5. Panel Data and Fixed Effects

- When we have longitudinal data we can try to tackle OVB when the unobserved omitted factors are *stable over time*
- Setting:
  - We can measure the outcome variable for a set of objects (people, firms, ...) at several point in time
  - The key variable of interest (the „treatment“) changes over time
  - We study the association between the change in the treatment variable and the change in the outcome variable
- Here:
  Consider *Fixed Effects Models* as one important approach

Fixed Effects

- Recall the potential outcome framework (now with a time index $t = 1, \dots, T$)

$$Y_{C_{it}it} = \begin{cases} Y_{1it} & if \quad C_{it} = 1 \\ Y_{0it} & if \quad C_{it} = 0 \end{cases}$$

- Let X_{it} be again a vector of **observed time varying** covariates
- Consider a situation where there is a vector of **unobserved** factors A_i that affect Y_{Cit} but **do not depend on time**
- This may for instance be a person‘s ability or personality, or a firm‘s location, or the quality of its management
- When these unobservable factors A_i are correlated with C_{it} such that

$$Cov[A_i, C_{it}] \neq 0$$

we have an omitted variable bias when regressing Y_{it} on C_{it}

But: Assume now that $E[Y_{0it}|A_i, X_{it}, t, C_{it}] = E[Y_{0it}|A_i, X_{it}, t]$

We thus make the following key assumption

$$E[Y_{0it}|A_i, X_{it}, t, C_{it}] = E[Y_{0it}|A_i, X_{it}, t]$$

Note:

- The assumption states that C_{it} is as good as randomly assigned conditional on A_i and X_{it}
- That is, if we compare two individuals who share the same values for X_{it} (observable) and A_i (unobservable), we assume that their potential outcomes do not differ
- This is a sensible identifying assumption whenever any unobserved determinants of the treatment (that also may affect the outcomes beyond the treatment) are **constant over time**

- Consider now the following linear model

$$E[Y_{it}|C_{it}, A_i, X_{it}, t] = \alpha + \rho C_{it} + X_{it}'\beta + A_i'\gamma + \lambda_t + \epsilon_{it}$$

- Thus the causal effect of the treatment C is again a constant ρ, i.e.

$$E[Y_{1it}|A_i, X_{it}, t] - E[Y_{0it}|A_i, X_{it}, t] = \rho$$

- We can now replace $\alpha_i = \alpha + A_i'\gamma$ and thus rewrite the model as

$$Y_{it} = \alpha_i + \lambda_t + \rho C_{it} + X_{it}'\beta + \epsilon_{it}$$

- Then running a regression will estimate the causal effect ρ of C on Y as $E[Y_{0it}|A_i, X_{it}, t, C_{it} = 1] = E[Y_{0it}|A_i, X_{it}, t, C_{it} = 0]$

- This is a **fixed effects model**:
  - The α_i are parameters to be estimated (i.e. estimating a dummy for every person/firm/object)
  - The γ_t are time effects that are also estimated (i.e. estimating a dummy for every period)

Study

Lazear’s (2000) study on Performance Pay at Safelite

- Safelite is a large auto glass company in the US
- Business: replace broken windshields.
- New compensation scheme in January 1994: Piece rate scheme (PPP) replaced hourly-wage scheme in 1994
- The piece rate scheme was phased in over 19 months, starting from the headquarter town.
- The gradual implementation of piece rate allows for within-worker variation identifying the incentive effect of piece rate on effort.
- But: also high turnover rates; many workers also hired after the introduction of the PPP
- In the following:
  - Unit of observation = Worker in a given month;
  - Productivity measure: Average windshields installed by the worker on a given day.

Safelite: Regression analysis

TABLE 3—REGRESSION RESULTS

Regression number	Dummy for PPP person-month observation	Tenure	Time since PPP	New regime	R^2	Description
1	0.368 (0.013)				0.04	Dummies for month and year included
2	0.197 (0.009)				0.73	Dummies for month and year; worker-specific dummies included (2,755 individual workers)
3	0.313 (0.014)	0.343 (0.017)	0.107 (0.024)		0.05	Dummies for month and year included
4	0.202 (0.009)	0.224 (0.058)	0.273 (0.018)		0.76	Dummies for month and year; worker-specific dummies included (2,755 individual workers)
5	0.309 (0.014)	0.424 (0.019)	0.130 (0.024)	0.243 (0.025)	0.06	Dummies for month and year included

Notes: Standard errors are reported in parentheses below the coefficients.
Dependent variable: ln output-per-worker-per-day.
Number of observations: 29,837.

Safelite (continued): What do the worker fixed effects do here?

- Regression without worker fixed effects (row 1)
  - this gives us an estimate of the causal effect of the treatment on the *average performance* of all workers working at a given point in time
  - (when believe that the treatment is as good as randomly assigned conditional on the time period which seems very plausible here)

- However: if we are interested in the causal effect of the treatment on the performance of an *average given worker* this is a „biased“ estimate
  - This is the case when the ability of workers depends on the treatment
  - For instance, when the PPP allows to hire better workers
  - Then $E[Y_{0it}|t, C_{it} = 1] > E[Y_{0it}|t, C_{it} = 0]$

    i.e. workers hired under the PPP would be better even without the PPP
  - In this respect the conditional independence assumption is violated
  - There is a classical selection bias and the PPP dummy should give a too high estimate for the causal effect of the PPP on a *given* worker

What do the worker fixed effects do here?

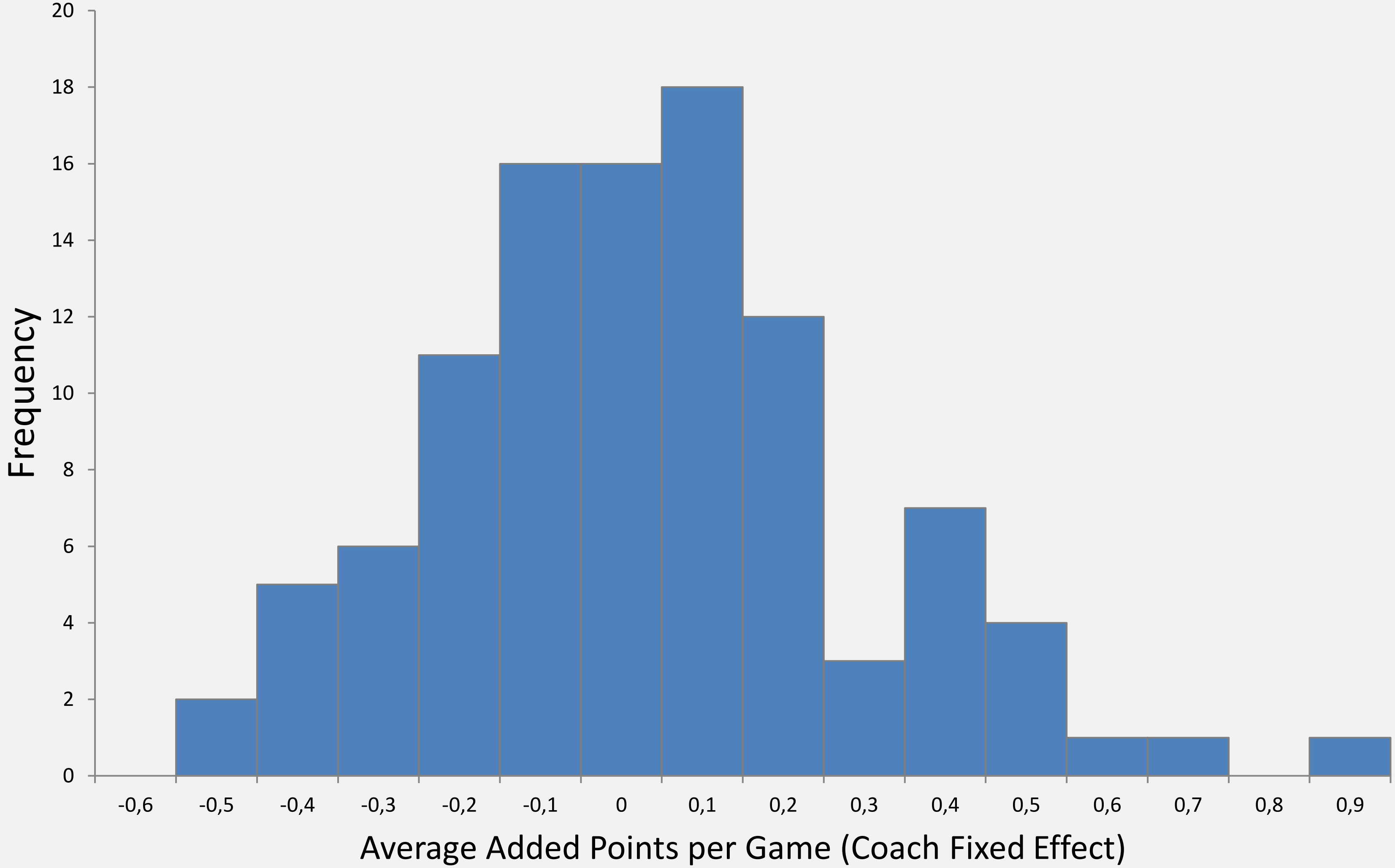
- The worker fixed effects model (row 2) takes this problem into account
- It imposes the weaker assumption that
$$E[Y_{0it}|A_i, t, C_{it}] = E[Y_{0it}|A_i, t]$$
- When A_i captures the workers unobserved ability this assumption states that for workers *of the same ability* the counterfactual performance is independent of the treatment
- The fixed effects model estimates the unoberved abilities of the workers (using that a worker‘s performance is observed over many months)
- It thus estimates the causal effect of the PPP conditional on worker‘s abilities
- Note: The model without fixed effects is here not wrong, it estimates something different
  – Without worker fixed effects it estimates the total effect on performance which includes a *selection* and an *incentive effect*
  – With worker fixed effects it estimates the pure incentive effect

Study

Estimating the Value Added of Managers

- How much do individual managers matter for firm behavior and performance?
- Bertrand and Schoar (2003) use
  - Manager-firm matched panel dataset, where they can track individual top managers across different firms over time.
  - Estimate how much of the unexplained variation in firm practices can be attributed to manager fixed effects (controlling for firm fixed effects and time-varying firm characteristics)
- Mühlheusser/Schneemann/Sliwka/Wallmeier (2017) consider the case of soccer coaches
  - Coaches frequently move between teams
  - Allows to disentangle the effect of the coach from the strength of the club by estimating models with manager and team fixed effects
  - Data on 20 seasons of the German Bundesliga

Estimated Soccer Coach Fixed Effects for the Bundesliga



Estimating Fixed Effects Models

- Estimating the coefficients of individual dummy variables seems demanding in large panels (1000 employees = 1000 fixed effects)
- However, if we are not interested in knowing the specific values of the individual fixed effects, we can estimate the model in a simpler manner
- Consider

$$Y_{it} = \alpha_i + \lambda_t + \rho C_{it} + X_{it}'\beta + \epsilon_{it}$$

- Now take the average across all time periods $\overline{Y_i} = \frac{1}{T}\sum_{t=1}^{T} Y_{it}$

$$\overline{Y}_i = \alpha_i + \overline{\lambda} + \rho\overline{C}_i + \overline{X}_i'\beta + \overline{\epsilon}_i$$

and substract this from Y_{it}

$$Y_{it} - \overline{Y}_i = \lambda_t - \overline{\lambda} + \rho\left(C_{it} - \overline{C}_i\right) + (X_{it}' - \overline{X}_i')\beta + \epsilon_{it} - \overline{\epsilon}_i$$

→ The α_i are eliminated!

$$Y_{it} - \overline{Y}_i = \lambda_t - \overline{\lambda} + \rho\left(C_{it} - \overline{C}_i\right) + (X'_{it} - \overline{X}'_i)\beta + \epsilon_{it} - \overline{\epsilon}_i$$

- Hence,
  - replace the outcome variable by its deviation from the mean over time
  - replace the explanatory variables by their deviations from their means over time
  - Regress the „**de-meaned**" outcome on the „de-meaned" explanatory variables

  $\rightarrow$ This gives us an estimate of ρ

  $\rightarrow$ We can estimate ρ and β without having to estimate the α_i

- This model is sometimes also called the **within-estimator**:

  It estimates the effect of ρ on Y from the within person variation in C

Python

Fixed Effects Regressions in Python

- Panel regressions in Python can be done with library linearmodels
- Install by !pip install linearmodels
- Import by from linearmodels import PanelOLS
- In order to run a panel regression use a MultiIndex DataFrame that is a DataFrame that uses two indices
  - one index for the entity variable (the omitted time constant variable)
  - one index for the time variable

```
df=df.set_index([‘entity', 'year’])
```

- Then fit the model by

```
reg = PanelOLS.from_formula('y ~ x + EntityEffects + TimeEffects', data=df).fit()
```

- Then print the output with print(reg)
  (Note the different notation to statsmodels: can directly print the results)

Your Task | Fixed Effects

- Open the notebook in which you estimated the association between Management Practices and ROCE
- For a part of the observations the data set contains panel data (consider the variables account_id containing a firm identifier and year)
- Bloom et al. (2012) report the following table, where the third colums shows the result of a fixed effects regression
- Replicate the regression using PanelOLS
- You may also replicate it using a simple OLS with dummies for each firm (recall +C(..))
- Note: Further relevant variables are
  - emp: number of employees
  - ppent capital (property, plant & equipment)
  - You can generate logs by using np.log(x) directly in the formula

	(1)	(2)	(3)
Sector	Manufact.	Manufact.	Manufact.
Dependent variable	Log (Sales)	Log (Sales)	Log (Sales)
Management	0.523*** (0.030)	0.233*** (0.024)	0.048** (0.022)
Ln(Employees)	0.915*** (0.019)	0.659*** (0.026)	0.364*** (0.109)
Ln(Capital)		0.289*** (0.020)	0.244*** (0.087)
Country controls	No	Yes	NA
Industry controls	No	Yes	NA
General controls	No	Yes	NA
Firm fixed effects	No	No	Yes
Organizations	2,927	2,927	1,453
Observations	7,094	7,094	5,561

Your Task — Fixed Effects (Simulated Sales Training Evaluation VII)

Generate the following notebook

```
n=2000
df1=pd.DataFrame(index=range(n))
df1['ability']=np.random.normal(100,15,n)
df1['year']=1
df1['persnr']=df1.index
df1['training']=0
## Now copy the DataFrame (i.e. generate observations for second year)
df2=df1.copy()
df2['year']=2
## Training only in year 2:
df2['training']=(df2.ability+np.random.normal(0,10,n)>=100)
## Generate DataFrame that spans both years by appending the two data frames
df=pd.concat([df1,df2], sort=False)
df['sales']= 10000 + df.training*5000 + df.ability*100 + df.year*2000
                 + np.random.normal(0,4000,2*n)
```

Note:

- The script generated a data frame simulating two years of data in which
  - Sales of each subject are observed in each year
  - training is affected by ability
  - subjects are only trained in year 2

Now analyze the generated data:

- Run an OLS regression of sales on training and year
- Define the time and entity indices
- Run a fixed effects regression

But note important caveats:

1. When you want to interpret the results of a Fixed Effects regression causally, a key underlying assumption is the so-called **common trend assumption**
   - That is „treatment“ and „control“ units follow the same underlying time trend
   - This is a key identifying assumption
2. When the treatment C_{it} hardly varies over time it is hard to evaluate the causal effect effect ρ
   - In the extreme when C_{it} is completely stable then $C_{it} = \overline{C}_{it}$
   - Not identifying a significant effect in the data then does not necessarily imply that there is no such effect
3. Fixed effects can **only eliminate time constant omitted** variables
   - If the treatment is correlated with time varying unobserved variables omitted variable issues remain

Difference-in-Difference Estimation

- Sometimes the regressor of interest varies only at a more aggregate level (say a state, or a firm) which we index by s
- Moreover, sometimes we do not observe the same individuals repeatedly but have different samples at different points in time t
- We can then use a so-called **difference-in-difference** estimation strategy
- This is like a fixed effects model with a fixed effect at a more aggregate level
- The underlying idea is again that there is an additive structure in the potential outcomes that is

$$E[Y_{0ist}|s,t] = \gamma_s + \lambda_t$$

- Assume that the causal effect of the treatment is a constant

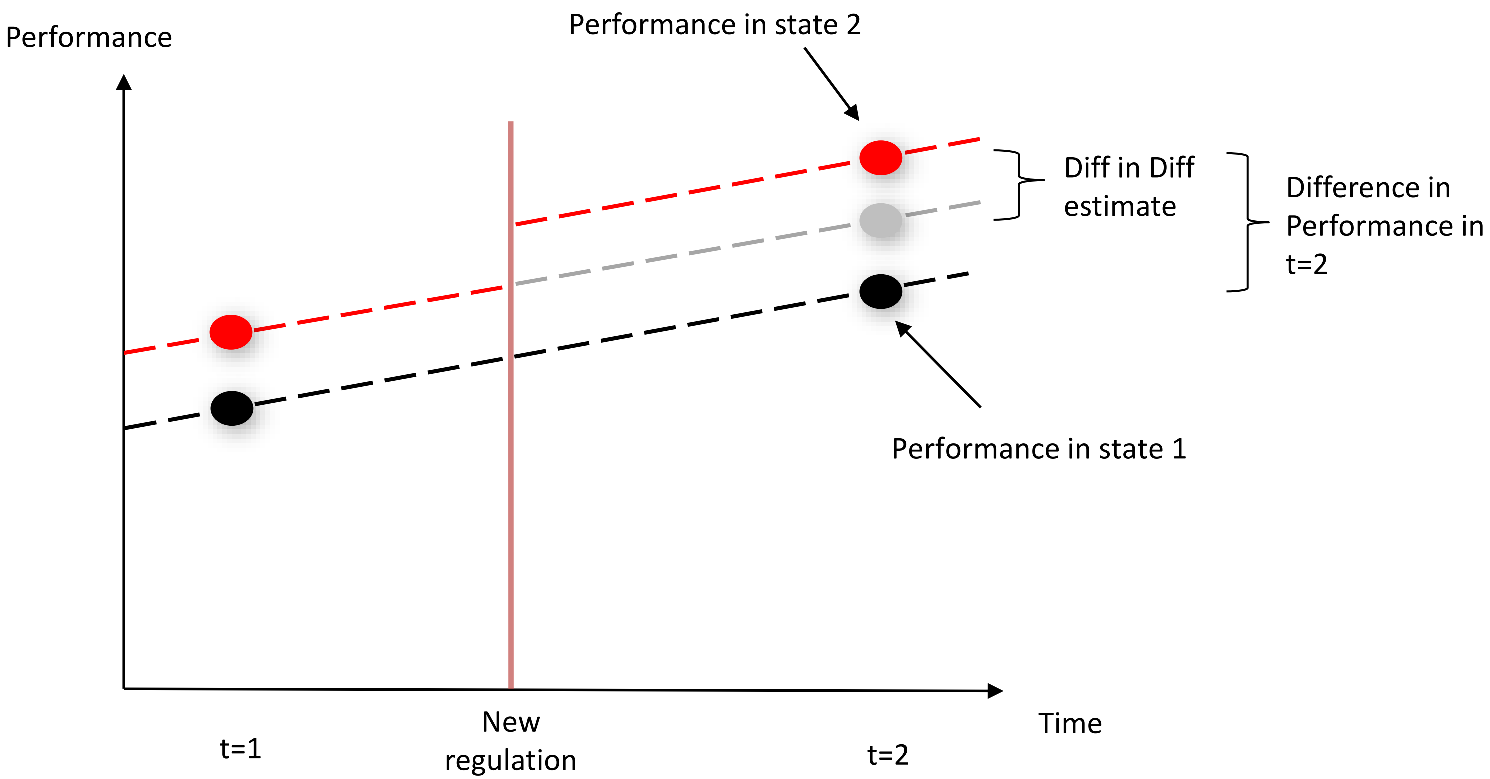
$$E[Y_{1ist} - Y_{0ist}|s,t] = \rho$$

- Then

$$Y_{ist} = \gamma_s + \lambda_t + \rho \cdot C_{st} + \epsilon_{ist}$$

Diff-in-Diff: Graphical illustration

- Suppose that there are two states (regions, firms, departments,…)
- A new regulation is introduced in state 2 but not in state 1
- We want to study the effect on firm performance



Consider

$$Y_{ist} = \gamma_s + \lambda_t + \rho \cdot C_{st} + \epsilon_{ist}$$

- Say we have two periods $t = 1,2$ and two states $s = 1,2$
- A new policy is adopted in state 2 in period 2 such that
  - $C_{11} = C_{21} = C_{21} = 0$ and $C_{22} = 1$
- Then
  - $E[Y_{ist}|s = 1, t = 2] - E[Y_{ist}|s = 1, t = 1] = \lambda_2 - \lambda_1$ and
  - $E[Y_{ist}|s = 2, t = 2] - E[Y_{ist}|s = 2, t = 1] = \lambda_2 - \lambda_1 + \rho$
- The difference-in-difference is

$$(E[Y_{ist}|s = 2, t = 2] - E[Y_{ist}|s = 2, t = 1])$$
$$- \quad (E[Y_{ist}|s = 1, t = 2] - E[Y_{ist}|s = 1, t = 1]) \quad = \rho$$

which is the causal effect of interest

Regression Diff-in-Diff

- Note: we could estimated the causal effect δ from just working with the differences and replace the expectations with the respective averages
- Typically it is more convenient to simply run a regression
  - Let $TREAT_i$ be a dummy indicating whether an observation comes from the treated region
  - Let $POST_t$ be a dummy indicating whether an observation comes from a period after the treatment has been implemented
- Then we can regress

$$Y_{it} = \alpha + \beta \cdot TREAT_i + \gamma \cdot POST_t + \rho \cdot (TREAT_i \cdot POST_t) + \epsilon_{it}$$

- The coefficient $\tilde{\rho}$ of the interaction term $TREAT_i \cdot POST_t$ yields an estimate of the causal effect
- Note:
  - Regression DiD also provides statistical tests
  - And it can be applied if there are more than two periods

Study

Dismissal Protection and Hiring

- Bauernschuster (2013) studies the effect of a firm-size threshold of the German dismissal protection law in 2004 on the hiring behavior of small firms
- From 1999 until the end of 2003 the dismissal protection law applied only to establishments with more than five (full time equivalent) workers
- In 2004, this threshold was shifted up to ten full-time equivalent workers
- Dismissal protection regulation was abandoned for workers hired after December 31, 2003 by establishments with 6 to 9 FTE employees
- Bauernschuster studies the effect on hiring applying a diff-in-diff strategy
  - „Treatment group“: establishments with 6-10 employees
  - „Control group“: establishments with 11-20 employees
- Uses data from the IAB establishment panel (on which the LPP is based)
- Dependent variables are hiring rates and total number of hirings per establishment in the first half of the year

Dismissal Protection and Hiring (continued)

The Dynamics of Hiring

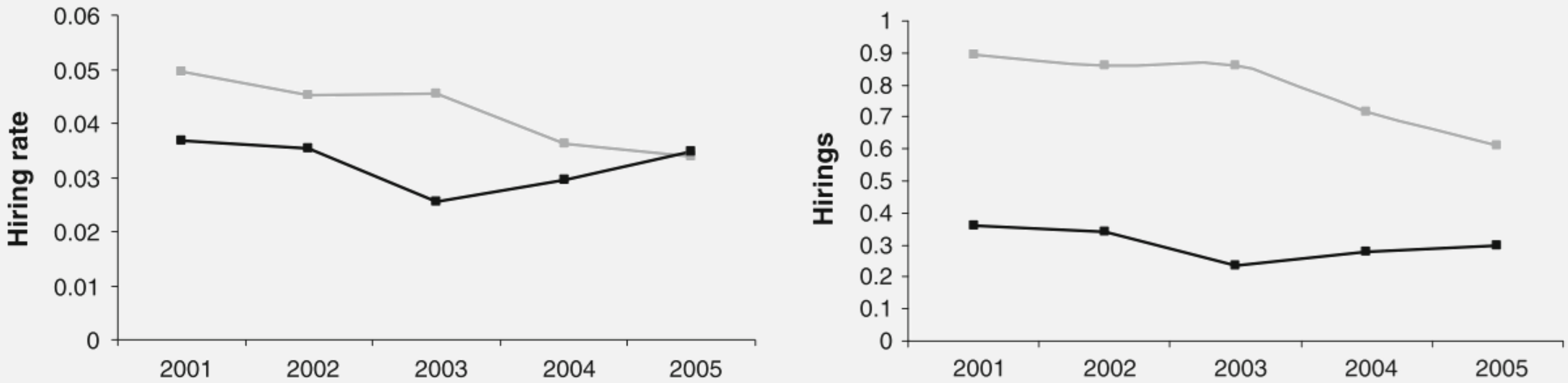


Fig. 2 Dynamics of hirings in treatment and control group. The *left figure* shows average hiring rates for treatment (*black line*) and control groups (*grey line*) over time. The *right figure* shows average absolute hirings for treatment (*black line*) and control groups (*grey line*) over time. The treatment group comprises establishments with more than five and up to ten full-time equivalent workers while the control group consists of establishments with more than ten and up to 20 full-time equivalent employees. Data source: IAB Establishment Panel

Bauernschuster (2013)

Dismissal Protection and Hiring (continued)

Table 1 DiD estimates on hirings

Parameter	Hiring rate (1)		Hiring rate (2)		Hiring rate (3)	
	Coefficient	Standard error	Coefficient	Standard error	Coefficient	Standard error
DiD 2004	0.013**	0.007	0.016**	0.007	0.020**	0.009
DiD 2005	0.021**	0.009	0.021**	0.009	0.020**	0.009
Treatment group	−0.020***	0.006	−0.019***	0.007	−0.024***	0.006
Year 2004	−0.009*	0.004	−0.013***	0.004	−0.014***	0.006
Year 2005	−0.012**	0.005	−0.013***	0.005	−0.013*	0.006
Control set 1	No		Yes		Yes	
Control set 2	No		No		Yes	
N	1,749		1,658		1,285	
R^2	0.0059		0.0725		0.1197	

The table reports the results of OLS difference-in-differences regressions with hiring rates as the dependent variable. The treatment group comprises establishments with more than five and up to ten full-time equivalent workers while the control group consists of establishments with more than ten and up to 20 full-time equivalent employees. The baseline year is 2003. Specification (1) includes no further controls. In specification (2), we additionally control for capital stock, works council, collective labor agreement, age, and industry (control set 1). In specification (3), we add the ratio of female workers, ratio of unqualified workers, ratio of apprentices, wage per worker in the previous year, value added per worker in the previous year as well as net hirings in the previous year as further controls (control set 2). Standard errors are clustered at the establishment level. ***, **, * denote significance at the 1, 5, and 10% levels, respectively. Data source: IAB Establishment Panel

Bauernschuster (2013)

The Common Trend Assumption

- Crucial for both a Diff-in-Diff and a Fixed Effects estimation strategy:
  Treatment and control group follow the same underlying time trend!
- If this is violated, then both approaches yield biased estimates
- It is called the *common trend assumption*
- Again this is an identifying assumption: We can claim that we identify a causal effect when the common trend assumption holds
- It can be very useful to check the trends in several periods
  – before the change occurs
  – after the change occurs
- If trends differ already before the intervention both strategies are problematic to identify causal effects