ps5_code

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```
library("patchwork")
library("tidyverse")
```

7.9

```
one_mh_iteration <- function(current, w, obs_value, tau, sigma){
  # Step 1: propose next location in chain
  proposal <- runif(1, min = current - w, current + w)

  # Step 2: decide whether or not to go there
  proposal_plausibility <- dnorm(proposal, 3, tau) * dnorm(obs_value, propos
  current_plausibility <- dnorm(current, 3, tau) * dnorm(obs_value, current,
  alpha <- min(1, proposal_plausibility / current_plausibility)
  next_stop <- sample(c(proposal, current),
                      size = 1, prob = c(alpha, 1 - alpha))

  # Return the results as a data frame
  return(data.frame(proposal, alpha, next_stop))
}
```

a

```
set.seed(320)
one_mh_iteration(w = 0.01, current = 3,
                 obs_value = 6.25, tau = 1, sigma = 0.75)
```

```
  proposal     alpha next_stop
1 2.990375 0.9457828  2.990375
```

There was about a 95% acceptance probability, and the proposed location was accepted.

b

```
set.seed(320)
one_mh_iteration(w = 0.5, current = 3,
                 obs_value = 6.25, tau = 1, sigma = 0.75)
```

```
  proposal      alpha next_stop
1 2.518728 0.04494218         3
```

There was about a 4% acceptance probability, and the proposed location was rejected.

c

```
set.seed(320)
one_mh_iteration(w = 1, current = 3,
                 obs_value = 6.25, tau = 1, sigma = 0.75)
```

```
  proposal       alpha next_stop
1 2.037457 0.001061415         3
```

There was a virtually zero acceptance probability, and the proposed location was rejected.

d

```
set.seed(320)
one_mh_iteration(w = 3, current = 3,
                 obs_value = 6.25, tau = 1, sigma = 0.75)
```

```
   proposal        alpha next_stop
1 0.1123701 5.303685e-13         3
```

There was a virtually zero acceptance probability, and the proposed location was rejected.

7.10

```
mh_tour <- function(N, current, w, obs_value, tau, sigma){
  # N: chain length
  # initialize vector
  mu <- rep(0, N)

  # simulate N Markov chain stops
  for(i in 1:N){
    # simulate one iteration
    this_iteration <- one_mh_iteration(current, w, obs_value, tau, sigma)

    # record next location
    mu[i] <- this_iteration$next_stop

    # update current location
    current <- this_iteration$next_stop
  }

  # return the chain locations
  return(data.frame(iteration = c(1:N), mu))
}

bayesrules::summarize_normal_normal(
```

```
  # from prior
  mean = 3, sd = 1,

  # from observations
  y_bar = 6.25, sigma = 0.75, n = 1
) |>
  mutate_if(is.numeric, round, digits = 4)
```

```
      model mean mode  var  sd
1     prior 3.00 3.00 1.00 1.0
2 posterior 5.08 5.08 0.36 0.6
```

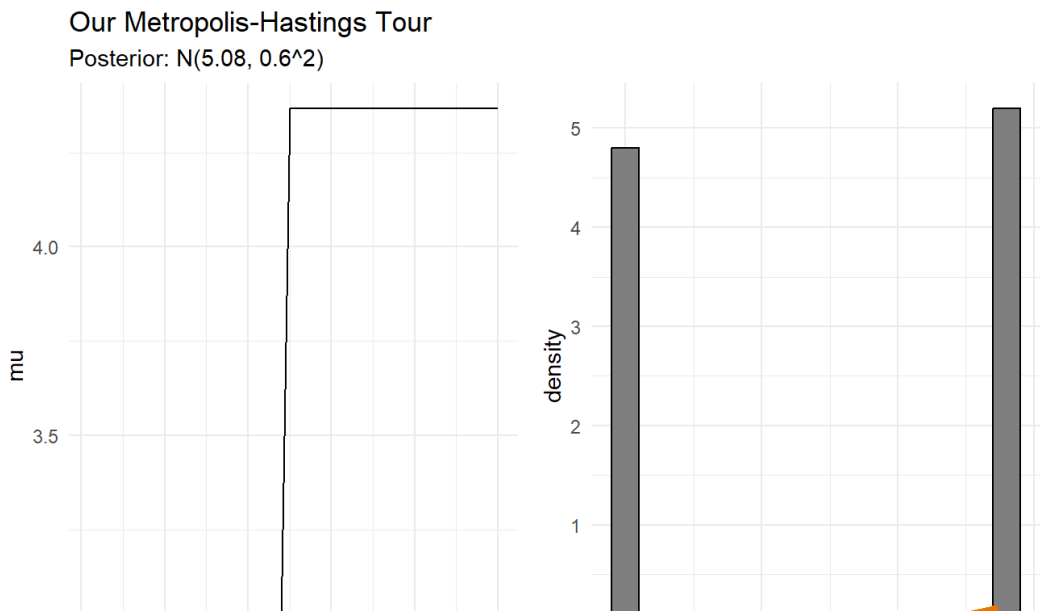
a

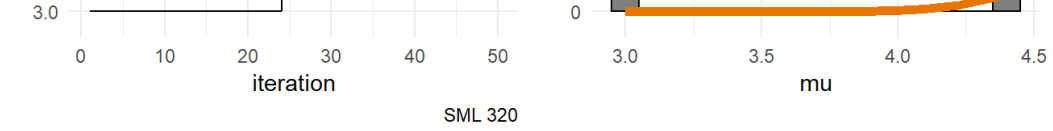
```
set.seed(320)
our_mh_tour <- mh_tour(N = 50, current = 3, w = 50,
                       obs_value = 6.25, tau = 1, sigma = 0.75)

p1 <- ggplot(our_mh_tour, aes(x = iteration, y = mu)) +
  geom_line() +
  labs(title = "Our Metropolis-Hastings Tour",
       subtitle = "Posterior: N(5.08, 0.6^2)",
       caption = "SML 320") +
  theme_minimal()

p2 <- ggplot(our_mh_tour, aes(x = mu)) +
  geom_histogram(aes(y = after_stat(density)),
                 binwidth = 0.1,
                 color = "black", fill = "gray50") +
  stat_function(fun = dnorm, args = list(5.08, 0.36),
                color = "#E77500",
                linewidth = 2) +
  theme_minimal()

# patchwork
p1 + p2
```





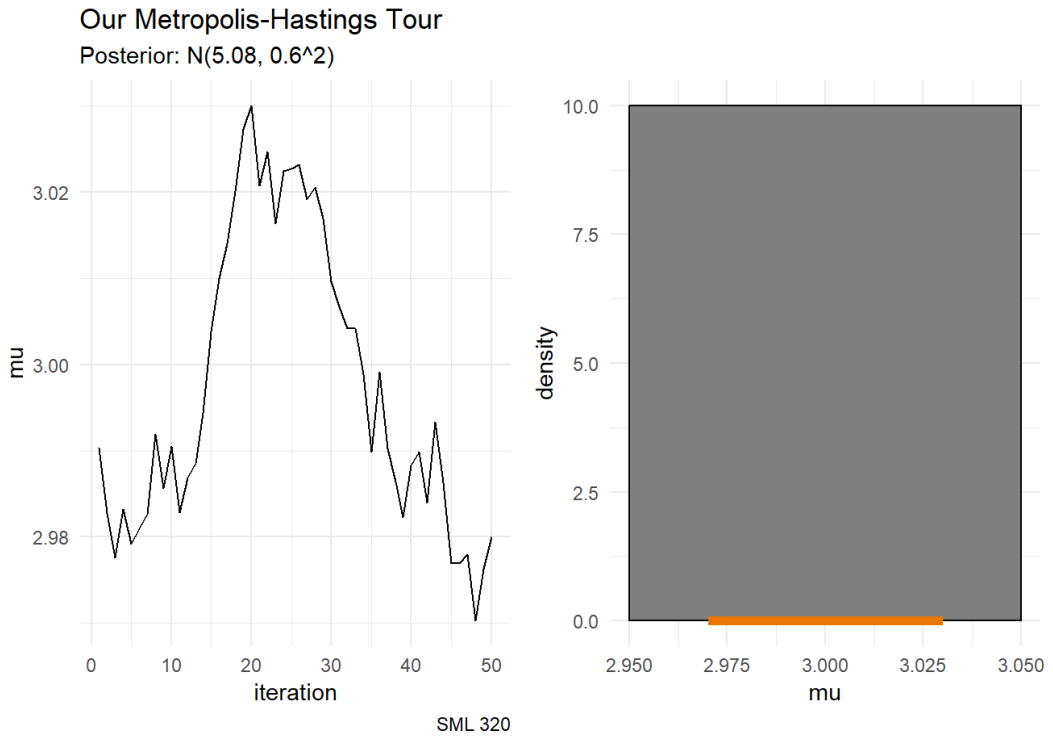
b

```
set.seed(320)
our_mh_tour <- mh_tour(N = 50, current = 3, w = 0.01,
                       obs_value = 6.25, tau = 1, sigma = 0.75)

p1 <- ggplot(our_mh_tour, aes(x = iteration, y = mu)) +
  geom_line() +
  labs(title = "Our Metropolis-Hastings Tour",
       subtitle = "Posterior: N(5.08, 0.6^2)",
       caption = "SML 320") +
  theme_minimal()

p2 <- ggplot(our_mh_tour, aes(x = mu)) +
  geom_histogram(aes(y = after_stat(density)),
                 binwidth = 0.1,
                 color = "black", fill = "gray50") +
  stat_function(fun = dnorm, args = list(5.08, 0.36),
                color = "#E77500",
                linewidth = 2) +
  theme_minimal()

# patchwork
p1 + p2
```



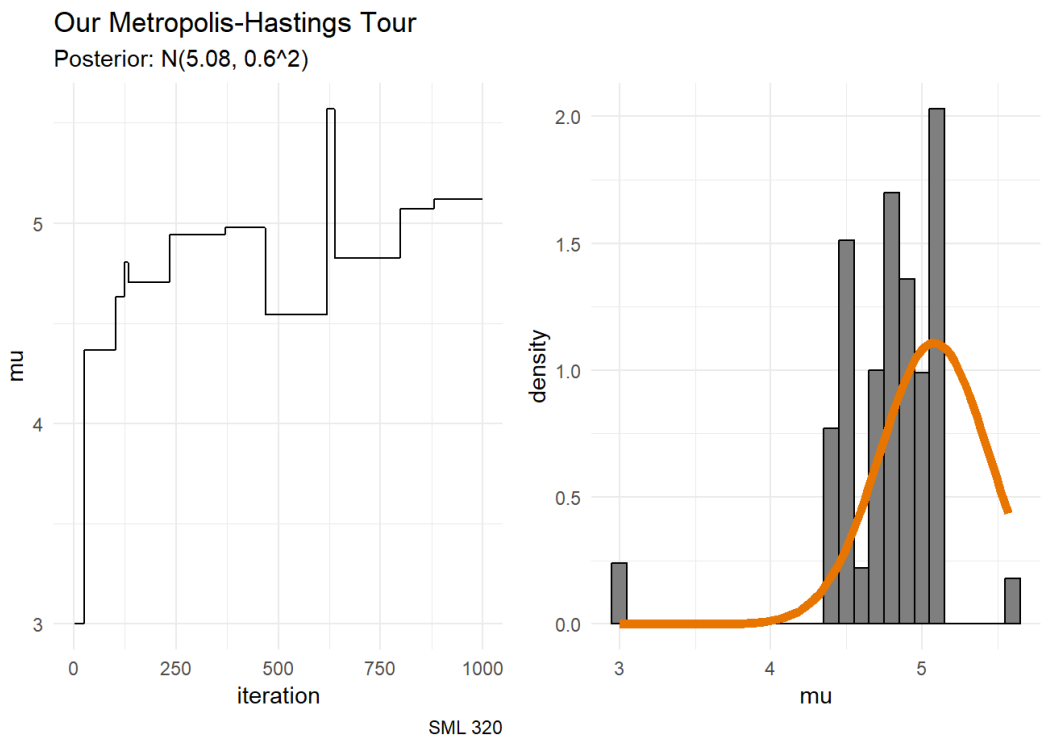
c

```
set.seed(320)
our_mh_tour <- mh_tour(N = 1000, current = 3, w = 50,
                       obs_value = 6.25, tau = 1, sigma = 0.75)

p1 <- ggplot(our_mh_tour, aes(x = iteration, y = mu)) +
  geom_line() +
  labs(title = "Our Metropolis-Hastings Tour",
       subtitle = "Posterior: N(5.08, 0.6^2)",
       caption = "SML 320") +
  theme_minimal()

p2 <- ggplot(our_mh_tour, aes(x = mu)) +
  geom_histogram(aes(y = after_stat(density)),
                 binwidth = 0.1,
                 color = "black", fill = "gray50") +
  stat_function(fun = dnorm, args = list(5.08, 0.36),
                color = "#E77500",
                linewidth = 2) +
  theme_minimal()

# patchwork
p1 + p2
```



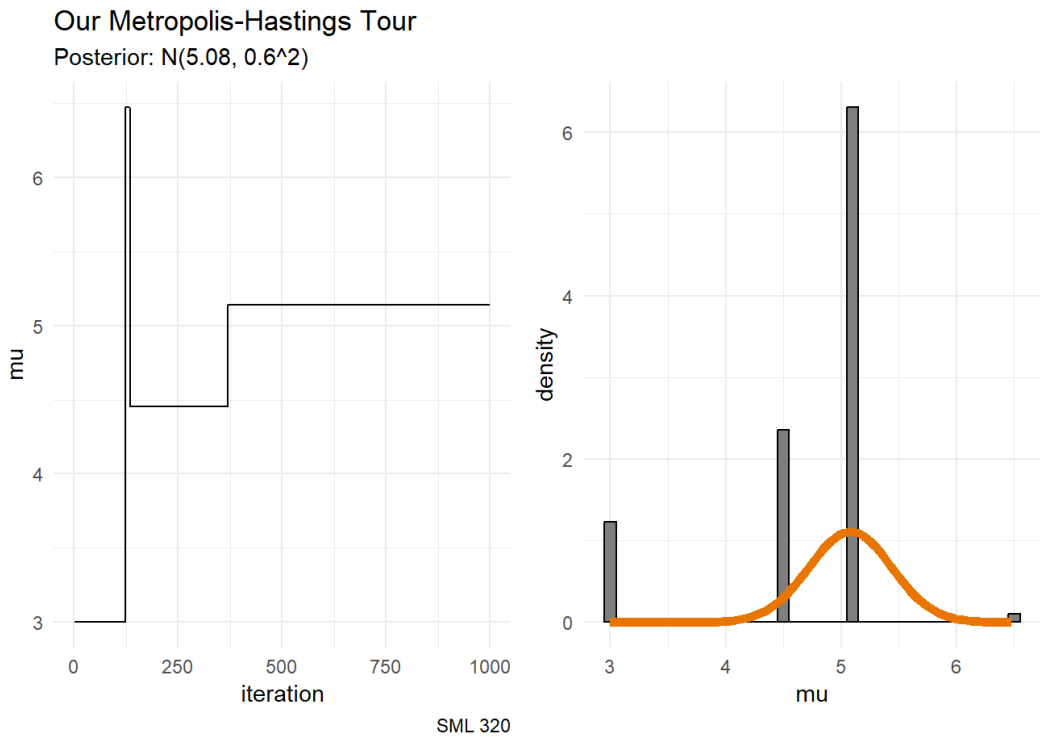
d

```
set.seed(320)
our_mh_tour <- mh_tour(N = 1000, current = 3, w = 1000,
                       obs_value = 6.25, tau = 1, sigma = 0.75)
```

```
p1 <- ggplot(our_mh_tour, aes(x = iteration, y = mu)) +
  geom_line() +
  labs(title = "Our Metropolis-Hastings Tour",
       subtitle = "Posterior: N(5.08, 0.6^2)",
       caption = "SML 320") +
  theme_minimal()

p2 <- ggplot(our_mh_tour, aes(x = mu)) +
  geom_histogram(aes(y = after_stat(density)),
                 binwidth = 0.1,
                 color = "black", fill = "gray50") +
  stat_function(fun = dnorm, args = list(5.08, 0.36),
                color = "#E77500",
                linewidth = 2) +
  theme_minimal()

# patchwork
p1 + p2
```



e

Comparing the results from parts (a) and (b), we observe that having a half-width w that is too large can create a situation that is nearly constant because of the very low acceptance probabilities.

f

Comparing the results from parts (a) and (b), we observe that having a half-width w that is too large can create situation that are nearly constant for several iterations

because of the very low acceptance probabilities.

7.11

```
one_mh_iteration_normal <- function(current, s, obs_value, tau, sigma){
  # Step 1: propose next location in chain
  proposal <- current + rnorm(1, mean = 0, sd = s)

  # Step 2: decide whether or not to go there
  proposal_plausibility <- dnorm(proposal, 3, tau) * dnorm(obs_value, propos
  current_plausibility <- dnorm(current, 3, tau) * dnorm(obs_value, current,
  alpha <- min(1, proposal_plausibility / current_plausibility)
  next_stop <- sample(c(proposal, current),
                      size = 1, prob = c(alpha, 1 - alpha))

  # Return the results as a data frame
  return(data.frame(proposal, alpha, next_stop))
}
```

a

```
set.seed(320)
one_mh_iteration_normal(s = 0.01, current = 3,
                        obs_value = 6.25, tau = 1, sigma = 0.75)
```

```
  proposal     alpha next_stop
1 2.979192 0.8861912  2.979192
```

The acceptance probability was about 87%, and the proposed location was accepted.

b

```
set.seed(320)
one_mh_iteration_normal(s = 0.5, current = 3,
                        obs_value = 6.25, tau = 1, sigma = 0.75)
```

```
  proposal        alpha next_stop
1 1.959624 0.0005451953         3
```

There was a virtually zero acceptance probability, and the proposed location was rejected.

c

```
set.seed(320)
one_mh_iteration_normal(s = 1, current = 3,
                        obs_value = 6.25, tau = 1, sigma = 0.75)
```

```
   proposal        alpha next_stop
1 0.9192488 1.470104e-08         3
```

There was a virtually zero acceptance probability, and the proposed location was rejected.

d

```
set.seed(320)
one_mh_iteration_normal(s = 3, current = 3,
                        obs_value = 6.25, tau = 1, sigma = 0.75)
```

```
   proposal        alpha next_stop
1 -3.242254 6.807206e-40         3
```

There was a virtually zero acceptance probability, and the proposed location was rejected.

7.12

```
mh_tour_normal <- function(N, current, s, obs_value, tau, sigma){
  # N: chain length
  # initialize vector
  mu <- rep(0, N)

  # simulate N Markov chain stops
  for(i in 1:N){
    # simulate one iteration
    this_iteration <- one_mh_iteration_normal(current, s,
                                              obs_value, tau, sigma)

    # record next location
    mu[i] <- this_iteration$next_stop

    # update current location
    current <- this_iteration$next_stop
  }

  # return the chain locations
  return(data.frame(iteration = c(1:N), mu))
}
```

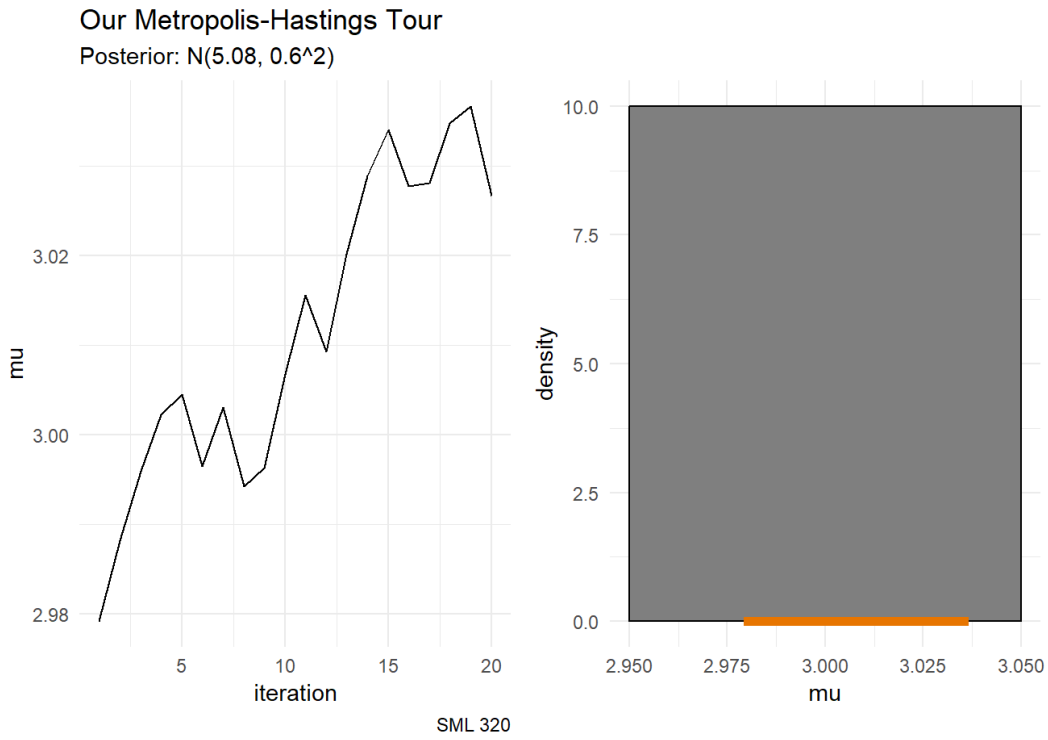
a

```
set.seed(320)
our_mh_tour <- mh_tour_normal(N = 20, current = 3, s = 0.01,
                  obs_value = 6.25, tau = 1, sigma = 0.75)
```

```
p1 <- ggplot(our_mh_tour, aes(x = iteration, y = mu)) +
  geom_line() +
  labs(title = "Our Metropolis-Hastings Tour",
       subtitle = "Posterior: N(5.08, 0.6^2)",
       caption = "SML 320") +
  theme_minimal()

p2 <- ggplot(our_mh_tour, aes(x = mu)) +
  geom_histogram(aes(y = after_stat(density)),
                 binwidth = 0.1,
                 color = "black", fill = "gray50") +
  stat_function(fun = dnorm, args = list(5.08, 0.36),
                color = "#E77500",
                linewidth = 2) +
  theme_minimal()

# patchwork
p1 + p2
```



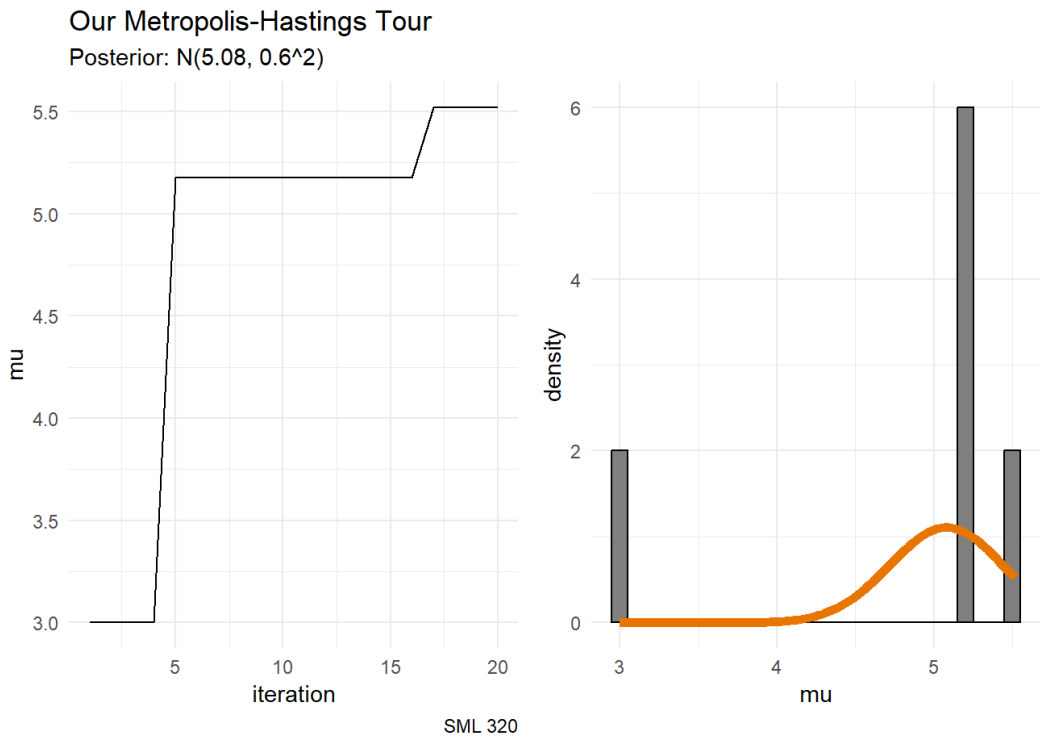
b

```
set.seed(320)
our_mh_tour <- mh_tour_normal(N = 20, current = 3, s = 10,
                 obs_value = 6.25, tau = 1, sigma = 0.75)

p1 <- ggplot(our_mh_tour, aes(x = iteration, y = mu)) +
  geom_line() +
  labs(title = "Our Metropolis-Hastings Tour",
       subtitle = "Posterior: N(5.08, 0.6^2)",
       caption = "SML 320") +
  theme_minimal()
```

```
p2 <- ggplot(our_mh_tour, aes(x = mu)) +
  geom_histogram(aes(y = after_stat(density)),
                 binwidth = 0.1,
                 color = "black", fill = "gray50") +
  stat_function(fun = dnorm, args = list(5.08, 0.36),
                color = "#E77500",
                linewidth = 2) +
  theme_minimal()

# patchwork
p1 + p2
```



C

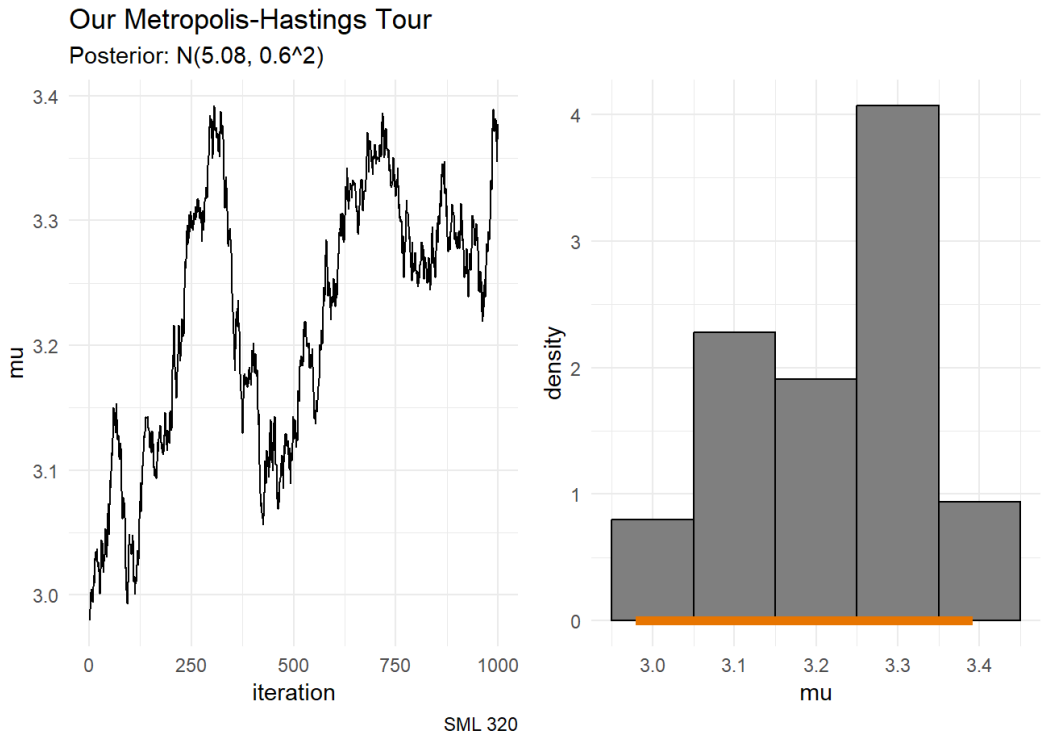
```
set.seed(320)
our_mh_tour <- mh_tour_normal(N = 1000, current = 3, s = 0.01,
                 obs_value = 6.25, tau = 1, sigma = 0.75)

p1 <- ggplot(our_mh_tour, aes(x = iteration, y = mu)) +
  geom_line() +
  labs(title = "Our Metropolis-Hastings Tour",
       subtitle = "Posterior: N(5.08, 0.6^2)",
       caption = "SML 320") +
  theme_minimal()

p2 <- ggplot(our_mh_tour, aes(x = mu)) +
  geom_histogram(aes(y = after_stat(density)),
                 binwidth = 0.1,
                 color = "black", fill = "gray50") +
  stat_function(fun = dnorm, args = list(5.08, 0.36),
                color = "#E77500",
```

```
                linewidth = 2) +
  theme_minimal()

# patchwork
p1 + p2
```



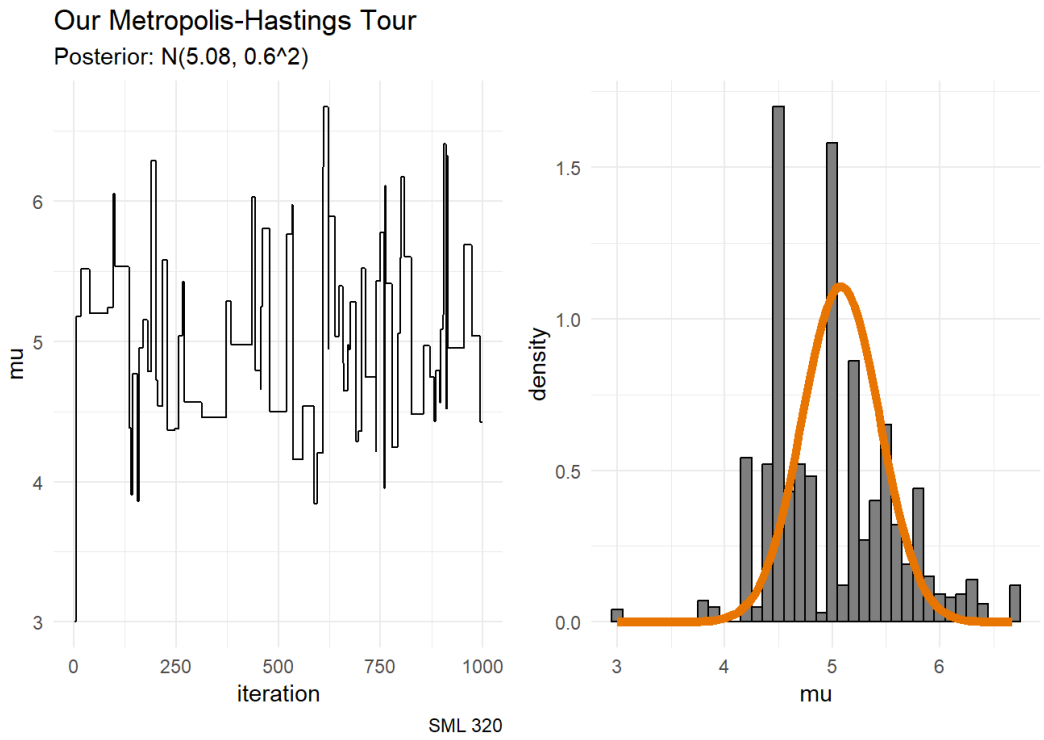
d

```
set.seed(320)
our_mh_tour <- mh_tour_normal(N = 1000, current = 3, s = 10,
                 obs_value = 6.25, tau = 1, sigma = 0.75)

p1 <- ggplot(our_mh_tour, aes(x = iteration, y = mu)) +
  geom_line() +
  labs(title = "Our Metropolis-Hastings Tour",
       subtitle = "Posterior: N(5.08, 0.6^2)",
       caption = "SML 320") +
  theme_minimal()

p2 <- ggplot(our_mh_tour, aes(x = mu)) +
  geom_histogram(aes(y = after_stat(density)),
                 binwidth = 0.1,
                 color = "black", fill = "gray50") +
  stat_function(fun = dnorm, args = list(5.08, 0.36),
                color = "#E77500",
                linewidth = 2) +
  theme_minimal()

# patchwork
p1 + p2
```



e

Comparing the results from parts (a) and (b), we observe that having a standard deviation s that is too large can create a situation that is nearly constant because of the very low acceptance probabilities.

f

Perhaps we should try a standard deviation that is closer to those in the prior and likelihood models. Here, with $s = 0.8$,

```
set.seed(320)
our_mh_tour <- mh_tour_normal(N = 1000, current = 3, s = 0.8,
                 obs_value = 6.25, tau = 1, sigma = 0.75)

p1 <- ggplot(our_mh_tour, aes(x = iteration, y = mu)) +
  geom_line() +
  labs(title = "Our Metropolis-Hastings Tour",
       subtitle = "Posterior: N(5.08, 0.6^2)",
       caption = "SML 320") +
  theme_minimal()

p2 <- ggplot(our_mh_tour, aes(x = mu)) +
  geom_histogram(aes(y = after_stat(density)),
                 binwidth = 0.1,
                 color = "black", fill = "gray50") +
  stat_function(fun = dnorm, args = list(5.08, 0.36),
                color = "#E77500",
                linewidth = 2) +
  theme_minimal()
```

```
# patchwork
p1 + p2
```