Upwind scheme

Monday, September 26, 2016 9:03 AM

$$u_i = \bar{u}_i = \frac{1}{\Delta x}\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} u\,dx$$

$$\frac{du_i}{dt} = \frac{1}{\Delta x}\left(f_{i-\frac{1}{2}} - f_{i+\frac{1}{2}}\right)$$

exact if

$$f_{i+\frac{1}{2}} = f(u(x_{i+\frac{1}{2}}))$$

need to approximate $f_{i+\frac{1}{2}}$ from cell averages of u

Upwind: fix the oscillations around shocks

$$f_{i+\frac{1}{2}} = \begin{cases} f(u_i) \\ f(u_{i+1}) \end{cases} \qquad \frac{f(u_i)-f(u_{i+1})}{u_i - u_{i+1}} \to 0 \quad < 0$$

Burgers Eqn

$$f(u) = \frac{u^2}{2}$$

$$\frac{f(u_i)-f(u_{i+1})}{u_i - u_{i+1}} = \frac{u_i + u_{i+1}}{2}$$

Discrete conservation of finite volume

Monday, September 26, 2016 9:03 AM

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0$$

$$\frac{d}{dt}\int_a^b u\,dx = f(u(a)) - f(u(b))$$

$$\Delta x \frac{du_i}{dt} = f_{i-\frac{1}{2}} - f_{i+\frac{1}{2}}$$

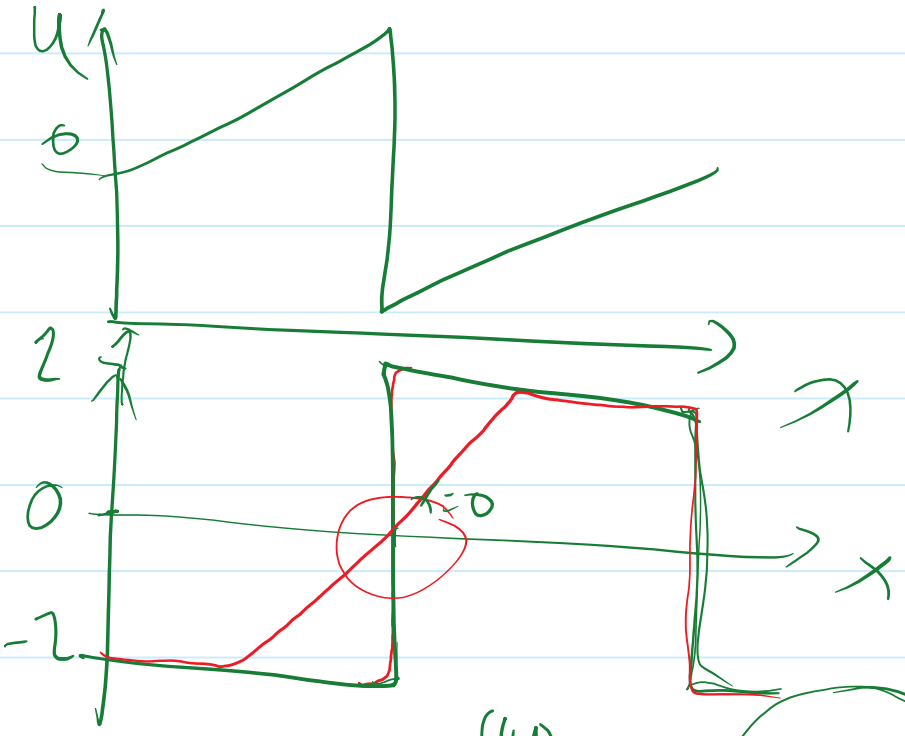
$$\Delta x \frac{du_{i+1}}{dt} = f_{i+\frac{1}{2}} - f_{i+1\frac{1}{2}}$$

$$\vdots$$

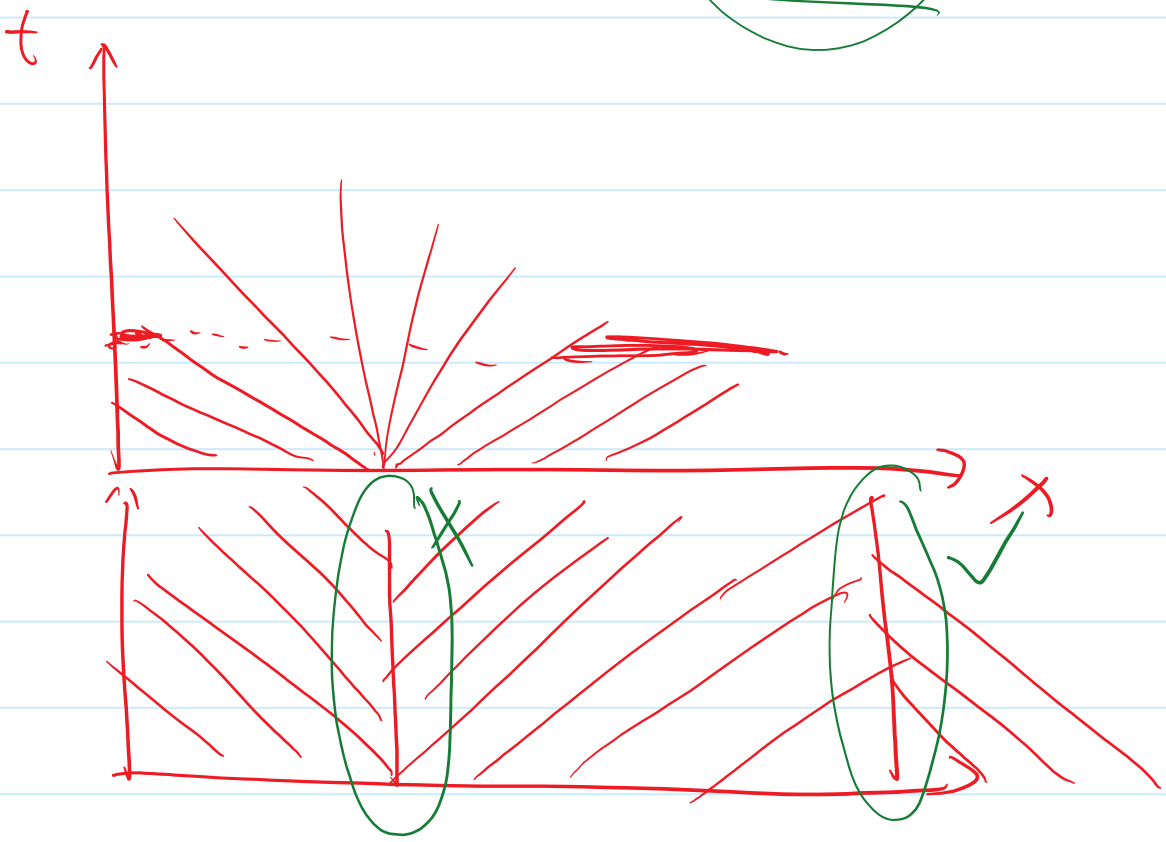
$$\Delta x \frac{f_N}{dt} = f_{N-\frac{1}{2}} - f_{N+\frac{1}{2}}$$

Non-uniqueness of solution and the Entropy Condition

Monday, September 26, 2016 9:03 AM

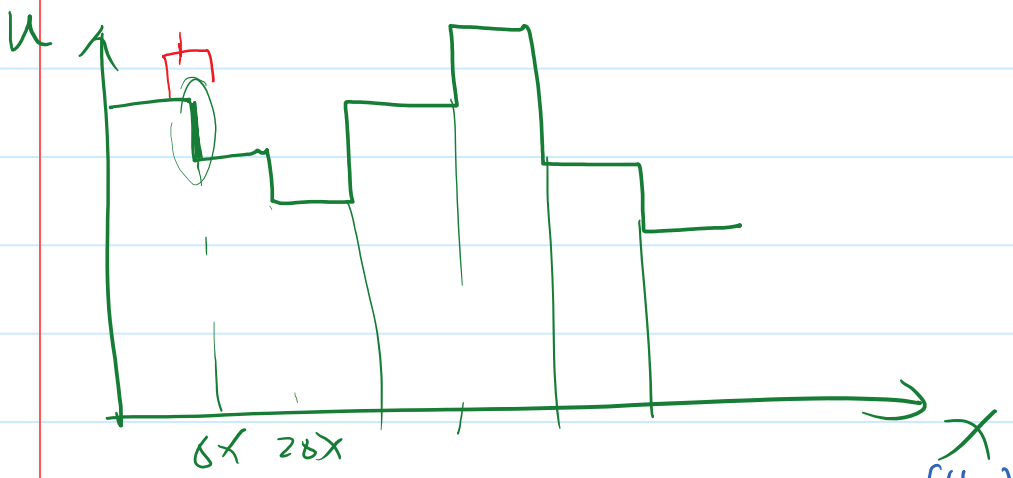


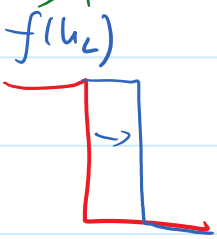
$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = \varepsilon \frac{\partial^2 u}{\partial x^2}$$

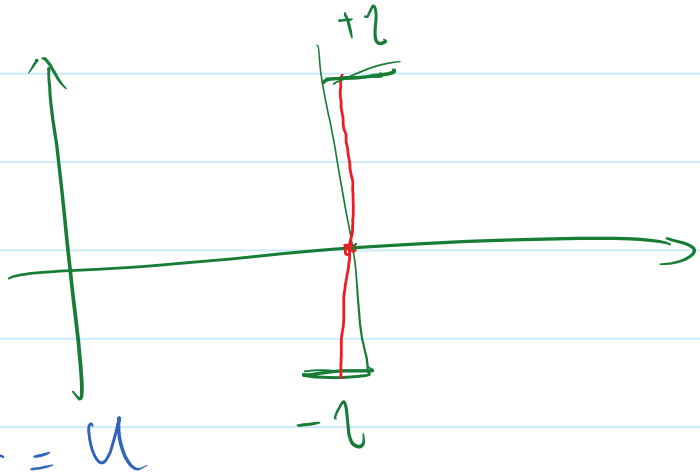


Godunov Numerical Flux

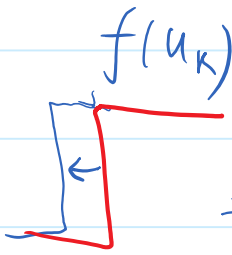
Monday, September 26, 2016 9:03 AM





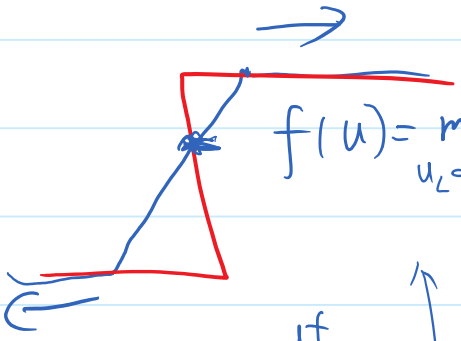


$$\frac{df}{du} = u$$

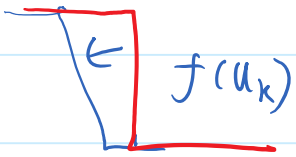


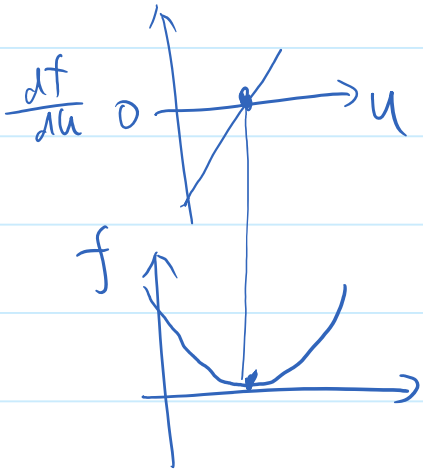
$u_R > u_L$

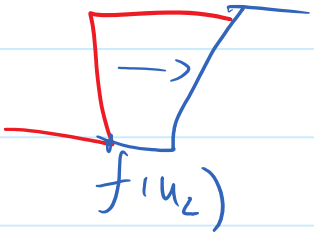
$$\frac{f(u_R) - f(u_L)}{u_R - u_L} < 0$$

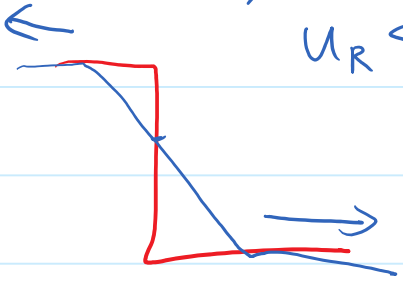


$$f(u) = \min_{u_L \le u \le u_R} f$$



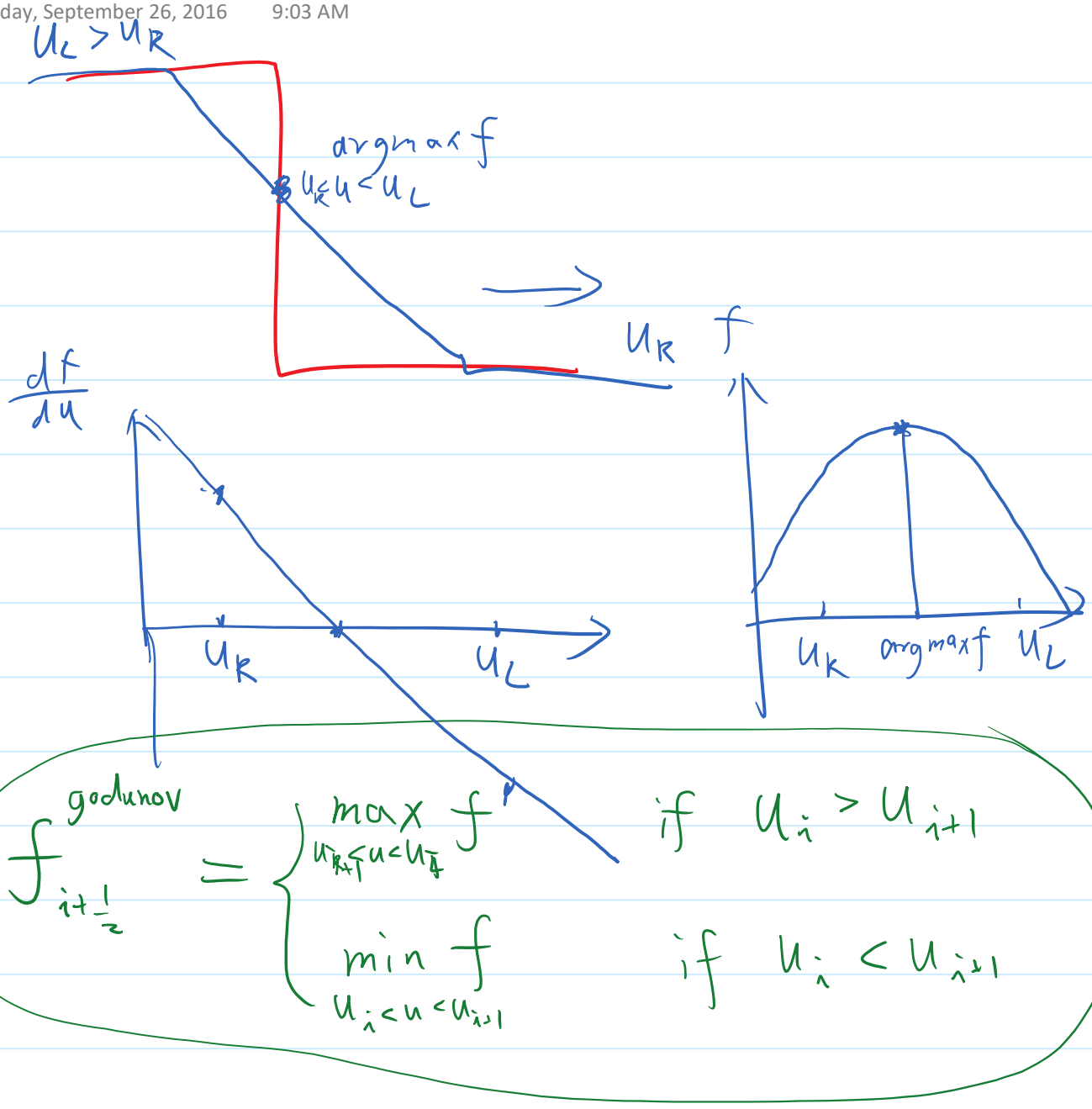






Godunov Numerical Flux

Monday, September 26, 2016 9:03 AM



$$f^{\text{godunov}}_{i+\frac{1}{2}} = \begin{cases} \max\limits_{u_R \le u \le u_L} f & \text{if } u_i > u_{i+1} \\ \min\limits_{u_i \le u \le u_{i+1}} f & \text{if } u_i < u_{i+1} \end{cases}$$

Second order scheme using Godunov Numerical Flux

Monday, September 26, 2016 9:03 AM