Complete the following tasks. Show your work/code where applicable. Assemble your work into one PDF document and upload the PDF back into our CatCourses page. In this set of exercises, when asked for a credible interval, use the "middle" approach. For instance, to obtain a 95 percent credible interval, we would refer to the 2.5 and 97.5 percentile locations from the simulated posterior distribution.

1. (*Bayes Rules!* Exercises 8.6 and 8.7) For each situation, find the appropriate credible interval

   (a) a 95 percent credible interval for π with $\pi|y \sim \text{Beta}(4, 5)$

   (b) a 60 percent credible interval for π with $\pi|y \sim \text{Beta}(4, 5)$

   (c) a 95 percent credible interval for λ with $\lambda|y \sim \text{Gamma}(1, 8)$

   (d) a 99 percent credible interval for λ with $\lambda|y \sim \text{Gamma}(1, 5)$

   (e) a 95 percent credible interval for μ with $\mu|y \sim \text{N}(10, 2^2)$

   (f) a 80 percent credible interval for μ with $\mu|y \sim \text{N}(-3, 1^2)$

2. (*Bayes Rules!* Exercises 8.9) For parameter π, suppose you have a Beta(1,0.8) prior model and a Beta(4,3) posterior. You wish to test the null hypothesis that $\pi \leq 0.4$ versus the alternative that $\pi > 0.4$

   (a) What is the posterior probability for the alternative hypothesis?

   (b) Calculate and interpret the Bayes Factor

3. (*Bayes Rules!* Exercises 8.10) For parameter μ, suppose you have a $\text{N}(10, 10^2)$ prior model and a $\text{N}(5, 3^2)$ posterior. You wish to test $H_o : \mu \geq 5.2$ versus $H_a : \mu < 5.2$

   (a) What is the posterior probability for the alternative hypothesis?

   (b) Calculate and interpret the Bayes Factor

4. (*Bayes Rules!* Exercises 8.14) Let π denote the proportion of U.S. adults that do not believe in climate change. To learn about π, we'll use survey data on n adults and count up the number of these that don't believe in climate change, Y

   (a) Explain which Bayesian model is appropriate for this analysis: Beta-Binomial, Gamma-Poisson, or Normal-Normal.

   (b) Specify and discuss your own prior model for π

   (c) For the remainder of the exercise, we'll utilize the authors' Beta(1,2) prior for π. How does your prior understanding differ from that of the authors?

   (d) Using the `pulse_of_the_nation` data from the `bayesrules` package, report the sample proportion of surveyed adults with the opinion that `climate_change` is "Not Real At All"

   (e) In light of the Beta(1,2) prior and data, calculate and interpret a (middle) 95% posterior credible interval for π

5. (*Bayes Rules!* Exercises 8.15) Continuing the analysis from Exercise 8.14, suppose you wish to test a researcher's claim that more than 10% of people believe in climate change: $H_o : \pi \leq 0.1$ versus $H_a : \pi > 0.1$

   (a) What decision might you make about these hypotheses utilizing the credible interval from the previous exercise?

   (b) Calculate and interpret the posterior probability of H_a

   (c) Calculate and interpret the Bayes Factor for your hypothesis test.

6. (*Bayes Rules!* Exercises 8.16) In the next exercises, you'll repeat and build upon your climate change analysis using MCMC simulation.

   (a) Simulate the posterior model of π, the proportion of U.S. adults that do not believe in climate change, with rstan using 4 chains and 10000 iterations per chain.

   (b) Produce and discuss trace plots, overlaid density plots, and autocorrelation plots for the four chains.

   (c) Report the effective sample size ratio and R-hat values for your simulation, explaining what these values mean in context.

7. (*Bayes Rules!* Exercises 8.17)

   (a) Utilize your MCMC simulation to approximate a (middle) 95% posterior credible interval for π

   (b) Utilize your MCMC simulation to approximate the posterior probability that $\pi > 0.1$

8. (*Bayes Rules!* Exercises 8.18)

   (a) Suppose you were to survey 100 more adults. Use your MCMC simulation to approximate the posterior predictive model of Y, the number that don't believe in climate change. Construct a histogram visualization of this model.

   (b) Summarize your observations of the posterior predictive model of Y.

   (c) Approximate the probability that at least 20 of the 100 people don't believe in climate change.

Here are some incomplete and approximate answers.

1. (a) (0.1570, 0.7551)
   (b) (0.3032, 0.5837)
   (c) (0.0032, 0.4611)
   (d) (0.0010, 1.0597)
   (e) (6.0801, 13.9199)
   (f) (-4.2816, -1.7184)
2. (a) 0.8208
   (b) 2.3122
3. (a) 0.5266
   (b) 2.4119
4. (a)
   (b)
   (c)
   (d) 0.15
   (e) (0.1291, 0.1733)
5. (a)
   (b) 0.9999
   (c) 750017.6
6.
7. (a) (0.1291, 0.1730)
   (b) virtually 100 percent
8. (a)
   (b)
   (c) 0.1221