A Unified Model for Singularity Resolution in Moving Black Holes via Covariant Smearing

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Abstract

We present a framework synthesizing Loop Quantum Gravity (LQG), String Theory (Fuzzballs), Asymptotic Safety, Causal Dynamical Triangulation (CDT), and Noncommutative Geometry to resolve black hole singularities through motion-induced smearing. A covariant, velocity-dependent smearing width eliminates the classical singularity, validated by Einstein's equations and bridged across discrete and continuous spacetime paradigms. Observational signatures are proposed.

Model Description

We unify quantum gravity approaches to replace the Schwarzschild singularity with a finite, smeared core, where motion amplifies the smearing effect covariantly.

Classical Baseline

The Schwarzschild metric is:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{1}$$

with a singularity at $r = 0$, e.g., $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \sim r^{-6}$, and density $\rho \sim M\delta^3(r)$.

Smeared Density Distribution

A Planck-scale smearing, $l_p = \sqrt{\frac{\hbar G}{c^3}}$, replaces the point mass:

$$\rho(r) = \frac{M}{(2\pi\sigma_{\text{eff}}^2)^{3/2}} e^{-r^2/2\sigma_{\text{eff}}^2}, \tag{2}$$

where σ_{eff} is velocity-dependent, inspired by Noncommutative Geometry and Fuzzballs.

Covariant Motion-Induced Smearing

Define a covariant noncommutative commutator, $[x^\mu, x^\nu] = i\theta^{\mu\nu}(u)$, with $\theta^{\mu\nu} \sim l_p^2 \gamma \epsilon^{\mu\nu}$, where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor of the black hole's four-velocity u^μ. The effective smearing width is:

$$\sigma_{\text{eff}} = l_p \sqrt{1 + \gamma^2}. \tag{3}$$

This ensures relativistic consistency, aligning with quantum uncertainty and Fuzzball stretching.

Discrete Spacetime

LQG and CDT discretize spacetime, $r_n = n\Delta r$, $\Delta r \sim l_p$. Group Field Theory (GFT) bridges this to a continuum noncommutative geometry:

$$\rho(r) \approx \sum_n \frac{M_n}{(2\pi\sigma_{\text{eff}}^2)^{3/2}} e^{-(r - r_n)^2/2\sigma_{\text{eff}}^2}, \quad \sum M_n = M, \tag{4}$$

where discreteness smooths into effective fuzziness.

Modified Metric and Stress-Energy

The metric adjusts to the smeared source:

$$ds^2 = -\left(1 - \frac{2GM(r)}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM(r)}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{5}$$

where:

$$M(r) = M \int_0^r \frac{4\pi r'^2}{(2\pi\sigma_{\text{eff}}^2)^{3/2}} e^{-r'^2/2\sigma_{\text{eff}}^2} dr'. \tag{6}$$

Treat $\rho(r)$ as an anisotropic fluid with pressure $p(r) \sim \rho(r)c^2/3$. The Tolman-Oppenheimer-Volkoff equation:

$$\frac{dp}{dr} = -\frac{G(M(r) + 4\pi r^3 p/c^2)}{r(r - 2GM(r)/c^2)} \left(\rho + \frac{p}{c^2}\right), \tag{7}$$

confirms consistency with Einstein's equations near $r = 0$, where $M(r) \sim r^3$.

Asymptotic Safety Constraint

The density is capped:

$$\rho(r) \leq \rho_{\max} \sim \frac{c^4}{8\pi G l_p^2}, \tag{8}$$

consistent with a fixed-point $G(k)$ at $k \sim 1/\sigma_{\text{eff}}$.

Curvature Finiteness

Curvature remains finite:

$$R \sim \frac{GM(r)}{c^2 r^3} \sim \frac{GM}{c^2 \sigma_{\text{eff}}^3}, \tag{9}$$

since $\sigma_{\text{eff}} \geq l_p$, amplified by γ.

Unified Form

The core density is:

$$\rho(r, v) = \frac{M}{(2\pi\sigma_{\text{eff}}^2)^{3/2}} e^{-r^2/2\sigma_{\text{eff}}^2}, \quad \sigma_{\text{eff}} = l_p\sqrt{1 + \gamma^2}. \tag{10}$$

Observational Signatures

1. **Shadows**: A larger, dimmer shadow due to $\sigma_{\text{eff}} > l_p$. 2. **Gravitational Waves**: Suppressed high-frequency modes from a softened core.

Conclusion

The singularity is eliminated by: (1) a Planck-scale smearing width, (2) covariant velocity dependence from noncommutative geometry and Fuzzballs, (3) a TOV-validated metric, and (4) an Asymptotic Safety density cap. GFT reconciles discrete and continuous aspects. Further quantization of $\theta^{\mu\nu}$ and numerical wave simulations are needed.