DATA 604 Assignment 4: First Simio Models

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Chapter 2, Problem 9

A deterministic distribution D has a constant value d – this can be represented as having a mean $\mu = d$ with a standard deviation of zero $\sigma = 0$.

The utilization can be calculated simply using

$$\rho = \frac{\lambda}{\mu} = \frac{\lambda}{d}$$

Using the formula for an $M/G/1$ system (where the general distribution G is in this case the deterministic distribution D):

$$W_q = \frac{\lambda\left(\sigma^2 + \frac{1}{\mu}\right)}{2\left(1 - \frac{\lambda}{\mu}\right)} = \frac{\lambda\left(0^2 + \frac{1}{d}\right)}{2\left(1 - \frac{\lambda}{d}\right)} = \frac{\lambda}{2d - 2\lambda}$$

Using Little's Law, the remaining metrics can be calculated:

$$L_q = \lambda W_q = \frac{\lambda^2}{2d - 2\lambda}$$

$$W = W_q + E(S) = \frac{\lambda}{2d - 2\lambda} + \frac{1}{d}$$

$$L = \lambda W = \frac{\lambda^2}{2d - 2\lambda} + \frac{\lambda}{d}$$

For these results to be valid, the constant service rate must not be less than the interarrival rate (i.e. $d \geq \lambda$) to avoid utilization greater than 1 or negative queue lengths or times.

If the service time distribution were replaced with one with an equal mean, but with some variation (i.e. $\mu = d$; $\sigma > 0$), the utilization would remain the same, as it is based only on λ and μ.

However, W_q would increase:

$$W_q = \frac{\lambda\left(\sigma^2 + \frac{1}{\mu}\right)}{2\left(1 - \frac{\lambda}{\mu}\right)} = \frac{\sigma^2\lambda d + \lambda}{2d - 2\lambda}$$

This value has increased by $\sigma^2\lambda d/(2d - 2\lambda)$. The relationships laid out by Little's Law necessitate that each of the remaining metrics would also increase.

Chapter 4, Problem 15

Expected Values

With $\lambda = 1$ and $d = 1/0.9 \approx 1.111$, the expected values for the deterministic distribution D are

$$\rho = \frac{\lambda}{d} = \frac{1}{1/0.9} = 0.9$$

$$W_q = \frac{\lambda}{2d - 2\lambda} = \frac{1}{2 \times 1.111 - 2} = 4.5$$

$$L_q = \frac{\lambda^2}{2d - 2\lambda} = \frac{1^2}{2 \times 1.111 - 2} = 4.5$$

$$W = \frac{\lambda}{2d - 2\lambda} + \frac{1}{d} = \frac{1}{2 \times 1.111 - 2} + 0.9 = 5.4$$

$$L = \frac{\lambda^2}{2d - 2\lambda} + \frac{\lambda}{d} = \frac{1^2}{2 \times 1.111 - 2} + 0.9 = 5.4$$

Simulated Values

An $M/D/1$ model is set up with the following parameters:

- Source with interarrival time `Random.Exponential(1)`
- Server with processing time `0.9` (times are given as *services per minute* not *minutes per service*)

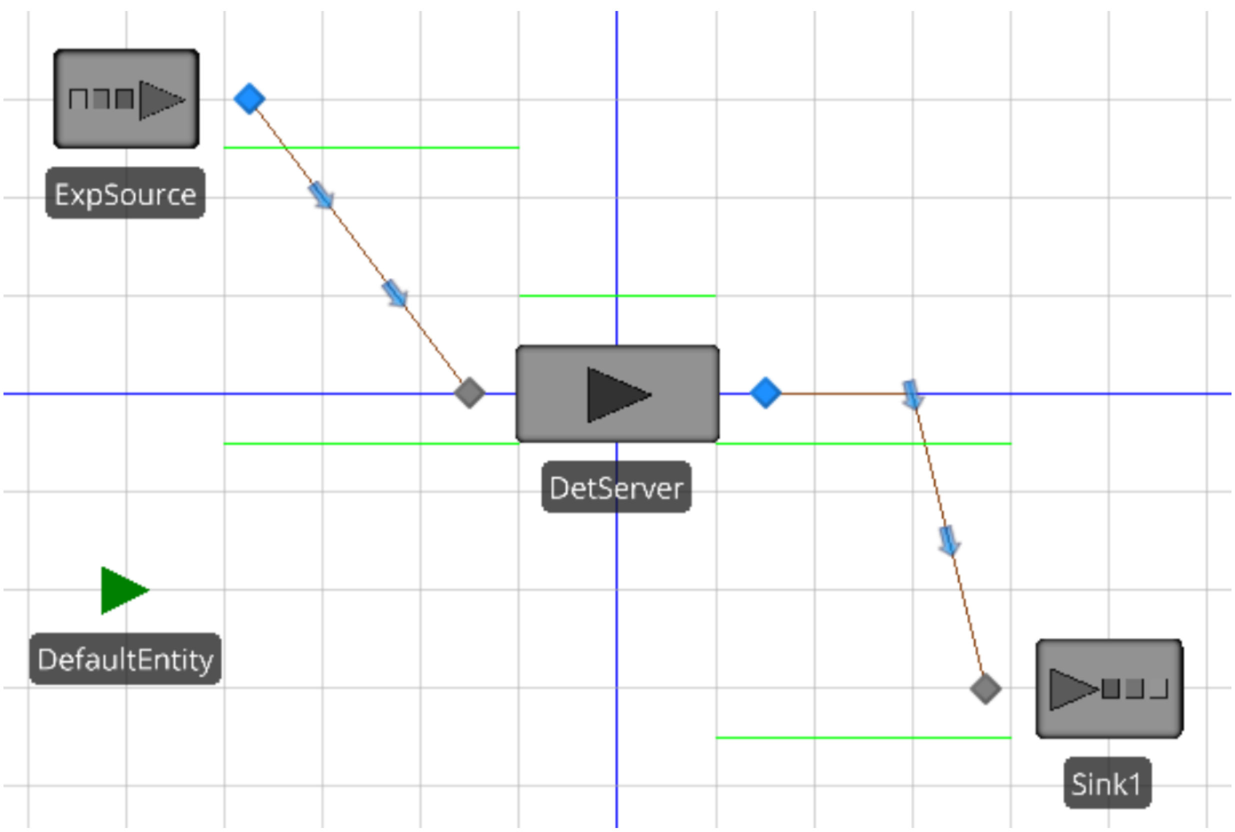

Figure 1:

An experiment is run using this model with the following parameters:

- 250 replications
- 30 hour run time
- 20 hour warm-up period
- 5 responses:
  - ρ = `DetServer.ResourceState.PercentTime(1)`
  - W_q = `DetServer.InputBuffer.Contents.AverageTimeWaiting`
  - L_q = `DetServer.InputBuffer.Contents.AverageNumberWaiting`
  - W = `DefaultEntity.Population.TimeInSystem.Average`
  - L = `DefaultEntity.Population.NumberInSystem.Average`

This experiment produces the following results:

Metric	Queueing	Simulated
ρ	0.9	0.903 ± 0.005
W_q	4.5	4.190 ± 0.293
L_q	4.5	4.274 ± 0.327
W	5.4	5.089 ± 0.293
L	5.4	5.177 ± 0.331

Each of these confidence intervals includes the estimated mean for the metric, so it can be said with strong confidence that the model is verified.

Model with Variation

A separate model is created for an $M/G/1$ system with the following parameters

- Source with interarrival time `Random.Exponential(1)`
- Server with processing time `Random.Normal(0.9, 0.2)` *normal distribution with same mean*

An experiment is run using this model with the same conditions and responses outlined above. The results of this experiment are presented below:

Metric	Simulated
ρ	0.898 ± 0.004
W_q	4.073 ± 0.264
L_q	4.111 ± 0.285
W	4.974 ± 0.264
L	5.009 ± 0.288

In the above discussion, it was theorized that the introduction would raise the time in queue W_q by $\sigma^2\lambda d/(2d - 2\lambda)$ (in this case 0.2) and a corresponding increase in all other metrics besides ρ. However, the results generated by the simulation showed a decrease in each of these metrics – this result is a bit confusing and may indicate the need for tweaking of the experimental parameters.