Essentials of Analytical Geometry and Linear Algebra 1

Preparation to midterm contest

INNOPOLIS UNIVERSITY

Goals

1. To prepare to midterm:
    a. Work in stress situation
    b. Refresh all needed topics
2. Make stronger relationship among group mates
3. Have fun
4. Show some possible tasks on midterm

Rules

What can be used: almost everything (laptops, neighbors, etc), *except asking you friends from other groups.*

1 round - 15 min, provide solution - 10 min

Guideline:

1. Choose first team (rock-scissors-paper), other teams we be in clockwise order
2. Solve tasks during one round
3. Current team should answer on all questions. I am asking who should answer, even more I can interrupt one guy from the team and ask other. If he cannot continue, task won’t count.
4. If other teams (in clockwise order) can give additional solution or worthy comment, extra score will be given.

Prices for winners

1. Our contest *should be active and interesting (for me)*, otherwise - no prices for everybody
2. If all teams will be good -> I am buying sweeties for everyone
3. If one team -> pizza for them

1st round

Derive a formula for the distance from point $M(x_0; y_0)$ to a line $ax + by + c = 0$ in the plane (the coordinate system is Cartesian).

Find the equation of the plane which passes through the intersection of the planes $2x + 3y + 10z - 8 = 0$, $2x - 3y + 7z - 2 = 0$ and is perpendicular to the plane $3x - 2y + 4z - 5 = 0$.

$2y + z - 2 = 0$

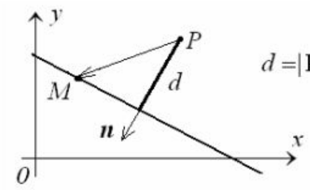
The plane $x - 2y + 3z = 0$ is rotated through a right angle about its line of intersection with the plane $2x + 3y - 4z + 2 = 0$. Find the equation of the plane in its new position.

$22x + 5y - 42 + 14 = 0$

1st round answers (1): in english

Consider a line in the x, y–plane.

Let $\boldsymbol{n}$ be a normal vector to the line and $M(x_0, y_0)$ be any point on the line. Then the distance d from a point P not on the line is equal to the absolute value of the projection of $\vec{PM}$ on $\boldsymbol{n}$:



$$d = |\mathrm{Proj}_{\boldsymbol{n}} \vec{PM}| = \left| \frac{\vec{PM} \cdot \boldsymbol{n}}{|\boldsymbol{n}|} \right|$$

In particular, if the line is given by the equation

$$Ax + By + C = 0,$$

and the coordinates of the point P are x_1 and y_1, that is,

$$\boldsymbol{n} = \{A, B\} \text{ and } \vec{PM} = \{x_1 - x_0, y_1 - y_0\},$$

then the distance from the point $P(x_1, y_1)$ to the line is calculated according to the following formula:

$$d = \frac{|A(x_1 - x_0) + B(y_1 - y_0)|}{\sqrt{A^2 + B^2}}.$$

Since $M(x_0, y_0)$ is a point on the line,

$$Ax_0 + By_0 + C = 0.$$

Therefore, we obtain

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

1st round answers (1): in russian

$$\vec{a} = \left\| \begin{matrix} a_x \\ a_y \\ a_z \end{matrix} \right\|$$

$$\vec{r} = \vec{r}_0 + \tau \vec{a}.$$

$$\frac{x - x_0}{a_x} = \frac{y - y_0}{a_y} = \frac{z - z_0}{a_z},$$

$$[\vec{a}, \vec{r} - \vec{r}_0] = \vec{o}$$

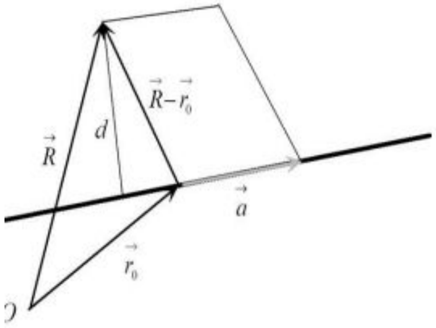


Рис. 3.4.1

Наконец, расстояние d в пространстве от некоторой точки с радиусом-вектором $\vec{R}$ до прямой $\vec{r} = \vec{r}_0 + \tau \vec{a}$ можно найти, воспользовавшись свойством, что S – площадь параллелограмма, построенного на паре векторов, равна длине векторного произведения этих векторов. Из рис. 3.4.1 получаем

$$d = \frac{S}{|\vec{a}|} = \frac{|[\vec{R} - \vec{r}_0, \vec{a}]|}{|\vec{a}|}.$$

1st round answers (2)

The equation of any plane passing through the intersection of the planes $2x + 3y + 10z - 8 = 0$ and $2x - 3y + 7z - 2 = 0$ is $2x + 3y + 10z - 8 + \lambda (2x - 3y + 7z - 2) = 0$.

The direction ratios of the normal to this plane are $2 + 2\lambda$, $3 - 3\lambda$, $10 + 7\lambda$. The direction ratios of the plane $3x - 2y + 4z - 5 = 0$ are 3, –2, 4. Since these two planes are perpendicular, $3(2 + 2\lambda) - 2 (3 - 3\lambda) + 4(10 + 7\lambda) = 0$.

$$6 + 6\lambda - 6 + 6\lambda + 40 + 28\lambda = 0 \text{ or } 40\lambda = -40 \text{ or } \lambda = -1$$

Therefore, the required plane is $2x + 3y + 10z - 8 - (2x - 3y + 7z - \lambda) = 0$.

$$\therefore\ 6y + 3z - 6 = 0 \text{ or } 2y + z - 2 = 0$$

1st round answers (3)

The plane $x - 2y + 3z = 0$ is rotated about the line of intersection of the planes

$$x - 2y + 3z = 0 \qquad (12.58)$$

$$2x + 3y - 4z + 2 = 0 \qquad (12.59)$$

The new position of the plane (12.58) passes through the line of intersection of the two given planes. Therefore, its equation is

$$x - 2y + 3z + \lambda(2x + 3y - 4z + 2) = 0 \qquad (12.60)$$

The plane (12.60) is perpendicular to the plane (12.58).

$$1(1 + 2\lambda) - 2(-2 + 3\lambda) + 3(3 - 4\lambda) = 0 \text{ or } -16\lambda + 14 = 0 \text{ or } \lambda = \frac{7}{8}.$$

Therefore, the equation of the plane (12.58) in its new position is

$$x - 2y + 3z + \frac{7}{8}(2x + 3y - 4z + 2) = 0. \qquad 22x + 5y - 42 + 14 = 0$$

2nd round

Find the altitude of tetrahedron $ABCD$ dropped from vertex C given coordinates of all its vertices in some Cartesian coordinate system: $A(1;\ 2;\ 1)$, $B(2;\ 0;\ 0)$, $C(-2;\ 4;\ 3)$, $D(4;\ 1;\ -1)$.

$h = \frac{1}{\sqrt{35}}$

Show that the lines $\frac{x+1}{1} = \frac{y+1}{2} = \frac{z+1}{3}$ and $x + 2y + 3z - 8 = 0$, $2x + 3y + 4z - 11 = 0$ are coplanar. Find the equation of the plane containing these two lines.

$4x + y - 2z + 3 = 0$

Find the equation of the straight lines through the origin each of which intersects the straight line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ and are inclined at an angle of 60° to it.

$\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ $\quad$ $\frac{x}{1} = \frac{y}{1} = \frac{z}{2}$.

2nd round answers (1)

2 points

5) $A(1;2;1)$, $B(2;0;0)$, $C(-2;4;3)$, $D(4;1;-1)$

$\overrightarrow{AB}(1;-2;-1)$, $\overrightarrow{AC}(-3;2;2)$, $AD(3;-1;-2)$

$$\overrightarrow{AB}\times\overrightarrow{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & -1 \\ 3 & -1 & -2 \end{vmatrix} = 3\vec{i} - \vec{j} + 5\vec{k}, \quad |\overrightarrow{AB}\times\overrightarrow{AD}| = \sqrt{35}$$

$$(\overrightarrow{AB}\times\overrightarrow{AD})\cdot\overrightarrow{AC} = -9-2+10 = -1.$$

$$V_{ABCD} = \frac{1}{6}, \quad S_{ABD} = \frac{\sqrt{35}}{2}; \quad \frac{1}{6} = \frac{1}{3}h\cdot\frac{\sqrt{35}}{2} \Rightarrow h = \frac{1}{\sqrt{35}}.$$

2nd round answers (2)

$$\frac{x+1}{1}=\frac{y+1}{2}=\frac{z+1}{3}=r \qquad (13.62)$$

$$x+2y+3z-8=0$$

$$2x+3y+4z-11=0 \qquad (13.63)$$

Any plane containing the second line is

$$x+2y+3z-8+\lambda(2x+3y+4z-11)=0 \qquad (13.64)$$

If the line given by (13.62) lies on this plane then the point (−1, −1, −1) also lies on the plane.

$$-1-2-3-8+\lambda(-2-3-4-11)=0$$

$$\Rightarrow -14+\lambda(-20)=0$$

$$\therefore \lambda=\frac{-7}{10}$$

$$x+2y+3z-8-\frac{7}{10}(2x+3y+4z-11)=0$$

$$10x+20y+30z-80-14x-21y-28z+77=0$$

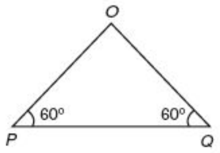
or

$$4x+y-2z+3=0 \qquad (13.65)$$

2nd round answers (3)

The equations of the line PQ are

$$\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}=r$$



The point P on this line is $P(2r + 3, r + 3, r)$.

The direction ratios of OP are $2r + 3$, $r + 3$, r. Since $\lfloor POQ = \frac{\pi}{3}$,

$$\cos\frac{\pi}{3}=\frac{2(2r+3)+1(r+3)+1\cdot(r)}{\sqrt{(2r+3)^2+(r+3)^2+r^2}\cdot\sqrt{4+1+1}}$$

$$\text{(i.e.)}\quad \frac{6r+9}{\sqrt{(6r^2+18r+18)}\sqrt{6}}=\frac{1}{2}$$

$$\text{(i.e.)}\quad \frac{9(2r+3)^2}{36(r^2+3r+3)}=\frac{1}{2}\Rightarrow\frac{4r^2+12r+9}{4(r^2+3r+3)}=\frac{1}{4}$$

$$\text{(i.e.)}\quad 4r^2+12r+9=r^2+3r+3$$

$$\text{(i.e.)}\quad 3r^2+9r+6=0$$

or $r^2 + 3r + 2 = 0$ or $r = -1, -2$. Therefore, the coordinates of P and Q are $(1, -2, -1)$ and $(-1, 1, -2)$.

Hence the equations of the lines OP and OQ are $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ and

$\frac{x}{1}=\frac{y}{1}=\frac{z}{2}$.

3rd round

Derive a formula for evaluating cross product of vectors $\mathbf{a}(\alpha_1; \alpha_2; \alpha_3)$ and $\mathbf{b}(\beta_1; \beta_2; \beta_3)$ in some affine coordinate system that has a triple $(\mathbf{i}; \mathbf{j}; \mathbf{k})$ for a basis.

Find the shortest distance and the equation to the line of shortest

distance between the two lines $\frac{x+7}{3} = \frac{y+4}{4} = \frac{z+3}{-2}$ and $\frac{x-21}{6} = \frac{y+5}{-4} = \frac{z-2}{-1}$.

$$\frac{x+1}{4} = \frac{y+4}{3} = \frac{z+7}{12}.$$

Find the equations of the line passing through the point (1, 2, 3) and perpendicular to the planes $x - 2y - z + 5 = 0$ and $x + y + 3z + 6 = 0$.

$$\frac{x-1}{5} = \frac{y-2}{4} = \frac{z-3}{-3}.$$

3rd round answers (1): in english

Theorem: Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be two non-parallel vectors. Then

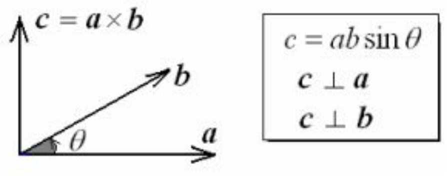
i) the vector $\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$ is orthogonal to both $\boldsymbol{a}$ and $\boldsymbol{b}$;

ii) the length of $\boldsymbol{c}$ is expressed by the formula

$$c = ab\sin\theta,$$

where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$;

iii) the set of vectors $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$ is a right-handed triplet as it is shown in the figure below.



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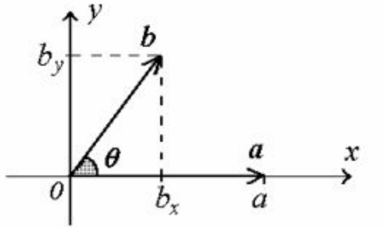
Proof: Let the rectangular coordinate system be chosen such that both vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ lie in the x,y-plane, and the x-axis is directed along $\boldsymbol{a}$.



Then $\boldsymbol{a} = \{a, 0, 0\}$ and $\boldsymbol{b} = \{b\cos\theta, b\sin\theta, 0\}$.

Therefore,

$$\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a & 0 & 0 \\ b\cos\theta & b\sin\theta & 0 \end{vmatrix} = ab\sin\theta\,\boldsymbol{k}.$$

Therefore, $|\boldsymbol{c}| = a\sin\theta$ and $\boldsymbol{c}$ is directed along the z-axis which is perpendicular to the x,y-plane. Hence, the theorem.

3rd round answers (2): in russian, from Beklemishev book

Предложение 3. *Каковы бы ни были векторы* $\mathbf{b}$ *и* $\mathbf{c}$, *найдется единственный (не зависящий от* $\mathbf{a}$*) вектор* $\mathbf{d}$ *такой, что при любом* $\mathbf{a}$ *выполнено равенство*

$$(\mathbf{a}, \mathbf{b}, \mathbf{c}) = (\mathbf{a}, \mathbf{d}). \tag{8}$$

Доказательство Докажем сначала существование вектора $\mathbf{d}$, а потом установим, что такой вектор возможен только один. Пусть векторы $\mathbf{b}$ и $\mathbf{c}$ коллинеарны. Тогда при любом $\mathbf{a}$ векторы $\mathbf{a}$, $\mathbf{b}$ и $\mathbf{c}$ компланарны и $(\mathbf{a}, \mathbf{b}, \mathbf{c}) = 0$. Поэтому мы можем положить $\mathbf{d} = \mathbf{0}$. Рассмотрим неколлинеарные векторы $\mathbf{b}$ и $\mathbf{c}$ и предположим сначала, что $\mathbf{a}$, $\mathbf{b}$ и $\mathbf{c}$ не компланарны. Построим на них ориентированный параллелепипед и примем за его основание параллелограмм, построенный на $\mathbf{b}$ и $\mathbf{c}$ (рис. 15). Введем ориентацию на прямой OH, перпендикулярной основанию. Мы зададим ее с помощью вектора $\mathbf{n}$ длины 1, составляющего с $\mathbf{b}$ и $\mathbf{c}$ правую тройку $\mathbf{n}, \mathbf{b}, \mathbf{c}$. (Тройка $\mathbf{b}, \mathbf{c}, \mathbf{n}$ также правая.)

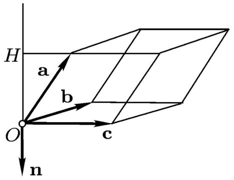


Рис. 15. Здесь тройка $\mathbf{a}, \mathbf{b}, \mathbf{c}$ левая

$(\mathbf{a}, \mathbf{n})$ — скалярная проекция вектора $\mathbf{a}$ на $\mathbf{n}$. По модулю она равна высоте параллелепипеда OH, а знак ее определяется ориентацией тройки $\mathbf{a}, \mathbf{b}, \mathbf{c}$. Действительно, $(\mathbf{a}, \mathbf{n}) > 0$ тогда и только тогда, когда концы векторов $\mathbf{a}$ и $\mathbf{n}$ лежат в одном полупространстве, т. е. тройка $\mathbf{a}, \mathbf{b}, \mathbf{c}$ правая так же, как $\mathbf{n}, \mathbf{b}, \mathbf{c}$. Таким образом, $(\mathbf{a}, \mathbf{n})$ положительно для правой тройки $\mathbf{a}, \mathbf{b}, \mathbf{c}$ и отрицательно для левой.

Пусть положительное число S — площадь основания параллелепипеда. Тогда произведение $(\mathbf{a}, \mathbf{n})S$ по модулю равно объему параллелепипеда, а знак его совпадает со знаком $(\mathbf{a}, \mathbf{n})$. Это значит, что $(\mathbf{a}, \mathbf{b}, \mathbf{c}) = S(\mathbf{a}, \mathbf{n})$. Полученное равенство совпадает с (8), если

$$\mathbf{d} = S\mathbf{n}. \tag{9}$$

Осталось рассмотреть случай, когда $\mathbf{b}$ и $\mathbf{c}$ не коллинеарны, а $\mathbf{a}$, $\mathbf{b}$ и $\mathbf{c}$ компланарны. В этом случае $\mathbf{a}$ лежит в плоскости векторов $\mathbf{b}$ и $\mathbf{c}$ и, следовательно, ортогонален вектору $\mathbf{d}$, вычисленному по формуле (9). Поскольку $(\mathbf{a}, \mathbf{b}, \mathbf{c}) = 0$ и $(\mathbf{a}, \mathbf{n}) = 0$, вектор (9) удовлетворяет равенству (8) и в этом случае. Итак, мы нашли вектор, который удовлетворяет (8) при любом $\mathbf{a}$ и определяется только по $\mathbf{b}$ и $\mathbf{c}$.

Допустим, что для фиксированных $\mathbf{b}$ и $\mathbf{c}$ нашлось два вектора $\mathbf{d}_1$ и $\mathbf{d}_2$ таких, что для любого $\mathbf{a}$ выполнено $(\mathbf{a}, \mathbf{b}, \mathbf{c}) = (\mathbf{a}, \mathbf{d}_1)$ и $(\mathbf{a}, \mathbf{b}, \mathbf{c}) = (\mathbf{a}, \mathbf{d}_2)$. Отсюда следует, что $(\mathbf{a}, \mathbf{d}_1) = (\mathbf{a}, \mathbf{d}_2)$ или $(\mathbf{a}, \mathbf{d}_1 - \mathbf{d}_2) = 0$. Поэтому вектор $\mathbf{d}_1 - \mathbf{d}_2$ ортогонален каждому вектору пространства и, следовательно, равен нулевому вектору. Это доказывает, что вектор $\mathbf{d}$, определяемый формулой (8), может быть только один. Предложение полностью доказано.

Опишем еще раз, как вектор $\mathbf{d}$ определяется по $\mathbf{b}$ и $\mathbf{c}$.

1. Если $\mathbf{b}$ и $\mathbf{c}$ коллинеарны, то $\mathbf{d} = \mathbf{0}$.
2. Если $\mathbf{b}$ и $\mathbf{c}$ не коллинеарны, то:

а) $|\mathbf{d}| = S = |\mathbf{b}||\mathbf{c}| \sin\varphi$, где φ — угол между $\mathbf{b}$ и $\mathbf{c}$;
б) вектор $\mathbf{d}$ ортогонален векторам $\mathbf{b}$ и $\mathbf{c}$;
в) тройка векторов $\mathbf{b}, \mathbf{c}, \mathbf{d}$ имеет положительную ориентацию.

При нашем выборе ориентации пространства — это правая тройка.

Определение. Вектор $\mathbf{d}$, определенный перечисленными выше условиями, или, что то же, формулой (8), называется *векторным произведением* векторов $\mathbf{b}$ и $\mathbf{c}$.

3rd round answers (2)

The two given lines are $\frac{x+7}{3}=\frac{y+4}{4}=\frac{z+3}{-2}=r$ and $\frac{x-21}{6}=\frac{y+5}{-4}=\frac{z-2}{-1}=r_1$.

The coordinates of any point P on the first line are $(3r-7, 4r-4, -2r-3)$.

The coordinates of any point Q on the second line are $(6r_1+21, -4r_1-5, -r+2)$.

The direction ratios of the line PQ are $3r-6r_1-28$, $4r+4r_1+1$, $-2r+r_1-5$.

If PQ is the line of the shortest distance then the two lines are perpendicular. The direction ratios of the two lines are 3, 4, −2 and 6, −4, −1. Then $3(3r-6r_1-28)+4(4r+4r_1+1)-2(-2r+r_1-5)=0$ and $6(3r-6r_1-2r)-4(4r+4r_1+1)-1(-2r+r_1-8)=0$

$$\text{(i.e.)}\quad 29r-4r_1-90=0$$
$$4r-53r_1-167=0$$

Solving for r and r_1, we get

$$\frac{r}{-3042}=\frac{r_1}{4563}=\frac{1}{-1521}\Rightarrow\frac{r}{-2}=\frac{r_1}{3}=\frac{1}{-1}$$
$$\therefore r=2 \text{ and } r_1=-3.$$

The coordinates of P and Q are given by $P(-1, 4, -7)$ and $Q(3, 7, 5)$.

$$\therefore PQ^2=(3+1)^2+(7-4)^2+(5+7)^2=16+9+144=169.$$
$$\therefore PQ=13 \text{ units}$$

The equations of the line of the shortest distance are $\frac{x+1}{3+1}=\frac{y+4}{7-4}=\frac{z+7}{5+7}$

(i.e.) $\frac{x+1}{4}=\frac{y+4}{3}=\frac{z+7}{12}$.

3rd round answers (3)

Let l, m, n be the direction ratios of the line of intersection of the planes $x - 2y - z + 5 = 0$ and $x + y + 3z + 6 = 0$.

Then $\quad l - 2m - n = 0$

and $\quad l + m + 3n = 0$

$$\therefore \frac{l}{-6+1} = \frac{m}{-1-3} = \frac{n}{1+2}$$

$$\text{(i.e.)} \quad \frac{l}{-5} = \frac{m}{-4} = \frac{n}{3}.$$

Since the line also passes through the point (−1, 2, 3), its equations

is $\frac{x-1}{5} = \frac{y-2}{4} = \frac{z-3}{-3}$.

4th round

Prove that the lines $x = ay + b = cz + d$ and $x = \alpha y + \beta = \gamma z + \delta$ are coplanar if $(r - c)(\alpha\beta - bd) - (\alpha - a)(\alpha\delta - \delta\gamma) = 0$.

Find the equation of plane passing through the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$

and parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$.

(i.e.) $11x + 2y - 5z + 6 = 0$

It is known that $AB = 2\sqrt{13}$, $AC = 4$, $BC = 2\sqrt{7}$, and vertices of triangle ABC have coordinates $A(1;\,1)$, $B(5;\,3)$, $C(3;\,5)$ in some affine coordinate system. Find the lengths of basis vectors of this system and the angle between them.

4th round answers (1)

First let us express the given lines in symmetrical form. The two

given lines are $\frac{x}{ac} = \frac{y+\frac{b}{a}}{c} = \frac{z+\frac{d}{c}}{a}$ and $\frac{x}{\alpha\gamma} = \frac{y+\frac{\beta}{\alpha}}{\gamma} = \frac{z+\frac{\delta}{\gamma}}{\alpha}$.

Then two lines are coplanar if $\begin{vmatrix} 0 & \frac{b}{a}-\frac{\beta}{\alpha} & \frac{d}{c}-\frac{\delta}{\gamma} \\ ac & c & a \\ \alpha\gamma & \gamma & \alpha \end{vmatrix} = 0.$

$$\text{(i.e.)} \quad -\left(\frac{b}{a}-\frac{\beta}{\alpha}\right)(ac\alpha - a\alpha\gamma) + \left(\frac{d}{c}-\frac{\delta}{\gamma}\right)(ac\gamma - \alpha c\gamma) = 0$$

$$\text{(i.e.)} \quad -\left(\frac{b\alpha - a\beta}{a\alpha}\right)a\alpha(c-\gamma) + \left(\frac{d\gamma - e\delta}{c\gamma}\right)c\gamma(a-\alpha) = 0$$

$$\text{(i.e.)} \quad (\gamma - c)\,(a\beta - b\alpha) - (a\delta - d\gamma)\,(\alpha - a) = 0$$

4th round answers (2)

$$\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{4} \qquad (13.42)$$

is

$$A(x-1)+B(y+1)+C(z-3)=0 \qquad (13.43)$$

where

$$2A-B+4C=0 \qquad (13.44)$$

Also the line is parallel to the plane

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{3} \qquad (13.45)$$

$$\therefore A+2B+3C=0 \qquad (13.46)$$

Solving for A, B and C from (13.44) and (13.46), we get

$\frac{A}{-3-8}=\frac{B}{4-6}=\frac{C}{4+1}$ or $\frac{A}{11}=\frac{B}{2}=\frac{C}{-5}$

Therefore, the equation of the required plane is $11(x-1)+2(y+1)-5(z-3)=0$.

(i.e.) $11x+2y-5z+6=0$

4th round answers (3)

$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=-161.$

6. Let $\vec{i}$ and $\vec{j}$ be basis vectors. Then $\vec{AB}=4\vec{i}+2\vec{j}$, $\vec{AC}=2\vec{i}+4\vec{j}$, $\vec{BC}=-2\vec{i}+2\vec{j}$.

$$|\vec{AB}|^2=16|\vec{i}|^2+16\,\vec{i}\cdot\vec{j}+4|\vec{j}|^2=52,$$
$$|\vec{AC}|^2=4|\vec{i}|^2+16\,\vec{i}\cdot\vec{j}+16|\vec{j}|^2=16,$$
$$|\vec{BC}|^2=4|\vec{i}|^2-8\,\vec{i}\cdot\vec{j}+4|\vec{j}|^2=28.$$

This is a system of linear equations with variables $|\vec{i}|^2$, $\vec{i}\cdot\vec{j}$, $|\vec{j}|^2$. Solving it yields $|\vec{i}|^2=4$, $|\vec{j}|^2=1$, $\vec{i}\cdot\vec{j}=-1$, therefore $|\vec{i}|=2$, $|\vec{j}|=1$, $\angle(\vec{i},\vec{j})=120^\circ$.

Feedback form (get a link by clicking on qr)

Deserve “A” grade!

– Oleg Bulichev

o.bulichev@innopolis.ru

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Room 105 (Underground robotics lab)