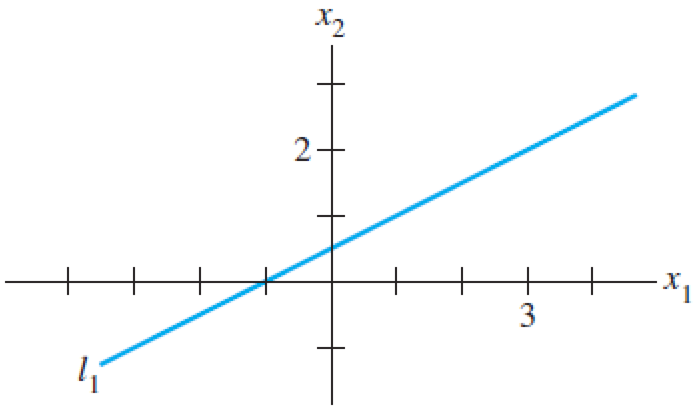
Practice Problems

1. The equation $x_1 = 2\sqrt{x_2} - 6$ is Not Linear.
2. The equation $x_2 = 2\left(\sqrt{6} - x_1\right) + x_3$ is Linear.

3. The following systems have Infinitely many solutions.

$$\begin{aligned} x_1 - 2x_2 &= -1 \\ -x_1 + 2x_2 &= 1 \end{aligned}$$



4.The following systems have One Solution.

$$\begin{aligned} 3x_1 + 6x_2 &= -3 \\ 5x_1 + 7x_2 &= 10 \end{aligned} \qquad \begin{bmatrix} 3 & 6 & -3 \\ 5 & 7 & 10 \end{bmatrix}$$

Scale R1 by 1/3 and obtain:

$$\begin{aligned} x_1 + 2x_2 &= -1 \\ 5x_1 + 7x_2 &= 10 \end{aligned} \qquad \begin{bmatrix} 1 & 2 & -1 \\ 5 & 7 & 10 \end{bmatrix}$$

Replace R2 by R2 + (–5)R1:

$$\begin{aligned} x_1 + 2x_2 &= -1 \\ -3x_2 &= 15 \end{aligned} \qquad \begin{bmatrix} 1 & 2 & -1 \\ 0 & -3 & 15 \end{bmatrix}$$

Scale R2 by –1/3:

$$\begin{aligned} x_1 + 2x_2 &= -1 \\ x_2 &= -5 \end{aligned} \qquad \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -5 \end{bmatrix}$$

Replace R1 by R1 + (–2)R2:

$$\begin{aligned} x_1 &= 9 \\ x_2 &= -5 \end{aligned} \qquad \begin{bmatrix} 1 & 0 & 9 \\ 0 & 1 & -5 \end{bmatrix}$$

The solution is $(x_1, x_2) = (9, -5)$, or simply $(9, -5)$.

Practice Problems

5. The following systems have One Solution.

The point of intersection satisfies the system of two linear equations:

$$\begin{aligned} x_1 + 2x_2 &= -13 \\ 3x_1 - 2x_2 &= 1 \end{aligned} \qquad \begin{bmatrix} 1 & 2 & -13 \\ 3 & -2 & 1 \end{bmatrix}$$

Replace R2 by R2 + (–3)R1 and obtain:

$$\begin{aligned} x_1 + 2x_2 &= -13 \\ -8x_2 &= 40 \end{aligned} \qquad \begin{bmatrix} 1 & 2 & -13 \\ 0 & -8 & 40 \end{bmatrix}$$

Scale R2 by –1/8:

$$\begin{aligned} x_1 + 2x_2 &= -13 \\ x_2 &= -5 \end{aligned} \qquad \begin{bmatrix} 1 & 2 & -13 \\ 0 & 1 & -5 \end{bmatrix}$$

Replace R1 by R1 + (–2)R2:

$$\begin{aligned} x_1 \quad &= -3 \\ x_2 &= -5 \end{aligned} \qquad \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -5 \end{bmatrix}$$

The point of intersection is $(x_1, x_2) = (-3, -5)$.

6. The following system is Inconsistent

First, swap R1 and R2. Then replace R3 by R3 + (–2)R1. Finally, replace R3 by R3 + (1)R2.

$$\begin{bmatrix} 0 & 1 & 5 & -4 \\ 1 & 4 & 3 & -2 \\ 2 & 7 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 2 & 7 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & -1 & -5 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

The system is inconsistent, because the last row would require that $0 = -2$ if there were a solution. The solution set is empty.

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7. Consider the following matrix as a row echelon form matrix of a linear system. Is the system consistent?

$$\begin{bmatrix} 1 & -4 & -3 & 0 & 7 \\ 0 & 1 & 4 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & -5 \end{bmatrix}$$

The system is Inconsistent, because of the last row would require 0 = -5 if there were a solution. The solution set is empty.

8. For what values of *h* and *k* is the following syst

$$\begin{aligned} 2x_1 - x_2 &= h \\ -6x_1 + 3x_2 &= k \end{aligned}$$

When the second equation is replaced by its sum with 3 times the first equation, the system becomes

$$\begin{aligned} 2x_1 - x_2 &= h \\ 0 &= k + 3h \end{aligned}$$

If $k + 3h$ is nonzero, the system has no solution. The system is consistent for any values of h and k that make $k + 3h = 0$.