

Starting from the Minorization-Maximization Formula

$$\gamma_i = \frac{W_i}{\sum_{j=1}^N \frac{C_{ij}}{E_j}},$$

we get

$$\gamma_i \cdot \sum_{j=1}^N \frac{C_{ij}}{E_j} = W_i.$$

We add an additional game  $M = N + 1$  with  $W_i^M$  beeing the number of wins of player  $i$  in this game. In the standard game this can be 1 or 0, as we usually only take one winner and a number of losers. With this definition

$$\gamma_i^M = \frac{W_i + W_i^M}{\sum_{j=1}^N \frac{C_{ij}}{E_j} + \frac{C_{iM}}{E_M}},$$

with the definition  $A_i = \sum_{j=1}^N \frac{C_{ij}}{E_j}$  the result is

$$\gamma_i^M = \frac{A_i \gamma_i + W_i^M}{A_i + \frac{C_{iM}}{E_M}},$$

calling  $r = W_i^M$  and  $x = \frac{C_{iM}}{E_M}$

$$\gamma_i^M = \frac{A_i \gamma_i + r}{A_i + x},$$

we can argue that after a number of games  $r \ll A_i \gamma_i$  and  $x \ll A_i$ . Therefor a Taylor series gives

$$\gamma_i^M = \gamma_i - \frac{\gamma_i}{A_i} x + \frac{1}{A_i} r.$$

The value of  $A_i$  depends on how many games allready played. As it is proportional to typical values of  $x$  it might be a good approach if one introduces

$$Bx = A_i,$$

with  $B$  being constant, if one has no way to get exact values of  $A_i$ :

$$\gamma_i^M = \gamma_i - \frac{\gamma_i}{B} + \frac{1}{B} \frac{r}{x},$$

or written differently

$$\gamma_i^M = \gamma_i + \frac{1}{B} \left( \frac{r}{x} - \gamma_i \right).$$

In the program

$$\frac{1}{B} = \text{params-} > \text{learn\_delta},$$

$$C_{iM} \cdot \gamma_i = C\_i M\_gamm\_i,$$

$$E_M = sum\_gammas.$$

giving

$$\gamma_i^M = \gamma_i + \frac{1}{B} \left( \frac{r \cdot sum\_gammas_M \cdot \gamma_i}{C\_iM\_gamm\_i} - \gamma_i \right).$$