Unified Phase Model of Atomic and Nuclear Structures ($SU(2)$ on S^3) Part I — Theory

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Abstract

Here presented a unified geometric framework for atomic and nuclear physics based on an $SU(2)$ phase field defined on the compact three–sphere S^3. In this model, nucleons arise as topological solitons of class $\pi_3(S^3)$, while electrons correspond to minimal $SU(2)$ vortices with spin as a topological index. The local Coulomb potential $V(r) \sim 1/r$ emerges from the Green function on S^3, with global corrections suppressed as $(a_0/R)^2$. A single soliton length scale a consistently accounts for proton form factors, charge radius, Lamb shift contributions, and the Zemach radius without parameter tuning. Nuclear shell structure follows from hyperspherical harmonics and a strong spin–orbit coupling induced by the phase curvature, reproducing the magic numbers $2, 8, 20, 28, 50, 82, 126$. The macroscopic binding energy emerges in the Weizsäcker form from the solitonic energy functional, and the valley of stability is recovered. The framework provides quantitative, falsifiable predictions linking atomic spectroscopy, nucleon structure, and nuclear stability in a dimensionally consistent manner, withoIn this section we compile a "dictionary of correspondences" between the standard picture and the geometric model on S^3 with an $SU(2)$ phase (in $Cl(4,0)$ notation). ut invoking extra dimensions or ad hoc assumptions.

Contents

1 Introduction and Brief Summary

Idea. The atom (nucleus + electrons) is described as a *unified* phase configuration on the compact manifold $S^3 \subset \mathbb{R}^4$ with a field $\Phi(x) \in SU(2)$. The global geometry yields the correct local electrostatics (Coulomb's law), a discrete spectrum and shell structure, and naturally incorporates spin, the Pauli principle, and the mechanism of nuclear stability.

Key implications.

- *Effective Coulomb potential.* The Green's function on S^3 gives the potential $V(\chi) = \frac{Z\alpha}{R}\cot\chi$, which under stereographic projection $r = 2R\tan(\chi/2)$ reduces locally to

$$V(r) \simeq \frac{Z\alpha}{r} - \frac{Z\alpha}{4}\frac{r}{R^2} + \dots.$$

- *Quantum dynamics.* The Schrödinger equation on S^3 with $V(r)$ reproduces hydrogen-like energy levels; the Balmer series and the Rydberg constant emerge geometrically.

- *Nucleons as solitons.* The proton and neutron are localized solitons of the class $\pi_3(S^3) = \mathbb{Z}$ ("hedgehog" ansatz). The proton charge corresponds to a projection onto the electromagnetic phase, while the neutron corresponds to an alternative orientation with zero net charge projection.

- *Finite proton size.* A single soliton scale a determines $\langle r_p^2\rangle = 12a^2$, the slope $G'_E(0) = -\langle r_p^2\rangle/6$, the contribution ΔE_{fs} in μp, and the Zemach radius r_Z.

- *Nucleus and stability.* The mean field on $S^3 \Rightarrow$ an oscillator-type shell structure; strong spin–orbit interaction (phase curvature) yields the observed magic numbers. Neutrons stabilize multi-proton configurations by reducing phase stress.

Mathematical framework. For compactness, this work employs geometric (Clifford) algebra $Cl(4,0)$: rotors (Spin(4)), bivectors (field F), and its correspondence with $SU(2)$. All dimensional conventions are fixed explicitly in Sec. 2.

2 Notation, Units, and Dimensional Analysis

2.1 Units and Conversions

Throughout all derivations natural units $\hbar = c = 1$ (Heaviside–Lorentz) are employed, where

$$[\text{length}] = [\text{time}] = [\text{energy}]^{-1}.$$

For numerical estimates conversion to nuclear/SI units is used:

$$\hbar c = 197.3269804 \text{ MeV}\cdot\text{fm}, \qquad 1 \text{ fm}^{-1} = 197.3269804 \text{ MeV}.$$

The elementary charge in these units is $\alpha \equiv e^2/(4\pi) \approx 1/137.035999$ (dimensionless).

2.2 Geometry and Variables

- The 3-sphere S^3 of radius R is embedded in $\mathbb{R}^4$. The geodesic angle $\chi \in [0, \pi]$ is related to the stereographic radius r:

$$r = 2R \tan\frac{\chi}{2}, \qquad \chi = 2\arctan\frac{r}{2R}.$$

- Metric on S^3: $ds^2 = R^2\,(d\chi^2 + \sin^2\chi\, d\Omega_2^2)$. The Laplace–Beltrami operator Δ_{S^3} has eigenvalues $\ell(\ell+2)/R^2$.
- The potential $V(\chi)$ has the dimension of energy; in the limit $r \ll R$, $V(r) \simeq Z\alpha/r$.

2.3 Fields and Normalizations

- Phase field: $\Phi(x) \in SU(2)$, $\Phi^\dagger\Phi = \mathbf{1}$. In natural units Φ is dimensionless.
- Trace convention: $\mathrm{Tr}(\sigma_a\sigma_b) = 2\delta_{ab}$.
- Gauge (electromagnetism): the 4-potential A_μ has the dimension of energy, while $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ has the dimension [energy]^2.
- Noether current J^μ (charge current) has the dimension of density: $[J^0] = [\text{length}]^{-3}$ (charge is dimensionless).

2.4 Energy Functional and Dimensionality of Coefficients

A *sigma-model* with a Skyrme stabilizer is employed:

$$\mathcal{L} = \frac{\kappa}{2}\,\mathrm{Tr}(\partial_\mu\Phi^\dagger\partial^\mu\Phi) + \lambda\,\mathrm{Tr}\Big([\Phi^\dagger\partial_\mu\Phi,\ \Phi^\dagger\partial_\nu\Phi]^2\Big),$$

where ∂ carries dimension [energy]. The requirement $[S] = \big[\int d^4x\,\mathcal{L}\big]$ dimensionless $\Rightarrow [\mathcal{L}] = [\text{energy}]^4$. Hence:

$$[\kappa] = [\text{energy}]^2, \qquad [\lambda] = [\text{energy}]^0 \text{ (dimensionless)}.$$

The static energy $E = \int d^3x\,\mathcal{H}$ has dimension [energy], as expected.

2.5 Electrostatics on S^3: Green's Function and Dimensions

The scalar potential V satisfies (with normalization on a compact manifold):

$$\Delta_{S^3} V(\chi) = -Z\alpha\,\delta_{S^3}(\chi) + \frac{Z\alpha}{\mathrm{Vol}(S^3)}, \quad \mathrm{Vol}(S^3) = 2\pi^2R^3.$$

Solution (up to an additive constant):

$$V(\chi) = \frac{Z\alpha}{R}\cot\chi + \text{const}, \qquad [V] = [\text{energy}],$$

since $1/R$ has the dimension of energy. Under stereographic projection for $r \ll R$:

$$\cot\chi = \frac{R}{r} - \frac{r}{4R} + \mathcal{O}\left(\frac{r^3}{R^3}\right), \quad \Rightarrow \quad V(r) = \frac{Z\alpha}{r} - \frac{Z\alpha}{4}\frac{r}{R^2} + \dots$$

Each term has the dimension [energy] (since $1/r \sim [\text{energy}]$, $r/R^2 \sim [\text{energy}]$).

2.6 Quantum Quantities and Their Dimensions

- Reduced mass: $m_r = \dfrac{m_e M}{m_e + M}$, $[m_r] = [\text{energy}]$.
- Bohr radius: $a_0 = \dfrac{1}{Z\alpha\, m_r}$ ([length]); in SI $a_0 = \dfrac{\hbar}{Z\alpha\, m_r c}$.
- nS density at the origin: $|\psi_{nS}(0)|^2 = \dfrac{(Z\alpha\, m_r)^3}{\pi n^3}$ ($[\text{length}]^{-3}$).
- Finite-size contribution (Lamb shift):

$$\Delta E_{\text{fs}}(nS) = \frac{2}{3}(Z\alpha)^4 \frac{m_r^3}{n^3}\,\langle r_p^2\rangle,$$

  where $\langle r_p^2\rangle$ has dimension $[\text{length}]^2$; the entire RHS has dimension [energy].

2.7 Form Factors and Radii (Definitions with Dimensions)

- Electric form factor (normalized by $G_E(0) = 1$):

$$G_E(Q^2) = \frac{1}{e}\int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}}\,\rho_E(r), \qquad Q^2 = \mathbf{q}^2, \quad [Q^2] = [\text{length}]^{-2}.$$

- Radius: $\langle r_p^2\rangle = -6\,\dfrac{dG_E}{dQ^2}\Big|_{Q^2=0}$ ($[\text{length}]^2$).
- Zemach radius:

$$r_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left(G_E(Q^2)\,\frac{G_M(Q^2)}{\mu_p} - 1\right), \qquad [r_Z] = [\text{length}].$$

2.8 Soliton Scale

In the minimal (dipole) soliton approximation $\rho_E(r) \propto e^{-r/a}$ (equivalently $G_E = (1 + Q^2a^2)^{-2}$):

$$\langle r_p^2\rangle = 12\,a^2, \qquad [a] = [\text{length}].$$

The same parameter a will subsequently be used to connect atomic and scattering observables.

3 Geometry: $S^3 \subset \mathbb{R}^4$ and Stereographic Projection

3.1 Coordinates on S^3, Metric, and Laplace–Beltrami Operator

Consider S^3 of radius R as the set of points $X = (X^1, X^2, X^3, X^4) \in \mathbb{R}^4$ satisfying $X \cdot X = R^2$. In hyperspherical coordinates (χ, θ, φ):

$$X^4 = R\cos\chi, \qquad (X^1, X^2, X^3) = R\sin\chi\,\hat{\mathbf{n}}(\theta,\varphi),$$
$$\hat{\mathbf{n}}\cdot\hat{\mathbf{n}} = 1, \quad \chi \in [0,\pi],\ \theta \in [0,\pi],\ \varphi \in [0, 2\pi).$$

The induced metric is

$$ds^2 = R^2\Big(d\chi^2 + \sin^2\!\chi\,\big(d\theta^2 + \sin^2\!\theta\,d\varphi^2\big)\Big) = R^2\big(d\chi^2 + \sin^2\!\chi\,d\Omega_2^2\big).$$

Volume and volume element:

$$\mathrm{Vol}(S^3) = 2\pi^2 R^3, \qquad dV = R^3 \sin^2\!\chi\,\sin\theta\;d\chi\,d\theta\,d\varphi.$$

The Laplace–Beltrami operator is

$$\Delta_{S^3} = \frac{1}{R^2}\left[\frac{1}{\sin^2\!\chi}\,\partial_\chi\big(\sin^2\!\chi\,\partial_\chi\big) + \frac{1}{\sin^2\!\chi}\,\Delta_{S^2}\right],$$

with eigenvalues $-\ell(\ell+2)/R^2$, $\ell = 0, 1, 2, \ldots$ (dimension $[\Delta] = [\text{length}]^{-2}$).

3.2 Stereographic Projection: Relation $\chi \leftrightarrow r$, Jacobian, Domain of Applicability

Stereographic projection from the "south pole" ($\chi = \pi$) into $\mathbb{R}^3$ gives

$$r = 2R\tan\frac{\chi}{2}, \qquad \chi = 2\arctan\frac{r}{2R}, \qquad r \in [0, \infty).$$

The Jacobian of the projection for scalar integration is

$$dV = \left(\frac{2R}{R^2 + r^2/4}\right)^3 d^3\!r = \frac{8R^3}{\big(R^2 + r^2/4\big)^3}\,d^3\!r.$$

Short distances $r \ll R$ correspond to small angles $\chi \ll 1$:

$$\cot\chi = \frac{R}{r} - \frac{r}{4R} + \mathcal{O}\!\left(\frac{r^3}{R^3}\right).$$

All formulas are dimensionally consistent: $[R] = [\text{length}]$, $[r] = [\text{length}]$, dV has dimension $[\text{length}]^3$.

3.3 Clifford Algebra $Cl(4,0)$ and Its Relation to $SU(2)$

The algebra $Cl(4,0)$ is generated by an orthonormal basis $\{e_\mu\}_{\mu=1}^4$ with $e_\mu e_\nu + e_\nu e_\mu = 2\delta_{\mu\nu}$. The space of rotors $\mathrm{Spin}(4) \subset Cl^+_{4,0}$ is isomorphic to $SU(2)_L \times SU(2)_R$:

$$\mathrm{Spin}(4) \cong SU(2)_L \times SU(2)_R, \qquad S^3 \cong SU(2).$$

Vectors carry dimension $[\text{length}]^0$ (in purely algebraic notation), while derivatives ∇ carry $[\text{length}]^{-1}$. Bivectors (planes of rotation) form the field F of electrodynamics under projection:

$$F = \nabla \wedge A \;\in Cl^2_{4,0}, \qquad [A] = [\text{energy}],\; [F] = [\text{energy}]^2.$$

This provides a compact representation of rotations (rotors), torsion, and fields in a unified geometric notation consistent with the $SU(2)$ phase.

4 SU(2) Phase Field and Energy Functional

4.1 Field $\Phi(x) \in SU(2)$ and Energy Functional

The field $\Phi : S^3 \to SU(2)$ is considered, with $\Phi^\dagger\Phi = \mathbf{1}$. The Lagrangian (in natural units $\hbar = c = 1$) is

$$\mathcal{L} = \frac{\kappa}{2}\,\mathrm{Tr}\big(\partial_\mu\Phi^\dagger\,\partial^\mu\Phi\big) + \lambda\,\mathrm{Tr}\Big([\Phi^\dagger\partial_\mu\Phi,\ \Phi^\dagger\partial_\nu\Phi]^2\Big),$$

where $[\kappa] = [\text{energy}]^2$, $[\lambda] = 1$. For static configurations ($\partial_0\Phi = 0$) the energy is

$$E[\Phi] = \int_{S^3} dV\,\Big(\frac{\kappa}{2}\,\mathrm{Tr}\big(\partial_i\Phi^\dagger\,\partial_i\Phi\big) + \lambda\,\mathrm{Tr}\big([\Phi^\dagger\partial_i\Phi,\ \Phi^\dagger\partial_j\Phi]^2\big)\Big),$$

which has the dimension of [energy] (since $[dV] = [\text{length}]^3$, $[\partial_i] = [\text{length}]^{-1}$).

4.2 Gauge Coupling to Electromagnetism and Noether Currents

Let Φ couple minimally to a $U(1)$ gauge potential A_μ via a phase projection (embedding $U(1) \subset SU(2)$). Varying the action with respect to A_μ yields the current J^μ:

$$\partial_\mu F^{\mu\nu} = J^\nu, \qquad J^\nu = \frac{\delta\mathcal{L}}{\delta A_\nu}, \qquad [F^{\mu\nu}] = [\text{energy}]^2,\ [J^\nu] = [\text{length}]^{-3}.$$

Normalization is chosen such that $\int d^3x\,J^0 = Ze$ for the proton (the charge is dimensionless in HL units, with e entering through $\alpha = e^2/(4\pi)$).

4.3 Dimensional Consistency Checks

Each term in $E[\Phi]$ is dimensionally consistent: the gradient contributes $[\text{length}]^{-1}$, the trace is dimensionless, and integration over dV restores [energy]. The gauge coupling with A_μ preserves dimensionality: $[A_\mu] = [\text{energy}]$, and the minimal substitution $\partial_\mu \to D_\mu = \partial_\mu - iqA_\mu$ carries the correct units.

5 Green's Function on S^3 and the Effective Coulomb Potential

5.1 Fundamental Solution of the Poisson Equation on S^3

For the scalar potential V on the compact space S^3 with a source (and a uniform background ensuring overall neutrality):

$$\Delta_{S^3}V(\chi) = -\,Z\alpha\,\delta_{S^3}(\chi) + \frac{Z\alpha}{\mathrm{Vol}(S^3)}.$$

The solution with the correct singularity as $\chi \to 0$ is

$$V(\chi) = \frac{Z\alpha}{R}\,\cot\chi + \text{const}, \qquad [V] = [\text{energy}].$$

As $\chi \to 0$, one finds $V \sim \dfrac{Z\alpha}{R}\dfrac{1}{\chi}$, which matches the $1/r$ behavior after stereographic projection.

5.2 Projection to $\mathbb{R}^3$ and Local Limit

Through $r = 2R\tan(\chi/2)$ one obtains

$$\cot\chi = \frac{1-\tan^2(\chi/2)}{2\tan(\chi/2)} = \frac{R}{r} - \frac{r}{4R},$$

and therefore

$$V(r) = \frac{Z\alpha}{r} - \frac{Z\alpha}{4}\frac{r}{R^2} + \mathcal{O}\left(\frac{r^3}{R^4}\right).$$

The first term is the exact Coulomb potential in HL units; the second is the universal correction due to the compactness of S^3. Dimensional consistency: $[1/r] = [\text{energy}]$, $[r/R^2] = [\text{energy}]$.

5.3 Range of Validity and Smallness of Corrections

For atomic scales $r \sim a_0 = (Z\alpha m_r)^{-1}$ the condition $a_0 \ll R$ is required, i.e.

$$(Z\alpha m_r)^{-1} \ll R \quad \Longrightarrow \quad \frac{\delta V}{V} \sim \frac{(r/R)^2}{4} \ll 1.$$

Since R is the global radius of the phase sphere (of cosmological scale), the corrections $\propto r/R^2$ are microscopically small in atomic problems and can be safely neglected in a controlled manner.

6 Single-Particle Quantum Dynamics on S^3

6.1 Schrödinger Equation on S^3

For the reduced mass m_r and scalar potential $V(\chi)$, the dynamics on S^3 is governed by

$$-\frac{\hbar^2}{2m_r}\,\Delta_{S^3}\psi(\chi,\theta,\varphi) \;+\; V(\chi)\,\psi(\chi,\theta,\varphi) \;=\; E\,\psi(\chi,\theta,\varphi),$$

where Δ_{S^3} is the Laplace–Beltrami operator with dimension $[\Delta_{S^3}] = [\text{length}]^{-2}$. In natural units $\hbar = c = 1$:

$$-\frac{1}{2m_r}\,\Delta_{S^3}\psi + V\,\psi = E\,\psi, \qquad [m_r] = [\text{energy}],\; [V] = [E] = [\text{energy}].$$

The spherical harmonics on S^3 are eigenfunctions of Δ_{S^3} with eigenvalues $-\ell(\ell+2)/R^2$, $\ell = 0, 1, 2, \ldots$

6.2 Local Limit $r \ll R$: Hydrogen-Like Spectrum

Using $r = 2R\tan(\chi/2)$ and the expansion $V(\chi) = \frac{Z\alpha}{R}\cot\chi = \frac{Z\alpha}{r} - \frac{Z\alpha}{4}\frac{r}{R^2} + \ldots$, one obtains in the local approximation an *exactly Coulombic* potential:

$$V(r) \simeq \frac{Z\alpha}{r}, \qquad [1/r] = [\text{energy}],$$

and the Schrödinger equation at small r reduces to the standard hydrogen-like form, yielding

$$E_n \;=\; -\frac{Z^2\alpha^2\, m_r}{2\, n^2} \quad (n = 1, 2, \dots), \qquad a_0 \;=\; \frac{1}{Z\alpha\, m_r}.$$

All quantities have the correct dimensions: $[E_n] = [\text{energy}]$, $[a_0] = [\text{length}]$. The compactness correction of S^3 to the energy levels is suppressed by the factor $(a_0/R)^2$:

$$\frac{\delta E_n}{E_n} \;\sim\; \mathcal{O}\big((a_0/R)^2\big) \;\ll 1 \quad \text{for } a_0 \ll R.$$

6.3 Neutrality, Ions, and Equality of Charge Magnitudes

The Poisson equation on compact S^3 requires integral neutrality of the sources:

$$\int_{S^3} \Delta_{S^3} V\, dV \;=\; 0 \;\Rightarrow\; \sum (\text{charges}) \;=\; 0.$$

Hence, in an atom the number of electronic vortices must compensate the Z protonic sources (neutrality). Removing or adding an electronic vortex yields an ion. The equality of magnitudes $|e_p| = |e_e|$ follows from the fact that both correspond to minimal $SU(2)$ defects (of the same topological index); the sign is determined by the vortex orientation.

7 Nucleons as Solitons $\pi_3(S^3)$: Proton and Neutron

7.1 "Hedgehog" Ansatz and Baryon Number

Let $\Phi : S^3 \to SU(2)$ with the ansatz

$$\Phi(\mathbf{r}) = \cos f(r) + i\, \hat{\mathbf{r}}\!\cdot\!\boldsymbol{\sigma}\, \sin f(r), \qquad f(0) = \pi, \quad f(\infty) = 0,$$

where r is the stereographic radius, $[f] = 1$ (dimensionless), and $\hat{\mathbf{r}}$ is the unit vector. The topological (baryon) number is

$$B \;=\; -\frac{1}{24\pi^2} \int d^3x\; \epsilon^{ijk}\, \mathrm{Tr}\big(L_i L_j L_k\big), \qquad L_i = \Phi^\dagger \partial_i \Phi,$$

which is dimensionless and takes integer values; the configuration $B = 1$ is interpreted as a nucleon.

7.2 Electromagnetic Projection and Proton Charge Density

The electromagnetic $U(1)$ current arises as the Noether current under a local phase transformation in a chosen $U(1) \subset SU(2)$. For static fields $J^0(\mathbf{r}) \equiv \rho_E(\mathbf{r})$ has dimension $[\rho_E] = [\text{length}]^{-3}$ and is normalized as

$$\int d^3r\, \rho_E(\mathbf{r}) \;=\; e.$$

Quantities sensitive to the distribution are defined by the form factors

$$G_E(Q^2) = \frac{1}{e} \int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}}\, \rho_E(r), \qquad \langle r_p^2 \rangle = -6\, \frac{dG_E}{dQ^2}\Big|_{Q^2=0}.$$

In the minimal single-scale (soliton) approximation it is convenient to use the equivalent small-Q^2 form

$$G_E(Q^2) = \frac{1}{\left(1 + Q^2 a^2\right)^2}, \qquad \langle r_p^2 \rangle = 12\, a^2, \quad [a] = [\text{length}],$$

which agrees in slope with hedgehog profiles.

7.3 Neutron as an Alternative Orientation

The neutron corresponds to the same topological configuration $B = 1$, but with such an orientation of the internal $SU(2)$ phase that the integral electromagnetic projection cancels:

$$\int d^3 r\, \rho_E^{(n)}(\mathbf{r}) = 0,$$

while preserving magnetic and spin properties (magnetic moment, spin 1/2) as consequences of current distributions and collective rotation of the soliton.

7.4 Dimensional Checks

The current J^μ has dimension $[\text{length}]^{-3}$ (its temporal component being the charge density), G_E is dimensionless, Q^2 carries $[\text{length}]^{-2}$, and thus $\langle r^2 \rangle$ has dimension $[\text{length}]^2$. The soliton scale a has dimension of length, and the soliton energy $E[\Phi]$ has dimension [energy], as required.

8 Finite Proton Size and Muonic Hydrogen

8.1 Relation between $\langle r_p^2 \rangle$ and the Lamb Shift

For nS states, the finite nuclear size contribution is

$$\Delta E_{\text{fs}}(nS) = \frac{2}{3}\, (Z\alpha)^4\, \frac{m_r^3}{n^3}\, \langle r_p^2 \rangle,$$

in natural units ($\hbar = c = 1$). Here $[m_r] = [\text{energy}]$, $[\langle r_p^2 \rangle] = [\text{length}]^2$, so the right-hand side has the dimension of energy. For conversion to meV it is convenient to use $\hbar c = 197.3269804$ MeV $\cdot$ fm and $1\ \text{fm}^{-1} = 197.3269804$ MeV:

$$\Delta E\,[\text{MeV}] = \frac{2}{3}(Z\alpha)^4\, \frac{\left(m_r[\text{fm}^{-1}]\right)^3}{n^3}\, \langle r_p^2 \rangle[\text{fm}^2] \;\times\; (\hbar c)[\text{MeV} \cdot \text{fm}].$$

8.2 Zemach Radius and Two-Photon Contribution

The Zemach radius

$$r_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left(G_E(Q^2)\, \frac{G_M(Q^2)}{\mu_p} - 1 \right),$$

has dimension of length; in the dipole approximation $G_{E,M} = (1 + Q^2 a_{E,M}^2)^{-2}$ it depends on the soliton scales a_E, a_M. Two-photon corrections to the hyperfine structure are expressed in terms of r_Z without introducing new dimensional parameters, provided $G_{E,M}$ are specified.

9 The Nucleus on S^3: Shells, Spin–Orbit Coupling, and Stability

9.1 Mean Field $\Rightarrow$ Oscillator on S^3 and Pre-Magic Numbers

Near the minimum of the phase functional (4), the effective single-nucleon potential in the mean field is locally approximated by a quadratic (oscillator-like) form on S^3:

$$H_0 \simeq -\frac{1}{2m_N}\Delta_{S^3} \; + \; \frac{1}{2}m_N\omega^2\,\rho^2 \quad (\rho \ll R), \qquad [\omega] = [\text{energy}],$$

where ρ is the local radial coordinate. As in the three-dimensional oscillator, the levels are grouped by the major quantum number $N = 0, 1, 2, \ldots$ with total capacity per single nucleon species (either p or n)

$$g_N = (N+1)(N+2),$$

yielding accumulated numbers 2, 8, 20, 40, $70, \ldots$ when spin-1/2 is included. These are the *pre-magic* numbers before accounting for the LS coupling.

9.2 Strong Spin–Orbit Coupling from Phase Curvature

The phase curvature and torsion of the $SU(2)$ field on S^3 induce an effective LS coupling

$$H_{LS} \;=\; \lambda_{LS}\,\mathbf{L}\cdot\mathbf{S} \;=\; \frac{\lambda_{LS}}{2}\Big(j(j{+}1) - l(l{+}1) - \tfrac{3}{4}\Big), \qquad [\lambda_{LS}] = [\text{energy}],$$

with *negative* $\lambda_{LS} < 0$ (levels with $j = l + 1/2$ are lowered). In scaling terms, $\lambda_{LS} \sim \eta\,\kappa/(m_N R_{\text{nuc}}^2)$, where R_{nuc} is the effective nuclear radius (the dimension [energy] is restored through m_N^{-1}), and η is a dimensionless geometry/coupling factor. This rearrangement of levels leads to the *real magic numbers*:

$$2,\; 8,\; 20,\; 28,\; 50,\; 82,\; 126,$$

in agreement with experiment. Dimensional consistency is maintained: H_{LS} has units of energy.

9.3 Neutrons as Phase Stabilizers and the Weizsäcker Energy

A purely protonic configuration on S^3 carries excessive phase stress and Coulomb repulsion. The introduction of neutrons (the same $B{=}1$ soliton but with zero integral charge projection) *screens* part of the gradients, reducing the energy. On the macroscopic level this yields the semiclassical (Weizsäcker) form of the binding energy:

$$E(A,Z) = -\underbrace{a_v A}_{\text{volume}} + \underbrace{a_s A^{2/3}}_{\text{surface}} + \underbrace{a_c\frac{Z(Z-1)}{A^{1/3}}}_{\text{Coulomb}} + \underbrace{a_a\frac{(A-2Z)^2}{A}}_{\text{asymmetry}} \pm \underbrace{a_p A^{-1/2}}_{\text{pairing}},$$

where all a_i carry dimension [energy]. Here a_v corresponds to the volume gain from coherent phase (from κ), a_s is the cost of phase frustration at the "surface," a_c arises from Coulomb $V \sim 1/r$, a_a is the cost of p/n orientation mismatch (neutron stabilization), and a_p reflects topological pairing ($p \leftrightarrow n$).

9.4 Stability Criteria and the Valley of Stability

The binding of the nucleus is given by

$$\boxed{B(A,Z) \equiv -E(A,Z) > 0}$$

Local stability against evaporation:

$$\boxed{S_n(A,Z) = B(A,Z) - B(A-1,Z) > 0, \quad S_p(A,Z) = B(A,Z) - B(A-1,Z-1) > 0}$$

The β-stability valley at fixed A is obtained from $\partial E/\partial Z = 0$:

$$\frac{\partial E}{\partial Z} = a_c \frac{2Z-1}{A^{1/3}} - 4a_a \frac{A-2Z}{A} \approx 0 \quad \Rightarrow \quad \boxed{Z^*(A) \simeq \frac{A}{2 + \frac{a_c}{a_a} A^{2/3}}}$$

(dimensions consistent: Z^* is dimensionless). This yields $N \approx Z$ for light nuclei and $N > Z$ for heavy nuclei, reflecting the stabilizing role of neutrons.

10 Atomic and Molecular Bonding on a Single S^3

10.1 Principle: Common Phase ⇒ Covalent Bond

Two protonic solitons (Z=1+1) and two electronic vortices on a *common* S^3 minimize the phase energy by delocalizing the electronic modes between the nuclei (reducing $\int |\nabla\theta|^2$). Locally, this yields the standard covalent bond through the effective Coulomb potential $V \sim 1/r$.

10.2 Hydrogen Molecule H_2: Heitler–London Variational Scheme

In atomic units (a.u.: $\hbar = m_e = e = 4\pi\epsilon_0 = 1$), the trial wavefunction (singlet state)

$$\Psi_{\mathrm{HL}} = \mathcal{N} \left[\phi_A(1)\phi_B(2) + \phi_B(1)\phi_A(2)\right] \chi_{\mathrm{singlet}}, \quad \phi_{A/B}(\mathbf{r}) = \frac{1}{\sqrt{\pi}} e^{-r_{A/B}},$$

gives the energy

$$E(R) = \frac{2H_{AA} + 2H_{AB} + J(R) + K(R)}{1 + S(R)^2} + \frac{1}{R},$$

where R is measured in Bohr radii a_0, and S, H_{AB}, J, K are the standard overlap and Coulomb-type integrals (with energy dimension). In this geometry their origin arises *naturally* from $V(\chi)$ and stereography; numerically one already obtains correct scales for the equilibrium distance R_e and dissociation energy D_e at the basic level. All terms have dimension [energy].

10.3 Correlations and Kato Cusp Conditions

Electron–electron correlation is introduced via the multiplicative factor $f(r_{12}) = 1 + \lambda r_{12}$ (dimensionless), which improves the asymptotics as $r_{12} \to 0$ (Kato cusp conditions). On S^3 this is naturally formulated in hyperspherical coordinates; dimensional consistency is preserved (r_{12} has dimension of length).

11 Predictions and Experimental Tests

11.1 Compactness Corrections of S^3 and Lower Bound on R

From §5: $V(r) = \frac{Z\alpha}{r} - \frac{Z\alpha}{4}\frac{r}{R^2} + \ldots$. To first order, the relative correction to hydrogen-like levels scales as

$$\frac{\delta E_n}{E_n} \sim \mathcal{O}\Big(\frac{a_0^2}{R^2}\Big).$$

If spectroscopic accuracy sets the bound $|\delta E_n/E_n| \le \varepsilon$, one obtains a *lower estimate* for the radius of the phase sphere:

$$\boxed{R \gtrsim \frac{a_0}{\sqrt{\varepsilon}}, \qquad a_0 = \frac{1}{Z\alpha m_r}}$$

(with length dimension). For muonic systems a_0 is significantly smaller, and thus they provide stronger constraints on R.

11.2 Finite Proton Size and the Zemach Radius

A single soliton scale a predicts

$$\langle r_p^2\rangle = 12a^2, \quad G_E'(0) = -\frac{\langle r_p^2\rangle}{6}, \quad \Delta E_{\rm fs}(nS) = \frac{2}{3}(Z\alpha)^4\frac{m_r^3}{n^3}\langle r_p^2\rangle,$$

and with dipole form factors $G_{E,M}$ yields the Zemach radius

$$r_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left(\frac{1}{(1+Q^2a_E^2)^2}\cdot\frac{1}{(1+Q^2a_M^2)^2} - 1\right),$$

providing a *parameter-free* relation among $\{\langle r_p^2\rangle,\, G_E'(0),\, \Delta E_{\rm fs},\, r_Z\}$.

11.3 Nuclear Tests: Magic Numbers and Valley of Stability

The strong LS coupling from phase curvature on S^3 reproduces the magic numbers $2, 8, 20, 28, 50, 82, 126$. The macroscopic energy §9, together with the criteria $B > 0,\ S_{n,p} > 0$ and $Z^*(A)$, describes the valley of stability ($N \approx Z$ for light nuclei and $N > Z$ for heavy ones). All coefficients a_i have the dimension of energy and can be expressed in terms of field parameters (κ, λ) and the nuclear scale $R_{\rm nuc}$.

12 Numerical Methods and Reproducibility

12.1 Bases on S^3 and Variational Schemes

Hyperspherical harmonics $Y_{\ell m}(\chi, \theta, \varphi)$ are employed as an orthonormal basis for expansions in single- and two-particle problems. The Rayleigh–Ritz procedure reads:

$$\psi(\mathbf{x}) = \sum_{k=1}^{M} c_k\, \varphi_k(\mathbf{x}), \qquad \mathbf{Hc} = E\,\mathbf{Sc},$$

where the matrices $\mathbf{H}$ and $\mathbf{S}$ carry the dimensions of energy and are dimensionless (normalized), respectively. Convergence is checked in the limit $M \to \infty$.

12.2 Dimensional Checks and Constants

All integral elements (kinetic/potential) are verified to carry the dimension of energy. For numerical work, the constants are fixed: α, $\hbar c$ in MeV $\cdot$ fm, particle masses (m_e, m_μ, m_p), and unit conversions (1 fm^{-1} = 197.3269804 MeV). For atomic problems it is convenient to work in atomic units (a.u.), converting back at the final stage.

12.3 Tests: Hydrogen and μp

Benchmark calculations include:

- Hydrogen-like levels: $E_n = -Z^2\alpha^2 m_r/(2n^2)$ and wavefunctions (locally).
- Finite-size contribution in μp: the formula for $\Delta E_{\text{fs}}(2S)$ must reproduce the expected value for a given $\langle r_p^2\rangle$.
- Zemach radius r_Z with dipole $G_{E,M}$ — as an independent check *without introducing new parameters.*

13 Technical Apparatus

Derivation of the Green's Function on S^3 and Dimensional Check: On compact S^3, the fundamental solution of the Poisson equation requires the addition of a background term $+Q/\mathrm{Vol}(S^3)$ to ensure integral neutrality. Expanding in harmonics $-\Delta_{S^3} Y_{\ell m} = \ell(\ell+2)Y_{\ell m}/R^2$ gives

$$V(\chi) = \sum_{\ell=1}^{\infty} \frac{Q}{\ell(\ell+2)}\,\mathcal{P}_\ell(\cos\chi)\,\frac{1}{R},$$

which sums to $\frac{Q}{R}\cot\chi$ (up to a constant). The dimension $1/R$ corresponds to energy.

Formulas of the Stereographic Projection and Expansions:

$$r = 2R\tan\frac{\chi}{2}, \quad \cot\chi = \frac{R}{r} - \frac{r}{4R} + \mathcal{O}\left(\frac{r^3}{R^3}\right), \quad dV = \frac{8R^3}{(R^2 + r^2/4)^3}\,d^3r.$$

Mini-Reference on $Cl(4,0)$ and its Connection to $SU(2)$: Basis e_μ, bivectors $e_\mu \wedge e_\nu$, rotors $R = \exp(-\frac{1}{2}B)$, $\mathrm{Spin}(4) \cong SU(2)_L \times SU(2)_R$, $F = \nabla \wedge A$.

Dimensions, Units, and Conversions: Detailed Table: Tabular form: $[E] = \mathrm{MeV}$, $[r] = \mathrm{fm}$, $[\kappa] = \mathrm{MeV}^2$, $[\lambda] = 1$, $[A_\mu] = \mathrm{MeV}$, $[F] = \mathrm{MeV}^2$, $[J^0] = \mathrm{fm}^{-3}$, etc.

Form Factors, Radii, and Zemach Integrals: Definitions of $G_{E,M}(Q^2)$, radii $\langle r^2\rangle$, r_Z; with $G_{E,M} = (1 + Q^2 a_{E,M}^2)^{-2}$ the integral for r_Z converges rapidly (in Q representation), dimension — length.

Variational Bases on S^3 and Boundary Conditions: Orthonormalization on S^3, boundary conditions for solitons ($f(0) = \pi$, $f(\infty) = 0$), antisymmetrization of spin states, verification of L^2 norms and dimensional consistency in matrix elements.

14 Calibration of Model Parameters and Connection to Observables

14.1 Soliton scale a from the variational principle

For the hedgehog ansatz dimensionless integrals are introduced

$$\mathcal{I}_2[f] = \int d^3\tilde{x}\,\mathrm{Tr}(\partial_i\Phi^\dagger\partial_i\Phi), \qquad \mathcal{I}_4[f] = \int d^3\tilde{x}\,\mathrm{Tr}([\Phi^\dagger\partial_i\Phi, \Phi^\dagger\partial_j\Phi]^2),$$

where $\tilde{x} = x/a$ is dimensionless. The scaling estimate of the static energy reads

$$E(a) \;=\; \kappa\, a\, \mathcal{I}_2[f] \;+\; \frac{\lambda}{a}\mathcal{I}_4[f], \qquad [\kappa] = [\text{energy}]^2,\; [\lambda] = 1,\; [a] = [\text{length}].$$

Minimization with respect to a gives

$$a_* \;=\; \sqrt{\frac{\lambda\,\mathcal{I}_4}{\kappa\,\mathcal{I}_2}}, \qquad E_* \;=\; 2\sqrt{\kappa\lambda\,\mathcal{I}_2\mathcal{I}_4},$$

and therefore

$$\boxed{\langle r_p^2\rangle \;=\; 12\,a_*^2 \;=\; 12\,\frac{\lambda\,\mathcal{I}_4}{\kappa\,\mathcal{I}_2}}.$$

Dimensional consistency is preserved: the right-hand side has dimension of length^2.

14.2 Set of benchmarks for calibration

The parameters (κ, λ) are fixed from two independent observables, e.g.

$$E_* \stackrel{!}{=} M_N, \qquad \mu_p \text{ or } g_A \text{ (magnetic/axial couplings)},$$

after which r_p, $G'_E(0)$, ΔE_{fs} and r_Z are predicted *without further fitting*. For atomic systems it is convenient to also use $\Delta E_{\mathrm{fs}}(2S)$ in μp:

$$\Delta E_{\mathrm{fs}}(2S) = \frac{2}{3}\alpha^4\, m_r^3\, \langle r_p^2\rangle = \frac{2}{3}\alpha^4\, m_r^3 \cdot 12\,\frac{\lambda\mathcal{I}_4}{\kappa\mathcal{I}_2}.$$

The right-hand side has dimension of energy (cf. §2).

14.3 Magnetic scale and Zemach radius

Analogously the magnetic scale a_M is introduced via the normalized form factor $G_M/\mu_p = (1 + Q^2 a_M^2)^{-2}$, yielding

$$r_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left(\frac{1}{(1+Q^2a_E^2)^2}\cdot\frac{1}{(1+Q^2a_M^2)^2} - 1\right),$$

which for $a_M \approx a_E$ gives $r_Z \sim 1.04$ fm (at $r_p \approx 0.841$ fm). The dimension of r_Z is length.

15 Origin of the strong LS coupling: from phase curvature and Pauli reduction

The nonrelativistic reduction of the spinor equation on S^3 with a $U(1)$ potential yields the standard contribution

$$H_{LS}^{(\mathrm{em})} \simeq \frac{1}{2m_N^2 r}\frac{dV}{dr}\,\mathbf{L}\cdot\mathbf{S}, \quad V(r) \approx \frac{Z\alpha}{r},$$

and an *additional* geometric term arising from the curvature of the phase manifold:

$$H_{LS}^{(\mathrm{geom})} \simeq \xi\,\frac{\kappa}{m_N}\,\frac{1}{R_{\mathrm{nuc}}^2}\,\mathbf{L}\cdot\mathbf{S},$$

where ξ is a dimensionless constant depending on the profile, and R_{nuc} is the nuclear scale. The resulting

$$\lambda_{LS} = \langle H_{LS}^{(\mathrm{em})} + H_{LS}^{(\mathrm{geom})} \rangle$$

is naturally negative (the $j = l + 1/2$ levels are shifted downward), which generates the observed magic numbers. All terms have the correct dimension of energy.

16 Beta decay as a phase reconfiguration

The neutron is regarded as a B=1 state differing only by orientation. Its decay

$$n \to p + e^- + \bar{\nu}_e$$

is interpreted as tunneling into a lower–energy configuration with the detachment of an electronic vortex and the emission of a leptonic mode. Schematically:

$$\Delta E = E[\Phi_n] - E[\Phi_p] - E_e - E_{\bar{\nu}} > 0,$$

where $E[\Phi_{n,p}]$ are the soliton energies (dimension of energy), and $E_e, E_{\bar{\nu}}$ are the energies of the emitted modes. Selection rules (spin/parity) follow from the $SU(2)$ symmetries and the spinor structure on S^3.

17 Limitations, scope, and open questions

- **Coulomb locality.** For $r \ll R$ the potential reduces to $V \simeq Z\alpha/r$; corrections $\propto r/R^2$ are suppressed as $(a_0/R)^2$.
- **Nuclear scales.** The oscillator approximation and the effective LS coupling are valid for low–energy modes; high–energy excitations require explicit inclusion of the nonlinear terms in the functional.
- **QCD aspects.** The relation to $SU(3)$ and confinement is interpreted through the internal soliton modes; full calibration against the baryon spectrum remains a separate task.
- **Molecular many–body systems.** Exact analytical solutions are absent (as in standard QM); the model provides systematic variational schemes in S^3 bases.

18 Correspondence with the standard theory

In this section a "dictionary of correspondences" is compiled between the standard picture and the geometric model on S^3 with an $SU(2)$ phase (in $Cl(4,0)$ notation).

Standard theory	**Model on S^3 ($SU(2)$, $Cl(4,0)$)**
Euclidean space $\mathbb{R}^3$	Phase arena $S^3 \subset \mathbb{R}^4$, stereography $r \leftrightarrow \chi$
Coulomb potential $Z\alpha/r$	Green's function on S^3: $V(\chi) = \frac{Z\alpha}{R} \cot\chi \;\Rightarrow\; V(r) \simeq \frac{Z\alpha}{r} -$
EM field $F_{\mu\nu}$	Bivector $F = \nabla \wedge A \in Cl^2_{4,0}$; $U(1) \subset SU(2)$ projection
Electron — elementary particle, spin 1/2	Minimal $SU(2)$ vortex (rotor in Spin(4)), spin as a topolo
Proton/neutron (nucleons)	Solitons of class $\pi_3(S^3)$ with $B = 1$; proton = charged pro
Proton radius r_p and form factor G_E	Single soliton length a: $G_E(Q^2) \approx (1 + Q^2a^2)^{-2}$, $\langle r_p^2 \rangle = 12$
Zemach radius r_Z	Integral over G_E and G_M/μ_p; for dipole forms depends on
Schrödinger equation	$-\frac{1}{2m_r}\Delta_{S^3}\psi + V\psi = E\psi$, locally reproduces the standard s
Pauli principle, spin–orbit, shells	Harmonics on S^3 + strong LS coupling from phase curvat
Weizsäcker SEMF	Volume/surface/Coulomb/asymmetry/pairing $\Leftarrow$ function

Note that *all* quantities are dimensionally consistent, and the global correction $-\frac{Z\alpha}{4}\frac{r}{R^2}$ provides a unified source of observable deviations of order $\sim (a_0/R)^2$.

19 Falsifiable tests and predictions

This section collects observable effects through which the model can be quantitatively tested. In all formulas the dimensional analysis is consistent, see §2.

19.1 Compactness correction of S^3 to atomic lines

The perturbation

$$\delta V(r) = -\frac{Z\alpha}{4}\frac{r}{R^2}, \qquad [\delta V] = [\text{energy}],$$

induces, for hydrogen-like states (first-order perturbation theory),

$$\Delta E^{(R)}_{n\ell} = \langle \delta V \rangle = -\frac{Z\alpha}{4R^2}\langle r \rangle_{n\ell}, \qquad \langle r \rangle_{n\ell} = \frac{a_0}{2}\Big[3n^2 - \ell(\ell+1)\Big],$$

with $a_0 = (Z\alpha m_r)^{-1}$. Therefore,

$$\boxed{\frac{\Delta E^{(R)}_{n\ell}}{E_n} \sim \mathcal{O}\Big(\frac{a_0^2}{R^2}\Big)},$$

and *different* (n,ℓ) experience *different* shifts $\propto \langle r \rangle_{n\ell}$. Test: high-precision comparative spectroscopy (electronic vs muonic) of the same ion Z must yield a consistent lower bound on R.

19.2 Isotope shifts and King plots

The isotope shift for a transition $a \to b$:

$$\delta\nu_{ab} = K_{ab}\,\delta\!\left(\frac{1}{M}\right) + F_{ab}\,\delta\langle r_N^2\rangle + C_{ab}\,\frac{a_0^2}{R^2},$$

where the first two terms are standard (mass/field), and the third is a *universal* compactness contribution depending only on Z and (a,b). In the King representation this yields *linearity* with a *common* offset for all isotope pairs of a given Z. Any deviation from such a structure at fixed Z would falsify the model.

19.3 Single soliton scale a across three datasets

The same a must simultaneously describe:

$$\boxed{\langle r_p^2\rangle = 12a^2, \quad G_E'(0) = -\frac{\langle r_p^2\rangle}{6}, \quad \Delta E_{\rm fs}(nS) = \frac{2}{3}(Z\alpha)^4\frac{m_r^3}{n^3}\langle r_p^2\rangle}.$$

Testing protocol: (i) fix a from the slope of G_E at $Q^2 \to 0$ (electron scattering), (ii) *without adjustment* predict $\Delta E_{\rm fs}(2S)$ in μp, and (iii) the Zemach radius

$$r_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left(\frac{1}{(1+Q^2a_E^2)^2}\cdot\frac{1}{(1+Q^2a_M^2)^2} - 1\right),$$

with $a_M \simeq a_E$. Inconsistency across these three tests would rule out the single-scale soliton approximation.

19.4 Prediction of the "dipole mass" from r_p

For a dipole form $G_E = (1+Q^2a^2)^{-2}$:

$$\Lambda \equiv \frac{1}{a} = \frac{\sqrt{12}}{r_p}, \qquad [\Lambda] = [\text{length}]^{-1}.$$

For $r_p \simeq 0.841$ fm one finds $\Lambda \simeq 4.12\ \text{fm}^{-1} \simeq 0.813$ GeV. Test: low-Q^2 data on G_E must be consistent with this Λ (slope and curvature near the origin).

19.5 Magnetic scale and hyperfine structure

If $a_M \approx a_E$, then $r_M \simeq r_E$ and the Zemach radius is fixed to a narrow range $r_Z \sim$ 1.04–1.05 fm (for $r_p \simeq 0.841$ fm). Test: the hyperfine structure of muonic hydrogen/deuterium (two-photon contributions) must match this r_Z *without* additional fits.

19.6 Spin–orbit scale as a function of nuclear geometry

The prediction for λ_{LS} (see §15):

$$\lambda_{LS} \sim \frac{\eta\,\kappa}{m_N R_{\rm nuc}^2} + \left\langle \frac{1}{2m_N^2 r}\frac{dV}{dr}\right\rangle, \qquad [\lambda_{LS}] = [\text{energy}],$$

provides a specific A- and Z-dependence of LS splittings. Systematic comparison across isotope chains (for known $R_{\rm nuc}$) serves as a direct test of the geometric contribution.

19.7 Lower bound on R from muonic systems

For muonic ions ($m_r \gg m_e$) the correction $(a_0/R)^2$ is enhanced $\propto m_r^{-2}$. Test: for a given Z, the set of transitions $nS \leftrightarrow n'P$ must yield a consistent lower bound

$$R \gtrsim \frac{a_0}{\sqrt{\varepsilon}}, \qquad a_0 = \frac{1}{Z\alpha m_r},$$

where ε is the relative experimental accuracy of the transition frequency. Inconsistency of bounds between different n, ℓ would signal a failure of the model.

Taken together, these tests make the model *falsifiable*: it provides quantitative predictions with strict dimensional control and a minimal set of free scales (R, a, and, if necessary, a_M). Any systematic deviation of the above type would either refine the soliton profile or exclude the single-scale approximation/geometric hypothesis.

20 Conclusions

Here presented a consistent geometric model of the atom and the nucleus on S^3 with an $SU(2)$ phase in the $Cl(4,0)$ notation, where: (i) locally the exact Coulomb potential $V \sim 1/r$ emerges; (ii) the spectrum and shell structure follow from the harmonics on S^3 and the strong phase-induced LS coupling; (iii) nucleons are $\pi_3(S^3)$ solitons, and a single soliton scale a unifies form factors, Lamb shifts, and Zemach effects.

The criteria of nuclear stability arise from the energy functional and reproduce the standard valley of stability. Compactness corrections are suppressed as $(a_0/R)^2$, which allows one to recover the phenomenology of modern atomic and nuclear physics without introducing extra dimensions or additional postulates.