Dmitry Beresnev B20-02.

N6

Prove: $(R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1}$

let's suppose $R_1 = \{(a,b), (c,d), \dots\}$
$R_2 = \{(b,e), (d,f), \dots\}$.

$R_1 \circ R_2 = \{(a,e), (c,f), \dots\}$ $R_2^{-1} = \{(e,b), (f,d), \dots\}$
$(R_1 \circ R_2)^{-1} = \{(e,a), (f,c), \dots\}$. $R_1^{-1} = \{(b,a), (d,c), \dots\}$
$R_2^{-1} \cdot R_1^{-1} = \{(e,a), (f,c), \dots\}$

So, $(R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1}$ M

N7

Let M_R be a matrix on R and M_S be a matrix on S
we know that:

$$M_{R\cap S} = M_R \wedge M_S$$

Matrix on Relation called antisymmetric then $\neg(m_{ij} = 1 \ \& \ m_{ji} = 1)$
we have ~~2~~ cases: (actually 3, but for as order is does not matter)

$$\left[\begin{array}{l} \begin{cases} m_{ij} = 1 \\ m_{ji} = 0 \end{cases} \\ \begin{cases} m_{ij} = 0 \\ m_{ji} = 0 \end{cases} \end{array}\right.$$

Let's construct table: (without loss of generality)

$M_{R_{ij}}$	$M_{R_{ji}}$	$M_{S_{ij}}$	$M_{S_{ji}}$	$M_{R\cap S_{ij}}$	$M_{R\cap S_{ji}}$
1	0	1	0	1	0
0	0	1	0	0	0
1	0	0	0	0	0
0	0	0	0	0	0

As we can see, $M_{R\cap S}$ is ~~~~ antisymmetric too

M

№ 9

$X = \{1, 2, 3, 6\}$ $R = \{(x, y) \mid x, y \in X$, x is a divisor of y$\}$

$X_R = \{(1,1), (2,1), (2,2), (3,1), (3,3), (6,1), (6,2),$
$(6,3), (6,6)\}$

R is reflexible, antisymmetric, transitive $\Rightarrow$
$\Rightarrow$ R is a <u>non-strict order relation</u>

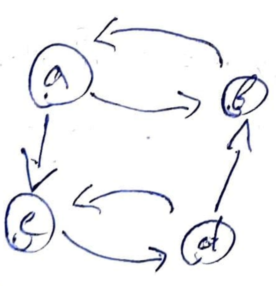
Therefore, we can not define min & max elements of X_R
$\min(X) = 1$; $\max(X) = 6$

№ 10

$X = \{0, 1, 2, 3\}$
$R = \{(0,1), (1,1), (1,2), (2,0), (2,2), (3,0)\}$

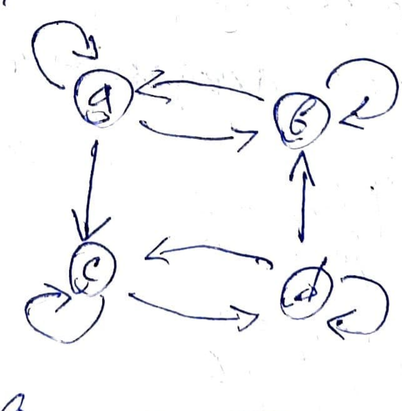
a) $\{(0,1), (0,0), (1,1), (1,2), (2,0), (2,2), (3,0), (3,3)\}$
b) $\{(0,1), (1,0), (1,1), (1,2), (2,1), (2,0), (0,2), (2,2), (3,0), (0,3)\}$

№ 11

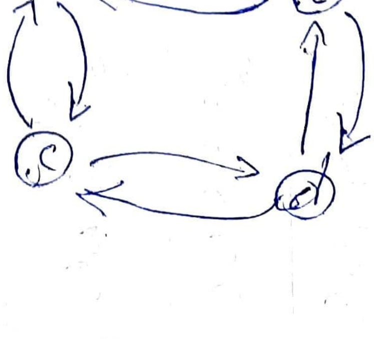


$\{(a,b), (b,a), (d,b),$
$(a,c), (c,d), (d,c)\}$

a)



b)



N12

Prove:

a) $(S \cup T)^{-1} = (S^{-1}) \cup (T^{-1})$

Let S be $\{(x_1, x_2), (x_3, x_4), \ldots, (x_{n-1}, x_n)\}$ and
T be $\{(y_1, y_2), (y_3, y_4), \ldots, (y_{k-1}, y_k)\}$

So, $(S \cup T)^{-1} = \{(x_2, x_1), (y_2, y_1), \ldots (x_n, x_{n-1}), (y_k, y_{k-1})\}$. (i)

And $S^{-1} = \{(x_2, x_1), (x_4, x_3), \ldots (x_n, x_{n-1})\}$.
$T^{-1} = \{(y_2, y_1), (y_4, y_3), \ldots, (y_k, y_{k-1})\}$

$S^{-1} \cup T^{-1} = \{(x_2, x_1), (y_2, y_1), \ldots, (x_n, x_{n-1}), (y_k, y_{k-1})\}$ (ii)

(i) ≡ (ii)

b) See Ex 6.

N13

a) non-reflexive, asymmetric, antisymmetric, transitive
b) reflexive, symmetric, transitive
c) reflexive, symmetric, transitive
d) reflexive, symmetric.

N14

Reflexive: 1, 3, 4, 6, 7, 8, 9
Symmetric: 1, 2, 4, 5, 7, 8
Transitive: 4, 5, 6, 7, 8, 9

N17

(on $\mathbb{N}$)

R1: non-reflexive, non-transitive, symmetric.
R2: non-reflexive, non-transitive, asymmetric, antisymmetric
R3: reflexive, transitive, symmetric
R4: irreflexive, transitive, asymmetric, antisymmetric
R5: irreflexive, non-transitive, symmetric.

N 18

$M_{\bar{R}}$ - ? $R, \bar{R}, M_R$

We should change all "1" to "0" and all "0" to "1"

N 19

$R = \{ (a,b) \mid a > b \}$

$\underset{\sim}{R \cup R^{-1}} = Z^{+2} / \{ (a,a) \mid a \in Z^{+} \}$

symmetric closure