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N1

Formula of propositional logic – it is a proposition which consists of logic operations ($\wedge$, $\vee$, $\to$, $\equiv$, $\neg$) and variables and has a truth value.

Prove $A \vee (B \& C) = (A \vee B) \& (A \vee C)$

Truth table.

A	B	C	$A \vee (B \& C)$	$(A \vee B) \& (A \vee C)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

As we can see from the truth table, given formulas are equal

Q.E.D.

N2

A, B, C – sets.

Prove: $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Proof. Suppose $x \in A \times (B \cup C) \Leftrightarrow$

$\Leftrightarrow (x \in A) \times (x \in B \cup x \in C) \Leftrightarrow$

$\Leftrightarrow (x \in A \times x \in B) \vee (x \in A \times x \in C) \Leftrightarrow$
(distribution)

$\Leftrightarrow x \in (A \times B) \cup (A \times C)$ Q.E.D.

N4

A, B - sets. Prove $|A \cup B| = |A| + |B|$ if $A \cap B = \varnothing$

Let $A = \{a_1, a_2, a_3, \dots, a_n\}$ And $\forall 1 \le i \le n, 1 \le j \le m: a_i \ne b_j$

$B = \{b_1, b_2, b_3, \dots, b_m\}$ (According to the statement $A \cap B = \varnothing$)

As A have n elements, $|A| = n$

As B have m elements, $|B| = m$

So let's calculate $A \cup B$:

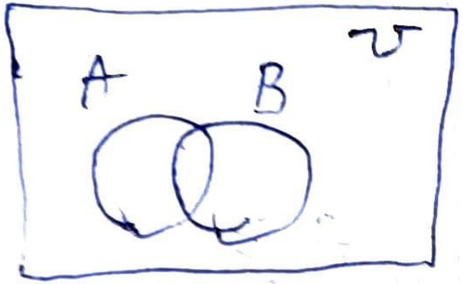
As $\forall 1 \le i \le n, 1 \le j \le m: a_i \ne b_j$ (given) we should add all elements of A (n) and of B (m) to our new set:

$$A \cup B = \{\underbrace{a_1, a_2, a_3, \dots, a_n}_{n \text{ elements}}, \underbrace{b_1, b_2, \dots, b_m}_{m \text{ elements}}\}.$$

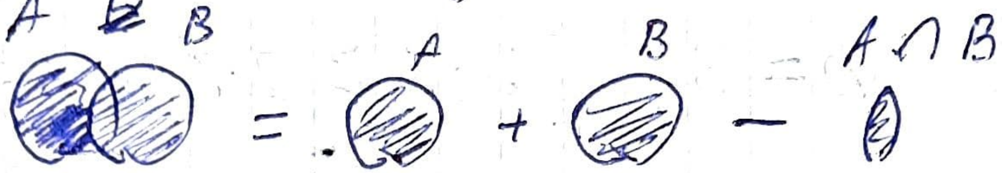
As $A \cup B$ have $n+m$ elements, $|A \cup B| = n+m$

Hence, $|A \cup B| = |A| + |B|$ Q.E.D.

Another way to prove is drawing Euler's diagramm:



We can easily notice that



Or $A \cup B = A + B - A \cap B$

~~What is true~~ So, $|A \cup B| = |A| + |B| - |A \cap B|$ is also true

But $A \cap B = \emptyset \Rightarrow |A \cap B| = |\emptyset| = 0$ So, we have for our case $|A \cup B| = |A| + |B|$ Q.E.D.

№3

Prove by induction $\forall n \in \mathbb{N}\ n \geq 3,\ n^2 \geq 2n+1$

Base: $n=3$: $3 \cdot 3 \geq 2 \cdot 3 + 1 \iff 9 \geq 7$ It is true

Induction case: Let's prove that if $P(n) = n^2 \geq 2n+1$ is true then $P(n+1)$ is also true.

$n^2 \geq 2n+1$

$n^2 + 2n + 1 \geq 2n + 1 + 2n + 1$, as $n \in \mathbb{N} \Rightarrow n > 0$

$(n+1)^2 \geq 4n+2 \overset{*}{>} 2n+3$

$* \ 4n+2 > 2n+3$
$2n > 1$
$n > \frac{1}{2}$ it is true for $\forall n \in \mathbb{N}$

$\Downarrow$

$(n+1)^2 > 2n+3 = 2(n+1)+1$

It is exactly $P(n+1)$. Hence, $P(n) \to P(n+1)$

Therefore, $n^2 \geq 2n+1$ is true for $\forall n \in \mathbb{N}$, $n \geq 3$.

Q.E.D.

№5

$R: S \to S$ $\quad S = \{3, 4, 8, 9\}$

$xRy \iff x \bmod y$ is odd

1) $R = \{(3,4), (3,8), (3,9), (4,3), (9,4), (9,8)\}$

2) R is irreflexive, because $\forall x \in S\ \neg(xRx)$

R is not transitive, because $\forall x, y, z \in S$
$\neg(xRy \,\&\, yRz \to xRz)$

R is not symmetric

R is not asymmetric

R is not antisymmetric

Answer: irreflexive
not transitive
not symmetric
not antisymmetric
not asymmetric.

3) Relation is called equivalence if it is
a) reflexive
b) symmetric
c) transitive.

But R is not neither a, b nor c.

So, R is not equivalence