DATA 604 Discussion 6: Random Variate Generation

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$$F(x) = \begin{cases} 0 & x \leq -3 \\ \frac{1}{2} + \frac{x}{6} & -3 < x \leq 0 \\ \frac{1}{2} + \frac{x^2}{32} & 0 < x \leq 4 \\ 1 & x > 4 \end{cases}$$

The first and fourth sections, setting $F(X) = R$ is trivial, yielding $R = 0$ and $R = 1$, respectively. For the function in the range $-3 < x \leq 4$ yields

$$\begin{array}{llll} R = \frac{1}{2} + \frac{X}{6} & \rightarrow & X + 3 = 6R & \rightarrow & X = 6R - 3 \\ R = \frac{1}{2} + \frac{X^2}{32} & \rightarrow & X^2 + 16 = 32R & \rightarrow & X = 4\sqrt{2R - 1} \end{array}$$

For the first of these sections, $-3 < x \leq 0$ equates to $0 < R \leq \frac{1}{2}$. For the second of these sections, $0 < x \leq 4$ equates to $\frac{1}{2} < R \leq 1$. This gives the final generator

$$X = \begin{cases} 6R - 3 & 0 < R \leq \frac{1}{2} \\ 4\sqrt{2R - 1} & \frac{1}{2} < R \leq 1 \end{cases}$$

The `runif` function in R can now be used to generate 1000 random samples in the interval $[0, 1]$ from the given distribution:

```
R <- runif(1000)
X <- ifelse(R < 0.5, 6 * R - 3, 4 * sqrt(2 * R - 1))
```

Histogram of 1000 generated samples