DATA605_w3_ Eigenvalues and Eigenvectors

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```
library(matlib)
library(pracma)
```

```
##
## Attaching package: 'pracma'
```

```
## The following objects are masked from 'package:matlib':
##
##     angle, inv
```

// Problem set 1

(1) What is the rank of the matrix A?

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{bmatrix}$$

```
A = matrix(c(1,-1,0,5, 2,0,1,4, 3,1,-2,-2, 4,3,1,-3),4)
A
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]   -1    0    1    3
## [3,]    0    1   -2    1
## [4,]    5    4   -2   -3
```

```
print('- RREF')
```

```
## [1] "- RREF"
```

```
rref(A)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    0
## [2,]    0    1    0    0
## [3,]    0    0    1    0
## [4,]    0    0    0    1
```

```
print('- matrix rank')
```

```
## [1] "- matrix rank"
```

```
Rank(A)
```

```
## [1] 4
```

The rank of matrix A is = 4

(2) Given an mxn matrix where m > n, what can be the maximum rank? The minimum rank, assuming that the matrix is non-zero?

The maximum rank of the matrix is n and the minimum rank is 1

(3) What is the rank of matrix B?

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$$

The 3 rows in the matrix are scaled equivalent there for the rank is 1

```
B = matrix(c(1,3,2, 2,6,4, 1,3,2),3)
B
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    3    6    3
## [3,]    2    4    2
```

```
#- RREF
rref(B)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    0    0    0
## [3,]    0    0    0
```

```
#- matrix rank
Rank(B)
```

```
## [1] 1
```

The rank of matrix A is = 1

// Problem Set 2

(4)Compute the eigenvalues and eigenvectors of the matrix A. You'll need to show your work. You'll need to write out the characteristic polynomial and show your solution.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

The eigenvalues of a triangular matrix are the diagonal 1,4,6. We can confirm that using R as follows

```
A <- matrix(c(1,0,0, 2,4,0, 3,5,6), 3)
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]    0    4    5
## [3,]    0    0    6
```

```
#eigen values / vectors'
e <- eigen(A)
print(e$values)
```

```
## [1] 6 4 1
```

lambda 1

$$1A \rightarrow (I - A) = \begin{bmatrix} 1-1 & -2 & -3 \\ 0 & 1-4 & -5 \\ 0 & 0 & 1-6 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -3 \\ 0 & -3 & -5 \\ 0 & 0 & -5 \end{bmatrix} RREF \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- R3*(-1/5)
- R2 + (5)*R3
- R2*(-1/3)
- R1 + (-3)*R3 *and R1 + (-2)*R2*

$$nullspace \rightarrow 1A \rightarrow (I - A) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- x1=1, x2=0, x3=0

$$N(I - A) \rightarrow span(s * \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix})$$

```
#lambda 1
A1 <- 1*diag(3) - A
A1
```

```
##      [,1] [,2] [,3]
## [1,]    0   -2   -3
## [2,]    0   -3   -5
## [3,]    0    0   -5
```

```
rref(A1)
```

```
##      [,1] [,2] [,3]
## [1,]    0    1    0
## [2,]    0    0    1
## [3,]    0    0    0
```

lambda 4

$$4A \rightarrow (4I - A) = \begin{bmatrix} 4-1 & -2 & -3 \\ 0 & 4-4 & -5 \\ 0 & 0 & 4-6 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -3 \\ 0 & 0 & -5 \\ 0 & 0 & -2 \end{bmatrix} RREF \begin{bmatrix} 1 & -2/3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- R3*(-1/2)
- R2 + (5)*R3 *and R1 + (3)*R3*
- R2*(1/3)

$$nullspace \rightarrow 1A \rightarrow (I - A) = \begin{bmatrix} 1 & -2/3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- x1=2/3, x2=1, x3=0

$$N(I - A) \rightarrow span(s * \begin{bmatrix} 2/3 \\ 1 \\ 0 \end{bmatrix})$$

```
#lambda 4
A2 <- 4*diag(3) - A
A2
```

```
##      [,1] [,2] [,3]
## [1,]    3   -2   -3
## [2,]    0    0   -5
## [3,]    0    0   -2
```

```
rref(A2)
```

```
##      [,1]          [,2] [,3]
## [1,]    1 -0.6666667    0
## [2,]    0  0.0000000    1
## [3,]    0  0.0000000    0
```

lambda 6

$$4A \rightarrow (4I - A) = \begin{bmatrix} 6-1 & -2 & -3 \\ 0 & 6-4 & -5 \\ 0 & 0 & 6-6 \end{bmatrix} = \begin{bmatrix} 5 & -2 & -3 \\ 0 & 2 & -5 \\ 0 & 0 & 0 \end{bmatrix} RREF \begin{bmatrix} 1 & 0 & -8/5 \\ 0 & 1 & -5/2 \\ 0 & 0 & 0 \end{bmatrix}$$

- R2*(1/2)
- R1 + (2)*R2
- R1*(1/5)

$$nullspace \rightarrow 1A \rightarrow (I - A) = \begin{bmatrix} 1 & 0 & -8/5 \\ 0 & 1 & -5/2 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- x1=8/5, x2=5/2, x3=1

$$N(I - A) \rightarrow span(s * \begin{bmatrix} 8/5 \\ 5/2 \\ 1 \end{bmatrix})$$

```
#lambda 6
A3 <- 6*diag(3) - A
A3
```

```
##      [,1] [,2] [,3]
## [1,]    5   -2   -3
## [2,]    0    2   -5
## [3,]    0    0    0
```

```
rref(A3)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0 -1.6
## [2,]    0    1 -2.5
## [3,]    0    0  0.0
```

Please show your work using an R-markdown document. Please name your assignment submission with your first initial and last name.