Complete the following tasks. Show your work/code where applicable. Assemble your work into one PDF document and upload the PDF back into our CatCourses page.

In the next set of exercises, return to the Bayesian model setting [from the *Bayes Rules!* textbook]

$$\begin{aligned} \mu &\sim \text{N}(0, 1^2) \\ Y|\mu &\sim \text{N}(\mu, 0.75^2) \end{aligned}$$

Assume we observe $Y = 6.25$ and wish to construct a Metropolis-Hastings simulation of the corresponding posterior model for μ.

1. (*Bayes Rules!* exercise 7.9) **(One iteration with a Uniform proposal model)** The function `one_mh_iteration()` from the text utilizes a Uniform proposal model,

   $$\mu^{'}|\mu \sim \text{Unif}(\mu - w, \mu + w)$$

   with half-width $w = 1$. Starting from a current value of $\mu = 3$ and using `set.seed(320)`, run the code below and comment on the returned `proposal`, `alpha`, and `next_stop` values.

   (a) `one_mh_iteration(w = 0.01, current = 3)`

   (b) `one_mh_iteration(w = 0.5, current = 3)`

   (c) `one_mh_iteration(w = 1, current = 3)`

   (d) `one_mh_iteration(w = 3, current = 3)`

2. (*Bayes Rules!* exercise 7.10) **(An entire tour with a Uniform proposal model)** Implement the Metropolis-Hastings function `mh_tour()` defined in Section 7.3 to construct tours of μ under each of the following scenarios. Construct trace plots and histograms for each tour.

   (a) 50 iterations, $w = 50$

   (b) 50 iterations, $w = 0.01$

   (c) 1000 iterations, $w = 50$

   (d) 1000 iterations, $w = 1000$

   (e) Contrast the trace plots in parts a and b. Explain why changing w has this effect.

   (f) Consider the results in parts c and d. Is the w value as important when the number of iterations is much larger?

3. (*Bayes Rules!* exercise 7.11) **(Changing the proposal model)** For this exercise, modify `one_mh_iteration()` to create a new function, `one_mh_iteration_normal()`, which utilizes a symmetric Normal proposal model, centered at the current chain value μ with standard deviation s:

$$\mu^{'}|\mu \sim \text{N}(\mu, s^2)$$

Subsequently, starting from a current value of $\mu = 3$ and `set.seed(320)`, run this function under each setting below. Comment on the returned `proposal`, `alpha`, and `next_stop` values.

   (a) `one_mh_iteration_normal(s = 0.01, current = 3)`

   (b) `one_mh_iteration_normal(s = 0.5, current = 3)`

   (c) `one_mh_iteration_normal(s = 1, current = 3)`

   (d) `one_mh_iteration_normal(s = 3, current = 3)`

4. (*Bayes Rules!* exercise 7.12) **(Metropolis-Hastings tour with Normal proposals)** Upon completing the previous exercise, `modify mh_tour()` to create a new function, `mh_tour_normal()`, which constructs a chain of μ values using a Normal proposal model with standard deviation s. Subsequently, `using set.seed(320)`, run this function under each setting below and construct a trace plot of the chain.

   (a) 20 iterations, $s = 0.01$

   (b) 20 iterations, $s = 10$

   (c) 1000 iterations, $s = 0.01$

   (d) 1000 iterations, $s = 10$

   (e) Contrast the trace plots in a and b. Explain in simple terms why changing the standard deviation of the Normal proposal model causes these differences.

   (f) Reflecting on the above results, tune your Metropolis-Hastings algorithm. That is, identify a reasonable value for standard deviation s and provide a trace plot as evidence.