Exam 2

CSC/MTH 228

Spring 2020

Instructions: Write all your answers on blank sheets of paper and submit via Blackboard. You do not need to rewrite the questions. Please show all your work clearly and make sure that your scanned answers are legible.

Problem 1. Using modular arithmetic, find:
a. the last digit of 91097^{2024}
b. the last two digits of 3999^{2020}
c. the last two digits of 3999^{2021}
d. the last digit of 91097^{2022}

Problem 2. a. Using the Euclidean Algorithm, find the GCD of 78 and 33.

b. Are 78 and 33 relatively prime? How do you know?

c. Now, write the gcd(78,33) as a linear combination of 78 and 33.

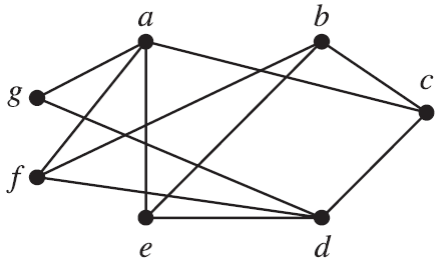
FOR PROBLEMS 3-5, PICK ANY 2 OUT OF 3.

Problem 3. Prove using mathematical induction that for all natural numbers n,
$6 + 6 \cdot 5 + 6 \cdot 5^2 + ... + 6 \cdot 5^n = \frac{6(5^{n+1}-1)}{4}$

Problem 4. Prove using mathematical induction that for all natural numbers n,
$4^n \geq 1 + 3^n$

Problem 5. Prove using mathematical induction that for all natural numbers n,
$f_1 + f_3 + ... + f_{2n+1} = f_{2n+2}$.
Reminder, Fibonacci sequence is defined as $f_1 = 1, f_2 = 1, ..., f_n = f_{n-1} + f_{n-2}$

Problem 6. For the graph G given below, answer the following questions.



a. What is E?

b. What is $|V|$?

c. What is $deg(f)$?

d. What is $N(\{a, d, f\})$?

e. Are a and d adjacent? Justify

f. Is G a simple graph? Justify.

g. Is G a connected graph? JUstify.

Problem 7. Draw a simple undirected graph with the given degree sequence, if possible. Be sure to label the vertices of the graph. If it is not possible, explain why.
a. 2, 2, 2, 3, 3, 3, 3

b. 1, 1, 1, 1, 2, 2, 2, 2

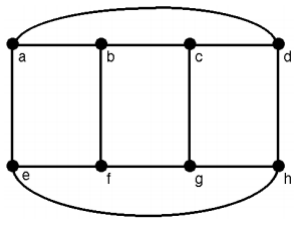
c. 2, 3, 3, 4, 5, 6, 7

d. 4, 4, 4, 4, 4

Problem 8. a. Draw a labeled undirected graph with the given adjacency matrix.

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

b. Write the adjacency matrix for the given graph:



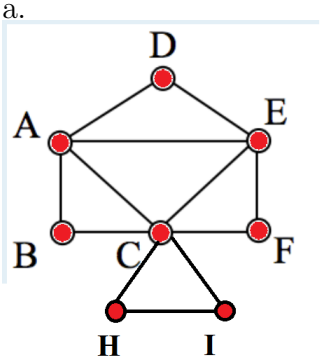
Problem 9. a. Consider the graph that you constructed in in problem 8 part a (i.e. the first part of the previous question). Is the graph bipartite? Justify using 2 coloring. If it is bipartitite, partition the vertices into V_1 and V_2 such that every edge in G has one endpoint

in V_1 and the other endpoint in V_2.

b. Consider the graph in problem 8 part b (i.e. the first part of the previous question). Is the graph bipartite? Justify using 2 coloring. If it is bipartitite, partition the vertices into V_1 and V_2 such that every edge in G has one endpoint in V_1 and the other endpoint in V_2

Problem 10. Determine if the given graph has an Euler circuit. If it does, list the sequence of vertices in the circuit. If not, give reasons to justify why the graph does not contain an Euler circuit.

a.



b.