

Bookcase

Problem category: Sorting, Binary Search, Dynamic Programming

Expected difficulty: 1800

Solution

Consider the following algorithm:

1. Sort the books by width, then by height. Now we can safely ignore the width. Notice that the heights form a sawtooth pattern.
2. Take each book from this series, in order, and add it to the shelf whose last added book is closest in height to, but no larger than, the current one.
3. Once the current book height becomes smaller than the smallest among all shelves so far, we need to create another shelf.

This algorithm is known as the longest increasing subsequence (LIS), in this case using the operator $>$.

Complexity

Since we need to sort the books, as well as perform binary searches, the overall time complexity is $O(n \log n)$.