

# The Design Structure Matrix (DSM)

## Advanced Numerical DSM techniques

### Eigen-Structure Analysis

One method by which DSMs may be analyzed is by analogy to dynamic linear systems.

Let  $u$  be a vector of remaining work on each task in a design process, and let  $A$  be a numerical DSM. Since a DSM shows the relationship between task rework, we can assume that work completed in the  $(t+1)$ st iteration is a linear function of work completed in the  $(t)$ th iteration, with the linear weights being the numerical values in the DSM. Therefore work in the  $(t+1)$ st iteration is  $A^*u$ .

At time zero the initial work remaining on all tasks is 1. (There is 100% of work remaining for every task.) Therefore  $u_0$  is a vector of 1s.  $u_0$  is known as the initial work vector. After the first iteration the work remaining is  $A^*u_0$ . After the second iteration the work remaining is  $A^*A^*u_0$ . After the  $n$ -th iteration the work remaining is  $A^n^*u_0$ . Each of these terms is known as a work vector.

If we sum up all of the work vectors we obtain the total work completed during a design process. This sum can be calculated directly using  $\text{inv}(I-A)^*u_0$ . (The sum only exists if the largest eigenvalue of the matrix  $A$  is less than unity.)

Perhaps more interesting than calculating the total work is looking at the eigenstructure of the matrix  $A$ . The eigenstructure refers to the eigenvalues and eigenvectors of  $A$ , which are important ways to characterize any square matrix. Eigenvalues and eigenvectors are very useful in understanding the behavior of linear dynamic systems. Each eigenvalue is associated with an eigenvector. For more information about the calculation of eigenvalues and eigenvectors consult a text on linear algebra.

Each eigenvalue tell us about the convergence rate of a dynamic mode. We need to pay attention to modes that have slow convergence rates, since these will dominate the iterative behavior and time taken. The slow convergence rate modes are those that have large magnitude eigenvalue.

Since eigenvectors are associated with eigenvalues, the eigenvectors associated with the large-magnitude eigenvalues are of particular interest. The entries that are heavily weighted in these eigenvectors describe those tasks that will be the primary components of the slowly-converging iterations. From a management perspective these are interesting since this tells us about which are the most important of the iterative, coupled tasks. These are the tasks that will have to be rework a significant number of times.

Of course, the argument above mixes linear algebra with design management, which is an unusual combination. Nevertheless, the eigenstructure approach is simple to apply and powerful in its ability to make useful insights.

For further information about and an application of the eigenstructure approach, compare *Smith, R. P., Eppinger, S. D.: Identifying controlling features of engineering design iteration, Manage. Sci., vol. 43, pp. 276-293, 1997.*

