Essentials of Analytical Geometry and Linear Algebra. Lecture 6.

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End of Lecture #5

- Part 1. Matrix inverse recap. General method
- Part 2. Applications

Lecture 6. Outline

- Part 1. Straight line in plane
- Part 2. Equations of a line
- Part 3. Pair of lines
- Part 4. Applications

Locus

Definition

"When a point moves so as to satisfy some geometrical condition or conditions, the path traced out by the point is called the **locus** of the point."

From: P. R. Vittal. "Analytical Geometry: 2D and 3D".

Example

Suppose a point $P(x, y)$ moves such that its distance from two fixed points $A(2, 3)$ and $B(5, -3)$ are equal. Then the geometrical law is $PA = PB \Rightarrow PA^2 = PB^2$

$$(x-2)^2 + (y-3)^2 = (x-5)^2 + (y+3)^2 \Rightarrow$$

$$2x - 4y - 7 = 0$$

(locus is a straight line)

Other examples

$$ax^2 + bxy + cy^2 = 0$$

$$ax^2 + by^2 = r^2$$

Part 1. Straight line in plane

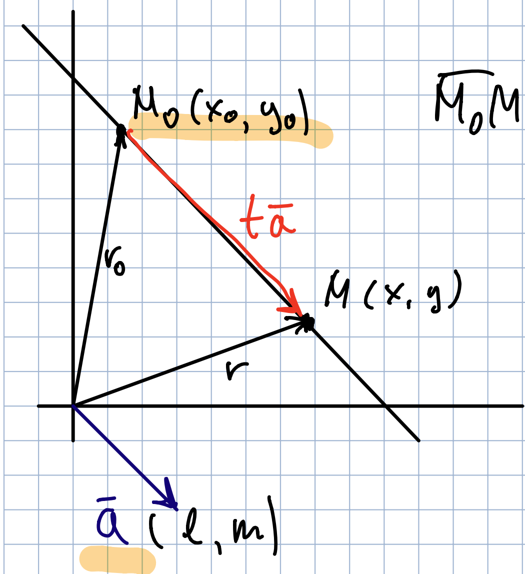
Definition

Given a point M_0 and a vector $\mathbf{a}$, set of all points M for which:

$$\overline{M_0 M} = t\mathbf{a}$$

$t \in \mathbb{R}$

$$\overline{M_0M} = t\bar{a}$$



$$\overline{M_0M} = t\bar{a} = \bar{r} - \bar{r}_0$$

$$\bar{r} = \bar{r}_0 + t\bar{a} \qquad t \in \mathbb{R}$$

$$\begin{cases} x = x_0 + tl \\ y = y_0 + tm \end{cases}$$

$$t = \frac{x - x_0}{l} \qquad t = \frac{y - y_0}{m}$$

$$\boxed{\frac{x - x_0}{l} = \frac{y - y_0}{m}}$$

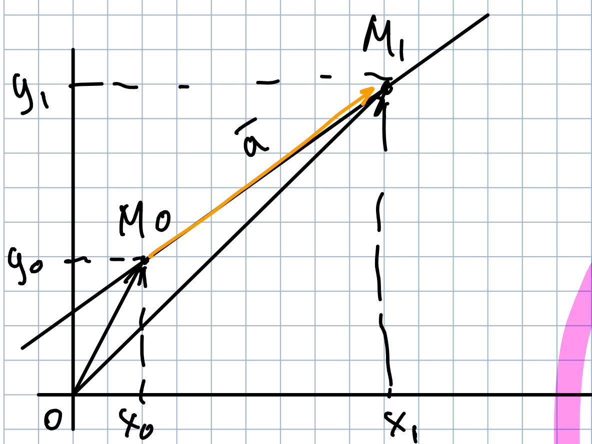
Canonical eq-n of a line

$$\underline{m}(x - x_0) = \underline{l}(y - y_0)$$

$$\underbrace{\boxed{\frac{m}{l}}}_{k}(x - x_0) = y - y_0 \Rightarrow y = kx + b$$

slope $\downarrow$ (k) intercept $\downarrow$ (b)

$$b = y_0 - \frac{m}{l}x_0$$



$M_0(x_0, y_0)$

$M_1(x_1, y_1)$

$$\bar{a} = \overline{OM_1} - \overline{OM_0}$$

$$\begin{cases} l = \underline{x_1 - x_0} \\ m = \underbrace{y_1 - y_0} \end{cases}$$

$$\bar{a} = \begin{bmatrix} l \\ m \end{bmatrix}$$

$$\frac{x - x_0}{l} = \frac{y - y_0}{m}$$

V

$$\boxed{\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0}}$$

$$\underline{A} = -m \qquad \underline{B} = l$$

$$\frac{x - x_0}{l} = \frac{y - y_0}{m} \qquad \frac{x - x_0}{B} = \frac{y - y_0}{-A}$$

$$\underbrace{A(x - x_0) + B(y - y_0) = 0}$$

$$\boxed{Ax + By + C = 0}$$

$A = -m$

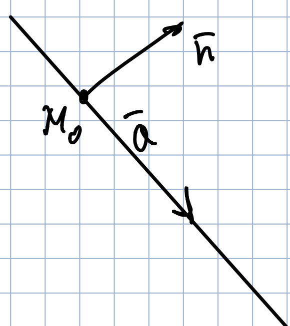
$B = l$

$$C = -Ax_0 - By_0$$

$\bar{a} = [l, m]$

$\bar{n} = [A, B]$

$$\bar{a} \cdot \bar{n} = 0$$



$$\begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = d$$

$$Ax + By = d$$

$$d = -C$$

$$\begin{bmatrix} A & B & C \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

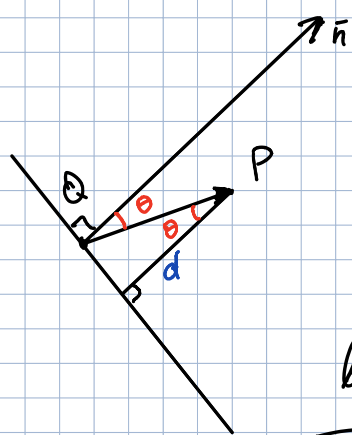
$P(1,2)$ $\quad\quad$ $Q(-2,5)$

$\bar{a} = [1, 2]$ $\quad\quad$ $O(0,0)$

find eq-n of line That

1. meets point $Q(-2,5)$ and parallel to line defined by $\bar{a}$, $O(0,0)$

(2.) Find distance between point P and line defined by point Q and directing vector $\bar{a}$



$d = ?$

Let $\bar{n}$ be a normal vector to the line.

$$\overline{QP} \cdot \bar{n} = |\overline{QP}| |\bar{n}| \cdot \cos\theta$$

But $|\overline{QP}| \cdot \cos\theta = d \Rightarrow$

$$\overline{QP} \cdot \bar{n} = |\bar{n}| \cdot d \Rightarrow$$

$$d = \frac{\overline{QP} \cdot \bar{n}}{|\bar{n}|}$$

if $\overline{QP}$ has coordinates (x_1, y_1) and $\bar{n}$ has coordinates (a, b) $\Rightarrow$

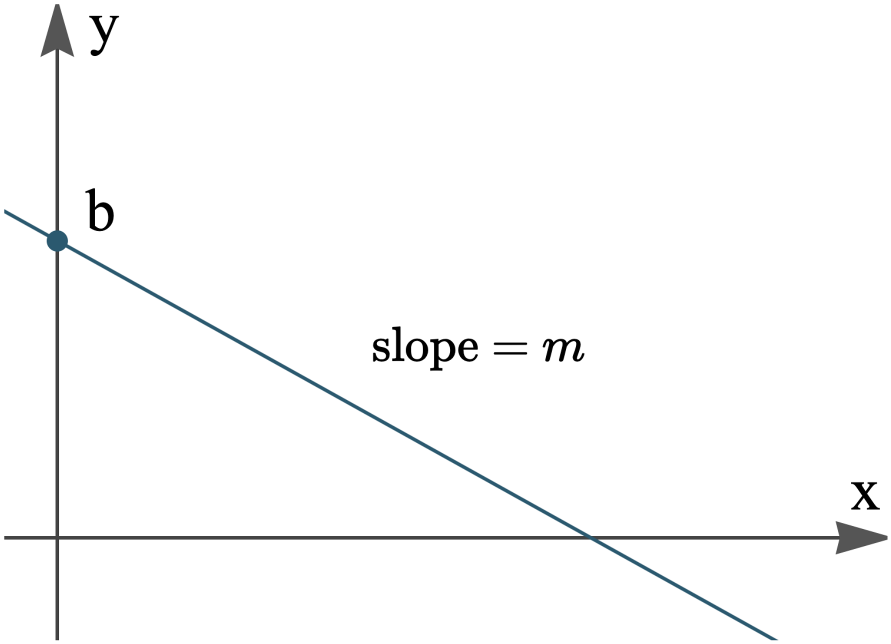
$$d = \frac{x_1 a + y_1 b}{\sqrt{a^2 + b^2}}$$

Note, than coordinates of $\bar{n}$ can be found using $\bar{a}$.

Slope-Intercept Equation

$$y = mx + b$$

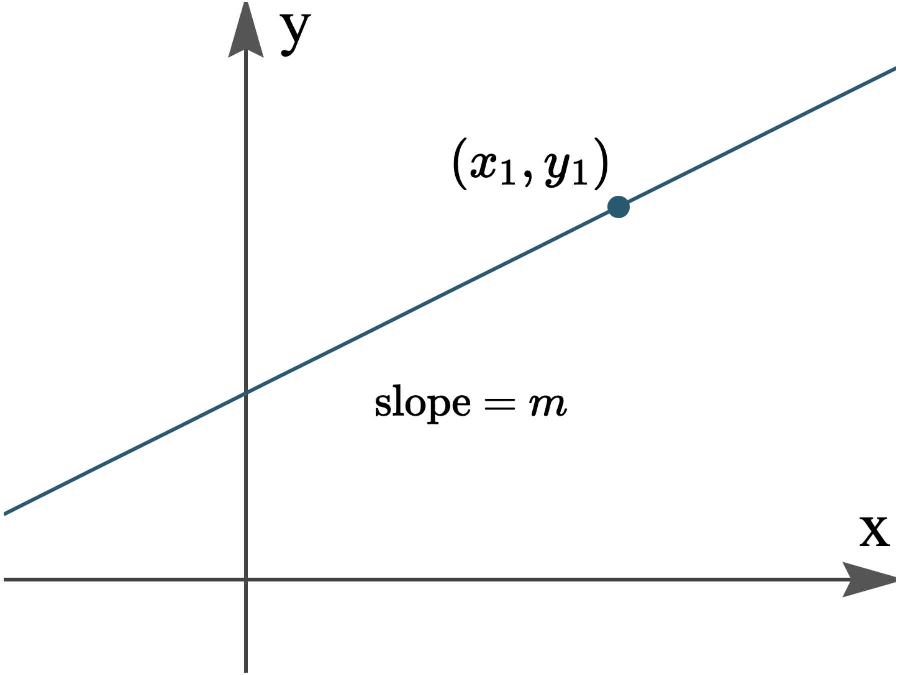
Example



Point-Slope Equation

$$y - y_1 = m(x - x_1)$$

Example



Part 2. Equations of a line (Forms of equations)

Parametric Equations

$$\overline{M_0 M} = t\mathbf{a}$$

General form

$$ax + by + c = 0$$

Problem solving

1) Find general equation of a line through (0,0).
2) Find equation of a line through (0,0) and (h,k).
3) Find equation of a line parallel to $y = -\frac{2}{3}x + 2$ and passing through point (9,-3).

Part 3. Pair of lines in a plane

Part 4. Applications

4.1. Calculation of distances

Distance between point and line

Find the perpendicular distance from the point (5, 6) to the line $-2x + 3y + 4 = 0$, 4.2. Linear classifiers

Useful links

- https://www.geogebra.org
- https://youtu.be/fNk_zzaMoSs
- http://immersivemath.com/ila