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B20-02

1. $\underbrace{\binom{13}{1}}_{\text{because we have 13 suits}} \underbrace{\binom{4}{3}}_{\text{we need 3 cards of one suit}} \cdot \underbrace{\binom{12}{1}}_{\text{then we have only 12 suits}} \cdot \underbrace{\binom{4}{2}}_{\text{and need 2 cards of this suit}} = \cancel{4}, 3744$

2. We have $\binom{n}{k}$ combinations.
Suppose now that we already pick "1" and "2"
Then $\binom{n-2}{k-2}$ is number of ~~pairs~~ combinations
which contains n elements and includes "1" and "2"
So answer is $\binom{n}{k} - \binom{n-2}{k-2}$

3. $100/4 = 25$ ~~in~~ integers are divisible by 4
$100/6 = 16$ integers are divisibly by 6
$100/12 = 8$ integers are divisible by 12 (by 4 and by 6)
Answer is $25 + 16 - 8 = 33$

4. a) $\left(\binom{4}{4} + \binom{4}{3} + \binom{4}{2} + \binom{4}{1}\right)^{10} =$
$= (1 + 4 + 6 + 4)^{10} = 15^{10}$

b) $\left(\binom{4}{4} + \binom{4}{3} + \binom{4}{2} + \binom{4}{1} + \binom{4}{0}\right)^{10} =$
$= (1 + 4 + 6 + 4 + 1)^{10} = 16^{10}$

5. $x_1 + x_2 + x_3 + x_4 = 15$
$$\begin{cases} x_1 \geq 2 \\ x_2 \geq 3 \\ x_3 \geq 10 \\ x_4 \geq -3 \end{cases}$$
$x_4 = 0 \to 1$ variant
$x_4 = -1 \to \binom{3}{1}$ variants
$x_4 = -2 \to \binom{3}{1} + \binom{3}{2}$ variants
$x_4 = -3 \to \binom{3}{1} + 2\cdot\binom{3}{2} + \binom{3}{3}$ variants
$x_4 > 0 \to 0$ variants $x_4 = 15 - (x_1 + x_2 + x_3) \leq 0$

Total: $1 + 3\cdot\binom{3}{1} + 3\binom{3}{2} + \binom{3}{3} =$
$= 1 + 9 + 9 + 1 = 20$

Answer: 20

6. If we want to sample each type at least once, lets include ~~it~~ them in our combinations.

Now we have $13-5=8$ "free" positions for slices.

As we have 5 types of pizza, we have 4 separators.

In total $4+8=12$ elements.

So the answer is $\binom{12}{4} = \frac{12!}{4!\,8!} = \underline{495}$ variants

7. 8 types

(a) $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 = \underline{\underline{8^6}} = \underline{\underline{2^{18}}}$

(b) $A: 8 \cdot 8 \cdot 8$
$B: 8 \cdot 8 \cdot 8$ $= 8^6 = \underline{\underline{2^{18}}}$

(c) $A: 8 \cdot 8 \cdot 8$ but
$8 \cdot 8 \cdot 8.$ we count same sets twice so, $8^6/2 = \underline{\underline{2^{17}}}$