Accelerating Eulerian fluid simulation with convolutional networks.

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CS 7180

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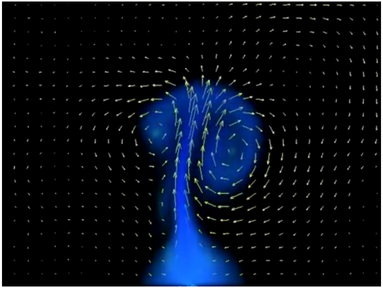
- We assume fluid is *incompressible* (like water)
- And *has no viscosity.*

wikipedia.org

Mathematical Representation for Fluid Flow

Represent the fluid's *velocity* at every time t and point in space $\vec{x}$ as vector field

$$\vec{u}(t, \vec{x})$$



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Flow evolves with time $\frac{\partial \vec{u}}{\partial t}$, by Navier-Stokes equations.
First: Newton's second law

$$\vec{F} = m\vec{a}$$

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Uniform density $\implies m = 1$. So...

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$$\vec{a} = \vec{F}$$

$\vec{u}$ is a vel., so $\frac{\partial \vec{u}}{\partial t}$ is accel. So

$$\frac{\partial \vec{u}}{\partial t} = \vec{F} = \text{ sum of all forces}$$

Navier-Stokes for no viscosity (AKA Euler)

- External Forces
  (gravity, bouyancy, ...)

$$\frac{\partial \vec{u}}{\partial t} = \boxed{\vec{F}_{ext}} - \vec{u} \cdot \nabla_{\vec{x}} \vec{u} - \boxed{\nabla_{\vec{x}} p}$$

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- Pressure

Both equations

- Newton's 2nd

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$$\nabla_{\vec{x}} \cdot u \stackrel{\rightarrow}{=} 0 \qquad \text{(I)}$$

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- Two equations, two unknowns $\vec{u}$ and p

Numerical Solving

Discretize

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$p_{n-1} \implies \vec{u}_{n-1} \implies \hat{u}_n$ **is easy and fast,**
But $\hat{u}_n \implies p_n$ **is hard and slow!**

$\hat{u}_{n+1} \implies p_{n+1}$

By (I),

$$\Delta p = \nabla \cdot \hat{u}_n$$

$(b = \nabla \cdot \hat{u}_n)$

$$\Delta p = b$$

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Replace derivatives with finite differences

$$\frac{\frac{p_{i+1,j} - p_{i,j}}{\Delta x} - \frac{p_{i,j} - p_{i-1,j}}{\Delta x}}{\Delta x} + \frac{\frac{p_{i,j+1} - p_{i,j}}{\Delta y} - \frac{p_{i,j} - p_{i,j-1}}{\Delta y}}{\Delta y} = b_{i,j}$$

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$$\frac{1}{l^2}\left(p_{i+1,j} + p_{i-1,j} + p_{i,j-1} + p_{i,j+1} - 4p_{i,j}\right) = b_{i,j}$$

Note each $b_{i,j}$ depends is a linear combination of 5 values of $p_{i,j}$ (7 in 3D).

The (very sparse) matrix is then

$$\begin{pmatrix} \vdots & & & \vdots & & & & \vdots & & \vdots \\ \cdots & 0 & 1 & 1 & -4 & 1 & 1 & 0 & 0 & \cdots \\ \cdots & 1 & 0 & 0 & 1 & -4 & 0 & 1 & 1 & \cdots \\ \vdots & & & \vdots & & & & \vdots & & \vdots \end{pmatrix} \begin{pmatrix} \vdots \\ p_{i,j-1} \\ p_{i-1,j} \\ p_{i,j} \\ p_{i+1,j} \\ p_{i,j+1} \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ \vdots \\ \vdots \\ b_{i,j} \\ b_{i+1,j} \\ \vdots \\ \vdots \end{pmatrix} l^2$$

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(This is the graph Laplacian for a grid graph.)

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We need to solve this equation $Lp = b$ for p in terms of b! AHH!

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- This work
  - Use unsupervised training
  - Use multi-resolution CNN
  - Use long-term loss function

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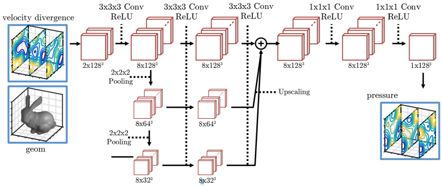
- Don't need true p_{n+1}, just use $|\nabla_{\vec{x}} \cdot \vec{u}| = |\mathrm{div}(\vec{u})|$ as loss.

Multi-Resolution CNN and Long-term Loss

- CNN



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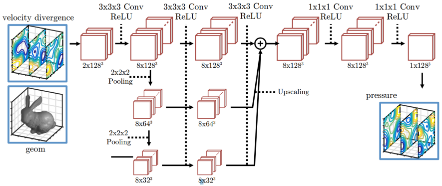
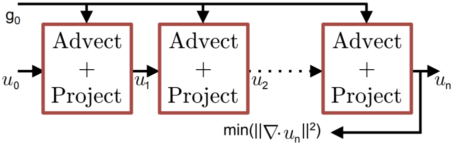


- Long-term Loss



Results

- Fast runtimes (for capped loss)

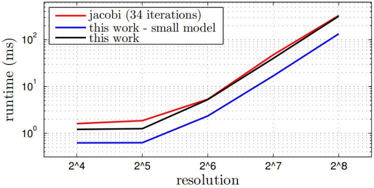


Figure 5. Pressure projection time (ms) versus resolution (PCG not shown for clarity).

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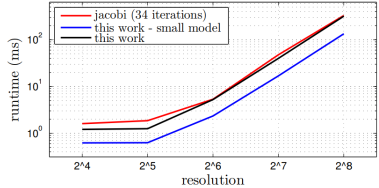

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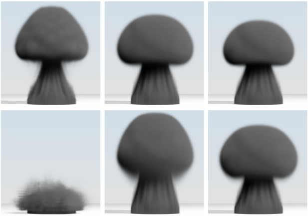
- Visually comparable results



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3. Comparison to Yang et al. fails

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4. “vorticity confinement?”

More Piazza Discussion.