Homework 6

1. Read textbook Kenneth H. Rosen "Discrete Mathematics and Its Applications, 7th Edition“ – paragraphs 9.2, 9.3, 9.4
2. Complete exercises 6-7, 9-19 (ex 15-16 and graphs in ex 17 are optional) and submit on Moodle by 10pm on Friday 16 October.

Exercise 6

Prove that for any binary relations R_1 and R_2 the following holds:

$(R_1 \circ R_2)^{-1} = {R_2}^{-1} \circ {R_1}^{-1}$

Exercise 7

Prove that if R and S are antisymmetric, then $R \cap S$ is antisymmetric as well.

Exercise 9

For the set $X = \{1, 2, 3, 6\}$ and the relation $R = \{(x, y) \mid x, y \in X, x \text{ is a divisor of } y\}$ show that the relation is the relation of order. Are there minimal and maximal elements in set X?

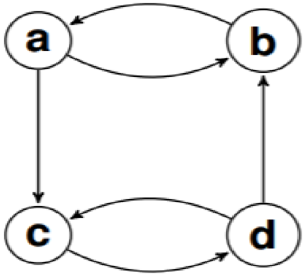
Exercise 10

Let R be the relation on the set {0, 1, 2, 3} containing the ordered pairs (0, 1),(1, 1),(1, 2),(2, 0),(2, 2),(3, 0). Find:

a) reflexive closure of R
b) symmetric closure of R

Exercise 11

For this directed graph



a) Find the reflexive closure (draw a graph)
b) Find the symmetric closure (draw a graph)

Exercise 12

Let S and T be binary relations on some set. Prove that:

a) $(S \cup T)^{-1} = (S^{-1}) \cup (T^{-1})$

b) $(S \circ T)^{-1} = (T^{-1}) \circ (S^{-1})$

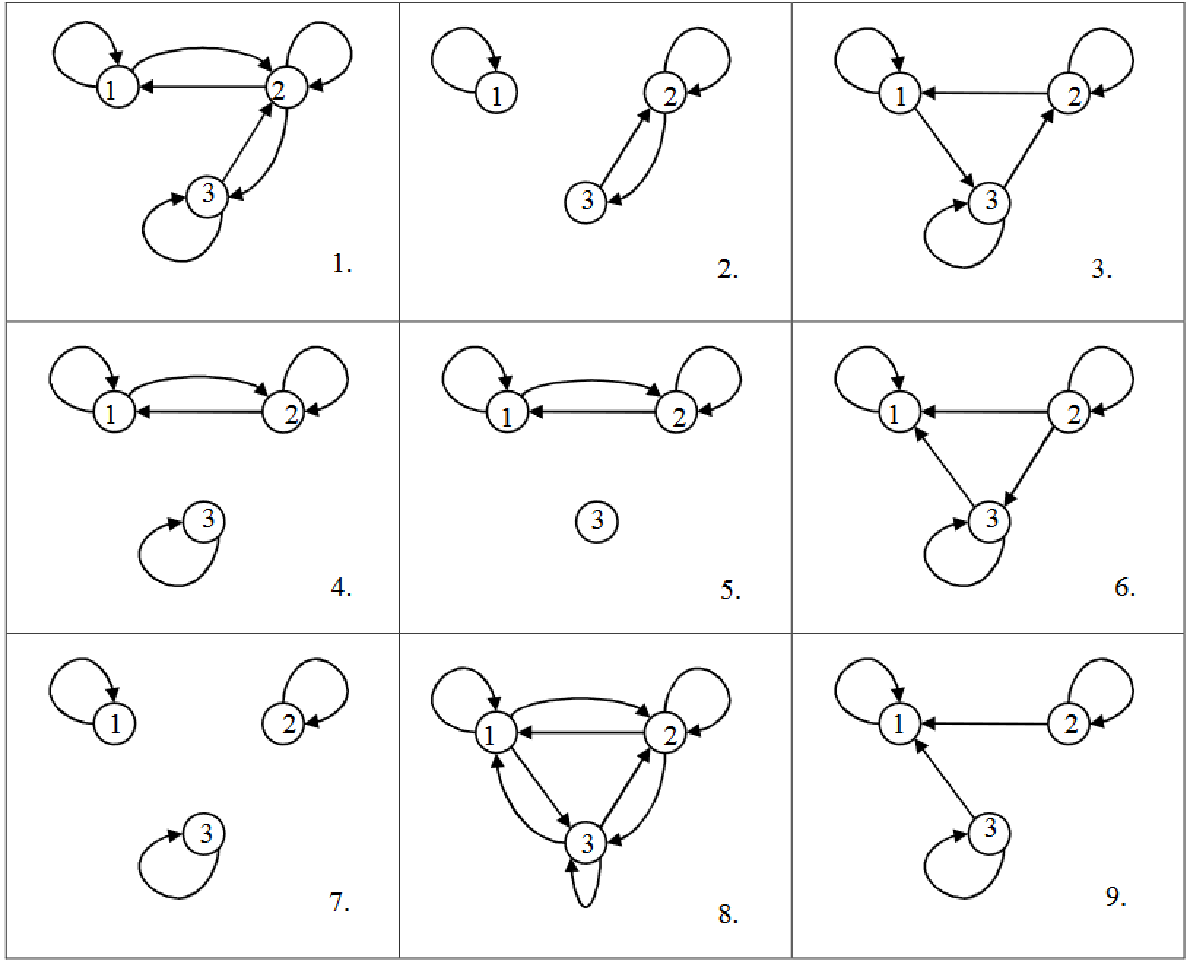
Exercise 13

Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if

A) a is taller than b.
B) a and b were born on the same day.
C) a has the same first name as b.
D) a and b have a common grandparent

Exercise 14

Which of the following relations are reflexive, symmetric, transitive?



Exercise 15 (optional)

Let us fix a finite set with n>0 elements and represent a binary relation R on this set by boolean matrices R[i,j]= if iRj then True else False.
Write boolean expressions for elements of matrices for

1. $(X \cup Y)$,
2. $(X \circ Y)$,
3. $(X)^{-1}$,

assuming that matrices for binary relations X and Y are given;
Describe algorithm that computes matrix for (1),(2),(3).

Exercise 16 (optional)

Let P be any property (e.g. reflexivity, symmetry, transitivity, etc.) of binary relations on a set. For any binary relation R on a set let P-closure of R be the smallest (the least) binary relation S that contains R. For a given binary relation R express in terms of R and operations on binary relations (including the inverse):

- symmetric closure of R,
- reflexive closure of R,
- transitive closure of R

Exercise 17

Are the following relations on N reflexive, transitive, (a/anti)symmetric?

$R_1 \colon a\ R_1\ b \ \leftrightarrow |a-b| = 1$

$R_2 \colon a\ R_2\ b \ \leftrightarrow \ 0 < a-b < 3$

$R_3 \colon a\ R_3\ b \ \leftrightarrow \ a+b \text{ – } even$

$R_4 \colon a\ R_4\ b \ \leftrightarrow \ a \geq b^2$

$R_5 \colon a\ R_5\ b \leftrightarrow greatest\ common\ divisor(a,b) = 1$

(optional) Draw graph for:

a) $R_1 \cap R_2$;

b) $R_1 \cup R_2$;

c) R_2^{-1};

d) $R_2 \circ R_4$;

e) $R_4 \circ R_2$;

f) $R_5 \backslash R_4^{-1}$

Exercise 18

How can the matrix for $\bar{R}$, the complement of the relation R , be found from the matrix representing R, when R is a relation on a finite set A?

Exercise 19

What is the symmetric closure of the relation $R = \{(a, b) \mid a > b\}$ on the set of positive integers?