# Lab Exercise

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R code for Metropolis-Hastings sampler using a bad proposal density:
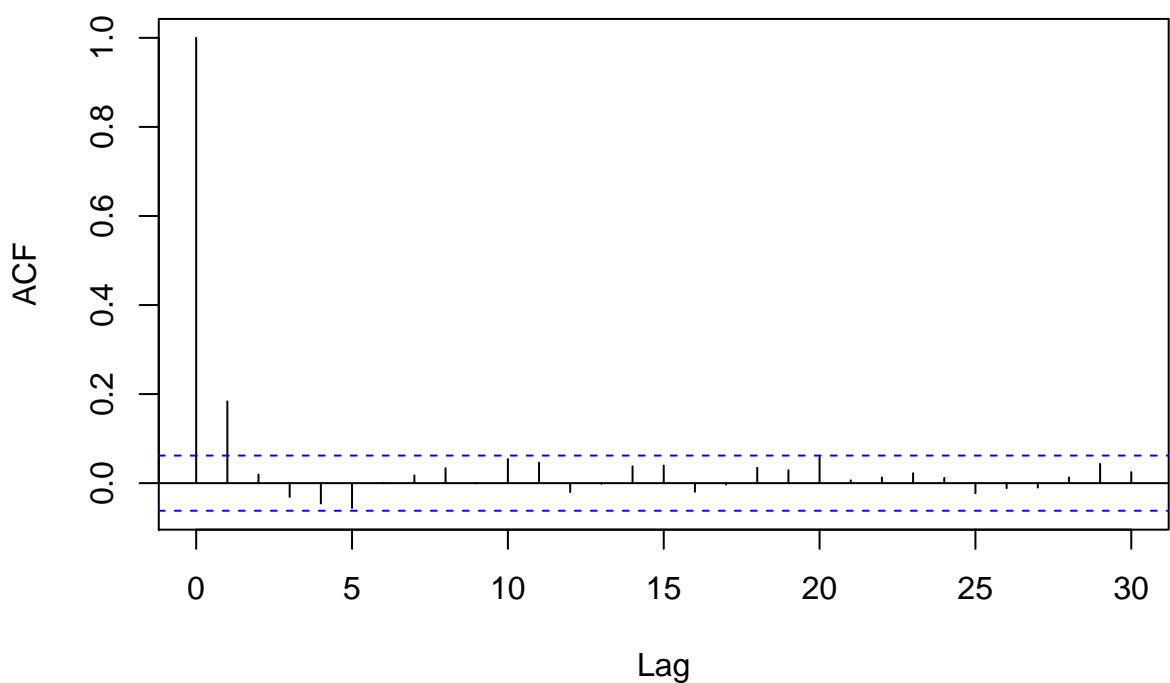
```r
target <- function ( theta , n =20 , ybar =0.0675){
  dens <- exp ( - n * ( theta - ybar )^2 / 2) * (1 / (1+ theta ^2))
  return ( dens )
}
proposal <- function ( theta_new , theta_old ){
  dens <- dnorm ( theta_new ,0.0675 ,0.2) # theta _ old is unused
}
accept_prob <- function ( theta_new , theta_old ){
  alpha_mh <- ( target ( theta_new ) * proposal ( theta_old , theta_new )) /
    ( target ( theta_old ) * proposal ( theta_new , theta_old ))
  alpha_mh <- ifelse ( alpha_mh >1 ,1 , alpha_mh )
  return(alpha_mh)
}
MH_sampler <- function ( theta_init , niter ){
  theta_samples <- rep (0 , niter )
  theta_samples [1] <- theta_init
  for ( i in 1: niter ){
    theta_old = theta_samples [ i ]
    theta_new <- rnorm (1, 0.0675, 0.2)
    u <- runif (1)
    alpha_mh = accept_prob ( theta_new , theta_old )
    if ( u <= alpha_mh ){
      theta_samples [ i +1] = theta_new
    } else {
      theta_samples [ i +1] = theta_old
    }
  }
  return ( theta_samples )
}
samples <- MH_sampler(0, 1000)
plot(samples, type = "l", main = "Trace plot for starting point at 0")
```

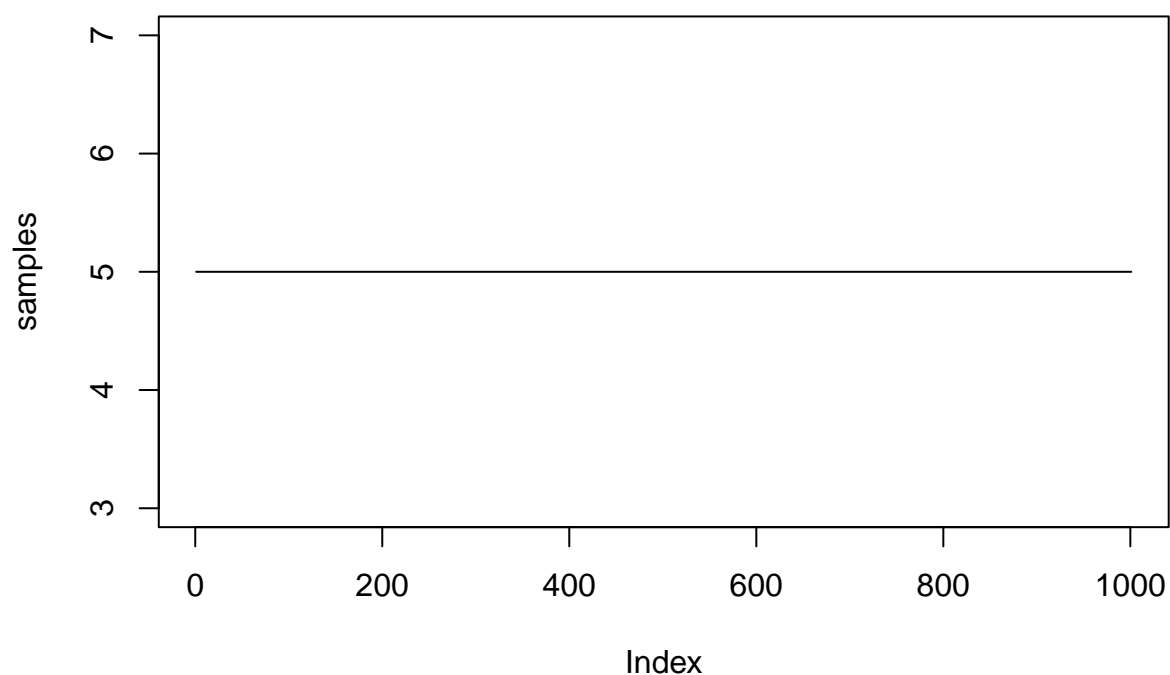## Trace plot for starting point at 0
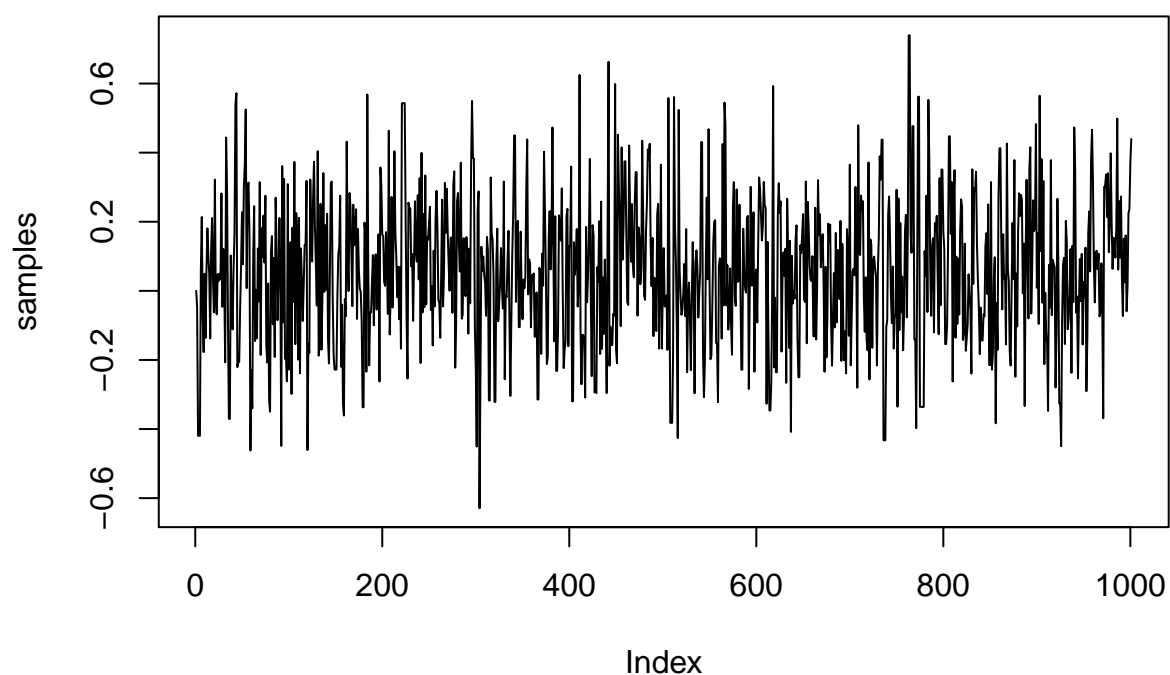

```r
acf(samples, type = "correlation", main = "Autocorrelation for starting point at 0")
```

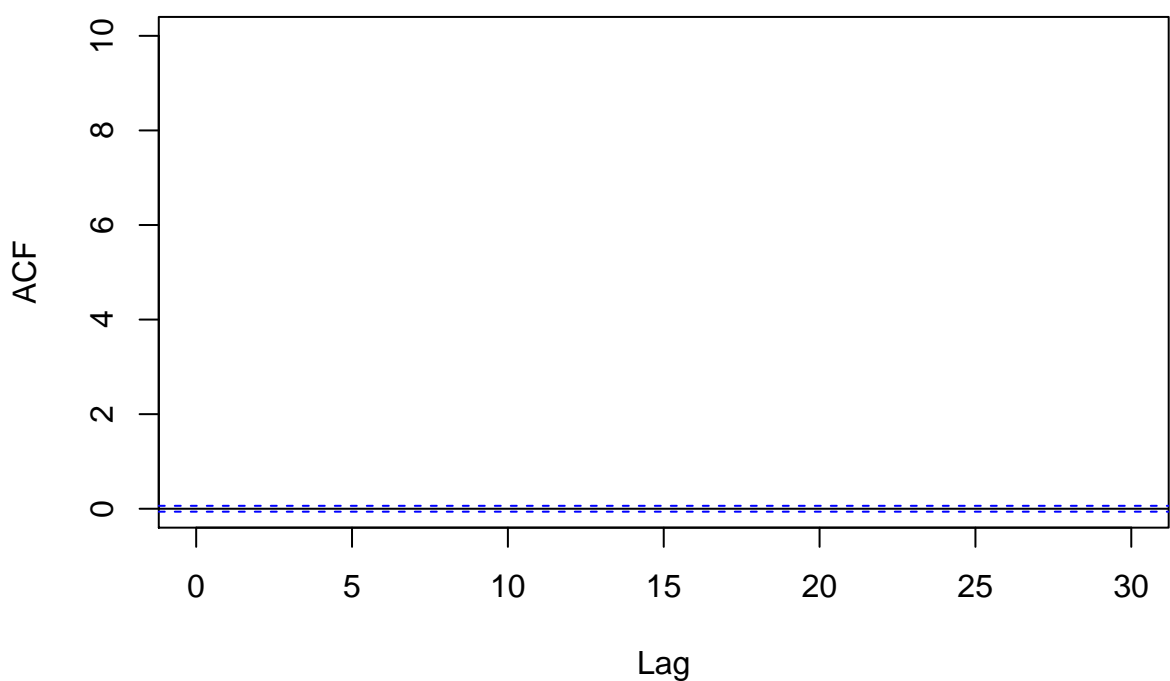## Autocorrelation for starting point at 0


```r
samples <- MH_sampler(5, 1000)
plot(samples, type = "l", main = "Trace plot for starting point at 5")
```

Trace plot for starting point at 5


```r
acf(samples, type = "correlation", main = "Autocorrelation for starting point at 5", ylim = c(0,10))
```
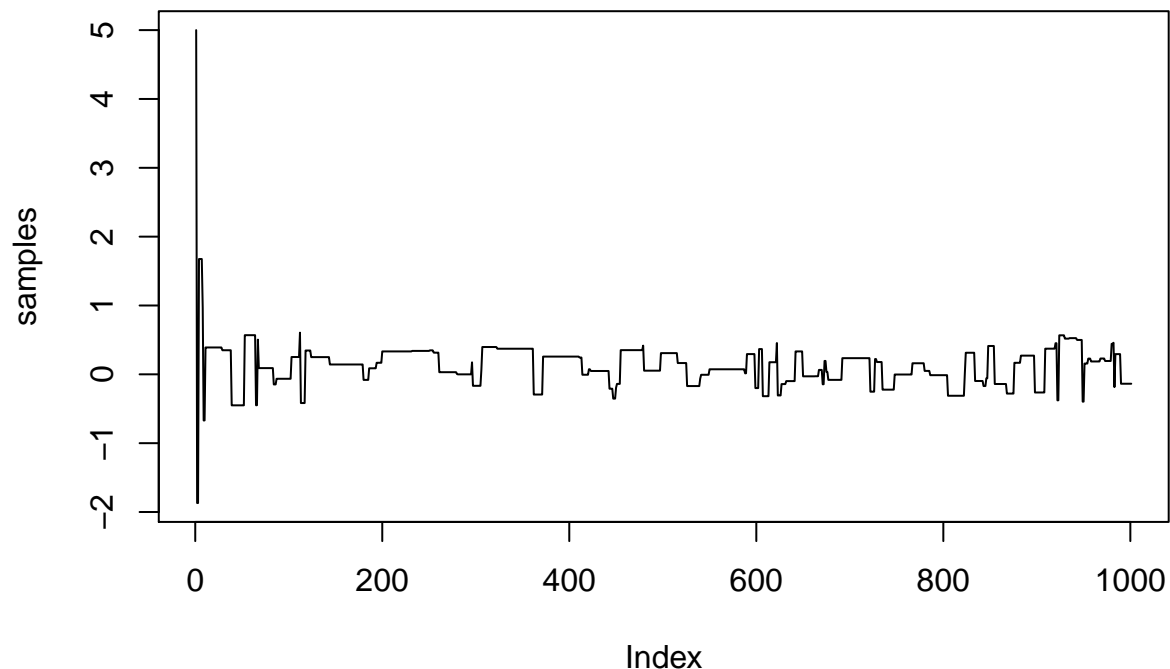
Autocorrelation for starting point at 5

N(0.0675, 0.2) is a bad proposal density because it has a very narrow distribution, which gives us a very narrow selection of starting points. Starting points that are far from the center of the distribution will result in a Markov chain that never moves, since the probability of moving to any nearby point is very low.

As you increase σ_0, the algorithm works with a wider range of starting points, since the proposal distribution is wider. For instance, if we take N(0.0635, 3) as our proposal distribution, we can use 5 as our initial starting point. See plots below:
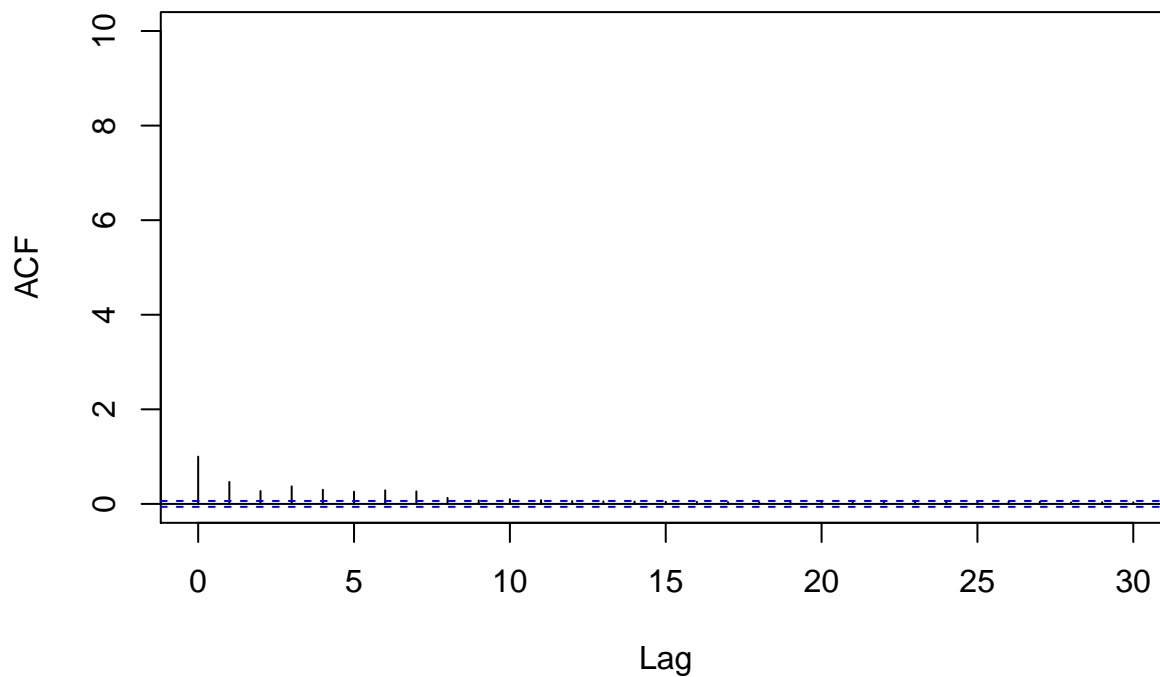
```r
proposal <- function ( theta_new , theta_old ){
  dens <- dnorm ( theta_new ,0.0675 ,3) # theta _ old is unused
}
MH_sampler <- function ( theta_init , niter ){
  theta_samples <- rep (0 , niter )
  theta_samples [1] <- theta_init
  for ( i in 1: niter ){
    theta_old = theta_samples [ i ]
    theta_new <- rnorm (1, 0.0675, 3)
    u <- runif (1)
    alpha_mh = accept_prob ( theta_new , theta_old )
    if ( u <= alpha_mh ){
      theta_samples [ i +1] = theta_new
    } else {
      theta_samples [ i +1] = theta_old
    }
  }
  return ( theta_samples )
}
samples <- MH_sampler(5, 1000)
plot(samples, type = "l", main = "Trace plot for starting point at 5")
```

Trace plot for starting point at 5


```r
acf(samples, type = "correlation", main = "Autocorrelation for starting point at 5", ylim = c(0,10))
```

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Autocorrelation for starting point at 5

However, as the variance of the proposal density increases, the autocorrelation also increases. This can be seen by making our starting point at 0, and comparing our plots with a N(0.0675, 3) density with our plots with a N(0.0675, 0.2) density.
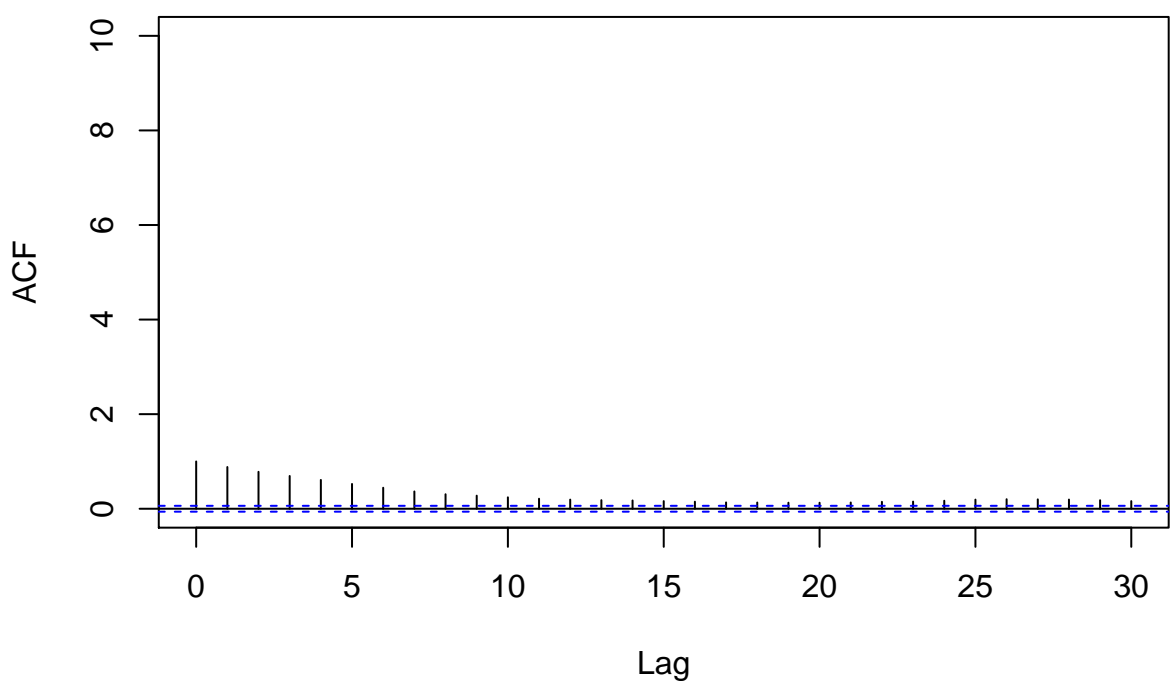
```
samples <- MH_sampler(0, 1000)
plot(samples, type = "l", main = "Trace plot for starting point at 5")
```

Trace plot for starting point at 5


```
acf(samples, type = "correlation", main = "Autocorrelation for starting point at 5", ylim = c(0,10))
```
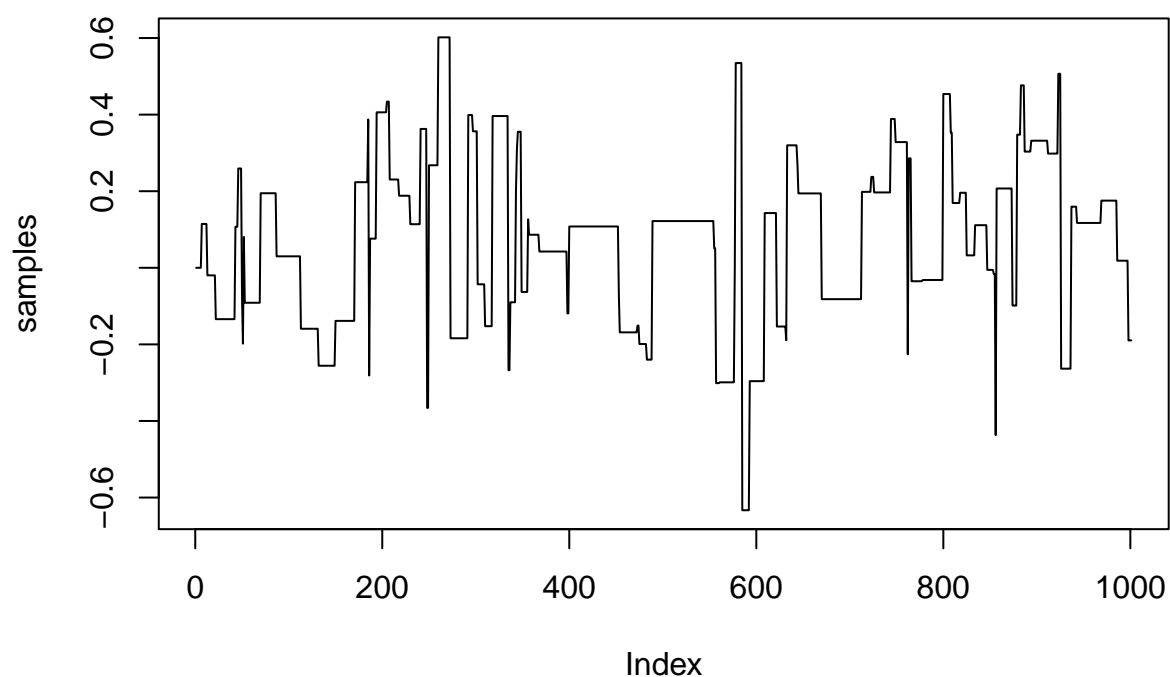
Autocorrelation for starting point at 5

The reason the autocorrelation increases when the proposal density has a larger variance is that the algorithm is more likely to reject a proposed step and stay in the same place.