Discrete Math

Lab 2 - September, 15

Agenda

Course Information

- HW submission process update

Main Part

- HW review
- Induction exercises
- Strong Induction

Homework submission

- On Moodle
- Strictly by the deadline
- Pdf file named gg-surname.pdf (01-Ivanov.pdf), where gg is your group number

Common mistakes in proofs

- Operations with inequalities (subtraction, multiplication)
- Integer, rational and irrational numbers (exercise - draw a Vienne diagram)
- Missing brackets
- Missing explanations of intermediate steps
- Missing induction hypothesis
- Missing explanation how the proof follows from the last statement obtained
- The proof is too complicated
- Spelling: to prove, but a proof; divide and divisible by.

Induction Exercises

1. Each of n famous scientists who meet at a conference (where $n \geq 2$) wants to shake hands with all the others. Work out how many handshakes there will be and prove by induction

2. What is the maximum number of regions into which a plane can be divided by n straight lines. Work out a formula and prove by induction.

Strong induction

STRONG INDUCTION To prove that $P(n)$ is true for all positive integers, where $P(n)$ is a propositional function, we complete two steps:
BASIS STEP: We verify that $P(1)$ is true.
INDUCTIVE STEP: We show that the conditional statement $[P(1) \land P(2) \land \cdots \land P(k)] \rightarrow P(k+1)$ is true for all positive integers k.

As we can use all k statements $P(1), P(2), \ldots, P(k)$ to prove $P(k+1)$, rather than just the statement $P(k)$ as in a proof by mathematical induction, strong induction is a more flexible proof technique. Because of this, some mathematicians prefer to always use strong induction instead of mathematical induction, even when a proof by mathematical induction is easy to find.

Strong induction (generalized)

STRONG INDUCTION To prove that $P(n)$ is true for all integers $n \geq b$ (b: fixed integer), where $P(n)$ is a propositional function, we complete two steps:

BASIS STEP: Verify that $P(b); P(b + 1); \ldots ; P(b + j)$ are true (j: a fixed positive integer)

INDUCTIVE STEP: We show that the conditional statement $[P(b) \wedge P(b+1) \wedge \cdots \wedge P(k)] \rightarrow P(k + 1)$ is true for all positive integers $k \geq b + j$

Strong induction - Example

Exercise Prove that every amount of postage of 8 cents or more can be formed using just 3-cent and 5-cent stamps.

Solution: Let *P (n)* be the statement that postage of *n* cents can be formed using 3-cent and 5-cent stamps

<u>BASIS STEP</u>: Show that the statements P(8); P(9); and P(10) are true

$8 = 3 \cdot 1 + 5 \cdot 1$

$9 = 3 \cdot 3 + 5 \cdot 0$

$10 = 3 \cdot 0 + 5 \cdot 2$

This completes the basis step.

Strong induction – Example (continued)

<u>INDUCTIVE STEP</u>: For the inductive hypothesis, we assume that any value *j* (8 ≤ *j* ≤ *k*) where k ≥ 10, can be expressed as j = 3a + 5b with a and b being non-negative integers

To carry out the inductive step using this assumption, we must show that we can express k + 1 as 3a + 5b with a and b being nonnegative integers.

Since we want to show P(k+1), we can use P(k-2), which is true by inductive hypothesis since 8 ≤ k - 2 ≤ k.

k - 2 = 3a + 5b
k - 2 + 3 = 3a + 4b + 3
k +1 = 3(a + 1) + 5b

We have completed both the basis step and the inductive step, so by the principle of strong induction, the statement is true for every integer *n* ≥ 8.

Strong Induction Exercises

1. Let the “Tribonacci sequence” be defined by $T_1 = T_2 = T_3 = 1$ and $T_n = T_{n-1} + T_{n-2} + T_{n-3}\ for\ n \geq 4$. Prove that $T_n < 2^n$ for all $n \in Z_+$.

2. Let a_n be the sequence defined by $a_1 = 1; a_2 = 8;\ a_n = a_{n-1} + 2a_{n-2} (n \geq 3)$ Prove that $a_n = 3 * 2^{n-1} + 2(-1)^n$ for all $n \in Z_+$.

Homework

1. Read textbook Kenneth H. Rosen "Discrete Mathematics and Its Applications, 7th Edition“ – paragraphs 1.1, 1.3, 1.4, 2.1, 2.2, 5.2

2. Submit on Moodle by 10pm September 18 (Friday) **(late submissions will be penalized or rejected) the pdf file named gg-surname.pdf,** where gg is your group number. **Incorrectly named files might be rejected.**

    1. Prove by induction that the number of non-empty subsets of a set of n elements is $2^n - 1$, for any positive integer n.

    2. Prove by strong induction that every integer greater than 1 is a product of primes