$$y = \frac{x^2 + x - 1}{x^2 - 2x + 1} \qquad D(f): (-\infty; 1) \cup (1; +\infty)$$

1) Vertical assymptotes

$$\lim_{x \to 1+0} \frac{x^2+x-1}{x^2-2x+1} = \lim_{x \to 1+0} \frac{1 + \frac{1}{x} - \frac{1}{x^2}}{1 - \frac{2}{x} + \frac{1}{x^2}} = \lim_{x \to 1+0} \frac{1+1-1}{1-2+1} = \lim_{x \to 1+0} \frac{1}{0} = \infty$$

$$\lim_{x \to 1-0} \frac{x^2+x-1}{x^2-2x+1} = \lim_{\varepsilon \to 0} \frac{\cancel{(\varepsilon+1)} + (\varepsilon+1)^2 + \varepsilon}{\varepsilon^2} = \lim_{\varepsilon \to 0} \frac{1}{0} = \infty$$

$\varepsilon = x - 1$

Therefore, $x = 1$ - assymptote

2) Horizontal assymptote

$$\lim_{x \to \infty} \frac{x^2+x-1}{x^2-2x+1} = \lim_{x \to \infty} \frac{1 + \overbrace{\frac{1}{x}}^{\to 0} - \overbrace{\frac{1}{x^2}}^{\to 0}}{1 - \underbrace{\frac{2}{x}}_{\to 0} + \underbrace{\frac{1}{x^2}}_{\to 0}} = \lim_{x \to \infty} \frac{1}{1} = 1$$

$$\lim_{x \to -\infty} = \lim_{x \to \infty} \frac{1}{1} = 1$$

Therefore $y = 1$ - assymptote.

3.) Line assymptote : $y = kx + b$

$$k = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{x^2 + x - \cancel{x}1}{x^3 - 2x^2 + x} = \cancel{\lim_{x \to \infty} \frac{x^2(1 + \frac{1}{x} - \frac{1}{x^2})}{x^2(x - 2 + \frac{1}{x})}} =$$

$$= \cancel{\lim_{x \to \infty} \frac{1 + \frac{1}{x} - \frac{1}{x^2}}{x - 2 + \frac{1}{x}} = \lim_{x \to \infty} \frac{1}{x-2}} = \lim_{x \to \infty} \frac{\frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3}}{1 - \frac{2}{x} + \frac{1}{x^2}} = 0$$

$k = 0 \Rightarrow$ No Line assymptote

Answer: $x = 1$ - vertical assymptote
$y = 1$ - horizontal assymptote
no line assymptota.

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