Problem Set 5—Key

CENG 340–Introduction to Environmental Engineering
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Problems

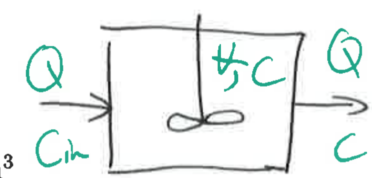
1. **(12 pt)** Aeration to convert $[Fe^{2+}]$ to $[Fe^{3+}]$

   Given:

   $[Fe^{2+}]_{in} = 7.0$ mg/L
   $[Fe^{2+}]_{out} = 0.25$ mg/L
   $k = 0.175\,\text{min}{-1}$
   $Q = 40{,}000\ \frac{m^3}{d}$

   - CMFR

     $$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out} - \dot{m}_{rxn}$$

     $$0 = QC_{in} - QC - kCV$$

     $$V = \frac{QC_{in} - QC}{kC} = \frac{40{,}000\ \frac{m^3}{day} \times (7 - 0.25)\ \frac{mg}{L}}{0.25\ \frac{mg}{L} \times 0.175\,\text{min}^{-1} \times \frac{60\,\text{min}}{h} \times \frac{24\,h}{day}} = 4286\,m^3$$

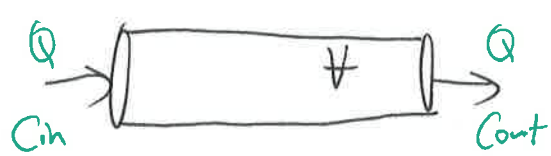
     For CMFR, V = 4286 m^3

     $$\theta = \frac{V}{Q} = \frac{4286\,m^3}{40{,}000\ \frac{m^3}{day}} \times \frac{24\,h}{day} = 2.6\,h$$

     For CMFR, $\theta = 2.6$ h

   - PFR

     $$C_{out} = C_{in} \times e^{-k\frac{V}{Q}}$$

     $$\frac{C_{out}}{C_{in}} = e^{-k\frac{V}{Q}}$$

     $$\ln\left(\frac{C_{out}}{C_{in}}\right) = -k\frac{V}{Q}$$

$$V = -\ln\left(\frac{C_{out}}{C_{in}}\right) \times \frac{40{,}000\,\frac{m^3}{day}}{0.175\,min^{-1}} \times \frac{1\,day}{24\,h} \times \frac{1\,h}{60\,min} = 529\,m^3$$

For PFR, $V = 529\ m^3$

$$\theta = \frac{V}{Q} = \frac{529\,m^3}{40{,}000\,\frac{m^3}{day}} \times \frac{24\,h}{day} = 0.32\,h$$

For PFR, $\theta = 0.32$ h

2. **(12 pt)** Alum and Alkalinity Requirements for Coagulation

$$Al_2(SO_4)_3 \bullet 14\,H_2O + 6\,HCO_3^- \longrightarrow 2\,Al(OH)_{3(s)} + 3\,SO_4^{2-} + 14\,H_2O + 6\,CO_2$$

Alkalinty: 6 moles of HCO_3^- (or 6 equivalents of alkalinity) are consumed per mole of alum added.

$$\text{Alkalinity Required} = \frac{12.5\,\text{mg Alum}}{L} \times \frac{1\,\text{mole alum}}{594\,\text{g alum}} \times \frac{6\,\text{eq alk}}{1\,\text{mole alum}} \times \frac{50\,\text{g}\,CaCO_3}{1\,\text{eq alk}}$$

$$\text{Alkalinity Required} = 6.3\,mg/L$$

Alum

$$\text{Alum Required} = 12.5\,\frac{mg}{L} \times 50 \times 10^6\,\frac{gal}{day} \times \frac{3.78\,L}{gal} \times 365\,\frac{day}{year} \times \frac{1\,kg}{10^6\,mg}$$

$$\text{AlumRequired} = 862,300\,kg/year$$

3. **WATER SOFTENING**

   (a) **(10 pt)** *Problem 10.3 from the textbook:*
   Given:

   $[Ca^{2+}] = 70$ mg/L

   $[Mg^{2+}] = 40$ mg/L

   $[HCO_3^-] = 40$ mg/L

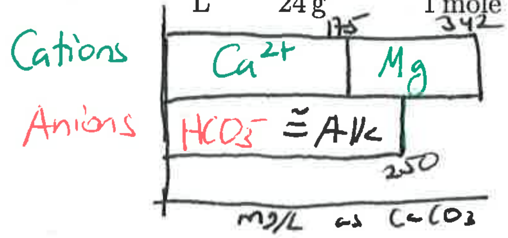
Convert concentrations of Ca^{2+} and Mg^{2+} to mg/L as $CaCO_3$.

$MW_{Ca^{2+}} = 40$ g/mole

$MW_{Mg^{2+}} = 24$ g/mole

$$Ca^{2+} = 70\,\frac{mg}{L} \times \frac{1\,mole}{40\,g} \times \frac{100\,g\,CaCO_3}{1\,mole} = 175\,mg/L\text{ as }CaCO_3$$

$$Mg^{2+} = 40\,\frac{mg}{L} \times \frac{1\,mole}{24\,g} \times \frac{100\,g\,CaCO_3}{1\,mole} = 166\,mg/L\text{ as }CaCO_3$$



$$\text{Total Hardness} = [Ca^{2+}] + [Mg^{2+}] = 175 + 166 = 342\,mg/L\text{ as }CaCO_3$$

Carbonate Hardness equals total hardness or concentration of HCO_3^-, whichever is smaller.

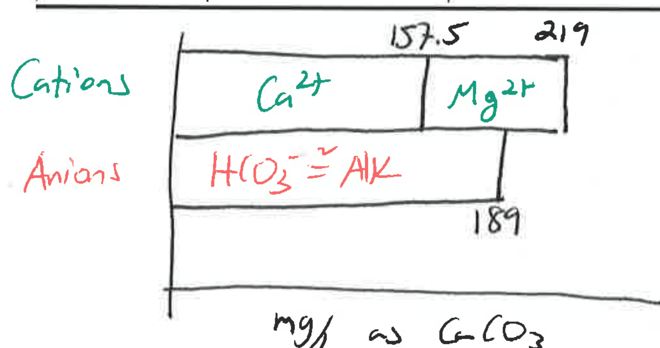
Carbonate Hardenss = 250 mg/L as $CaCO_3$

Noncarbonate Hardness = Total Hardness - Carbonate Hardness = 342- 250 = 92 mg/L as $CaCO_3$

(b) **(12 pt)** *Problem 10.5 from the Textbook:*

Convert CO_2 and the relevant ions to units of as $CaCO_3$:

Chemical	**Conc., mg/L**	**MW $CaCO_3$/MW Ion**	**Conc., mg/L as $CaCO_3$**
CO_2	17.6	100/44	40
Ca^{2+}	63	100/40	157.5
Mg^{2+}	15	100/24	61.5
HCO_3^-			189



Total Hardness = $[Ca^{2+}] + [Mg^{2+}] = 157.5 + 61.5 = 219$ mg/L as $CaCO_3$

Carbonate Hardness = $[HCO_3^-] = 189$ mg/L as $CaCO_3$

Calculate Lime Required for Softening:

i. To remove CO_2 (needed to raise pH for lime precipitation): 40 mg/L as $CaCO_3$
ii. To remove $Ca^{2+} = 157.5$ as $CaCO_3$
iii. Total Lime Added = $157.5 + 40 = 197.5$ mg/L as $CaCO_3$

Remaining Hardness consists of $[Mg^{2+}]$, since no magnesium was removed.

Remaining Hardness = 61.5 mg/L as $CaCO_3$

(c) **(12 pt)** FE Style Question

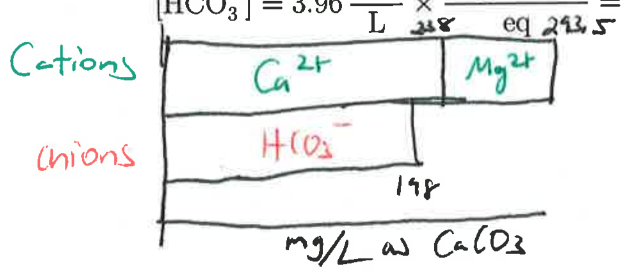
Convert all concentrations from meq/L to mg/L as $CaCO_3$

$$[CO_2] = 0.44\,\frac{\text{meq}}{\text{L}} \times \frac{50\,\text{g CaCO}_3}{\text{eq}} = 22\,\text{mg/L asCaCO}_3$$

$$[Ca^{2+}] = 4.76\,\frac{\text{meq}}{\text{L}} \times \frac{50\,\text{g CaCO}_3}{\text{eq}} = 238\,\text{mg/L asCaCO}_3$$

$$[Mg^{2+}] = 1.11\,\frac{\text{meq}}{\text{L}} \times \frac{50\,\text{g CaCO}_3}{\text{eq}} = 55.5\,\text{mg/L asCaCO}_3$$

$$[HCO_3^-] = 3.96\,\frac{\text{meq}}{\text{L}} \times \frac{50\,\text{g CaCO}_3}{\text{eq}} = 198\,\text{mg/L asCaCO}_3$$



Note: Noncarbonate Hardness that is due to $[Ca^{2+}]$ cannot be removed with lime. So, need lime to remove CO_2, carbonate hardness (which is all Ca^{2+}), and noncarbonate hardness that is Mg^{2+}.

Lime Requirements (in units of as $CaCO_3$ = $[CO_2] + [HCO_3^-] + [Mg^{2+}] = 198 + 55.5 + 22 = 275.5$ mg/L as $CaCO_3$

Convert lime requirements from as mg/L $CaCO_3$ to concentraion of lime (CaO).

Molecular weight of CaO = 56 g/mole.

$$275.5\,\frac{\text{mg CaCO}_3}{\text{L}} \times \frac{56\text{ g CaO}}{100\text{ g CaCO}_3} = 154\text{ mg/L CaO}$$

Calculate tons (metric) per day required:

$$\text{Tons per day of CaO} = 154\,\frac{\text{mg}}{\text{L}} \times 5 \times 10^6\,\frac{\text{gal}}{\text{day}} \times 3.78\,\frac{\text{L}}{\text{gal}} \times \frac{1\text{ ton}}{10^9\text{ mg}}$$

CaO Requirements = 2.92 ton/day
3.2 ton/day also acceptable answer (works for English tons, or for metric tons, assuming bulk lime is 90% lime).

4. **(12 pt)** Problem 10.8 in the Textbook:

Size the rapid-mix (coagulation) tank:

Given Q = 50 $\frac{m^3}{day}$

Assume θ = 1 to 2 min

For θ = 1 min:

$$V = Q \times \theta = 50\frac{m^3}{day} \times 1\text{ min} \times \frac{1\text{ day}}{24\text{ h}} \times \frac{1\text{ h}}{60\text{ min}}$$

V = 0.035 m^3

For θ = 2 min
V = 0.07 m^3

Acceptable values of V = 0.035–0.07 m^3

Calculate the power needed for mixing:

Assume $\bar{G}$ = 600 to 1000 s^{-1}

For 12 ^{0}C:

Use the table inside the cover of the text book and interpolate to calculate the dynamic viscosity of water at 12 ^{0}C.
For water at T = 12 ^{0}C, $\mu = 1.24\times10^{-3}\ \frac{kg}{m\times s} = \frac{N\times s^2}{m^2\times s} = \frac{N\times s}{m^2}$

$P = \bar{G}^2 \times \mu \times V$

For $\bar{G} = 800\ s^{-1}$:

$$P = (800s^{-1})^2 \times 1.24 \times 10^{-3}\,\frac{N\times s}{m^2} \times 0.035\,m^3 = 28\,\frac{N\times m}{s} = 28\,\frac{J}{s} = 28\,W$$

Acceptable valuse of P: 15.6 to 87 Watts

To calculate Power at 24 ^{0}C:

Use the table inside the cover of the text book and interpolate to calculate the dynamic viscosity of water at 24 ^{0}C:

$\mu = 9.13\times10^{-4}$

Same calculations as for 24 ^{0}C.

Acceptable value of P: 11.5 to 64 Watts.