

# Essentials of Analytical Geometry and Linear Algebra I, Class #2

Innopolis University, September 2020

## 1 Dot Product

1. Find  $|\mathbf{a}|^2 - 2\sqrt{3}\mathbf{a} \cdot \mathbf{b} - 7|\mathbf{b}|^2$  given that  $|\mathbf{a}| = 4$ ,  $|\mathbf{b}| = 1$ ,  $\angle(\mathbf{a}, \mathbf{b}) = 150^\circ$ .
2. Find the angle<sup>1</sup> between  $\mathbf{a} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$  and  $\mathbf{b} = \begin{bmatrix} -5 \\ -1 \\ -1 \end{bmatrix}$ .
3. Prove that vectors  $\mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$  and  $\mathbf{a}$  are perpendicular to each other.
4. All three vectors  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  have length of 3 and  $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$ . Find  $\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}$ .
5. The edges of cube  $ABCD A_1 B_1 C_1 D_1$  have length of 1.  $P$  is a midpoint of  $CC_1$ , and  $Q$  is a center of face  $AA_1 B_1 B$ . Points  $M$  and  $N$  belong to lines  $AD$  and  $A_1 B_1$  respectively, and at that  $MN$  intersects with  $PQ$  and is perpendicular to it. Find  $MN$ .
6. Find an angle between  $\mathbf{a}$  and  $\mathbf{b}$  if:
  - (a)  $\mathbf{a} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ ,  $\mathbf{b} = \begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix}$ ;
  - (b)  $\mathbf{a} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ ,  $\mathbf{b} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$ ;
  - (c)  $\mathbf{a} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$ ,  $\mathbf{b} = \begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix}$ .
7. There are two vectors on some basis  $\mathbf{a} = \begin{bmatrix} x \\ 1-x \end{bmatrix}$ ,  $\mathbf{b} = \begin{bmatrix} x^2-2x \\ x^2-2x+1 \end{bmatrix}$ . It is needed to find  $x$ , when:
  - (a) vectors are collinear;
  - (b) they have the same direction.
8. There are two vectors  $\mathbf{a} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$  and  $\mathbf{b} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$ .  $\mathbf{c}$  length equal to 1. The vector is perpendicular to  $\mathbf{a}$ . The angle between  $\mathbf{b}$  and  $\mathbf{c}$  is  $\arccos(\sqrt{\frac{2}{27}})$ . Find the coordinates of  $\mathbf{c}$ . How many solutions the task have?

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<sup>1</sup>If not stated otherwise, the coordinate system in this section is supposed to be Cartesian.

## 2 Cross Product

1. Find the cross product<sup>2</sup> of:

(a) vectors  $\mathbf{a} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$  and  $\mathbf{b} = \begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix}$ ;

(b) vectors  $\mathbf{a} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$  and  $\mathbf{b} = \begin{bmatrix} -18 \\ 12 \\ -6 \end{bmatrix}$ .

2. Simplify the expressions:

(a)  $(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - \mathbf{b})$ ;

(b)  $(3\mathbf{a} - \mathbf{b} - \frac{1}{3}\mathbf{c}) \times (2\mathbf{a} + \frac{3}{2}\mathbf{b} - 3\mathbf{c})$ .

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<sup>2</sup>If not stated otherwise, the coordinate system in this assignment is supposed to be Cartesian.