How do we represent the meaning of a word?

Definition: **meaning** (Webster dictionary)

- the idea that is represented by a word, phrase, etc.
- the idea that a person wants to express by using words, signs, etc.
- the idea that is expressed in a work of writing, art, etc.

Commonest linguistic way of thinking of meaning:

signifier (symbol) ⟺ signified (idea or thing)

= denotational semantics

tree ⟺ {, , , …}

How do we have usable meaning in a computer?

Previously commonest NLP solution: Use, e.g., WordNet, a thesaurus containing lists of **synonym sets** and **hypernyms** (“is a” relationships)

e.g., synonym sets containing “good”:

```
from nltk.corpus import wordnet as wn
poses = { 'n':'noun', 'v':'verb', 's':'adj (s)', 'a':'adj', 'r':'adv'}
for synset in wn.synsets("good"):
    print("{}: {}".format(poses[synset.pos()],
          ", ".join([l.name() for l in synset.lemmas()])))
```

```
noun: good
noun: good, goodness
noun: good, goodness
noun: commodity, trade_good, good
adj: good
adj (sat): full, good
adj: good
adj (sat): estimable, good, honorable, respectable
adj (sat): beneficial, good
adj (sat): good
adj (sat): good, just, upright
…
adverb: well, good
adverb: thoroughly, soundly, good
```

e.g., hypernyms of “panda”:

```
from nltk.corpus import wordnet as wn
panda = wn.synset("panda.n.01")
hyper = lambda s: s.hypernyms()
list(panda.closure(hyper))
```

```
[Synset('procyonid.n.01'),
Synset('carnivore.n.01'),
Synset('placental.n.01'),
Synset('mammal.n.01'),
Synset('vertebrate.n.01'),
Synset('chordate.n.01'),
Synset('animal.n.01'),
Synset('organism.n.01'),
Synset('living_thing.n.01'),
Synset('whole.n.02'),
Synset('object.n.01'),
Synset('physical_entity.n.01'),
Synset('entity.n.01')]
```


Problems with resources like WordNet

- A useful resource but missing nuance:
  - e.g., “proficient” is listed as a synonym for “good”
    This is only correct in some contexts
  - Also, WordNet list offensive synonyms in some synonym sets without any coverage of the connotations or appropriateness of words
- Missing new meanings of words:
  - e.g., wicked, badass, nifty, wizard, genius, ninja, bombest
  - Impossible to keep up-to-date!
- Subjective
- Requires human labor to create and adapt
- Can’t be used to accurately compute word similarity (see following slides)

Representing words as discrete symbols

In traditional NLP, we regard words as discrete symbols:
hotel, conference, motel – a localist representation

Means one 1, the rest 0s

Such symbols for words can be represented by one-hot vectors:

motel = [0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]

hotel = [0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]

Vector dimension = number of words in vocabulary (e.g., 500,000+)

Problem with words as discrete symbols

Example: in web search, if a user searches for "Seattle motel", we would like to match documents containing "Seattle hotel"

But:

motel = [0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
hotel = [0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]

These two vectors are orthogonal

There is no natural notion of **similarity** for one-hot vectors!

Solution:

- Could try to rely on WordNet's list of synonyms to get similarity?
  - But it is well-known to fail badly: incompleteness, etc.
- **Instead: learn to encode similarity in the vectors themselves**

Representing words by their context

- Distributional semantics: **A word's meaning is given by the words that frequently appear close-by**
  - *"You shall know a word by the company it keeps"* (J. R. Firth 1957: 11)
  - One of the most successful ideas of modern statistical NLP!
- When a word *w* appears in a text, its **context** is the set of words that appear nearby (within a fixed-size window).
- We use the many contexts of *w* to build up a representation of *w*

…government debt problems turning into ***banking*** *crises as happened in 2009…*
…saying that Europe needs unified ***banking*** *regulation to replace the hodgepodge…*
…India has just given its ***banking*** *system a shot in the arm…*

These context words will represent ***banking***

Word vectors

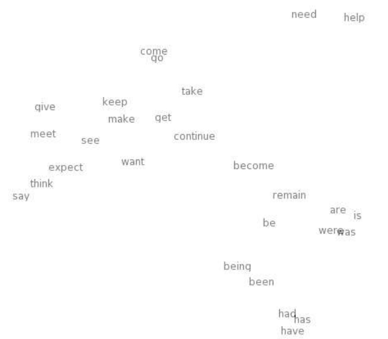
We will build a dense vector for each word, chosen so that it is similar to vectors of words that appear in similar contexts, measuring similarity as the vector dot (scalar) product

$$banking = \begin{pmatrix} 0.286 \\ 0.792 \\ -0.177 \\ -0.107 \\ 0.109 \\ -0.542 \\ 0.349 \\ 0.271 \end{pmatrix} \qquad monetary = \begin{pmatrix} 0.413 \\ 0.582 \\ -0.007 \\ 0.247 \\ 0.216 \\ -0.718 \\ 0.147 \\ 0.051 \end{pmatrix}$$

Note: word vectors are also called (word) embeddings or (neural) word representations
They are a distributed representation

Word meaning as a neural word vector – visualization

$$expect = \begin{pmatrix} 0.286 \\ 0.792 \\ -0.177 \\ -0.107 \\ 0.109 \\ -0.542 \\ 0.349 \\ 0.271 \\ 0.487 \end{pmatrix}$$



3. Word2vec: Overview

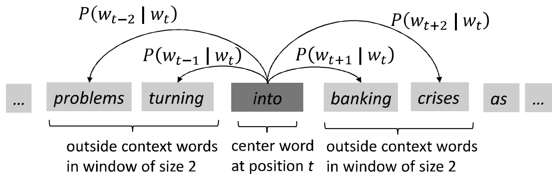
Word2vec (Mikolov et al. 2013) is a framework for learning word vectors

Idea:

- We have a large corpus ("body") of text: a long list of words
- Every word in a fixed vocabulary is represented by a vector
- Go through each position *t* in the text, which has a center word *c* and context ("outside") words *o*
- Use the similarity of the word vectors for *c* and *o* to calculate the probability of *o* given *c* (or vice versa)
- Keep adjusting the word vectors to maximize this probability

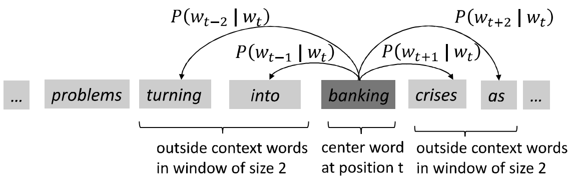
Word2Vec Overview

Example windows and process for computing $P(w_{t+j} \mid w_t)$



Word2Vec Overview

Example windows and process for computing $P(w_{t+j} \mid w_t)$



Word2vec: objective function

For each position $t = 1, \ldots, T$, predict context words within a window of fixed size *m*, given center word w_t. Data likelihood:

$$\text{Likelihood} = L(\theta) = \prod_{t=1}^{T} \prod_{\substack{-m \le j \le m \\ j \ne 0}} P(w_{t+j} \mid w_t; \theta)$$

θ is all variables to be optimized

sometimes called a *cost* or *loss* function

The objective function $J(\theta)$ is the (average) negative log likelihood:

$$J(\theta) = -\frac{1}{T} \log L(\theta) = -\frac{1}{T} \sum_{t=1}^{T} \sum_{\substack{-m \le j \le m \\ j \ne 0}} \log P(w_{t+j} \mid w_t; \theta)$$

Minimizing objective function ⟺ Maximizing predictive accuracy

Word2vec: objective function

- We want to minimize the objective function:

$$J(\theta) = -\frac{1}{T}\sum_{t=1}^{T}\sum_{\substack{-m\le j\le m \\ j\ne 0}} \log P\left(w_{t+j} \mid w_t; \theta\right)$$

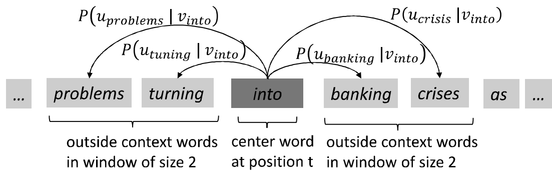
- **Question:** How to calculate $P\left(w_{t+j} \mid w_t; \theta\right)$?
- **Answer:** We will *use two* vectors per word *w*:
  - v_w when *w* is a center word
  - u_w when *w* is a context word
- Then for a center word *c* and a context word *o*:

$$P(o|c) = \frac{\exp(u_o^T v_c)}{\sum_{w\in V}\exp(u_w^T v_c)}$$

Word2Vec with Vectors

- Example windows and process for computing $P\left(w_{t+j} \mid w_t\right)$
- $P\left(u_{problems} \mid v_{into}\right)$ short for $\mathrm{P}\left(problems \mid into \,; u_{problems}, v_{into}, \theta\right)$

All words vectors θ appear in denominator



Word2vec: prediction function

② Exponentiation makes anything positive

① Dot product compares similarity of *o* and *c*.
$u^T v = u.v = \sum_{i=1}^{n} u_i v_i$
Larger dot product = larger probability

$$P(o|c) = \frac{\exp(\boxed{u_o^T v_c})}{\boxed{\sum_{w\in V}\exp(u_w^T v_c)}}$$

③ Normalize over entire vocabulary to give probability distribution

- This is an example of the **softmax function** $\mathbb{R}^n \to (0,1)^n$ Open region

$$\mathrm{softmax}(x_i) = \frac{\exp(x_i)}{\sum_{j=1}^{n}\exp(x_j)} = p_i$$

- The softmax function maps arbitrary values x_i to a probability distribution p_i
  - "max" because amplifies probability of largest x_i
  - "soft" because still assigns some probability to smaller x_i
  - Frequently used in Deep Learning

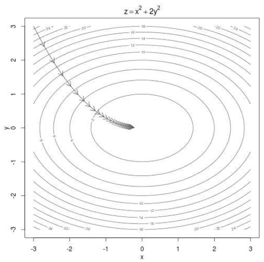
But sort of a weird name because it returns a distribution!

To train the model: Optimize value of parameters to minimize loss

To train a model, we gradually adjust parameters to minimize a loss

- Recall: θ represents **all** the model parameters, in one long vector
- In our case, with *d*-dimensional vectors and *V*-many words, we have →
- Remember: every word has two vectors

$$\theta = \begin{bmatrix} v_{aardvark} \\ v_a \\ \vdots \\ v_{zebra} \\ u_{aardvark} \\ u_a \\ \vdots \\ u_{zebra} \end{bmatrix} \in \mathbb{R}^{2dV}$$



- We optimize these parameters by walking down the gradient (see right figure)
- We compute all vector gradients!

4. Objective Function

Maximize $J'(\theta) = \prod_{t=1}^{T} \prod_{\substack{-m \le j \le m \\ j \neq 0}} p(w'_{t+j} \mid w_t; \theta)$

Or minimize ave. neg. log likelihood

$$J(\theta) = -\frac{1}{T} \sum_{t=1}^{T} \sum_{\substack{-m \le j \le m \\ j \neq 0}} \log p(w'_{t+j} \mid w_t)$$

[negate to minimize; log is monotone]

Text length

window size

where

$$p(o \mid c) = \frac{\exp(u_o^T v_c)}{\sum_{w=1}^{V} \exp(u_w^T v_c)}$$

word IDs

Each word type (vocab entry) has two word representations: as center word and context word

We now take derivatives to work out minimum

$$\frac{\partial}{\partial v_c} \log \frac{\exp(u_o^T v_c)}{\sum_{w=1}^{V} \exp(u_w v_c)}$$

$$= \underbrace{\frac{\partial}{\partial v_c} \log \exp(u_o^T v_c)}_{1} - \underbrace{\frac{\partial}{\partial v_c} \log \sum_{w=1}^{V} \exp(u_w^T v_c)}_{2}$$

1 $\frac{\partial}{\partial v_c} \underbrace{\log \exp}_{\text{inverses}} (u_o^T v_c) = \frac{\partial}{\partial v_c} u_o^T v_c = u_o$

Vector! Not high school single variable calculus

You can do things one variable at a time, and this may be helpful when things get gnarly.

$$\forall j \quad \frac{\partial}{\partial (v_c)_j} u_o^T v_c = \frac{\partial}{\partial (v_c)_j} \sum_{i=1}^{d} (u_o)_i (v_c)_i = (u_o)_j$$

Each term is zero except when $i = j$

2 $\frac{\partial}{\partial v_c} \underbrace{\log}_{f} \underbrace{\sum_{w=1}^{V} \exp(u_w^T v_c)}_{z = g(v_c)}$

$$= \underbrace{\frac{1}{\sum_{w=1}^{V} \exp(u_w^T v_c)}} \cdot \underbrace{\frac{\partial}{\partial v_c} \sum_{x=1}^{V} \exp(u_x^T v_c)}$$

← Important to change index

$\frac{\partial}{\partial v_c} f(\overbrace{g(v_c)}^{z}) = \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial v_c}$ Use chain rule

$$= \frac{1}{\sum_{w=1}^{V} \exp(u_w^T v_c)} \cdot \left(\sum_{x=1}^{V} \frac{\partial}{\partial v_c} \underbrace{\exp}_{f} (\underbrace{u_x^T v_c}_{z = g(v_c)}) \right)$$

Move deriv inside sum

$$\left(\sum_{x=1}^{V} \exp(u_x^T v_c) \frac{\partial}{\partial v_c} u_x^T v_c \right)$$

Chain rule

$$\left(\sum_{x=1}^{V} \exp(u_x^T v_c)\, u_x \right)$$

$$\frac{\partial}{\partial v_c} \log(p(o \mid c)) = u_o - \frac{1}{\sum_{w=1}^{V} \exp(u_w^T v_c)} \cdot \left(\sum_{x=1}^{V} \exp(u_x^T v_c)\, u_x \right)$$

$$= u_o - \sum_{x=1}^{V} \frac{\exp(u_x^T v_c)}{\sum_{w=1}^{V} \exp(u_w^T v_c)} u_x$$

Distribute term across sum

$$= u_o - \underbrace{\sum_{x=1}^{V} p(x \mid c)\, u_x}$$

This is an expectation: average over all context vectors weighted by their probability

= observed − expected

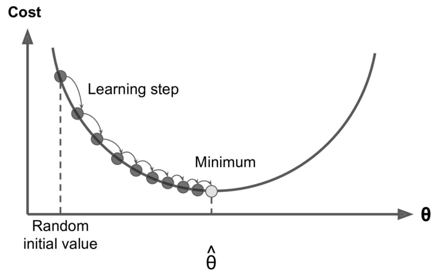
This is just the derivatives for the center vector parameters

Also need derivatives for output vector parameters (they're similar)

Then we have derivative w.r.t. all parameters and can minimize

5. Optimization: Gradient Descent

- We have a cost function $J(\theta)$ we want to minimize
- **Gradient Descent** is an algorithm to minimize $J(\theta)$
- **Idea:** for current value of θ, calculate gradient of $J(\theta)$, then take small step in direction of negative gradient. Repeat.



Note: Our objectives may not be convex like this ☹

But life turns out to be okay ☺

Gradient Descent

- Update equation (in matrix notation):

$$\theta^{new} = \theta^{old} - \alpha \nabla_\theta J(\theta)$$

α = step size or *learning rate*

- Update equation (for single parameter):

$$\theta_j^{new} = \theta_j^{old} - \alpha \frac{\partial}{\partial \theta_j^{old}} J(\theta)$$

- Algorithm:

```
while True:
    theta_grad = evaluate_gradient(J,corpus,theta)
    theta = theta - alpha * theta_grad
```


Stochastic Gradient Descent

- **Problem**: $J(\theta)$ is a function of **all** windows in the corpus (potentially billions!)
  - So $\nabla_\theta J(\theta)$ is very expensive to compute
- You would wait a very long time before making a single update!

- **Very** bad idea for pretty much all neural nets!
- **Solution: Stochastic gradient descent (SGD)**
  - Repeatedly sample windows, and update after each one
- Algorithm:

```
while True:
    window = sample_window(corpus)
    theta_grad = evaluate_gradient(J,window,theta)
    theta = theta - alpha * theta_grad
```


Lecture Plan

1. The course (10 mins)
2. Human language and word meaning (15 mins)
3. Word2vec introduction (15 mins)
4. Word2vec objective function gradients (25 mins)
5. Optimization basics (5 mins)
6. Looking at word vectors (10 mins or less)
   - See Jupyter Notebook