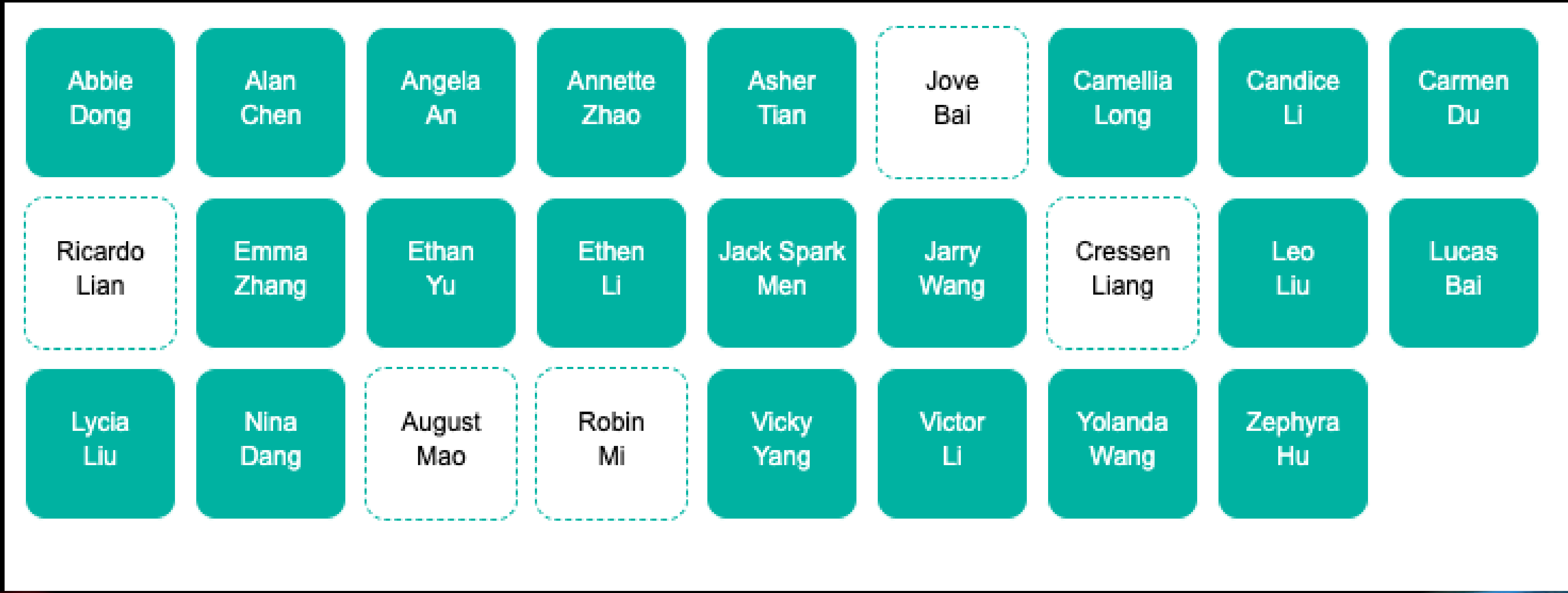
F2

Angelina Wang	Azura Cheng	Bonnie Zhao	Cecilia Jia	Diana Li	Oscar Liu	Zavier Liu	Tom Ou	Mia Peng
Joshua Hui	Julius Lv	Justin Jia	Lucas Li	Lyra Zhang	Iris Shao	Dylan Suo	PeytonAson Wang	Struck Wang
Selene Hou	Serena Feng	Silas Lv	Simon Wang	Stella Sun	Yola Wang	Mark Xiao	Elsa Ye	Emily Zhang
Zinnia Dong	Flora Zhang							

F3

Allen Peng	Aurora Yuan	Brittney Wei	Cynthia Liu	Erya Hu	Eva Gai	Felicity Pan	Fiona Ding	Freya Fan
Gordon Yao	Helios Luo	Honey Ruan	Iris Xie	Jasper Wu	Kaven Zhang	Kevin Gao	Leo Yang	Linger Li
Lydia Wei	Maggie Gao	Micheal Zhao	Ray Meng	Rose Jiang	Ross Ma	Roy Liu	Ryan Wang	Sky Bai
Star Su	Stella Xi							

F5


Abbie Dong
Alan Chen
Angela An
Annette Zhao
Asher Tian
Jove Bai
Camellia Long
Candice Li
Carmen Du
Ricardo Lian
Emma Zhang
Ethan Yu
Ethen Li
Jack Spark Men
Jarry Wang
Cressen Liang
Leo Liu
Lucas Bai
Lycia Liu
Nina Dang
August Mao
Robin Mi
Vicky Yang
Victor Li
Yolanda Wang
Zephyra Hu

F4

Aris Cheng	Tracy Dang	Jason Du	Alanna Fan	Iris Gao	Eason Gong	Vardy Hai	Edmund He	Liora Li
Miranda Li	Leo Liu	Francis Lv	Dorothy Qian	Dary Song	Jerry Tu	Belinda Wang	Winnie Wang	Elena Wei
James Xu	Mike Yan	Simon Yang	Coco Zhang	Patrick Zhang	Sophia Zhang	Molly Zheng	Carrie Zhou	Rudy Zhu

F1

Annie Bai	Franklin Chen	Charlie Cheng	Zoey Cheng	Harry Cui	King Deng	Aisling Fu	Ken He	Ivy Jing
Lee Li	Ziheng Li	Lewis Liu	Sam Liu	Ava Peng	Peter Shen	Sylvia Song	Coco Wang	Eileen Wang
Jason Wang	Luna Wang	Cathy Yan	Melody You	Allen Zhang	Lucas Zhang	Lynette Zhang	Ryder Zhang	Flora Zhou
Cyntina Zuo								

F6

Lucas Chang	Chloe Fang	Bruce Gao	Wyatt Hou	Jackson Jiang	Ivan Lei	Eric Li	Keira Li	Vivian Li
Jeremy Lin	Orange Liu	Serika Ren	Yumiko Shi	Hannah Si	Eric Song	Eric Tan	Silas Wang	Niki Wei
Kiki Wen	Miyu Wu	Tia Wu	Kumi Yuan	Estelle Zhang	Soren Zhang	Viakey Zhang	Jack Zhao	

2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

In this section, you will:

- Find the domains of rational functions.
- Identify vertical asymptotes.
- Identify horizontal asymptotes.

Reminders
Project Deadline – 9th Jan. Everyone upload on Jupiter
Complex Number Quiz – 9th Jan

TURN AND TALK

$$f(x) = \frac{x^2 - 1}{x^2 - x}$$

- Identify the **domain restrictions**
- Predict whether the graph has **vertical asymptotes or holes**
- Find any x-**intercepts**

TURN AND TALK

Step 1: Factor

$$f(x) = \frac{(x-1)(x+1)}{x(x-1)}$$

Step 2: Domain restrictions

$$x \neq 0, \ x \neq 1$$

Step 3: Hole or asymptote

- $x = 1$: factor cancels → **hole**
- $x = 0$: denominator remains → **vertical asymptote**

Step 4: x-intercepts

$$x = -1$$

I SAY YOU SAY

1. **Rational function** – 有理函数 (yǒu lǐ hán shù)
2. **Numerator** – 分子 (fēn zǐ)
3. **Denominator** – 分母 (fēn mǔ)
4. **Polynomial** – 多项式 (duō xiàng shì)
5. **Degree of a polynomial** – 多项式的次数 (duō xiàng shì de cì shù)
6. **Asymptote** – 渐近线 (jiàn jìn xiàn)

A **rational function** is a quotient of polynomial functions. It can be written in the form

$$f(x) = \frac{N(x)}{D(x)}$$

where $N(x)$ and $D(x)$ are polynomials and $D(x)$ is not the zero polynomial.

The *domain* of a rational function of x includes all real numbers except x-values that make the denominator zero. Much of the discussion of rational functions will focus on the behavior of their graphs near x-values excluded from the domain.

2-07 Asymptotes of Rational Functions

- Rational Function
  - Fraction

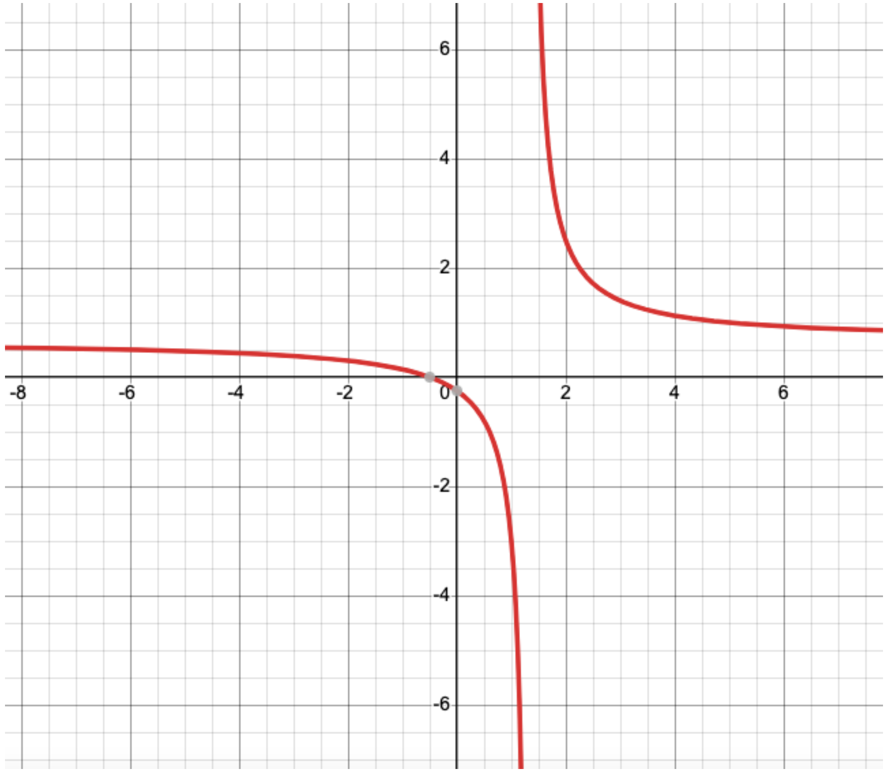
- $f(x) = \frac{2x+1}{3x-4}$

- Domain:
  - Denominator $\neq 0$

2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

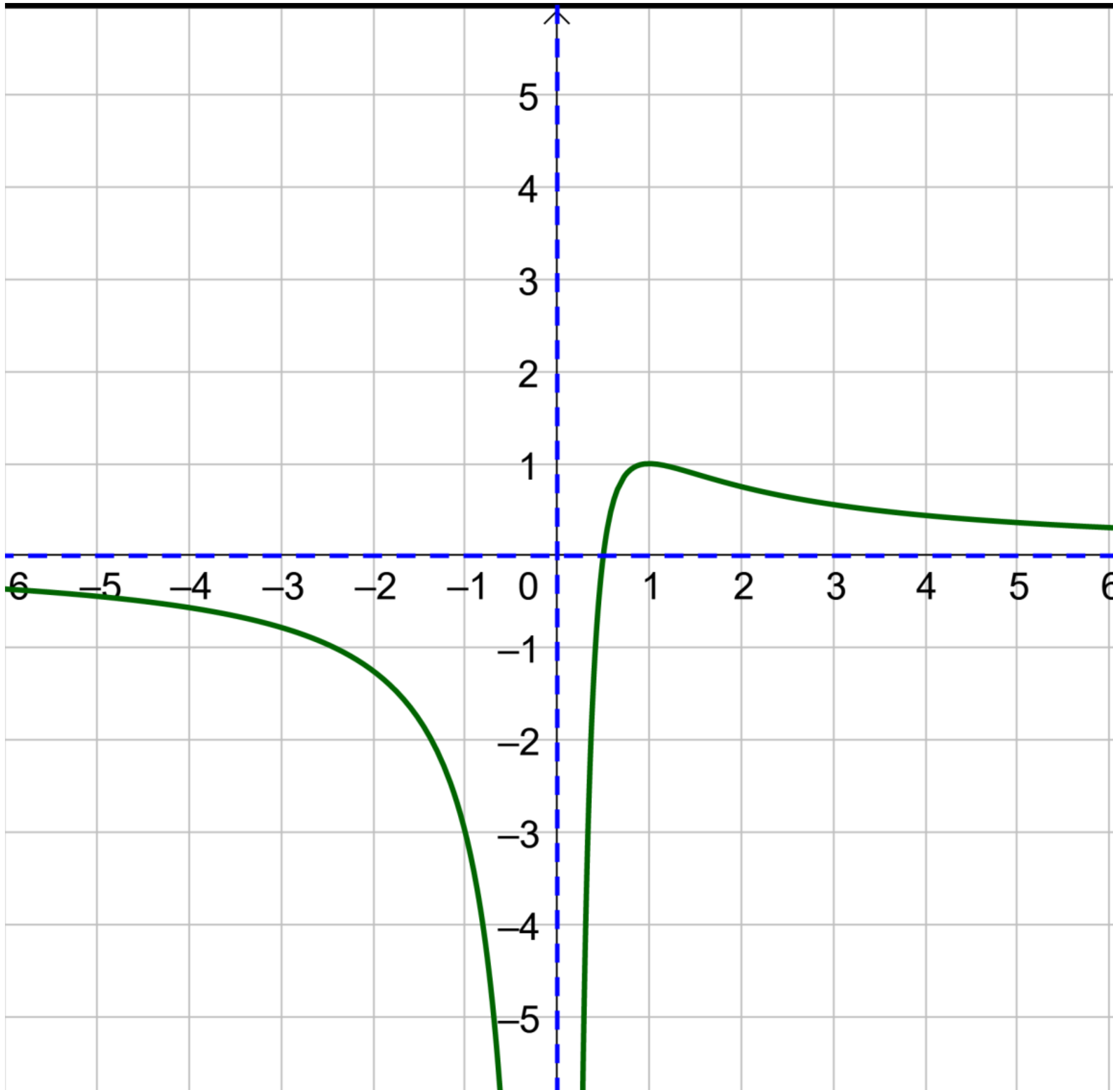
- Rational Function
  - Fraction
- $f(x) = \frac{2x+1}{3x-4}$

- Domain:
  - Denominator $\neq 0$



2-07 Asymptotes of Rational Functions

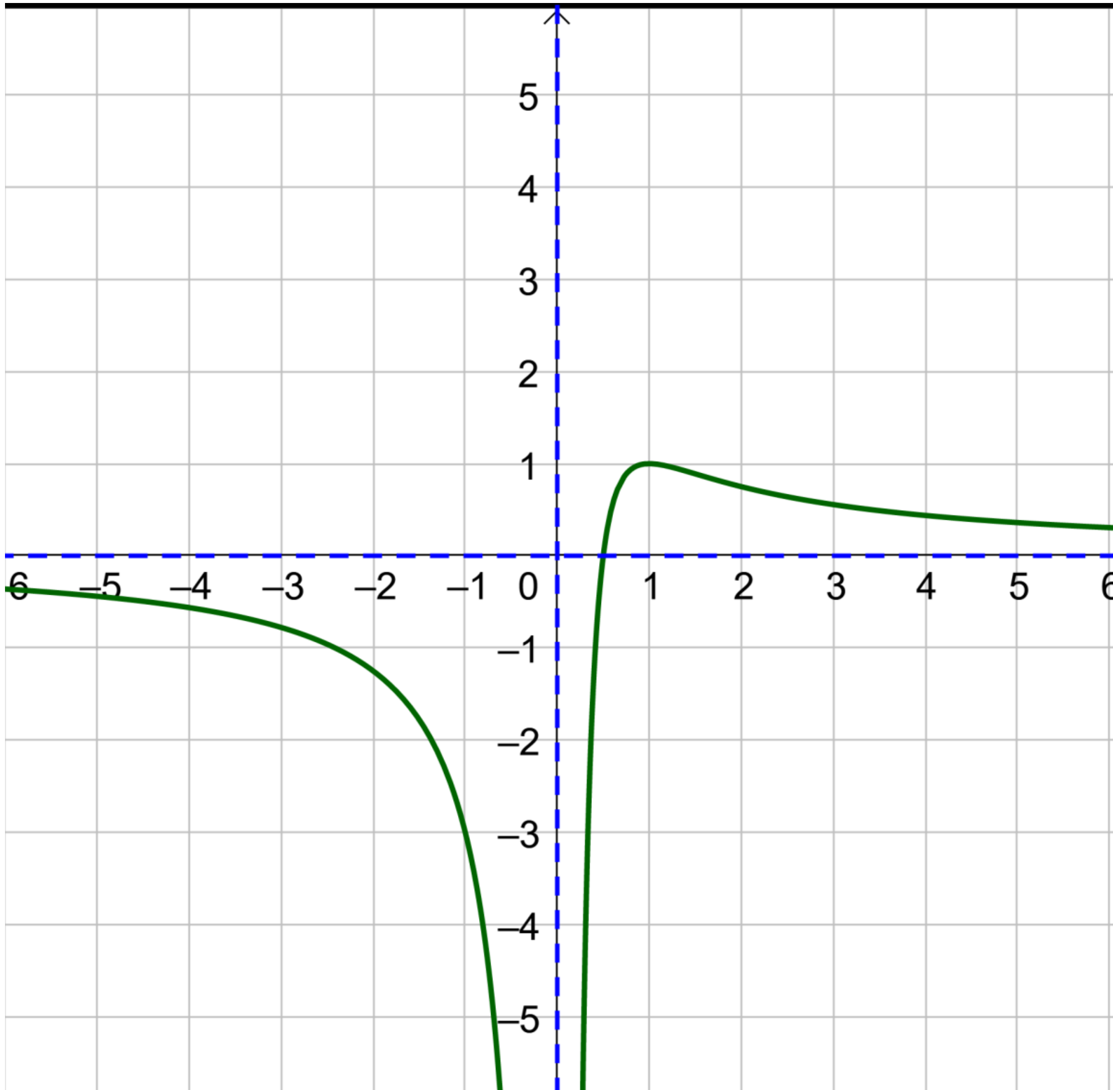
- Asymptotes describe behavior of the graph at the edges



2-07 Asymptotes of Rational Functions

- Asymptotes describe behavior of the graph at the edges

$x = 0 \;,\; y = 0$



VERTICAL AND HORIZONTAL ASYMPTOTES

Definitions of Vertical and Horizontal Asymptotes

1. The line $x = a$ is a **vertical asymptote** of the graph of f when

$$f(x) \to \infty \quad \text{or} \quad f(x) \to -\infty$$

as $x \to a$, either from the right or from the left.

2. The line $y = b$ is a **horizontal asymptote** of the graph of f when

$$f(x) \to b$$

as $x \to \infty$ or $x \to -\infty$.

2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

- Vertical Asymptotes
  - Factor and KILL
  - Set denominator = 0 and solve for *x*

$$y = \frac{\cancel{(x-1)}(x-3)}{\cancel{(x-1)}(x-2)}$$

2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

Horizontal Asymptotes

Find degree of numerator (N) and denominator(D)

1)N<D, y = 0

2) If N=D, y = leading coeff

3)If N>D, No HA

$$y = \frac{x}{(x-1)(x-2)}$$

$$y = \frac{x^3}{(x-1)(x-2)}$$

$$y = \frac{3x^2-3}{(x-1)(x-2)}$$

Vertical and Horizontal Asymptotes

Let f be the rational function

$$f(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdot \cdot \cdot + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdot \cdot \cdot + b_1 x + b_0}$$

where $N(x)$ and $D(x)$ have no common factors.

1. The graph of f has *vertical* asymptotes at the zeros of $D(x)$.

2. The graph of f has at most one *horizontal* asymptote determined by comparing the degrees of $N(x)$ and $D(x)$.

a. When $n < m$, the graph of f has the line $y = 0$ (the x-axis) as a horizontal asymptote.

b. When $n = m$, the graph of f has the line $y = \frac{a_n}{b_m}$ (ratio of the leading coefficients) as a horizontal asymptote.

c. When $n > m$, the graph of f has no horizontal asymptote.

2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

- Find the asymptotes of

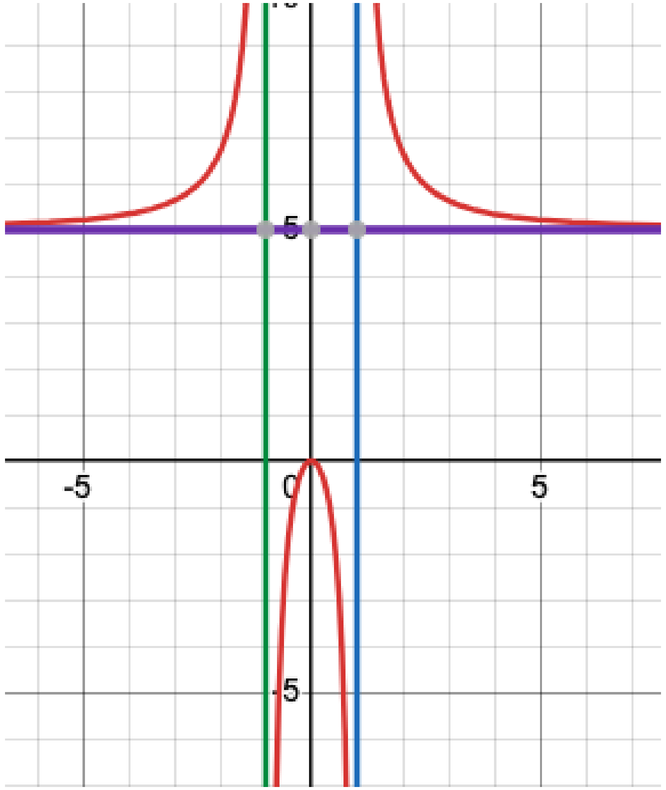
$$f(x) = \frac{5x^2}{x^2-1}$$

2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

- Find the asymptotes of $f(x) = \frac{5x^2}{x^2-1}$

x= 1 or x = -1

y=5



2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

- For $f(x) = \frac{2x^2 - x}{2x^2 + x - 1}$
  a. Find the domain
  b. Find the removable discontinuity
  c. Find the asymptotes

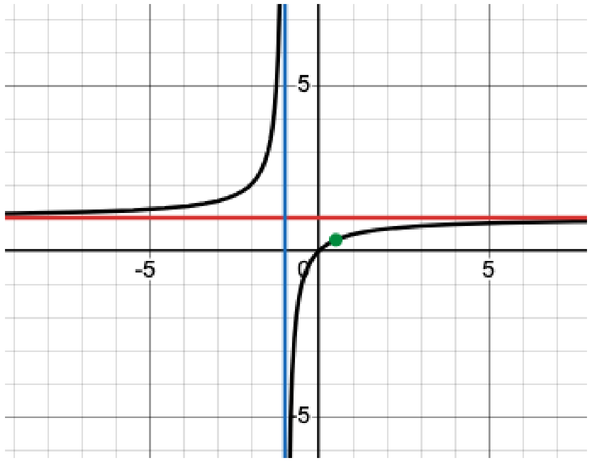
(2x-1)(x+1)

Hole at (0.5, 1/3)

VA: x= -1

HA:

y= 1



2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

- For $f(x) = \frac{2x^2 - x}{2x^2 + x - 1}$
  a. Find the domain
  b. Find the removable discontinuity
  c. Find the asymptotes

2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

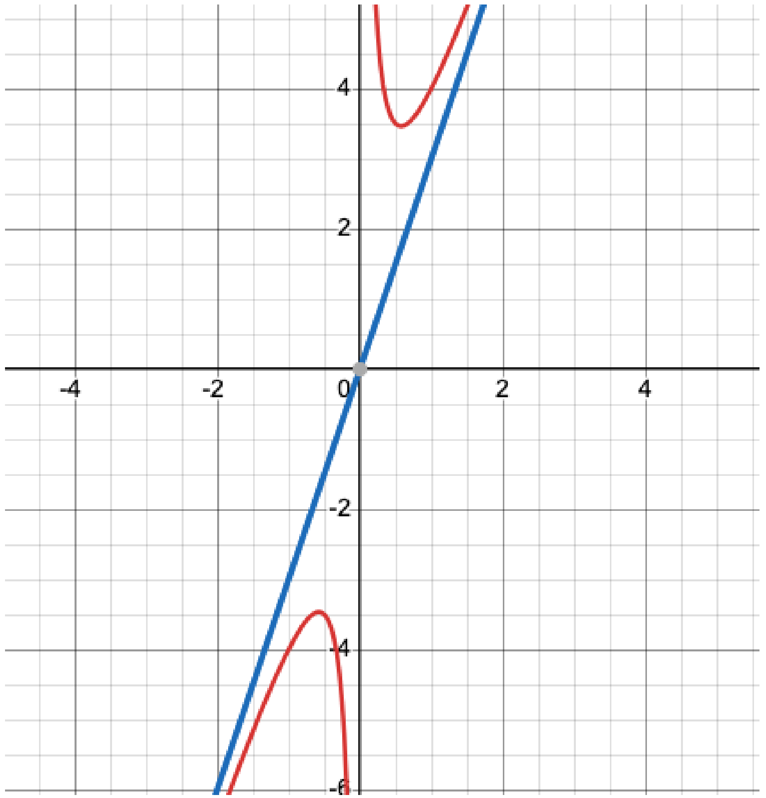
- Slant Asymptote
  - If $N = D + 1$, Divide and ignore remainder
- Find the asymptotes of $f(x) = \frac{3x^2+1}{x}$

2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

- Slant Asymptote
  - If $N = D + 1$, Divide and ignore remainder
- Find the asymptotes of

$$f(x) = \frac{3x^2+1}{x}$$

- y=3x



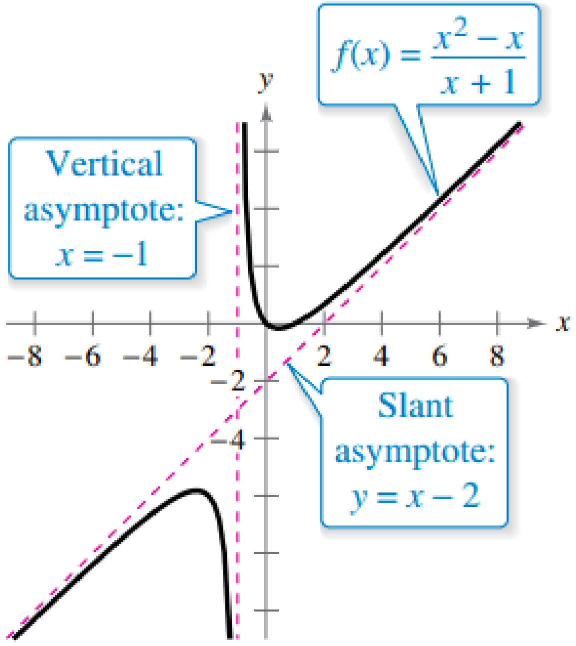


Figure 2.26

Slant Asymptotes

Consider a rational function whose denominator is of degree 1 or greater. If the degree of the numerator is exactly *one more* than the degree of the denominator, then the graph of the function has a **slant** (or **oblique**) **asymptote.** For example, the graph of

$$f(x) = \frac{x^2 - x}{x + 1}$$

has a slant asymptote, as shown in Figure 2.26. To find the equation of a slant asymptote, use long division. For example, by dividing $x + 1$ into $x^2 - x$, you obtain

$$f(x) = \frac{x^2 - x}{x + 1} = \underbrace{x - 2}_{\substack{\text{Slant asymptote} \\ (y = x - 2)}} + \frac{2}{x + 1}.$$

SLANT OR HORIZONTAL ASYMPTOTE

1. $f(x) = \frac{2x^3+x^2-5}{x^2+4}$
   - Does the function have a horizontal asymptote or slant asymptote?

SLANT

2. $g(x) = \frac{5x^2+2x-1}{3x^2+7x+2}$
   - Does the function have a horizontal asymptote or slant asymptote?

HA

3. $h(x) = \frac{x^3+3x+1}{x^2-4}$
   - Does the function have a horizontal asymptote or slant asymptote?

SLANT

Guidelines for Graphing Rational Functions

Let $f(x) = \dfrac{N(x)}{D(x)}$, where $N(x)$ and $D(x)$ are polynomials and $D(x)$ is not the zero polynomial.

1. Simplify f, if possible. List any restrictions on the domain of f that are not implied by the simplified function.
2. Find and plot the y-intercept (if any) by evaluating $f(0)$.
3. Find the zeros of the numerator (if any). Then plot the corresponding x-intercepts.
4. Find the zeros of the denominator (if any). Then sketch the corresponding vertical asymptotes.
5. Find and sketch the horizontal asymptote (if any) by using the rule for finding the horizontal asymptote of a rational function on page 168.
6. Plot at least one point *between* and one point *beyond* each x-intercept and vertical asymptote.
7. Use smooth curves to complete the graph between and beyond the vertical asymptotes.

2-08 GRAPHS OF RATIONAL FUNCTIONS

In this section, you will:

- Find the intercepts of rational functions.
- Graph rational functions.
- Solve applied problems involving rational functions.

2-08 Graphs of Rational Functions

- Intercepts
  - x-int: let $y = 0$

$$\left(\frac{\sqrt{3}}{3},0\right), \left(-\frac{\sqrt{3}}{3},0\right)$$

  - y-int: let $x = 0$

none

- Find the intercepts of $f(x) = \frac{3x^2-1}{x}$

GRAPHS OF RATIONAL FUNCTIONS

- To graph rational functions

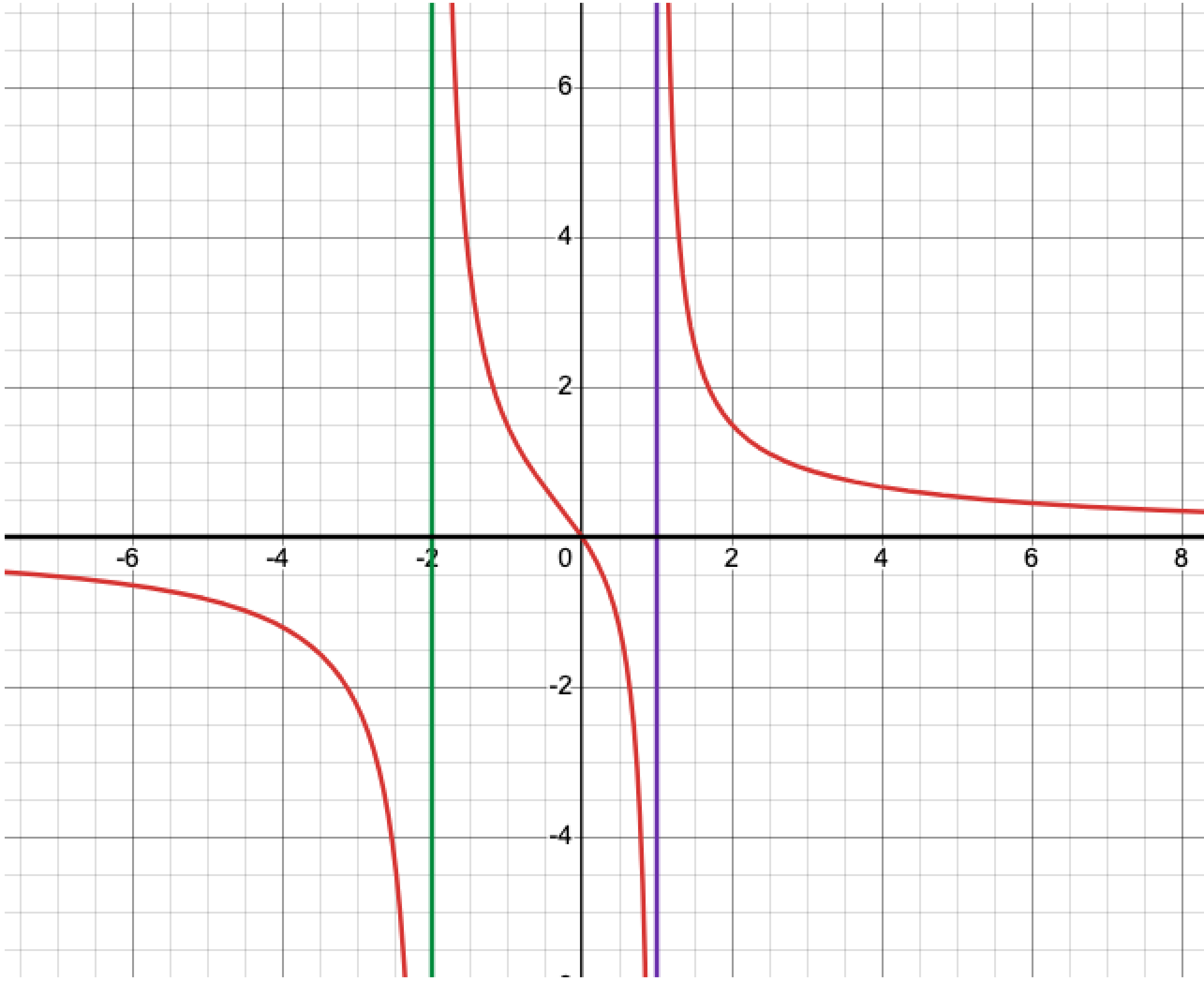
1. Find **asymptotes**
2. Find **x –intercept** and **y-intercept**

2-08 GRAPHS OF RATIONAL FUNCTIONS

- Graph $f(x) = \frac{3x}{x^2+x-2}$

2-08 GRAPHS OF RATIONAL FUNCTIONS

- Graph $f(x) = \frac{3x}{x^2+x-2}$



2-08 GRAPHS OF RATIONAL FUNCTIONS

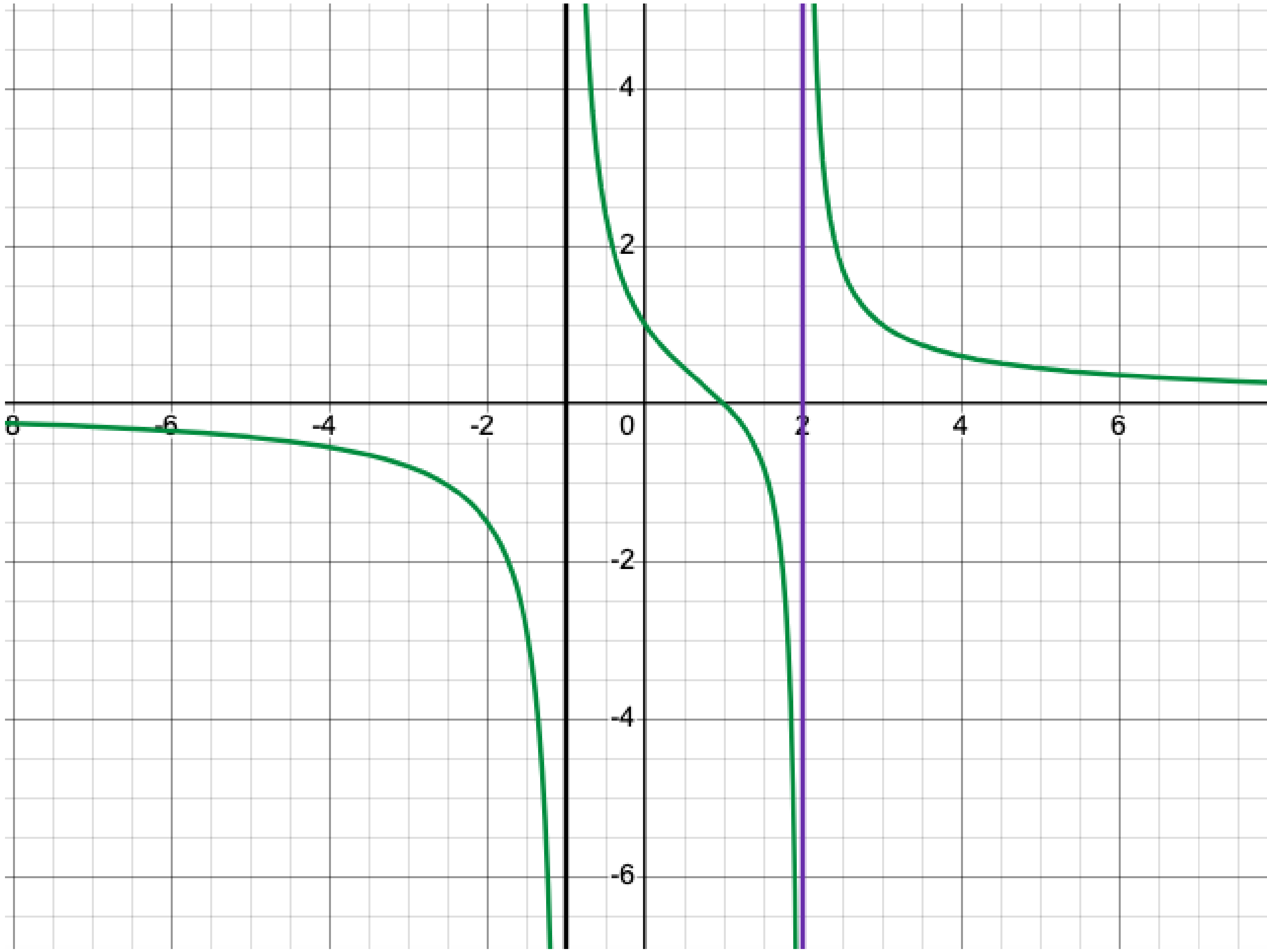
- Sketch the function

$$y = \frac{2(x-1)}{(x-2)(x+1)}$$

2-08 Graphs of Rational Functions

- Find the function

$$y = \frac{2(x-1)}{(x-2)(x+1)}$$



TRUE OR FALSE

- The domain of a rational function includes all real numbers.

TRUE OR FALSE

- The domain of a rational function includes all real numbers.

FALSE

TRUE OR FALSE

- If a factor cancels between the numerator and denominator, the graph has a hole.

TRUE OR FALSE

- If a factor cancels between the numerator and denominator, the graph has a hole.

TRUE

TRUE OR FALSE

- Vertical asymptotes occur where the numerator equals zero.

TRUE OR FALSE

- Vertical asymptotes occur where the numerator equals zero.

FALSE

Rational Functions – Worksheet

Consider the rational function: $h(x) = \dfrac{x^2 - 1}{x - 2}$

Answer the following questions carefully. Show all necessary working where appropriate.

#Vertical Asymptote

(a) Find the vertical asymptote(s) of the function.

Horizontal / Slant Asymptote

(a) Determine whether the function has a horizontal or slant (oblique) asymptote.

(b) Write the equation of the asymptote.

Slant

Rational Functions – Worksheet

Consider the rational function: $h(x) = \dfrac{x^2 - 1}{x - 2}$

Answer the following questions carefully. Show all necessary working where appropriate.

#<u>Vertical Asymptote</u>

(a) Find the vertical asymptote(s) of the function.

x = 2

<u>Horizontal / Slant Asymptote</u>

(a) Determine whether the function has a horizontal or slant (oblique) asymptote.

(b) Write the equation of the asymptote.

Slant

$$\frac{(x^2 - 2)}{x - 2} = x + 2 + \frac{2}{x - 2}$$

$y = x + 2$

Intercepts

(a) Find the x-intercept(s).

(b) Find the y-intercept.

Behavior:

(a) As $x \to 2^-$, describe the behavior of $h(x)$.

(b) As $x \to 2^+$, describe the behavior of $h(x)$.

Intercepts
(a) Find the x-intercept(s).

(1,0)

(b) Find the y-intercept.

(0,0.5)

Behavior:
(a) As $x \to 2^-$, describe the behavior of h(x).

h(x) gets smaller

(b) As $x \to 2^+$, describe the behavior of h(x).

h(x) gets bigger

<u>Behavior Relative to the Slant Asymptote</u>:

a) For large positive values of x, does the graph approach the slant asymptote from **above**
below?

(b) For large negative values of x, does the graph approach the slant asymptote from **abov**
or **below**?

<u>Sketching the Graph</u>

Using all the information above, sketch a neat and labelled graph of h(x).

(Use the space below)

Behavior Relative to the Slant Asymptote:

a) For large positive values of x, does the graph approach the slant asymptote from **above**
below?

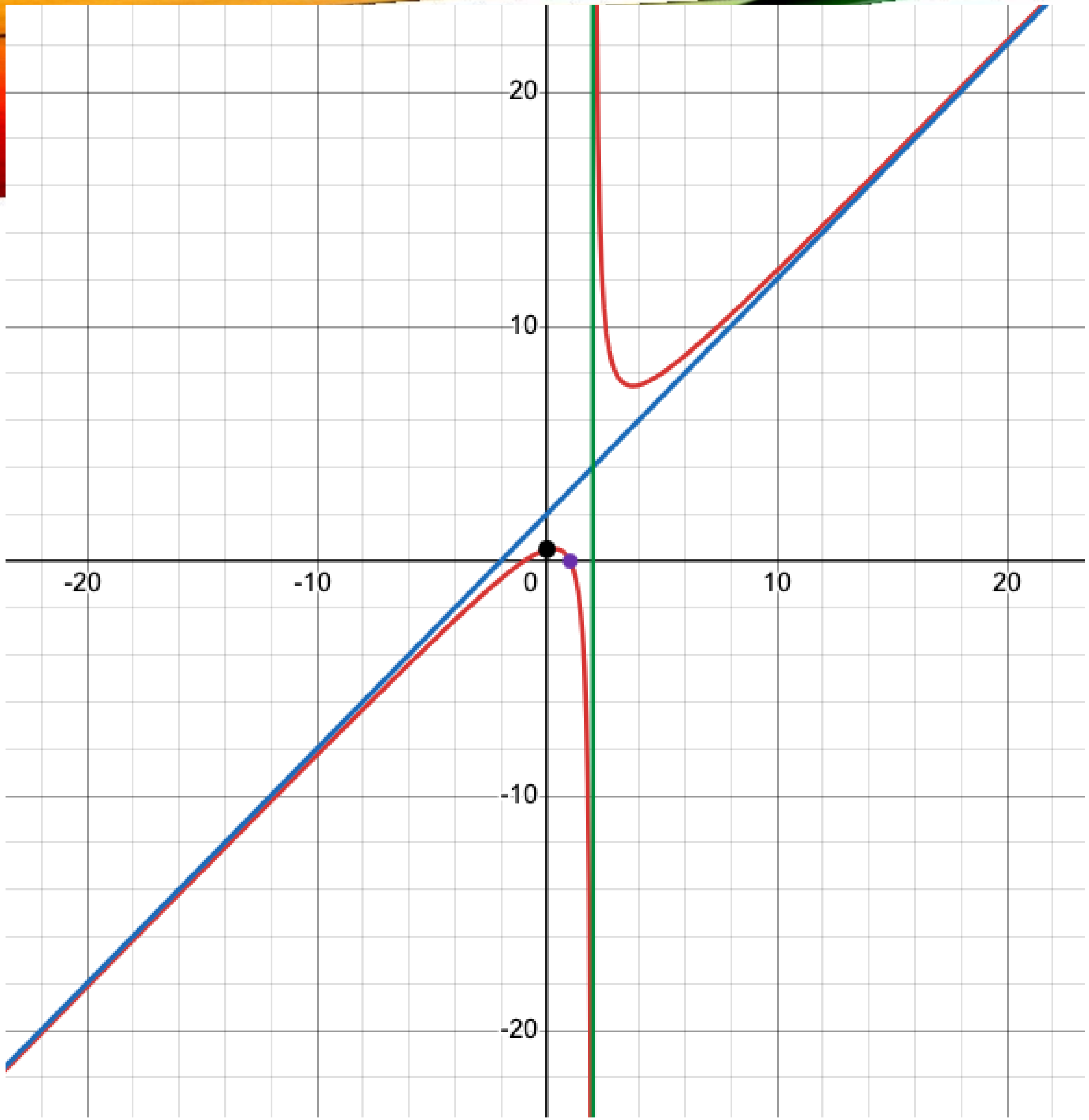
Above

(b) For large negative values of x, does the graph approach the slant asymptote from **abov**
or **below**?

Below

Sketching the Graph

Using all the information above, sketch a neat and labelled graph of h(x).
(Use the space below)


20
10
-10
-20
-20
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