Line Integrals

Background

line integral is denoted

$\int_C f(x,y)ds$

\(\int_{C} f(x,y) \sqrt{{dx \over dt}^2 + {dy \over dt}^2 \)

$\int_C f(x,y)||r'(t)||^2$

For a line integral with respect to arc length if we change the direction, the integral does not change.
$\int_C f(x,y)ds = \int_{-C} f(x,y)ds$

Example

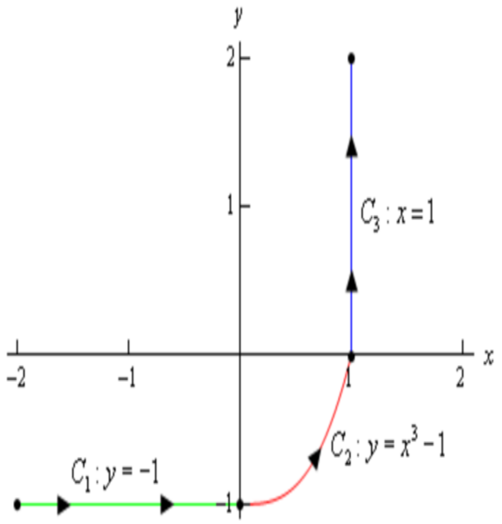
Evaluate

$\int_C xy^4ds$ where C is the right half of the circle $x^2 + y^2 = r^2$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} xy^4ds = \frac{8192}{5}$

Example 2

Evaluate $\int_C 4x^3ds$ where C is the curve shown below



$\int_{C_1} 4x^3ds + \int_{C_2} 4x^3ds + \int_{C_3} 4x^3ds = -5.732$

Example 2

Evaluate $\int_C 4x^3 ds$ where C is the line segment from (-2,-1) to (2,1)

$\vec{r} =< -2 + 3t, -1 + 3t$\rt

$\int_0^1 4x^3 ds = -21.213$

Example 3

Evaluate $\int_C 4x^3 ds$ where C is the line segment from (2,1) to (-2,-1)

$\vec{r} =< 1 - 3t, 2 - 3t$\rt

$\int_0^1 4x^3 ds = -21.213$