DATA 604 Discussion 9: Variance Reduction

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The inverse cdf $F^{-1}(u)$ can be obtained by integrating $f(x)$, setting the result equal to u, and solving for x:

$$f(x) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}$$
$$F(x) = \int f(x)dx = -e^{-x^2/2\sigma^2} = u$$
$$F^{-1}(u) = \sigma\sqrt{-2\ln(u)}$$

For a given σ, the value can be estimated using antithetic variables. When generating m samples, X represents values generated from $\frac{m}{2}$ samples u from the uniform distribution, and X' represents values generated from the antithetic sample $u' = 1 - u$. X_2 represents a second set of values generated from $\frac{m}{2}$ samples u_2.

```
antithetic <- function(m, sigma = 1) {
  # get first set of random numbers & variates
  u <- runif(m / 2)
  x <- sigma * sqrt(-2 * log(u))
  # get antithetic numbers & variates
  uprime <- 1 - u
  xprime <- sigma * sqrt(-2 * log(uprime))
  # get second set of numbers & variates
  u2 <- runif(m / 2)
  x2 <- sigma * sqrt(-2 * log(u2))
  # get variances of (x + xprime) / 2 and (x + x2) / 2
  var_prime <- (var(x) + var(xprime) + 2 * cov(x, xprime)) / 4
  var_2 <- (var(x) + var(x2) + 2 * cov(x, x2)) / 4
  # return pecent variance reduction
  return((var_2 - var_prime) / var_2)
}
```

Running this function for $m = 10000$ 100 times, the average variance reduction is about 95%.