5. Diff-in-Diff and Panel Data

- The fundamental problem of causal inference is that we do not know the counterfactual outcome
- When we have longitudinal data: can assess the counterfactual outcome with data from the past (from before the change that we want to evaluate)
  - We thus tackle selection bias/OVB by comparing changes over time
  - Can work quite well when unobserved confounders are *stable over time*
- Setting:
  - We can measure the outcome variable for a set of objects (people, firms, ...) at least two points in time
  - The key variable of interest (the „treatment“) changes over time
  - We study the association between the *change* in the treatment variable and the *change* in the outcome variable

5.1 Difference-in-Difference Estimation

- Consider again the potential outcome framework and introduce a time dimension $t = 1{,}2$ such that potential outcomes are $Y_{1i}(t)$ and $Y_{0i}(t)$
- Suppose no one is treated at time $\mathrm{t} = 1$ and thus $Y_{0i}(1)$ is observed for all
- But some people receive the treatment at date $\mathrm{t} = 2$

$$Y_{C_i i}(2) = \begin{cases} Y_{1i}(2) & if \quad C_i = 1 \\ Y_{0i}(2) & if \quad C_i = 0 \end{cases}$$

- We again want to estimate the Average Treatment Effect on the Treated

$$E[Y_{1i}(2) - Y_{0i}(2) | C_i = 1]$$

Note:

- We still cannot observe the counterfactual outcome $Y_{0i}(2)$ for the treated
- But we can observe $Y_{0i}(1)$, that is their outcome before the treatment
- When does this help us the estimate the causal effect?

Now impose the following assumption:

The Parallel Trends Assumption

When

$$E[Y_{0i}(2) - Y_{0i}(1)|C_i = 1] = E[Y_{0i}(2) - Y_{0i}(1)|C_i = 0]$$

then the treated and untreated units have parallel time trends.

Intuition:

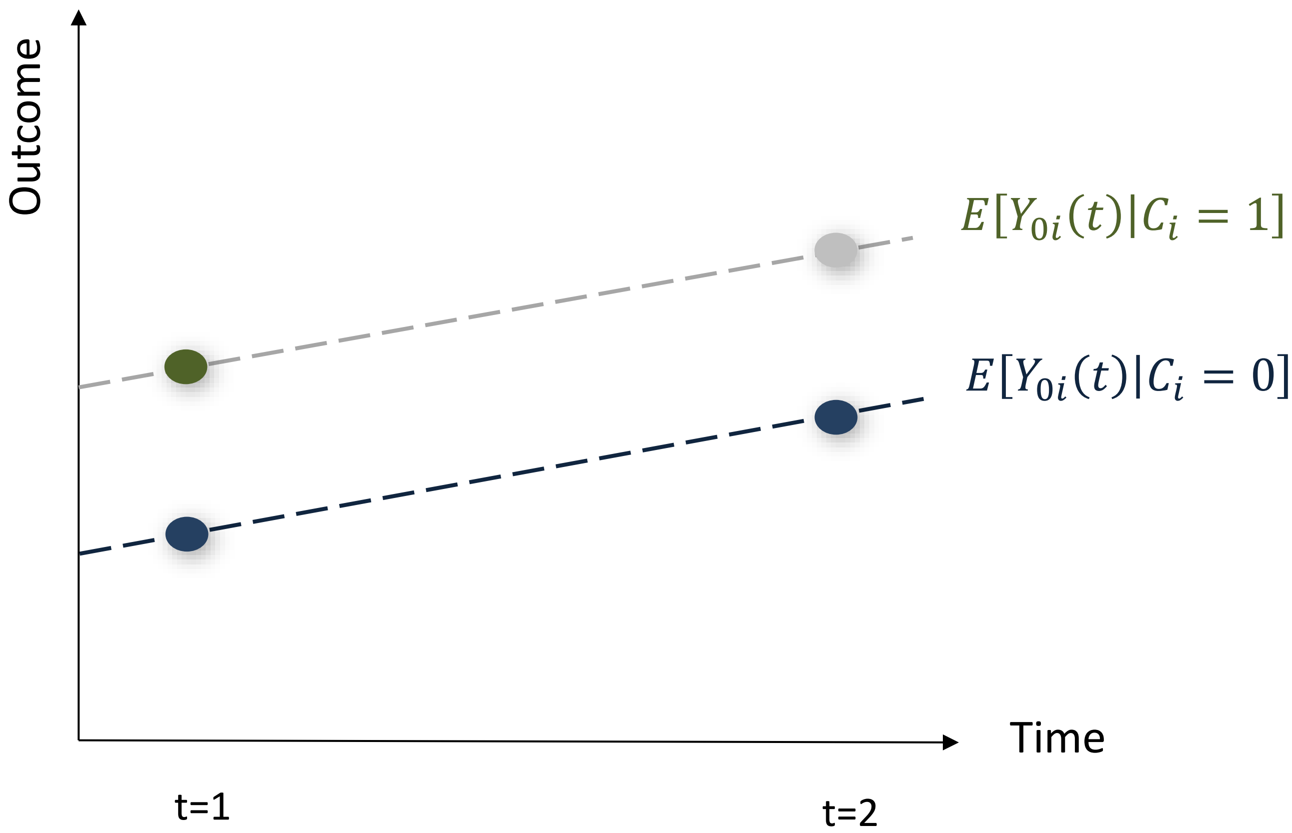
- The treated and untreated units may differ in their levels of performance
- But (without receiving the treatment) expected outcomes increase by the same amount on average from period 1 to period 2

When this assumption holds…

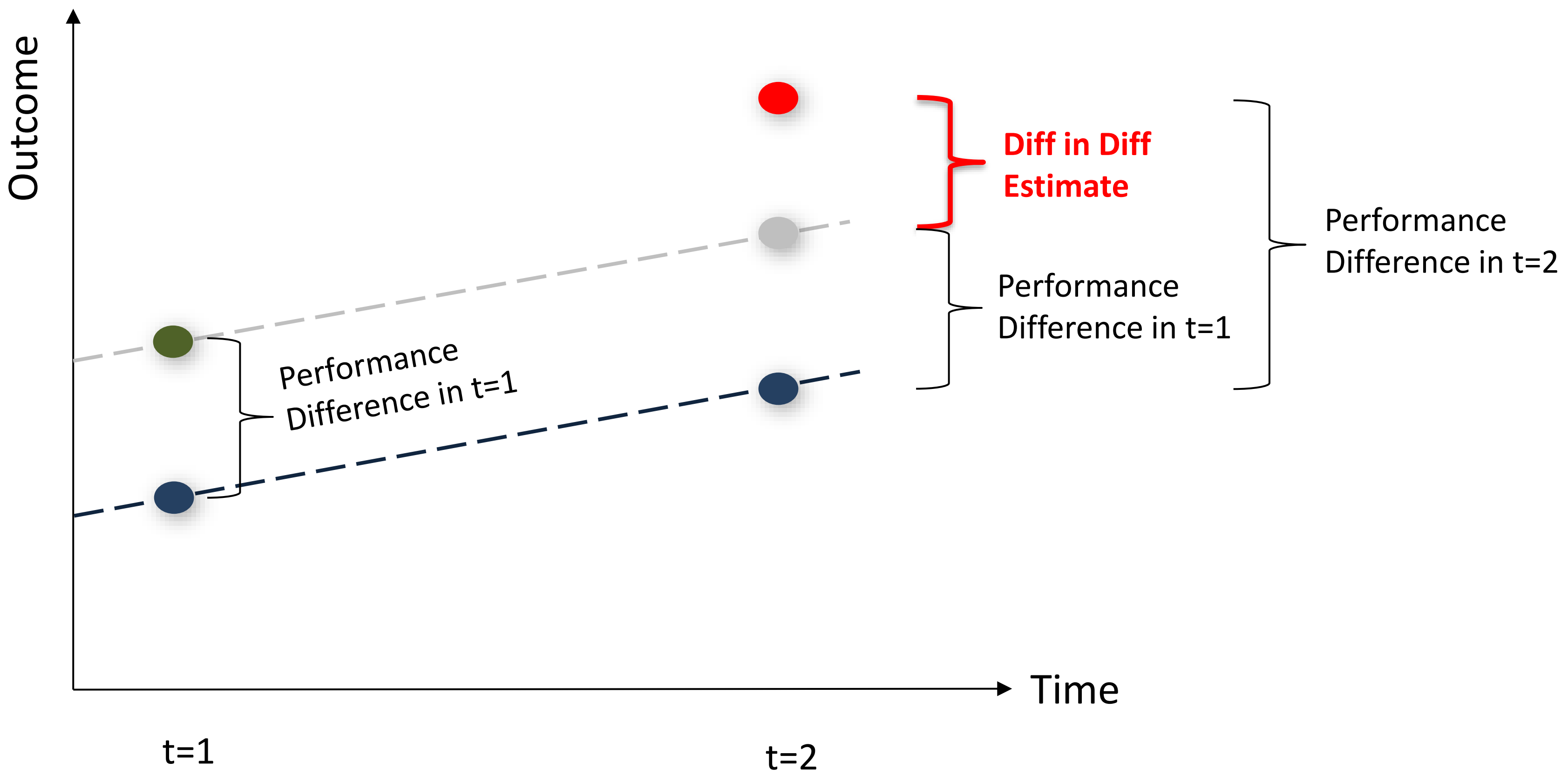
- we can estimate the ATT even with non-random assignment!

The Parallel Trends Assumption:

The same trends without the treatment



The Diff-in-Diff Estimator



We can also show this formally:

- Use the common trend assumption

$$E[Y_{0i}(2) - Y_{0i}(1)|C_i = 1] = E[Y_{0i}(2) - Y_{0i}(1)|C_i = 0]$$

… to obtain the expected counterfactual outcome in period 2

$$E[Y_{0i}(2)|C_i = 1] = E[Y_{0i}(1)|C_i = 1] + E[Y_{0i}(2) - Y_{0i}(1)|C_i = 0]$$

- Such that the causal effect is

$$E[Y_{1i}(2) - Y_{0i}(2)|C_i = 1]$$
$$= E[Y_{1i}(2) - Y_{0i}(1)|C_i = 1] - E[Y_{0i}(2) - Y_{0i}(1)|C_i = 0]$$

→ The causal effect is thus just the difference between the

- performance increase in the group that receive $C_i = 1$ and
- performance increase in the group that receive $C_i = 0$

- Key idea: Estimate the counterfactual outcome by assuming that the treated units follow the same time trend

To illustrate the effect it is convenient to rearrange

$$E[Y_{1i}(2) - Y_{0i}(2)|C_i = 1]$$

$$= E[Y_{1i}(2) - Y_{0i}(1)|C_i = 1] - E[Y_{0i}(2) - Y_{0i}(1)|C_i = 0]$$

$$= (E[Y_i(2)|C_i = 1] - E[Y_i(2)|C_i = 0])$$

$$-(E[Y_i(1)|C_i = 1] - E[Y_i(1)|C_i = 0])$$

→ The causal effect is just the difference between

- the difference in performance between the groups in $t = 2$
- the difference in performance between the groups in $t = 1$

- Therefore, it is called the **difference-in-difference estimator** (“Diff-in-Diff”, “DiD”)

Your Task — Diff-in-Diff (Simulated Sales Training Evaluation VIII)

- Download the following notebook:
  https://github.com/dsliwka/EEMP2024/blob/main/SalesSimDiD.ipynb
- Go through the simulation code and understand how the data is generated
- Note:
  - tgroup is the group to be trained (it will have value 1 in both periods for those agents who are trained in period 2)
  - training only has value 1 when the agent is indeed trained (in period 2)
- Now plot sales by tgroup and year

  Plot a barplot with x='year', y='sales', hue='tgroup'
- Compute average sales by year and group & compute Diff-in-Diff

  (convenient to use df.groupby(['tgroup','year']).sales.mean())
- Save the notebook in your Google Drive

Regression Diff-in-Diff

- Note: we can estimated the causal effect ρ from just working with the differences and replace the expectations with the respective averages
- Typically it is more convenient to simply run a regression
  - Let $TREAT_i$ be a dummy indicating whether an observation comes from the treated group (dummy=1 also before the change!)
  - Let $POST_t$ be a dummy indicating whether an observation comes from a period after the treatment has been implemented
- Then we can regress

$$Y_{it} = \alpha + \beta \cdot TREAT_i + \gamma \cdot POST_t + \rho \cdot (TREAT_i \times POST_t) + \epsilon_{it}$$

- The coefficient $\tilde{\rho}$ of the interaction term $TREAT_i \times POST_t$ yields an estimate of the causal effect
- Note:
  - Regression DiD also provides statistical tests
  - And it can be applied if there are more than two periods

Your Task — Diff-in-Diff (Simulated Sales Training Evaluation VIII)

- Open the notebook that simulated the sales panel data (SalesSimDiD.ipynb)
- Now perform a regression DiD
- Note:
  - It is convenient to generate a dummy variable post which takes value 1 only in year 2
  - Be careful when replicating the specification stated on the previous slide: $TREAT_i$ is a dummy indicating that an obs is in the treatment group
- Interpret the regression coefficients
- Save the notebook

5.2 Fixed Effects

- In the DiD approach we controlled for the group that the agent belonged to (i.e. the treated or untreated)
- The key idea was to at the same time take out
  - "level effects", i.e. differences between the groups that are there at the outset and stay constant over time
  - and "time effects", i.e. differences between the periods that are driven by the same underlying time trends in the groups
- We can also apply the same logic, but control for
  - level effects at the level of an individual subject rather than a group of subjects
  - time effects across multiple periods
  - as well as time-varying further control variables
- Then we estimate a so-called ***Fixed Effects Model***

- Suppose that A_i is a vector of unobserved characteristics of a unit (i.e. person, firm, state..) that are constant over time

- Consider the following linear model

$$Y_{it} = \alpha + \rho C_{it} + X_{it}'\beta + A_i'\gamma + \lambda_t + \epsilon_{it}$$

where λ_t are dummies for each period that capture the time trend

- Thus the causal effect of the treatment C is again a constant ρ, i.e.

$$E[Y_{1it}|A_i, X_{it}, t] - E[Y_{0it}|A_i, X_{it}, t] = \rho$$

- We can now replace $\alpha_i = \alpha + A_i'\gamma$ and thus rewrite the model as

$$Y_{it} = \alpha_i + \lambda_t + \rho C_{it} + X_{it}'\beta + \epsilon_{it}$$

$$Y_{it} = \alpha_i + \lambda_t + \rho C_{it} + X_{it}'\beta + \epsilon_{it}$$

- We can estimate this by including
  - a dummy α_i for each unit and
  - a dummy λ_t for each time period
- Then running a regression will estimate the causal effect ρ of C on Y as $E[Y_{0it}|A_i, X_{it}, t, C_{it} = 1] = E[Y_{0it}|A_i, X_{it}, t, C_{it} = 0]$
- This is a **fixed effects model**:
  - The α_i are parameters to be estimated (i.e. estimating a dummy for every person/firm/object)
  - The λ_t are time effects that are also estimated (i.e. estimating a dummy for every period)

Python

Fixed Effects Regressions in Python

- It is very easy to estimate a fixed effects model in pyfixest

  `reg = pf.feols('y ~ x | f1 + f2 ', data=dfp)`

- Here you specify the variables for which fixed effects are estimated in `f1` & `f2` (of course you can only specify only one type fixed effect or even more than two)

- Note: When you estimate a fixed effects model
  - It is likely that observations of the same entity (firm, person etc.) are not independent
  - Hence, it typically makes sense to estimate clustered standard errors where the cluster is determined by the respective entity with `vcov={“CRV1”: “f1”}`

Your Task **Fixed Effects**

- Open the notebook in which you estimated the association between Management Practices and ROCE
- For a part of the observations the data set contains panel data (consider the variables account_id containing a firm identifier and year)
- Bloom et al. (2012) report the following table, where the third colums shows the result of a fixed effects regression
- Replicate the regression using pyfixest
- Note: Further relevant variables are
  - emp: number of employees
  - ppent capital (property, plant & equipment)
  - You can generate logs by using np.log(x) directly in the formula

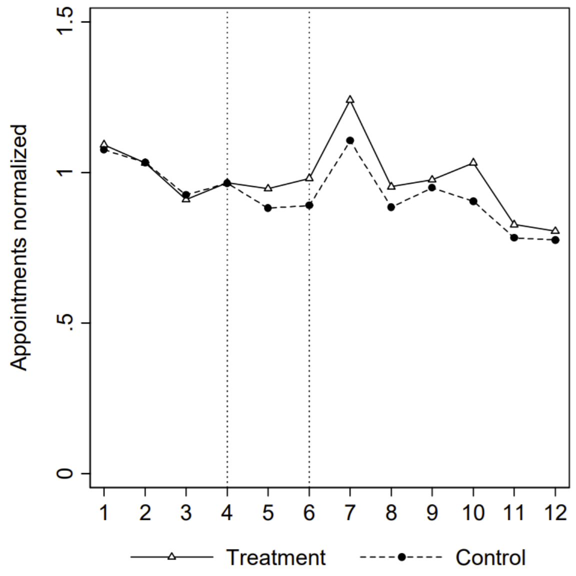
	(1)	(2)	(3)
Sector	Manufact.	Manufact.	Manufact.
Dependent variable	Log (Sales)	Log (Sales)	Log (Sales)
Management	0.523*** (0.030)	0.233*** (0.024)	0.048** (0.022)
Ln(Employees)	0.915*** (0.019)	0.659*** (0.026)	0.364*** (0.109)
Ln(Capital)		0.289*** (0.020)	0.244*** (0.087)
Country controls	No	Yes	NA
Industry controls	No	Yes	NA
General controls	No	Yes	NA
Firm fixed effects	No	No	Yes
Organizations	2,927	2,927	1,453
Observations	7,094	7,094	5,561

Your Task — Fixed Effects (Simulated Sales Training Evaluation IX)

- Open again the SalesSimDiD notebook
- Run a fixed effects regression including person and year fixed effects
- Compare the results with the DiD results obtained before (display them side-by-side with etable)
- Save the notebook

A note on the common trend assumption

- The common trend assumption can of course not be tested in the treatment period (we don‘t know the counterfactual)
- But: if you have more periods before the intervention it is very useful to check whether it holds in these periods



From Manthei/Sliwka (2019)

Outlook: Synthetic Control Method/Synthetic Difference-in-Differences

See Abadie (2021), Arkhangelsky et al. (2021)

- In Synthetic Control methods you create an artificial control group
- Basic idea: for each “treated“ unit generate a “synthetic” control unit
  - The synthetic control unit’s “outcome” is the weighted average of several other (real) units
  - Weights are derived by minimizing the difference in time trends prior to intervention

Note: Using the estimated fixed effects

Sometimes the α_i estimated for the individuals/firms are useful themselves

Examples:

- How much do individual managers matter for firm behavior and performance? Bertrand and Schoar (2003) use
  - Manager-firm matched panel dataset, where they can track individual top managers across different firms over time.
  - Estimate how much of the variation in firm practices can be attributed to manager fixed effects
- Are there Team-Players, i.e. people that makes teams systematically better? Weidmann/Deming (2021) run an experiment where they
  - Repeatedly randomly assign people to group tasks
  - Estimate team-player fixed effects controlling for subjects‘ ability for the tasks
  - Find that there are people who by their social skills systematically make teams better

Conclusion

1. When you want to interpret the results of a DiD or Fixed Effects regression causally, a key underlying assumption is the **common trend assumption**
   - That is „treatment“ and „control“ units follow the same underlying time trend
   - This is a key identifying assumption
2. When the treatment C_{it} hardly varies over time it is hard to evaluate the causal effect effect ρ
   - In the extreme when C_{it} is completely stable then $C_{it} = \overline{C}_{it}$
   - Not identifying a significant effect in the data then does not necessarily imply that there is no such effect
3. Fixed effects can **only eliminate time-constant omitted** variables
   - If the treatment is correlated with time varying unobserved variables omitted variable issues remain