Smoothed particle hydrodynamics

24. januar 2022

Mathias Chunnoo

1 2D Kernels

1.a Polynomial kernel

$$W_{poly}(\vec{r}, h) = \begin{cases} \frac{4}{\pi h^8} \left(h^2 - \|\vec{r}\|^2\right)^3 & \text{if } 0 \leqslant \|\vec{r}\| \leqslant h \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

1.b Spiky kernel

$$\nabla W_{spiky}(\vec{r}, h) = \begin{cases} -\frac{30}{\pi h^5} \frac{\vec{r}}{\|\vec{r}\|} \left(h - \|\vec{r}\|\right)^2 & \text{if } 0 \leqslant \|\vec{r}\| \leqslant h \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

1.c Viscosity kernel

$$\nabla^2 W_{visc}(\vec{r}, h) = \begin{cases} -\frac{20}{\pi h^5} \left(h - \|\vec{r}\|\right) & \text{if } 0 \leqslant \|\vec{r}\| \leqslant h \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

2 Navier-Stokes

$$\frac{\mathrm{d}\vec{v}_i}{\mathrm{d}t} = -\frac{1}{\rho_i} \nabla p + \frac{\mu}{\rho_i} \nabla^2 \vec{v} + \vec{f} \tag{4}$$

$$\rho_i \approx \sum_j m_j W_{poly}(\vec{r}_i - \vec{r}_j, h) \tag{5}$$

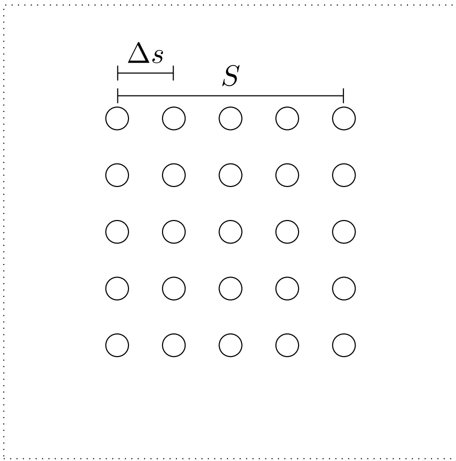
$$p_i = k(\rho_i - \rho_0) \tag{6}$$

$$\frac{\nabla p_i}{\rho_i} \approx \sum_j m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} \right) \nabla W_{spiky}(\vec{r}_i - \vec{r}_j, h) \tag{7}$$

$$\frac{\mu}{\rho_i} \nabla^2 \vec{v}_i \approx \frac{\mu}{\rho_i} \sum_j m_j \left(\frac{\vec{v}_j - \vec{v}_i}{\rho_j} \right) \nabla^2 W_{visc}(\vec{r}_i - \vec{r}_j, h) \tag{8}$$

3 Values

3.a Initial particle grid and area



$$\Delta s = \frac{S}{\sqrt{n} - 1} \tag{9}$$

$$A = S^2 \tag{10}$$

3.b Gravity

$$g = \begin{bmatrix} 0 \\ -9.81 \end{bmatrix} \tag{11}$$

3.c Rest density

$$\rho_0 = 1000 \tag{12}$$

3.d Viscosity term

$$\mu = ? \tag{13}$$

3.e Mass

$$M = A\rho_0 \tag{14}$$

$$m = \frac{M}{n} \tag{15}$$

$$n = \rho_0 \frac{A}{m} \tag{16}$$

3.f Support radius

$$x = \frac{n}{A}\pi h^2 \tag{17}$$

$$h = \sqrt{\frac{Ax}{\pi n}} \tag{18}$$

3.g Time step and Courant condition

$$\xi = 4.9633 \tag{19}$$

$$\Delta t = \min_i \left[\frac{0.5h}{c_s + \frac{2\xi\mu}{h\rho_i}} \right] \tag{20}$$

$$\lambda \approx 0.4 \tag{21}$$

$$\Delta t \leqslant \lambda \frac{h}{\|\mathbf{v}_{max}\|} \tag{22}$$