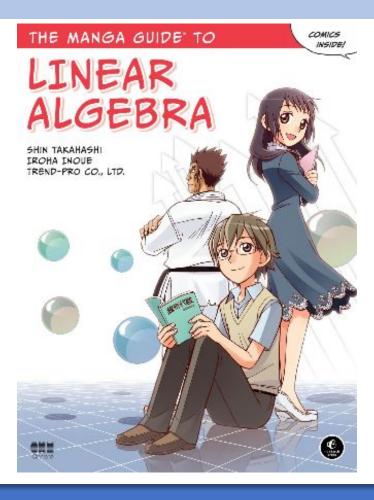


# Essentials of Analytical Geometry and Linear Algebra 1

Strategy for efficient exam solving Test 1 solutions







#### Exam / test

- 1. Preparation before test
- 2. Preparation in a day of test
- 3. Behavior on exam



# Strategy for efficient exam solving

**Problem:** During exam, spend too much time on finding the solution

**Solution:** Find the right strategy for *preparation* and *behavior during an exam*.

#### **Approx time consument:**

Preparation: 3-8 hours in overall

Exam:

o find the idea how to solve — 10 sec - 2 min

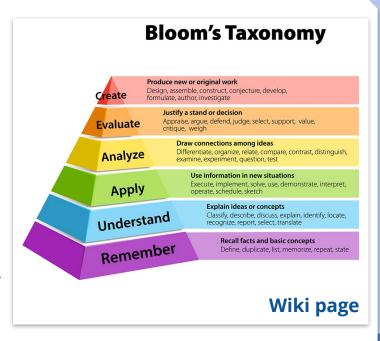
implement it — 10-20 min



# **Preparation strategy**

- Understand the concept of a new topic (<u>Apply</u> or <u>Analyze</u> in terms of Bloom's Taxonomy)
  - a. Look at slides and videos
  - Play with concept (suggest some ideas and prove it or disprove via computer or hand calculations)
- 2. Take a book (material) with exercises and solutions for it.
  - a. Look at the task, imagine how to solve it.
  - b. Check it from solutions. If you sure that your solution is also applicable check it.

For some tasks *practical skills* are important (find not only an idea, but implement it)!





#### Behavior on and before exam

Before: Prepare your brain (skim the material) and mentality (by self hypnosis techniques) (science russian book "Преодолей себя! Психическая подготовка в спорте")

#### During an exam:

- 1. Ranking tasks by doing speed
  - a. Can solve on-a-fly (expect max grade)
  - b. Easy concept tough implementation (expect that some computational mistakes can be done)
  - c. Tough concept (cannot find the solution on-a-fly) (time consuming tasks)
- 2. Solve it in these order
- 3. **.**..
- 4. Profit

- (a) Rank is a number of rows of a matrix. True / False
- (b) For any matrix A there exists only one inverse matrix. True / False
- (c) The determinant of a matrix is always exists. True / False
- (d) Two vectors always form a basis. True / False
- (e) Result of matrix multiplication operation is always defined. True / False
- (d) Inverse matrix  $(A^{-1})$  is always exists. True / False
- (e) It is always possible to change one basis to any other basis of the same space. True / False
- (c) For a square matrix A:  $det A^{\top} = -det A$ . True / False



- (a) Find the determinant of the following matrix:  $\begin{bmatrix} 1 & 3 & 3 & 1 \\ 3 & 0 & 1 & 3 \\ 3 & 2 & 2 & 2 \end{bmatrix}$
- (b) Let A be a square matrix. Show that its left and right inverses are the same matrix.



(2 points) Find angles between vectors **a** and **b**, **b** and **c**, **a** and **c**.

$$\mathbf{a} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \ \mathbf{c} = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$



(2 points) For which values x, vectors **a** and **b** are basis of some space? Prove your answers.

$$\mathbf{a} = \begin{bmatrix} 1 - x \\ -x \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 1 - x \\ 1 \end{bmatrix}$$

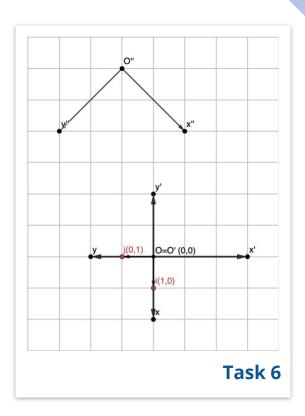


(2 points) Prove that the result of a cross product will not changes if to one of the vectors add vector  $\mathbf{x}$  such that  $\mathbf{x}$  is a collinear to another vector.

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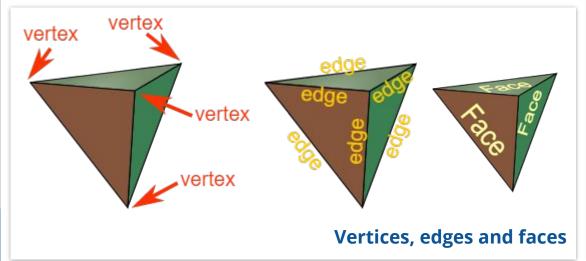
(2 points) Find a transformation matrix from x''O''y'' to x'O'y'.

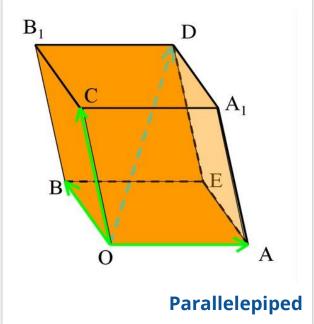


# Task 7 (1)

(2 points) Find all face areas of a parallelepiped, if its edges are:

$\lceil 1 \rceil$		$\lceil 2 \rceil$		$\lceil 2 \rceil$
5	,	1	,	2
2	Ů	0		2





(2 points) Find rank of the following matrix for all possible values of parameter  $\alpha$ ,  $\alpha \in R$ . Explain your answers.

$$\begin{bmatrix} 1 & 3 & 3 & \alpha \\ \alpha & 6 & 6 & 3 \\ 1 & \alpha & 3 & 1 \end{bmatrix}$$

