

# Homework

①  $(p \rightarrow q) \rightarrow r = p \rightarrow (q \rightarrow r) ?$

p	q	r	$(p \rightarrow q) \rightarrow r$	$p \rightarrow (q \rightarrow r)$
0	0	0	0	1
0	0	1	1	1
0	1	0	0	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	1	1

These statements are not equivalent

② a)  $(p \& q) \rightarrow p$

p	q	$(p \& q) \rightarrow p$
0	0	1
0	1	1
1	0	1
1	1	1

Last column is all "1".

Therefore,  $(p \& q) \rightarrow p$  is a tautology.

b)  $p \rightarrow (p \vee q)$

p	q	$p \rightarrow (p \vee q)$
0	0	1
0	1	1
1	0	1
1	1	1

Last column is always "1".

Therefore,  $p \rightarrow (p \vee q)$  is a tautology.

③  $\neg(p \& q) \equiv \neg p \vee \neg q ?$

p	q	$\neg(p \& q)$	$\neg p \vee \neg q$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Two last columns are the same, so

$\neg(p \& q) \equiv \neg p \vee \neg q$  is true.

$$4. \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

(1) Let  $x \in A \cap (B \cup C)$  then  $\Rightarrow$

$$\Rightarrow x \in A \text{ and } x \in (B \cup C)$$

$$\Rightarrow x \in A \text{ and } (x \in B \text{ or } x \in C)$$

$$\Rightarrow (x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C)$$

$$\Rightarrow x \in (A \cap B) \cup (A \cap C)$$

$$\Rightarrow (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$$

(2) Let  $y \in (A \cap B) \cup (A \cap C) \Rightarrow$

$$\Rightarrow (y \in A \text{ and } y \in B) \text{ or } (y \in A \text{ and } y \in C)$$

$$\Rightarrow y \in A \text{ and } (y \in B \text{ or } y \in C)$$

$$\Rightarrow y \in A \cap (B \cup C)$$

$$\Rightarrow (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$$

From (1) and (2) we have  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

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$$3. \quad A = \{1, 2, 3, 4, 5\} \quad B = \{0, 3, 6\}$$

a)  $A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$

b)  $A \cap B = \{3\}$

c)  $A \setminus B = \{1, 2, 4, 5\}$

d)  $B \setminus A = \{0, 6\}$