Pset 7

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1 Strongly Independent Set

To prove that `SIS` is NP-complete, we show that `IS` $\leq_p$ `SIS` where `IS` is the `Independent-Set` problem as discussed in lecture.

Proof. Let $G = (V, E)$ an undirected graph that is a valid input to the `IS` problem. We will form $G' = (V', E')$ by replacing every edge $(u, v) \in E$ with two edges, (u, w) and (w, v). Then, we connect all the w nodes together.

If G is `yes` on the `IS` problem, then G' will also be `yes` on the `SIS` problem with the same k. This is because any nodes in the independent set S of size $|S| \geq k$ will have a distance between each other of at least 2 by definition. Due to the adding of the (u, w) and (w, v) edges, this distance will become at least 4. So the nodes in S are a valid strong independent set in G' with size $|S| \geq k$. Otherwise, if G does not have an independent set of size $|S| \geq k$, it cannot have a strong independent set of the same size as by definition, we can't form a set of the same size where there is no path of length 1.

If G' is `yes` on the `SIS` problem, then G will also be `yes` on the `IS` problem with the same k. Let S be the strong independent set of size $|S| \geq k$ in G'. We know that there can be no w nodes in S as all the w nodes are connected to each other so you can reach any node from a w node within 2 edges. That means we can remove the w nodes from G' and re-connect the (u, v) nodes to get G. The nodes in the strong independent set S are the same nodes in the independent set for G. Otherwise, if G' does not have a strong independent set of size $|S| \geq k$, it cannot have an independent set of the same size in G. In this case, is at least one pair of nodes u, v whose distance is ≤ 2. If it is 1, then we have already violated the independent set principle. If the distance is 2, then we know the path from u to v is $(u, w) \rightarrow (w, v)$ by construction. Once we remove the w node and reconnect the (u, v), they will be neighbors and violate the independent set principle.

We have shown `IS` $\leq_p$ `SIS`. Since `IS` is NP-complete, `SIS` is NP-complete as well. □

2 Dominating Set