

# Discrete Math Test 1 Variant 1

1. Prove directly **and** by contradiction that if  $x + y$  is even, then  $x$  and  $y$  have the same parity (are both equal or both odd) (1 point)
2. Suppose  $x \in \mathbb{R}$ . Prove that if  $x^5 + 7x^3 + 5x \geq x^4 + x^2 + 8$ , then  $x \geq 0$  (2 points)
3. Prove by induction that  $6^n - 1$  is divisible by 5 for all positive integers  $n$  (2 points)
4. Fibonacci numbers,  $f_0, f_1, \dots$ , are defined by the equations  $f_0 = 0, f_1 = 1$  and  $f_n = f_{n-1} + f_{n-2}$  for  $n = 2, 3, 4, \dots$ . Prove for  $n \in \mathbb{Z}_+$  that  $f_1^2 + f_2^2 + \dots + f_n^2 = f_n f_{n+1}$  (2 points)
5.  $A \cup B = \{1, 2, 3, 4, 5\}, (A \setminus B) \cup (B \setminus A) = \{1, 2, 5\}$ . Find  $A \cap B$  (1 point)
6. How many elements does have the set:  $\wp(\{\emptyset, b, \{a, b\}\})$  (1 point)
7. Is this proposition true? Explain your answer.  $(A \setminus B = A) \rightarrow B \subset A$  (1 point)