DATA 604 Discussion 2: Queueing Theory

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Problem Setup

For an $M/M/1$ queue with mean interarrival time of 1.25 minutes and mean service time of 1 minute, $\lambda = 1/1.25 = 0.8$ arrivals per minute and $\mu = 1/E(S) = 1/1 = 1$ service per minute.

Mathematical Solution

Using the equations provided in the text,

$$\rho = \frac{\lambda}{c\mu} = \frac{0.8}{1 \times 1} = 0.8$$

The utilization is 0.8 – the single server is occupied 80% of the time.

$$L = \frac{\rho}{1 - \rho} = \frac{0.8}{1 - 0.8} = 4$$

For the steady-state system, the time weighted average number of entities in the system is 4.

$$W = \frac{L}{\lambda} = \frac{4}{0.8} = 5$$

The average time in the system is 5 minutes.

$$W_q = W - \frac{1}{\mu} = 5 - \frac{1}{1} = 4$$

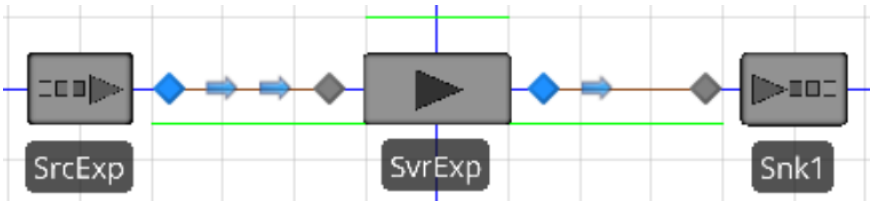
The average time in the queue is 4 minutes.

$$L_q = \lambda W_q = 0.8 \times 4 = 3.2$$

There are an average of 3.2 entities in the queue.

Simulation Solution

The below model was created



with the following parameters (using minutes as the time unit):

- SrcExp:
  - Arrival Logic: Interarrival Time
  - Interarrival Time: `Random.Exponential(1.25)`
- SvrExp:
  - Initial Capacity: 1
  - Ranking Rule: First In First Out
  - Processing Time: `Random.Exponential(1)`

The results of a simulation of 240 hours produced the following results; the screenshot indicates which simulated value corresponds to which calculated value:

Object Type	Object Name	Data Source	Category	Data Item	Statistic	Average Total	
ModelEntity	DefaultEntity	[Population]	Content	NumberInSystem	Average	4.1060	L
			FlowTime	TimeInSystem	Average (Minutes)	5.1508	W
Server	SvrExp	[Resource]	Capacity	UnitsUtilized	Average	0.8030	P
		InputBuffer	Content	NumberInStation	Average	3.2602	L_q
			HoldingTime	TimeInStation	Average (Minutes)	4.0895	W_q

These results are quite similar to the mathematical results calculated above.