Triple Integration

Example

Integrate the following:

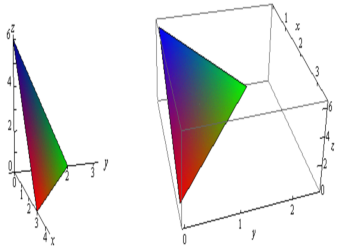
$\iiint_B (8xyz)dV$ where $B = [2,3] \times [1,2] \times [0,1]$

$\int_2^3 \int_1^2 \int_0^1 (8xyz)dxdydz = 15$

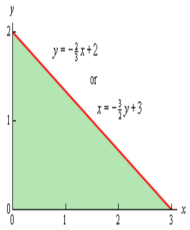
Example

Integrate the following:

$\iiint_E (2x)dV$ where E is the region under the plane $2x + 2y + z = 6$ in the first octant



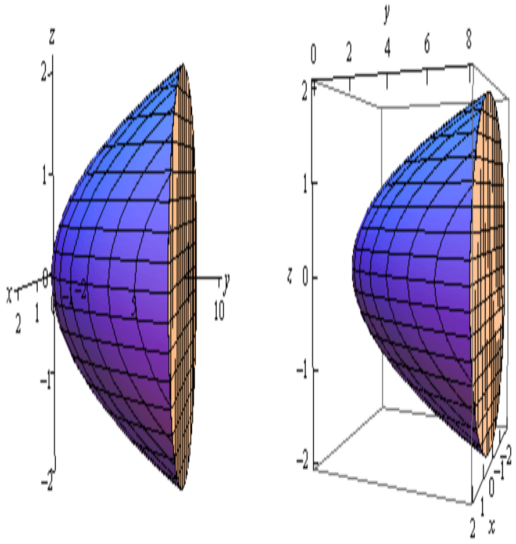
We now need to determine the region D in the xy-plane. We can get a visualization of the region by pretending to look straight down on the object from above. What we see will be the region D in the xy-plane. So D will be the triangle with vertices at $(0,0)$, $(3,0)$, and $(0,2)$. Here is a sketch of D.



$\int_0^3 \int_0^{\frac{-2}{3}x+2} \int_0^{6-2x-2y} 2x dz dy dx = 9$

Example

Evaluation $\iiint_E \sqrt{3x^2 + 3z^2} dV$ where E is the solid bounded by the plane $y = 2x^2 + 2z^2$ and y=8



$$\iint \int_{3x^2+3z^2}^{8} \sqrt{3x^2+3z^2} dydxdz$$

$$\iint \sqrt{3x^2+3z^2}(8-(2x^2+2z^2))dA$$

$$\sqrt{3}\int_0^{2\pi}\int_0^2 (8r-2r^3)rdrd\theta = \frac{256\sqrt{3}\pi}{15}$$