Essentials of Analytical Geometry and Linear Algebra 1

Midterm explanation

Innopolis University

Task 1

1. (4 points) Find the distance from the point $(1, 1, -1)$ to the line of intersection of the planes $x + y + z = 1$ and $2x - y - 5z = 1$.

Task 1 solution tips

1. Obtain a line from general to canonical form
2. Use equation of a distance between line and point

Task 2

2. (4 points) Two vertices of a triangle are $(4, -3)$ and $(-2, 5)$. If the orthocenter (intersection of altitudes) of the triangle is at $(1, 2)$, find the coordinates of the third vertex.

Task 2 solution tips

1. triangle ABC and orthocenter D. We know A, B, D coords.
2. We have 2 var -> need to equations dot(AD,BC)=0 and dot(BD,AC)=0

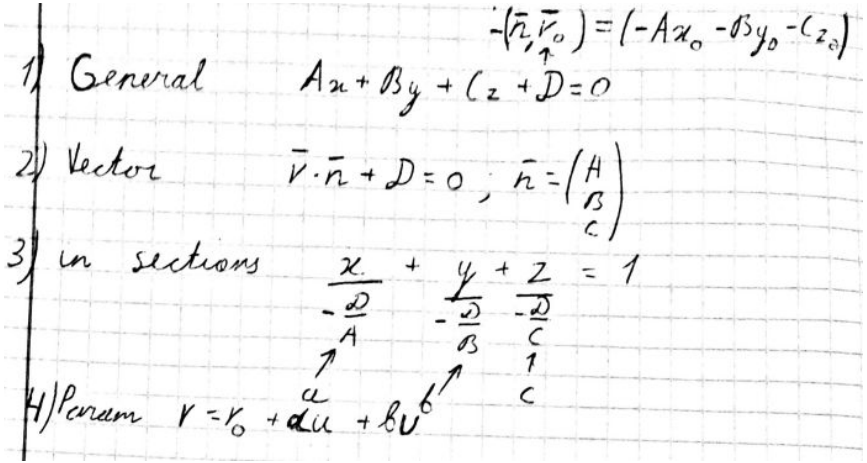
Task 3

(a) Solve the system $\mathbf{Aw} = \mathbf{b}$, where $\mathbf{w} = (x,y,z)$. $\mathbf{A} = \begin{bmatrix} 1 & -3 & 1 \\ -1 & 2 & -5 \\ 5 & -13 & 13 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 4 \\ 3 \\ 8 \end{bmatrix}$

(b) Draw relative positions of the planes that correspond to equations.

Task 3 solution tips

1. Using gauss elimination or whatever - solve (a): it can be either no solutions, or 1 exact solution or infinite number of solutions
2. (b) is about drawing 3 planes Aw=b -> A1x+A2y+A3z+D=0, D = -b
3. It’s easier to draw using in sections form because you need to take denominators as coordinates of the plane



Task 4

Find the distance between the parallel planes $2x-y+2z+2=0$, $6x-3y+6z-4=0$.

Task 4 solution tips (different variant, idea the same)

Find the distance between the parallel planes $2x - 2y + z + 3 = 0$, $4x - 4y + 2z + 5 = 0$.

Solution

Let (x_1, y_1, z_1) be a point on the plane $2x - 2y + z + 3 = 0$

$$\therefore \quad 2x_1 - 2y_1 + z_1 + 3 = 0 \qquad (12.62)$$

Then the distance between the parallel planes is equal to the distance from (x_1, y_1, z_1) to the other plane.

$$= \pm\frac{4x_1 - 4y_1 + 2z_1 + 5}{\sqrt{4^2 + 4^2 + 2^2}} = \pm\frac{(-6+5)}{6} = \frac{1}{6} \quad \text{using (12.62)}$$

The distance between the parallel planes $ax + by + cz + d = 0$ and $ax + by + cz + d_1 = 0$ is $\dfrac{|d - d_1|}{\sqrt{a^2 + b^2 + c^2}}$.

On dividing the equation $4x - 4y + 2z + 5 = 0$ by 2, we get

$$2x - 2y + z + \frac{5}{2} = 0$$

Distance between the planes $= \dfrac{3 - \frac{5}{2}}{\sqrt{9}} = \dfrac{\left(\frac{1}{2}\right)}{3} = \dfrac{1}{6}$.

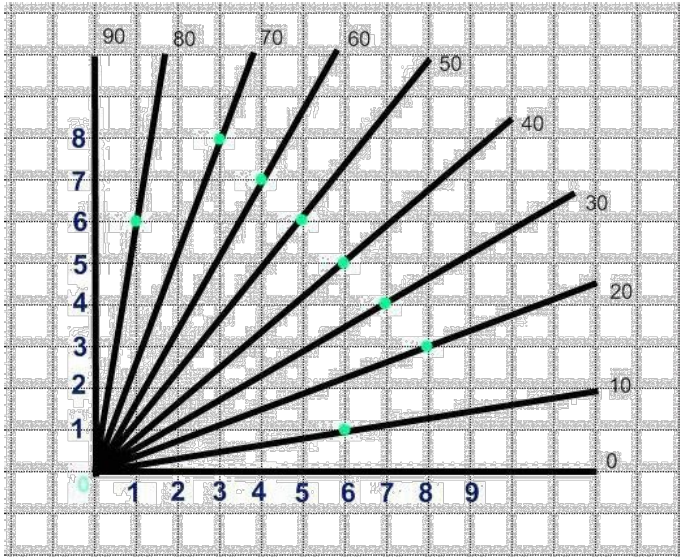
Task 5

(5 points) Diagonals of a rhombus intersect at point $M(1;2)$, the longest of them being parallel to a horizontal axis. The side of the rhombus equals 2 and its obtuse angle is $120°$. Compose the equations of the sides of this rhombus in canonical form.

Task 5 solution tips

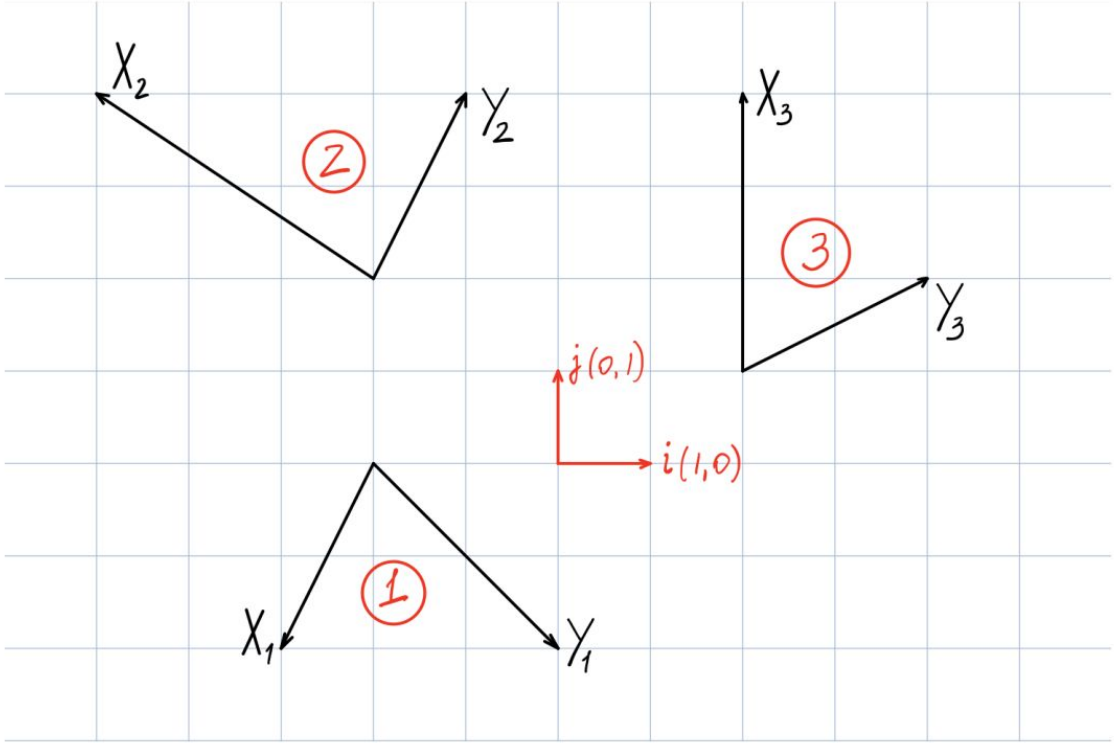
Can be solved by:

1. pure graphical (if you have cells in notebook) and a ruler;
2. using analytical way (cosine and sin rule).



Task 6

(6 points) Find the transition matrix from basis $(\mathbf{x_1}, \mathbf{y_1})$ to the basis $(\mathbf{x_2}, \mathbf{y_2})$.



Task 6 solution tips

1. Based on additional material from 4th lecture link
2. Vector is basis independent

Task 7

(6 points) Apex S of a regular quadrilateral pyramid (four edges form a base, all base edges are equal) $SABCD$ has coordinates $(0, 0, 0)$ in some Cartesian coordinate system. It's also known that vertices A and C have coordinates $(-3, 2, -6)$ and $(1, 4, 2)$ respectively.

(a) Find coordinates of vertices B and C.

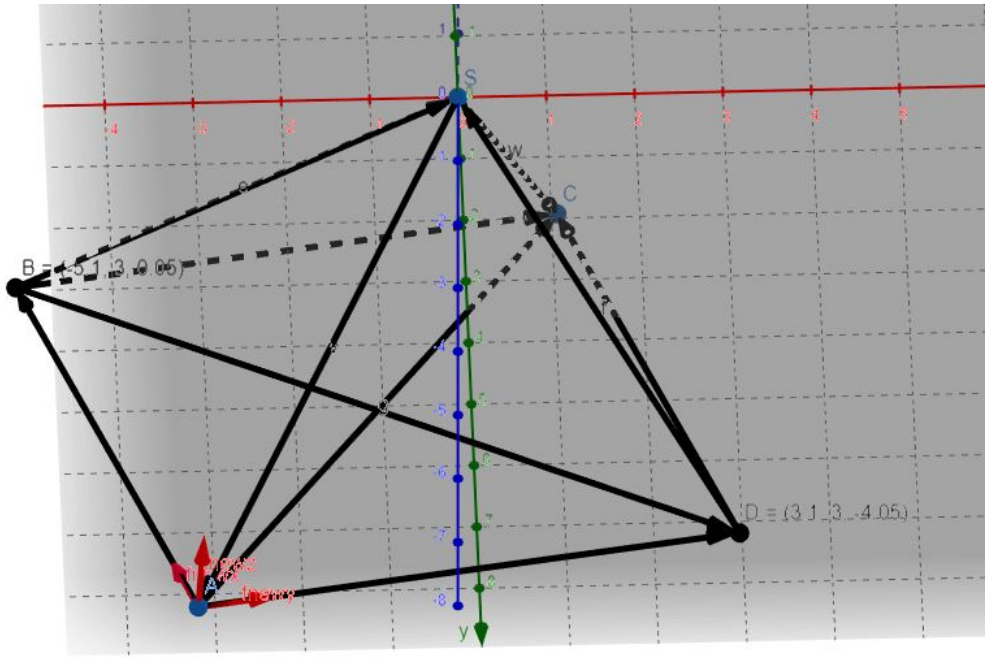
(b) Find the volume of the pyramid.

(c) Find the distance from vertex A to the plane CDS.

(d) Find the distance between lines AB and DS.

Task 7 solution

[Matlab code]
[Geogebra link]



Deserve “A” grade!

– Oleg Bulichev

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Room 105 (Underground robotics lab)