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Graphs of Basic Functions

Properties of Exponents

$b^m \cdot b^n = b^{m+n}$ $\quad b^0 = 1$

$\dfrac{b^m}{b^n} = b^{m-n} \quad (b \neq 0) \quad b^{-n} = \dfrac{1}{b^n} \quad (b \neq 0)$

$(b^m)^n = b^{m \cdot n}$

$(ab)^m = a^m b^m \quad \left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m} \quad (b \neq 0)$

Properties of Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real,

$a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[m \cdot n]{a}$

$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab} \quad \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}} \quad (b \neq 0)$

Factoring and Special Case Products

$a^2 + 2ab + b^2 = (a + b)^2$
$a^2 - 2ab + b^2 = (a - b)^2$
$a^2 - b^2 = (a + b)(a - b)$
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Complex Numbers

$i = \sqrt{-1}$ and $i^2 = -1$

For a real number $b > 0$, $\sqrt{-b} = i\sqrt{b}$

The complex numbers $a + bi$ and $a - bi$ are conjugates, and $(a + bi)(a - bi) = a^2 + b^2$.

Quadratic Formula

Given $ax^2 + bx + c = 0$, $a \neq 0$, the solutions are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Absolute Value Equations and Inequalities

If $k \geq 0$, then
$|u| = k$ is equivalent to $u = k$ or $u = -k$.
$|u| = |w|$ is equivalent to $u = w$ or $u = -w$.
$|u| < k$ is equivalent to $-k < u < k$.
$|u| > k$ is equivalent to $u < -k$ or $u > k$.

Distance Formulas

The distance between two points a and b on a number line is given by $|a - b|$ or $|b - a|$.

The distance between (x_1, y_1) and (x_2, y_2) is
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

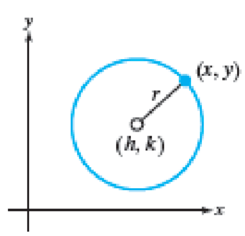
Midpoint Formula

The midpoint of the line segment between (x_1, y_1) and (x_2, y_2) is

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

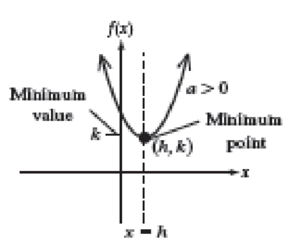
Circles

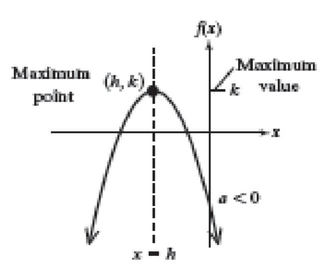
$(x - h)^2 + (y - k)^2 = r^2$ where $r > 0$



Quadratic Functions

$f(x) = ax^2 + bx + c \; (a \neq 0)$ $\quad$ Vertex: $\left[\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right]$
$f(x) = a(x - h)^2 + k \; (a \neq 0)$ $\quad$ Vertex: (h, k)





Slope and Average Rate of Change

Slope of a line through (x_1, y_1) and (x_2, y_2): $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

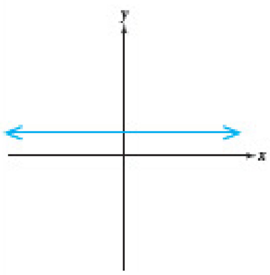
Average rate of change of $f(x)$ between (x_1, y_1) and (x_2, y_2): $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

Difference quotient: $\dfrac{f(x + h) - f(x)}{h}$

Graphs of Basic Functions

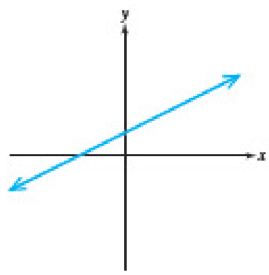
Constant Function

$f(x) = b$



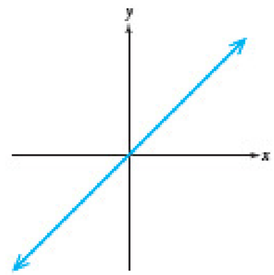
Linear Function

$f(x) = mx + b$



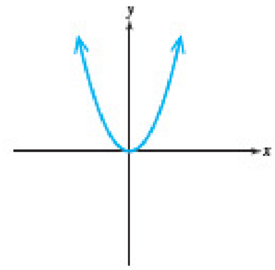
Identity Function

$f(x) = x$



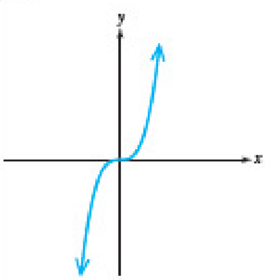
Quadratic Function

$f(x) = x^2$



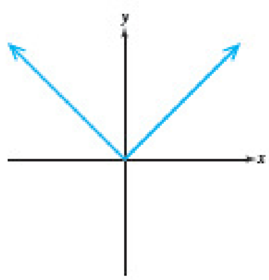
Cubic Function

$f(x) = x^3$



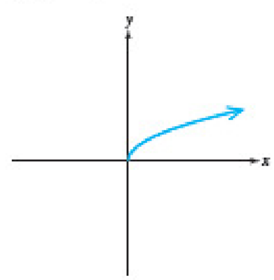
Absolute Value Function

$f(x) = |x|$



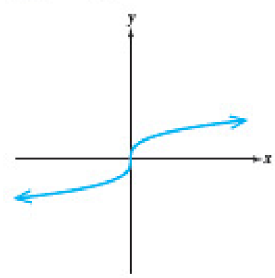
Square Root Function

$f(x) = \sqrt{x}$



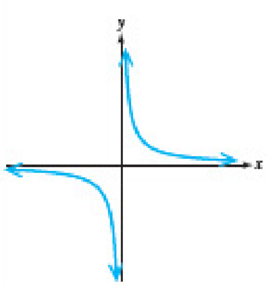
Cube Root Function

$f(x) = \sqrt[3]{x}$



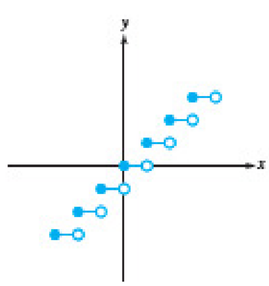
Reciprocal Function

$f(x) = \dfrac{1}{x}$



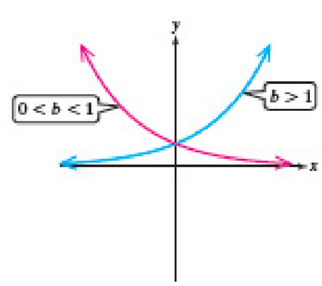
Greatest Integer Function

$f(x) = [\![x]\!]$



Exponential Function

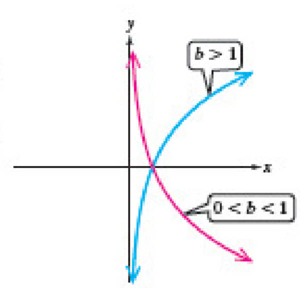
$f(x) = b^x$, where $b > 0$ and $b \neq 1$



Logarithmic Function

$f(x) = \log_b x$, where $b > 0$ and $b \neq 1$

$y = \log_b x \Leftrightarrow b^y = x$



Transformations of Graphs

Given $c > 0$ and $h > 0$, the graph of the given function is related to the graph of $y = f(x)$ as follows:

$y = f(x) + c$ Shift the graph of $y = f(x)$ up c units.
$y = f(x) - c$ Shift the graph of $y = f(x)$ down c units.

$y = f(x - h)$ Shift the graph of $y = f(x)$ to the right h units.
$y = f(x + h)$ Shift the graph of $y = f(x)$ to the left h units.

$y = -f(x)$ Reflect the graph of $y = f(x)$ over the x-axis.
$y = f(-x)$ Reflect the graph of $y = f(x)$ over the y-axis.

$y = af(x)$ If $a > 1$, stretch the graph of $y = f(x)$ vertically by a factor of a.
If $0 < a < 1$, shrink the graph of $y = f(x)$ vertically by a factor of a.

$y = f(ax)$ If $a > 1$, shrink the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{a}$.
If $0 < a < 1$, stretch the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{a}$.

Tests for Symmetry

Consider the graph of an equation in x and y. The graph of the equation is

- Symmetric to the **y-axis** if substituting $-x$ for x results in an equivalent equation.
- Symmetric to the **x-axis** if substituting $-y$ for y results in an equivalent equation.
- Symmetric to the **origin** if substituting $-x$ for x and $-y$ for y results in an equivalent equation.

Even and Odd Functions

- f is an **even function** if $f(-x) = f(x)$ for all x in the domain of f.
- f is an **odd function** if $f(-x) = -f(x)$ for all x in the domain of f.

Properties of Logarithms

$\log_b 1 = 0$ $\quad \log_b (xy) = \log_b x + \log_b y$

$\log_b b = 1$ $\quad \log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

$\log_b b^x = x$ $\quad \log_b x^p = p \log_b x$

$b^{\log_b x} = x$

Change-of-base formula: $\quad \log_b x = \dfrac{\log_a x}{\log_a b}$

$b^x = b^y$ implies that $x = y$.

$\log_b x = \log_b y$ implies that $x = y$.

Variation

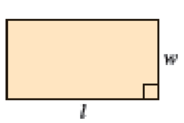
y varies **directly** as x.
y is **directly** proportional to x. $\Big\}\ y = kx$

y varies **inversely** as x.
y is **inversely** proportional to x. $\Big\}\ y = \dfrac{k}{x}$

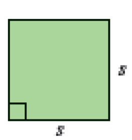
y varies **jointly** as w and x.
y is **jointly** proportional to w and x. $\Big\}\ y = kwx$

Perimeter and Circumference

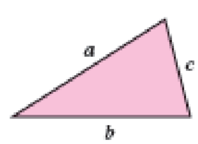
Rectangle
$P = 2l + 2w$



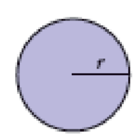
Square
$P = 4s$



Triangle
$P = a + b + c$

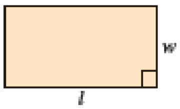


Circle
Circumference: $C = 2\pi r$

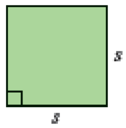


Area

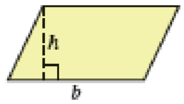
Rectangle
$A = lw$



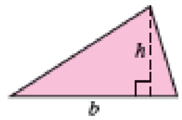
Square
$A = s^2$



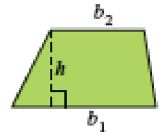
Parallelogram
$A = bh$



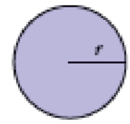
Triangle
$A = \frac{1}{2}bh$



Trapezoid
$A = \frac{1}{2}(b_1 + b_2)h$

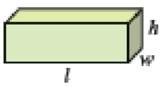


Circle
$A = \pi r^2$

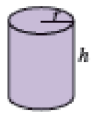


Volume

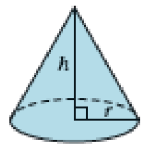
Rectangular Solid
$V = lwh$



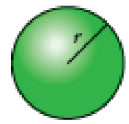
Right Circular Cylinder
$V = \pi r^2 h$



Right Circular Cone
$V = \frac{1}{3}\pi r^2 h$



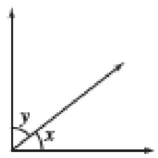
Sphere
$V = \frac{4}{3}\pi r^3$



Angles

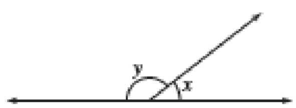
- Two angles are complementary if the sum of their measures is 90°.

$x + y = 90°$



- Two angles are supplementary if the sum of their measures is 180°.

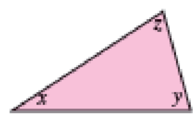
$x + y = 180°$



Triangles

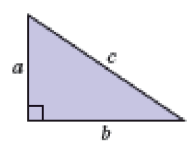
- The sum of the measures of the angles of a triangle is 180°.

$x + y + z = 180°$



- Given a right triangle with legs of length a and b, and hypotenuse of length c, the Pythagorean theorem indicates that

$a^2 + b^2 = c^2$



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Trigonometric Functions

$\sin t = y$ $\qquad \csc t = \frac{1}{y}(y \neq 0)$

$\cos t = x$ $\qquad \sec t = \frac{1}{x}(x \neq 0)$

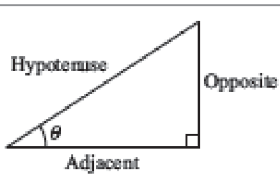
$\tan t = \frac{y}{x}(x \neq 0)$ $\qquad \cot t = \frac{x}{y}(y \neq 0)$

Right Triangle Trigonometry

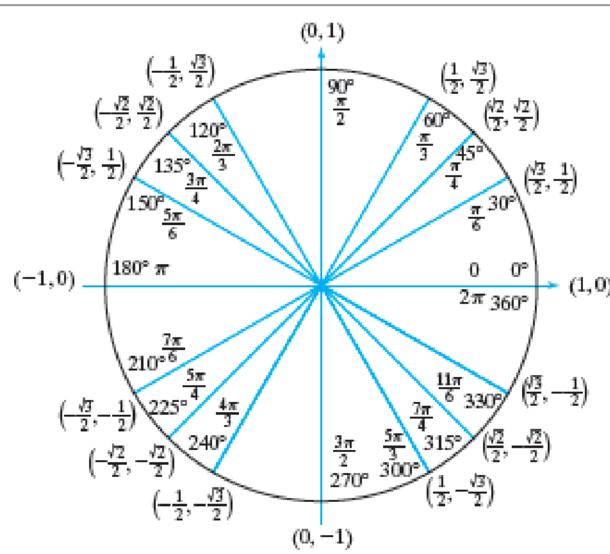
$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}}$ $\qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$



Unit Circle



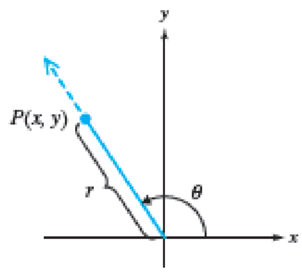
Trigonometric Functions of Any Angle

$\sin\theta = \dfrac{y}{r}$ $\qquad \csc\theta = \dfrac{r}{y}(y \neq 0)$

$\cos\theta = \dfrac{x}{r}$ $\qquad \sec\theta = \dfrac{r}{x}(x \neq 0)$

$\tan\theta = \dfrac{y}{x}(x \neq 0)$

$\cot\theta = \dfrac{x}{y}(y \neq 0)$



Inverse Trigonometric Functions

$y = \sin^{-1}x \quad \Leftrightarrow \quad \sin y = x$

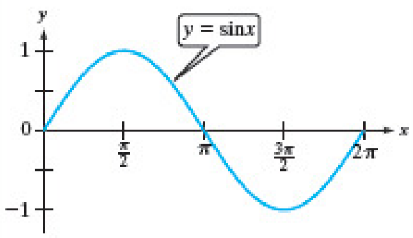
$-1 \leq x \leq 1$ and $-\dfrac{\pi}{2} \leq y \leq \dfrac{\pi}{2}$

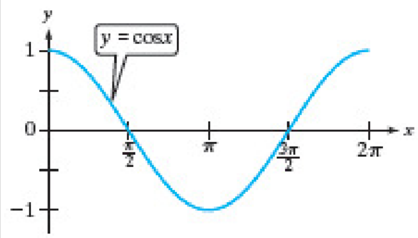
$y = \cos^{-1}x \quad \Leftrightarrow \quad \cos y = x$

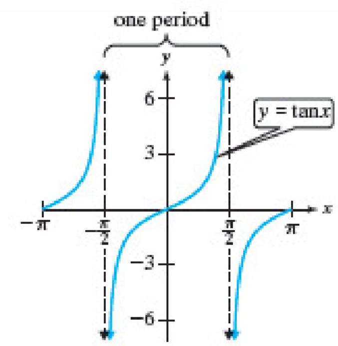
$-1 \leq x \leq 1$ and $0 \leq y \leq \pi$

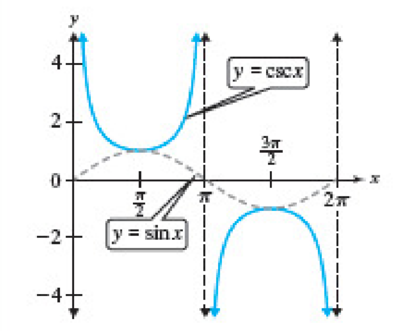
$y = \tan^{-1}x \quad \Leftrightarrow \quad \tan y = x$

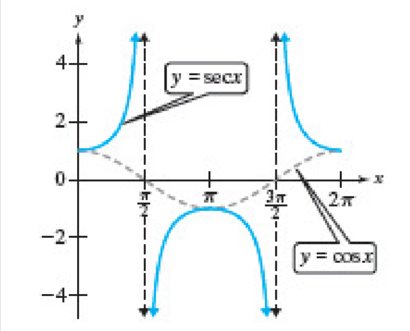
$x \in \mathbb{R}$ and $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$

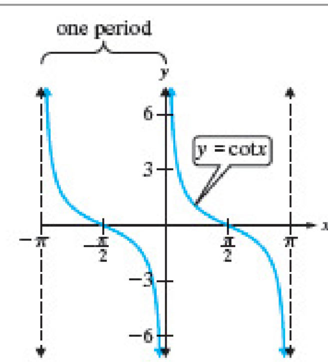












Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Reciprocal Identities

$\csc\theta = \dfrac{1}{\sin\theta}$ $\qquad \sec\theta = \dfrac{1}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$

Quotient Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Sum and Difference Formulas

$\sin(u+v) = \sin u\cos v + \cos u\sin v$

$\sin(u-v) = \sin u\cos v - \cos u\sin v$

$\cos(u+v) = \cos u\cos v - \sin u\sin v$

$\cos(u-v) = \cos u\cos v + \sin u\sin v$

$\tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u\tan v}$

$\tan(u-v) = \dfrac{\tan u - \tan v}{1 + \tan u\tan v}$

Even and Odd Identities

$\cos(-\theta) = \cos\theta$ $\qquad \sec(-\theta) = \sec\theta$

$\sin(-\theta) = -\sin\theta$ $\qquad \csc(-\theta) = -\csc\theta$

$\tan(-\theta) = -\tan\theta$ $\qquad \cot(-\theta) = -\cot\theta$

Double-Angle Formulas

$\sin 2\theta = 2\sin\theta\cos\theta$ $\qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ $\qquad = 1 - 2\sin^2\theta$

$\qquad = 2\cos^2\theta - 1$

Cofunction Identities

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right) \quad \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$
$$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right) \quad \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$
$$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right) \quad \csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$$

Power-Reducing Formulas

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2} \qquad \cos^2\theta = \frac{1 + \cos 2\theta}{2}$$
$$\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

Half-Angle Formulas

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}} \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$$
$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}} = \frac{1 - \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 + \cos\alpha}$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$
$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$
$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$
$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

Sum-to-Product Formulas

$$\cos x - \cos y = -2\sin\frac{x + y}{2}\sin\frac{x - y}{2}$$
$$\cos x + \cos y = 2\cos\frac{x + y}{2}\cos\frac{x - y}{2}$$
$$\sin x + \sin y = 2\sin\frac{x + y}{2}\cos\frac{x - y}{2}$$
$$\sin x - \sin y = 2\cos\frac{x + y}{2}\sin\frac{x - y}{2}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$