Discrete Mathematics and Logic
Tutorial 6

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Binary relations

Definitions

A binary relation R on a set X is called

- reflexive if $\forall x \in X \; xRx$,
- irreflexive if $\forall x \in X \; \neg(xRx)$,
- symmetric if $\forall x, y \in X \; (xRy \to yRx)$,
- asymmetric if $\forall x, y \in X \; \neg(xRy \to yRx)$,
- antisymmetric if $\forall x, y \in X \; (xRy \,\&\, yRx \to x = y)$,
- transitive if $\forall x, y, z \in X \; (xRy \,\&\, yRz \to xRz)$.

Binary relations

$A = \{a, b, c, d\}$

R_1	a	b	c	d
a	1	1	1	0
b	1	1	0	1
c	1	0	1	0
d	1	0	1	1

R_2	a	b	c	d
a	1	1	1	0
b	1	1	0	1
c	1	0	0	0
d	1	0	1	1

- reflexive if $\forall x \in X \;\; xRx$,

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- transitive if $\forall x, y, z \in X \ (xRy \,\&\, yRz \rightarrow xRz)$.

Functions

$A = \{a, b, c, d\}$, $f : A \to A$

f	a	b	c	d
a	0	1	0	0
b	1	0	0	0
c	0	0	1	0
d	1	0	0	0

$f(A) = \{a, b, c\}$, f is not bijection.

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$f(A) = \{a, b, c, d\}$, f is bijection.

$g : \mathbb{N} \to \mathbb{N}$, $g(x) = x + 1$. Is it bijection?

Dirichlet Drawer Principle

Theorem

If $k+1$ or more objects are placed into k boxes, then there is at least one box containing two or more objects.

Example

For 367 or more people , at least two of them must have been born on the same date.

Cardinality

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Example

There are 20 cakes: 15 cakes with nuts and 12 cakes with cream. How many cakes have the both topics?

$|A \cup B| = 20$

$|A| = 15$

$|B| = 12$

$|A \cap B| = |A| + |B| - |A \cup B| = 15 + 12 - 20 = 7$

Cardinality

$$|A \cup B| = |A| + |B|, \text{ if, } A \cap B = \emptyset$$

$$|A \times B| = |A| \cdot |B|$$

Exercises

How many passwords can be created with the following constraints:

1) The password is three characters long and contains two letters ("a, b, c, ..., z") and one digit in some order.
2) The password is four characters long and contains three letters and one digit. All of the letters must come before the digit in the password.
3) The password is eight or nine characters long and contains only digits.

Cardinality

$$|A \cup B| = |A| + |B|, \text{ if, } A \cap B = \emptyset$$

$$|A \times B| = |A| \cdot |B|$$

Exercises

How many passwords can be created with the following constraints:

1) The password is three characters long and contains two letters ("a, b, c, . . ., z") and one digit in some order.

$A = \{a, b, \ldots, z, 0, 1, \ldots, 9\}$, $|A| = 26 + 10 = 36$

$|Pas| = |A \times A \times A| = 36 \cdot 36 \cdot 36 = 46656$

Cardinality

$$|A \cup B| = |A| + |B|, \text{ if, } A \cap B = \emptyset$$
$$|A \times B| = |A| \cdot |B|$$

Exercises

How many passwords can be created with the following constraints:

2) The password is four characters long and contains three letters (“a, b, c, ..., z”) and one digit. All of the letters must come before the digit in the password.

$A = \{a, b, \ldots, z\}, B = \{0, 1, \ldots, 9\}, \ |A| = 26, |B| = 10$

$|Pas| = |A \times A \times A \times B| = 26 \cdot 26 \cdot 26 \cdot 10 = 175760$

Cardinality

$$|A \cup B| = |A| + |B|, \text{ if, } A \cap B = \emptyset$$
$$|A \times B| = |A| \cdot |B|$$

Exercises

How many passwords can be created with the following constraints:

3) The password is eight or nine characters long and contains only digits.

$B = \{0, 1, \ldots, 9\}$, $|B| = 10$

$|Pas| = |B^8 \cup B^9| = |B^8| + |B^9| = 100000000 + 1000000000 = 1100000000$

Midterm Exam

Midterm Examination on Discrete Mathematics & Logic (October 19, 2020)

It is 90-minutes in-class written examination. The purpose of the examination is to evaluate understanding of the basic concepts, to develop short proofs, and to solve practice problems.

- 14:20-15:50 — B20-01, B20-02, B20-03
- 16:00-17:30 — B20-04, B20-05, B20-06

- Logic
- The naive set theory
- Functions & Relations

Midterm Exam

Examples

1. Logic

- Prove $\neg(A \,\&\, B) = \neg A \vee \neg B$

2. The naive set theory

- Prove $|A \times B| = |A| \times |B|$

3. Functions & Relations

- Is a function bijection?
- Is a relation R equivalence? If the relation R on the set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is defined as: $xRy \leftrightharpoons$ "$|x - y|$ is even".

Thank you for your attention!