Discrete Math

Lab 1 - September, 8

Agenda

Course Introduction

- TA introduction, office hours
- Grading policy
- Textbook

Main Part

- Introduction to proofs
- Proof by contradiction
- Proof by induction

TA introduction

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Research interest:

Requirements engineering

TA introduction

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Research interest:

Software metrics

TA introduction

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Research interest:

Project Management

Requirements engineering

Cyber Physical Systems

Grading

Course points:

- Labs – 20 points (see below)
- 3 in-class tests – 10 points each
- Mid-term – 25 points
- Final exam – 25 points

Labs points:

- In-class participation 0.5 point for each individual contribution in a class but not more than 0.5 points a week (i.e. 7 points in total for 14 study weeks),
- Home assignments 1 point for each homework (i.e. 13 points in total).
- Attendance is mandatory, we will check it at every lab

Textbook

- Kenneth H. Rosen "Discrete Mathematics and Its Applications, 7th Edition“

Introduction to proofs

Proof is a valid argument that establishes the truth of a mathematical statement.

Argument is a sequence of statements that end with a conclusion.

By **valid**, we mean that the conclusion, or final statement of the argument, must follow from the truth of the preceding statements, or premises, of the argument.

Statements that can be shown to be true are called **propositions** or **theorems.**

Basic proof techniques

- Direct proof
- Proof by construction
- Proof by contradiction
- Proof by induction

Direct proof

Way of showing the truth or falsehood of a given statement by a straightforward combination of established facts, usually axioms, existing lemmas and theorems, without making any further assumptions

The example of direct proof was shown in the tutorial

Proof by construction

A method of proof that demonstrates the existence of a mathematical object by creating or providing a method for creating the object

Example Prove that there exist nonzero integers x, y, z such that $x^2 + y^2 = z^2$.

Proof by contradiction

Show that if a proposition was false, then some false fact would be true. Since a false fact by definition can't be true, the proposition must be true

Example. Prove by contradiction that $\sqrt{2}$ is irrational

Theorem $\sqrt{2}$ is irrational

Proof. (see next slide)

Proof by contradiction example

Theorem $\sqrt{2}$ is irrational

Proof. Proof by contradiction. Suppose the claim is false, and $\sqrt{2}$ is rational.

Then we can write $\sqrt{2}$ as a fraction a/b in lowest terms.

Squaring both sides gives $2 = \frac{a^2}{b^2}$ and so $2b^2 = a^2$

This implies that a is a multiple of 2. Therefore a^2 must be a multiple of 4.

But since $2b^2 = a^2$, we know $2b^2$ is a multiple of 4 and so b^2 is a multiple of 2. This implies that b is a multiple of 2.

So, a and b have 2 as a common factor, which contradicts the fact that a=b is in lowest terms. Thus, $\sqrt{2}$ must be irrational.

Exercises

1. Give a direct proof and a proof by contradiction of the statement: If n is even, then n + 4 is even.

2. Prove that at least one of the real numbers $a_1, a_2, \ldots, a_n$ is greater than or equal to the average of these numbers. What kind of proof did you use?

3. Prove or disprove that if m and n are integers such that m*n = 1, then either m = 1 and n = 1, or else m = −1 and n = −1.

4. Show that at least ten of any 64 days chosen must fall on the same day of the week.

Proof by induction

Propositional function P(n) is a statement containing one or more variables that becomes a proposition when each of its variables is assigned a value or is bound by a quantifier

PRINCIPLE OF MATHEMATICAL INDUCTION To prove that P(n) is true for all positive integers, where P(n) is a propositional function, we complete two steps:
BASIS STEP: We verify that P(1) is true.
INDUCTIVE STEP: We show that the conditional statement $P(k) \rightarrow P(k + 1)$ is true for all positive integers k.

The assumption that P(k) is true is called the **inductive hypothesis**

Proof by induction - Example

Exercise Use mathematical induction to show that $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$ for all nonnegative integers n

Solution: Let *P(n)* be the proposition that $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$
for the integer *n*.
BASIS STEP: *P* (0) is true because $2^0 = 1 = 2^1 - 1$. This completes the basis step.
INDUCTIVE STEP: For the inductive hypothesis, we assume that *P(k)* is true for an arbitrary integer *k* ≥ 0. That is, we assume that $1 + 2 + 2^2 + \cdots + 2^k = 2^{\mathrm{k}+1} - 1$

To carry out the inductive step using this assumption, we must show that when we assume that *P(k)* is true, then *P*(*k* + 1) is also true.

Proof by induction – Example (continued)

That is, we must show that $1 + 2 + 2^2 + \cdots + 2^k + 2^{k+1} = 2^{\mathrm{k+2}} - 1$
assuming the inductive hypothesis P (k).

Under the assumption of P(k), we see that

$$1 + 2 + 2^2 + \cdots + 2^k + 2^{k+1} = \left(1 + 2 + 2^2 + \cdots + 2^k\right) + 2^{k+1} =$$

using the induction hypothesis, we get:

$$\left(1 + 2 + 2^2 + \cdots + 2^k\right) + 2^{k+1} = \left(2^{k+1} - 1\right) + 2^{k+1} = 2 * 2^{k+1} - 1 = 2^{k+2} - 1$$

We have completed the inductive step.

As we have completed the basis step and the inductive step, by mathematical induction we have shown that $1 + 2 + 2^2 + \cdots + 2^k + 2^{k+1} = 2^{\mathrm{k+2}} - 1$ is true for all nonnegative integers n.

Exercises II

1. Find and prove by induction a formula for $\sum_{i=1}^{n}(2i-1)$ (i.e., the sum of the first n odd numbers), where $n \in Z_+$

2. Find and prove by induction a formula for $\sum_{i=1}^{n}\frac{1}{i(i+1)}$, where $n \in Z_+$.

3. Prove that for any real number $x > -1$ and any positive integer x,

   $$(1+x)^n \geq 1+nx$$

Homework

1. Read textbook Kenneth H. Rosen "Discrete Mathematics and Its Applications, 7th Edition“ – paragraphs 1.7, 5.1

2. Create a folder on google drive, share it with your TA and upload the solutions to the following exercises by 10pm September 11 (Friday)

    1. Prove by contradiction that there exist no integers a and b for which $21a + 30b = 1$

    2. Prove by induction that $n! > 2^n$ for $n \geq 4$

    3. (Optional) Prove by induction that a convex n-gon has $n(n - 3)/2$ diagonals