

Questions 3rd March 2017

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1) Consider a plane-wave solution:

$$\phi(t, \vec{x}) = N e^{-i(Et - \vec{p} \cdot \vec{x})} \quad (1)$$

Show that it *positive* and *negative* energy solutions
How can we interpret this?

2) By considering the three cases where $\mu = \nu = 0$, $\mu = \nu \neq 0$, and $\mu \neq \nu$, show that:

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \quad (2)$$

3) Prove that the angular momentum operator $\hat{\mathbf{L}}$ does not commute with the Hamiltonian of the Dirac equation:

$$\hat{H}_D = \alpha \cdot \hat{\mathbf{p}} + \beta m \quad (3)$$

4) Verify that the helicity operator

$$\hat{h} = \frac{\hat{\Sigma} \cdot \hat{\mathbf{p}}}{2p} = \frac{1}{2p} \begin{pmatrix} \hat{\sigma} \cdot \hat{\mathbf{p}} & 0 \\ 0 & \hat{\sigma} \cdot \hat{\mathbf{p}} \end{pmatrix} \quad (4)$$

commutes with the Dirac Hamiltonian of (2)

4i) What the *Helicity* operator represent?

4ii) Why is *Helicity* a good quantum number for **massless** particles?

4ii) What does a *Parity* transformation correspond to?