Complete the following tasks. Show your work where applicable. Some answers have been provided. Assemble your work into one PDF document and upload the PDF back into our CatCourses page.

1. **FizzBuzz and the Conditional Probabilities** In this setting, the universal set is the set of natural numbers from 1 to 32

$$\{1, 2, 3, 4, ..., 32\}$$

Let T be the subset of numbers that are divisible by 3 and let F be the subset of numbers that are divisible by 5.

(a) Write out the elements of each of the following sets: F, T, $F \cap T$, $F \cup T$

- $F = \{5, 10, 15, 20, 25, 30\}$
- $T = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30\}$
- $F \cap T = \{15, 30\}$
- $F \cup T = \{3, 5, 6, 9, 10, 12, 15, 18, 20, 21, 24, 25, 27, 30\}$

(b) Compute $P(F \cup T)$

$$\frac{14}{32}$$

(c) Compute $P(F \cap T)$

$$\frac{2}{32}$$

(d) Compute $P(F|T)$

$$P(F|T) = \frac{F \cap T}{P(T)} = \frac{\frac{2}{32}}{\frac{10}{32}} = \frac{2}{10}$$

(e) Compute $P(T|F)$

$$P(T|F) = \frac{F \cap T}{P(F)} = \frac{\frac{2}{32}}{\frac{6}{32}} = \frac{2}{6}$$

(f) Does $P(F|T) = P(T|F)$?

No

2. This data came from a pre-employment drug screening.

	Positive Test Result (drug use is indicated)	**Negative Test Result** (drug use is not indicated)
Subject Uses Drugs	44 (true positive)	6 (false negative)
Subject is Not a Drug User	90 (false positive)	860 (true negative)

(a) **False positive** Find the probability of selecting a subject with a positive test result given that the subject does not use drugs.

If D is the event of drug usage and T is the event of a positive drug test, then

$$P(T|D^c) = \frac{P(D^c \cap T)}{P(D^c)} = \frac{\frac{90}{1000}}{\frac{950}{1000}} = \frac{90}{950}$$

(b) **False negative** Find the probability of selecting a subject with a negative test result given that the subject uses drugs.

If D is the event of drug usage and T is the event of a positive drug test, then

$$P(T^c|D) = \frac{P(D \cap T^c)}{P(D)} = \frac{\frac{6}{1000}}{\frac{50}{1000}} = \frac{6}{50}$$

(c) **Predictive value**

i. Find the positive predictive value for the test. That is, find the probability that a subject uses drugs, given that the test yields a positive result.

$$\frac{44}{134}$$

ii. Find the negative predictive value for the test. That is, find the probability that a subject does not use drugs, given that the test yields a negative result.

$$\frac{860}{866}$$