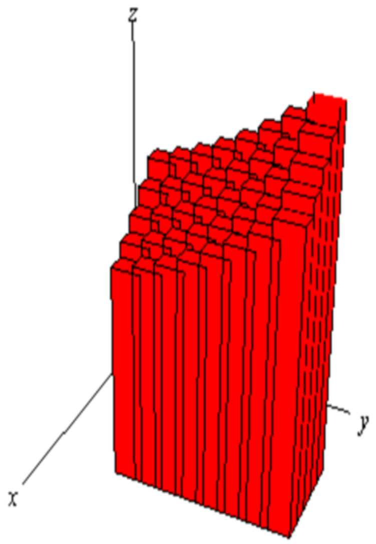
Double Integration

Background

$V \approx \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i^*, y_j^*) \Delta A$

$V = \lim_{n,m \to \infty} \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i^*, y_j^*) \Delta A$



$V = \iint_R f(x, y) dx dy$

Example

Use the mid-point rule to estimate the volume under f(x,y) =x^{2} + y and above the rectangle given by $-1 \leq x \leq 3$, $0 \leq y \leq 4$ in the xy plane. Use 4 subdivisions in the x direction and 2 divisions in the y direction.

$V \approx \sum_{i=1}^{4} \sum_{j=1}^{1} f(x *_i, y *_j)(2)(1) = 68$

The exact volume (using integration) $= 69.3333$

Example

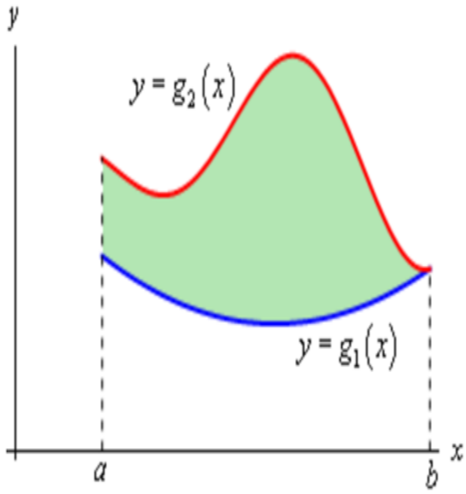
Integrate the following firstly with respect to x, then again but firstly with respect to y

$\iint_R (12x - 18y) dA \; R = [-1, 4] \times [2, 3]$

$\int_2^3 \int_{-1}^4 (12x - 18y) dx dy = -135$

$\int_{-1}^4 \int_2^3 (12x - 18y) dy dx = -135$

Example



Find the area of this region.

$A = \iint_R dxdy$

$A = \int \int_{g1(x)}^{g2(x)} dydx$

$A = \int [g_2(x) - g_1(x)]dx$

Example- polar coordinates

$\iint_D 2xydA$ between the circles with radius 2 and radius 5, centred around the origin and lies in the first quadrant.

$\int_2^5 \int_0^{\frac{\pi}{2}} 2rcos(\theta)rsin(\theta)rdrd\theta = \frac{609}{4}$

Example- polar coordinates

$\iint_D e^{x^2+y^2}dA$, unit disk centered around the origin.

$\int_0^{2\pi} \int_0^1 e^{r^2}rdrd\theta = \pi(e-1)$