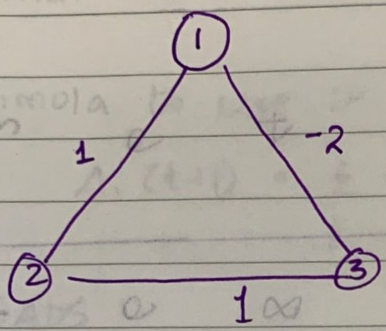
NOTE

- symmetric network with $w_{ij} = w_{ji}$

- $w_{ii} = 0$

HOPFIELDS NETWORK - TUTORIAL 5

EXERCISE 11



NOTE

1) Question can be attempted with both $\{-1,1\}$ and $\{0,1\}$ as initial activation states

2) Hamming distance = 1

I. Let us do this with $\{-1,1\}$ activation state.

- To attempt the transition diagram, we need a state transition table. (TABLE 1)

- Number of states can be found using $2^N = 2^3 = 8$ (N = no of nodes)

- ACTIVATION FUNCTION CALCULATION

$$A_i(t+1) = \text{sign}\left(\sum_j w_{ij} A_j(t)\right) \quad \text{— (1)}$$

Select a neuron at random to update its value (asynchronous) at $A_i(t+1)$.

$$\text{sign}(x) = 1 \quad \text{if } x > 0$$
$$\text{sign}(x) = -1 \quad \text{if } x < 0$$
$$\text{sign}(x) = A_i(t) \quad \text{if } x = 0$$

STATE NAME	x_1	x_2	x_3	FIRING x_1/ NEW STATE	FIRING x_2/ NEW STATE	FIRING x_3/ NEW STATE	ENERGY FUNCTIONAL
1	1	1	1	4	1	2	0
2	1	1	-1	2	2	2	-2
3	1	-1	-1	3	3	3	-2
4	-1	1	1	4	4	4	-2
5	-1	1	-1	2	8	4	4
6	1	-1	1	6	6	6	-2
7	-1	-1	1	6	1	3	4
8	-1	-1	-1	3	8	6	0

Let us take state 1 and decide to fire neuron 1.

Initial state = {1,1,1}

Formula to use :-

$$A_i(t+1) = \sum_j W_{ij} \cdot A_j(t)$$

READS

The sum of [weights between i and j multiplied by activation function of j].

∴ For firing node 1, (which is connected to node 2 and 3)

$$A_i(t+1) = \left[W_{12} \cdot A_j(2) + W_{13} \cdot A(3) \right]$$
$$= [(1)(1) + (-2)(1)]$$
$$= 1 - 2$$
$$= -1.$$

According to the sign function the result is less than zero which means new state is -1.

∴ By firing node 1, the new state transition is

{1,1,1} to {-1,1,1}

↳ In the table {-1,1,1} is state 4

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- For firing node 2 with initial ~~state~~ activation $\{1,1,1\}$.

$$A_i(t+1) = \left[W_{23} \cdot A(3) + W_{21} \cdot A(1) \right]$$

$$= (1)(1) + (1)(1)$$

$$= 2.$$

Activation function is $2 > 0$ therefore the new state is

$\{1,1,1\}$ to $\{1,1,1\}$

↳ In the table it is state 1.

- For firing node 3 with initial activation $\{1,1,1\}$

$$A_i(t+1) = \left[W_{31} \cdot A(1) + W_{32} \cdot A(2) \right]$$

$$= (-2)(1) + (1)(1)$$

$$= -2+1$$

$$= -1$$

Activation function is less than zero so the new state is

$\{1,1,1\}$ to $\{1,1,-1\}$

↳ In the table this is state 2.

ENERGY FUNCTION FOR STATE {1,1,1}

$$E(W, A) = -\frac{1}{2} \sum_{i,j} W_{ij} \cdot A_i \cdot A_j \quad \rightarrow (2)$$

2 ways of calculating

1) Write down all the terms by combination of i and j.

2) Remove the $\frac{1}{2}$ and only use 3 terms within the sum function i.e [symmetrical network]

$$E = -[W_{i1} \cdot A_i \cdot A_1 + W_{i2} \cdot A_i \cdot A_2 + W_{i3} \cdot A_i \cdot A_3]$$

Applying method 2, For initial state {1,1,1} the energy function is

$$E = -[W_{12} \cdot A_1 \cdot A_2 + W_{13} \cdot A_1 \cdot A_3 + W_{23} \cdot A_2 \cdot A_3]$$

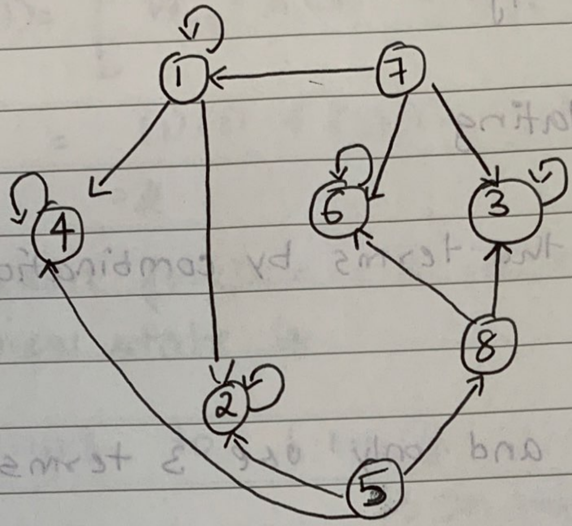
$$= -[(1)(1)(1) + (-2)(1)(1) + (1)(1)(1)]$$

$$= -[1 + (-2) + 1]$$

$$= 0$$

[Pls continue to calculate it this way]

STATE TRANSITION DIAGRAM $\{-1,1\}$



The stable states are
$\{1,1,-1\}$
$\{1,-1,-1\}$
$\{-1,1,1\}$
$\{-1,-1,1\}$
with Energy function = -2

CALCULATION OF HOPFIELD NET FOR - $\{0,1\}$

- The activation function is given by.

Act $\geq 0 \rightarrow 1$
$< 0 \rightarrow 0$

- The formula for activation function is the same

$$A_i(t+1) = \sum_j W_{ij} \cdot A_j(t)$$

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Note
→ Model answer starts from state 0 to state 7

The state transition table is

STATE NAME	x_1	x_2	x_3	FIRING x_1	FIRING x_2	FIRING x_3	ENERGY FUNCTION
0	0	0	0	4	2	1	0
1	0	0	1	1	3	1	0
2	0	1	0	6	2	3	0
3	0	1	1	3	3	3	-1
4	1	0	0	4	6	4	0
5	1	0	1	1	7	4	2
6	1	1	0	6	6	6	-1
7	1	1	1	3	7	6	0

For initial activation state $\{0,0,0\}$

Firing x_1:-

$= \sum_j W_{ij} \cdot A_j(t)$

$= [W_{12} \cdot A(2) + W_{13} \cdot A(3)]$

$= [(1)(0) + (-2)(0)]$

$= 0$

As per activation function, the answer $= 0$ ≤ 0
the new state will change from

$\{0,0,0\} \rightarrow \{1,0,0\}$

Firing x_2:-

$= [\sum_j W_{ij} \cdot A_j(t)]$

$= [-2(0) + (1)(0)]$

$= 0$

New state is $\{0,0,0\} \rightarrow \{0,1,0\}$

Firing x_3:

$= [(-2)(0) + (1)(0)]$

$= 0$

New state is $\{0,0,0\} \rightarrow \{0,0,1\}$

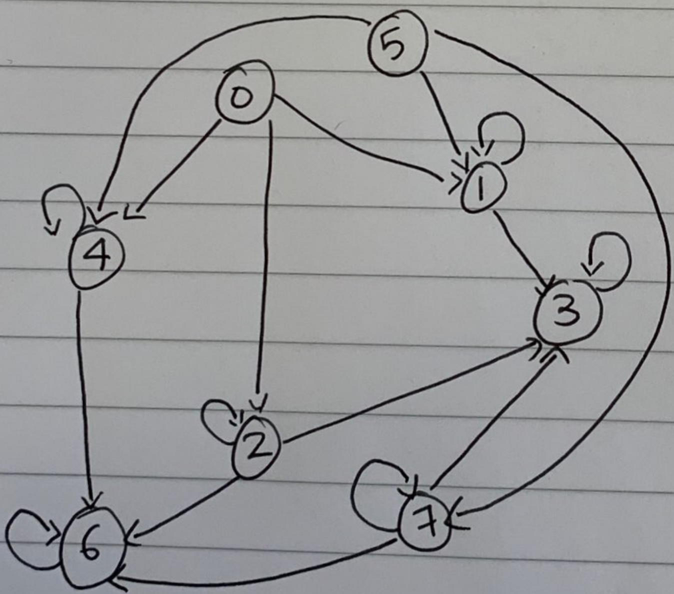
ENERGY FUNCTION CALCULATION FOR $\{0,0,0\}$
(Refer formula ②)

$$\therefore E = -[W_{12} \cdot A_1 \cdot A_2 + W_{13} \cdot A_1 \cdot A_3 + W_{23} \cdot A_2 \cdot A_3]$$

$$= -[(1)(0)(0) + (-2)(0)(0) + (1)(0)(0)]$$

$$= 0$$

STATE TRANSITION DIAGRAM



The stable states are:

$\{0,1,1\}$
and
$\{1,1,0\}$
with Energy $= -1$

NOTE:- Order of the states do not matter as far as you have the correct Energy value.