Piggy on the Track

Problem category: Logic
Expected difficulty: 1000

Solution

First, let's consider the problem of finding Piggy. We can do it in at most $n-1$ queries:

1. Fix a position p and ask the oracle if Piggy is at that position $n-1$ times;
2. If Piggy was not found, then it will be at position p after all those queries.

However, this does not help us solve the original problem, since in the worst case there would be a single query left, and that query alone cannot be used to determine Piggy's direction.

Then let's save one query and see what we can do with it. Suppose that we make $n-2$ queries at position 1. There would be four cases in which Piggy is not found:

Direction	Starting position	Final position
forwards	2	n
forwards	3	1
backwards	$n-1$	1
backwards	n	2

Suppose we now query for position n. If this fails, and Piggy were moving forwards, then it was at position 1 and would be at 2 after the query. So we query for position 2 and, if this succeeds, we can conclude that Piggy is indeed moving forwards.

If Piggy were moving backwards, none of the above queries would find it: in the second-to-last query, it would move from 1 to n or from 2 to 1, neither of which is the last query's position.