KEY

EE 5356 – Spring 2016

MID EXAM – I (Solution)

2/18/16

1. Entropy (H) is the minimum theoritical bit rate at which a group of N samples can be coded and is given by

5 marks

$$\text{Entropy } (H) = -\sum_{i=1}^{N} p(a_i) \log_2 p(a_i) \quad \text{bits/symbol}$$

N = # of symbols

$p(a_i)$ = probability of symbol a_i

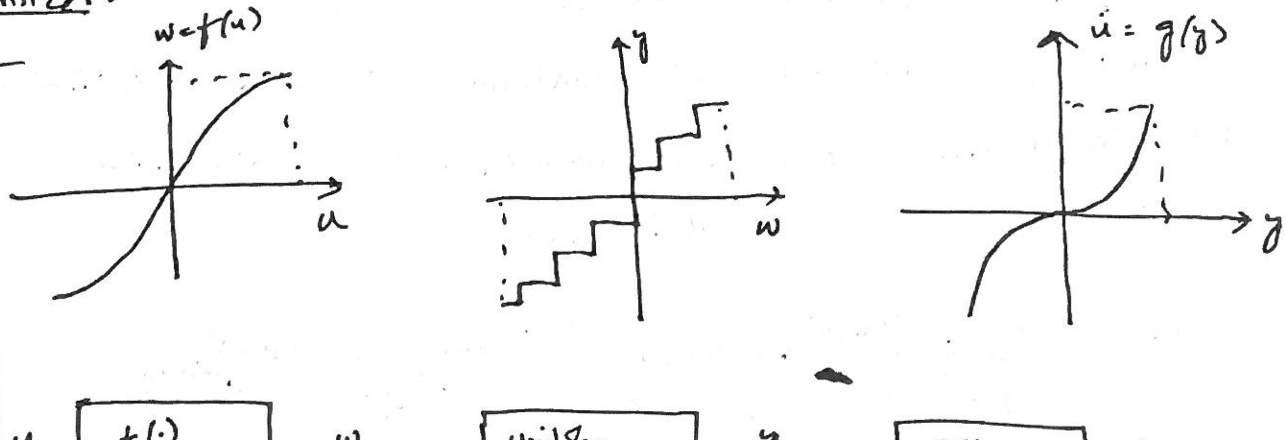
(Also) Entropy of a source is defined as the average information generated by the source

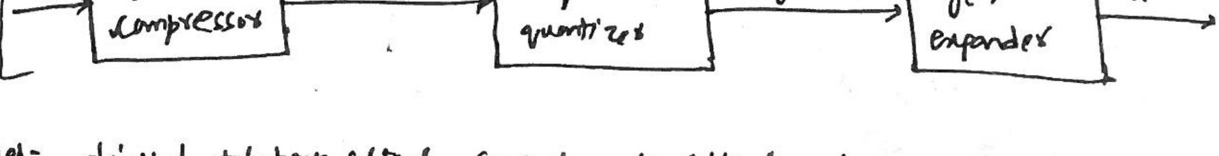
2. Compandor (compressor – expander) is a uniform quantizer preceded and succeeded by non linear transformations.

The input random variable u is first passed through a non linear memoryless transformation $f(\cdot)$ to yeild another random variable w. This random variable is uniformly quantized to give $y \in \{y_i\}$, which is non linearly transformed by $g(\cdot)$ to give output u'. The overall ~~output~~ transformation from u to u' is a non-uniform quantizer.

3 marks

5 marks





Examples: digital telephony systems, concert audio systems etc.

2 marks

μ law compander; North America & Japan

A law compander: Rest of the World

3. <u>Halftone image generation</u>: Halftone images are binary images that give a gray scale rendition.

7 marks

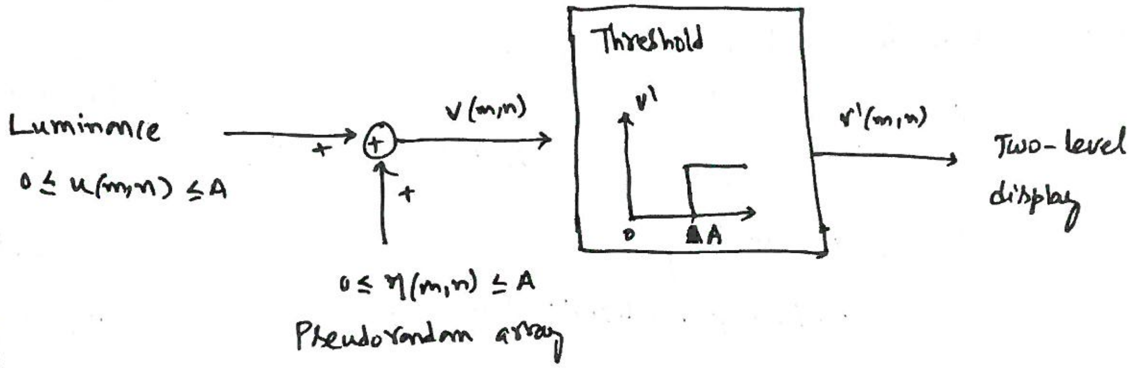


Fig. Digital halftone generation.

3 marks For each image sample, (representing a luminance value) a random number (halftone screen) is added, and the resulting signal is quantized by a 1-bit quantizer. The output (0 & 1) then represents a black & white dot. The gray level rendition in half tones is due to local spatial averaging performed by the eye.

4. <u>Pseudorandom Noise Quantization (Dithering)</u>:

→ 3 marks

To ~~suppress~~ <u>suppress contouring effects</u>, a small amount of uniformly distributed pseudorandom noise is added to the luminance samples before quantization. This " " " also called dither.

5 marks

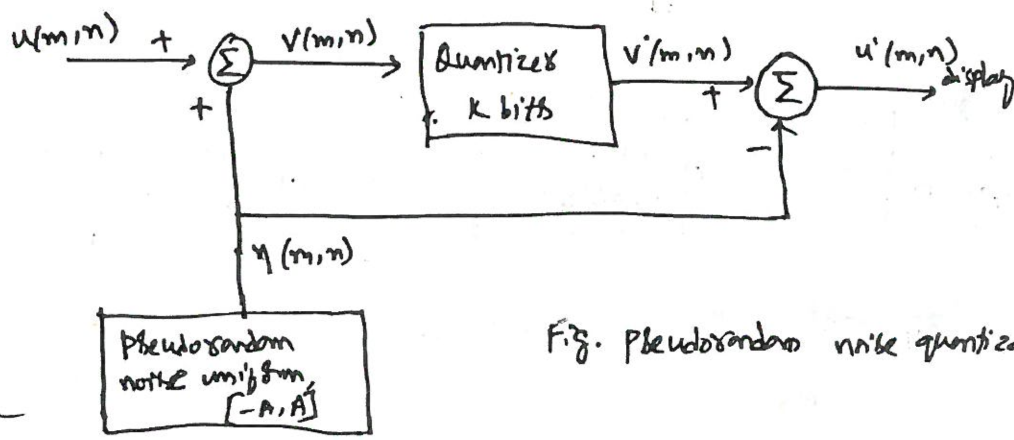


Fig. Pseudorandom noise quantization.

To display the image, the same (or another) pseudorandom sequence is subtracted from the quantizer output. The effect is that in the regions of low-luminance gradients (which are the regions of contours), the input noise causes the pixels to go above & below the original decision level, thereby breaking the contours. However, the average value of the quantized pixels is about the same with and without the additive noise. During display, the noise tends to fill in the regions of contours in such a way that the spatial average is unchanged. The amount of dither added should be kept small enough to maintain the spatial resolution but large enough to allow the luminance values to vary randomly about the quantizer decision levels. The noise should usually affect the least significant bit of the quantizer. Reasonable image quality is achievable by a 3-bit quantizer. For 8 bit PCM image (0 – 255 levels), A is generally chosen as 16.

2 marks

5. Contrast Quantization:

Since visual sensitivity is nearly uniform to just noticeable changes in contrast, it is more appropriate to quantize the contrast function as shown in fig below.

3 marks

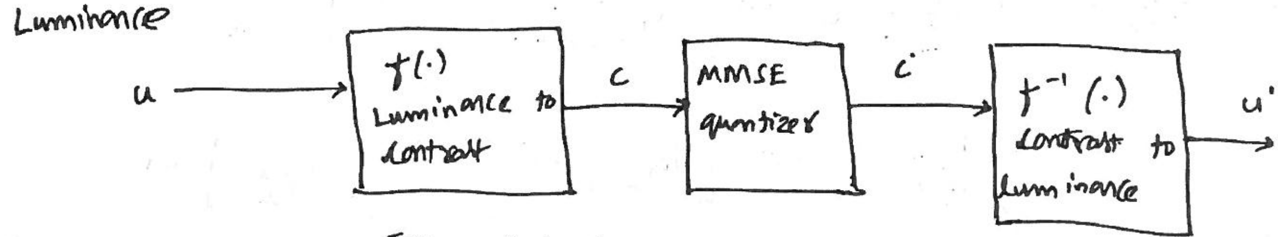


Fig. Contrast quantization

Two non-linear transformations that have been used for representation of contrast c are

$$c = \alpha \ln(1 + \beta u) \qquad 0 \le u \le 1 \qquad \text{— eq I}$$

$$c = \alpha u^{\beta} \qquad \text{— II}$$

where α and β are constants and u represents the luminance.

In equation I, $\alpha = \dfrac{\beta}{\ln(1+\beta)}$, α lying between 6 and 18

In equation II, $\alpha = 1$ and $\beta = 1/3$ have been suggested.

2 marks

For the given contrast representation we simply use the minimum mean square error (MMSE) quantizer for the contrast field. To display (reconstruct) the image, the quantized contrast is transformed back to luminance value by the inverse transformation. Experimental studies indicate that a 2% change in contrast is just noticeable. Therefore, if uniformly quantized, the contrast scale needs 50 levels, or about 6 bits. However, with the optimum mean square quantizer, 4 to 5 bits/pixel could be sufficient.

6. Leibnitz's rule:

10 marks

$$\frac{\partial}{\partial t}\int_{a(t)}^{b(t)} f(x,t)\,dx = \int_{a(t)}^{b(t)} \frac{\partial f(x,t)}{\partial t}\,dx + f(b(t),t)\frac{\partial b(t)}{\partial t} - f(a(t),t)\frac{\partial a(t)}{\partial t}$$

7.

20 marks

$$H(g) = -\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}} \log_2\left[\frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-x^2}{2\sigma^2}}\right] dx$$

$$= -\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-x^2}{2\sigma^2}}\left[\log_2 \frac{1}{\sqrt{2\pi}\,\sigma} - \frac{x^2}{2\sigma^2}\log_2 e\right] dx$$

$$= \log_2(\sqrt{2\pi}\,\sigma)\underbrace{\int_{-\infty}^{\infty}\frac{e^{-x^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma}\,dx}_{=1} + \frac{\log_2 e}{2\sigma^2}\underbrace{\int_{-\infty}^{\infty}\frac{x^2}{\sqrt{2\pi}\,\sigma} e^{\frac{-x^2}{2\sigma^2}}\,dx}_{=\sigma^2}$$

$$= \frac{1}{2}\log_2(2\pi\sigma^2) + \frac{1}{2}\log_2 e$$

$$= \frac{1}{2}\log_2(2\pi e\sigma^2) \quad //$$

8. In minimizing MSE in the region of large Pdf, step size is <u>small</u> and vice versa.

3 marks

Since a uniform quantizer can be easily implemented, it is of interest to know how to best quantize a non uniformly distributed random variable by an L-level uniform quantizer. For simplicity, let $p_u(u)$ be an even function and let L be an even integer.

For a fixed L, the optimum uniform quantizer is determined completely by the quantization step size q. Define

$$2a \triangleq Lq$$

where q has to be determined so that the MSE is minimized. In terms of these parameters,

$$MSQE = \sum_{j=2}^{L-1} \int_{t_j}^{t_{j+1}} (u-r_j)^2 p_u(u)\, du + 2\int_{a-q}^{\infty} (u-r_L)^2 p_u(u)\, du$$

Since $\{t_j, r_j\}$ come from a uniform quantizer, this simplifies to

$$MSQE = \underbrace{2\sum_{j=1}^{L/2-1} \int_{(j-1)q}^{jq} \left(u - \frac{(2j-1)q}{2}\right)^2 p_u(u)\,du}_{\text{Granular noise}} + \underbrace{2\int_{(L/2-1)q}^{\infty} \left(u - \frac{(L-1)q}{2}\right)^2 p_u(u)\,du}_{\text{overload noise}}$$

$\therefore$ MSQE = Granular noise + overload noise

The problem now is to minimize MSQE as a function of q, ie

$$\frac{d\,MSQE}{dq} = 0$$

So, we get

$$\frac{\delta \sigma_q^2}{\delta \Delta} = -\sum_{i=1}^{\frac{L}{2}-1} (2i-1) \int_{(i-1)\Delta}^{i\Delta} \left(u - \frac{2i-1}{2}\Delta\right) p_u(u)\, du$$

$$- (L-1)\int_{(\frac{L}{2}-1)\Delta}^{\infty} \left(u - \frac{L-1}{2}\Delta\right) p_u(u)\, du = 0$$

continued

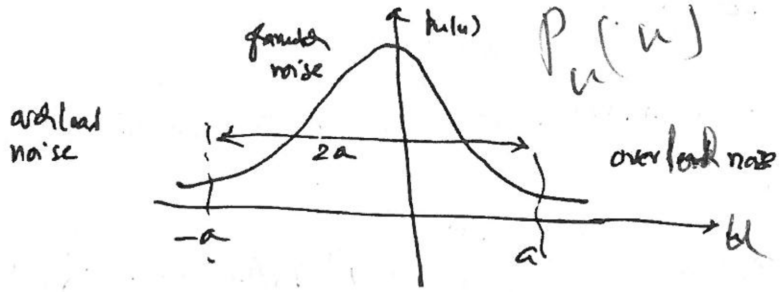
See notes

Forcing uniform quantizer on a non uniform pdf for a given L, increasing step size Δ, increases $\left(\frac{L}{2}-1\right)\Delta$. Hence decrease in overload noise, but increases granular noise

Decreasing step size Δ, decreases $\left(\frac{L}{2}-1\right)\Delta$. Hence increase in overload noise but decreases granular noise.

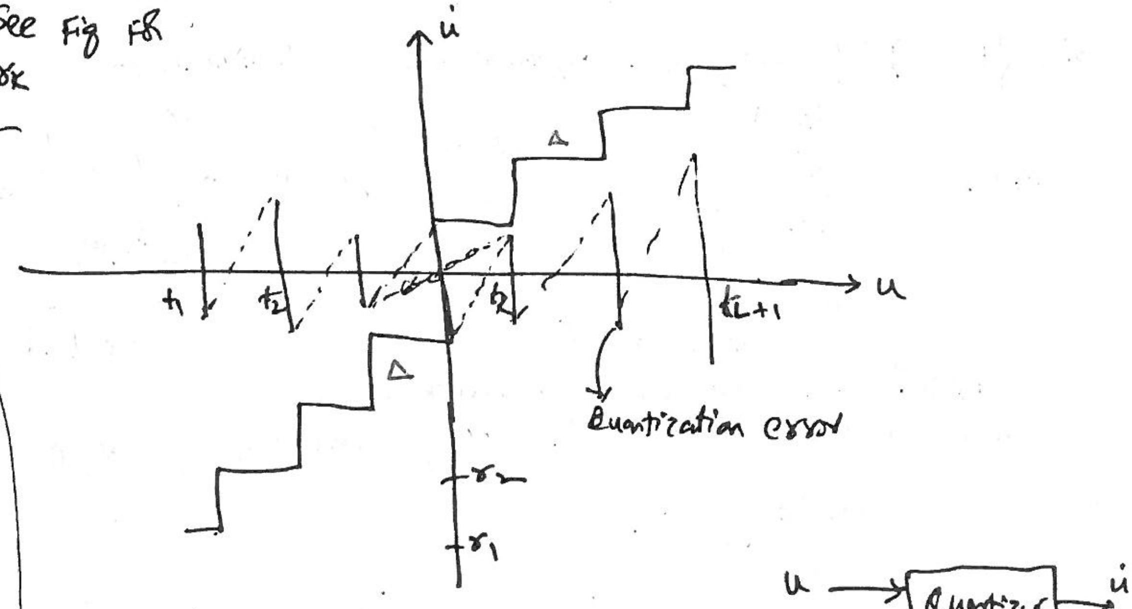
10 marks

∴ Choice of Δ is a balance between overload and granular noise Δ.



→ See Fig for r_k

7 marks



(Uniform quantizer)

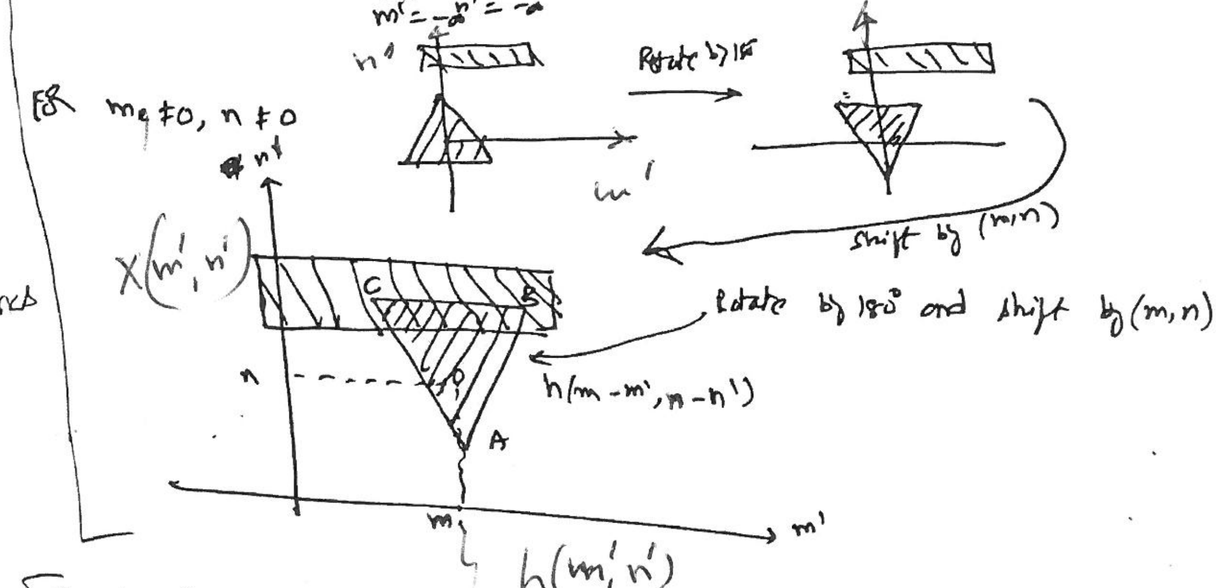
9. For shift invariant systems, given

$$y(m,n) = h(m,n) \circledast x(m,n)$$

$$\triangleq \sum_{m'=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} h(m-m', n-n')\, x(m', n')$$

For $m \neq 0$, $n \neq 0$

7 marks



3 marks

The impulse response ~~rotated~~ array is rotated by 180° about origin and then shifted by (m,n) and overlayed on the array $x(m', n')$.
The sum ~~of~~ of the product of the arrays $\{x(\cdot,\cdot)\}$ and ~~of~~ $\{h(\cdot,\cdot)\}$ in the overlapping regions gives the result at (m,n).

For continuous and discrete case the equations are

Analog $\quad g(x,y) = h(x,y) \circledast f(x,y) \triangleq \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x-x', y-y')\, f(x', y')\, dx'\, dy'$

Digital (Discrete) $\quad y(m,n) = h(m,n) \circledast x(m,n) \triangleq \sum_{m',n'=-\infty}^{\infty} \sum^{\infty} h(m-m', n-n')\, x(m', n')$