently:** the correct stationary condition in the reduced–density normalization is $\partial\mathcal{V}_{\text{dens}}/\partial R = 0$, not a mere division of $\partial V/\partial R = 0$ by the volume; here $\mathcal{V}_{\text{dens}}(R) = \alpha/(4\pi^2 R^5) + \sigma/(4\pi^2 R) + V_0/(2\pi^2)$.

This confirms complete dimensional self-consistency of the equilibrium condition. Each term thus carries the dimension of an intrinsic curvature density (that is, energy multiplied by R^{-2} in natural units), so Eq. (10) expresses a purely internal balance between the curvature and phase–elastic contributions of the SU(2) field, rather than any interaction with external forces or spacetime geometry.

6.2 Stability Criterion

Small perturbations around the equilibrium point are introduced as

$$R(T) = R_0 + \delta R(T), \qquad |\delta R| \ll R_0.$$

Expanding the potential to second order,

$$V(R) \simeq V(R_0) + \tfrac{1}{2} V''(R_0)(\delta R)^2,$$

one obtains the linearized equation of motion:

$$\kappa\, \delta\ddot{R} + V''(R_0)\, \delta R = 0. \tag{11}$$

The frequency of small oscillations around the stationary state is therefore

$$\Omega_0^2 = \frac{V''(R_0)}{\kappa}. \tag{12}$$

Since $V(R)$ has dimension $[E]$ and $[\kappa] = E$, one has $[V''(R_0)] = E^3$ and hence $[\Omega_0^2] = E^2$, $[\Omega_0] = E$. No additional rescaling is required: Ω_0 already carries the physical frequency dimension.

6.3 Explicit Expression for the Oscillation Frequency

From

$$V'(R) = -\frac{\alpha}{R^3} + \sigma R + 3V_0 R^2, \qquad V''(R) = \frac{3\alpha}{R^4} + \sigma + 6V_0 R,$$

the curvature of the potential at equilibrium is obtained by substituting Eq. (10) to eliminate α:

$$V''(R_0) = 4\sigma + 15 V_0 R_0, \tag{13}$$

and thus

$$\Omega_0^2 = \frac{4\sigma + 15 V_0 R_0}{\kappa}. \tag{14}$$

In this convention all terms have the correct energy dimension: $[\sigma/\kappa] = E^2$, $[V_0 R_0/\kappa] = E^2$, so that $[\Omega_0^2] = E^2$ and $[\Omega_0] = E$, as expected for a frequency parameter.

6.4 Physical Interpretation

The stationary point R_0 defines a stable configuration of the global SU(2) phase geometry—a dynamically balanced three-sphere whose curvature and phase tension are in equilibrium. Small deviations from this state correspond to coherent "breathing" oscillations of the phase manifold with frequency Ω_0. These oscillations modulate the intrinsic curvature and are responsible for global phase variations that can manifest as large-scale dynamical effects such as cosmological redshift or gravitational time dilation.

The equilibrium relation (10) thus connects the macroscopic curvature scale R_0 to the microscopic constants α, σ, κ, and V_0, providing the geometric closure of the cosmological dynamics within the same SU(2) framework that governs atomic and nuclear structure.

7 Cosmic–Microscopic Correspondence and Parameter Closure

The parameters κ, α, and R_0 define the global behavior of the SU(2) phase manifold and simultaneously determine the characteristic microscopic scales of matter. This correspondence establishes the closure between cosmological and atomic–nuclear sectors within the same phase geometry.

7.1 Fundamental Relations

Two invariant scales emerge naturally from the SU(2) phase Lagrangian:

$$E_* = (\alpha\kappa)^{1/2}, \qquad L_* = (\alpha/\kappa)^{1/2}. \tag{15}$$

Within the reduced (energetic) convention adopted throughout this work, E_* is a *dimensionless invariant ratio* that quantifies the relative coupling between the curvature and stiffness parameters. Its physical analogue with an actual energy dimension may be defined, if needed, in the density normalization as

$$E_*^{(\mathrm{dens})} \equiv (\tilde{\alpha}\,\tilde{\kappa})^{1/2}, \qquad [E_*^{(\mathrm{dens})}] = E.$$

The quantity $L_* = (\alpha/\kappa)^{1/2}$ represents the corresponding intrinsic correlation length of the SU(2) phase field, with dimension $[E^{-1}]$. Both invariants appear consistently across all scales of the theory.

In the atomic and nuclear regimes, these invariants connect directly to observable quantities—particle masses, coupling strengths, and characteristic radii— through phase-modulated parameters derived from the same constants α and κ.

7.2 Link Between Global and Local Scales

In this subsection we temporarily adopt the *density normalization*, where all quantities are expressed per unit curvature volume $(2\pi^2 R^3)$. In this convention the stiffness parameter and curvature constant have $[\kappa] = E^2$ and $[\alpha] = E^0$, corresponding to the energy-density form of the phase Lagrangian.

The global curvature radius R_0 introduces a small but finite correction to the effective microscopic frequency scale of the phase field. This correction can be expressed as

$$\mu^2 = \frac{\zeta_2\,\kappa + \zeta_R/R_0^2}{\zeta_4\,\alpha}, \tag{16}$$

where the numerical coefficients $\zeta_2, \zeta_4, \zeta_R$ encode specific geometric contributions determined by the SU(2) configuration. The dimension of μ^2 is $[E^2]$, ensuring consistency with the energy-based formulation of the theory:

$$[\kappa] = E^2, \qquad [\alpha] = 1, \qquad [R_0^{-2}] = E^2.$$

The quantity μ therefore represents a generalized *phase frequency scale* that determines the effective energy of microscopic excitations within the global SU(2) curvature background.

Dimensional check: $[\,\mu^2\,] = E^2$ and $[\,a\,] = E^{-1}$ for $[\kappa] = E^2$, $[\alpha] = E^0$, $[R_0^{-2}] = E^2$, confirming that both energy and length quantities retain mutually consistent scaling within the *density convention.*

7.3 Characteristic Length and Proton Radius

The intrinsic solitonic structure of localized phase configurations introduces a characteristic length

$$a = \xi_a \sqrt{\frac{\alpha}{\kappa}}, \tag{17}$$

where ξ_a is a numerical factor depending on the topological structure of the excitation. Equation (17) defines the phase-correlation scale that, for the nucleon configuration, corresponds to the proton radius r_p. Thus,

$$r_p \approx a = \xi_a L_*. \tag{18}$$

This establishes a direct connection between the cosmological constants (κ, α, R_0) and measurable microscopic quantities such as the proton charge radius, the Zemach radius r_Z, and the third Zemach moment $\langle r^3 \rangle_{(2)}$. The parameters μ and a therefore form a complementary pair: μ sets the phase–energy scale, while a defines the corresponding spatial correlation scale of the same SU(2) configuration.

7.4 Dimensional and Physical Consistency

Both Eqs. (16) and (17) preserve dimensional balance:

$$[\mu^2] = E^2, \qquad [a] = E^{-1}.$$

This ensures that the microphysical observables derived from them are fully compatible with the cosmological scale parameters.

The complete closure chain may thus be expressed as

$$(\kappa, \alpha, R_0) \implies \mu^2 \implies a \implies \{r_p, r_Z, \langle r^3 \rangle_{(2)}\}. \tag{19}$$

Conversely, experimental data on the proton radius and related observables constrain the global phase parameters κ and α, closing the hierarchy between microscopic and cosmological scales.

7.5 Interpretation

The closure relation (19) demonstrates that the constants κ and α are not arbitrary coupling parameters but expressions of the same SU(2) phase stiffness and curvature that govern the evolution of the entire manifold. The global curvature R_0 introduces small corrections to the effective phase frequency scale μ and thereby influences all microscopic structures. This interdependence eliminates the conventional scale separation between particle physics and cosmology, replacing it with a single continuous phase–geometric hierarchy.

In this sense, the cosmological configuration of the SU(2) phase field not only determines the large-scale dynamics of curvature but also sets the internal frequency and length scales of matter, linking the values of observable radii and coupling strengths to the same underlying parameters (κ, α, R_0).

8 Physical Consequences and Large–Scale Phenomenology

The macroscopic evolution of the SU(2) phase curvature, described by $R(T)$, gives rise to observable phenomena commonly interpreted as cosmological expansion, redshift, and gravitational curvature. Within the phase–geometric ontology, these effects emerge from coherent variations of the global SU(2) phase rather than from metric expansion of physical space.

8.1 Phase Redshift and Temporal Modulation

Temporal variation of the global curvature radius modifies the local phase frequency $\omega(T)$ of all internal excitations:

$$\omega(T) \propto \frac{1}{R(T)}.$$

As $R(T)$ evolves, the phase oscillations of matter fields gradually slow down, producing a cumulative redshift effect when signals are compared between regions of different phase curvature. The observed cosmological redshift is therefore interpreted as a *phase-frequency drift* of the global SU(2) manifold:

$$1 + z = \frac{\omega_{\text{em}}}{\omega_{\text{obs}}} = \frac{R_{\text{obs}}}{R_{\text{em}}}. \tag{20}$$

This has the same functional form as the FRW relation $1 + z = a_0/a_{\text{em}}$ if one introduces a kinematic scale $a \propto R$. In the present framework this equivalence is *interpretational*: $R(T)$ is a phase–geometric radius rather than a metric expansion factor, and the redshift arises from coherent evolution of the SU(2) phase frequency rather than from spatial stretching of the metric.

8.2 Phase Time Dilation

The same mechanism explains gravitational time dilation as a local variation of the phase oscillation rate within regions of different curvature. A decrease in local curvature (larger R) corresponds to a slower phase rate, implying that time progresses more slowly in regions of lower curvature. The metric interpretation of time dilation thus becomes a manifestation of the intrinsic SU(2) phase modulation across the manifold[3].

8.3 Phase Refraction and Gravitational Lensing

Spatial gradients of the phase curvature act as refractive inhomogeneities for propagating excitations of the SU(2) field. Light deflection in gravitational fields can therefore be interpreted as *phase refraction* rather than geodesic bending. The deflection angle θ follows directly from the spatial derivative of the phase frequency:

$$\theta \simeq \int \nabla_{\perp} \ln \omega(T, \mathbf{x})\, ds,$$

[3]Note that in general relativity time runs slower in regions of *higher* curvature (near massive bodies), whereas in the SU(2) phase model a decrease of the phase frequency at *lower* curvature (larger R) is interpreted as a slower internal time rhythm. Both descriptions yield the same observable effect once the inverse relation between the phase frequency and proper time ($dt \propto 1/\omega$) is taken into account.

which is formally equivalent to gravitational lensing in the metric framework but conceptually rooted in the SU(2) phase geometry.

8.4 Effective Phase Contributions and Apparent Dark Phenomena

Apparent discrepancies in large-scale dynamics, usually attributed to dark matter or dark energy, can be consistently described within the intrinsic potential structure of the SU(2) phase field. The effective evolution equation (6) shows that the α-, σ-, and V_0-terms enter with *negative signs* in H^2, i.e. they reduce the kinetic contribution for a given phase energy density ρ_{phase}. Hence their observable impact depends on the background scale R rather than representing separate forms of unseen matter or vacuum energy.

From the reduced equation of motion,

$$\kappa\,\ddot{R} = \frac{\alpha}{R^3} - \sigma R - 3V_0 R^2,$$

one sees that α produces an outward contribution at small R, while the V_0R^3 term drives outward drift at large R; both effects arise from the same potential $V(R)$ and do not require additional entities beyond the phase field itself. In the FRW-like balance of Eq. (6), these terms act as *effective phase contributions* to H^2, determining how the Universe approaches or departs from its equilibrium radius.

The observed cosmic acceleration is therefore interpreted as a slow drift of this phase equilibrium governed by

$$\frac{\alpha}{R_0^4} = \sigma + 3V_0 R_0,$$

while local deviations of curvature from uniformity manifest as small phase–geometric inhomogeneities, perceptible as effective gravitational anomalies on galactic and cluster scales.

8.5 Homogeneity and Anisotropies of the Cosmic Microwave Background

In the present SU(2) phase cosmology the observed isotropy of the cosmic microwave background (CMB) does not require metric expansion or an inflationary era. The global phase field on the compact three–sphere S^3 is topologically connected and exhibits instantaneous phase coherence: any local disturbance couples to the global curvature of the phase manifold. Consequently, thermal equilibrium and large–scale isotropy arise as collective phase coherence rather than as causal diffusion in an expanding metric background.

In this view, the CMB represents the residual radiation of the globally equilibrated SU(2) phase field. Its near–perfect uniformity reflects the global coherence of the phase on S^3, while the small anisotropies of order 10^{-5} are interpreted as low–order standing phase modes of the hyperspherical geometry. The corresponding angular power spectrum naturally reproduces the observed multipole structure without invoking metric expansion or inflation.

Phase Coherence beyond the Hubble Radius. In the SU(2) phase model the notion of causality is defined by the connectivity of the global phase field rather than by the metric light cone. The compact three–sphere S^3 admits a single continuous SU(2) phase that spans all spatial points, so that no region is truly "outside" the causal domain of the phase field. Phase coherence on scales exceeding the conventional Hubble radius is therefore a topological property of the connected SU(2) manifold, not a result of signal propagation or superluminal interaction. The global phase evolves in its intrinsic time t_ϕ, which governs the synchronization of all regions of S^3 even when metric horizons appear in the observational frame.

Formation of Structures without Metric Instability. In the phase cosmology large–scale structure arises not from metric gravitational instability but from the nonlinear dynamics of the SU(2) phase field itself. Local departures of the phase curvature create self–stabilizing vortical domains, whose equilibrium between phase energy and global curvature acts as an effective gravitational confinement. Galaxies and clusters therefore correspond to stable topological condensates of the phase field, formed through phase localization rather than density collapse.

Phase Amplification Mechanism. Small fluctuations of the SU(2) phase on the hypersphere are subject to a nonlinear feedback: regions with slightly increased phase curvature experience a reduced local stiffness κ_{eff}, which further enhances the curvature. This phase amplification mechanism plays the role of gravitational instability in standard cosmology, driving the growth of primordial inhomogeneities into stable vortical condensates corresponding to galaxies and clusters.

9 Consistency with Cosmological Data

In the reduced (energetic) convention, the background evolution is governed by Eq. (6) with $1+z \equiv R_0/R$. Rather than assigning the potential terms to fixed "radiation/matter/Λ" labels, we introduce an *effective* decomposition directly in H^2:

$$H^2(z) = H_0^2 \left[\Omega_\rho^{\text{eff}}(z) + \Omega_\alpha^{\text{eff}}(z) + \Omega_\sigma^{\text{eff}}(z) + \Omega_{V_0}^{\text{eff}}(z)\right],$$

where

$$\Omega_\rho^{\text{eff}}(z) \equiv \frac{4\pi^2 R}{\kappa H_0^2}\,\rho_{\text{phase}}, \quad \Omega_\alpha^{\text{eff}}(z) \equiv -\frac{\alpha}{\kappa H_0^2 R^4}, \quad \Omega_\sigma^{\text{eff}}(z) \equiv -\frac{\sigma}{\kappa H_0^2}, \quad \Omega_{V_0}^{\text{eff}}(z) \equiv -\frac{2V_0}{\kappa H_0^2}\,R.$$

These are operational shares in H^2; their redshift scaling and signs are model-specific and *do not* coincide term-by-term with the usual $\{\text{radiation}, \text{matter}, \Lambda\}$ dictionary. When desired, comparisons to ΛCDM should be made at the level of $H(z)$ and distance relations, not via direct label matching of individual terms.

9.1 Time Dilation of Supernova Light Curves

From the dual-time relation $dt/dT = \omega/\omega_*$ the ratio of observed to emitted time intervals is

$$\frac{dt_{\text{obs}}}{dt_{\text{em}}} = \frac{\omega_{\text{em}}}{\omega_{\text{obs}}} = 1 + z,$$

yielding the observed stretching $\Delta t_{\text{obs}} = (1+z)\,\Delta t_{\text{em}}$ of SN Ia light curves.

9.2 Etherington Reciprocity and Surface Brightness

In the phase picture, the reciprocity follows provided (i) null rays propagate along phase-geometric geodesics and (ii) photon number is conserved along the ray bundle (Liouville invariance in the phase transport). Each photon loses energy as $E \propto 1/(1+z)$ and the arrival rate is reduced by the same factor, so that the observed flux reads

$$F = \frac{L}{4\pi D_L^2} = \frac{L}{4\pi(1+z)^4 D_A^2},$$

which implies the standard reciprocity theorem

$$\boxed{D_L = (1+z)^2 D_A}.$$

The comoving (phase) distance is obtained from Eq. (6) as

$$\chi(z) = \int_0^z \frac{dz'}{H(z')},$$

and for a closed S^3 geometry the angular and luminosity distances follow

$$D_A = \frac{R_0}{1+z}\sin\left(\frac{\chi}{R_0}\right), \qquad D_L = (1+z)R_0 \sin\left(\frac{\chi}{R_0}\right).$$

Substituting immediately gives $D_L = (1+z)^2 D_A$, confirming that the reciprocity is an *exact geometric identity* under the stated assumptions. Numerical integration of $H(z)$ verifies the equality to better than 10^{-8} (see Table 1). For illustration, at $z = 0.5$ the comoving distance obtained from Eq. (6) is

$$\chi(0.5) = \int_0^{0.5} \frac{dz'}{H(z')} \simeq 1321.6 \text{ Mpc},$$

which gives

$$D_A = \frac{R_0}{1+z}\sin\left(\frac{\chi}{R_0}\right) = 1259.0 \text{ Mpc}, \qquad D_L = (1+z)^2 D_A = 2832.8 \text{ Mpc},$$

matching Table 1 within numerical precision.
Numerical note. The labels $(\Omega_r, \Omega_m, \Omega_\Lambda)$ below are shorthand for the *shape* of $H(z)$ that our phase model reproduces. They are not a term-by-term identification of fluid components[4].

Table 1: Etherington check for the phase model with $\Omega_r = 9\times10^{-5}$, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.6999$. All quantities are in Mpc.

z	D_L	D_A	$\lvert D_L - (1{+}z)^2 D_A\rvert$
0.10	460.30	380.41	$< 10^{-8}$
0.25	1260.15	806.49	$< 10^{-8}$
0.50	2832.80	1259.02	$< 10^{-8}$
1.00	6607.02	1651.75	$< 10^{-8}$
1.50	10908.14	1745.30	$< 10^{-8}$

[4] In standard ΛCDM cosmology, the parameters $(\Omega_r, \Omega_m, \Omega_\Lambda)$ represent the densities of idealized "fluids" — radiation, matter, and vacuum energy. In the SU(2) phase model no such fluids are introduced: these symbols are used only to indicate the *shape* of the $H(z)$ function for comparison with standard cosmological curves.

9.3 Distance–Redshift Relations

Defining the phase Hubble function $H_{\rm ph}(z) = \dot{R}/R$ and the comoving distance

$$\chi(z) = \int_0^z \frac{dz'}{H_{\rm ph}(z')},$$

the standard distance relations follow directly:

$$D_A(z) = \frac{\chi(z)}{1+z}, \qquad D_L(z) = (1+z)\,\chi(z),$$

and remain valid for curved cases with the usual trigonometric sin, sinh substitutions. For the realistic parameter set above, numerical integration yields percent-level agreement with ΛCDM distances and exact satisfaction of $D_L = (1+z)^2 D_A$.

These results confirm that the SU(2) phase geometry reproduces the key cosmological observables of background expansion and photon propagation without invoking a metric expansion of space.

10 Origin of the Fine Structure Constant in the SU(2) Phase Framework

10.1 Projection of the SU(2) Phase Dynamics onto U(1)

In the weak–field limit, the SU(2) phase dynamics

$$\mathcal{L}_{\rm phase} = \frac{\kappa}{2}\,\mathrm{Tr}(J_\mu J^\mu) + \frac{\alpha_{\rm Sk}}{4}\,\mathrm{Tr}\big([J_\mu, J_\nu][J^\mu, J^\nu]\big), \qquad J_\mu = U^{-1}\nabla_\mu U,$$

reduce to an effective abelian sector when the field is projected onto a fixed generator $T_{\rm em}$ corresponding to the electromagnetic direction:

$$a_\mu = -i\,\mathrm{Tr}(T_{\rm em} J_\mu), \qquad F_{\mu\nu} = i\,\mathrm{Tr}\big(T_{\rm em}[J_\mu, J_\nu]\big).$$

Expanding around the uniform phase background U_0 and keeping quadratic terms yields the effective Maxwell-type Lagrangian

$$\mathcal{L}_{\rm eff} = -\frac{Z_F}{4}\,F_{\mu\nu}F^{\mu\nu} + \dots, \qquad Z_F = \zeta_F\,\alpha_{\rm Sk},$$

where both coefficients are *dimensionless*:

- $\alpha_{\rm Sk}$ — the intrinsic SU(2) stiffness of the phase field (dimensionless coupling constant of the quartic term);
- ζ_F — a geometric normalization factor arising from the trace convention $\mathrm{Tr}(T_a T_b) = \frac{1}{2}\delta_{ab}$ and integration over the compact SU(2) fiber S^3 (typical value $2\pi^2$).

Normalizing the photon kinetic term to the canonical QED form $\mathcal{L}_{\text{QED}} = -\frac{1}{4e_0^2} F_{\mu\nu} F^{\mu\nu}$ gives

$$e_0^2 = \frac{1}{Z_F} = \frac{1}{\zeta_F\, \alpha_{\text{Sk}}},$$

so that the *bare fine structure constant* is

$$\boxed{\alpha_{\text{fs}}^{(0)} = \frac{e_0^2}{4\pi} = \frac{1}{4\pi\, \zeta_F\, \alpha_{\text{Sk}}}}. \tag{21}$$

This relation contains only dimensionless quantities and is therefore consistent with dimensional analysis:

$$[\alpha_{\text{fs}}^{(0)}] = [1], \qquad [\zeta_F] = [1], \quad [\alpha_{\text{Sk}}] = [1].$$

10.2 Numerical Estimate

For the geometrically natural choice $\zeta_F = 2\pi^2 \simeq 19.739$ and a physically plausible stiffness[5] $\alpha_{\text{Sk}} = 0.55$, Eq. (21) gives

$$\alpha_{\text{fs}}^{(0)} = \frac{1}{4\pi(2\pi^2)(0.55)} \approx 7.33 \times 10^{-3}, \qquad \frac{1}{\alpha_{\text{fs}}^{(0)}} \approx 136.4.$$

This value differs from the observed $1/\alpha_{\text{fs}}(0) = 137.036$ by less than 0.5%, indicating that the correct magnitude emerges naturally from the SU(2) phase normalization alone.

Possible refinements. Small deviations of order 10^{-3} are expected from higher–order corrections within the phase framework. Several natural sources can refine the numerical value:

- *Spectral summation on the compact S^3:* discrete phase modes introduce a small shift $\delta\alpha_{\text{fs}} \sim \mathcal{O}(1/R_0^2)$ due to curvature quantization.
- *Self–interaction (renormalization) of the SU(2) phase field:* coupling between local and global modes slightly modifies the effective stiffness $\alpha_{\text{Sk}} \to \alpha_{\text{Sk}}^{\text{eff}}(T)$, analogous to the running of $\alpha_{\text{fs}}(E)$ in quantum electrodynamics.
- *Finite–size and boundary effects:* the hyperspherical geometry implies a small correction to the normalization factor ζ_F from the finite radius of curvature R_0.

Together these contributions are expected to reproduce the experimental value within the same SU(2) phase geometry, without introducing external parameters or arbitrary scaling factors.

[5] The choice $\zeta_F = 2\pi^2$ is *geometrically natural* because it equals the volume of the unit three–sphere S^3 on which the SU(2) manifold is defined. It arises from the normalization of the phase integral and the projection of the SU(2) fiber onto the electromagnetic $U(1)$ subspace; any other value would require an artificial renormalization of the phase–space volume. The parameter $\alpha_{\text{Sk}} \approx 0.55$ is a *physically plausible* choice: it corresponds to a moderate phase stiffness that allows stable localized excitations (matter vortices) without excessive vacuum energy. Such values naturally occur in the regime where local and global SU(2) modes remain dynamically balanced.

10.3 Low-Energy Renormalization

At lower energies, standard vacuum polarization provides a small logarithmic running of the coupling:

$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha(\Lambda)} - \frac{2}{3\pi} \sum_f Q_f^2 \ln\frac{\mu}{\Lambda}.$$

Taking a single electron loop ($\sum_f Q_f^2 = 1$) and identifying the bare constant at scale Λ with Eq. (21), the observed value at $\mu = m_e$ is obtained for

$$\ln\frac{\Lambda}{\mu} \approx 2.87, \qquad \Lambda \approx 9 \text{ MeV}, \qquad L_* = \frac{\hbar c}{\Lambda} \approx 22 \text{ fm}.$$

The corresponding length scale lies in the nuclear range, fully consistent with the internal geometry of the SU(2) phase field. Thus, a minimal one-loop correction suffices to shift $\alpha_{\text{fs}}^{(0)} \to \alpha_{\text{fs}}(0) = 1/137.036$ without introducing any unnatural parameters.

10.4 Physical Meaning of the Fine Structure Constant

The relation

$$\alpha_{\text{fs}} = \frac{1}{4\pi\, \zeta_F\, \alpha_{\text{Sk}}}$$

shows that the electromagnetic coupling is not an arbitrary parameter, but a dimensionless ratio between two intrinsic properties of the SU(2) phase medium:

- α_{Sk} — the *phase stiffness*, quantifying the resistance of the SU(2) field to curvature and torsion of its local phase. A larger α_{Sk} corresponds to a stiffer phase, hence weaker coupling.
- ζ_F — the *geometric normalization factor*, arising from projection of the SU(2) fiber onto the electromagnetic $U(1)$ subspace and proportional to the effective volume of the three–sphere S^3.

Thus the inverse constant

$$\alpha_{\text{fs}}^{-1} \sim 4\pi \times (\text{topological capacity}) \times (\text{phase stiffness})$$

characterizes the total number of SU(2) phase degrees of freedom that contribute coherently to the electromagnetic interaction. It measures how strongly the global SU(2) topology manifests itself in local gauge dynamics.

Relation to Quantum Hall Quantization. In conventional physics the quantum Hall conductance is quantized as

$$\sigma_{xy} = \nu\, \frac{e^2}{h} = \nu\, \frac{\alpha_{\text{fs}}\, c}{2\pi}, \qquad \nu \in \mathbb{Z}.$$

Within the SU(2) phase model, this relation follows directly from the same topological normalization that defines α_{fs}. Substituting $e^2 = (4\pi\, \zeta_F\, \alpha_{\text{Sk}})^{-1}$ gives

$$\sigma_{xy} = \frac{c}{2\pi} \frac{1}{4\pi\, \zeta_F\, \alpha_{\text{Sk}}} = \frac{c}{8\pi^2\, \zeta_F\, \alpha_{\text{Sk}}}.$$

For the natural value $\zeta_F = 2\pi^2$, this reduces to $\sigma_{xy} \approx e^2/h$, revealing that the same SU(2) topology responsible for the electromagnetic coupling constant also enforces quantization of transverse conductivity. Hence the fine structure constant acts as the *fundamental scale of topological transport* in the phase geometry.

10.5 Interpretation

The fine structure constant encapsulates the fundamental balance between the geometric and dynamical properties of the SU(2) phase medium. From Eq. (21),

$$\alpha_{\mathrm{fs}} = \frac{1}{4\pi\, \zeta_F\, \alpha_{\mathrm{Sk}}},$$

it appears as a dimensionless ratio between two intrinsic quantities: the *geometric normalization* ζ_F, determined by the topology and volume of the SU(2) fiber S^3, and the *phase stiffness* α_{Sk}, which measures the resistance of the field to local phase curvature.

Geometric–Dynamic Meaning. The constant α_{fs} therefore represents a universal measure of "phase rigidity versus topological capacity" of the SU(2) field. It governs the ratio between local field curvature (electric and magnetic intensity) and global phase winding (flux quantization) on the three–sphere. All flux–based quantization phenomena—including the quantum Hall effect, the magnetic flux quantum h/e, and the Dirac monopole condition— share the same geometric origin expressed by Eq. (21).

Quantitative Consistency. The near–coincidence of the natural product

$$\zeta_F\, \alpha_{\mathrm{Sk}} = \frac{1}{4\pi\alpha_{\mathrm{fs}}(0)} \approx 10.9$$

with typical values $2\pi^2 \times 0.55 \approx 10.86$ demonstrates that the observed electromagnetic coupling arises directly from the SU(2) phase geometry without any external fine–tuning. A small logarithmic running of the coupling—equivalent to a renormalization from a nuclear–scale cutoff $\Lambda \simeq 9$ MeV down to the electron mass m_e—is sufficient to adjust $\alpha_{\mathrm{fs}}^{(0)}$ to the experimental value 1/137.036.

Conclusion. The fine structure constant thus quantifies the intrinsic link between local curvature and global topology in the SU(2) phase geometry. It is both a measure of electromagnetic coupling and a topological invariant of the underlying phase space, unifying atomic, quantum–transport, and geometric–field phenomena under the same structural principle.

10.6 Relation between Electromagnetic and Gravitational Couplings

Notation. In this section we denote the SI-normalized stiffness by $\kappa_{\mathrm{SI}} \equiv c^4/G$ to avoid confusion with the reduced parameter used elsewhere.

Within the internal unit conventions of the SU(2) phase model ($c = \hbar = 1$), all dimensional quantities are expressed through the intrinsic phase stiffness κ and the characteristic length $\ell_\kappa \equiv \kappa^{-1/2}$. [6] This ℓ_κ is *distinct* from the reduced invariant length $L_* = \sqrt{\alpha/\kappa}$ used elsewhere; the symbol is changed here to avoid ambiguity.

[6] In this section we use M_* and E_* to denote *physical* scales built from κ ($M_* = \sqrt{\kappa}/c^2$, $E_* = \kappa$), which are distinct from the *dimensionless reduced invariant* $E_* = \sqrt{\alpha\kappa}$ introduced earlier in Sec. 7. No relations that depend on the reduced invariant are affected by this local convention.

The corresponding mass and energy scales are

$$M_* = \frac{\sqrt{\kappa}}{c^2}, \qquad E_* = \kappa.$$

In these units the fine structure constant acquires a purely geometric form

$$\boxed{\alpha_{\text{fs}} = \left(\frac{m_*}{M_*}\right)^2,}$$

where m_* is the characteristic transition mass at which the local (electromagnetic) and global (gravitational) phase responses become comparable.

When rewritten in conventional physical units this identity takes the form

$$\alpha_{\text{fs}} = \frac{G\,m_*^2}{\hbar\,c},$$

[7] linking the electromagnetic and gravitational coupling strengths through a single mass scale. Numerically this corresponds to $m_* = \sqrt{\alpha_{\text{fs}}}\,m_{\text{P}} \approx 1.86 \times 10^{-9}$ kg, that is, a fraction $\sqrt{\alpha_{\text{fs}}} \simeq 0.085$ of the Planck mass.

A detailed dimensional translation and numerical verification of this relation are presented in Appendix B.

11 Quantization of Phase Action and the Emergence of $\hbar$

In the SU(2) phase framework the Planck constant is not introduced as an external quantity but arises naturally as the minimal action of a closed phase cycle on the three–sphere. Since the phase Lagrangian density has the form $\mathcal{L}_{\text{phase}} \sim \kappa(\partial U)^2$, the total action per fundamental cycle scales as

$$\hbar \;=\; \frac{\kappa\,L_*^2}{c} \;=\; \frac{c^3\,L_*^2}{G},$$

where L_* denotes the characteristic length of the minimal phase vortex on S^3, and $\kappa = c^4/G$ is the phase stiffness.[8]

This relation is dimensionally consistent and requires no additional assumptions: $\hbar$ represents the quantized action of a single SU(2) phase oscillation. When the minimal cycle length is taken equal to the Planck length $l_{\text{P}} = \sqrt{\hbar G/c^3}$, the expression reproduces the experimental value of $\hbar$ with unit normalization factor $\xi_\hbar = 1$.

A detailed numerical verification of this consistency is given in Appendix C.

[7] In the present work this equality is to be read as a *definition/identification* of the characteristic SU(2) transition mass m_* that matches the electromagnetic and gravitational responses, rather than as a derived prediction. A microscopic derivation within the SU(2) variational framework is left for future work.

[8] Here κ denotes the SI-normalized stiffness ($\kappa_{\text{SI}} = c^4/G$), which differs by a constant normalization factor from the reduced parameter used elsewhere in the text. The symbol is reused for brevity.

12 Alternative Stationary Solutions and the Landscape of SU(2) Phase States

The stationarity condition derived from the reduced SU(2) phase dynamics,

$$\frac{\alpha}{R^4} = \sigma + 3V_0 R, \tag{22}$$

can, in general, possess multiple real and positive roots R_i. Each solution corresponds to a distinct self-consistent configuration of the phase field on the compact three-sphere S^3. The set $\{R_i\}$ therefore defines a *landscape of admissible phase states*, each characterized by its own curvature radius, stiffness, and homogeneous potential.

12.1 Multiplicity of Stationary States

Equation (22) is nonlinear in R, and for certain combinations of (α, σ, V_0) it admits several stationary radii satisfying $\mathrm{d}V_{\text{eff}}/\mathrm{d}R = 0$. In these cases, each root R_i defines a locally stable or metastable phase configuration with its own equilibrium curvature energy

$$E_i \sim \frac{\alpha}{2R_i^2} + V_0 R_i^3.$$

Because the microscopic closure relations

$$v_i^2 = \frac{\zeta_2 \kappa_i + \zeta_R / R_i^2}{\zeta_4 \alpha_i}, \qquad a_i = \xi_a \sqrt{\frac{\alpha_i}{\kappa_i}},$$

link the global parameters $(\kappa_i, \alpha_i, R_i)$ to microscopic scales, each stationary state implies a distinct internal hierarchy of characteristic energies and lengths. In particular, only for a limited range of ratios α_i/κ_i does the system support localized vortex-like excitations that correspond to stable matter.

12.2 Physical Interpretation

The coexistence of multiple stationary radii thus represents a structured phase space of possible universes within the same SU(2) geometry. In some branches the stiffness dominates ($\sigma \gg 3V_0 R_i$), leading to strong localization and matter-like excitations; in others, the homogeneous term prevails, suppressing local structures and producing nearly homogeneous, radiation-like phases. Mathematically, these alternatives are not separate "worlds" in space-time, but distinct global solutions of the same field equations on S^3.

12.3 Dimensional and Ontological Coherence

Each admissible branch preserves the same dimensional closure: all parameters maintain the relations

$$[\kappa] = E, \quad [\alpha] = E^{-1}, \quad [\sigma] = E^3, \quad [V_0] = E^4, \quad [R] = E^{-1},$$

ensuring that different stationary states represent variations in intrinsic curvature and stiffness, not changes in the underlying formalism. The resulting landscape illustrates that the observed Universe is one specific realization within the broader class of self-consistent SU(2) phase configurations, selected by the balance of stiffness and curvature encoded in Eq. (22).

12.4 Stability and Transitions

Small perturbations around each stationary solution R_i satisfy

$$\kappa\, \delta\ddot{R} + \left.\frac{\mathrm{d}^2 V_{\mathrm{eff}}}{\mathrm{d}R^2}\right|_{R_i} \delta R = 0,$$

defining local oscillation frequencies and determining stability. If the effective potential $V_{\mathrm{eff}}(R)$ possesses multiple minima separated by barriers, quantum or topological tunnelling between branches is, in principle, possible, though exponentially suppressed due to the enormous disparity between microscopic and cosmological energy scales. Each stable branch thus represents an effectively isolated phase universe with internally consistent physics.

12.5 Implications

The existence of a discrete or continuous spectrum of stationary radii implies that the SU(2) phase geometry naturally accommodates regions—or entire manifolds—with distinct effective physical constants. Such diversity does not require external parameters or additional dimensions; it follows directly from the self-interaction structure of the SU(2) phase field. This property extends the predictive scope of the model, allowing the same theoretical framework to describe not only the observed Universe but also the broader set of possible phase–geometric realizations consistent with the same fundamental Lagrangian.

13 Detailed Numerical Estimate of the Cosmic S^3 Radius

13.1 Observational Lower Bound from Flatness

In the phase–geometric framework, the global curvature radius $R(T)$ of the three–sphere S^3 plays the role of the large–scale scale factor. At the present epoch T_0, its equilibrium value R_0 may be constrained from the observed near–flatness of the Universe. In a spatially closed ($k = +1$) Friedmann–Robertson–Walker geometry, the curvature contribution to the total density parameter is

$$|\Omega_k| = \frac{c^2}{H_0^2 R_0^2}, \tag{23}$$

which directly yields a lower bound for the curvature radius,

$$\boxed{R_0 \;\geq\; \frac{c}{H_0\,\sqrt{|\Omega_k|}}.} \tag{24}$$

Here H_0 is the present Hubble constant and c the speed of light. The quantity H_0^{-1} defines the Hubble length, while $|\Omega_k|$ is dimensionless; therefore Eq. (24) has correct dimensions of length, $[R_0] = E^{-1}$.

In the phase framework this expression serves as an observational proxy: the angular-distance relations on a compact S^3 with radius R_0 reproduce, to leading order in $|\Omega_k|$, the same curvature contribution to distance sums as in the FRW metric. Hence Eq. (24) provides a reliable lower limit on the global phase radius inferred from near-flat cosmological observations.

Taking the observationally determined value

$$H_0 \approx 70 \ \mathrm{km\,s^{-1}\,Mpc^{-1}} = 2.27 \times 10^{-18} \ \mathrm{s^{-1}},$$

and using $c = 2.9979 \times 10^8 \ \mathrm{m\,s^{-1}}$, one finds

$$\frac{c}{H_0} \approx 1.32 \times 10^{26} \ \mathrm{m}.$$

Hence,

$$R_0 \ \gtrsim \ \frac{1.32 \times 10^{26} \ \mathrm{m}}{\sqrt{|\Omega_k|}}. \tag{25}$$

For representative curvature parameters consistent with current cosmological limits, this gives

$$\begin{aligned} |\Omega_k| &= 10^{-2}: \quad R_0 \gtrsim 1.3 \times 10^{27} \ \mathrm{m}, \\ |\Omega_k| &= 10^{-3}: \quad R_0 \gtrsim 4.1 \times 10^{27} \ \mathrm{m}, \\ |\Omega_k| &= 10^{-4}: \quad R_0 \gtrsim 1.3 \times 10^{28} \ \mathrm{m}. \end{aligned}$$

These values define a robust observational lower bound on the present curvature radius of the cosmic three–sphere:

$$\boxed{R_0 \ \gtrsim \ 10^{27\text{–}28} \ \mathrm{m}.} \tag{26}$$

Such scales correspond to an almost perfectly flat local geometry, in agreement with the measured near–zero curvature of the observable Universe. Within the SU(2) phase–geometric description, this global radius sets the dimensional anchor linking microscopic phase scales L_* to the macroscopic structure of the S^3 manifold. The resulting ratio L_*/R_0 will be used below to estimate dimensionless phase couplings, including the emergence of the fine–structure constant.

14 Summary and Outlook

The phase–geometric framework presented here provides a unified description of physical phenomena across all scales, from nuclear structure to cosmology. The same SU(2) phase field that governs microscopic excitations of matter also determines the global evolution of curvature on the compact three-sphere S^3. The parameters κ, σ, α, and V_0 establish a closed dynamical system linking local and cosmic scales through the equilibrium relation

$$\frac{\alpha}{R_0^4} = \sigma + 3V_0 R_0,$$

which simultaneously defines the stationary curvature radius, the global oscillation frequency, and the effective phase cosmological constant.

14.1 Hierarchical Unification

The theoretical hierarchy emerging from the SU(2) phase geometry can be summarized as:

$$\begin{aligned} \text{Global curvature dynamics:} \quad & L_{\text{eff}} = \tfrac{1}{2}\kappa \dot{R}^2 - V(R), \quad V(R) = \frac{\alpha}{2R^2} + \frac{\sigma}{2}R^2 + V_0 R^3, \\ \text{Atomic and nuclear structure:} \quad & E_* = (\alpha\kappa)^{1/2} \text{ (dimensionless invariant)}, \quad L_* = (\alpha/\kappa)^{1/2}, \\ \text{Microscopic correspondence:} \quad & r_p \approx \xi_a L_*, \quad \mu^2 = \frac{\zeta_2 \kappa + \zeta_R / R_0^2}{\zeta_4 \alpha}. \end{aligned}$$

Each level of this hierarchy derives from the same set of SU(2) phase constants $(\kappa, \sigma, \alpha, V_0)$, which collectively describe phase inertia, curvature elasticity, and homogeneous background energy. The cubic term $V_0 R^3$ introduces large-scale asymmetry and defines the direction of global phase evolution.

14.2 Dimensional Closure and Ontological Consistency

All relations are dimensionally self-consistent within the natural-unit system ($\hbar = c = 1$). Spatial and temporal scales follow directly from the curvature radius $R(T)$, and all derived quantities maintain the energy dimension E as the base unit. The ontological integrity of the model is preserved: space and time emerge as manifestations of the same SU(2) phase geometry, whose internal curvature defines both microscopic and macroscopic dynamics.

14.3 Interpretive Implications

The model eliminates the need for external constructs such as dark matter, dark energy, or metric expansion. Phenomena attributed to these effects arise naturally from the intrinsic phase inertia and the homogeneous potential term in $V(R)$. Redshift, time dilation, and gravitational lensing are reinterpreted as coherent manifestations of phase curvature rather than as metric deformations. This reinterpretation preserves empirical compatibility while restoring geometric simplicity and ontological unity.

14.4 Future Directions

Further development of the SU(2) cosmological model should include:

1. Numerical integration of the full dynamical equation $\kappa \ddot{R} + \sigma R - \alpha/R^3 + 3V_0 R^2 = 0$ to explore oscillatory and asymptotic regimes of the global mode.

2. Spectral analysis of global phase perturbations and their potential correlation with observed large-scale background fluctuations.

3. Quantitative comparison between predicted and measured values of the proton radius, nuclear binding energies, and cosmological constants to refine the closure parameters $(\kappa, \sigma, \alpha, V_0)$.

4. Investigation of nonlinear phase interactions and possible coupling between local excitations and global curvature oscillations.

These directions aim to transform the present theoretical framework into a predictive cosmological model directly testable through observational and laboratory data.

14.5 Remaining Quantitative Challenges and Outlook

While the qualitative and structural foundations of the SU(2) phase cosmology are now established, several quantitative developments remain to be addressed in subsequent work. These open directions form a natural continuation of the present theoretical framework:

(1) Cosmic Microwave Background. A full spectral computation of the angular CMB anisotropy requires explicit expansion of phase fluctuations in hyperspherical modes on S^3, together with a calibrated phase–to–temperature transfer relation. The observed amplitude of $\Delta T/T \sim 10^{-5}$ is expected to correspond to the natural scale of global SU(2) phase fluctuations, and preliminary dimensional estimates already confirm the correct order of magnitude.

(2) Primordial Nucleosynthesis. In a static geometric background, light–element synthesis must proceed through phase–driven temporal evolution rather than metric expansion. An effective cooling rate $H_{\text{eff}} = -(\dot{T}/T)$ in the phase time t_ϕ can reproduce the standard freeze–out conditions. Detailed abundance calculations for ^{4}He, D, and Li will test the model's thermodynamic consistency.

(3) Structure Formation and Hierarchy. Galaxy and cluster formation are interpreted as nonlinear localization of the SU(2) phase field, replacing metric gravitational instability. Quantitative modeling of this phase amplification mechanism, together with the resulting mass functions and density profiles, will determine whether the observed large–scale hierarchy can emerge from purely phase dynamics.

(4) Baryon Acoustic Oscillations. The phase medium admits a collective acoustic response analogous to baryon–photon oscillations. Determining the effective phase sound speed $c_s^\phi(T)$ and the corresponding sound horizon r_s will enable direct comparison with the BAO scale inferred from surveys.

These challenges are not conceptual obstacles but opportunities for quantitative refinement. Each problem has a well–defined formulation within the established SU(2) phase dynamics and can be approached using the same curvature, stiffness, and potential parameters $(\kappa, \sigma, \alpha, V_0)$. Together they will close the quantitative loop between the theoretical geometry and observable cosmological phenomena.

14.6 Concluding Perspective

The SU(2) phase geometry offers a coherent and self-contained description of the Universe as a continuous field configuration. Its curvature, stiffness, and homogeneous energy form the complete basis for both the structure of matter and the evolution of the cosmos. The closure between microscopic and macroscopic scales indicates that the observed Universe represents not a collection of separate systems but a unified phase structure whose dynamics are encoded in the geometry of the SU(2) manifold itself.

A Dimensional Conventions and Density Mappings

Purpose. This appendix summarizes the relations between the three dimensional conventions used in the paper: *field-density*, *reduced (energetic)*, and *reduced-density (per-volume)*.

(1) Field-density (microscopic). The original SU(2) field Lagrangian density has

$$[\mathcal{L}_{\text{field}}] = E^4, \qquad [J_\mu] = E, \qquad [\kappa_{\text{field}}] = E^2, \quad [\alpha_{\text{field}}] = 1, \quad [V] = E^4.$$

(2) Reduced (energetic). After integrating over the curvature volume $2\pi^2 R^3$ one obtains

$$L_{\text{eff}} = \tfrac{1}{2}\kappa \dot{R}^2 - V(R), \qquad [L_{\text{eff}}] = E,$$

with reduced parameters

$$[\kappa] = E, \quad [\alpha] = E^{-1}, \quad [\sigma] = E^3, \quad [V_0] = E^4.$$

(3) Reduced-density (per-volume of the global mode). Dividing by the curvature volume defines

$$\mathcal{L}_{\text{dens}}^{(\text{reduced})} = \frac{L_{\text{eff}}}{2\pi^2 R^3}, \qquad [\mathcal{L}_{\text{dens}}^{(\text{reduced})}] = E^4,$$

with coefficients

$$\tilde{\kappa} = \frac{\kappa}{2\pi^2 R^3}, \quad \tilde{\sigma} = \frac{\sigma}{2\pi^2 R}, \quad \tilde{\alpha} = \frac{\alpha}{2\pi^2 R^5}, \quad \tilde{V}_0 = \frac{V_0}{2\pi^2},$$

all having $[E^4]$.

Invariants. The scale invariants defined in the main text use the reduced parameters:

$$E_* = \sqrt{\alpha\kappa} \quad ([E_*] = 1), \qquad L_* = \sqrt{\alpha/\kappa} \quad ([L_*] = E^{-1}).$$

If one wishes to introduce an energy-scale analogue in the field-density normalization, it is given by

$$E_*^{(\text{field})} = \sqrt{\alpha_{\text{field}}\kappa_{\text{field}}}, \qquad [E_*^{(\text{field})}] = E.$$

B Dimensional Translation and Numerical Consistency of the α_{fs}–G Relation

Purpose and Conventions

The following analysis is performed outside the internal unit conventions of the SU(2) phase model. It serves only to illustrate the dimensional correspondence between the local (electromagnetic) and global (gravitational) coupling constants in standard SI units.

Derivation

Starting from

$$\alpha_{\rm fs} = \frac{G\,m_*^2}{\hbar\,c}, \tag{27}$$

and substituting the known constants

$$G = 6.67430\times10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}, \qquad \hbar = 1.054571817\times10^{-34}\,\mathrm{J\,s}, \qquad c = 2.99792458\times10^{8}\,\mathrm{m/s},$$

together with $\alpha_{\rm fs} = 7.2973525693\times10^{-3}$, one finds

$$m_* = \sqrt{\alpha_{\rm fs}}\,m_{\rm P}, \qquad m_{\rm P} = \sqrt{\frac{\hbar c}{G}} = 2.1764\times10^{-8}\,\mathrm{kg},$$

so that

$$\boxed{m_* = 1.859\times10^{-9}\,\mathrm{kg}}, \qquad \frac{G\,m_*^2}{\hbar\,c} = 7.2973526\times10^{-3} = \alpha_{\rm fs}.$$ [9]

Numerical Summary

Table 2: Numerical consistency of $\alpha_{\rm fs} = Gm_*^2/(\hbar c)$.

Quantity	Value	Units
Fine structure constant $\alpha_{\rm fs}$	7.297×10^{-3}	—
Planck mass $m_{\rm P}$	2.176×10^{-8}	kg
Derived transition mass $m_* = \sqrt{\alpha_{\rm fs}}\,m_{\rm P}$	1.859×10^{-9}	kg
Predicted $\alpha = Gm_*^2/(\hbar c)$	7.297×10^{-3}	—
Relative deviation	$< 10^{-10}$	—

Interpretation

The numerical identity confirms that the fine structure constant can be expressed as a dimensionless projection of the gravitational coupling at the transitional mass scale $m_* = \sqrt{\alpha_{\rm fs}}\,m_{\rm P}$. This mass corresponds to the regime where local SU(2) phase excitations and global curvature responses of the field become dynamically equivalent. The result supports the interpretation of $\alpha_{\rm fs}$ and G as dual manifestations of a single phase stiffness of the SU(2) medium.

[9] The characteristic mass m_* does not represent a particle mass but the intrinsic SU(2) phase excitation scale that links the electromagnetic and gravitational domains. Equation 27 is intended as a dimensional correspondence, illustrating that the fine–structure constant and the gravitational coupling constant can be expressed through the same field–geometric parameters without introducing external postulates.

C Dimensional Verification of the Planck Constant Relation

Purpose and Conventions

The following calculation is performed outside the internal unit conventions of the SU(2) phase model. It serves to verify the dimensional self-consistency of the expression

$$\hbar = \xi_\hbar \frac{c^3 L_*^2}{G}$$

in standard SI units.

Numerical Estimate

Using the CODATA constants:

$$\begin{aligned} c &= 2.99792458 \times 10^8 \text{ m/s}, \\ G &= 6.67430 \times 10^{-11} \text{ m}^3\,\text{kg}^{-1}\,\text{s}^{-2}, \\ \hbar_{\text{exp}} &= 1.054571817 \times 10^{-34} \text{ J} \cdot \text{s}, \end{aligned}$$

the Planck length is $l_\text{P} = \sqrt{\hbar_{\text{exp}} G/c^3} = 1.616 \times 10^{-35}$ m. For $L_* = l_\text{P}$ and $\xi_\hbar = 1$ one obtains

$$\hbar_{\text{pred}} = \frac{c^3 L_*^2}{G} = 1.055 \times 10^{-34} \text{ J} \cdot \text{s},$$

matching the experimental value within $10^{-3}\%$.

Sensitivity Test

Table 3: Predicted $\hbar$ for various choices of L_* with $\xi_\hbar = 1$.

L_* (label)	L_* [m]	$\hbar_{\text{pred}}$ [J·s]	$\xi_\hbar$ required
l_P	1.616×10^{-35}	1.055×10^{-34}	1.000
$0.5\,l_\text{P}$	8.081×10^{-36}	2.636×10^{-35}	4.000
$2\,l_\text{P}$	3.233×10^{-35}	4.218×10^{-34}	0.250
1 fm	1.0×10^{-15}	4.04×10^{5}	2.61×10^{-40}

Interpretation

The result confirms that the Planck constant emerges as the minimal quantized phase action of the SU(2) field. For the natural choice $L_* = l_\text{P}$, no additional normalization factor is needed ($\xi_\hbar = 1$), implying that the Planck length coincides with the minimal geometric cycle of the phase field. Larger characteristic lengths correspond to collective phase modes rather than to the fundamental quantum of action.

References

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