For the convergence analysis of high order CS, we use an asymmetric, infinitely differentiable function for an initial condition that is the superposition of three Gaussian bells:

$$f(t=0,x) \equiv f_0(x) = \tfrac{3}{4}\exp(-\left(\tfrac{x+0.25}{0.03}\right)^2) + \exp(-\left(\tfrac{x}{0.06}\right)^2) + \tfrac{1}{2}\exp(-(\tfrac{x-0.25}{0.1}^2)$$

Writing $a = 3/4, c = 1/2, w_a = 0.03, w_b = 0.06, w_c = 0.1, x_a = 0.25, x_c = -0.25$, the above is equivalent to

$$f_0(x) = a\exp(-\left(\tfrac{x+x_a}{w_a}\right)^2) + \exp(-\left(\tfrac{x}{w_b}\right)^2) + c\exp(-(\tfrac{x+x_c}{w_c})^2)$$

The first few derivatives are given by:

$$\begin{aligned}
f_0^{(1)}(x) &= a\left(-\frac{2}{w_a^2}\right)(x+x_a)\exp\left[-\left(\frac{x+x_a}{w_a}\right)^2\right] \\
&\quad \left(-\frac{2}{w_b^2}\right)x\exp\left[-\left(\frac{x}{w_b}\right)^2\right] \\
&\quad c\left(-\frac{2}{w_c^2}\right)(x+x_c)\exp\left[-\left(\frac{x+x_c}{w_c}\right)^2\right]
\end{aligned}$$

$$\begin{aligned}
f_0^{(2)}(x) &= a\left(-\frac{2}{w_a^2}\right)\left[1-\frac{2}{w_a^2}(x+x_a)^2\right]\exp\left[-\left(\frac{x+x_a}{w_a}\right)^2\right] \\
&\quad \left(-\frac{2}{w_b^2}\right)\left[1-\frac{2}{w_b^2}x^2\right]\exp\left[-\left(\frac{x}{w_b}\right)^2\right] \\
&\quad c\left(-\frac{2}{w_c^2}\right)\left[1-\frac{2}{w_c^2}(x+x_c)^2\right]\exp\left[-\left(\frac{x+x_c}{w_c}\right)^2\right]
\end{aligned}$$

$$\begin{aligned}
f_0^{(3)}(x) &= a\left(-\frac{2}{w_a^2}\right)^2\left[3(x+x_a)-\frac{2}{w_a^2}(x+x_a)^3\right]\exp\left[-\left(\frac{x+x_a}{w_a}\right)^2\right] \\
&\quad \left(-\frac{2}{w_b^2}\right)^2\left[3x-\frac{2}{w_b^2}x^3\right]\exp\left[-\left(\frac{x}{w_b}\right)^2\right] \\
&\quad c\left(-\frac{2}{w_c^2}\right)^2\left[3(x+x_c)-\frac{2}{w_c^2}(x+x_c)^3\right]\exp\left[-\left(\frac{x+x_c}{w_c}\right)^2\right]
\end{aligned}$$

$$
\begin{aligned}
f_0^{(4)}(x) &= a\left(-\frac{2}{w_a^2}\right)^2\left[3+6\left(-\frac{2}{w_a^2}\right)(x+x_a)^2+\left(-\frac{2}{w_a^2}\right)^2(x+x_a)^4\right]\exp\left[-\left(\frac{x+x_a}{w_a}\right)^2\right] \\
&\quad \left(-\frac{2}{w_b^2}\right)^2\left[3+6\left(-\frac{2}{w_b^2}\right)x^2+\left(-\frac{2}{w_b^2}\right)^2x^4\right]\exp\left[-\left(\frac{x}{w_c}\right)^2\right] \\
&\quad c\left(-\frac{2}{w_c^2}\right)^2\left[3+6\left(-\frac{2}{w_a^2}\right)(x+x_c)^2+\left(-\frac{2}{w_c^2}\right)(x+x_c)^4\right]\exp\left[-\left(\frac{x+x_c}{w_c}\right)^2\right]
\end{aligned}
$$

$$
\begin{aligned}
f_0^{(1)}(x) &= a\frac{d}{dx}\left[-\left(\frac{x+x_a}{w_a}\right)^2\right]\exp(-\left(\frac{x+x_a}{w_a}\right)^2)+\frac{d}{dx}\left[-\left(\frac{x}{w_b}\right]\right)^2\left(-\frac{2}{w_b}\right)\exp(-\left(\frac{x}{w_b}\right)^2) \\
&\quad +c\frac{d}{dx}\left[-\left(\frac{x+x_c}{w_c}\right)^2\right]\exp(-(\frac{x+x_c}{w_c})^2)
\end{aligned}
$$

$$
\begin{aligned}
f_0^{(1)}(x) &= a\left(-\frac{2}{w_a}\right)\left(\frac{x+x_a}{w_a}\right)\exp(-\left(\frac{x+x_a}{w_a}\right)^2)+\left(-\frac{2}{w_b}x\right)\exp(-\left(\frac{x}{w_b}\right)^2) \\
&\quad +c\left(-\frac{2}{w_c}\right)\left(\frac{x+x_c}{w_c}\right)\exp(-(\frac{x+x_c}{w_c})^2)
\end{aligned}
$$

$$
\begin{aligned}
f_0^{(2)}(x) &= a\left(-\frac{2}{w_a}\right)\frac{d}{dx}\left[\left(\frac{x+x_a}{w_a}\right)\right]\exp(-\left(\frac{x+x_a}{w_a}\right)^2)+\left(\frac{x+x_a}{w_a}\right)\frac{d}{dx}\left[\exp(-\left(\frac{x+x_a}{w_a}\right)^2)\right]+ \\
& \frac{d}{dx}\left[\left(-\frac{2}{w_b}x\right)\right]\exp(-\left(\frac{x}{w_b}\right)^2)+\left(-\frac{2}{w_b}x\right)\frac{d}{dx}\left[\exp(-\left(\frac{x}{w_b}\right)^2)\right] \\
& +c\left(-\frac{2}{w_c}\right)\frac{d}{dx}\left[\left(\frac{x+x_c}{w_c}\right)\right]\exp(-(\frac{x+x_c}{w_c})^2)+c\left(-\frac{2}{w_c}\right)\left(\frac{x+x_c}{w_c}\right)\frac{d}{dx}\left[\exp(-(\frac{x+x_c}{w_c})^2)\right] \\
&= a\left(-\frac{2}{w_a}\right)\left[\frac{1}{w_a}+\frac{x+x_a}{w_a}\frac{d}{dx}\left(-\left(\frac{x+x_a}{w_a}\right)^2\right)\right]\exp(-\left(\frac{x+x_a}{w_a}\right)^2) \\
& +\left(-\frac{2}{w_b}\right)\exp(-\left(\frac{x}{w_b}\right)^2)+\left(-\frac{2}{w_b}x\right)\frac{d}{dx}\left[-\left(\frac{x}{w_b}\right)^2\right]\exp(-\left(\frac{x}{w_b}\right)^2) \\
& +c\left(-\frac{2}{w_c}\right)\left[\frac{1}{w_c}+\left(\frac{x+x_c}{w_c}\right)\frac{d}{dx}\left[-(\frac{x+x_c}{w_c})^2\right]\right]\exp(-(\frac{x+x_c}{w_c})^2) \\
&= a\left(-\frac{2}{w_a}\right)\left[\frac{1}{w_a}+\left(-\frac{2}{w_a}\right)\left(\frac{x+x_a}{w_a}\right)^2\right]\exp(-\left(\frac{x+x_a}{w_a}\right)^2) \\
& +\left(-\frac{2}{w_b}\right)\exp(-\left(\frac{x}{w_b}\right)^2)+\left(-\frac{2}{w_b}x\right)^2\exp(-\left(\frac{x}{w_b}\right)^2) \\
& +c\left(-\frac{2}{w_c}\right)\left[\frac{1}{w_c}+\left(-\frac{2}{w_c}\right)\left(\frac{x+x_c}{w_c}\right)^2\right]\exp(-(\frac{x+x_c}{w_c})^2)
\end{aligned}
$$