Smooth Particle Hydrodynamics (SPH):
A New Feature in LS-DYNA

Jean Luc LACOME
DYNALIS
Immeuble AEROPOLE - Bat 1
5, Avenue Albert Durand
31700 BLAGNAC
Tel : +33-5 61 16 42 18
dynalis@wanadoo.fr

Keywords:
Particle methods, S.P.H., Smoothed Particle Hydrodynamics

ABSTRACT

A new particle element has been added to LS-DYNA. It is based on Smoothed Particle Hydrodynamics theory. SPH is a meshless lagrangian numerical technique used to model the fluid equations of motion. SPH has proved to be useful in certain class of problems where large mesh distortions occur such as high velocity impact, crash simulations or compressible fluid dynamics.

First, we introduce the basis principles of the SPH method. Then the coupling of this technique to LS-DYNA is presented and the input needed for such analysis is provided.

INTRODUCTION

Meshless methods have known important developments these last years for resolving conservation laws.

S.P.H. (Smoothed Particle Hydrodynamics) is a meshless lagrangian method developed initially to simulate astrophysical problems. But, the easy way with which it is possible to introduce sophisticated phenomena, has made of SPH a very interesting tool to resolve other physic problems: resolution of continuum mechanics, crash simulations, ductile and brittle fracture in solids.

The easy use of SPH allows the resolution of many problems that are hardly reproducible with classical methods. It is very easy to obtain a first approach of the kinematics of the problem to study. For instance, due to the absence of mesh, one can calculate problems with large irregular geometry.

From a computational point of view, we represent a fluid with a set of moving particles evolving at the flow velocity. Each SPH particle represents an interpolation point on which all the properties of the fluid are known. The solution of the entire problem is then calculated on all the particles with a regular interpolation function, the so-called smoothing length. The equations of conservation are then equivalent to terms expressing fluxes or inter-particular forces.

In this paper, we give the basis principles of the method. The important development of the method is the definition of a new consistent approximation that leads to a new class of renormalization formulations. We then provide the philosophy used to implement the particle element into LS-DYNA. Finally, sample examples illustrating this new capability are included.

BASIS PRINCIPLES OF THE SPH METHOD

Particle methods are based on quadrature formulas on moving particles $(x_i(t), w_i(t))_{i \in P}$. P is the set of the particles, $x_i(t)$ is the location of particle i and $w_i(t)$ is the weight of the particle. We classically move the particles along the characteristic curves of the field v and also modify the weigths with the divergence of the flow to conserve the volume :

$$(i)\frac{d}{dt}x_i = v(x_i, t) \qquad (ii)\frac{d}{dt}w_i = div(v(x_i, t))w_i \qquad (1.1)$$

We can then write the following quadrature formula :

$$\int_{\Omega} f(x)dx \approx \sum_{j\in P} w_j(t)\, f(x_j(t)) \qquad (1.2)$$

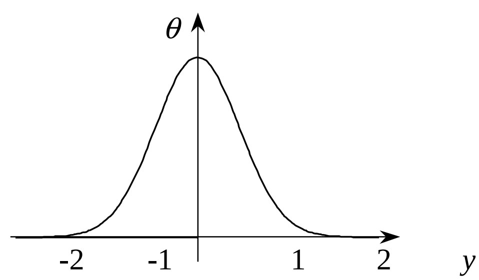
Particle Approximation of Function
The previous quadrature formula together with the notion of smoothing kernel leads to the definition of the particle approximation of a function.

To define the smoothing kernel, we need first to introduce an auxiliary function θ. The most useful function used by the SPH community is the cubic B-spline which has some good properties of regularity.

It is defined by:

$$\theta(y) = C \times \begin{cases} 1-\frac{3}{2}y^2+\frac{3}{4}y^3 & \text{for} \quad y \le 1 \\ \frac{1}{4}(2-y)^3 & \text{for} \quad 1 < y \le 2 \\ 0 & \text{for} \quad y > 2 \end{cases} \qquad (2.1)$$

where C is the constant of normalization that depends on the space dimension.



We have then enough elements to define the smoothing kernel W:

$$W(x_i - x_j, \bar{h}) = \frac{1}{\bar{h}}\theta\left(\frac{x_i - x_j}{\bar{h}}\right) \qquad (2.2)$$

$W(x_i - x_j, \bar{h}) \longrightarrow \delta$ when $\bar{h} \longrightarrow 0$, where δ is the Dirac function. $\bar{h}$ is a function of x_i and x_j and is the so-called smoothing length of the kernel.

We can now define the particle approximation $\Pi^h u$ of the function u, by approximating the integral (1.2):

$$\Pi^h u(x_i) = \sum_{j\in\Omega} w_j(t) u(x_j) W(x_i - x_j, \bar{h}) \qquad (2.2)$$

The approximation of gradients is obtained by applying the operator of derivation on the smoothing length. We then obtain :

$$\nabla\Pi^h u(x_i) = \sum_{j\in\Omega} w_j(t) u(x_j) \nabla W(x_i - x_j, \bar{h}) \qquad (2.2)$$

Gather Approximation
In finite differences or finite elements, we have only one spatial discretization parameter. One major difference of SPH with these classical technics is that we have two parameters to determine the spatial resolution: the smoothing length $\bar{h}$ and the characteristic length of the mesh Δx .

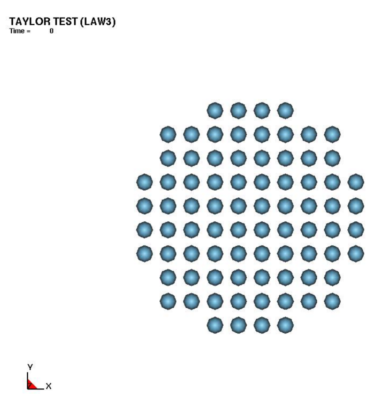
At the really beginning of SPH at the end of the 70's, the smoothing length was choosen to remain constant during all the simulation. Recent developments showed the advantage for each particle to have its own smoothing length, which depends on the local number of particles.

The smoothing length $\bar{h}$ represents the mean value of the smoothing length between x_i and x_j. In earlier formulations, the choice of $\bar{h}$ was given by $\bar{h} = (h(x_i) + h(x_j))/2$, where $h(x_i)$ and $h(x_j)$ are the smoothing length values respectively for particle x_i and x_j. The reason of this choice is that it easily leads to conservative schemes.

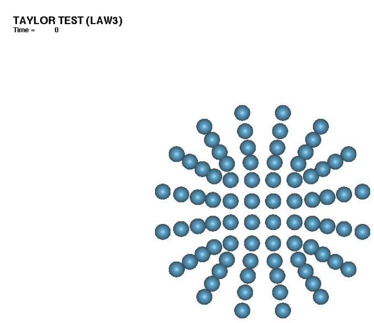
Recent works proved that this approximation is not consistent and can lead to unstable results. A new class of methods has been rising since 1996: renormalization, moving least square,… They lead to a better understanding of stability and convergence problems. One good choice is to define $\bar{h} = h(x_i)$. This is called the gather formulation. This formulation means that the neighbour particles of a given particle are the particles that are included in a sphere centered in x_i with a radius of $h(x_i)$. This choice has been introduced in LS-DYNA. The convergence criteria are known for this approximation and leads to a better precision for SPH calculation.

SPH/LS-DYNA COUPLING

Because of the lack of a numerical grid, the SPH processor requires some conditions in setting the initial particle masses and coordinates. The particle mesh needs to be enough regular. It means that all the particles of a given neighbourhood need to have the same mass. As a consequence, the particles of a same material, which have the same initial density, need to have the same initial volume. To preserve this, they need to be distributed on a uniform mesh. For instance, when meshing a cylinder it is better to use Mesh 1 than Mesh 2 to guarantee stability and convergence of the method.



Mesh 1



Mesh2

User defines some initial conditions for the SPH elements (initial smoothing length, part id, eos, material law, mass of the particle). Some new keywords have been defined to use the new SPH option.

*CONTROL_SPH : which defines the general control parameters needed for the calculation.

*SECTION_SPH :	which defines parameters for every part of SPH particles.
*ELEMENT_SPH:	which defines every particle, assigns its part ID and mass.

The declaration of a PART with SPH elements is done classically.

*NODE	NID	X	Y	Z		
*ELEMENT_SPH	NID	PID	MASS			
*PART	PID	SID	MID	EOSID		
*SECTION_SPH	SID	CSLH				
*MAT_ELASTIC	MID	RO	E	PR	DA	DB
*EOS	EOSID					

The SPH processor has been developed as an extra layer of LS-DYNA. Therefore, all the actual features of LS-DYNA can be used with the particles. Initial velocities, contacts, rigid walls, … are defined by using classical keywords of LS-DYNA.

When running a SPH calculation, a flag is activated indicating the presence of SPH particles. Then for these special elements, we call the SPH processor. The nodal forces between the particles are computed as well as the energy, pressure and the deviatoric stresses. This is done by using the particle approximations of the equations of conservation (mass, momentum, energy). Once we have all the mechanical quantities for the particles we return to the LS-DYNA main program. The connection of the particles to the brick and shell elements is realised by using the classical nodes to surface contacts of LS-DYNA.

CONCLUSIONS

A particle element method has been implemented in LS-DYNA. We presented that new technique based on Smoothed Particle Hydrodynamics. We provided the data needed for its use in LS-DYNA. The SPH processor can be an alternative to resolve problems with large deformation and mesh tangling.

REFERENCES

RANDLES, P.W. and LIBERSKY, L.D. (1996) Smoothed Particle Hydrodynamics: Some recent improvements and applications, Comput. Meth. Appl. Mech. Engrg, pp. 375-408

VILA, J.P. Meshless methods for conservation laws, M3AS, to appear.