Innopolis University
Essentials of Analytical Geometry and Linear Algebra I
Test I.

October 2, 2020.

VARIANT 1

Full name:	Group:

Task:	1	2	3	4	5	6	7	8	Total
Score:									

1. (1 point) For each of the following statements mark it as True or False. Justify each answer.

   (a) $\det \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix} = 0$ True / False

   (b) The result of Dot product operation is a vector. True / False

   (c) Rank is a number of columns of a matrix. True / False

   (d) Inverse matrix (A^{-1}) is always exists. True / False

   (e) It is always possible to change one basis to any other basis of the same space. True / False

2. (2 points)

   (a) Find the determinant of the following matrix: $\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 0 & 2 & 4 \\ 2 & 1 & 3 & 1 \end{bmatrix}$

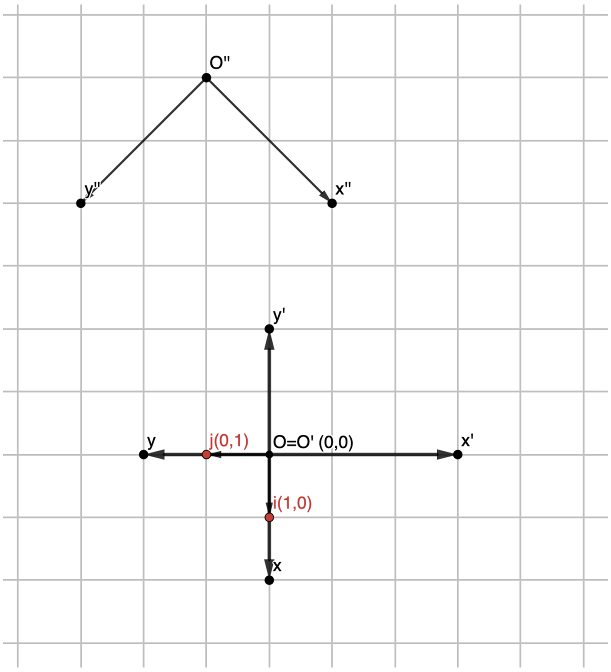
   (b) Let A be a square matrix. Show that its left and right inverses are the same matrix.

3. (2 points) Find angles between vectors $\mathbf{a}$ and $\mathbf{b}$, $\mathbf{b}$ and $\mathbf{c}$, $\mathbf{a}$ and $\mathbf{c}$.
   $\mathbf{a} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$

4. (2 points) For which values x, vectors $\mathbf{a}$ and $\mathbf{b}$ are basis of some space? Prove your answers.
   $\mathbf{a} = \begin{bmatrix} x \\ 1-x \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} x \\ 2 \end{bmatrix}$

5. (2 points) Prove that the result of a cross product will not changes if to one of the vectors add vector $\mathbf{x}$ such that $\mathbf{x}$ is a collinear to another vector.

6. (2 points) Find a transformation matrix from xOy to $x'O'y'$.



7. (2 points) Find all face areas of a parallelepiped, if its edges are:
$\begin{bmatrix}2\\7\\0\end{bmatrix}, \begin{bmatrix}1\\2\\4\end{bmatrix}, \begin{bmatrix}3\\3\\3\end{bmatrix}$

8. (2 points) Find rank of the following matrix for all possible values of parameter α, $\alpha \in R$. Explain your answers.
$\begin{bmatrix}1 & 2 & 2 & \alpha\\ \alpha & 4 & 4 & 2\\ 1 & \alpha & 2 & 1\end{bmatrix}$

End of Test 1

VARIANT 2

Full name:	Group:

Task:	1	2	3	4	5	6	7	8	Total
Score:									

1. (1 point) For each of the following statements mark it as True or False. Justify each answer.
   (a) Rank is a number of rows of a matrix. True / False
   (b) For any matrix A there exists only one inverse matrix. True / False
   (c) The determinant of a matrix is always exists. True / False
   (d) Two vectors always form a basis. True / False
   (e) Result of matrix multiplication operation is always defined. True / False

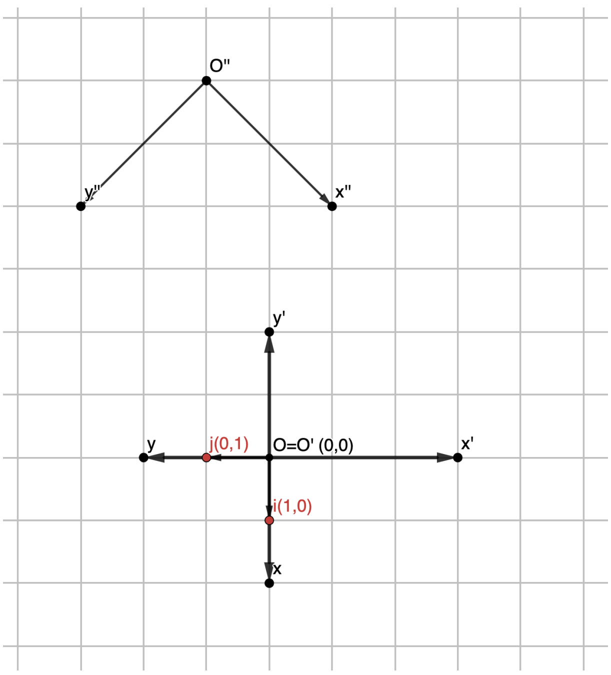
2. (2 points)
   (a) Find the determinant of the following matrix: $\begin{bmatrix} 1 & 3 & 3 & 1 \\ 3 & 0 & 1 & 3 \\ 3 & 2 & 2 & 2 \end{bmatrix}$
   (b) Let A be a square matrix. Show that its left and right inverses are the same matrix.

3. (2 points) Find angles between vectors $\mathbf{a}$ and $\mathbf{b}$, $\mathbf{b}$ and $\mathbf{c}$, $\mathbf{a}$ and $\mathbf{c}$.
$\mathbf{a} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$

4. (2 points) For which values x, vectors $\mathbf{a}$ and $\mathbf{b}$ are basis of some space? Prove your answers.
$\mathbf{a} = \begin{bmatrix} 1-x \\ -x \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1-x \\ 1 \end{bmatrix}$

5. (2 points) Prove that the result of a cross product will not changes if to one of the vectors add vector $\mathbf{x}$ such that $\mathbf{x}$ is a collinear to another vector.

6. (2 points) Find a transformation matrix from $x''O''y''$ to $x'O'y'$.



7. (2 points) Find all face areas of a parallelepiped, if its edges are:
$\begin{bmatrix}1\\5\\2\end{bmatrix}, \begin{bmatrix}2\\1\\0\end{bmatrix}, \begin{bmatrix}2\\2\\2\end{bmatrix}$

8. (2 points) Find rank of the following matrix for all possible values of parameter α, $\alpha \in R$. Explain your answers.
$\begin{bmatrix}1 & 3 & 3 & \alpha\\ \alpha & 6 & 6 & 3\\ 1 & \alpha & 3 & 1\end{bmatrix}$

End of Test 1

VARIANT 3

Full name:	Group:

Task:	1	2	3	4	5	6	7	8	Total
Score:									

1. (1 point) For each of the following statements mark it as True or False. Justify each answer.
   (a) Rank can be greater than a number of rows of a matrix. True / False
   (b) The result of Cross product operation is a vector. True / False
   (c) For a square matrix A: $detA^\top = -detA$. True / False
   (d) Every vector space has a basis True / False
   (e) Any 2D plane is a subspace of R^3 True / False

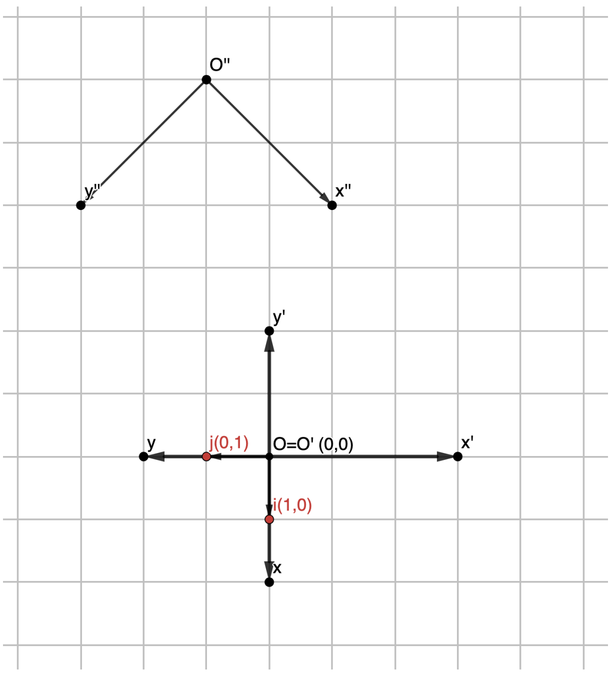
2. (2 points)
   (a) Find the determinant of the following matrix: $\begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 3 & 0 & 3 \\ 2 & 2 & 1 & 1 \end{bmatrix}$
   (b) Let A be a square matrix. Show that its left and right inverses are the same matrix.

3. (2 points) Find angles between vectors **a** and **b**, **b** and **c**, **a** and **c**.
   $\mathbf{a} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

4. (2 points) For which values x, vectors **a** and **b** are basis of some space? Prove your answers.
   $\mathbf{a} = \begin{bmatrix} x \\ 3 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} x \\ 2-x \end{bmatrix}$

5. (2 points) Prove that the result of a cross product will not changes if to one of the vectors add vector **x** such that **x** is a collinear to another vector.

6. (2 points) Find a transformation matrix from xOy to $x''O''y''$.



7. (2 points) Find all face areas of a parallelepiped, if its edges are:
$\begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$

8. (2 points) Find rank of the following matrix for all possible values of parameter α, $\alpha \in R$. Explain your answers.
$\begin{bmatrix} 2 & \alpha & 1 & 4 \\ \alpha & 3 & 3 & 2 \\ 2 & 1 & \alpha & 4 \end{bmatrix}$

End of Test 1

VARIANT 4

Full name:	Group:

Task:	1	2	3	4	5	6	7	8	Total
Score:									

1. (1 point) For each of the following statements mark it as True or False. Justify each answer.
   (a) Rank of a matrix is always defined. True / False
   (b) If B is produced by interchanging two rows of A, then $detB = detA$. True / False
   (c) The set of vectors $\begin{bmatrix}1\\2\end{bmatrix}$, $\begin{bmatrix}1\\3\end{bmatrix}$ form a basis that spans some 2D plane. True / False
   (d) Any subset of vectors form a subspace. True / False
   (e) Multiplication by scalar operation is always applicable. True / False

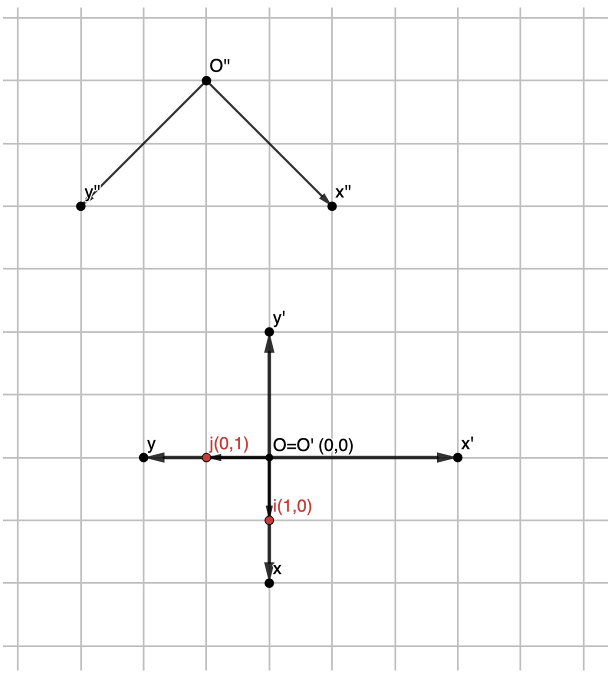
2. (2 points)
   (a) Find the determinant of the following matrix: $\begin{bmatrix}2 & 3 & 1 & 3\\1 & 1 & 3 & 1\\1 & 2 & 0 & 2\end{bmatrix}$
   (b) Let A be a square matrix. Show that its left and right inverses are the same matrix.

3. (2 points) Find angles between vectors **a** and **b**, **b** and **c**, **a** and **c**.
   $\mathbf{a} = \begin{bmatrix}1\\2\\1\end{bmatrix}$, $\mathbf{b} = \begin{bmatrix}-2\\-1\\2\end{bmatrix}$, $\mathbf{c} = \begin{bmatrix}-1\\3\\1\end{bmatrix}$

4. (2 points) For which values x, vectors **a** and **b** are basis of some space? Prove your answers.
   $\mathbf{a} = \begin{bmatrix}-x\\3-x\end{bmatrix}$, $\mathbf{b} = \begin{bmatrix}-x\\1\end{bmatrix}$

5. (2 points) Prove that the result of a cross product will not changes if to one of the vectors add vector **x** such that **x** is a collinear to another vector.

6. (2 points) Find a transformation matrix from $x'O'y'$ to xOy.



7. (2 points) Find all face areas of a parallelepiped, if its edges are:
$$\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

8. (2 points) Find rank of the following matrix for all possible values of parameter α, $\alpha \in R$. Explain your answers.
$$\begin{bmatrix} 1 & \alpha & 2 & 1 \\ \alpha & 4 & 3 & 4 \\ 1 & 1 & \alpha & 2 \end{bmatrix}$$

End of Test 1