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1) Using the properties of the γ matrices, and the definiton of $\gamma^5 \equiv \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$, show that:

$$(\gamma^5)^2 = 1 \tag{1}$$

$$\gamma^{5\dagger} = \gamma^5 \tag{2}$$

$$\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5 \tag{3}$$

2) Show that the chiral projection operators, $P_R = \frac{1}{2}(1+\gamma^5)$ and $P_L = \frac{1}{2}(1-\gamma^5)$ satisy:

$$P_R + P_L = 1 \tag{4}$$

$$P_R P_R = P_R \tag{5}$$

$$P_L P_L = P_L \tag{6}$$

$$P_L P_R = 0 \tag{7}$$

3) Write the lowest-order Feynman diagram for $e^+e^- \to \mu^-\mu^+$. 3i) Write some high-order Feynman diagrams.

4) Discuss why high-energy electron-positron colliders must also have high instantaneuous luminosity.

BONUS Q) For a spin-1 system, the eigenstate of the operator $\hat{S_n} = \mathbf{n}.\hat{\mathbf{S}}$ with eigenvalue $+1$ corresponds to the spin being in the direction $\hat{\mathbf{n}}$. Writing this state in terms of the eigenstates of $\hat{S_z}$ i.e.

$$|1,+1>_\theta = \alpha|1,-1> + \beta|1,0> + \gamma|1,+1> \tag{8}$$

and taking $\mathbf{n} = (\sin\theta, \cos\theta)$ show that

$$|1,+1>_\theta = \frac{1}{2}(1-cos\theta)|1,-1> + \frac{1}{\sqrt{2}}sin\theta|1,0> + \frac{1}{2}(1+cos\theta)|1,+1> \tag{9}$$

Hint, write $\hat{S_x}$ in terms of the spin ladder operators.