$$\lim_{x\to 0}\frac{1+x\cos x-\sqrt{1+2x}}{\ln(1+x)-x}=?$$

$\cancel{\cos x}$ $x_0=0$

$f(x)=\cos x \qquad f(x_0)=1 \qquad \cos x=1-\frac{x^2}{2}+\frac{x^4}{24}+o(x^4)$

$f'(x)=-\sin \qquad f'(x_0)=0$

$f''(x)=-\cos x \qquad f''(x_0)=-1$

$f'''(x)=\sin x \qquad f'''(x_0)=0$

$f^{iv}(x)=\cos x \qquad f^{iv}(x_0)=1$

$x_0=0$

$f(x)=\sqrt{1+2x} \qquad f(x_0)=1 \qquad \sqrt{1+2x}=1+x-\frac{x^2}{2}+\frac{x^3}{2}-\frac{5}{8}x^4+o(x^4)$

$f'(x)=\frac{1}{\cancel{2}\sqrt{1+2x}} \qquad f'(x_0)=1$

$f''(x)=-\frac{1}{(1+2x)^{+\frac{3}{2}}} \qquad f''(x_0)=-1$

$f'''(x)=\frac{3}{(1+2x)^{+\frac{5}{2}}} \qquad f'''(x_0)=3$

$f^{iv}(x)=-\frac{15}{(1+2x)^{+\frac{7}{2}}} \qquad f^{iv}(x_0)=-15$

$\ln(1+x)$ $\cancel{=x}$, $x_0=0$

$f(x)=\ln(1+x) \qquad f(x_0)=0 \qquad \ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+o(x^4)$

$f'(x)=\frac{1}{1+x} \qquad f'(x_0)=1$

$f''(x)=-\frac{1}{(1+x)^2} \qquad f''(x_0)=-1$

$f'''(x)=+\frac{2}{(1+x)^3} \qquad f'''(x_0)=2$

$f^{iv}(x)=-\frac{6}{(1+x)^4} \qquad f^{iv}(x_0)=-6$

$$\lim_{x\to 0}\frac{1+x-\frac{x^3}{2}+o(x^4)-1-x+\frac{x^2}{2}-\frac{x^3}{2}+\frac{5}{8}x^4-o(x^4)}{x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+o(x^4)-x}=$$

$$\lim_{x\to 0}\frac{\frac{x^2}{2}-x^3+\frac{5}{8}x^4}{-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+o(x^4)} = \cancel{\lim_{x\to 0}\frac{\frac{5}{8}-\frac{1}{x}+\frac{1}{2x^2}}{-\frac{1}{4}+\underbrace{\frac{1}{3x}}_{\to 1}-\underbrace{\frac{1}{2x^2}}_{\to 0}+\underbrace{\frac{o(x^4)}{x^4}}_{\to 0}}}$$

$$= \lim_{x\to 0}\frac{\frac{1}{2}-\underbrace{x}_{\to 0}+\underbrace{\frac{5}{8}x^2}_{\to 0}}{-\frac{1}{2}+\underbrace{\frac{x}{3}}_{\to 0}-\underbrace{\frac{x^2}{4}}_{\to 0}+\underbrace{\frac{o(x^4)}{x^2}}_{\to 0}} = \lim_{x\to 0}\frac{1}{2}\cdot(-2) = -1$$

Answer $= -1$

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$\boxed{\text{V1}}$