2.1.

a) $u(m,n) = (m+n)^3$

$$u(m,n)\,\delta(m-1,n-2) = (1+2)^3\,\delta(m-1,n-2)$$
$$= 27\,\delta(m-1,n-2)$$

$$u(m,n) \circledast \delta(m-1,n-2) = \sum\sum u(m-m', n-n')\,\delta(m'-1, n'-2)$$

Let $m-m' = \ell$, $n-n' = k$

$$= \sum\sum u(\ell, k)\,\delta(m-1-\ell, n-2-k)$$
$$= u(m-1, n-2)$$
$$= (m-1+n-2)^3$$
$$= (m+n-3)^3$$

b) $f(x,y) = (x+y)^3$

$$f(x,y)\,\delta(x-1,y-2) = 27\,\delta(x-1,y-2)$$
$$f(x,y) \circledast \delta(x-1,y-2) = f(x-1,y-2)$$
$$= (x+y-3)^3$$

c) $$\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{\pm jn\theta}\,d\theta = \frac{1}{2\pi}\left(\pm\frac{1}{jn}\right)e^{\pm jn\theta}\Big|_{-\pi}^{\pi}$$

$$= \pm\frac{1}{2\pi jn}\left(e^{\pm jn\pi} - e^{\mp jn\pi}\right)$$

$$= \frac{\sin n\pi}{n\pi} = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$= \delta(n)$$

2.2.

a) $$h(m,n) \circledast u(m,n) = \sum\sum_{m',n'=-\infty}^{\infty} h(m-m', n-n')\,u(m',n')$$

Let $k = m-m'$, $\ell = n-n'$

then $m' = m-k$, $n' = n-\ell$

$$h(m,n) \circledast u(m,n) = \sum_{m-k,\, n-\ell=-\infty}^{\infty}\sum h(k,\ell)\, u(m-k,\, n-\ell)$$

m, n fixed

$$= \sum_{k,\ell=-\infty}^{\infty}\sum u(m-k,\, n-\ell)\, h(k,\ell)$$

$$= u(m,n) \circledast h(m,n)$$

b) $h(m,n) \circledast \left[a_1 u_1(m,n) + a_2 u_2(m,n)\right]$

$$= \sum_{m',n'=-\infty}^{\infty}\sum h(m-m',\, n-n')\left[a_1 u_1(m',n') + a_2 u_2(m',n')\right]$$

$$= \sum_{m',n'-\infty}^{\infty}\sum \left[a_1 h(m-m',\, n-n')\, u_1(m',n') + a_2 h(m-m',\, n-n')\, u_2(m',n')\right]$$

$$= a_1 \sum_{m',n'=-\infty}^{\infty}\sum h(m-m',\, n-n')\, u_1(m',n') + a_2 \sum_{m',n'=-\infty}^{\infty}\sum h(m-m',\, n-n')\, u_2(m',n')$$

$$= a_1\left[h(m,n) \circledast u_1(m,n)\right] + a_2\left[h(m,n) \circledast u_2(m,n)\right]$$

c) $h(m,n) \circledast u(m-m_0,\, n-n_0)$

$$= \sum_{m',n'=-\infty}^{\infty}\sum h(m-m',\, n-n')\, u(m'-m_0,\, n'-n_0)$$

put $k = m'-m_0$, $\ell = n'-n_0$

$$= \sum_{k,\ell=-\infty}^{\infty}\sum h\left((m-m_0)-k,\, (n-n_0)-\ell\right) u(k,\ell)$$

$$= h(m-m_0,\, n-n_0) \circledast u(m,n)$$

$\underline{\underline{d}}$ $h(m,n) \circledast [u_1(m,n) \circledast u_2(m,n)]$

$$= h(m,n) \circledast \sum_{m',n'=-\infty}^{\infty}\sum u_1(m-m', n-n')\, u_2(m',n')$$

$$= \sum_{m',n'=-\infty}^{\infty}\sum h(m,n) \circledast u_1(m-m', n-n')\, u_2(m',n')$$

from property $\underline{\underline{b}}$

$$= \sum_{m',n'=-\infty}^{\infty}\sum \sum_{k,\ell=-\infty}^{\infty}\sum h(m-k, n-\ell)\, u_1(k-m', \ell-n')\, u_2(m',n')$$

$$= \sum_{m',n'=-\infty}^{\infty}\sum \left[h(m-m', n-n') \circledast u_1(m',n')\right] u_2(m',n')$$

$$= [h(m,n) \circledast u_1(m,n)] \circledast u_2(m,n)$$

$\underline{\underline{e}}$ $h(m,n) \circledast \delta(m,n) = \sum_{m',n'=-\infty}^{\infty}\sum h(m-m', n-n')\, \delta(m',n')$

$$= \begin{cases} h(m-0, n-0) & , m'=n'=0 \\ 0 & , \text{otherwise} \end{cases}$$

$$= h(m,n)$$

$\underline{\underline{f}}$ $\sum_{m,n=-\infty}^{\infty}\sum v(m,n) = \sum_{m,n=-\infty}^{\infty}\sum \sum_{m',n'=-\infty}^{\infty}\sum h(m-m', n-n')\, u(m',n')$

$$= \sum_{m',n'=-\infty}^{\infty}\sum \left[\sum_{m,n=-\infty}^{\infty}\sum h(m-m', n-n')\right] u(m',n')$$

$$= \left[\sum_{m,n=-\infty}^{\infty}\sum h(m,n)\right]\left[\sum_{m',n'=-\infty}^{\infty}\sum u(m',n')\right]$$

2.3.

a) $$h(x,y) \circledast u(x,y) = \iint_{-\infty}^{\infty} h(x-x', y-y')\, u(x',y')\, dx'dy'$$

Let $\omega = x - x'$, $z = y - y'$

then $x' = x - \omega$, $y' = y - z$

$$= \iint_{-\infty}^{\infty} h(\omega, z)\, u(x-\omega, y-z)\, d\omega\, dz$$

$$= u(x,y) \circledast h(x,y)$$

b) $$h(x,y) \circledast [a_1 u_1(x,y) + a_2 u_2(x,y)]$$

$$= \iint_{-\infty}^{\infty} h(x-x', y-y')[a_1 u_1(x',y') + a_2 u_2(x',y')]\, dx'dy'$$

$$= a_1 \iint_{-\infty}^{\infty} h(x-x', y-y')\, u_1(x',y')\, dx'dy' + a_2 \iint_{-\infty}^{\infty} h(x-x', y-y')\, u_2(x',y')\, dx'dy'$$

$$= a_1 h(x,y) \circledast u_1(x,y) + a_2 h(x,y) \circledast u_2(x,y)$$

c) $$h(x,y) \circledast u(x-x_0, y-y_0)$$

$$= \iint_{-\infty}^{\infty} h(x-x', y-y')\, u(x'-x_0, y'-y_0)\, dx'dy'$$

Let $\omega = x' - x_0$, $z = y' - y_0$

$$= \iint_{-\infty}^{\infty} h(x-x_0-\omega, y-y_0-z)\, u(\omega, z)\, d\omega\, dz$$

$$= h(x-x_0, y-y_0) \circledast u(x,y)$$

d. $h(x,y) \circledast [u_1(x,y) \circledast u_2(x,y)]$

$$= h(x,y) \circledast \iint_{-\infty}^{\infty} u_1(x-x', y-y')\, u_2(x', y')\, dx'dy'$$

$$= \iint_{-\infty}^{\infty} h(x-x'', y-y'') \left[\iint_{-\infty}^{\infty} u_1(x''-x', y''-y')\, u_2(x', y')\, dx'dy' \right] dx''dy''$$

Let $w = x''-x'$, $z = y''-y'$

$$= \iint_{-\infty}^{\infty} \left[\iint_{-\infty}^{\infty} h(x-w-x', y-z-y')\, u_1(w, z)\, dw\, dz \right] u_2(x', y')\, dx'dy'$$

$$= \iint_{-\infty}^{\infty} \left[h(x-x', y-y') \circledast u_1(x-x', y-y') \right] u_2(x', y')\, dx'dy'$$

$$= \left[h(x,y) \circledast u_1(x,y) \right] \circledast u_2(x,y)$$

e. $h(x,y) \circledast \delta(x,y) = \iint_{-\infty}^{\infty} h(x-x', y-y')\, \delta(x', y')\, dx'dy'$

$$= \iint_{-\infty}^{\infty} h(x', y')\, \delta(x-x', y-y')\, dx'dy'$$

$$= \iint_{-\infty}^{\infty} h(x,y)\, \delta(x-x', y-y')\, dx'dy'$$

$$= h(x,y) \iint_{-\infty}^{\infty} \delta(x-x', y-y')\, dx'dy'$$

$= h(x,y)$ area of δ Function $= 1$

f $\iint_{-\infty}^{\infty} v(x,y)\,dx\,dy = \iint_{-\infty}^{\infty}\iint_{-\infty}^{\infty} h(x-x', y-y')\,u(x', y')\,dx'\,dy'\,dx\,dy$

$$= \iint_{-\infty}^{\infty}\left[\iint_{-\infty}^{\infty} h(x-x', y-y')\,dx\,dy\right]u(x', y')\,dx'\,dy'$$

$$= \iint_{-\infty}^{\infty}\left[\iint_{-\infty}^{\infty} h(x,y)\,dx\,dy\right]u(x', y')\,dx'\,dy'$$

$$= \left[\iint_{-\infty}^{\infty} h(x,y)\,dx\,dy\right]\left[\iint_{-\infty}^{\infty} u(x', y')\,dx'\,dy'\right]$$

2.4.

a $y(m,n) = 3x(m,n) + 9$

$h(m,n;m',n') = 3\,\delta(m-m', n-n') + 9$

Non linear (super position fails)

shift invariant

IIR

b $y(m,n) = m^2n^2\,x(m,n)$

$h(m,n;m',n') = m^2n^2\,\delta(m-m', n-n')$

Linear

~~Shift invariant~~ Variant ?

FIR

c $y(m,n) = \sum_{m'=-\infty}^{m}\sum_{n'=-\infty}^{n} x(m', n')$

$h(m,n;m'',n'') = \sum_{m',n'=-\infty}^{m}\sum^{n} \delta(m'-m'', n'-n'')$

$= \text{step}(m-m'', n-n'')$

Linear
shift invariant
IIR

d. $y(m,n) = x(m-m_0, n-n_0)$

$h(m,n; m',n') = \delta(m-m'-m_0, n-n'-n_0)$

Linear
shift invariant
FIR

e. $y(m,n) = \exp\left[-|x(m,n)|\right]$

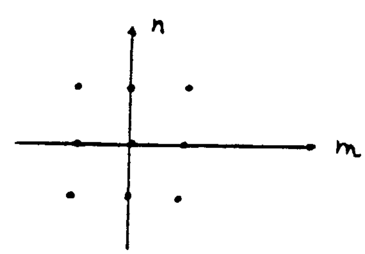
$h(m,n; m',n') = \exp\left[-|\delta(m-m', n-n')|\right]$

Non Linear
shift invariant
IIR $\left(e^0 = 1 \quad e^{-1} = 1/e \quad \text{Infinite region of support}\right)$

f. $y(m,n) = \sum_{m',n'=-1}^{1}\sum x(m',n')$

$h(m,n; m'',n'') = \sum_{m',n'=-1}^{1}\sum \delta(m'-m'', n'-n'')$

Linear
shift ~~invariant~~ variant ?
FIR



Finite region of support

g. $y(m,n) = \sum_{m',n'=0}^{M-1}\sum^{N-1} x(m',n')\, e^{-j\frac{2\pi m m'}{M}}\, e^{-j\frac{2\pi n n'}{N}}$

$$h(m,n;m'',n'') = \sum_{m',n'=0}^{M-1}\sum^{N-1} \delta(m'-m'',n'-n'')\, e^{\frac{-j2\pi m m'}{M}}\, e^{\frac{-j2\pi n n'}{N}}$$

$$= \begin{cases} e^{\frac{-j2\pi m^2}{M}}\, e^{\frac{-j2\pi n^2}{N}} & 0 \leqslant m \leqslant M-1 \\ & 0 \leqslant n \leqslant N-1 \\ 0 & \text{otherwise} \end{cases}$$

Linear
shift variant
FIR

2.5.

a

$$x(m,n):\quad \begin{array}{ccc} 1 & 4 & 1 \\ \boxed{2} & 5 & 3 \end{array}$$

$$\sum\sum x(m,n) = 16$$

i.

$$h(m,n):\quad \begin{array}{c|ccc} 0 & -1 & 1 \\ -1 & \boxed{4} & -1 \\ 0 & -1 & 0 \end{array}$$

$$\begin{array}{ccccc} 0 & -1 & -3 & +3 & 1 \\ -1 & -2 & 11 & +2 & +2 \\ -2 & \boxed{2} & 11 & 6 & -3 \\ 0 & -2 & -5 & -3 & 0 \end{array}$$

$$\sum\sum h(m,n) = 1 \qquad\qquad \sum\sum y(m,n) = 16$$

ii.

$$\begin{array}{ccc} 1 & \boxed{2} & 3 \end{array}$$

$$\begin{array}{ccccc} 1 & 6 & 12 & 14 & 3 \\ 2 & \boxed{9} & 16 & 21 & 9 \end{array}$$

$$\sum\sum h(m,n) = 6 \qquad\qquad \sum\sum y(m,n) = 96$$

iii

$$\begin{array}{c} -2 \\ \boxed{3} \\ -1 \end{array} \qquad \begin{array}{ccc} -2 & -8 & -2 \\ -1 & 2 & -3 \\ \boxed{5} & 11 & 8 \\ -2 & -5 & -3 \end{array}$$

$\sum\sum h(m,n) = 0 \qquad \sum\sum y(m,n) = 0$

b// The convolution of two arrays will yield a non zero output value as long as the shift of the two arrays with respect to each other has at least a single overlapping element.

This is true in the m direction for $M_1 + M_2 - 1$ and in the n direction for $N_1 + N_2 - 1$

b// Another approach

$h(m,n) \quad , \quad 0 \leq m \leq M_1 - 1 \quad , \quad 0 \leq n \leq N_1 - 1$

$x(m,n) \quad , \quad 0 \leq m \leq M_2 - 1 \quad , \quad 0 \leq n \leq N_2 - 1$

$y(m,n) = x(m,n) \circledast h(m,n)$

$Y(z_1, z_2) = X(z_1, z_2)\, H(z_1, z_2)$

Both $H(z_1, z_2)$ and $X(z_1, z_2)$ are polynomials of finite degree in z_1^{-1} and z_2^{-1}. The highest and lowest degree of (z_1^{-1}) in $H(z_1 - z_2)$ are 0 and $M_1 - 1$ respectively. For $X(z_1, z_2)$, these quantities are 0 and $M_2 - 1$. Since $Y(z_1, z_2)$ is the product of $H(z_1, z_2)$ and $X(z_1, z_2)$, then the lowest and highest degree of (z_1^{-1}) in $Y(z_1, z_2)$ will be the sum of those of $X(z_1, z_2)$ and $H(z_1, z_2)$ i.e. 0 and $M_1 + M_2 - 2$

A similar argument holds for powers of (z_2^{-1})

$\therefore$ $y(m,n)$ will be a finite array which is non zero for

$0 \leq m \leq M_1 + M_2 - 2 \quad , \quad 0 \leq n \leq N_1 + N_2 - 2$

2.6

$$g(x,y) = h(x,y) \circledast f(x,y)$$

$$G(\xi_1,\xi_2) = \iint_{-\infty}^{\infty} h(x,y) \circledast f(x,y)\, e^{-j2\pi(x\xi_1+y\xi_2)}\,dx\,dy$$

$$= \iint_{-\infty}^{\infty}\left[\iint_{-\infty}^{\infty} h(x-x',y-y')f(x',y')\,dx'dy'\right] e^{-j2\pi(x\xi_1+y\xi_2)}\,dx\,dy$$

$$= \iint_{-\infty}^{\infty} f(x',y')\left[\iint_{-\infty}^{\infty} h(x-x',y-y')\, e^{-j2\pi(x\xi_1+y\xi_2)}\,dx\,dy\right]dx'dy'$$

Let $x'' = x - x'$, $y'' = y - y'$
then $x = x'' + x'$, $y = y'' + y'$

$$= \iint_{-\infty}^{\infty} f(x',y')\, e^{-j2\pi(x'\xi_1+y'\xi_2)}\left[\iint_{-\infty}^{\infty} h(x'',y'')\, e^{-j2\pi(x''\xi_1+y''\xi_2)}\,dx\,dy\right]dx'dy'$$

$$= H(\xi_1,\xi_2)\; F(\xi_1,\xi_2)$$

Following the same steps we can prove the correlation relation with the substitution $x'' = x + x'$
and $y'' = y + y'$

2.7

$$\mathcal{F}\left[\delta(x,y)\right] = \iint_{-\infty}^{\infty} \delta(x,y)\, e^{-j2\pi(\xi_1 x + \xi_2 y)}\,dx\,dy$$

$= e^0$ using the sifting property of the δ function

$= 1$

$$\mathcal{F}\left[\delta(x \pm x_0, y \pm y_0)\right] = \iint_{-\infty}^{\infty} \delta(x \pm x_0, y \pm y_0)\, e^{-j2\pi(\xi_1 x + \xi_2 y)}\, dx\,dy$$

$$= e^{-j2\pi(\mp \xi_1 x_0 \mp \xi_2 y_0)}$$

$$= e^{\pm j2\pi x_0 \xi_1}\, e^{\pm j2\pi y_0 \xi_2}$$

$$\mathcal{F}\left\{e^{\pm j2\pi x \eta_1} e^{\pm j2\pi y \eta_2}\right\} = \iint_{-\infty}^{\infty} e^{\pm j2\pi(x\eta_1 + y\eta_2)}\, e^{-j2\pi(x\xi_1 + y\xi_2)}\, dx\,dy$$

$$= \iint_{-\infty}^{\infty} e^{-j2\pi\left[x(\xi_1 \mp \eta_1) + y(\xi_2 \mp \eta_2)\right]}\, dx\,dy$$

$$= \delta(\xi_1 \mp \eta_1, \xi_2 \mp \eta_2)$$

$$\mathcal{F}\left[e^{-\pi(x^2+y^2)}\right] = \iint_{-\infty}^{\infty} e^{-\pi(x^2+y^2)}\, e^{-j2\pi(\xi_1 x + \xi_2 y)}\, dx\,dy$$

Multiply by $e^{-\pi(\xi_1^2 + \xi_2^2)}\, e^{\pi(\xi_1^2 + \xi_2^2)}$

$$= e^{-\pi(\xi_1^2 + \xi_2^2)} \iint_{-\infty}^{\infty} e^{-\pi(x^2 + 2j\xi_1 x - \xi_1^2)}\, e^{-\pi(y^2 + 2j\xi_2 y - \xi_2^2)}\, dx\,dy$$

$$= e^{-\pi(\xi_1^2 + \xi_2^2)} \underbrace{\iint_{-\infty}^{\infty} e^{-\pi(x + j\xi_1)^2}\, e^{-\pi(y + j\xi_2)^2}\, dx\,dy}_{=1}$$

(Let $u = \sqrt{\pi}\,(x + j\xi_1)$ and $v = \sqrt{\pi}\,(y + j\xi_2)$)

$$= e^{-\pi(\xi_1^2 + \xi_2^2)}$$

$$\mathcal{F}\left\{\mathrm{rect}(x,y)\right\} = \iint_{-\infty}^{\infty} \mathrm{rect}(x,y)\, e^{-j2\pi(\xi_1 x + \xi_2 y)}\, dx\,dy$$

$$= \iint_{-1/2}^{1/2} e^{-j2\pi(\xi_1 x + \xi_2 y)} \, dx\, dy$$

$$= \frac{1}{-j2\pi\xi_1} \cdot \frac{1}{-j2\pi\xi_2} \, e^{-j2\pi\xi_1 x}\Big|_{-1/2}^{1/2} \, e^{-j2\pi\xi_2 y}\Big|_{-1/2}^{1/2}$$

$$= \frac{1}{j2\pi\xi_1}\left(e^{-j\pi\xi_1} - e^{j\pi\xi_1}\right) \cdot \frac{1}{j2\pi\xi_2}\left(e^{-j\pi\xi_2} - e^{j\pi\xi_2}\right)$$

$$= \frac{\sin \pi\xi_1}{\pi\xi_1} \cdot \frac{\sin \pi\xi_2}{\pi\xi_2}$$

$$= \mathrm{Sinc}(\xi_1, \xi_2)$$

$$\mathcal{F}\{\mathrm{tri}(x,y)\} = \mathcal{F}\{\mathrm{rect}(x,y) \circledast \mathrm{rect}(x,y)\}$$

$$= \mathcal{F}\{\mathrm{rect}(x,y)\}\, \mathcal{F}\{\mathrm{rect}(x,y)\}$$

$$= \mathrm{Sinc}(\xi_1, \xi_2)\, \mathrm{Sinc}(\xi_1, \xi_2)$$

$$= \mathrm{Sinc}^2(\xi_1, \xi_2)$$

$$\mathcal{F}\{\mathrm{comb}(x,y)\} = \mathcal{F}\Big\{\sum_{m,n=-\infty}^{\infty}\sum \delta(x-n, y-m).\Big\}$$

$\Updownarrow$ expand in a series

$$= \mathcal{F}\Big\{\sum_{m,n=-\infty}^{\infty}\sum e^{j2\pi(nx+my)}\Big\}$$

$$= \sum_{m,n=-\infty}^{\infty}\sum \mathcal{F}\Big\{e^{j2\pi(nx+my)}\Big\}$$

$$= \sum_{m,n=-\infty}^{\infty}\sum \delta(\xi_1 - n, \xi_2 - m)$$

$$\mathcal{F}\left[\mathrm{comb}(x, y)\right] = \mathrm{comb}\,(\xi_1, \xi_2)$$

$$\sin(2\pi x \eta_1)\cos(2\pi y \eta_2)$$

$$= \left(\frac{e^{j2\pi x \eta_1} - e^{-j2\pi x \eta_1}}{2j}\right)\left(\frac{e^{j2\pi y \eta_2} + e^{-j2\pi y \eta_2}}{2}\right)$$

$$= \frac{1}{4j}\left[e^{j2\pi(x\eta_1 + y\eta_2)} + e^{j2\pi(x\eta_1 - y\eta_2)} - e^{j2\pi(-x\eta_1 + y\eta_2)} - e^{j2\pi(-x\eta_1 - y\eta_2)}\right]$$

Fourier transform

$$= \frac{1}{4j}\left[\delta(\xi_1 - \eta_1, \xi_2 - \eta_2) + \delta(\xi_1 - \eta_1, \xi_2 + \eta_2) - \delta(\xi_1 + \eta_1, \xi_2 - \eta_2) - \delta(\xi_1 + \eta_1, \xi_2 + \eta_2)\right]$$

$$\cos 2\pi(x\eta_1 + y\eta_2)$$

$$= \frac{1}{2}\left[e^{j2\pi(x\eta_1 + y\eta_2)} + e^{-j2\pi(x\eta_1 + y\eta_2)}\right]$$

Fourier transform

$$= \frac{1}{2}\left[\delta(\xi_1 + \eta_1, \xi_2 + \eta_2) + \delta(\xi_1 - \eta_1, \xi_2 - \eta_2)\right]$$

2.8

Separability :

$$\mathcal{F}\left[x_1(m)\,x_2(n)\right] = \sum_{m,n=-\infty}^{\infty}\sum x_1(m)\,x_2(n)\,e^{-j(m\omega_1 + n\omega_2)}$$

$$= \sum_{m=-\infty}^{\infty} x_1(m)\,e^{-jm\omega_1} \sum_{n=-\infty}^{\infty} x_2(n)\,e^{-jn\omega_2}$$

$$= X_1(\omega_1)\, X_2(\omega_2)$$

Shifting :

$$\mathcal{F}\left\{x(m \pm m_0, n \pm n_0)\right\} = \sum_{m,n=-\infty}^{\infty}\sum x(m \pm m_0, n \pm n_0)\, e^{-j(m\omega_1 + n\omega_2)}$$

Let $m' = m \pm m_0$, $n' = n \pm n_0$

$$= \sum_{m',n'=-\infty}^{\infty}\sum x(m', n')\, e^{-j(m'\omega_1 + n'\omega_2) \pm j(m_0\omega_1 + n_0\omega_2)}$$

$$= e^{\pm j(m_0\omega_1 + n_0\omega_2)} \sum_{m',n'=-\infty}^{\infty}\sum x(m', n')\, e^{-j(m'\omega_1 + n'\omega_2)}$$

$$= e^{\pm j(m_0\omega_1 + n_0\omega_2)}\, X(\omega_1, \omega_2)$$

Modulation :

$$\mathcal{F}\left[e^{\pm j(\omega_{01} m + \omega_{02} n)}\, x(m, n)\right]$$

$$= \sum_{m,n=-\infty}^{\infty}\sum x(m, n)\, e^{-j(m(\omega_1 \mp \omega_{01}) + n(\omega_2 \mp \omega_{02}))}$$

$$= X(\omega_1 \mp \omega_{01},\ \omega_2 \mp \omega_{02})$$

Linearity :

$$\mathcal{F}\left\{a_1 x_1(m,n) + a_2 x_2(m,n)\right\}$$

$$= \sum_{m,n=-\infty}^{\infty}\sum \left[a_1 x_1(m,n) + a_2 x_2(m,n)\right] e^{-j(m\omega_1 + n\omega_2)}$$

$$= a_1 \sum_{m,n=-\infty}^{\infty}\sum x_1(m,n)\, e^{-j(m\omega_1 + n\omega_2)} + a_2 \sum_{m,n=-\infty}^{\infty}\sum x_2(m,n)\, e^{-j(m\omega_1 + n\omega_2)}$$