A New Novel Trap For Neutral Atoms*

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A modification to the standard Time-averaged Orbiting Potential (TOP) trap that allows for very isotropic potentials.

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I. INTRODUCTION

The mechanical arguments that Boltzmann used to derive the H-theorem incited a series of attacks against his theories. In response to an argument concerning the conflict between the time reversal symmetry inherent to mechanical systems and the irreversible nature of entropy, Boltzmann explicitly showed a number of situations where the initial conditions of a system could be artifically constructed to produce bizarre collective behavior. Among these specific cases was the fact that a gas might never reach equilibrium in a perfectly isotropic harmonic potential, and in particular the monopole mode of a gas would be *undamped*. Experimental study of this particular phenomenon has so far been prevented by the difficulty in generating a sufficiently isotropic harmonic potential.

The theoretical issues which fueled the debate stem from the Boltzmann equation. For a phase space distribution, $f \equiv f(\mathbf{r}, \mathbf{p}, t)$, the Boltzmann equation in its simplest form is

$$\frac{df}{dt} = I_{coll}[f], \tag{1}$$

where $I_{coll}[f]$ is a collision function, referred to as the *collision integral*. Essentially, the Boltzmann states that the phase space distribution of a gas approaches equilibrium at a rate proportional to the frequency of collisions. At equilibrium, the distribution becomes stationary and the collision integral vanishes. Expanding the derivative and including the explicit definition of $I_{coll}[f]$ leads to the more standard form of the Boltzmann equation

$$\begin{aligned}\frac{\partial f}{\partial t} + \mathbf{v}_1 \cdot \nabla_{\mathbf{r}} f + \frac{\mathbf{F}}{m} \cdot \nabla_{\mathbf{v}_1} f = \\ \frac{\sigma_0}{4\pi} \int d^2\Omega d^3\mathbf{v}_2 \left|\mathbf{v}_2 - \mathbf{v}_1\right| \left(f(\mathbf{v}_1')f(\mathbf{v}_2') - f(\mathbf{v}_1)f(\mathbf{v}_2)\right)\end{aligned} \tag{2}$$

where σ_0 is the collision cross section. Energy and momentum conservation are implicitly assumed, but could easily be included in the collision integral using a delta function. The other major assumption is that the atoms can be treated as hard spheres, but this approximation is valid for s-wave collisions when mean field effects are negligible and the cloud is non-degenerate.

Distributions of the form

$$\log f = \alpha + \beta \cdot \mathbf{v} + \gamma v^2 \tag{3}$$

cause the collision integral to vanish due to conservation of energy and momentum and are referred to as *collisionally invariant*. These distributions, generally called local equilibrium distributions, do not necessarily satisfy Eq. 2 and constraints must be placed on α, β, and γ. For a sufficiently general outside potential, these constants can be constrained to produce an equilibrium distribution equivalent to Maxwell-Boltzmann. For certain special potentials, collisionally invariant distributions exist that satisfy Eq. 2. For an isotropic harmonic potential, the solution is essentially an equilibrium distribution undergoing temperature oscillations where the spatial distribution varies identically such that the distribution satisfies the equilibrium constraint at each point in time. Persistence of monopole oscillations can be described in various ways, but can be elucidated by the two particle case.

In the presence of spherical symmetry, the problem can be treated as analogously one dimensional and the radial motion of a single particle of mass m, energy E and angular momentum L is governed by the effective potential

$$V_e = \frac{L^2}{2mr^2} + \frac{1}{2}m\omega^2 r^2. \tag{4}$$

The radial force,

$$m\frac{d^2r}{dt^2} = -\frac{d}{dr}V_e, \tag{5}$$

can be combined with the kinetic energy,

$$\frac{1}{2}m\left(\frac{dr}{dt}\right)^2 = E - V_e, \tag{6}$$

by integrating Eq. 5. This yields

$$\frac{d^2}{dt^2}r^2 = -\Omega^2\left(r^2 - r_0^2\right), \tag{7}$$

* A footnote to the article title

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where $\Omega \equiv 2\omega$ and $r_0^2 = E/(m\omega^2)$, so that the square radius undergoes sinusoidal oscillations around its mean value r_0^2 at a frequency of 2ω. If there are 2 particles, 1 and 2, each with individual values of E, L and r^2, each particle will oscillate at 2ω and taking the linear combination of their respective differential equations yields

$$\frac{d^2}{dt^2}r_t^2 = -\Omega^2\left(r_t^2 - r_{0t}^2\right) \tag{8}$$

so their combined square radius, $r_t^2 = r_1^2 + r_2^2$, oscillates around its mean value, $r_{0t}^2 = (E_1+E_2)/(m\omega^2)$. The magnitude of the monopole motion depends on the magnitude and relative phase of the individual particle trajectories. These individual quantities will abruptly change in the event of a collision. Assuming the collisions are local, r_1, r_2 and thus r_t^2 will not change from the instant before to the instant after the collision. Similarly, momentum and energy conservation imply that $\frac{d}{dt}r_t^2$ and r_{0t}^2 are unchanged by the collision. These three continuities imply that the parameters and boundary conditions of Eq. 8 are matched directly before and after a collision. This ensures that neither the magnitude or phase of the oscillation will change as the result of a pairwise collsion. Generalizing to N atoms, so that

$$r_t^2 = \sum_{i=1}^{N} r_i^2 \tag{9}$$

and

$$r_{0t}^2 = \frac{1}{m\omega^2}\sum_{i=1}^{N} E_i, \tag{10}$$

one can see that the monopole mode is left unperturbed – and in particular *undamped* – by local, pairwise, momentum- and energy-conserving collisions.

Experimentally, we evaporatively cool ^{87}Rb atoms in a new magnetic trap capable of generating an isotropic harmonic potential with trap frequencies that differ by less than 0.1%. Degeneracy effects are minimized by keeping cloud temperatures well above T_c. Monopole mode damping rates are compared against quadrupole mode damping rates, which are sufficiently fast and linearly dependent on collision frequency. We selectively drive monopole (quadrupole) motion by (a)symmetrically modulating the strength of the confinement about its mean value. The cloud is then allowed to propagate freely in the spherical trap before it is non-destructively imaged using phase contrast microscopy. Six images are taken of each cloud along two orthogonal axes with an interval of 17 ms in order to sample roughly 1.5 oscillation periods. Widths of the cloud along each dimension, σ_i, are determined from Gaussian surface fits of individual images and converted to monopole and quadrupole amplitudes through the following relations

$$A_M^i = \frac{\sigma_{xi}^2 + \sigma_{yi}^2 + \sigma_{zi}^2}{\langle \sigma_x^2 + \sigma_y^2 + \sigma_z^2 \rangle} - 1 \tag{11}$$

$$A_Q^i = \frac{\sigma_{xi}^2 + \sigma_{yi}^2 - 2\sigma_{zi}^2}{\langle \sigma_x^2 + \sigma_y^2 - 2\sigma_z^2 \rangle} - 1 \tag{12}$$

where i indicates the individual image and the average is taken over all images for a single run of the experiment. Analysis is simplified by fitting the oscillation obtained from each experimental cycle with a fixed frequency sine wave, from which the relative oscillation amplitude is extracted, as indicated by the solid lines in Fig. 1.

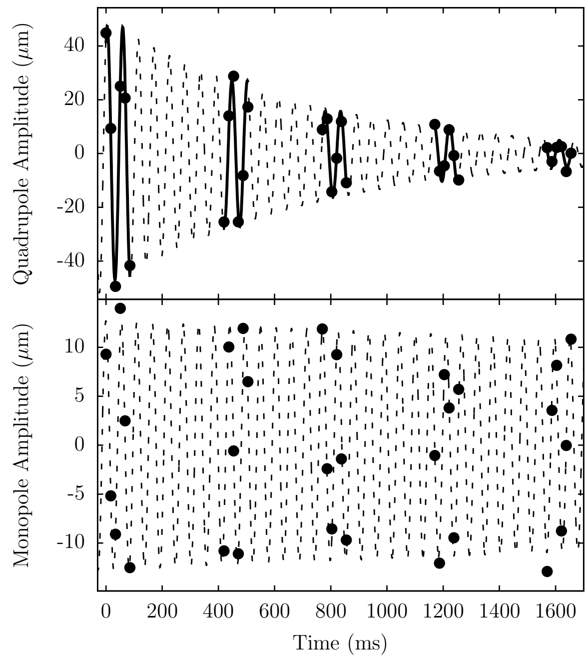

FIG. 1. Sample data for a driven quadrupole mode and monopole mode in a spherical trap with $|f_z - f_r|/f_z < 0.002$. Black lines on the quadrupole data indicate a typical fitting procedure where individual periods taken in a single run are fit with an undamped sine wave to extract the amplitude.

One sees the suppressed damping of the monopole mode from the sample data in Fig. 1. From Eq. 2, the damping rate is found to be linearly proportional to the collision rate. For the quadrupole mode, the damping rate is given by

$$\Gamma_Q = \frac{1}{10}n_{cl}\left(0\right)\sigma\bar{v}, \tag{13}$$

where $n_{cl}\left(0\right)$ is the peak atomic density, σ is the scattering cross section given by $8\pi a^2$, and $\bar{v}$ is the average thermal velocity. By varying the evaporation parameters, we can tune the collision rate of of the sample and alternately drive quadrupole or monopole modes. A direct comparison of quadrupole and monopole damping rates is shown in Fig. 2, where $|f_z - f_r|/f_z < 0.003$. While the linear dependence on quadrupole damping rate is in good agreement with Eq. 13, the monopole damping rate appears to be independent of collision rate.

The average monopole damping rate is small, but still finite, with a mean value of $\Gamma_M = 0.14(3)$. This nonzero

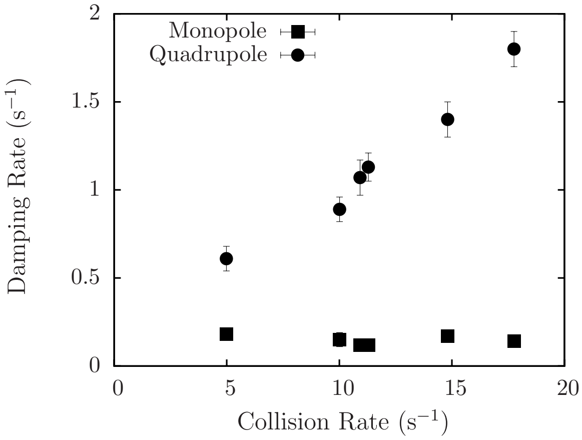

FIG. 2. Monopole and quadrupole damping rates as a function of interatomic collision rate. Each set was taken in very spherical traps where $|f_z - f_r|/f_z < 0.003$.

damping could be potentially due to small anisotropies in the trapping potential. With no collisions present, oscillations along the principal axes of the trap are fully decoupled and monopole- or quadrupole-like oscillations are totally undamped. If the principal trap frequencies differ such that $\omega_1 = \omega_2 \neq \omega_3$, dephasing occurs between oscillations along different principal axes and oscillations between monopole-like motion and purely quadrupole-like motion occur with a period given by

$$T_{MQ} = \frac{\pi}{|\omega_{1,2} - \omega_3|}. \tag{14}$$

With collisions present, monopole-quadrupole oscillations lead to damping when the quadrupole motion is nonzero. This effect can be seen when $T_{MQ} < 1/\Gamma_Q$.