Homework Berezneo B20-02

Problems 1-6

(1)

Person	1	2
Number of ways to take not respect has	(n-1)	(n-2)

(2) $P(\cup_{i=1}^{n} E_i) = \sum_{i=1}^{n} p(E_i)$

<u>Base</u> $n=1$: $P(E_1) = p(E_1)$ it is <u>true</u>

<u>Induction case</u>. Let's prove that if it is true for some k, then it is true for $k+1$.

We know that $|A_1 \cup A_2 \cup A_3 \cup .. \cup A_n| =$

$= \sum_{i=1}^{n} |A_i| - \sum_{i,j=1 (i<j)}^{n} |A_i \cap A_j| + \dots + (-1)^{n+1} |A_1 \cap A_2 \cap .. \cap A_n|$

So for our case (as all events are pairwise disjoint)

$|E_1 \cup E_2 \cup .. \cup E_k| = \sum_{i=1}^{k} |E_i|$

Suppose $|E_1 \cup E_2 \cup .. \cup E_k| = K$

then for $k+1$:

$|K \cup E_{k+1}| = |K| + |E_{k+1}| - |K \cap E_{k+1}|$ → 0 because K and E_{k+1} are disjoint

So $|K \cup E_{k+1}| = |K + E_{k+1}| \Leftrightarrow |\cup_{i=1}^{k+1} E_i| = \sum_{i=1}^{k+1} |E_i| \Leftrightarrow$

$\Leftrightarrow P(\cup_{i=1}^{k+1} E_i) = \sum_{i=1}^{k+1} p(E_i)$ as was to be proved prone to COVID | non-prone to COVID

(3) workers: $0,3$ | $0,7$

$p(\text{COVID})$ $0,4$ | $0,2$

a) $P = 0,3 \cdot 0,4 + 0,7 \cdot 0,2 = \underline{0,26} = P(c)$

b) $\cancel{P}$ $\cancel{P(}$

$\cancel{P(0,4 \cap 0,3) = \frac{P(0,3 | 0,4) \cdot 0,4}{0,26}}$

$$P(C | Pr) = P(Pr | C) \cdot \frac{P(C)}{P(Pr)} \approx \underline{\underline{0,347}}$$

$P(C) = 0,26$

$P(Pr) = 0,3$

$P(Pr | C) = 0,4$

Bias Theorem

(4) $A - 20\%$ $p(A) = 6\%$

$B - 30\%$ $p(B) = 7\%$

$C - 50\%$ $p(C) = 8\%$

$$p(E) = 0,2 \times 0,06 + 0,3 \times 0,07 + 0,5 \times 0,08 =$$
$$= 0,012 + 0,021 + 0,04 =$$
$$= \underline{\underline{0,073}}$$

(5) $\{1, 2, 3, 4\}$

a) $$\frac{1 \cdot \underbrace{3 \cdot 2 \cdot 1}_{\text{elements}}}{n!} + \frac{2 \cdot 1 \cdot 2 \cdot 1}{n!} + \frac{2 \cdot 1 \cdot 1 \cdot 4}{n!} =$$

$= \frac{12}{24} = \underline{0,5}$

b) $\underline{0,5}$ see above

c) $$\frac{4 \cdot 3 \cdot 2 \cdot 1}{n!} + \frac{1 \cdot 4 \cdot 2 \cdot 1}{n!} = \frac{8}{24} = \underline{\frac{1}{3}}$$

d) $\frac{4 \cdot 3 \cdot 2 \cdot 1}{n!} = \frac{6}{24} = \frac{1}{4} = 0{,}25$

e) $\frac{4 \cdot 3 \cdot 2 \cdot 1}{n!} + \frac{4 \cdot 2 \cdot 2 \cdot 1}{n!} + \frac{2 \cdot 4 \cdot 2 \cdot 1}{n!} +$

$\cancel{\frac{3 \cdot 4 \cdot 2 \cdot 1}{n!}} + \frac{2 \cdot 1 \cdot 4 \cdot 3}{n!} + \cancel{\frac{2 \cdot 1 \cdot 4 \cdot 1}{n!}} = \frac{6}{24} = \frac{1}{4}$

$= 0{,}25$

Notation: $n_1 \cdot C \cdot n_2 \cdot n_3$ - number of possible values on this fixed position

C - Constant on fixed positions

6. 4 . . . . . .

$P(3 \text{ Heads out of } 4) = C_4^3 \cdot \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^1 =$

$= \frac{4}{24} = \frac{1}{4} = 0{,}25$

1

$$P_n = \sum_{i=2}^{n} \frac{(-1)^i}{i!}$$

$$D_n = n! - \binom{n}{1}(n-1)! + \binom{n}{2}(n-2)! + (-1)^n \binom{n}{n} 0!$$