AP Calculus BC Syllabus

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Course Description:

AP Calculus BC is a college level course equivalent to a full year of calculus at most universities. The course is based on the College Board's Advanced Placement Calculus BC curriculum, whose purpose is "developing understanding of the concepts of calculus and providing experience with its methods and applications." The course will emphasize that most calculus concepts and problems can be viewed or represented several ways: graphically, numerically, algebraically and verbally. Graphing calculators are tools for moving between these representations, so we will use them regularly. The primary purpose of this course is to prepare students for the AP Exam as well as future college math courses Students will be required to work the problems from a graphical, numerical or analytical point of view and present their solutions both verbally and in writing. Students are expected to perform college level work.

The course is designed around the three "Big Ideas" of calculus, including: Big Idea #1: Change Big Idea #2: Limits Big Idea #3: Analysis of Functions

Textbook: Stewart, James. Single Variable Calculus. Belmont, California: Brooks/Cole – Thomson Learning, 2008, 6th Edition.

Expectations:

Students are expected to:

- Attend class on a daily basis.
- Keep a class notebook of notes, model problems, and homework assignments with noted corrections.
- Attempt and/or complete every problem on assigned homework.
- Read the assigned textbook readings.
- Participate in class, group discussions, and problem solving activities.
- Schedule make-up deadlines with the teacher when an absence occurs

Materials needed

- Notebook
- Folder or binder
- Graphing Calculator TI-83, TI-84, or TI-89 recommended
- Completed Homework

Technology Resources

Students are required to have a graphing calculator. The TI-89 is recommended and used on a daily basis along with its' companion view screen. A computer and SmartBoard are also located in the

classroom and are used to access and display many of the interactive websites available online.
Calculator Use

Students will use their graphing calculators to explore concepts and check their understanding. All Students will have access to graphing calculators.

Students will use graphing calculators to discover topics including, but not limited to the following:

• Developing an intuitive understanding of limits

• Investigating the Intermediate Value Theorem • Relationship between a function and its derivative

• Riemann sums and the definite integral

• Average value of a function

• Numerical integration

Assessment

Assessments used in this course include publisher and teacher made tests and quizzes, projects, popquizzes, homework assignments, classroom participation and class work.

•Tests are given at the end of each chapter of our primary text and are comprised of short-answer type questions from all previous chapters. The cumulative nature of these tests insures that each student is periodically exposed to each concept previously presented in the course. The questions are written to assess the students' ability to solve problems from a graphical, numerical, and analytical approach.

•Quizzes are given every other week and contain selected questions from the previous two weeks homework assignments. These quizzes help to insure that the students take their homework seriously, and clear up any questions they may have on homework assignments as soon as possible.

• Projects are assigned approximately every two weeks. Each project is a multipart free-response style question. The main goal of these projects is to familiarize each student with this type of question and what is expected from them in their solution. Each project is graded as it would be on an AP Exam. 7 This is a learning process designed to help students feel more comfortable with free-response style questions.

• Pop-quizzes are comprised of multiple-choice questions, and are given 1-2 times a week. The questions are chosen from our supplemental texts, and are designed to mimic multiple-choice questions that would be found on an AP Exam. The majority of pop-quizzes are given to the students as group quizzes. Here is where the verbal component of the course shines. The learning that goes on in these groups, and the communication that takes place is invaluable.

•Homework is assigned every day and is discussed the following day. Students are often called upon to present their solution to the class. The student will write their solution on the chalkboard, explain their thinking, and answer any questions there may be from others in the class (including the instructor). Here, again, the verbal component of learning calculus is strong.

Student Practice

Throughout each unit, Topic Questions will be provided to help students check their understanding. The Topic Questions are especially useful for confirming understanding of difficult or foundational topics before moving on to new content or skills that build upon prior topics. Topic Questions can be assigned before, during, or after a 2 lesson, and as in-class work or homework. Students will get rationales for each Topic Question that will help them understand why an answer is correct or incorrect, and their results will reveal misunderstandings to help them target the content and skills needed for additional practice.

At the end of each unit or at key points within a unit, Personal Progress Checks will be provided in class or as homework assignments in AP Classroom. Students will get a personal report with feedback on every topic, skill, and question that they can use to chart their progress, and their results will come with rationales that explain every question's answer. One to two class periods are set aside to re-teach skills based on the results of the Personal Progress Checks.

An extra lab period each week is devoted to an appropriate calculator activity, multistep word problems, Topic Questions, Personal Progress Checks, and/or free-response questions (FRQ's) from released AP Calculus BC Exams. Emphasis is placed on problem solving, using the calculus in new settings, and helping students to see the connections among the big ideas and the major themes in calculus. FRQs, which emphasize real-world applications of the calculus, are selected for discussion during this lab period. The course is also designed around the four Mathematical Practices in AP Calculus outlined in the 2019 CED including: Practice #1: Implementing Mathematical Processes Practice #2: Connecting Representations Practice #3: Justification Practice #4: Communication and Notation

Mathematical Practices

Reasoning with definitions and theorems- In problems where students practice applying the results of key theorems (e.g., Intermediate Value Theorem, Mean Value Theorems, and/or L'Hospital's Rule), students are required for each problem to demonstrate verbally and/or in writing that the conditions are met in order to use the theorem.

Connecting concepts and processes- Students can relate the concept of limits to other parts of the calculus. They can connect visual representations of concepts with and without technology, and identify common structures in different contexts. Students are provided with the graph of a function and a second function defined as the definite integral of the graphed function with a variable upper limit. Using differentiation and antidifferentiation, students evaluate specific values of the second function and then find the intervals where the integral function is increasing, decreasing, concave up, and concave down. They use this information to sketch a rough graph of the second function

Implementing algebraic/computational processes. Students can select appropriate algebraic and computational strategies. They can sequence and complete them correctly. They can apply technology, attend to precision analytically, numerically, graphically and verbally, and specify units of measure. Students will use multiple modes to calculate problems. Students are presented with a table of observations collected over time periods of different lengths. Students use Riemann sums to numerically approximate the average value of the readings over the given time period and interpret the meaning of that value

Engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them. Students are presented with numerous functions modeling velocity and time for objects in motion. These functions are presented numerically, graphically, analytically, and verbally. Given some initial conditions, students calculate or approximate displacement, total distance travelled, and acceleration for these functions (both by hand and with a graphing calculator).

Building notational fluency Students are given a variety of growth and decay word problems where the rate of change of the dependent variable is proportional to the same variable (e.g., population growth, radioactive decay, continuously compounded interest, and/or Newton's law of cooling). Students are asked to translate the problem situation into a differential equation using proper notation. Students show the steps in solving the differential equation, continuing to use proper notation for each step (e.g., when to keep or remove absolute value). In a later activity, students will vary initial conditions and use their calculators to graph the resulting solutions so that students can explore the effect of these changes.

Communicating Throughout the course, students are required to present solutions to homework problems both orally and on the board to the rest of the class. On at least one question on each quiz and test, students are explicitly instructed to include clearly written justifications in complete sentences for their solutions.

Chapter 2: Limits and Continuity (1 day)

- Continuity and one-sided limits
- Infinite limits
- Intermediate Value Theorem

Chapter 3: Differentiation (15 days)

- Derivatives and rates of change
- Approximating rates of change from graphs and tables of data
- The derivative as a function
- Relating the graphs of f , f ' , and f ''
- The relationship between differentiability and continuity
- Differentiation formulas (sums, differences, products, and quotients)
- The chain rule
- Derivatives of trigonometric functions
- Implicit differentiation
- Rate of change in the natural and social sciences, including position, velocity, acceleration, and rectilinear motion— includes an oral presentation of an example of how rates of change are used in the natural and social sciences.
- Related Rates – For each problem, students are required to distinguish, in writing, between the values that are constant and the values that are used only at time t.
- Linear approximations and differentials—includes a calculator exercise where students zoom in on a point on a curve, noting how it begins to look like a tangent line in order to demonstrate the concept of linearization

Chapter 4: Applications of Differentiation (18 days)

- Maximum and minimum values on an interval
- Extreme Value Theorem
- Rolle's Theorem and the Mean Value Theorem
- First and Second Derivative Tests
- Increasing/Decreasing Test Concavity Test and inflection points
- Limits at infinity and horizontal asymptotes
- Summary of curve sketching
- Graphing with calculus and calculators
- Optimization
- Newton's Method
- Anti-derivatives and indefinite integration following directly from derivatives of basic functions

Chapter 5: Integrals (12 days)

- Properties of the definite integral
- Definite integral as a limit of Riemann sums
- Area under a curve using Riemann sums (left, right, and midpoint) and trapezoidal sums
- Approximating definite integrals represented analytically, graphically, and by tables of data
- Fundamental Theorem of Calculus
- Using the Fundamental Theorem of Calculus to evaluate definite integrals
- Net Change Theorem and net change vs. total change
- The Substitution Rule and the use of substitution of variables to evaluate definite integrals
- Integrals of symmetric functions

Chapter 7: Inverse functions, Exponential, Logarithmic, and Inverse Trigonometric functions (14 days)

- Inverse functions
- Finding the derivative of the inverse of a function at a given value
- Natural logarithmic functions—differentiation and integration
- Natural exponential function—differentiation and integration
- General logarithmic and exponential functions—differentiation and integration
- Exponential growth and decay
- Inverse trigonometric functions—differentiation and integration
- Indeterminate forms and L'Hospital's Rule

Chapter 8&9.1: Techniques of Integration (17 days)

- Integration by parts
- Trigonometric integrals
- Trigonometric substitution
- Integration by partial fractions
- Strategy for integration
- Approximate integration using Midpoint Rule, Trapezoidal Rule, and Simpson's Rule
- Error bounds
- Improper Integrals Types 1 and 2

- Arc length

Chapter 10: Differential Equations (5 days)

- Modeling with differential equations
- Finding the solution of an initial-value problem
- Use of direction fields (slope fields) to interpret a differential equation geometrically
- Drawing slope fields and solutions curves for differential equations
- Euler's method as a numerical solution of a differential equation
- Solving separable differential equations

Chapter 11: Parametric Equations and Polar Coordinates (11 days)

- Curves defined by parametric equations
- Calculus with parametric curves
- Polar coordinates
- Areas and lengths in polar coordinates

Chapter 12: Infinite Sequences and Series (24 days)

- Sequences
- Monotonic sequences (increasing or decreasing)
- Series (convergent or divergent)
- Convergence of a series defined in terms of the limit of the sequence of partial sums of a series Test for divergence of a series
- Properties of a convergent series
- The Integral Test for convergence or divergence and its relationship to improper integrals and areas of rectangles
- The Comparison Tests and the Limit Comparison Test
- Alternating Series Test and the Alternating Series Estimation Theorem
- Absolute convergence and the Ratio and Root Tests
- Strategy for testing series
- Power series and radius and interval of convergence Taylor and Maclaurin series for a given function
- Maclaurin series for e^x ,$\frac{1}{1-x}$,sin x, cos x, and $\tan^{-1}(x)$
- Applications of Taylor Polynomials

From AP Website: Vectors (3 days)

REVIEW FOR THE AP EXAM*

- 2-3 Weeks
- Various Released Exams –Multiple Choice
- Various Free Response Questions Released on AP Central

Sample Instructional Activity

Unit 1 – limits and continuity (Big Ideas: Change, Limits, Analysis of Functions)

Notation Read Aloud

Begin by writing a limit expression in analytical form and then read the expression aloud to the class: "The limit of x cubed as x approaches 0 from the left." Do the same for 1–2 additional examples that use a variety of limit notations (e.g., the symbol for infinity). Then have students pair up and take turns reading aloud different limit expressions to one another.

Create Representations

Present students with a limit expression in analytical form and then have them translate that expression into a variety of representations: constructing a graph, creating a table of values, and writing it as a verbal expression. Then have students check their graphs and tables using technology.

Work Backward

Present students with a set of limit problems. Rather than determining the given limits, have them make a list of the various strategies that would be used to determine the limits (e.g., factoring, multiplying by conjugate, and simplify using trigonometric identities). After confirming their list is complete, have students work in pairs to create and write limit problems, each requiring one of the listed strategies. Then have them swap problems with another pair of students to complete each other's problems.

Discussion Groups

Give each group of students a piecewise-defined function, a graph paper, and a list of x-values. Have them graph the function, then discuss whether the function is continuous or discontinuous at each x-value, and explain why. Ask students to take turns recording the group's conclusion for each x-value. If continuous, have students discuss and show that all three continuity conditions are satisfied. If discontinuous, have students state which condition was not satisfied.

Think Aloud In small groups

Have students discuss the Intermediate Value Theorem and share ideas about real-world applications (e.g., speed of your car and weight of your kitten). Have groups post their ideas on a classroom wall using sticky notes.

Sort cards

In a classroom activity, students will sort cards pertaining to the graph of a function f consisting of vertical asymptotes, horizontal asymptotes, jump, removable, and nonremovable discontinuities.

Students will have to match selected portions of the graph to its written description and symbolic (notation) description. Here, students are learning how to express limits in both written and symbolic form to understand the behavior of a function f as f gets sufficiently close to a particular x-value.

Graphing Calculator Activity

Connecting Infinite Limits and Vertical Asymptotes Using a table of values for x, students will use a calculator to find values for a given function f(x). They will notice that the values for f(x) either approach positive or negative infinity. Then students will use their graphing calculator to explore the graph of the function so that they could verify the location of the vertical asymptote. Using the table of values, students will use limit notation to explain why the function has a vertical asymptote near that value of x.

Interpreting graphs

In a classroom activity, students will calculate the velocities (the average rate of change) of several automobiles using both functions given analytically and data presented in a table of time versus displacement. Students will use their information to approximate the instantaneous velocity of the automobile at a particular time t and to sketch a graph of velocity as a function of time. They will provide a verbal (that is, written in words) interpretation of the movement of each vehicle (such as "The car's velocity is positive and decreasing") and explain how their verbal interpretation is connected to the graph they have drawn.

Choose correct method

Students will complete an activity where they have to choose a method for determining a limit arranged in a chart. They will start with direct substitution; if they get 0/0, they will have to choose from Algebra, Table of Values, or a Graph as a means for finding the limit. Then, they will write a brief explanation why they chose that method for finding the limit. Students will also use a flow chart to help them find limits

Complete the table

The students will learn about three types of discontinuities by completing this table – removable, jump (piecewise), and asymptotic. They will also justify the type of discontinuity using correct notation.

Unit 2 – Differentiation and Fundamental Properties (Big Ideas: Change, Limits, Analysis of Functions)

Graph and Switch

Present students with two or three functions and the graph of each function. Have each student choose a random derivative question and one function. Questions could include: Find the average rate of change on an interval, instantaneous rate of change at a point, derivative as a function, derivative value at a point, or equations for tangent or normal lines at a point. Have students answer their question and place their answer onto the function's graph. Then have students share their solutions with each other to give and receive feedback.

Match Mine

Create cards containing graph images of functions with various continuous, discontinuous, differentiable, and nondifferentiable points or intervals. Provide each student in a pair with the same nine cards. Student A arranges their graphs in a 3 by 3 grid, which is not visible to Student B. Student A describes each of their graph's positions using information about continuity and differentiability to describe the graph. Based on the descriptions, Student B attempts to arrange their cards to match the grid of Student A.

Error Analysis

Assign a function to each student. Ask them to find the function's derivative using one or more derivative rules. Allow them to check their answers. Ask half of the class to redo their work to include an error, thus having the wrong answer. Ask students to record their correct or incorrect work on a card. Mix up the cards and redistribute, having students determine if the answer is correct or incorrect. If incorrect, they should explain what error was made, and find the correct answer. 4

Graphic Organizer

Provide students with colored paper, pens, and markers. Ask them to create a chart, a foldable card, or other creative method to organize all the derivative rules. For each rule, have them include the mathematical definition, examples, pictures, and helpful hints to understand and remember the rule.

Round Table

Provide each student with the same worksheet containing four functions that require the product rule or quotient rule when finding the derivative. Then have students sit in groups of four. Each student determines the derivative of function No. 1, and then they pass their papers clockwise to the next student. Each student checks the first problem and, if necessary, discusses any mistakes with the previous student. Each student now completes function No. 2 on the paper, and the process continues until each student has their original paper back.

Unit 3 – Differentiation: Composite, Implicit and Inverse Functions (Big Ideas: Change, Limits, Analysis of Functions)

Scavenger Hunt

Place a card with a starter question somewhere in the classroom, for example, "Find the derivative of $f(x) = \sin(4x)$." Place another card in the room with the solution to that card plus another question: "Solution: $4\cos(4x)$. Next problem: Find the derivative of $f(x) = (\sin(x))^4$." Continue posting solution cards with new problems until the final card presents a problem whose solution is on the original starter card (note that this solution should be added to the starter card above).

Work Backward

Provide a chain rule problem with several possible answers. Have students identify which piece of the original problem contributed its derivative to one of the factors in the answer they are looking at. For each possible answer, students should say whether the answer is correct. They should also identify which piece of the composition was differentiated incorrectly or skipped, or state which factor of that particular answer did not come from the original problem. Students often have trouble knowing when to stop differentiating when learning ex or trigonometric functions, so make sure you include these types of problems.

Round Table

In groups of four, each student has an identical paper with four different problems on it. Students complete the first problem on their paper and then pass the paper clockwise to another member in their group. That student checks the first problem and then completes the second problem on the paper. Students rotate again and the process continues until each student has their own paper back.

Quiz-Quiz-Trade

Give students a card containing a question and have them write the answer on the back. Students then circulate around the room and find a partner. One student quizzes the other by showing only the side of the card with the question on it, and then they reverse roles. They swap cards, find a new partner, quiz each other, and the process continues.

Graphing Calculator

In a class activity, students will use their graphing calculator to discover the power rule for derivatives. Students will enter functions such as, y = x, y = x2 , y = x3 into their calculators and graph the derivatives of the functions one at a time in order to explore the graphs and make a conjecture about the derivative of a power function. Then, students will use the strategy of turn and talk to try and generalize a rule for finding the derivative of a power function.

Unit 4 – Contextual Applications of Differentiation. (Big Ideas: Change, Limits, Analysis of Functions)

Quickwrite

Divide students into groups and give each group a context (outdoors, in a supermarket, in biology, in the government, at home, etc.). Students then write for a few minutes, listing things that are changing in that particular context.

Create Representations

Provide verbal descriptions of a roller coaster ride: at time 0, velocity is 0 but about to become positive; at time 2, velocity is positive and increasing; at time 5, velocity is 0 and decreasing, etc. Have students graph position (from start), velocity, acceleration, speed, and then draw arrows at each point depicting whether their body would lean forward, backward, or not at all.

Marking the Text

Have students read through a problem and highlight/underline the given quantities and directions in a problem, stating whether that information always applies or applies only at an instant.

Round Table

Give students different related rates problems and a paper divided into five sections, titled as following:

- Draw a picture
- Equation
- Derivative
- Specific information used
- Interpretation

Students first draw a picture of the situation and pass the papers clockwise. Students then critique the work in the previous section, complete the next section, and pass the papers again until all sections are completed.

Scavenger Hunt

A starter question is posted in the room, for example, "Approximate the value . . . ." Have students work through the problem to find the value and then look for that value at the top of another card posted in the room. Students then solve the problem on that card, for example, "Write the equation of the tangent line . . . " and look for that solution on a third card, etc. The solution to the last problem will be on the starter card.

Studying graphs

In a class activity, students will consider several graphs of functions – continuous and discontinuous. Using the graphs, students will take away that if a function is continuous on a closed interval, then the function has both a maximum and minimum. Students will understand that a function may still have a maximum or minimum even though it is not continuous on a closed interval.

Is the function differentiable?

students will use the strategy of turn and talk to discuss which functions are differentiable and why. They will discuss the essential condition for differentiability. After students learn that continuity is a requirement for differentiability, they will complete the second part of the activity, where they will consider graphs of continuous functions and draw tangent lines at various points along their graphs. The students should discover that slopes of tangents do not exist at corner points, cusps, or vertical lines.

Unit 5 – Analytical Applications of Differentiation (Big Ideas: Change, Limits, Analysis of Functions)

Critique Reasoning

Arrange students in groups of four to six, provide them with a function's derivative and ask them to determine if g(x) is increasing or decreasing at a specific x-value. Ask students to share the reasoning for their conclusion with classmates in their group. Members of the group can then provide feedback and suggestions.

Think-Pair-Share

Provide students with a graph of f′ and a graph of f′′. Ask them to identify relative extrema and practice writing justifications for relative extrema using the first or second derivative test. Once they've written their justification, ask them to pair with a partner and share their justifications. Students can then discuss similarities or differences in their justification wording.

Create a Plan

Provide students with a function represented analytically on a closed interval. Ask them to discuss and write x-values that are viable candidates for absolute extrema. Once they have established the viable candidates, ask them to design a method for analyzing the behavior of the function's graph at the candidates and for identifying the extrema.

Predict and Confirm

Provide students with the graph of a differentiable function but do not provide the rule for the function. Ask students to sketch a graph of the derivative of the function. Once students are done, reveal the rule for f(x). Ask students to calculate f'(x), and use technology to graph f'(x) and compare it to their sketched graph.

Interpreting the meaning of the derivative

In a class activity, students will start with a function G(x) without context and be asked to interpret G'(5). Then, context will be added to function G(t) to mean the amount of unprocessed gravel arriving at a processing plant, where G is measured in tons and t is measured in hours. Since students often struggle interpreting a derivative, students will be provided with a template. The template will be: At time t = _____, the function is increasing or decreasing at a rate of ____________ (units of y)/(units of x). This activity will require students to interpret different representations. In a final part of this activity, students will critique student samples from past free-response questions to learn both correct and incorrect ways of interpreting a derivative.

Unit 6 – Integration and Accumulation of Change

Quickwrite

Present the class with several examples of indefinite integrals set to Rieman sums in summation notation. Ask students to take 5 minutes to identity and write about all the common elements between the two expressions and why they think the two expressions are equivalent. After finishing the 5 minutes, ask students to share their observations with the class.

Look for a pattern

Present students with several indefinite integrals and proposed, yet incorrect, antiderivatives. Ask them to check the antiderivatives by differentiating each and comparing to the original integrands. As students see each integrand is incorrect, ask them to identify a pattern within the errors. Identifying this pattern will establish the foundation for integrating using substitution.

Odd One Out

To help students select a strategy, form groups of four, presenting each student an indefinite integral whose integrand is rational. For each group, include one integrand that requires long division or completing the square. Ask students to decide if their example fits with the group. Identifying the odd one out will help students connect integrand form to appropriate strategy.

Exploring graphs

Exploring Accumulations of Change (Skill 4.B) Class will start with a velocity vs. time graph, where v(t) = 70 miles per hour from t = 0 to t = 2 hours. From that information, students will easily know that the total distance traveled is 140 miles. What is important here is for the students to understand that 140 corresponds to the area under the graph of v(t) = 70 from t = 0 to t = 2 hours.

Unit 7 – Differential Equations (Big Ideas: Change, Limits, Analysis of Functions)

Match Mine

Give student pairs a blank 3 x 3 grid, nine graphs of slope fields, each on a separate card. Some students in terms of x only, some in terms of y only and some in terms of x and y only. Be sure to include some trigonometric functions. Students A arranges the graphs on the grid without showing student B and then describes the arrangement so student B can match it on their own.

Numbered Heads Together
Have each student complete the same problem individually. Make sure to use a variety of notation in whatever problem you pick. Then have students compare answers and procedures within groups. Students fix any mistakes until they all agree on the same answer.

Collaborative Poster
Assign each student a role within their group:

- Separating the variables
- Integrating both sides
- Finding C
- Writing the final particular solution

Then distribute a free-response question to each group and have them work on their assigned roles.

Unit 8- applications of integration (Big Ideas: Change, Limits, Analysis of Functions)

Scavenger Hunt
Post around the room 8-10 cards problem cards, each of which has a solution to the previous problem. Include average value problems with tables and functions.
Round Table

In groups of four, each student has an identical paper with the same free-response along with four labeled boxes representing steps in the problem:

- Identify all points of intersection
- Set up the integral(s)
- Integrate by hand
- Integrate using a calculator

Have students complete the first step on their paper, then pass the paper clockwise, to another member in the group. That student checks the first step and completes the second step on the paper. Students rotate again and the process continues until each student has their paper back.

Quiz Quiz trade

Create cards with problems revolving around the x- axis or y- axis and others revolving around other axis. Give each student a card and have them write their answers on the back. Students quiz a partner about their own card, then swith card and repeat the process with a new partner.

Connecting position, velocity and acceleration

Students will apply the Fundamental Theorem of Calculus to problems pertaining to position, velocity, and acceleration. In some problems, students will be given information about the velocity of an object in multiple representations and will answer questions about the object's position. In other problems, students will be given information about the acceleration of an object, the object's velocity and position at a time t, and will be asked to find particular solutions for the object's velocity and position.

Unit 9 – parametric equations, polar coordinates and vector-valued functions (Big Ideas: Change, Limits, Analysis of Functions)

Numbered Heads Together

Organise the class into groups of four and give each student a number. Glive the class one set of parametric equations and have students individually find the second derivative. When all the students in a group have finished, have them stand up to discuss their answers, make any necessary corrections, and then sit back down. Choose a group and call a specific student to share the answer with the class.

Scavenger Hunt

Post around the room 8-10 cards problem cards, each of which has a solution to the previous problem. Focus on giving students an initial value problem and asking for one or both components at a different t- or x-value.

Create representations

Give students polar equations of various types. Have students create a table of values, sketch the graph of the curve using rectangular coordinates, and sketch the graph using polar coordinates. Preface this activity by modeling the steps with one function on large paper, using wiki sticks to show the y-values as heights becoming the r-values as radii.

Paraphrasing

Give students a proof that derives the polar area formula. Ask them to restate the meaning and derivation of this formula using their own words. Have them also compare and contrast this formula to the integration used to find areas under functions in the Cartesian coordinate system.

Stand up, hand up, pair up

Give each pair of students a polar area question to solve. Once they have completed both roles obtaining only the integral set up, have them use a calculator to find the numeric solution and confirm with you before standing up to switch pairs.

Unit 10 – infinite sequences and series (Big Ideas: Change, Limits, Analysis of Functions)

Predict and confirm

Demonstrate a geometric series, the harmonic series and the alternative series by distribution pieces of a donut, pizza or licorice. Ask the class to predict how much the student(s) will receive in total.

Graph organiser

Put students in groups with poster paper and have them explain all the series tests using pictures, texts, flowcharts, cartoons or other drawings. Have them include each test's conditions and how to choose which test to apply.

Odd one out

Have students decide which series is the odd one out. For example, one series may have a different type of convergence or one series may only converge for all real numbers.

Collaborative poster

Have students create their own free-response questions. Give each student a basic series e.g cos(x), sin(x) ,.. and ask the first two members to choose a manipulation of the series and show work to complete the task. Ask the final two members to watch silently and confirm the first two steps. Then the final two members choose further actions to perform on the new series.