Complete the following tasks. Show your work/code where applicable. Some answers have been provided. Assemble your work into one PDF document and upload the PDF back into our CatCourses page.

1. (*Bayes Rules!* exercise 5.5) (**Text messages**) Let random variable λ represent the rate of text messages people receive in an hour. At first, you believe that the typical number of messages per hour is 5 with a standard deviation of 0.25 messages.

   (a) Tune and plot an appropriate Gamma(s, r) prior model for λ.

   (b) What is the prior probability that the rate of text messages per hour is larger than 10?

2. (*Bayes Rules!* exercise 5.6) (**Text messages with data**) [Continuing from the previous exercise], you collect data from six friends. They received 7, 3, 8, 9, 10, 12 text messages in the previous hour.

   (a) Plot the resulting likelihood function of λ.

   (b) Plot the prior pdf, likelihood function, and the posterior pdf of λ.

   (c) Calculate descriptive statistics for the prior and the posterior models of λ.

   (d) Comment on how your understanding about λ changed from the prior (in the previous exercise) to the posterior based on the data you collected from your friends.

3. (*Bayes Rules!* exercise 5.9) (**Investing in stock**) You just bought stock in FancyTech. Let random variable μ be the average dollar amount that your FancyTech stock goes up or down in a one-day period. At first, you believe that μ is 7.2 dollars with a standard deviation of 2.6 dollars.

   (a) Tune and plot an appropriate Normal prior model for μ.

   (b) According to your plot, does it seem plausible that the FancyTech stock would increase by an average of 7.6 dollars in a day?

   (c) Does it seem plausible that the FancyTech stock would increase by an average of 4 dollars in a day?

   (d) What is the prior probability that, on average, the stock price goes down?

   (e) What is the prior probability that, on average, your stock price goes up by more than 8 dollars per day?

4. (*Bayes Rules!* exercise 5.10) (**Investing in stock with data**) [Continuing from the previous exercise], it's reasonable to assume that the daily changes in FancyTech stock value are Normally distributed around an *unknown* mean of μ with a *known* standard deviation of $\sigma = 2$ dollars. On a random sample of 4 days, you observe changes in stock value of -0.7, 1.2, 4.5, and -4 dollars.

(a) Plot the resulting likelihood function of μ.

(b) Plot the prior pdf, likelihood function, and the posterior pdf of μ.

(c) Calculate descriptive statistics for the prior and the posterior models of μ.

(d) Comment on how your understanding about μ changed from the prior (in the previous exercise) to the posterior based on the observed data.

(e) What is the posterior probability that, on average, the stock price goes down?

(f) What is the posterior probability that, on average, your stock price goes up by more than 8 dollars per day?

5. (*Bayes Rules!* exercise 6.5) (**Beta-Binomial grid approximation**) Consider the Beta-Binomial model for π with $Y|\pi \sim \text{Bin}(n, \pi)$ and $\pi \sim \text{Beta}(3, 8)$. Suppose that in $n = 10$ independent trials, you observe $Y = 2$ successes.

   (a) Utilize grid approximation with grid values $\pi \in \{0, 0.25, 0.5, 0.75, 1\}$ to approximate the posterior model of π.

   (b) Repeat part a using a grid of 201 equally spaced values between 0 and 1.

6. (*Bayes Rules!* exercise 6.6) (**Gamma-Poisson grid approximation**) Consider the Gamma-Poisson model for λ with $Y_i|\lambda \sim \text{Pois}(\lambda)$ and $\lambda \sim \text{Gamma}(20, 5)$. Suppose you observe $n = 3$ independent data points $(Y_1, Y_2, Y_3) = (0, 1, 0)$.

   (a) Utilize grid approximation with grid values $\lambda \in \{0, 1, 2, \ldots, 8\}$ to approximate the posterior model of λ.

   (b) Repeat part a using a grid of 201 equally spaced values between 0 and 8.

7. (*Bayes Rules!* exercise 6.13) (**MCMC with RStan: Beta-Binomial**) Consider the Beta-Binomial model for π with $Y|\pi \sim \text{Bin}(n, \pi)$ and $\pi \sim \text{Beta}(3, 8)$. Suppose that in $n = 10$ independent trials, you observe $Y = 2$ successes.

   (a) Simulate the posterior model of with RStan using 3 chains and 12000 iterations per chain.

   (b) Produce trace plots for the three chains.

   (c) What is the range of values on the trace plot x-axis? Why is the maximum value of this range not 12000?

   (d) Create a density plot of the values for the three chains.

   (e) Harkening back to Chapter 5, specify the posterior model of π. How does your MCMC approximation compare?

8. (*Bayes Rules!* exercise 6.15) (**MCMC with RStan: Gamma-Poisson**) Consider the Gamma-Poisson model for λ with $Y_i|\lambda \sim \text{Pois}(\lambda)$ and $\lambda \sim \text{Gamma}(20, 5)$. Suppose you observe $n = 3$ independent data points $(Y_1, Y_2, Y_3) = (0, 1, 0)$.

(a) Simulate the posterior model of λ with RStan using 4 chains and 10000 iterations per chain.

(b) Produce trace and density plots for the chains.

(c) From the density plots, what seems to be the most posterior plausible value of λ?

(d) Harkening back to Chapter 5, specify the posterior model of λ. How does your MCMC approximation compare?

Here are some incomplete answers.