Essentials of Analytical Geometry and Linear Algebra. Lecture 10.

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November 13, 2020

Lecture 10. Outline

- Part 1. Recap on equations of conic sections
- Part 2. Problem solving

Part 1. Recap on equations of conic sections

Think – Pair – Share
(aka TPS)

Recall conic sections along with their equations. 1. Write a list on a paper (1-2 min.)

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4. Share results

1) $y = \underline{m}x + \underline{c}$ — line.

2) $b^2x^2 + a^2y^2 = a^2b^2$

$$b^2x^2 + a^2(mx+c)^2 = a^2b^2$$

$$(a^2m^2+b^2)x^2 + 2a^2mcx + a^2c^2 - a^2b^2 = 0$$

$$x_{1,2} = \frac{-a^2mc \pm ab\sqrt{a^2m^2+b^2-c^2}}{a^2m^2+b^2}$$

$D = a^2m^2 + b^2 - c^2$

$D < 0$

$D > 0$

$D = 0 \Rightarrow \underbrace{c^2 = a^2m^2 + b^2}$

$$X_0 = -\frac{a^2m}{c} \qquad y_0 = \frac{b^2}{c}$$

given (x_1, y_1) - is a tangent point

$$y - y_1 = m(x - x_1)$$

$$\frac{y_1}{x_1} = \frac{\frac{b^2}{c}}{-\frac{a^2 m}{c}} = -\frac{b^2}{a^2 m}$$

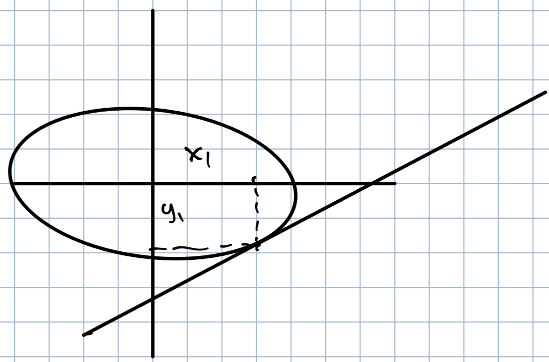
$$m = -\frac{b^2}{a^2} \cdot \frac{x_1}{y_1}$$

$$y - y_1 = -\frac{b^2 x_1}{a^2 y_1}(x - x_1)$$

$$\underbrace{a^2 y_1 y} - a^2 y_1^2 = +b^2 x_1^2 - \underbrace{b^2 x_1 x}$$

$$a^2 y_1 y + b^2 x_1 x = \underbrace{b^2 x_1^2 + a^2 y_1^2}_{a^2 b^2}$$

$$a^2 y_1 y + b^2 x_1 x = a^2 b^2 \quad : a^2 b^2 \implies \frac{x_1 x}{a^2} + \frac{y_1 y}{b^2} = 1$$



Assignment.

Let (k, l) – tangent point

$$\frac{x^2}{16} + \frac{y^2}{100} = 1$$

intercept points of the line

$$\begin{cases} A(X_A, 0) \\ B(0, Y_B) \end{cases} \qquad X_A, Y_B > 0$$

Find $\min X_A \cdot Y_B$

$$\frac{a+b}{2} \geq \sqrt{a \cdot b}$$

$$a = 4 \qquad b = 10$$

$$\frac{kx}{a^2} + \frac{ly}{b^2} = 1$$

$$A: \quad \frac{k \cdot x_A}{a^2} = 1 \qquad B: \quad \frac{l \cdot y_B}{b^2} = 1$$

$$x_A = \frac{16}{k} \qquad y_B = \frac{100}{l}$$

$$\boxed{x_A \cdot y_B = \frac{1600}{k \cdot l}}$$

$$1 = \frac{k^2}{16} + \frac{l^2}{100} = \left(\frac{k}{4}\right)^2 + \left(\frac{l}{10}\right)^2 \geq 2\sqrt{\frac{k^2 l^2}{4^2 \cdot 10^2}}$$

$$1 \geq 2\,\frac{kl}{40} \qquad \boxed{kl \leq 20}$$

$$\min(x_A \cdot y_B) = \frac{1600}{kl} = 80.$$

$\overset{x_1}{}\ \overset{y_1}{}$

$\underbrace{A\ (2, 7)}_{\text{tangent point}}$ $\quad x^2 + y^2 = 53$

Let the circle has a center at $(3, 0)$

and passes through both foci

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

Find equation of the circle.

1) eq. of circle:

$$(x-3)^2 + y^2 = r^2$$

2) Find coord-s of foci

$$c^2 = b^2 - a^2$$

$A(0, -\sqrt{7})$ $B(0, \sqrt{7})$

$r = 4.$ ....

Hyperbolas

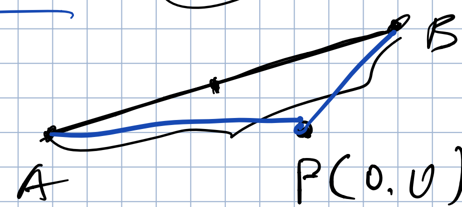
We know coordinates of foci of a hyperbola

1) $A(7, 12)$ $B(11, 8)$

2) the hyperbola passes through the origin: $P(0, 0)$

Find: ε (eccentricity) $\varepsilon = \frac{c}{a}$

$$|AB| = \sqrt{4^2+4^2} = \sqrt{32} = 2c$$



$$2a = AP - BP =$$

$$= \underbrace{\sqrt{49+144} - \sqrt{121+64}} = 2a$$

$$\varepsilon = \frac{2c}{2a} = \frac{c}{a} = \frac{\sqrt{32}}{\sqrt{49+144} - \sqrt{121+64}}$$

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Relation to Quadratic forms and Matrices

Conic sections are the sets of points whose coordinates satisfy a second-degree polynomial equation (A, B, C, D, E, F are numbers):

$$Q(x, y) = Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$$

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In matrix form (it is the **same** equation):

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} A & B/2 \\ B/2 & C \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} D & E \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + F = 0.$$

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This following expression is called the **quadratic form**: $Ax^2 + Bxy + Cy^2$.

Matrix of the **quadratic form** : $\begin{bmatrix} A & B/2 \\ B/2 & C \end{bmatrix}$

Given a conic

$$Q(x, y) = Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$$

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Discriminant of a conic is Δ

$$B^2 - 4AC = 4\Delta$$

As you can see,
$$\Delta = \begin{vmatrix} A & B/2 \\ B/2 & C \end{vmatrix}$$

Given a conic

$$Q(x, y) = Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$$

- $B^2 - 4AC < 0$, the equation represents an ellipse;
  $A = C$ and $B = 0$, the equation represents a circle,
- $B^2 - 4AC = 0$, the equation represents a parabola;
- $B^2 - 4AC > 0$, the equation represents a hyperbola;
  $A + C = 0$, the equation represents a rectangular hyperbola

Useful links

- https://www.geogebra.org
- https://youtu.be/fNk_zzaMoSs
- http://immersivemath.com/ila