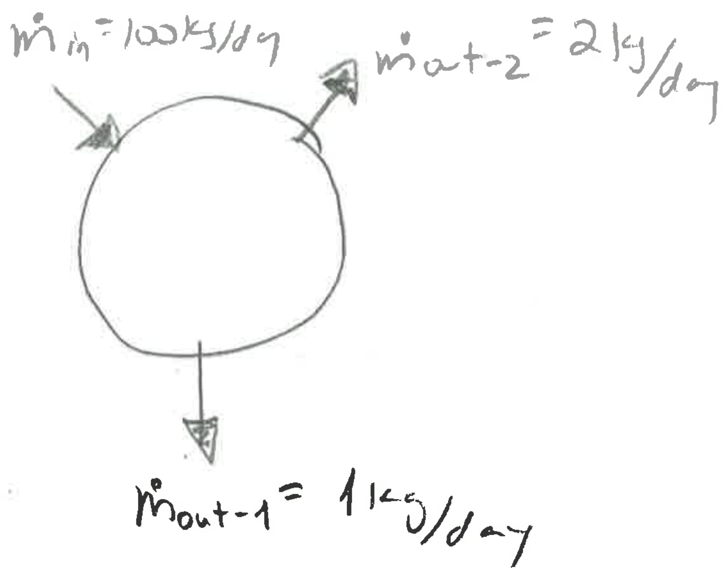
Pset 4 - KEY

CENG 340
Deborah Sills

1

11 pts



no rxn

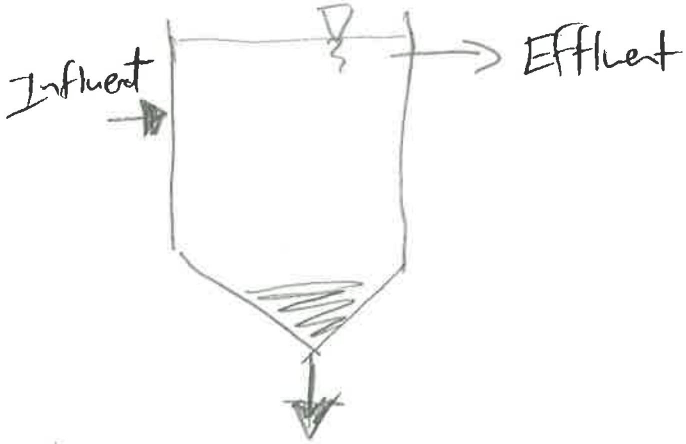
$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out} \pm \cancel{\dot{m}_{rxn}}$$

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out-1} - \dot{m}_{out-2}$$

$$\frac{dm}{dt} = 100\ kg/day - 1\ kg/day - 2\ kg/day$$

$$\boxed{\frac{dm}{dt} = 97\ kg/day}$$

(2)



11 pts

Assume that $Q_{in} = Q_{out} = Q$

b.c. $Q_{sludge} << Q_{out}$

Sludge production $= \frac{dm}{dt}$

MASS Balance on Sludge: no rxn

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out} \pm \cancel{\dot{m}_{rxn}}$$

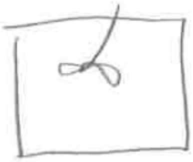
$$\frac{dm}{dt} = Q_{in} * C_{in} - Q_{out} * C_{out} = Q\left(C_{in} - C_{out}\right)$$

$$\frac{dm}{dt} = 3780\,m^3/day \left(220\,mg/L - 5\,mg/L\right) * \frac{1000\,L}{m^3} \times \frac{1\,kg}{10^6\,mg}$$

Sludge production $= \frac{dm}{dt} = 813\ kg/day$

3

BATCH REACTOR



Mass Balance on chloride

$$\frac{dm}{dt} = \cancel{\dot{m}_{in}} - \cancel{\dot{m}_{out}} - \dot{m}_{rxn}$$

no flow

11pt

$$V\frac{dC}{dt} = -V\left(\frac{dC}{dt}\right)_{rxn\ only}$$

$$\frac{dC}{dt} = -kC$$

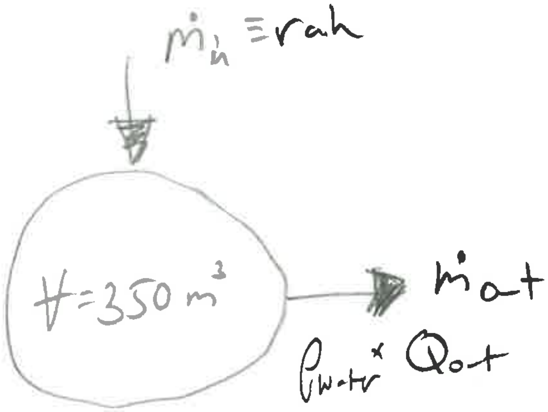
$$\int_{C_0}^{C} \frac{dC}{C} = \int_{0}^{t} -k\,dt$$

$$C = C_0 e^{-kt} = 1\,\text{mg/L}\; e^{-0.360\frac{1}{day} \cdot 1\,day}$$

$$\boxed{C = 0.698\ \text{mg/L}}$$

(4)

11 pt



$\rho_{H_2O} \equiv$ density of water

$V = 350\ m^3$

$t = 6\ h$

$Q_{out} = 320\ L/min$

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\cancel{\rho_{H_2O}} \frac{dV}{dt} = \cancel{\rho_{H_2O}} Q_{in} - \cancel{\rho_{H_2O}} Q_{out}$$

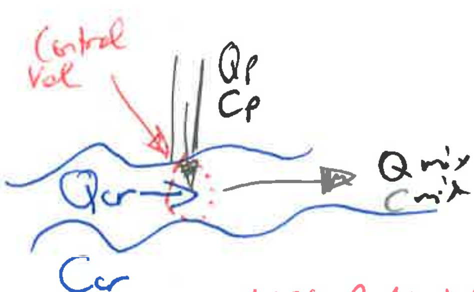
$$\frac{dV}{dt} = Q_{in} - Q_{out}$$

$$Q_{in} = \frac{dV}{dt} + Q_{out} = \frac{350\ m^3}{6\ h} \times \frac{1000\ L}{1\ m^3} \times \frac{1\ h}{60\ min} + 320\ L/min$$

$$\boxed{Q_{in} = 1290\ L/min}$$

(5)

11 pts



MASS BALANCE ON SALT

$$\frac{dm}{dt} = \dot{m}_{in} \; \dot{m}_{out} \pm \dot{m}_{rxn}$$

$$\frac{dm}{dt} = 0 \quad @ \text{ junction}$$

(5) cont...

no rxn

$$0 = \dot{m}_{in} - \dot{m}_{out} \pm \cancel{\dot{m}_{rxn}}$$

$$0 = Q_{cr}C_{cr} + Q_pC_p - Q_{mix}C_{mix}$$

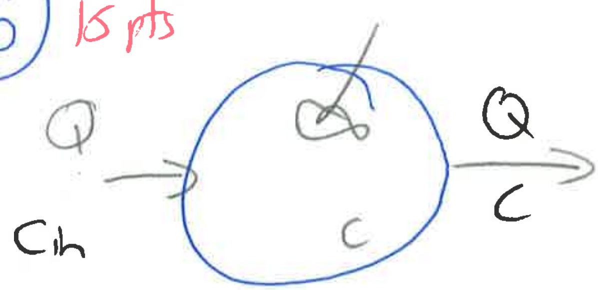
$$C_{mix} = \frac{Q_{cr}C_{cr} + Q_pC_p}{Q_{mix}}$$

but $Q_{mix} = Q_{cr} + Q_p = 0.5 + 2.8 = 3.3\ MGD$

$$C_{mix} = \frac{Q_{cr}C_{cr} + Q_pC_p}{Q_{cr} + Q_p} = \frac{2.8\ \cancel{MGD} \times 175\ mg/L + 0.5\ \cancel{MGD} \times 35{,}000\ mg/L}{3.3\ \cancel{MGD}}$$

$$C_{mix} = 5450\ mg/L = \underline{5.45 \times 10^3\ mg/L}$$

(6) 15 pts



$r = 0.001\ \frac{mg}{L \cdot day}$ zero order

$C_{in} = 3.4\ mg/L$

(a)

7.5 pts

$$\frac{d\cancel{m}}{dt} = \dot{m}_{in} - \dot{m}_{out} \pm \dot{m}_{rxn}$$

Steady state

$$0 = QC_{in} - QC - rV$$

$$C = \frac{QC_{in} - rV}{Q} = \frac{10^3\ m^3/day \times 3.4\ mg/L - 0.001\ \frac{mg}{\cancel{L} \cdot day} \times 10^6\ m^3}{10^3\ m^3/day}$$

$$\boxed{C = 3.4\ mg/L}$$ ← nothing happened :(

(6)(b)

$$\cancel{\frac{dm}{dt}} = \dot{m}_{in} - \dot{m}_{out} - \dot{m}_{rxn}$$

steady state 0

$$0 = QC_{in} - QC - kCV$$

$$C = \frac{QC_{in}}{Q + kV} = \frac{103\,m^3/day \times 3.4\,mg/L}{103\,m^3/day + 0.01\,\frac{1}{day} \times 106\,m^3}$$

~~Max~~ 5 pts

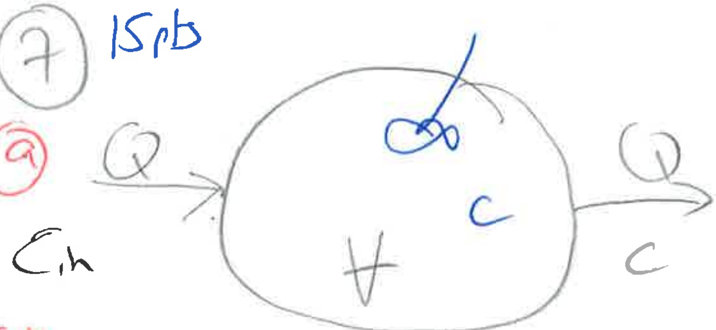
$$\boxed{C = 3.36\ mg/L}$$ ← nothing happened again :(

WHY?

Find out in lecture on Friday, 4 Oct!

(7) 15 pts

(a)



$$Area = 10\,ha \times 10{,}000\,m^2/ha = 100{,}000\,m^2$$

$$V = Area \times \underbrace{d}_{1m} = 100{,}000\,m^3$$

7.5 pts

(a)

$$\cancel{\frac{dm}{dt}} = \dot{m}_{in} - \dot{m}_{out} \pm \dot{m}_{rxn}$$

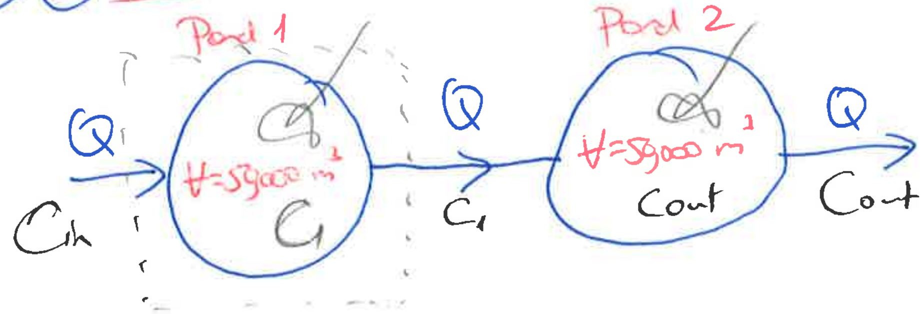
steady state 0

$$0 = QC_{in} - QC - kCV$$

$$k = \frac{Q\,(C_{in} - C)}{CV} = \frac{8640\,m^3/day\,(100-20)\,mg/L}{20\,mg/L \times 100{,}000\,m^3}$$

$$\boxed{k = 0.35\ day^{-1}}$$

7 b 15 pts



Mass Balance on Pond 1

$$\cancel{\frac{dm}{dt}} = \dot{m}_{in} - \dot{m}_{out} - \dot{m}_{rxn}$$

$$0 = QC_{in} - QC_1 - KC_1\forall$$

Eq. 1 $$C_1 = \frac{QC_{in}}{Q + KV}$$

Mass Balance on Pond 2

$$\cancel{\frac{dm}{dt}} = \dot{m}_{in} - \dot{m}_{out} - \dot{m}_{rxn}$$

$$0 = QC_1 - QC_{out} - KC_{out}\forall$$

Eq. 2 $$C_{out} = \frac{QC_1}{Q + KV}$$

Substitute C_1 from Eq. 1 into Eq. 2

$$C_{out} = \left(\frac{Q}{Q + KV}\right)^2 C_{in}$$

$$\left(\frac{Q}{Q + KV}\right)^2 = \frac{C_{out}}{C_{in}}$$

$$\frac{Q}{Q + KV} = \sqrt{\frac{C_{out}}{C_{in}}}$$

$$Q = (Q + KV)\sqrt{\frac{C_{out}}{C_{in}}}$$

$$K = \frac{Q\left(1 - \sqrt{\frac{C_{out}}{C_{in}}}\right)}{V\sqrt{\frac{C_{out}}{C_{in}}}} = \frac{8640\left(1 - \sqrt{\frac{20}{100}}\right)}{5000\sqrt{\frac{20}{100}}}$$

$$\boxed{K = 0.21\ day^{-1}}$$

7 c

5pts

k	0.35 day
Q	8640 m^3/day
V	100000 m^3

t days	Cout
0	20
1	12.9
2	8.4
3	5.4
4	3.5
5	2.3
6	1.5
7	0.9
8	0.6
9	0.4
10	0.3

When Sewage Stops Flowing – Non-Steady State Problem

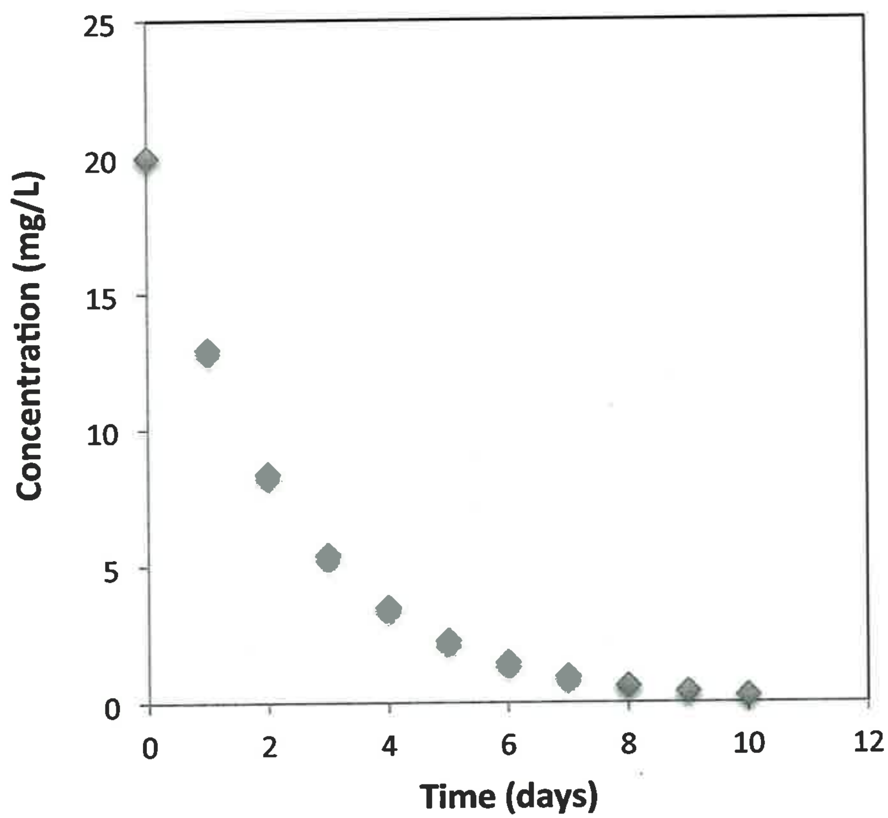
$c(0) = 20\,mg/L$

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out} - \dot{m}_{rxn}$$

$$V\frac{dC}{dt} = QC_{in} - QC - kCV$$

$$\int_{C(0)}^{C} \frac{dC}{C} = \int_0^t -\left(\frac{Q}{V}+k\right)dt$$

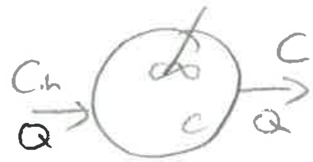
$$\boxed{C = C_0 e^{-\left(\frac{Q}{V}+k\right)t}}$$ – plot



15 pts ⑧

(a) Steady State CMFR

7.5 pts



96% removal

$C = 0.04\, C_{in}$

$k = 0.8\ {}^{1}/_{day}$

$$\frac{d\cancel{m}}{\cancel{dt}} = \dot{m}_{in} - \dot{m}_{out} - \dot{m}_{rxn}$$

$$0 = QC_{in} - QC - kC\forall$$

$$0 = C_{in} - C - kC\frac{V}{Q}$$

$$\text{HRT} \rightarrow \boxed{\frac{V}{Q} = \theta}$$

$$\theta = \frac{C_{in} - C}{kC} = \frac{(100-4)\,mg/L}{0.8\,{}^{1}/_{day} \cdot 4\,mg/L} = 30 \text{ days}$$

$$\boxed{\text{HRT} = 30 \text{ days}}$$

(b) Steady State PFR

7.5 pts

$$C_{out} = C_{in} \times e^{-k\theta}$$

$$e^{-k\theta} = \frac{C_{out}}{C_{in}}$$

$$\theta = \frac{-\ln\left(\frac{C_{out}}{C_{in}}\right)}{k} = \frac{-\ln\left(\frac{4}{100}\right)}{0.8\,{}^{1}/_{day}} = 4 \text{ day}$$

$$\boxed{\text{HRT} = 4 \text{ days}}$$

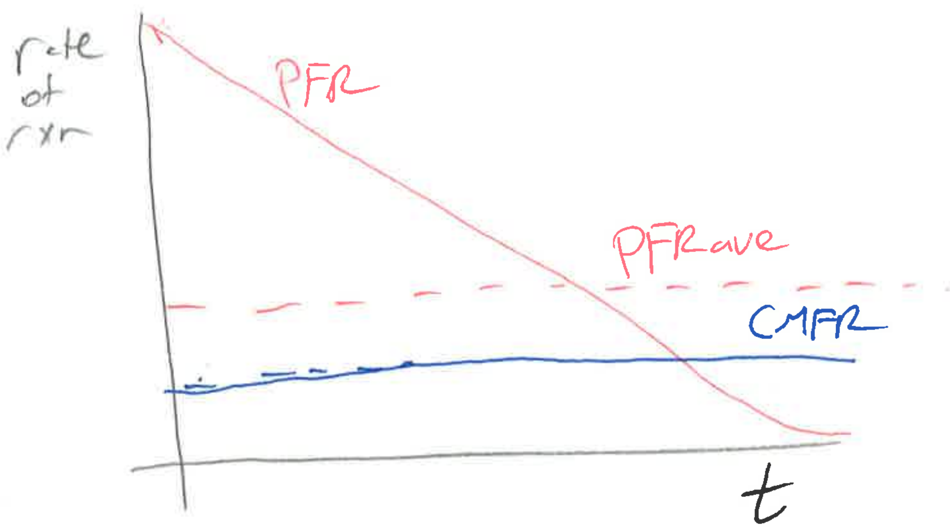
8 c For 1st order rxns

$$HRT_{PFR} < HRT_{CMFR}$$

5 pts extra credit.

Since the ~~concentration of~~ influent contaminant mixes throughout the CMFR as soon as the contaminant enters the reactor, the average rate of reaction in the CMFR is lower than in a PFR.

The influent contaminant in a PFR, on the other hand, does not mix with the fluid in the reactor, resulting in a higher reaction rate (compared to the CMFR) throughout more than half of the reactor.



If the rxn rate in PFR is higher, this results in a lower HRT (compared to CMFR) needed for the same performance! → $HRT_{PFR} < HRT_{CMFR}$