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1) The Mandelstam variables can be expressed as :

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \tag{1}$$

$$t = (p_1 - p_3)^2 = (p_{2-} - p_4)^2 \tag{2}$$

$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2 \tag{3}$$

In the centre of mass frame, how can s be re-written? What are the implications of s?

1i) For the process $1 + 2 \to 3 + 4$, show that $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$

2) From Fermi's Golden Rule:

$$\Gamma_{fi} = 2\pi |T_{fi}|^2 \rho(E_i) \tag{4}$$

what do Γ_{fi} and $\rho(E_i)$ represent?

2i) Show that $\rho(E_i)$ can be re-written as:

$$\rho(E_i) = \frac{dn}{dE} = \frac{dn}{dp}|\frac{dp}{dE}| \tag{5}$$

where $\frac{dn}{dp} = \frac{4\pi p^2}{(2\pi)^3}V$

2ii) The wavefunction normalisation of one particle per unit volume is not Lorentz invariant since it only applies to a particular frame of reference. For a general process, $a + b \to 1 + 2$, the Lorentz invariant matrix element, using wavefunctions with a Lorentz-invariant normalisation is defined as:

$$M_{fi} =< \psi_1^{'} \psi_2^{'} .. | \bar{H}^{'} | \psi_a^{'} \psi_b^{'} .. > \tag{6}$$

The Lorentz-invariant normalisation of the wavefunctions is defined as:

$$\frac{d^3\vec{p}}{(2\pi)^3 2E} \tag{7}$$

Prove this term is Lorentz-invariant by considering a Lorentz transformation alog the *z-axis*

3) For the decay $a \rightarrow 1 + 2$, show that the momenta of both daughter particles in the c.o.m frame, p^*, are:

$$p^* = \frac{1}{2m_a}\sqrt{[(m_a^2 - (m_1 + m_2)^2][m_a^2 - (m_1 - m_2)^2]} \tag{8}$$

4) In the context of interaction cross sections, what is flux defined as? 4i) Expand on this defintion to construct an equation for the interaction rate. 4ii) What is the significance of the σ experimentally?

5) The Lorentz invariant flux factor, $F = 4E_aE_b(v_a + v_b)$ is known as the Lorentz-invariant flux factor. Show that F is Lorentz invariant.