Observation of Suppressed Damping of the Breathe Mode in a Spherical Harmonic Potential*

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A modification to the standard Time-averaged Orbiting Potential (TOP) trap that allows for very isotropic potentials.

Usage: Secondary publications and information retrieval purposes.

PACS numbers: May be entered using the `\pacs{#1}` command.

Structure: You may use the `description` environment to structure your abstract; use the optional argument of the `\item` command to give the category of each item.

I. INTRODUCTION

The mechanical arguments that Boltzmann used to derive the H-theorem incited a series of attacks against his theories. In response to an argument concerning the conflict between the time reversal symmetry inherent to mechanical systems and the irreversible nature of entropy, Boltzmann explicitly showed a number of situations where the initial conditions of a system could be artifically constructed to produce bizarre collective behavior. Among these specific cases was the fact that a gas might never reach equilibrium in a perfectly isotropic harmonic potential, and in particular the monopole mode of a gas would be *undamped*. Experimental study of this particular phenomenon has so far been prevented by the difficulty in generating a sufficiently isotropic harmonic potential.

The theoretical issues which fueled the debate stem from the Boltzmann equation. For a phase space distribution, $f \equiv f(\mathbf{r}, \mathbf{p}, t)$, the Boltzmann equation in its simplest form is

$$\frac{df}{dt} = I_{coll}[f], \tag{1}$$

where $I_{coll}[f]$ is a collision function, referred to as the *collision integral*. Essentially, the Boltzmann states that the phase space distribution of a gas approaches equilibrium at a rate proportional to the frequency of collisions. At equilibrium, the distribution becomes stationary and the collision integral vanishes. Expanding the derivative and including the explicit definition of $I_{coll}[f]$ leads to the more standard form of the Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{v}_1 \cdot \nabla_{\mathbf{r}} f + \frac{\mathbf{F}}{m} \cdot \nabla_{\mathbf{v}_1} f = \frac{\sigma_0}{4\pi} \int d^2\Omega d^3\mathbf{v}_2 \, |\mathbf{v}_2 - \mathbf{v}_1| \left(f(\mathbf{v}_1') f(\mathbf{v}_2') - f(\mathbf{v}_1) f(\mathbf{v}_2) \right) \tag{2}$$

where σ_0 is the collision cross section. Energy and momentum conservation are implicitly assumed, but could easily be included in the collision integral using a delta function. The other major assumption is that the atoms can be treated as hard spheres, but this approximation is valid for s-wave collisions when mean field effects are negligible and the cloud is non-degenerate.

Distributions of the form

$$\log f = \alpha + \beta \cdot \mathbf{v} + \gamma v^2 \tag{3}$$

cause the collision integral to vanish due to conservation of energy and momentum and are referred to as *collisionally invariant*. These distributions, generally called *local equilibrium distributions*, do not necessarily satisfy Eq. 2 and constraints must be placed on α, β, and γ. For a sufficiently general outside potential, these constants can be constrained to produce an equilibrium distribution equivalent to Maxwell-Boltzmann. For certain special potentials, collisionally invariant distributions exist that satisfy Eq. 2. For an isotropic harmonic potential, the solution is essentially an equilibrium distribution undergoing temperature oscillations where the spatial distribution varies identically such that the distribution satisfies the equilibrium constraint at each point in time. Persistence of monopole oscillations can be described in various ways, but can be elucidated by the two particle case.

In the presence of spherical symmetry, the problem can be treated as analogously one dimensional and the radial motion of a single particle of mass m, energy E and angular momentum L is governed by the effective potential

$$V_e = \frac{L^2}{2mr^2} + \frac{1}{2} m\omega^2 r^2. \tag{4}$$

The radial force,

$$m\frac{d^2 r}{dt^2} = -\frac{d}{dr} V_e, \tag{5}$$

can be combined with the kinetic energy,

$$\frac{1}{2} m \left(\frac{dr}{dt} \right)^2 = E - V_e, \tag{6}$$

by integrating Eq. 5. This yields

$$\frac{d^2}{dt^2} r^2 = -\Omega^2 \left(r^2 - r_0^2 \right), \tag{7}$$

* A footnote to the article title

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where $\Omega \equiv 2\omega$ and $r_0^2 = E/(m\omega^2)$, so that the square radius undergoes sinusoidal oscillations around its mean value r_0^2 at a frequency of 2ω. If there are 2 particles, 1 and 2, each with individual values of E, L and r^2, each particle will oscillate at 2ω and taking the linear combination of their respective differential equations yields

$$\frac{d^2}{dt^2} r_t^2 = -\Omega^2 \left(r_t^2 - r_{0t}^2 \right) \tag{8}$$

so their combined square radius, $r_t^2 = r_1^2 + r_2^2$, oscillates around its mean value, $r_{0t}^2 = (E_1 + E_2)/(m\omega^2)$. The magnitude of the monopole motion depends on the magnitude and relative phase of the individual particle trajectories. These individual quantities will abruptly change in the event of a collision. Assuming the collisions are local, r_1, r_2 and thus r_t^2 will not change from the instant before to the instant after the collision. Similarly, momentum and energy conservation imply that $\frac{d}{dt} r_t^2$ and r_{0t}^2 are unchanged by the collision. These three continuities imply that the parameters and boundary conditions of Eq. 8 are matched directly before and after a collision. This ensures that neither the magnitude or phase of the oscillation will change as the result of a pairwise collsion. Generalizing to N atoms, so that

$$r_t^2 = \sum_{i=1}^{N} r_i^2 \tag{9}$$

and

$$r_{0t}^2 = \frac{1}{m\omega^2} \sum_{i=1}^{N} E_i, \tag{10}$$

one can see that the monopole mode is left unperturbed – and in particular *undamped* – by local, pairwise, momentum- and energy-conserving collisions.

Experimentally, we evaporatively cool ^{87}Rb atoms in a new magnetic trap capable of generating an isotropic harmonic potential with trap frequencies that differ by less than 0.1%. Degeneracy effects are minimized by keeping cloud temperatures well above T_c. The monopole mode damping rates are compared against quadrupole mode damping rates, which are sufficiently fast and linearly dependent on collision frequency. Monopole (quadrupole) motion is selectively driven by (a)symmetrically modulating the trap frequencies. After propagating in the spherical trap, the cloud is then imaged non-destructively using phase constrast microscopy. Six images are taken of each cloud and spaced evenly over one breathe period. This allows us to extract the amplitude of a single oscillation relative to the mean cloud size and removes any uncertainty caused by atom number fluctuations between experimental cycles. Typically we analyze the data by fitting a single oscillation with a fixed frequency sine wave in order to extract the amplitude as indicated by the solid lines in Fig. 1.

Sample data for driven quadrupole and monopole mode oscillations are shown in Fig. 1 where one sees that

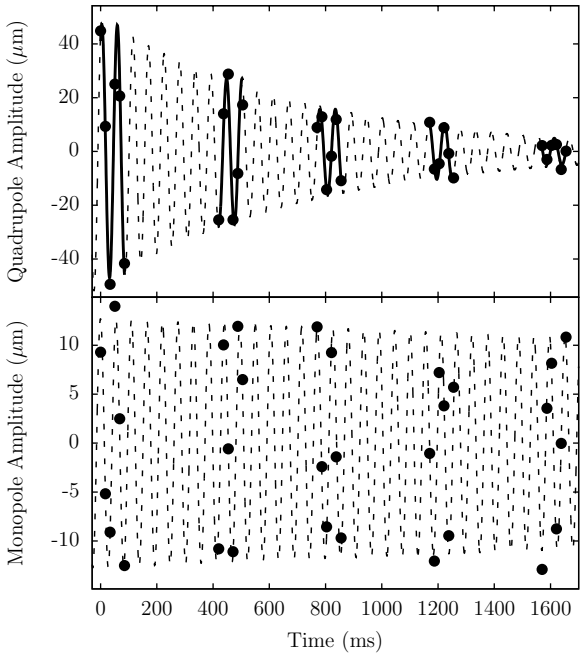


FIG. 1. Sample data for a driven quadrupole mode and monopole mode in a spherical trap. Black lines on the quadrupole data indicate a typical fitting procedure where individual periods taken in a single run are fit with an undamped sine wave to extract the amplitude. It should be noted that decay times for the monopole mode are determined from data taken over much longer times.

the monopole mode appears nearly undamped and the quadrupole mode exibits a fast decay. The quadrupole mode is expected to decay at a rate given by

$$\Gamma_Q = \frac{1}{5} n_{cl}(0) \sigma \bar{v} \tag{11}$$

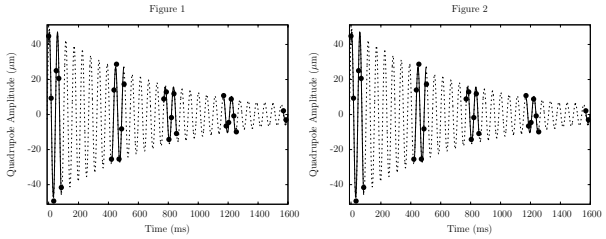


FIG. 2. Sample data for a driven quadrupole mode and monopole mode in a spherical trap. Black lines on the quadrupole data indicate a typical fitting procedure where individual periods taken in a single run are fit with an undamped sine wave to extract the amplitude.

a. A few notes on `tags` `\tag{#1}` requires the `amsmath` package. Place the `\tag{#1}` command before the `\label{#1}`, if any. The numbering produced by `\tag{#1}` *does not affect* the automatic numbering in

REVTEX; therefore, the number must be known ahead of time, and it must be manually adjusted if other equations are added. `\tag{#1}` works with both single-line and multiline equations. `\tag{#1}` should only be used in exceptional cases—do not use it to number many equations in your paper. Please note that this feature of the `amsmath` package is *not* compatible with the `hyperref` (6.77u) package.

Enclosing display math within `\begin{subequations}` and `\end{subequations}` will produce a set of equations that are labeled with letters, as shown in Eqs. (11b) and (11a) below. You may include any number of single-line and multiline equations, although it is probably not a good idea to follow one display math directly after another.

$$\mathcal{M} = ig_Z^2(4E_1E_2)^{1/2}(l_i^2)^{-1}(g^e_{\sigma_2})^2\chi_{-\sigma_2}(p_2) \times[\epsilon_i]_{\sigma_1}\chi_{\sigma_1}(p_1). \tag{12a}$$

$$\left\{abc123456abcdef\alpha\beta\gamma\delta1234556\alpha\beta\frac{1\sum_b^a}{A^2}\right\}, \tag{12b}$$

Giving a `\label{#1}` command directly after the `\begin{subequations}`, allows you to reference all the equations in the `subequations` environment. For example, the equations in the preceding subequations environment were Eqs. (11).

1. *Wide equations*

The equation that follows is set in a wide format, i.e., it spans the full page. The wide format is reserved for long equations that cannot easily be set in a single column:

$$\mathcal{R}^{(\mathrm{d})} = g^e_{\sigma_2}\left(\frac{[\Gamma^Z(3,21)]_{\sigma_1}}{Q_{12}^2-M_W^2}+\frac{[\Gamma^Z(13,2)]_{\sigma_1}}{Q_{13}^2-M_W^2}\right)+x_WQ_e\left(\frac{[\Gamma^\gamma(3,21)]_{\sigma_1}}{Q_{12}^2-M_W^2}+\frac{[\Gamma^\gamma(13,2)]_{\sigma_1}}{Q_{13}^2-M_W^2}\right). \tag{13}$$

This is typed to show how the output appears in wide format. (Incidentally, since there is no blank line between the `equation` environment above and the start of this paragraph, this paragraph is not indented.)

II. CROSS-REFERENCING

REVTEX will automatically number such things as sections, footnotes, equations, figure captions, and table captions. In order to reference them in text, use the `\label{#1}` and `\ref{#1}` commands. To reference a particular page, use the `\pageref{#1}` command.

The `\label{#1}` should appear within the section heading, within the footnote text, within the equation, or within the table or figure caption. The `\ref{#1}` command is used in text at the point where the reference is to be displayed. Some examples: Section I on page 1, Table I, and Fig. 3.

III. FLOATS: FIGURES, TABLES, VIDEOS, ETC.

Figures and tables are usually allowed to "float", which means that their placement is determined by LATEX, while the document is being typeset.

Use the `figure` environment for a figure, the `table` environment for a table. In each case, use the `\caption` command within to give the text of the figure or table caption along with the `\label` command to provide a key for referring to this figure or table. The typical content of a figure is an image of some kind; that of a table is an alignment.

Insert an image using either the `graphics` or `graphix` packages, which define the `\includegraphics{#1}` command. (The two packages differ in respect of the optional arguments used to specify the orientation, scaling, and translation of the image.) To create an alignment, use the `tabular` environment.

The best place to locate the `figure` or `table` environment is immediately following its first reference in text; this sample document illustrates this practice for Fig. 3,

TABLE I. A table that fits into a single column of a two-column layout. Note that REVTEX 4 adjusts the intercolumn spacing so that the table fills the entire width of the column. Table captions are numbered automatically. This table illustrates left-, center-, decimal- and right-aligned columns, along with the use of the `ruledtabular` environment which sets the Scotch (double) rules above and below the alignment, per APS style.

Left[a]	Centered[b]	Decimal	Right
1	2	3.001	4
10	20	30	40
100	200	300.0	400

[a] Note a.
[b] Note b.

TABLE II. This is a wide table that spans the full page width in a two-column layout. It is formatted using the `table*` environment. It also demonstates the use of `\multicolumn` in rows with entries that span more than one column.

	D^1_{4h}		D^5_{4h}	
Ion	1st alternative	2nd alternative	lst alternative	2nd alternative
K	$(2e) + (2f)$	$(4i)$	$(2c) + (2d)$	$(4f)$
Mn	$(2g)$[a]	$(a) + (b) + (c) + (d)$	$(4e)$	$(2a) + (2b)$
Cl	$(a) + (b) + (c) + (d)$	$(2g)$[a]	$(4e)$[a]	
He	$(8r)$[a]	$(4j)$[a]	$(4g)$[a]	
Ag		$(4k)$[a]		$(4h)$[a]

[a] The z parameter of these positions is $z \sim \frac{1}{4}$.

which shows a figure that is small enough to fit in a single column.

In exceptional cases, you will need to move the float earlier in the document, as was done with Table II: LaTeX's float placement algorithms need to know about a full-page-width float earlier.

Fig. 4 has content that is too wide for a single column, so the `figure*` environment has been used.

The content of a table is typically a `tabular` environment, giving rows of type in aligned columns. Column entries separated by `&`'s, and each row ends with \\. The required argument for the `tabular` environment specifies how data are aligned in the columns. For instance, entries may be centered, left-justified, right-justified, aligned on a decimal point. Extra column-spacing may be be specified as well, although REVTeX 4 sets this spacing so that the columns fill the width of the table. Horizontal rules are typeset using the `\hline` command. The doubled (or Scotch) rules that appear at the top and bottom of a table can be achieved enclosing the `tabular` environment within a `ruledtabular` environment. Rows whose columns span multiple columns can be typeset using the `\multicolumn{#1}{#2}{#3}` command (for example, see the first row of Table II).

Tables I, II, III, and IV show various effects. A table that fits in a single column employs the `table` environment. Table II is a wide table, set with the `table*` environment. Long tables may need to break across pages. The most straightforward way to accomplish this is to specify the `[H]` float placement on the `table` or `table*` environment. However, the LaTeX 2_ε package `longtable` allows headers and footers to be specified for each page of the table. A simple example of the use of `longtable` can be found in the file `summary.tex` that is included with the REVTeX 4 distribution.

There are two methods for setting footnotes within a table (these footnotes will be displayed directly below the table rather than at the bottom of the page or in the bibliography). The easiest and preferred method is just to use the `\footnote{#1}` command. This will automatically enumerate the footnotes with lowercase roman letters. However, it is sometimes necessary to have multiple entries in the table share the same footnote. In this case, there is no choice but to manually create the footnotes using `\footnotemark[#1]` and `\footnotetext[#1]{#2}`. `#1` is a numeric value. Each time the same value for `#1` is used, the same mark is produced in the table. The `\footnotetext[#1]{#2}` commands are placed after the `tabular` environment. Examine the LaTeX source and output for Tables I and IV for examples.

Video 1 illustrates several features new with REVTeX4.1, starting with the `video` environment, which is in the same category with `figure` and `table`. The `\setfloatlink` command causes the title of the video to be a hyperlink to the indicated URL; it may be used with any environment that takes the `\caption` command. The `\href` command has the same significance as it does in the context of the `hyperref` package: the second argument is a piece of text to be typeset in your document; the first is its hyperlink, a URL.

Physical Review style requires that the initial citation of figures or tables be in numerical order in text, so don't

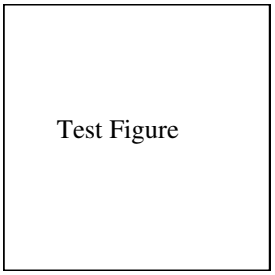


FIG. 3. A figure caption. The figure captions are automatically numbered.

TABLE III. Numbers in columns Three–Five are aligned with the "d" column specifier (requires the `dcolumn` package). Nonnumeric entries (those entries without a ".") in a "d" column are aligned on the decimal point. Use the "D" specifier for more complex layouts.

One	Two	Three	Four	Five
one	two	three	four	five
He	2	2.77234	45672.	0.69
C[a]	C[b]	12537.64	37.66345	86.37

[a] Some tables require footnotes.

[b] Some tables need more than one footnote.

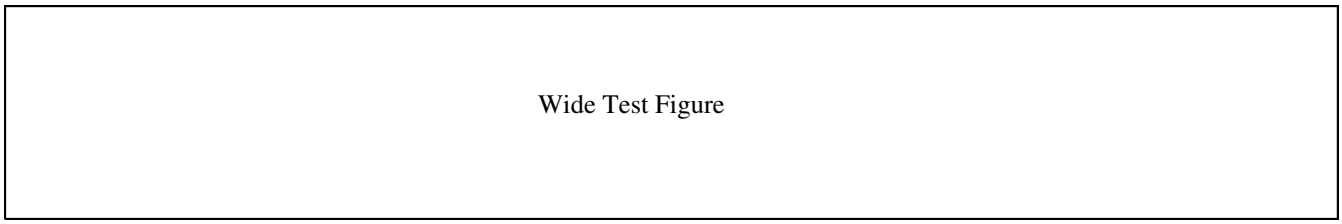
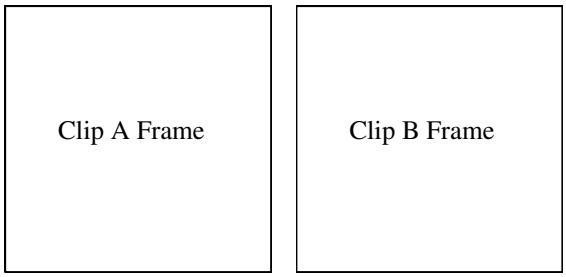


FIG. 4. Use the figure* environment to get a wide figure that spans the page in `twocolumn` formatting.



Video 1. Students explain their initial idea about Newton's third law to a teaching assistant. Clip (a): same force. Clip (b): move backwards.

cite Fig. 4 until Fig. 3 has been cited.

ACKNOWLEDGMENTS

We wish to acknowledge the support of the author community in using REVTEX, offering suggestions and encouragement, testing new versions, ...

TABLE IV. A table with numerous columns that still fits into a single column. Here, several entries share the same footnote. Inspect the LaTeX input for this table to see exactly how it is done.

	r_c (Å)	r_0 (Å)	κr_0		r_c (Å)	r_0 (Å)	κr_0
Cu	0.800	14.10	2.550	Sn[a]	0.680	1.870	3.700
Ag	0.990	15.90	2.710	Pb[b]	0.450	1.930	3.760
Au	1.150	15.90	2.710	Ca[c]	0.750	2.170	3.560
Mg	0.490	17.60	3.200	Sr[d]	0.900	2.370	3.720
Zn	0.300	15.20	2.970	Li[b]	0.380	1.730	2.830
Cd	0.530	17.10	3.160	Na[e]	0.760	2.110	3.120
Hg	0.550	17.80	3.220	K[e]	1.120	2.620	3.480
Al	0.230	15.80	3.240	Rb[c]	1.330	2.800	3.590
Ga	0.310	16.70	3.330	Cs[d]	1.420	3.030	3.740
In	0.460	18.40	3.500	Ba[e]	0.960	2.460	3.780
Tl	0.480	18.90	3.550				

[a] Here's the first, from Ref. **?** .
[b] Here's the second.
[c] Here's the third.
[d] Here's the fourth.
[e] And etc.

Appendix A: Appendixes

To start the appendixes, use the `\appendix` command. This signals that all following section commands refer to appendixes instead of regular sections. Therefore, the `\appendix` command should be used only once—to setup the section commands to act as appendixes. Thereafter normal section commands are used. The heading for a section can be left empty. For example,

```
\appendix
\section{}
```

will produce an appendix heading that says "APPENDIX A" and

```
\appendix
\section{Background}
```

will produce an appendix heading that says "APPENDIX A: BACKGROUND" (note that the colon is set automatically).

If there is only one appendix, then the letter "A" should not appear. This is suppressed by using the star version of the appendix command (`\appendix*` in the place of `\appendix`).

Appendix B: A little more on appendixes

Observe that this appendix was started by using

```
\section{A little more on appendixes}
```

Note the equation number in an appendix:

$$E = mc^2. \tag{B1}$$

1. A subsection in an appendix

You can use a subsection or subsubsection in an appendix. Note the numbering: we are now in Appendix B 1.

Note the equation numbers in this appendix, produced with the subequations environment:

$$E = mc, \tag{B2a}$$
$$E = mc^2, \tag{B2b}$$
$$E \gtrsim mc^3. \tag{B2c}$$

They turn out to be Eqs. (B2a), (B2b), and (B2c).