Physics from an SU(2) Phase Geometry on the 3-Sphere: Foundations

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Abstract

A unified geometric framework is formulated in which classical mechanics, electrodynamics, relativity, and quantum mechanics emerge as limiting cases of a single SU(2) phase dynamics on the compact three-sphere S^3. The internal phase field $U(x) \in \mathrm{SU}(2)$ defines curvature, spin, and temporal structure within a globally finite, locally continuous manifold. All physical entities — matter, radiation, and gravitation — are interpreted as manifestations of curvature and evolution of this phase geometry. The model introduces a dual notion of time, with local operational time $t(x)$ determined by the phase rate $\omega(x)$ relative to a global evolution parameter T, thereby unifying gravitational and kinematic time dilation. Newtonian dynamics, Maxwell's equations, and the Einstein field equations are recovered as successive limits of the same Lagrangian structure, while quantum discreteness arises from the compactness of S^3 and the SU(2) spin topology. The framework offers geometric interpretations of relativistic and quantum phenomena, including light deflection, entanglement, tunneling, and cosmological redshift, and predicts testable deviations from the ΛCDM relation between redshift and luminosity. The SU(2) phase geometry thus provides a coherent and self-contained foundation linking classical and quantum domains through a single compact phase manifold.

Contents

1 Introduction and Motivation

The purpose of this work is to establish a unified geometric framework in which all known physical interactions — gravitational, electromagnetic, and quantum — emerge from a single phase structure defined on the compact three-sphere S^3. The internal geometry of this phase space is described by the Lie group SU(2), which is topologically equivalent to S^3. Within this setting, matter, radiation, and spacetime curvature appear as different manifestations of a single underlying field — the *phase field* $U(x) \in \mathrm{SU}(2)$.

The choice of SU(2) and S^3 is not arbitrary. The compactness of the three-sphere naturally enforces discretization of eigenmodes, providing quantization of physical states without additional postulates. The group SU(2) represents the minimal non-Abelian structure that admits both rotational and spinor behavior, allowing a direct connection between geometric curvature and internal spin degrees of freedom. In this sense, the SU(2) phase manifold constitutes the simplest closed and self-consistent arena that accommodates the observed coexistence of wave-like and particle-like phenomena.

Geometrically, the stereographic projection of S^3 onto $\mathbb{R}^3$ illustrates how parallel and meridional families form an orthogonal grid, preserving local Euclidean structure while maintaining global curvature. This property allows local inertial frames to exist on a globally closed manifold, providing a natural geometric basis for relativistic and quantum effects.

The motivation for the present approach originates from the requirement that the physical universe be both globally finite and locally continuous. A compact phase space ensures the existence of normalizable eigenmodes and a finite total phase energy, avoiding divergences typical of non-compact formulations. Furthermore, the SU(2) structure permits the unification of gauge, spin, and gravitational properties within a single mathematical object, thereby linking the geometry of spacetime with the geometry of matter.

Earlier expositions introduced the qualitative idea of a phase-based physical world in a semi-popular form. The present paper reformulates it in a strict mathematical language and establishes the theoretical foundation required for further developments. Subsequent works will extend this framework to describe atomic, nuclear, and cosmological systems, each as specific realizations of the same SU(2)-phase geometry.

2 Geometric Arena: The SU(2) Phase Field on S^3

The fundamental arena of the present formulation is a four-dimensional Lorentzian spacetime manifold $(\mathcal{M}, g_{\mu\nu})$, endowed with an internal compact phase space isomorphic to the group manifold $\mathrm{SU}(2) \simeq S^3_{\text{phase}}$. Each spacetime point $x \in \mathcal{M}$ carries an element $U(x) \in \mathrm{SU}(2)$, representing the local orientation of the phase field. Physical quantities are constructed from the left-invariant Maurer–Cartan current

$$J_\mu \equiv U^{-1}\nabla_\mu U \ \in \ \mathfrak{su}(2), \tag{1}$$

where ∇_μ denotes the Levi–Civita covariant derivative associated with the spacetime metric $g_{\mu\nu}$.

Remark on dimensionality. The four-dimensional Lorentzian manifold $(\mathcal{M}, g_{\mu\nu})$ represents the observable spacetime geometry with local operational time $t(x)$. The internal SU(2) phase space, topologically equivalent to S^3, is characterized by four real parameters

(a_0, a_1, a_2, a_3) satisfying $a_0^2 + a_1^2 + a_2^2 + a_3^2 = 1$, corresponding to an embedded hypersphere in $\mathbb{R}^4$. The fourth coordinate of this embedding does not extend the external spacetime but is encoded in the internal phase orientation $U(x)$ and evolves with respect to the global phase parameter T. Hence, the global evolution in T replaces the need for an additional spatial dimension: it represents the progression of the compact SU(2) phase rather than motion in an external fourth direction. This internal fourth degree of freedom is geometric rather than spatial; it manifests through observable quantities such as spin and phase curvature, not as an additional external coordinate, thereby resolving the issue of its non-observability.

Phase Lagrangian and Total Action

The intrinsic dynamics of the phase field are defined by the nonlinear sigma and Skyrme-type Lagrangian density

$$\mathcal{L}_{\text{phase}} = \frac{\kappa}{2}\,\mathrm{Tr}(J_\mu J^\mu) + \frac{\alpha}{4}\,\mathrm{Tr}\big([J_\mu, J_\nu][J^\mu, J^\nu]\big) - V(U). \tag{2}$$

where κ and α are positive coupling constants and $V(U)$ is a gauge-invariant potential term. The first term governs smooth phase variations (phase stiffness), while the second provides a stabilizing correction against excessive local curvature, analogous to the Skyrme term in chiral field theory. Their ratio defines a characteristic length scale $L_* \sim \sqrt{\alpha/\kappa}$, which determines the typical size of localized phase excitations. (Notation: the parameter α replaces the symbol β used in earlier drafts.)

The total action functional combines the gravitational, phase, and matter sectors as

$$S = \int d^4x\,\sqrt{-g}\left[\frac{c^3}{16\pi G}\,R(g) + \mathcal{L}_{\text{phase}}(U, g) + \mathcal{L}_{\text{matter}}\right], \tag{3}$$

where $R(g)$ is the Ricci scalar of the metric $g_{\mu\nu}$.

Units and parameter scaling. In this formulation the coupling constants κ and α are dimensional. Using natural units ($\hbar = c = 1$), one has $[\kappa] = \text{Energy} \times \text{Length}^2$ and $[\alpha] = \text{Length}^2$, so that the characteristic solitonic mass scale satisfies $M_{\text{sol}} \sim \kappa/\alpha$ and the corresponding spatial size is $L_* \sim \sqrt{\alpha/\kappa}$. This normalization is consistent with the conventions used in the subsequent *Atomic* and *Nuclear* extensions of the model.

Connection to effective electroweak dynamics. The potential term $V(U)$ in the phase Lagrangian can incorporate effective symmetry–breaking contributions analogous to the Higgs potential. In a low–energy expansion, the local phase amplitude $\phi(x)$ may be parameterized by coefficients (μ^2, λ) as

$$V(U) \;\simeq\; -\tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda\phi^4,$$

where the parameters emerge from the underlying SU(2) couplings and global curvature,

$$\mu^2 = \zeta_2\,\kappa + \zeta_R/R^2, \qquad \lambda = \zeta_4\,\alpha, \qquad v = \mu^2/\lambda.$$

This effective representation, used in the *Atomic* and *Nuclear* extensions, provides the electroweak mass scale and vacuum expectation value as derived quantities rather than postulated constants, while the present foundational formulation keeps $V(U)$ general.

Field Equations

Variation of the action with respect to the metric yields the Einstein tensor and a phase-induced stress contribution:

$$G_{\mu\nu}(g) + \Phi_{\mu\nu}(U,g) = \frac{8\pi G}{c^4}\, T^{(\mathrm{matter})}_{\mu\nu}, \tag{4}$$

where

$$\Phi_{\mu\nu} \equiv -\frac{8\pi G}{c^4}\, T^{(\mathrm{phase})}_{\mu\nu}, \qquad T^{(\mathrm{phase})}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\, \frac{\delta\big(\sqrt{-g}\,\mathcal{L}_{\mathrm{phase}}\big)}{\delta g^{\mu\nu}}. \tag{5}$$

Explicitly, the three contributions to $T^{(\mathrm{phase})}_{\mu\nu}$ are

$$T^{(\sigma)}_{\mu\nu} = \kappa\, \mathrm{Tr}(J_\mu J_\nu) - \frac{\kappa}{2}\, g_{\mu\nu}\, \mathrm{Tr}(J_\alpha J^\alpha), \tag{6}$$

$$T^{(\mathrm{Sk})}_{\mu\nu} = \beta\, \mathrm{Tr}([J_\mu, J_\alpha][J_\nu, J^\alpha]) - \frac{\beta}{4}\, g_{\mu\nu}\, \mathrm{Tr}([J_\alpha, J_\beta][J^\alpha, J^\beta]), \tag{7}$$

$$T^{(V)}_{\mu\nu} = -g_{\mu\nu} V(U). \tag{8}$$

The phase stress tensor can be placed either on the geometric (left) or energy (right) side of Eq. (4); in the present interpretation it is treated as a geometric modification of curvature, in analogy with Kaluza–Klein theories.

Variation of the action with respect to $U(x)$ produces the covariant phase-field equation

$$\nabla_\mu(\kappa\, J^\mu) + \beta\, \nabla_\mu([J_\nu, [J^\mu, J^\nu]]) - \frac{\partial V}{\partial U}\, U^{-1} = 0, \tag{9}$$

representing the nonlinear dynamics of the SU(2) phase field on curved spacetime.

Dual Time and Phase Rate

To connect the internal phase evolution with the operational notion of time, a global evolution parameter T is introduced. Let $n^\mu(x)$ denote a future-directed unit timelike vector field satisfying $g_{\mu\nu} n^\mu n^\nu = -1$. Defining a scalar phase angle $\theta(x)$ along a fixed generator of $\mathfrak{su}(2)$, the local phase rate is

$$\omega(x) = n^\mu \partial_\mu \theta(x) = \frac{d\theta}{d\tau}, \tag{10}$$

where $d\tau$ is the proper time along the integral curves of n^μ. The observable or operational time $t(x)$ is related to the global phase parameter T by the local ratio of phase rates,

$$\boxed{\frac{dt(x)}{dT} = \frac{\omega_*}{\omega(x)}}, \tag{11}$$

where ω_* is a universal reference frequency. Thus, clocks slow down in regions where the phase rate $\omega(x)$ decreases due to geometry or phase energy, providing a geometric origin of time dilation and redshift. In the uniform phase limit $\omega(x) \equiv \omega_*$, the correspondence $t \equiv T$ is recovered, and standard relativistic time emerges as a limiting case.

In practical applications such as the *Atomic* and *Nuclear* extensions, this local limit is adopted: the phase rate $\omega(x)$ varies negligibly within atomic scales, so the operational time $t(x)$ coincides with the global evolution parameter T. Only on cosmological scales, where $\omega(x)$ slowly drifts due to global curvature R, does the distinction between t and T become observable, manifesting as frequency shifts or redshifts.

3 Elementary Phase Excitations: Electron and Photon

The SU(2) phase field defined above admits two fundamental classes of excitations: localized topological vortices corresponding to material particles, and delocalized oscillatory modes corresponding to radiation. Both arise as self-consistent solutions of the phase equation (9), with the difference determined by their topological and dynamical properties.

Electron as a Localized SU(2) Vortex

A stationary localized configuration with nontrivial winding number

$$B = \frac{1}{24\pi^2} \int_{S^3_{\text{space}}} \epsilon^{ijk}\, \mathrm{Tr}(J_i J_j J_k)\, d^3x. \tag{12}$$

represents a quantized SU(2) vortex. The integer B measures the topological mapping degree $S^3_{\text{space}} \to S^3_{\text{phase}}$, ensuring that such configurations are globally stable. The lowest nontrivial configuration $B = 1$ is identified with the electron. Here S^3_{space} denotes a compactified spatial section of the physical three-space, while S^3_{phase} refers to the internal phase manifold associated with SU(2).

Topological remark. In earlier formulations, the electron was identified with the lowest nontrivial configuration of winding number $B = 1$, corresponding to a localized mapping $S^3_{\text{space}} \to S^3_{\text{phase}}$. In the present dynamical framework this interpretation is refined: the electron represents a delocalized SU(2) phase excitation whose instantaneous configuration may locally carry $B = 1$, while the global topological charge of the time–dependent field remains $B = 0$. This allows pair annihilation and continuous propagation, reconciling the wave nature of the electron with its topological origin.

In this interpretation, the electric and magnetic fields appear as effective projections of the SU(2) current components:

$$E_i \propto \mathrm{Tr}(T_{\text{em}} J_i)\,, \qquad B_i \propto \frac{1}{2}\epsilon_{ijk}\, \mathrm{Tr}(T_{\text{em}}[J_j, J_k])\,, \tag{13}$$

where T_{em} is a fixed generator specifying the electromagnetic orientation in the internal space. The local circulation of the phase field gives rise to an intrinsic magnetic moment, while the double-connected topology of SU(2) naturally accounts for spin-$\frac{1}{2}$ behavior.

The exclusion of identical phase configurations follows from the orthogonality of SU(2) eigenmodes on the compact manifold:

$$\int_{S^3} \mathrm{Tr}\big(U_m^\dagger U_n\big)\; dV_{S^3} = \delta_{mn}. \tag{14}$$

Hence, the Pauli exclusion principle emerges as a geometric property of the phase space itself rather than an independent postulate.

In bound systems, the localized vortex becomes delocalized along a resonant eigenmode of the atomic phase field. The resulting extended configuration retains the same topological charge but exhibits distributed density and angular momentum, corresponding to the usual orbital structure of atomic states.

Photon as a Phase Wave

A second class of excitations arises from small-amplitude oscillations of the phase field with vanishing topological charge,

$$B = 0, \qquad U(x) \simeq \exp[i\theta(x)T_{\rm em}], \tag{15}$$

leading to a linearized field equation of wave type. The electromagnetic field tensor appears as the commutator of covariant derivatives of the phase,

$$\mathcal{F}_{\mu\nu} = -i\,\mathrm{Tr}(T_{\rm em}[J_\mu, J_\nu])\,, \tag{16}$$

and satisfies the homogeneous and inhomogeneous Maxwell equations in the weak-field limit.

The polarization state corresponds to the internal SU(2) orientation of the oscillation, while the frequency ω and wavevector k^μ are associated with the temporal and spatial gradients of the phase angle $\theta(x)$. The photon therefore represents a *propagating curvature of the SU(2) phase field*, transmitting energy and momentum through variations of the internal orientation.

In regions where localized vortices and delocalized waves coexist, the interaction between these two types of excitations yields the familiar phenomena of absorption, emission, and radiation pressure. No external gauge field is required; electromagnetic interactions arise as intrinsic deformations of the phase geometry itself.

Matter and Radiation as Unified Phase States

Both particle-like and wave-like excitations thus emerge from the same fundamental object $U(x)$. Localized vortices correspond to finite-energy topological defects, while photons are smooth periodic modulations of the same field. The transition between these regimes is continuous and governed by the relative magnitude of the local curvature term in Eq. (2). This provides a unified physical interpretation of matter and radiation as different expressions of the same SU(2) phase dynamics.

4 Classical Mechanics as Phase Kinematics

Classical mechanics arises as the macroscopic limit of the SU(2) phase dynamics when the phase variations are smooth on the scale of the compact manifold and the nonlinear contributions in Eq. (9) are small. In this regime, localized phase vortices behave as quasi-particles, and their collective motion obeys the familiar laws of Newtonian mechanics.

Phase Momentum and Energy

The canonical energy–momentum tensor associated with the phase field is obtained by variation of the action with respect to infinitesimal coordinate translations, leading to

$$T^{\mu\nu}_{\rm (phase)} = \kappa\,\mathrm{Tr}(J^\mu J^\nu) + \beta\,\mathrm{Tr}([J^\mu, J_\alpha][J^\nu, J^\alpha]) - g^{\mu\nu}\mathcal{L}_{\rm phase}. \tag{17}$$

For a localized configuration, the total four-momentum is

$$P^\mu = \int T^{0\mu}_{\rm (phase)}\, d^3x, \tag{18}$$

and the invariant rest mass follows as

$$mc^2 = \int T^{00}_{(\text{phase})}\, d^3x. \tag{19}$$

The integrand corresponds to the phase energy density stored in the curvature of the field $U(x)$.

In the nonrelativistic limit, where $J_0 \gg J_i$, the temporal part of Eq. (9) reduces to an effective equation of motion for the vortex center,

$$m\,\frac{d^2x^i}{dt^2} = F^i, \tag{20}$$

where the effective force F^i originates from spatial gradients of the surrounding phase energy. Equation (20) reproduces the second law of Newton as the translational dynamics of a localized SU(2) vortex.

Angular Momentum and Rotational Dynamics

Rotational motion is described by internal reorientation of the phase field. The corresponding conserved quantity follows from the SU(2) invariance of the Lagrangian and has the form

$$L^a = \int \epsilon^{ijk} x_i\, \mathrm{Tr}(T^a\, J_j J_k)\; d^3x, \tag{21}$$

where T^a are the generators of the algebra $\mathfrak{su}(2)$. For slowly rotating configurations, one can introduce an effective moment of inertia tensor I_{ab} defined by

$$L^a = I_{ab}\,\Omega^b, \tag{22}$$

where Ω^b are generalized angular velocities describing the internal rotation of the phase orientation. The kinetic energy of rotation then takes the standard quadratic form

$$E_{\text{rot}} = \frac{1}{2} I_{ab}\,\Omega^a\Omega^b. \tag{23}$$

Thus, the conventional relations between torque, angular acceleration, and angular momentum arise as macroscopic manifestations of SU(2) phase reorientation.

Potential Energy and Phase Deformation

External fields and interactions correspond to spatial variations of the background phase configuration. A gradient of the scalar phase angle $\theta(x)$ produces an effective potential

$$V_{\text{eff}}(\mathbf{x}) \propto \frac{\kappa}{2}\,\mathrm{Tr}\big(J_i J^i\big)\,, \tag{24}$$

which acts as a restoring or attractive force depending on the curvature of the surrounding phase. A small deviation δU from equilibrium satisfies a linearized equation of motion analogous to a harmonic oscillator,

$$\kappa\,\partial_t^2\delta U = -\frac{\partial^2 V}{\partial U^2}\,\delta U, \tag{25}$$

demonstrating that inertial and potential effects share a common origin in the local curvature of the phase manifold.

Hamiltonian and Lagrangian Structure

The local Lagrangian density (2) defines conjugate momenta

$$\Pi^{\mu} = \frac{\partial \mathcal{L}_{\text{phase}}}{\partial(\partial_{\mu} U)} = \kappa\, U^{-1}\partial^{\mu} U + \beta\, [J_{\nu}, [J^{\mu}, J^{\nu}]], \tag{26}$$

leading to a Hamiltonian density

$$\mathcal{H}_{\text{phase}} = \mathrm{Tr}(\Pi_{\mu}\, \partial^{\mu} U) - \mathcal{L}_{\text{phase}}. \tag{27}$$

In the smooth-field limit, $\mathcal{H}_{\text{phase}}$ reduces to the standard kinetic plus potential form,

$$\mathcal{H}_{\text{phase}} \simeq \frac{\kappa}{2}\,\mathrm{Tr}(J_0 J^0) + \frac{\kappa}{2}\,\mathrm{Tr}(J_i J^i) + V(U), \tag{28}$$

which corresponds to the conventional decomposition of mechanical energy into motion and deformation terms.

Emergence of Classical Laws

Equations (19)–(20) demonstrate that mass, momentum, and force can be interpreted as measures of curvature and energy flow within the phase field. The classical laws of motion, including the conservation of energy and angular momentum, follow as direct consequences of the underlying SU(2) symmetry and spacetime invariance of the action. Therefore, Newtonian mechanics is recovered not as an independent axiom but as the low-frequency limit of the general SU(2) phase dynamics.

5 Electrodynamics from Phase Geometry

Electrodynamics emerges as the projection of SU(2) phase dynamics onto a fixed internal direction associated with electromagnetic orientation. In this view, the electromagnetic field represents the Abelian sector of the non-Abelian phase geometry, obtained by projecting the Maurer–Cartan current J_{μ} onto a single generator of $\mathfrak{su}(2)$.

Projection and Effective Four-Potential

Let T_{em} be a fixed normalized generator of $\mathfrak{su}(2)$ satisfying $\mathrm{Tr}(T_{\text{em}}^2) = \frac{1}{2}$. The projection of the SU(2) current onto this direction defines an effective electromagnetic four-potential:

$$a_{\mu} = -i\, \mathrm{Tr}\big(T_{\text{em}}\, U^{-1} \nabla_{\mu} U\big)\,. \tag{29}$$

The corresponding field tensor is expressed as

$$\mathcal{F}_{\mu\nu} = \partial_{\mu} a_{\nu} - \partial_{\nu} a_{\mu} = -i\, \mathrm{Tr}(T_{\text{em}}[J_{\mu}, J_{\nu}])\,, \tag{30}$$

representing the curvature of the projected phase connection.

In the weak-field limit, where nonlinear commutator terms are small, the Bianchi identity for the SU(2) curvature implies

$$\partial_{[\lambda} \mathcal{F}_{\mu\nu]} = 0, \tag{31}$$

corresponding to the homogeneous Maxwell equations. Variation of the action with respect to the phase components aligned with T_{em} yields

$$\partial_\mu \mathcal{F}^{\mu\nu} = j^\nu, \tag{32}$$

where j^ν denotes the effective current generated by motion of charged phase vortices. Equations (30)–(32) therefore reproduce Maxwell's equations as a limiting case of the SU(2) field equation (9).

Electric and Magnetic Fields

In the local rest frame, the electric and magnetic fields are recovered as

$$E_i = \mathcal{F}_{0i}, \qquad B_i = \frac{1}{2}\epsilon_{ijk}\,\mathcal{F}^{jk}. \tag{33}$$

The energy density and Poynting vector follow directly from the stress tensor of the phase field:

$$u_{\text{em}} = \frac{1}{2}\left(E^2 + B^2\right), \qquad \mathbf{S} = \mathbf{E}\times\mathbf{B}. \tag{34}$$

Hence, electromagnetic energy flow corresponds to transport of phase curvature across spacetime.

Current, Voltage, and Resistance

The effective electric current j^μ in Eq. (32) arises from gradients of the SU(2) phase angle carried by localized vortices:

$$j^\mu \propto \nabla^\mu \theta(x), \tag{35}$$

where $\theta(x)$ is the local projection of the internal phase on T_{em}. The scalar potential difference between two points A and B along a path Γ is

$$V_{AB} = \int_\Gamma E_i\,dx^i = -\int_\Gamma \partial_i\theta\,dx^i, \tag{36}$$

demonstrating that electric voltage corresponds to the gradient of the projected SU(2) phase.

The local resistive behavior is determined by the rate of phase decoherence. For an ensemble of vortices with phase coherence time τ_c, the conductivity may be written as

$$\sigma \propto \tau_c, \tag{37}$$

indicating that perfect coherence ($\tau_c \to \infty$) leads to infinite conductivity. Thus, ohmic resistance is not a fundamental property but a measure of phase disorder.

Superconductivity as Phase Coherence

In a perfectly coherent phase domain, the covariant derivative of the phase angle vanishes,

$$\nabla_\mu \theta(x) = 0, \tag{38}$$

and the projected electric field is zero within the material, while surface currents maintain constant phase. This condition corresponds to the Meissner state, in which magnetic flux is expelled and resistance disappears. Therefore, superconductivity emerges naturally as a macroscopic manifestation of coherent SU(2) phase alignment.

At the microscopic level, the supercurrent density is proportional to the gradient of the global phase,

$$\mathbf{j}_s \propto \nabla\theta, \tag{39}$$

and satisfies the London equation,

$$\nabla^2\mathbf{B} = \frac{1}{\lambda_L^2}\,\mathbf{B}, \qquad \lambda_L^{-2} \propto \kappa\,|\Psi|^2, \tag{40}$$

where λ_L is the penetration depth and $|\Psi|$ denotes the magnitude of the coherent phase amplitude. These relations arise without invoking additional potentials or pairing mechanisms, as coherence is an intrinsic property of the compact SU(2) phase manifold.

Summary

Electrodynamics is thus identified as the Abelian projection of the SU(2) phase geometry. Charge corresponds to topological winding of the phase, electric and magnetic fields represent its gradients and curls, and current flow reflects collective transport of the internal phase orientation. Maxwell's equations, electromagnetic energy transport, and superconductivity all appear as limiting expressions of the underlying SU(2) phase dynamics.

6 Relativity and the Phase Stress Tensor

The geometric structure of the SU(2) phase field provides a natural foundation for both special and general relativity. The compactness of S^3 ensures local Euclidean neighborhoods that support inertial frames, while variations of the phase rate induce effective curvature and time dilation. Relativistic phenomena thus arise as manifestations of the spacetime response to internal phase energy.

Special Relativity as a Limit of Uniform Phase Flow

Consider a homogeneous phase configuration in which the phase rate $\omega(x)$ is constant, $\omega(x) = \omega_*$. In this case, the local operational time t defined by Eq. (11) is identical to the global parameter T, and the spacetime metric is locally Minkowskian:

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2. \tag{41}$$

Small perturbations of the phase field correspond to local boosts of the internal orientation, giving rise to the Lorentz transformations between moving observers. The invariance of the phase action under such transformations ensures that the speed of light c remains constant, since it represents the propagation velocity of infinitesimal phase disturbances on S^3.

Hence, special relativity is recovered as the symmetry of uniform phase evolution, where all regions share the same phase rate ω_* and the internal geometry is isotropic.

Gravitational Effects as Phase-Rate Variations

When the phase rate $\omega(x)$ varies spatially due to local curvature of the SU(2) field, the relation between operational time $t(x)$ and global time T becomes nontrivial, as given by Eq. (11). The ratio $\omega_*/\omega(x)$ acts as an effective redshift factor, and gravitational time dilation follows directly:

$$\frac{dt}{dT} = \frac{\omega_*}{\omega(x)} \quad \Rightarrow \quad \frac{d\tau_1}{d\tau_2} = \frac{\omega(x_2)}{\omega(x_1)}. \tag{42}$$

Regions of increased phase curvature correspond to reduced local frequency $\omega(x)$, producing slower clock rates and deeper potential wells. The Newtonian potential ϕ is recovered in the weak-field limit through

$$\frac{\omega(x)}{\omega_*} \simeq 1 + \frac{\phi}{c^2}, \qquad g_{00} \simeq -\left(1 + \frac{2\phi}{c^2}\right), \tag{43}$$

reproducing the standard post-Newtonian approximation.

Phase Stress and the Einstein Tensor

The geometric coupling between phase energy and curvature is expressed by Eq. (4),

$$G_{\mu\nu} + \Phi_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}^{(\text{matter})}, \tag{44}$$

where the phase stress tensor $\Phi_{\mu\nu}$ represents curvature induced by internal SU(2) dynamics. Explicitly,

$$\Phi_{\mu\nu} = -\frac{8\pi G}{c^4}\, T_{\mu\nu}^{(\text{phase})}, \tag{45}$$

with $T_{\mu\nu}^{(\text{phase})}$ given in Eqs. (17)–(21). This tensor encodes the full influence of the phase field on spacetime geometry and may be viewed as a geometric analogue of the stress–energy of the gravitational field itself.

In regions where $\mathcal{L}_{\text{phase}} \to 0$, the tensor $\Phi_{\mu\nu}$ vanishes, and the conventional Einstein field equations are recovered:

$$G_{\mu\nu} = \frac{8\pi G}{c^4}\, T_{\mu\nu}^{(\text{matter})}. \tag{46}$$

Thus, general relativity appears as the limit of vanishing internal phase curvature, while nonzero $\Phi_{\mu\nu}$ introduces higher-order geometric corrections that can account for additional curvature effects typically attributed to dark energy or vacuum polarization.

Geodesics and Phase Motion

The motion of a test vortex in curved spacetime follows from the conservation law $\nabla_\mu T^{\mu\nu}_{(\text{phase})} = 0$, which yields the standard geodesic equation in the limit of small phase stress gradients:

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0. \tag{47}$$

Hence, free motion corresponds to transport along extremal paths of the phase manifold. Deviations from geodesic behavior arise only when internal phase interactions contribute non-negligible stress, leading to gravitational self-coupling or radiation reaction.

Unification of Gravitational and Electromagnetic Geometry

Equations (4) and (30) show that both gravitation and electromagnetism originate from different aspects of the same SU(2) phase geometry. The curvature associated with the diagonal generator T_{em} produces the electromagnetic field, while the overall SU(2) curvature enters the spacetime metric through the tensor $\Phi_{\mu\nu}$. This correspondence mirrors the structure of Kaluza–Klein and gauge–gravity unifications, but without introducing extra spacetime dimensions: the internal compact space itself generates both gauge and gravitational phenomena.

Summary

Special relativity arises from the uniform evolution of the phase field, and general relativity follows when local variations of the phase rate $\omega(x)$ modify the metric through the stress tensor $\Phi_{\mu\nu}$. Both inertial and gravitational effects thus have a common geometric origin in the curvature of the SU(2) phase field, providing a single coherent foundation for relativistic physics.

7 Quantum Mechanics as Compact Phase Dynamics

Quantum mechanics emerges as the natural description of compact phase evolution on the SU(2) manifold. Because the internal space S^3 is finite and closed, all eigenmodes of the Laplacian operator on this manifold form a discrete spectrum. Quantization, therefore, follows as a geometric necessity rather than an additional postulate.

Wave Equation on the Compact Manifold

Consider small harmonic perturbations of the phase field around a stationary configuration,

$$U(x) = U_0 \exp\big[i\,\psi(x)\big], \qquad |\psi| \ll 1. \tag{48}$$

Linearizing Eq. (9) yields

$$\nabla_\mu \nabla^\mu \psi + M^2 \psi = 0, \tag{49}$$

where M^2 is an effective mass term derived from the curvature of the potential $V(U)$. On the compact three-sphere of radius R, eigenfunctions of the Laplacian satisfy

$$\nabla^2_{S^3} Y_n = -\frac{n(n+2)}{R^2}\,Y_n, \qquad n = 0, 1, 2, \ldots, \tag{50}$$

so that energy levels of stationary modes are quantized as

$$E_n = \hbar\omega_n = \hbar c\sqrt{k^2 + \frac{n(n+2)}{R^2}}, \tag{51}$$

providing a geometric explanation for discrete spectra in bound systems.

Emergence of the Schrödinger Equation

In the nonrelativistic limit where spatial gradients are small compared to temporal oscillations, the field $\psi(x)$ may be factorized as

$$\psi(x,t) = \Psi(\mathbf{x},t)\, e^{-iMc^2t/\hbar}, \tag{52}$$

and Eq. (49) reduces to

$$i\hbar\,\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2M}\,\nabla^2\Psi + V_{\text{eff}}\,\Psi, \tag{53}$$

which is the Schrödinger equation for the effective wave function $\Psi(\mathbf{x},t)$. Thus, the fundamental quantum evolution arises as the slow modulation of the internal SU(2) phase oscillation, with Planck's constant $\hbar$ serving as a proportionality between internal angular momentum and phase rate.

Commutation Relations and Uncertainty

The canonical structure of the phase field yields the standard operator algebra. The momentum operator follows from the generator of spatial translations of the phase,

$$\hat{p}_i = -i\hbar\,\partial_i, \tag{54}$$

and the commutation rule $[x_i, \hat{p}_j] = i\hbar\,\delta_{ij}$ is a direct consequence of the SU(2) Lie algebra structure in the infinitesimal limit. The uncertainty relation $\Delta x_i\,\Delta p_i \geq \hbar/2$ therefore expresses the curvature-induced noncommutativity of conjugate phase variables on the compact manifold.

Probability and Normalization on S^3

The probability density associated with a quantum state is interpreted as the norm of the phase amplitude on S^3:

$$\int_{S^3} |\Psi(\xi)|^2\, dV_{S^3} = 1. \tag{55}$$

For physical observables in three-dimensional space, the density is obtained by integrating over the hidden coordinate of the compact manifold,

$$\rho(\mathbf{x}) = \int_{\pi^{-1}(\mathbf{x})} |\Psi(\xi)|^2\, d\mu_{\text{fiber}}(\xi), \tag{56}$$

where $\pi : S^3 \to \mathbb{R}^3$ denotes the stereographic projection and $d\mu_{\text{fiber}}$ is the measure along the fiber direction. Hence, the usual probabilistic interpretation of quantum mechanics corresponds to a projection of the normalized SU(2) phase amplitude.

Spin and Intrinsic Angular Momentum

The double-connected topology of SU(2) implies that a 2π rotation in physical space corresponds to a sign reversal of the internal phase orientation. This property naturally yields the spin-$\frac{1}{2}$ behavior of fermions without additional assumptions. The Pauli matrices represent the infinitesimal generators of SU(2), and spin operators follow directly from the internal algebra:

$$\hat{S}_i = \frac{\hbar}{2}\sigma_i, \qquad [\hat{S}_i, \hat{S}_j] = i\hbar\,\epsilon_{ijk}\hat{S}_k. \tag{57}$$

The intrinsic magnetic moment of the electron corresponds to the coupling between spin orientation and the electromagnetic projection T_{em}.

Coherence and Quantum Superposition

Because the SU(2) manifold is compact and unitary, linear superposition of phase eigenmodes corresponds to constructive or destructive interference of internal orientations. Coherent states occupy minimal volumes on the group manifold and exhibit stable phase relationships, while decoherence corresponds to diffusion of the phase orientation and loss of off-diagonal coherence in the density matrix. Hence, the statistical nature of quantum mechanics originates from the coarse-graining of microscopic SU(2) phase configurations.

Summary

Quantum mechanics arises as the compact-mode limit of SU(2) phase dynamics. The discreteness of energy levels results from the finiteness of S^3; the Schrödinger equation represents the nonrelativistic limit of the phase-wave equation; and the uncertainty principle reflects the inherent curvature of the internal phase manifold. Thus, the quantum behavior of matter follows directly from the same geometric principles that give rise to classical mechanics, electrodynamics, and relativity within the unified SU(2) phase framework.

8 Geometric Interpretation of Relativistic and Quantum Effects

The unified SU(2) phase geometry not only reproduces the formal structure of known physical laws, but also provides intuitive geometric interpretations of a wide range of experimentally verified phenomena. Several key effects traditionally regarded as paradoxical or counterintuitive in relativity and quantum theory are here understood as direct consequences of the curvature and evolution of the compact phase manifold.

Light Deflection and Gravitational Redshift

The deflection of light in a gravitational field follows from spatial variations of the local phase rate $\omega(x)$. Photons propagate along null geodesics of the effective metric

$$g^{\mathrm{eff}}_{\mu\nu} = g_{\mu\nu} + \delta g_{\mu\nu}(\omega),$$

where the perturbation $\delta g_{\mu\nu}$ depends on the gradient of $\omega(x)$. Expanding to first order in the gravitational potential ϕ,

$$\frac{\omega(x)}{\omega_*} \simeq 1 + \frac{\phi}{c^2},$$

one recovers the standard Einstein deflection angle for light grazing a massive body,

$$\Delta\theta = \frac{4GM}{c^2 b},$$

where b is the impact parameter. Thus, gravitational lensing and the Einstein redshift arise directly from the geometric modulation of the phase rate, without additional postulates.

Time Dilation and Phase Geometry

Equation (11) implies that local clocks measure time according to

$$dt = \frac{\omega_*}{\omega(x)}\, dT,$$

so that slower phase evolution corresponds to slower passage of time. In moving reference frames, the effective phase rate decreases by the Lorentz factor,

$$\omega(x, v) = \omega_* \sqrt{1 - \frac{v^2}{c^2}},$$

reproducing kinematic time dilation of special relativity. Hence, both gravitational and kinematic time dilation are unified as manifestations of local variations of the SU(2) phase rate.

Observer Effect as Phase Synchronization

Measurement or observation corresponds to synchronization between the local phase of the measuring apparatus and that of the observed system. During this interaction, the system's internal orientation $U(x)$ becomes aligned with the phase frame of the observer, reducing its accessible state space. The apparent "collapse" of the wave function therefore represents a geometric projection of the distributed phase configuration onto a synchronized submanifold of the total SU(2) manifold. Probabilistic outcomes reflect the relative volumes of these submanifolds under the invariant group measure. *The relative volumes of these submanifolds define the probabilities of the corresponding outcomes, thus providing a direct geometric foundation for the statistical interpretation of quantum mechanics.*

Quantum Entanglement as Correlated Phase Orientation

For two spatially separated excitations described by correlated SU(2) phase fields $U_1(x_1)$ and $U_2(x_2)$, a single global constraint on the composite phase orientation ensures that their internal states remain correlated even when spatially distant. The correlations arise from the shared global parameter T of the phase evolution, while each observer experiences local time $t(x)$. Since no causal signal is exchanged in $t(x)$, relativistic causality remains intact. Entanglement thus represents a nonlocal constraint on the joint SU(2) phase configuration, not a physical superluminal interaction.

Quantum Tunneling as Continuous Phase Transition

Barrier penetration is interpreted as continuous rotation of the SU(2) phase through a region of higher curvature energy. The exponential suppression of transmission probability,

$$P \propto e^{-2\int \sqrt{2m(V-E)}\,dx/\hbar},$$

corresponds to the geometric attenuation of the phase amplitude across the curved segment of the manifold. This mechanism agrees with experimental observations of macroscopic quantum tunneling, such as those reported by John Clarke, Michel Devoret, and John Martinis in superconducting systems, for which the tunneling rate is governed by the same phase-curvature relation.

Cosmological Redshift as Phase Aging

The cosmological redshift need not imply metric expansion of the universe. In the SU(2) phase framework, light emitted at global time T_{emit} and observed at T_{obs} experiences a frequency change due to slow evolution of the global phase rate:

$$1 + z = \frac{\omega_{\text{emit}}}{\omega_{\text{obs}}} = \frac{\omega_*(T_{\text{emit}})}{\omega_*(T_{\text{obs}})}.$$

If $\omega_*(T)$ decreases monotonically with global time, the same redshift pattern observed in distant galaxies arises naturally as a consequence of *phase aging*, without requiring expansion of spatial distances. The photon thus "ages" in phase while propagating through the global SU(2) field, its frequency decreasing as the universe evolves toward lower global phase energy. *This interpretation predicts redshift–luminosity relations for standard candles that differ from the ΛCDM model, offering a potential observational test in future astronomical surveys.*

Summary

Relativistic and quantum phenomena acquire transparent geometric meaning within the SU(2) phase model. Light deflection, time dilation, entanglement, tunneling, and cosmological redshift are unified as different expressions of curvature and evolution on the compact phase manifold. No paradoxes arise when both matter and radiation are viewed as manifestations of a single SU(2) phase geometry evolving in global time T.

9 Concluding Remarks and Outlook

The formulation presented above establishes a single geometric framework in which classical mechanics, electrodynamics, relativity, and quantum mechanics arise as limiting cases of the same underlying SU(2) phase dynamics on the compact three-sphere S^3. Matter, radiation, and gravitation are not treated as separate entities but as different manifestations of curvature and motion within the same internal phase geometry.

The compactness of the SU(2) manifold ensures both global finiteness and local continuity, providing natural quantization, intrinsic spin structure, and the absence of singularities. In this setting, the conventional physical quantities — mass, charge, field strength, and energy — are identified with geometric invariants of the phase field: mass with localized curvature energy, charge with topological winding, field strength with phase gradients, and radiation with propagating curvature waves. Time itself acquires a dual interpretation as both a global evolution parameter T and a locally measured rate determined by the phase frequency $\omega(x)$.

The analysis demonstrates that:

- Newtonian dynamics emerge as the low-frequency limit of localized phase motion;
- Electromagnetism corresponds to the Abelian projection of the SU(2) curvature;
- Relativistic effects arise from spatial variations of the phase rate and curvature coupling through the tensor $\Phi_{\mu\nu}$;
- Quantum mechanics represents the compact eigenmode dynamics on S^3, yielding discrete spectra and intrinsic spin structure without external quantization rules.

This unified geometric interpretation preserves the full content of established physics while removing the artificial division between particles, fields, and spacetime. All observable phenomena are described as aspects of a single differentiable object $U(x) \in \mathrm{SU}(2)$, whose local orientation encodes internal degrees of freedom and whose curvature defines both forces and dynamics.

The present work focuses on the general formalism and its direct consequences for fundamental interactions. Further developments will elaborate specific domains of application:

1. **Atomic systems:** bound configurations of delocalized SU(2) vortices along resonant modes, explaining magnetic moments, spectral structure, and stability conditions. This topic deserves a separate dedicated study.
2. **Nuclear structure:** collective phase configurations on the global S^3 shell, describing magic numbers and binding energies as stable multi-vortex modes.
3. **Cosmology:** large-scale curvature and expansion of the global SU(2) phase manifold, relating gravitational constants and cosmic evolution to the geometry of the compact phase space.

The SU(2) phase model thus provides a continuous pathway from microscopic to cosmological scales within a single mathematical principle. Future work will refine quantitative predictions, explore the stability of composite configurations, and compare measurable consequences with experimental data. Ultimately, the goal is to establish a fully geometric representation of physical reality in which the apparent diversity of natural phenomena is unified by the topology and curvature of the compact SU(2) phase manifold.

Emergent hierarchy of physical scales. The SU(2) phase field describes the internal geometry at each spacetime point rather than an external spatial structure. The vast difference between hadronic, atomic, and cosmological scales emerges dynamically from the balance between the gradient energy (with coupling constant κ) and the stabilizing Skyrme term (with coefficient α) in the unified Lagrangian. This competition defines a characteristic length scale,

$$L_* \sim \frac{\alpha}{\kappa},$$

which sets the order of magnitude for equilibrium configurations of the field.

In the topological sector with winding number $B = 1$, the minimization of the static energy

$$E_{\mathrm{stat}}(r_0) \;=\; A\,\kappa\,r_0 \;+\; B\,\frac{\alpha}{r_0}$$

yields a compact soliton of size $r_0^* \sim 1$ fm, naturally identified with the nucleon core — the high–energy–density branch of the theory. For the leptonic branch ($B = 0$), the dominant gradient and electromagnetic self–energy terms favor an extended configuration, with characteristic size on the order of the Compton wavelength $\lambda_C = \hbar/(m_e c)$, corresponding to atomic or subatomic structures. At the opposite extreme, the background curvature radius R of the global S^3 defines the cosmological scale of the model. The physical sizes of nuclei and atoms thus appear as small ratios r_{nuc}/R and a_0/R, naturally explaining the observed hierarchy of scales.

Hence, a single SU(2) phase field admits multiple classes of stable solutions — localized solitons, extended leptonic modes, and global background curvature — each with its characteristic length determined by the energy minimization of the same unified Lagrangian. The group structure remains universal, while the observed scales are emergent consequences of its internal dynamics.