Discrete Mathematics and Logic
Tutorial 3

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The naive set theory

$$\overline{A \cap \overline{B} \cap C} \cap \overline{A}$$

$\overline{A \cap \overline{B} \cap C} \cap \overline{A} = (\overline{A} \cup B \cup \overline{C}) \cap \overline{A} = ?$

$*\overline{X \cap Y} = \overline{X} \cup \overline{Y}$

The naive set theory

$$\overline{A \cap \overline{B} \cap C} \cap \overline{A}$$

$$(\overline{A} \cup B \cup \overline{C}) \cap \overline{A} = (\overline{A} \cap \overline{A}) \cup (B \cap \overline{A}) \cup (\overline{C} \cap \overline{A}) =$$

$$\overline{A} \cup (B \cap \overline{A}) \cup (\overline{C} \cap \overline{A}) = ?$$

$$*X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$$

The naive set theory

$$\overline{A \cap \overline{B} \cap C} \cap \overline{A}$$

$$(\overline{A} \cup B \cup \overline{C}) \cap \overline{A} = \overline{A}$$

$$\overline{A} \cup (B \cap \overline{A}) \cup (\overline{C} \cap \overline{A}) = \overline{A}$$

$$\overline{A \cap \overline{B} \cap C} \cap \overline{A} = \overline{A}$$

$*X \cap (X \cup Y) = X$
$*X \cup (X \cap Y) = X$

Multiplication

Definition

$X \times Y = \{(x, y) \mid x \in X \,\&\, y \in Y\}$

Example

If $A = \{1, 2, 3\}$ and $B = \{a, b\}$, then

$$A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$$

$$B \times A = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$$

$$A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$

$$B \times B = \{(a, a), (a, b), (b, a), (b, b)\}$$

Multiplication

Properties

$$(A \cup B) \times C = (A \times C) \cup (B \times C)$$

Let $A = \{a, b\}$, $B = \{b, c\}$, and $C = \{1, 2\}$.
$A \cup B = \{a, b, c\}$

$A \times C = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$
$B \times C = \{(b, 1), (b, 2), (c, 1), (c, 2)\}$

$(A \cup B) \times C = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\} =$
$(A \times C) \cup (B \times C)$

Multiplication

Properties

$$(A \cap B) \times (X \cap Y) = (A \times X) \cap (B \times Y)$$

Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, and $X = \{1, 2, 3\}$, $Y = \{2, 3, 4\}$.

$A \cap B = \{b, c\} \quad X \cap Y = \{2, 3\}$

$(A \cap B) \times (X \cap Y) = \{(b, 2), (b, 3), (c, 1), (c, 2)\}$

Multiplication

Properties

$$(A \cap B) \times (X \cap Y) = (A \times X) \cap (B \times Y)$$

Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, and $X = \{1, 2, 3\}$, $Y = \{2, 3, 4\}$.

$A \times X =$
$\{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3), (c, 1), (c, 2), (c, 3)\}$

$B \times Y =$
$\{(b, 2), (b, 3), (b, 4), (c, 2), (c, 3), (c, 4), (d, 2), (d, 3), (d, 4)\}$

$(A \times X) \cap (B \times Y) = \{(b, 2), (b, 3), (c, 1), (c, 2)\}$

Power of a set

Definition

For a set A, the power of A is the set $2^A = \mathcal{P}(A) = \{B \mid B \subseteq A\}$

Examples

1) If $A = \{a\}$ then $\mathcal{P}(A) = \{\emptyset, \{a\}\}$
2) If $A = \{a, b\}$ then $\mathcal{P}(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

Cardinality

Properties

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Example

Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 3, 4, 5\}$.

$A \cup B = \{1, 2, 3, 4, 5\}$, $A \cap B = \{2, 3, 4\}$

$|A \cup B| = 5$, $|A| = 4$, $|B| = 4$, $|A \cap B| = 3$

Cardinality

Properties

$$|A \times B| = |A| \cdot |B|$$

Example

Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$.

$A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

$|A \times B| = 6, \ |A| = 3, \ |B| = 2$

Cardinality

$$|2^A| = 2^{|A|}$$

Example

$A = \{x, y, z\}$

x	y	z	*Subsets*
0	0	0	$\emptyset$
0	0	1	$\{z\}$
0	1	0	$\{y\}$
0	1	1	$\{y, z\}$
1	0	0	$\{x\}$
1	0	1	$\{x, z\}$
1	1	0	$\{x, y\}$
1	1	1	$A = \{x, y, z\}$

Thank you for your attention!