Essentials of Analytical Geometry and Linear Algebra I, Class #3

Innopolis University, September 2020

1 Operations with Matrices

1.1 Introduction to matices

1. Let $A = \begin{bmatrix} 3 & 1 \\ 5 & -2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$:
   (a) Find $A + B$;
   (b) Find $2A - 3B + I$;
   (c) Find AB and BA (make sure that, in general, $AB \neq BA$ for matrices);
   (d) Find AI and IA.

2. Let $A = \begin{bmatrix} 2 & -1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix}$:
   (a) Find AB and BA if they exist;
   (b) Find $A^T B$ and BA^T if they exist.

3. If solution exists, what the dimension of the result matrix. There are several matrices: $\underset{3\times3}{A}$, $\underset{2\times3}{B}$, $\underset{3\times2}{C}$, $\underset{3\times5}{D}$, $\underset{3\times5}{D}$, $\underset{1\times2}{E}$, $\underset{3\times1}{K}$.
   (a) ABC;
   (b) AB^TC^T;
   (c) $EBAE$;
   (d) $K^T \times K^TCE^T$.

1.2 Determinants

1. Find the determinants of the following matrices:
   (a) $A = \begin{bmatrix} 5 & -2 \\ 1 & 6 \end{bmatrix}$; (b) $B = \begin{bmatrix} 1 & -3 & -1 \\ -2 & 7 & 2 \\ 3 & 2 & -4 \end{bmatrix}$.

2. A triangle is constructed on vectors $\mathbf{a} = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$.
   (a) Find the area of this triangle.
   (b) Find the altitudes of this triangle.

3. Find the matrix product AB if $A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 7 & x \end{bmatrix}$, $B = \begin{bmatrix} 5 & -1 \\ x & 2 \\ -3 & -1 \end{bmatrix}$.
   Then find the largest possible value of $det(AB)$.

2 Scalar Triple Product

1. Find the scalar triple product of $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 7 \\ 3 \\ -5 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 3 \\ 4 \\ -3 \end{bmatrix}$.

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3 Operations with Matrices

3.1 Introduction to matices

1. Is it true that:

   (a) $(A+B)^2 + (A-B)^2 = 2A^2 + 2B^2$;

   (b) $(A+B)(A-B) = A^2 - B^2$;

   (c) $(A-I)^3 = A^3 - 3A^2 + 3A - I$.

3.2 Determinants

1. Solve the system of equations $\begin{cases} 3x - y = 11, \\ 5x - 2y = -1 \end{cases}$ using Cramer's rule.

2. Solve the equation $\det(A - xI) = 0$ if $A = \begin{bmatrix} 5 & -2 \\ -2 & 8 \end{bmatrix}$ and I is the identity matrix of the corresponding size.

4 Scalar Triple Product

1. Vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are not coplanar. Find all values of α such that vectors $\mathbf{a} + 2\mathbf{b} + \alpha\mathbf{c}$, $4\mathbf{a} + 5\mathbf{b} + 6\mathbf{c}$, $7\mathbf{a} + 8\mathbf{b} + \alpha^2\mathbf{c}$ are coplanar.

2. It is known that basis vectors $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ have lengths of 1, 2, $2\sqrt{2}$ respectively, and $\angle(\mathbf{e}_1, \mathbf{e}_2) = 120°$, $\angle(\mathbf{e}_1, \mathbf{e}_3) = 135°$, $\angle(\mathbf{e}_2, \mathbf{e}_3) = 45°$. Find the volume of a parallelepiped constructed on vectors with coordinates $(-1; 0; 2)$, $(1; 1\,4)$ and $(-2; 1; 1)$ in this basis.