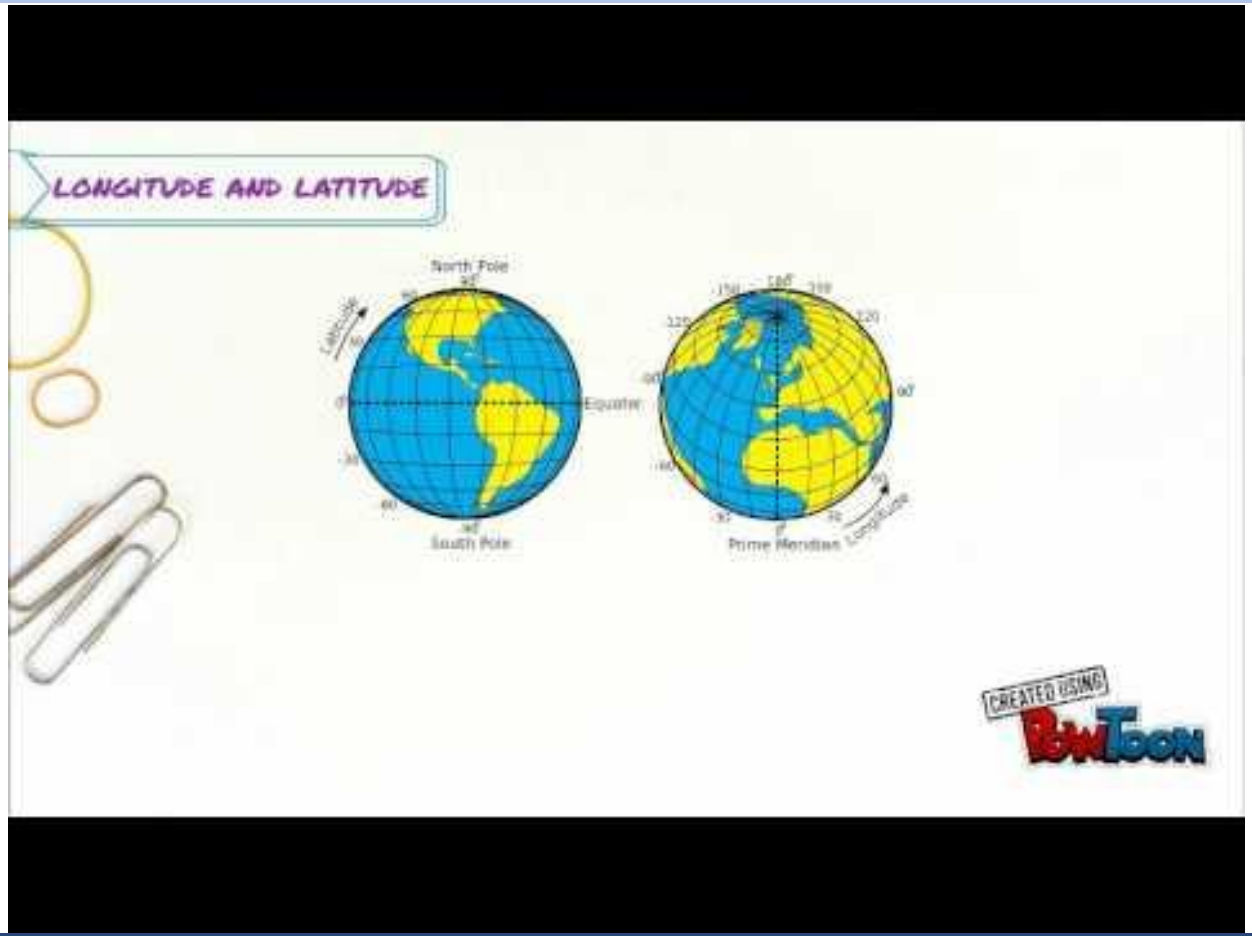
Essentials of Analytical Geometry and Linear Algebra 1

Line on the plane

Innopolis University


LONGITUDE AND LATITUDE
North Pole
Latitude
Equator
South Pole
Prime Meridian
Longitude
CREATED USING
BWTOON

Objectives

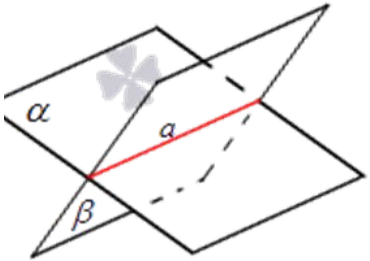
- To see the structure of all formulas which were covered
- To understand how to transform 1 form to another form
- How to use all this stuff

What elements do we know

Line in plane

Plane

Line in space



What elements do we know

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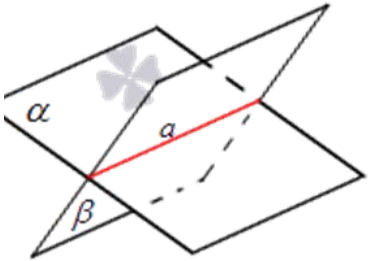
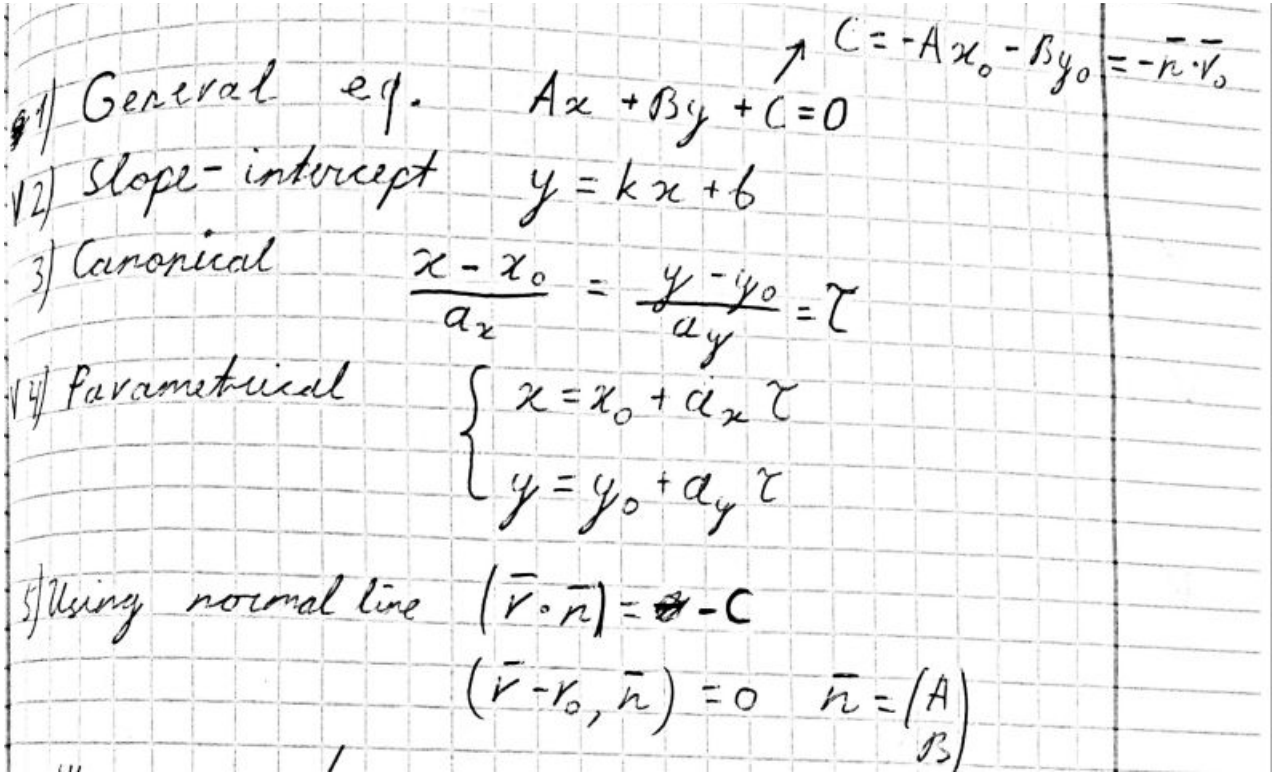


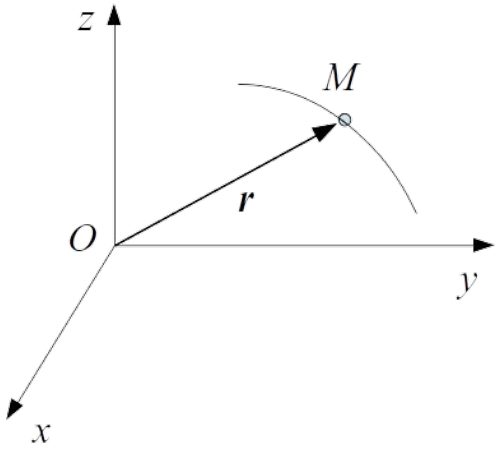
Line in plane (formulas)



Why do we need parametric form?

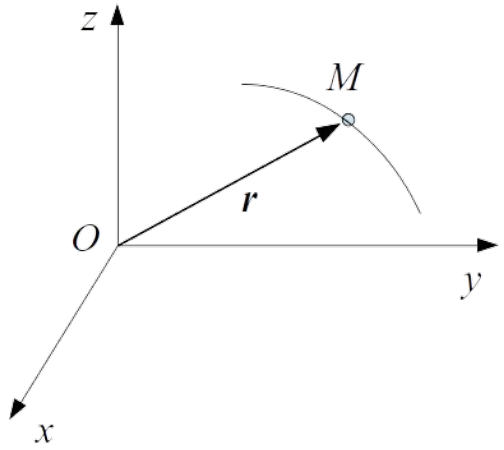
1. Robotics (kinematics)
2. CAD (computer aided design)

Methods of Describing Motion of a Particle



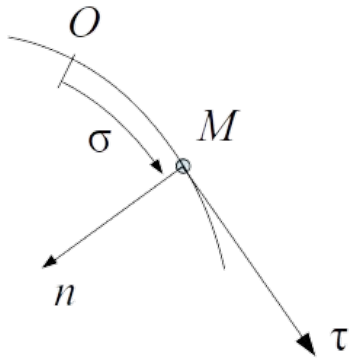
Vector

$$\boldsymbol{r}=\boldsymbol{r}(t)\ \boldsymbol{r}=x\,\boldsymbol{i}+\ y\,\boldsymbol{j}+\ z\,\boldsymbol{k}$$



Coordinate

$$x=x(t)\quad y=y(t)\quad z=z(t)\qquad \boldsymbol{r}=\begin{bmatrix}x(t)\\ y(t)\\ z(t)\end{bmatrix}$$



Natural

$$\sigma=\breve{OM}=\sigma(t)$$

Velocity of a particle (1)

Velocity direction is always tangent to the path

$$\boldsymbol{V}=\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}=\dot{\boldsymbol{r}}$$

$$\boldsymbol{V}=\frac{\mathrm{d}(x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k})}{\mathrm{d}t}=$$

$$\frac{\mathrm{d}(x\boldsymbol{i})}{\mathrm{d}t}+\frac{\mathrm{d}(y\boldsymbol{j})}{\mathrm{d}t}+\frac{\mathrm{d}(z\boldsymbol{k})}{\mathrm{d}t}=$$

$$\dot{x}\boldsymbol{i}+\dot{y}\boldsymbol{j}+\dot{z}\boldsymbol{k}$$

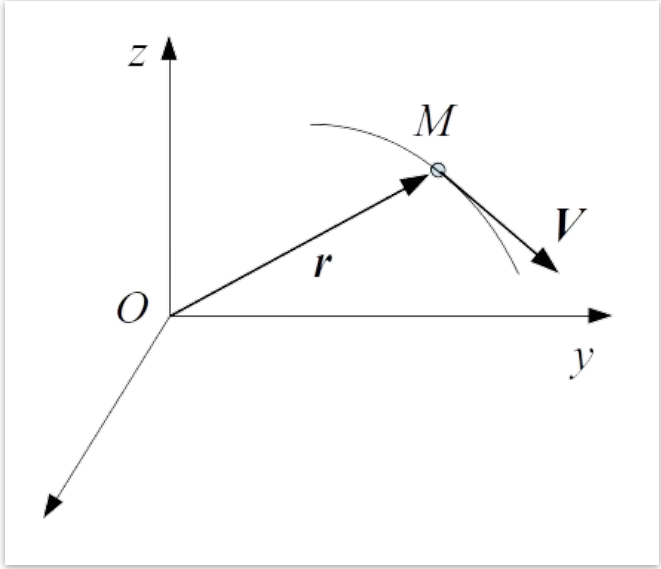
$$\boldsymbol{V}=\begin{bmatrix}\dot{x}(t)\\ \dot{y}(t)\\ \dot{z}(t)\end{bmatrix}$$

$$V_x=\dot{x}(t)$$

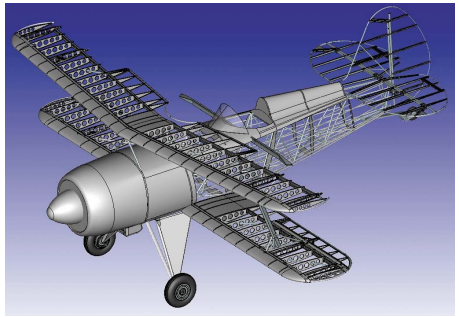
$$V_y=\dot{y}(t)$$

$$V_z=\dot{z}(t)$$

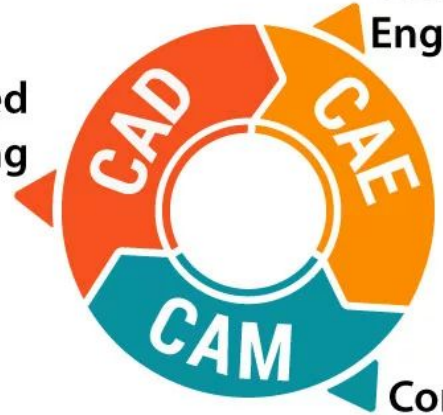
$$V=\sqrt{V_x^2+V_y^2+V_z^2}$$



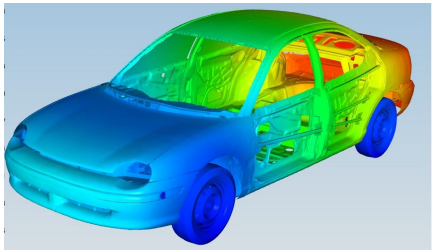
Computer Aided Design (1)



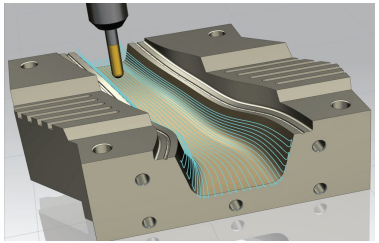
Computer-Aided Drafting



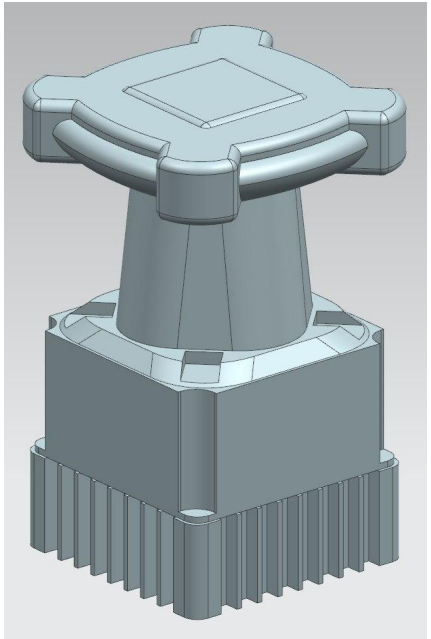
Computer-Aided Engineering



Computer-Aided Manufacturing



Computer Aided Design (2)



Solid Modeling

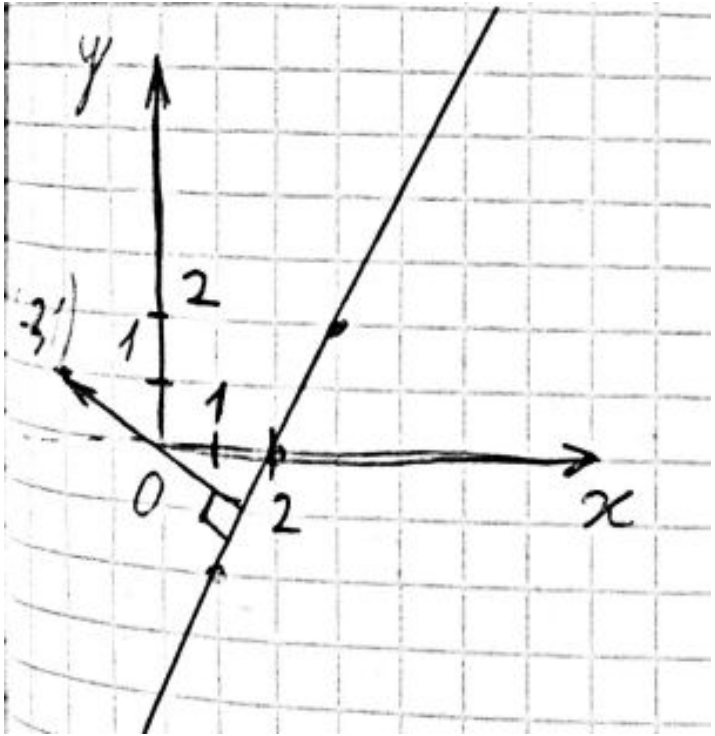
Surface Modeling

Computer Aided Design (3)

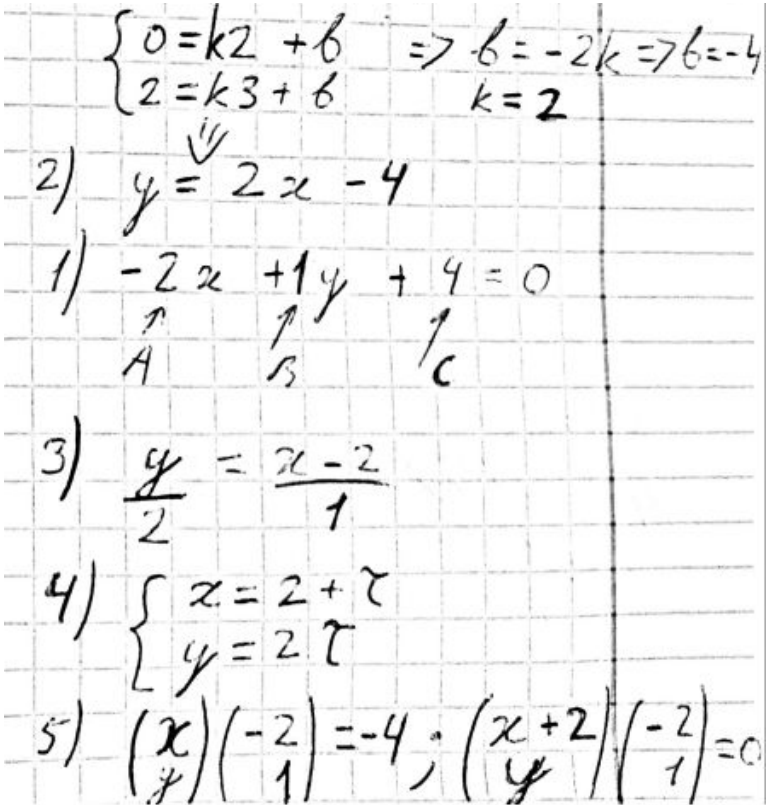
Type	Form	Example	Description
1. *Explicit*	$y = f(x)$	$y = mx + b$	Line
2. *Implicit*	$f(x, y) = 0$	$(x - a)^2 + (y - b)^2 = r^2$	Circle
3. *Parametric*	$x = \frac{g(t)}{w(t)}; y = \frac{h(t)}{w(t)}$	$x = a_0 + a_1 t; y = b_0 + b_1 t$	Line
		$x = a + r\ \cos t; y = b + r\ \sin t$	Circle

Line in plane (Task)

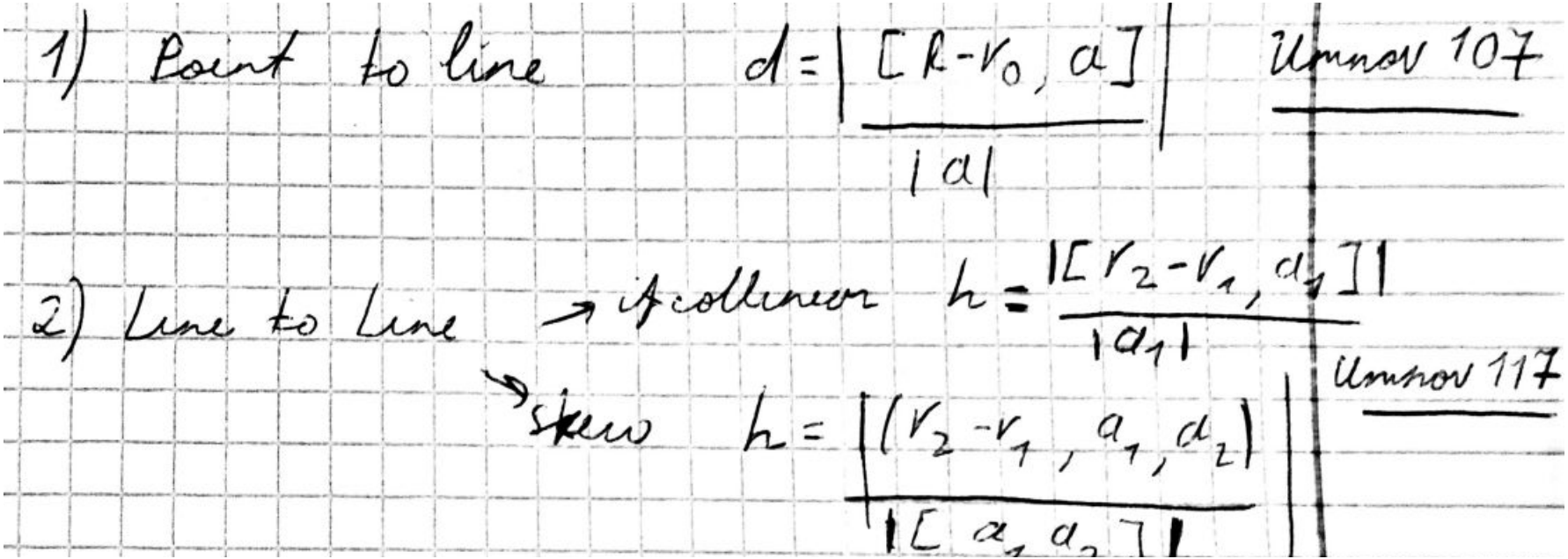
Write down all forms of the line



Line in plane (Answer)



Formulas of distances



Tasks 1 and 2

1. Find the slope of the line joining the points $(2, 3)$ and $(4, -5)$.
2. Find the slope of the line $2x - 3y + 7 = 0$.

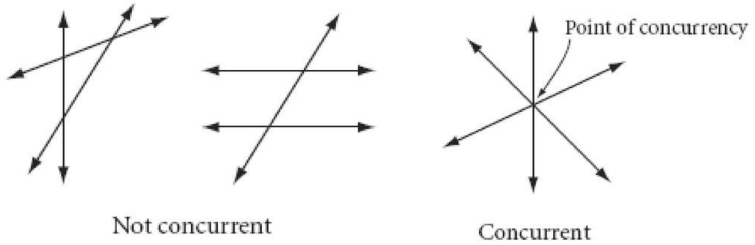
Task 3

3. Find the equation of the straight line, the portion of which between the axes is bisected at the point $(2, -5)$.

Task 4

5. Find the equation of the straight line concurrent with the lines $2x+3y=3$ and $x+2y=2$ and also concurrent with the lines $3x-y=1$ and $x+5y=11$.

- **Concurrent – Lines or segments that have three or more points in common**



Task 5

7. Find the centroid of the triangle formed by the lines given by the equations $12x^2 - 20xy + 7y^2 = 0$ and $2x - 3y + 4 = 0$.

Pair of straight lines

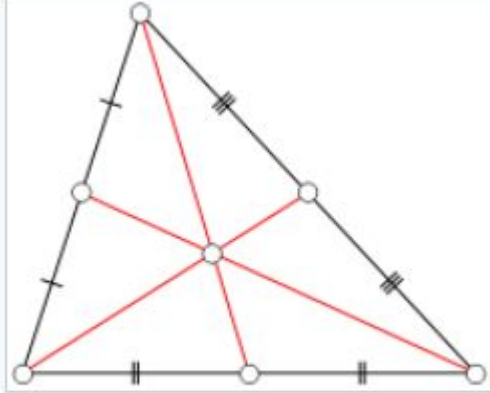
Task 5

7. Find the centroid of the triangle formed by the lines given by the equations $12x^2 - 20xy + 7y^2 = 0$ and $2x - 3y + 4 = 0$.

Task 5 (hints) (possible solution)

7. Find the centroid of the triangle formed by the lines given by the equations $12x^2 - 20xy + 7y^2 = 0$ and $2x - 3y + 4 = 0$.

$$C = \frac{1}{3}(L + M + N) = \left(\frac{1}{3}(x_L + x_M + x_N),\ \ \frac{1}{3}(y_L + y_M + y_N)\right).$$



Centroid of a triangle

Deserve “A” grade!

– Oleg Bulichev

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Room 105 (Underground robotics lab)