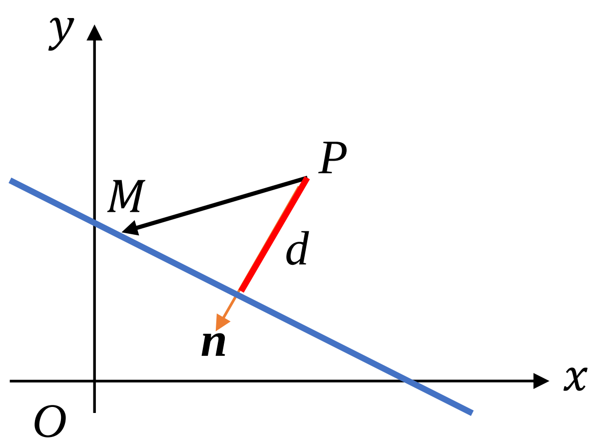
Distance From a Point to a Line

Consider a line in the x, y–plane.
Let $\boldsymbol{n}$ be a normal vector to the line and $M(x_0, y_0)$ be any point on the line. Then the distance d from a point P not on the line is equal to the absolute value of the projection of $\overrightarrow{PM}$ on $\boldsymbol{n}$:

$$d = \left|\text{Proj}_n\, \overrightarrow{PM}\right| = \left|\frac{\overrightarrow{PM} \cdot \boldsymbol{n}}{|\boldsymbol{n}|}\right|$$



In particular, if the line is given by the equation

$$Ax + By + C = 0\,,$$

and the coordinates of the point P are x_1 and y_1, that is,
$\boldsymbol{n} = \{A, B\}$ and $\overrightarrow{PM} = \{x_1 - x_0, y_1 - y_0\}$,
then the distance from the point $P(x_1, y_1)$ to the line is calculated according to the following formula:

$$d = \frac{|A(x_1 - x_0) + B(y_1 - y_0)|}{\sqrt{A^2 + B^2}}$$

Since $M(x_0, y_0)$ is a point on the line,

$$Ax_0 + By_0 + C = 0$$

Therefore, we obtain

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Example

Let ABC be a triangle in x, y–plane with the vertices at the points $A = \{2, -1\}$, $B = \{4, 4\}$ and $C = \{9, 7\}$.
Find the altitude from the vertex A.

Solution

The altitude from the vertex A equals the distance d from the point A to the line passing through the points B and C.
Find the equation of the line BC:

$$\frac{x - x_1}{p_x} = \frac{y - y_1}{p_y} \Rightarrow \frac{x - 4}{5} = \frac{y - 4}{3} \Rightarrow 3x - 5y + 8 = 0$$

Therefore, a normal vector to the line BC is $\mathbf{n} = \{3, -5\}$.

Since $\overrightarrow{AC} = \{7, 8\}$, we finally obtain

$$d = \left|\mathrm{Proj}_n \overrightarrow{AC}\right| = \left|\frac{\overrightarrow{AC} \cdot \boldsymbol{n}}{|\boldsymbol{n}|}\right| \Rightarrow \left|\frac{7 \cdot 3 - 5 \cdot 8}{\sqrt{25 + 9}}\right| = \frac{19}{\sqrt{34}} \approx 3.25$$