Mathematical Analysis. Assignment 7. Antiderivatives

1. Find the following integrals:

(a)
$$\int \frac{\sqrt{4+x^2}+2\sqrt{4-x^2}}{\sqrt{16-x^4}} dx;$$

(b)
$$\int 2^{2x} e^x dx$$
;

(c)
$$\int \frac{dx}{3x^2-x^4}$$
;

(d)
$$\int \sin^2 \frac{x}{2} dx$$
;

(e)
$$\int \frac{dx}{5-12x-9x^2}$$
;

$$(f) \int \frac{dx}{2x^2 - 5x + 7};$$

(g)
$$\int \frac{dx}{\sqrt{x^2 - 2x + 5}};$$

$$(h) \int \frac{dx}{\sqrt{17-4x-x^2}};$$

(i)
$$\int \frac{3x-2}{2-3x+5x^2} dx$$
;

(j)
$$\int \frac{x+3}{\sqrt{3+4x-4x^2}} dx$$
;

(k)
$$\int \sqrt{x^2 + 2x + 5} \, dx;$$

(l)
$$\int \frac{x^5}{x+1} \, dx;$$

(m)
$$\int x\sqrt{1+x}\,dx$$
;

(n)
$$\int \frac{\sqrt[3]{x} \, dx}{x(\sqrt{x} + \sqrt[3]{x})};$$

(o)
$$\int e^{2x^2+2x-1}(2x+1) dx$$
;

(p)
$$\int \frac{dx}{\cosh x}$$
;

(q)
$$\int \frac{\ln^2 x}{x} dx$$
;

(r)
$$\int \ln \frac{1+x}{1-x} \cdot \frac{dx}{x^2-1};$$

2. Find the following integrals:

(a)
$$\int \frac{\sin x}{\sqrt{\cos 2x}} dx$$
;

(b)
$$\int \frac{\ln \tan x}{\sin 2x} \, dx;$$

(c)
$$\int \frac{\sqrt[3]{\operatorname{arccot} x}}{1+x^2} dx;$$

(d)
$$\int (x^2 - 2x + 3) \ln(x + 1) dx$$
;

(e)
$$\int \arctan \sqrt{x} \, dx$$
;

(f)
$$\int (x^2 - 6x + 2) e^{3x} dx$$
;

(g)
$$\int \sin x \cosh x \, dx$$
;

(h)
$$\int x \arccos \frac{1}{x} dx$$
.

3. Find integrals of some rational functions:

(a)
$$\int \frac{x^5 - 2x^2 + 3}{x^2 - 4x + 4} dx$$
;

(b)
$$\int \frac{dx}{(x-2)^2(x+3)^3}$$
;

(c)
$$\int \frac{x^2 dx}{(x+1)(x^3+1)}$$
;

(d)
$$\int \frac{dx}{x^4 - x^3 - x + 1}.$$

4. Find integrals of some irrational functions:

(a)
$$\int \frac{\sqrt{x-1} - \sqrt{x+1}}{\sqrt{x-1} + \sqrt{x+1}} dx;$$

(b)
$$\int \sqrt[3]{\frac{x+1}{x-1}} \, dx;$$

(c)
$$\int \frac{dx}{\sqrt[6]{(x-7)^7(x-5)^5}};$$

(d)
$$\int \frac{dx}{\sqrt[3]{4x^2+4x+1}-\sqrt{2x+1}};$$

(e)
$$\int \frac{x^3 - 6x^2 + 11x - 6}{x^2 + 4x + 3} dx$$
;

(f)
$$\int x^2 \sqrt[3]{(x+1)^2} \, dx$$
;

(g)
$$\int \sqrt[3]{1 + \sqrt[4]{x}} \, dx$$
;

(h)
$$\int \frac{dx}{\sqrt[3]{1+x^3}}.$$

5. Find integrals of some transcendental functions:

(a)
$$\int \sin x \sin 2x \sin 3x \, dx$$
;

(b)
$$\int \frac{\sin 3x}{\cos x} dx$$
;

(c)
$$\int \cos^3 x \sin^8 x \, dx$$
;

(d)
$$\int \frac{\sin^4 x}{\cos x} dx$$
;

(e)
$$\int \frac{dx}{\sin^2 x \cos^3 x};$$

(f)
$$\int \frac{\cos^3 x}{\sin^5 x} dx$$
;

(g)
$$\int \frac{dx}{\sin^2 x \cos^4 x}$$

$$(h) \int \frac{\cos x \, dx}{\sin^2 x - 6\sin x + 5}$$

(g)
$$\int \frac{dx}{\sin^2 x \cos^4 x};$$
(h)
$$\int \frac{\cos x \, dx}{\sin^2 x - 6 \sin x + 5};$$
(i)
$$\int \frac{dx}{2 \cos^2 x + \sin x \cos x + \sin^2 x};$$

$$(j) \int \frac{dx}{7\cos x - 4\sin x + 8};$$

(k)
$$\int \frac{e^x + e^{3x}}{1 - e^{2x} + e^{4x}} dx;$$

(l)
$$\int \frac{dx}{\sqrt{1+e^x}+\sqrt{1-e^x}};$$

(m)
$$\int \ln \left(\sqrt{x+1} + \sqrt{x-1}\right) dx;$$

(n)
$$\int \frac{x \, dx}{\sin^2 x}$$
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