Age Zircon model

Main model parameters

Let us define a dataset of N dated zircons from S samples. We indicate with $\mathbf{z} = \{z_1, \ldots, z_N\}$ the (unknown) ages of the zircons and with $\mathbf{x} = \{x_1, \ldots, x_S\}$ the ages of the samples. The samples are ordered according on their depth and we assume their age strictly follows that order such that $x_i > x_{i-1}$.

The exact age of each zircon (z_i) is assumed to be unknown but linked to its measured age, which is expressed as a mean μ_i and a standard deviation σ_i. The mean ages μ and a standard deviations σ of all zircons represent the input data of the model. Since the age of zircons can be measured based on different methods (e.g. $m \in \{1, \ldots, M\}$, e.g. Zr-U-Pb, ZFT, Ar-Ar), the true age of a zircon is assumed to be further affected by the how it was measured. We compute the likelihood of the age of a zircon i based on dating method m as:

$$P(\mu_i | z_i, \sigma_i, \epsilon_m) \sim \mathcal{N}(z_i, \sigma_i + \epsilon_m) \tag{1}$$

where $\epsilon_m \in \mathbb{R}^+$ is the bias in the estimated error of the measurement method m. The parameters z_i, η_m and ϵ_m are considered as unknown and estimated from the data using a Bayesian algorithm.

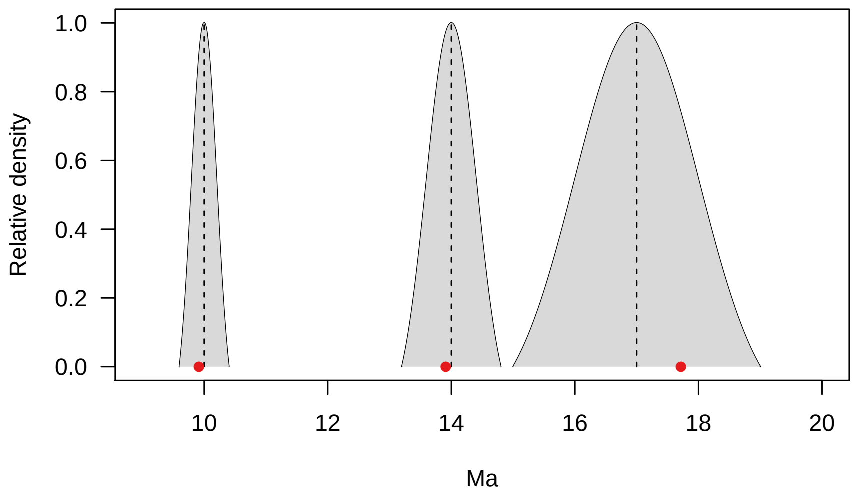

Figure 1: Example of three zircons with estimated ages of 10, 14, and 17 Ma (μ; indicated by the dashed vertical lines) and standard deviations ($\sigma + \epsilon$; shown by the gray shaded areas). The sampled true ages are indicated by the red circles.

The prior probability of $z_{i,j}$, i.e. the ith zircon in sample j is modeled by a half-Uniform-Cauchy distribution, which we define as

$$P(z_{i,j}|x_j, s_j) = \begin{cases} z_{i,j} \sim \mathcal{U}(0, 2x_j), & \text{if } z_{i,j} < x_j \\ z_{i,j} \sim \mathcal{C}(x_j, s_j), & \text{if } z_{i,j} \geq x_j \end{cases} \tag{2}$$

where x_j is the estimated age of the sample and s_j is a sample-specific scale parameter of the Causchy distribution.

The value of x_j, the sample age, is determined by two latent variables and constrained by sampled values of $\mathbf{z}$ such that $x_i > x_{i-1}$ for $i \in \{2, \ldots, S\}$. Specifically we define as

$$\zeta_j = \min(z_n), \text{ for } n \in \{j, \ldots, N\}, \tag{3}$$

the minimum age across all zircons included in sample j and in all older samples. Thus, ζ_j

represents the maximum boundary for the age of sample j. Under this notation we define the age of a sample as:

$$x_j = \begin{cases} r_j(\zeta_j), & \text{if } j = 1 \\ x_{j-1} + r_j(\zeta_j - x_{j-1}), & \text{if } j > 1 \end{cases} \tag{4}$$

where $r_j \in (0, 1)$ is a sample-specific latent parameter determining how close x_j is to its lower boundary x_{j-1} (or 0 if $j = 1$). We also account for the fact that a zircon can be younger then the sample it is assigned to, due to dating error or a later recrystallization of the zircon. To do that, we additionally sample a vector of identifiers $\mathbf{I} = \{I_1, \ldots, I_N\}$ that define which zircons are accounted for $(I = 1)$ or excluded $(I = 0)$ in determining the maximum boundary for the age of sample j:

$$\zeta_j = \min(z_n), \text{ for } n \in \{j, \ldots, N\}, \text{ if } z_j = 1. \tag{5}$$

Priors and hyper-priors

We use a gamma prior for the vector of scale parameters of the Cauchy distributions $\{s_1, \ldots, s_S\} \sim \Gamma(5, 5)$, a normal prior centered on 0 on the vector of bias parameters $\{\eta_1, \ldots, \eta_M\} \sim \mathcal{N}(0, 5)$ and a gamma prior on the vector of parameters $\{\epsilon_1, \ldots, \epsilon_M\} \sim \mathcal{N}(0, 5)$. We use a beta distribution as prior on the vector of parameters $\{r_1 \ldots, r_S\} \sim \mathcal{B}(a, b)$ and consider the shape parameters a and b themselves as unknown parameters and assign them gamma distributed hyper-priors, $a, b \sim \Gamma(1, 0.1)$. Finally we use a Bernoulli distribution as prior on the indicators $\{I_1, \ldots, I_N\}$ with parameter $p = 0.99$, thus assigning a 0.01 prior probability for a zircon to be younger than the sample it is assigned to.

Parameter estimation

The model includes $2N + 2M + 2S + 2$ parameters (z and I, η and ϵ, s and r, a and b, respectively). All parameters are estimated through Metropolis-Hastings Markov chain Monte Carlo (MCMC). We use a sliding window proposal with reflection at the boundaries for r, normal kernel proposals for z, binomial proposals for I and sliding window for η. We use multiplier proposals on all other parameters since they only span the positive range.

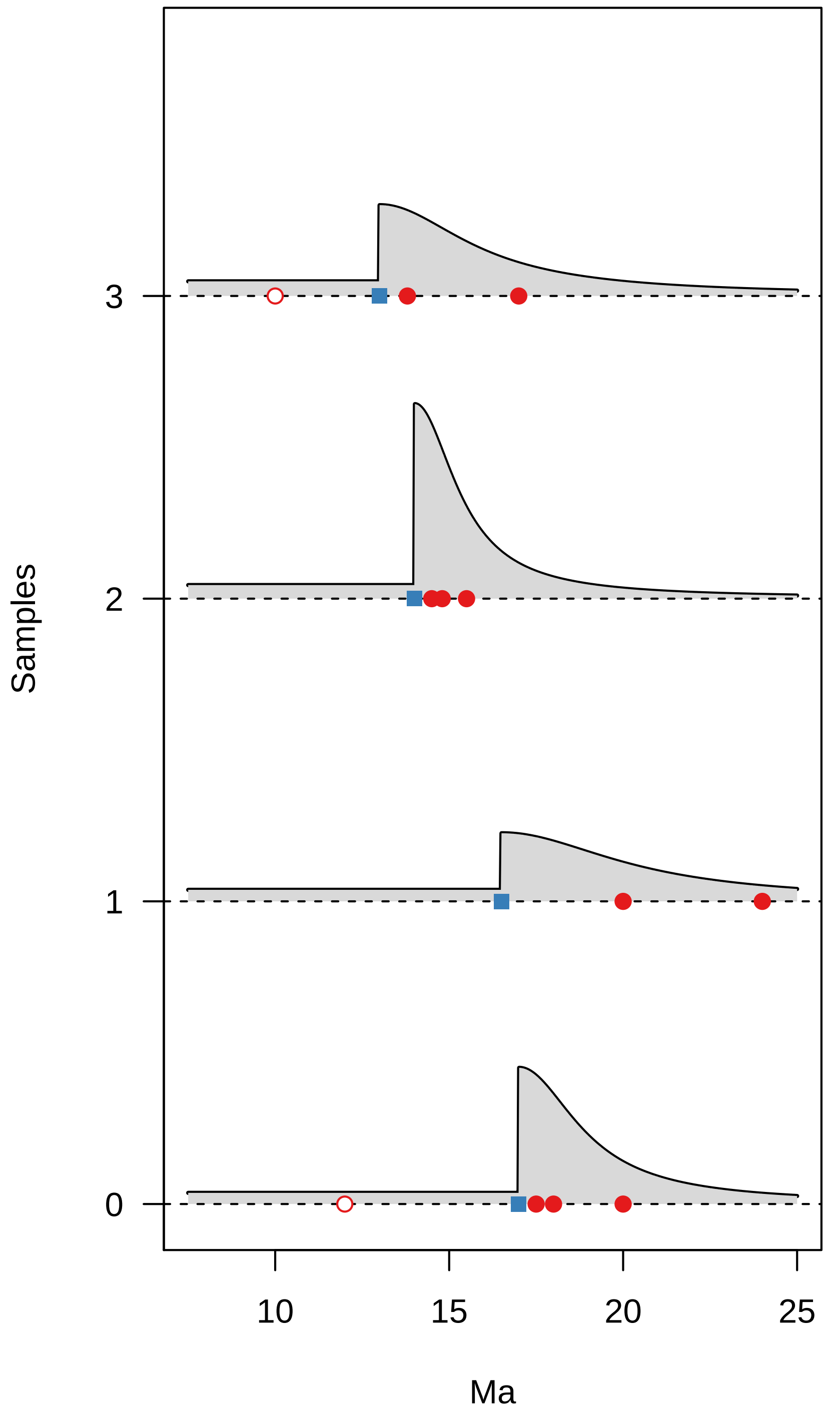

Figure 2: Example of three zircons with estimated ages of 10, 14, and 17 Ma (μ; indicated by the dashed vertical lines) and standard deviations ($\sigma + \epsilon$; shown by the gray shaded areas). The sampled true ages are indicated by the red circles.