Essentials of Analytical Geometry and Linear Algebra I, Class #10

Innopolis University, November 2020

1. Find the foci, latus rectum, vertices and directrices of the following parabola: $y^2 + 4x - 2y + 3 = 0$.
2. Find the equations of the tangent and normal to the parabola $y^2 = 4(x-1)$ at $(5, 4)$.
3. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$ one of whose vertices is at the vertex of the parabola. Find its side.
4. Find the equation of the ellipse whose foci are $(4, 0)$ and $(-4, 0)$ and $e = 1/3$
5. Find the eccentricity, foci and the length of the latus rectum of the ellipse $9x^2 + 4y^2 = 36$
6. Find the equation of the normal to the ellipse $3x^2 + 2y^2 = 5$ at $(-1, 1)$.
7. The equation $25(x^2 - 6x + 9) + 16y^2 = 400$ represents an ellipse. Find the centre and foci of the ellipse. How should the axis be transformed so that the ellipse is represented by the equation $\frac{x^2}{25} + \frac{y^2}{16} = 1$?
8. Find the locus of the poles with respect to the ellipse of the tangents to the parabola $y^2 = 4px$.

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1. Find the equation of the parabola with the following foci and directrices
   (a) $(1,2):\ x+y-2=0$,
   (b) $(1,-1):\ x-y=0$,
   (c) $(0,0):\ x-2y+2=0$
2. Find the foci, latus rectum, vertices and directrices of the following parabolas:
   $y^2-4x+2y-3=0$,
   $y^2-8x-9=0$.
3. Find the condition that the straight line $lx+my+n=0$ is a tangent to the parabola.
4. Find the equation of the ellipse whose foci, directrix and eccentricity are given below:
   (a) Focus is $(1,2)$, directrix is $2x-3y+6=0$ and eccentricity is $2/3$
   (b) Focus is $(0,0)$, directrix is $3x+4y-1=0$ and eccentricity is $5/6$
   (c) Focus is $(1,-2)$, directrix is $3x-2y+1=0$ and eccentricity is $1/\sqrt{2}$
5. Find the equation of the ellipse whose foci are $(3,0)$ and $(-3,0)$ and $e=\sqrt{\frac{3}{8}}$
6. Find the eccentricity, foci and the length of the latus rectum of the ellipse
   (a) $3x^2+4y^2-12x-8y+4=0$,
   (b) $25x^2+9y^2-150x-90y+225=0$
7. Find the equation of the tangent to the ellipse $x^2+2y^2=6$ at $(2,-1)$.
8. Find the angle subtended by a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passing through an end of the minor axis at the centre of the ellipse.
9. Find the locus of the point of intersection of normals at two points on an ellipse which are extremities of conjugate diameters.