Useful Formulae in Quantum Optics

Mint

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1 Universal formula

$$[\hat{A},\hat{B}^n]=n[\hat{A},\hat{B}]\hat{B}^{n-1}\quad if\quad [B,[A,B]]=0 \tag{1}$$

$$[\hat{A}^n,\hat{B}]=n\hat{A}^{n-1}[\hat{A},\hat{B}]\quad if\quad [A,[A,B]]=0 \tag{2}$$

$$\frac{d}{d\lambda}e^{\lambda\hat{A}}=\hat{A}e^{\lambda\hat{A}} \tag{3}$$

$$[\hat{A},e^{\lambda\hat{B}}]=\lambda[\hat{A},\hat{B}]e^{\lambda\hat{B}}\quad if\quad [B,[A,B]]=[A,[A,B]]=0 \tag{4}$$

$$e^{\hat{A}+\hat{B}}=e^{\hat{A}}e^{\hat{B}}e^{-\frac{1}{2}[\hat{A},\hat{B}]}\quad if\quad [B,[A,B]]=[A,[A,B]]=0 \tag{5}$$

$$e^{-\alpha\hat{A}}\hat{B}e^{\alpha\hat{A}}=\hat{B}-\alpha[\hat{A},\hat{B}]+\frac{\alpha^2}{2!}[A,[A,B]]+\cdots \tag{6}$$

2 Creation and annihilation operators(omit the hat of operator)

$$\begin{cases} e^{xa}a^\dagger e^{-xa}=a^\dagger+x \\ e^{-xa^\dagger}ae^{xa^\dagger}=a+x \end{cases} \tag{7}$$

$$\begin{cases} e^{xa^\dagger a}ae^{-xa^\dagger a}=ae^{-x} \\ e^{xa^\dagger a}a^\dagger e^{-xa^\dagger a}=a^\dagger e^{x} \end{cases} \tag{8}$$

$$\begin{cases} e^{xa}f(a,a^\dagger)e^{-xa}=f(a,a^\dagger+x) \\ e^{-xa^\dagger}f(a,a^\dagger)e^{xa^\dagger}=f(a+x,a^\dagger) \end{cases} \tag{9}$$

$$e^{xa^\dagger a}f(a,a^\dagger)e^{-xa^\dagger a}=f(ae^{-x},a^\dagger e^{x}) \tag{10}$$

$$\begin{cases} [a,f(a,a\dagger)]=\frac{\partial f}{\partial a^\dagger} \\ [a^\dagger,f(a,a^\dagger)]=-\frac{\partial f}{\partial a} \end{cases} \tag{11}$$

3 Coherent state

$$definition: \quad |\alpha\rangle = \alpha|\alpha\rangle \tag{12}$$

$$\begin{cases} |\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \\ |\alpha\rangle = D(\alpha)|0\rangle \quad where \quad D(\alpha) = e^{\alpha a^\dagger - \alpha^* a} \end{cases} \tag{13}$$

$$D^\dagger(\alpha) = D(-\alpha) = [D(\alpha)]^{-1} \tag{14}$$

$$\begin{cases} D^{-1}(\alpha) a D(\alpha) = a + \alpha \\ D^{-1}(\alpha) a^\dagger D(\alpha) = a^\dagger + \alpha^* \end{cases} \quad plug \quad \hat{I} = DD^{-1} \quad to \quad higher \quad order \tag{15}$$

$$\begin{cases} a^\dagger |\alpha\rangle\langle\alpha| = (\frac{\partial}{\partial\alpha} + \alpha^*)|\alpha\rangle\langle\alpha| \\ a|\alpha\rangle\langle\alpha| = \alpha|\alpha\rangle\langle\alpha| \end{cases} \tag{16}$$

4 Squeezed state

$$S(\xi) = exp(\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}) \quad where \quad \xi = re^{i\theta} \tag{17}$$

$$\begin{cases} r: squeezed \quad degree \\ \theta: squeezed \quad direction \end{cases} \tag{18}$$

$$S^\dagger(\alpha) = S(-\alpha) = [S(\alpha)]^{-1} \tag{19}$$

$$\begin{cases} S^{-1}(\xi) a S(\xi) = acoshr - a^\dagger e^{i\theta} sinhr \\ S^{-1}(\xi) a^\dagger S(\xi) = a^\dagger coshr - ae^{-i\theta} sinhr \end{cases} \quad plug \quad \hat{I} = SS^\dagger \quad to \quad higher \quad order \tag{20}$$