Mid-term examination on Discrete Mathematics & Logic

(Innopolis University, October 19, 2020)

Task 1 (4 points)

What is a formula of propositional logic? Prove $A \vee (B \,\&\, C) = (A \vee B) \,\&\, (A \vee C)$

Task 2 (5 points)

For sets $A,\ B,\ C$, prove $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Task 3 (5 points)

Prove by induction that, for any natural number $n \geq 3$, $n^2 \geq 2n + 1$

Task 4 (5 points)

For sets A, B, prove $|A \cup B| = |A| + |B|$, if $A \cap B = \varnothing$

Task 5 (6 points)

The relation R on the set $S = \{3, 4, 8, 9\}$ is defined as: $xRy \Leftrightarrow x \bmod y$ is odd. (Recall "$a \bmod b$" stands for the modulo operation that finds the remainder after division of a by b.)

1) Write R down as a set with all of its elements.
2) Check if R is reflexive, irreflexive, symmetric, anti-symmetric, asymmetric, and/or transitive.
3) Conclude whether R is a relation of equivalence or not.