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VI $\lim\limits_{x\to 0} \dfrac{\cos x - 1 + \frac{x^2}{2}}{x^4} - ?$

Let us decompose function $\cos x$ using Maclaurin formula:

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots + \frac{(-1)^n x^{2n}}{(2n)!} + o(x^{2n+1})$$

Now substitute it to our limit:

$$\lim_{x\to 0} \frac{\cos x - 1 + \frac{x^2}{2}}{x^4} = \lim_{x\to 0} \frac{1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \ldots - 1 + \frac{x^2}{2}}{x^4} =$$

$$= \lim_{x\to 0} \left(\frac{1}{24} - \underbrace{\frac{x^2}{6!}}_{\to 0} + \underbrace{\frac{x^4}{8!}}_{\to 0} + \ldots \right)_{\to 0} = \frac{1}{24}.$$

Answer: $\frac{1}{24}$.