A possible spatial entanglement measure

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Taking a two particle wave function in one dimension $\psi(x,y)$ we suggest an entanglement measure. The wave function is not entangled, if it can be written in the form

$$\psi_{unentangled}(x,y) = \phi_1(x)\phi_2(y) \ .$$

The spatial entanglement measure should be 0 in this case. To keep it simple we will restrict the wave function to real functions. It should be straight forward to handle complex wave functions.

We construct a unentangled part of the density of states from the general two particle wave function $\psi(x,y)$.

$$\rho_{unentangle,part}(x,y) = \int_{-\infty}^{\infty} \psi^2(x,y)dx \cdot \int_{-\infty}^{\infty} \psi^2(x,y)dy \ .$$

This unentangled part of the density of states can now be subtracted from the two particle density of states of the original function

$$\rho_{entangled,part}(x,y) = \psi^2(x,y) - \rho_{unentangled,part}(x,y) \ .$$

This can be positive or negative and we suggest the entanglement measure as the integral of its absolute value

$$SEM = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\rho_{entangled,part}(x,y)| \, dx \, dy \ .$$

We take two examples to check, if the SEM fulfills basic properties. It should be zero for a normalized unentangled wave function

$$\psi(x,y) = \sqrt{\frac{2}{\pi}} e^{-x^2} e^{-y^2} \ .$$

This results in a SEM value of 0.

Taking a fully entangled wave function

$$\psi(x,y) = \sqrt{\frac{1}{\pi}} \left(e^{-(x-2)^2} e^{-(y+2)^2} + e^{-(y-2)^2} e^{-(x+2)^2} \right) \ .$$

This results in a SEM value of 1.

We conclude, that the measure might be a possible entanglement measurement.