$= \cancel{-\frac{n}{6} + \frac{1}{2} + \frac{1}{6n}} \xrightarrow[n\to\infty]{} \frac{1}{2}$

(1) <u>Prove</u>: number of non-empty subsets of a set of n elements is $2^n - 1$ for $\forall n \in \mathbb{Z}_+$

Mistake in statement!

Proof by Induction:

1. <u>Base of induction:</u>

for $n = 1$, we have set $\{x\}$. It's obviously that there is only one non-empty subset – $\{x\}$.

$2^1 - 1 = 1$ So, suggestion is true for $n = 1$

2. <u>Induction Case:</u>

Suppose that suggestion is true for some $n \in \mathbb{Z}_+$.
Let's prove that then suggestion is also true.
Firstly, let me introduce notation $S(n)$ – number of subsets of a set of n elements. So, what we have:

a) $S(n+1)$ includes $S(n)$ subsets similar to $S(n)$ subsets

b) $S(n+1)$ includes $S(n)$ subsets, which are $S(n)$ subsets with new additional element.

c) $S(n+1)$ includes $\{x\}$, where x – is a new element.

Let me elaborate: $S_2^1, S_2^2, S_2^3 \dots = S_1^1, S_1^2, S_1^3 \dots, S_1^1 + \{x\}, S_1^2 + \{x\}, \dots, \{x\}$,
where S_2^i – all subsets of a set with $n+1$ elements
S_1^i – all subsets of set with n elements
x – new element

So, we have: $S(n+1) = S(n) + S(n) + 1 = 2S(n) + 1 = 2(2^n - 1) + 1 = \boxed{2^{n+1} - 1}$

as was to be proved

$e^{\frac{2}{2n}\cdot x}$

$e^{x - \frac{1}{2n}}$

$n^2 \geq \frac{1}{n}$

$\sum_{k=1}^{n} k^2 - \frac{n}{3}$

$\frac{{}^2 + 3n + 1}{6n}$

(2) Every ~~int~~ $\forall n \in \mathbb{Z}, n>1$ is a product of primes.

Proof by Induction:

1. Base of Induction:

for $n=2$: $n=2\cdot 1$, 2 and 1 are primes <u>true</u>

for $n=3$: $n=3\cdot 1$, 3 and 1 are primes <u>true</u>

for $n=4$: $n=2\cdot 2$, 2 is prime <u>true</u>

2. Case of Induction

Suppose that suggestion is true for $\forall n, n\in \mathbb{Z}_+$.
Let's then prove that suggestion is true for $n+1$.
$n+1$ is an integer. So, there are two possible cases:

a) $n+1$ is a prime, so $n+1 = 1\cdot(n+1)$ as was to be proved

b) $n+1$ is a composite number. So, we can represent $n+1$ as a product of some integer x and y, $x\neq 1, y\neq 1, x\neq n+1, y\neq n+1$. Hence, $n+1 = x\cdot y$. It's obviously that $x<(n+1)$ and $y<(n+1)$. But we have already proved that <u>all</u> intgeres less than $(n+1)$ are products of primes: $x = p_1^{k_1}\cdot p_2^{k_2}\cdots p_i^{k_i}$

$y = p_1^{t_1}\cdot p_2^{t_2}\cdots p_i^{t_i}$,

where p_i are primes less than x and their powers k_j.
likewise t_j for y.

So we have $n+1 = x\cdot y = p_1^{k_1+t_1}\cdot p_2^{k_2+t_2}\cdots p_i^{k_i+t_i}$. And we know that prime in any power is still prime. Therefore, $n+1$ is a product of primes.

as was to proved.