

Aivika 3 User Guide:  
Version for .NET Framework and Mono

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# Introduction

... A fews words about simulation in general and Aivika in particular ...



# Chapter 1

## Getting Started

Before we build our first simulation model, we have to introduce some basic concepts that lie in foundation of the Aivika simulation approach. They widely use a notion of abstract computation and their practical usability essential depends on the special feature of the F# programming language, which is known as *computation expressions*.

If you are already familiar with the F# asynchronous workflow then you can find the Aivika approach simple and easy to use. Otherwise, it is strongly recommended that you should be acquainted with the mentioned asynchronous workflow first.

### 1.1 Simulation

We can treat the simulation as a function of the simulation run:

```
namespace Simulation.Aivika

type Simulation<'a> = Simulation of (Run -> 'a)
```

Here the `Run` type denotes some object that contains the information about the current simulation run. Its definition is quite implementation dependent.

There is also a computation expression builder that allows us to create `Simulation` computations. The builder has name `simulation`:

```
let x : Simulation<'a> = simulation { .. }
```

Given the specified simulation `Specs`, we can create a simulation `Run` and then launch the simulation to receive the result:

```
module Simulation =

    val run : Specs -> Simulation<'a> -> 'a
    val runSeries : int -> Specs -> Simulation<'a> -> seq<Async<'a>>
```

The simulation specs can contain the information about the start time and final time of modeling. Since Aivika also allows us to integrate the differential equations, we must provide the integration time step and the method regardless of whether they are actually used. Also the specs can define the random number generator that we can use in the model.

```

type Time = float
type Method = Euler | RungeKutta2 | RungeKutta4

type Specs =
  { StartTime : Time;
    StopTime : Time;
    DT : Time;
    Method : Method;
    GeneratorType : GeneratorType }

```

The omitted `GeneratorType` type allows specifying the random number generator.

For example, we can use the same seed with the `SimpleGeneratorWithSeed` data constructor to get always a reproducible sequence of numbers, which can be helpful for testing. Below we will use the `StrongGenerator` data constructor to get a random number generator of high quality. But you can also use the `SimpleGenerator` data constructor. It is quite fast on every platform, but it returns random numbers of rather poor quality on Windows, while the random numbers generated on OS X seem to be quite good.

Returning to the main topic, the main idea is that many simulation models can be ultimately reduced to the `Simulation` computation. Hence they can be trivially simulated using the mentioned above run functions by the specified specs. At least, all models that we will build with help of Aivika are such ones.

Now the following concept may look difficult at a glance, but it is very important for understanding. All simulation computations in Aivika are interconnected. They can be either transformed to others or can be run within others. The transformation of computation is usually called *lifting* in functional programming.

Aivika defines in different modules a set of inline functions that all have a common name `lift`. The following function allows transforming an arbitrary `Simulation` computation to something equivalent but defined as another computation denoted below as awkward type with letter *m*.

```

module Simulation =
  val inline lift : Simulation<'a> -> ^m

```

It allows transforming the `Simulation` computation to anything else:

$$\begin{array}{c}
 \text{Simulation<'a>} \\
 \downarrow \text{Simulation.lift} \\
 \dots
 \end{array}$$

## 1.2 External Parameters

In practice many models depend on external parameters, which is useful for providing the Sensitivity Analysis.

To represent such parameters, Aivika uses almost the same definition that it uses for representing the `Simulation` computation.

```

type Parameter<'a> = Parameter of (Run -> 'a)

```

The corresponding computation builder is called `parameter`:

```
let x: Parameter<'a> = parameter { .. }
```

A key difference between two computations `Simulation` and `Parameter` is that the parameter can be memoized before running the simulation so that the resulting `Parameter` computation would return a constant value within every simulation run and then its value would be updated for other runs (in a thread-safe way).

```
module Parameter =
  val memo : Parameter<'a> -> Parameter<'a>
```

We usually have to memoize the parameter if its computation is impure and it depends on performing some side effect such as reading an external file or generating a random number.

It is natural to represent the simulation specs as external parameters when modeling.

```
module Parameter =

  val starttime : Parameter<Time>
  val stoptime : Parameter<Time>
  val dt : Parameter<Time>
```

Since we provide the random number generator with the simulation specs, it is also natural to generate the random numbers within the `Parameter` computation.

```
module Parameter =

  val randomUniform : float -> float -> Parameter<float>
  val randomNormal : float -> float -> Parameter<float>
  val randomExponential : float -> Parameter<float>
  val randomErlang : float -> int -> Parameter<float>
  val randomPoisson : float -> Parameter<int>
  val randomBinomial : float -> int -> Parameter<int>
```

To support the Design of Experiments (DoE), Aivika defines two additional computations that return the current simulation run index and the total run count respectively, when launching the Monte-Carlo simulation.

```
module Parameter =

  val runIndex : Parameter<int>
  val runCount : Parameter<int>
```

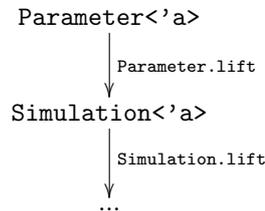
As before, there is the `lift` function that allows transforming an arbitrary `Parameter` computation.

```
module Parameter =
  val inline lift : Parameter<'a> -> ^m
```

For example, an arbitrary `Parameter` computation can be transformed to the `Simulation` one. It means that the former can be used in every piece of the code, where the `Simulation` computation is expected. It is just enough to call the `lift` function:

```
let x1 : Parameter<'a> = ...
let x2 : Simulation<'a> = x1 |> Parameter.lift
```

It allows using the external parameters within the simulation:



### 1.3 Ordinary Differential Equations

Although the main strength of the Aivika library is its orientation on discrete event simulation, it is simpler to demonstrate the method using another paradigm. So, our first simulation model will be described by a set of ordinary differential equations (ODEs).

Assuming that the `Point` type represents a modeling time point within the current simulation run, we can define a time varying function which would be suitable for approximating the integrals.

Let us call it the `Dynamics` computation to emphasize that it can model some dynamic processes defined usually with help of differential equations and difference equations of System Dynamics.

```
type Dynamics<'a> = Dynamics of (Point -> 'a)
```

The corresponding computation builder is called `dynamics`:

```
let x: Dynamics<'a> = dynamics { .. }
```

Since the modeling time is passed in to every part of the `Dynamics` computation, it is natural to define the following computation that would return the current time.

```
module Dynamics =
  val time : Dynamics<Time>
```

There are different functions that allow running the `Dynamics` computation within the simulation: in the start time, in the final time, in all integration time points and in arbitrary time points defined by their numeric values.

```
module Dynamics =

  val runInStartTime : Dynamics<'a> -> Simulation<'a>
  val runInStopTime : Dynamics<'a> -> Simulation<'a>

  val runInIntegTimes : Dynamics<'a> -> Simulation<seq<'a>>
  val runInTimes : seq<Time> -> Dynamics<'a> -> Simulation<seq<'a>>
```

A key feature of the `Dynamics` computation is that it allows approximating the integral by the specified derivative and initial value:

```
module SD =
  val integ : Lazy<Dynamics<float>> -> Dynamics<float> -> Dynamics<float>
```

The point is that the ordinary differential and difference equations can be defined declaratively almost as in maths and as in many commercial simulation software tools of System Dynamics such as Vensim[18], itthink/Stella[5], Berkeley-Madonna[6] and Simtegra MapSys<sup>1</sup>.

Aivika overloads arithmetic operators for type `Dynamics<float>`. It allows us to treat these computations as numbers.

Moreover, we can create new computations from real numbers:

```
module SD =
  val num : 'a -> Dynamics<'a>
```

Actually, this is a synonym of the `dynamics.Return` method but the former is more convenient for using within differential equations as it will be shown below.

To demonstrate the approach, we can rewrite a model from the 5-Minute Tutorial of Berkeley-Madonna[6] with the following equations.

$$\begin{aligned} \dot{a} &= -ka \times a, & a(t_0) &= 100, \\ \dot{b} &= ka \times a - kb \times b, & b(t_0) &= 0, \\ \dot{c} &= kb \times b, & c(t_0) &= 0, \\ ka &= 1, \\ kb &= 1. \end{aligned}$$

For example, we can return the integral values in the final simulation time. In the same way, we could return the integral values in arbitrary time points that we would specify using other run functions.

But there is another way. We can return a `ResultSet` object that encompasses all results we are interested in.

Assuming that the library implementation files `Simulation.Aivika.dll` and `Simulation.Aivika.Results.dll` are referenced properly, we can write the following complete script with the definition of our simulation model in file `Model.fsx`.

```
// File ChemicalReaction/Model.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"

#nowarn "40"

open System

open Simulation.Aivika
open Simulation.Aivika.Results
open Simulation.Aivika.SD

let specs = {

  StartTime=0.0; StopTime=13.0; DT=0.01;
  Method=RungeKutta4; GeneratorType=StrongGenerator
```

---

<sup>1</sup>In the past the author of Aivika developed visual simulation software tool `Simtegra MapSys`, but the software is unfortunately not available for the wide audience any more.

```

}

let model : Simulation<ResultSet> = simulation {

    let rec a = integ (lazy (- ka * a)) (num 100.0)
    and b = integ (lazy (ka * a - kb * b)) (num 0.0)
    and c = integ (lazy (kb * b)) (num 0.0)
    and ka = 1.0
    and kb = 1.0

    return
        [ResultSource.From ("A", a, "Var A");
         ResultSource.From ("B", b, "Var B");
         ResultSource.From ("C", c, "Var C")]
        |> ResultSet.create
}

```

To show the results in the final time point, for example, we can call the corresponding function.

We can write the code in another file `Run.fs`, which will allow us to use the model repeatedly for another goal as we will see soon.

```

// File ChemicalReaction/Run.fsx

#load "Model.fsx"

open Simulation.Aivika
open Simulation.Aivika.Results

ResultSet.printInStopTime Model.specs Model.model

```

It prints the following information in terminal:

```

-----

// time
t = 13

// Var A
A = 0,000226032940945024

// Var B
B = 0,00293842823104866

// Var C
C = 99,9968355388281

```

Like other computations, there is a transforming function for the `Dynamics` computation as well.

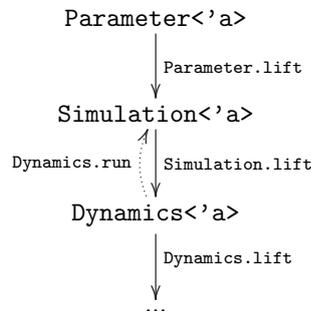
```

module Dynamics =
    val inline lift : Dynamics<'a> -> ^m

```

It allows using the `Dynamics` computation in those places, where something different is expected. It is worth noting that the F# compiler checks the types and it won't allow transforming the computation if it makes no sense. So, the lifting functions are quite safe.

The `Parameter` and `Simulation` computations can be transformed to the `Dynamics` one:



## 1.4 Simulation Experiment

The model constructing is very important by no means but it is not sufficient. To validate the model or to analyze it, we have to automate the process of displaying the most important simulation results. Aivika provides with such an ability.

The simulation library allows saving the results in CSV files that can be then opened in the Office application or R statistics tool for the further analysis. Also the library allows plotting the results on charts as well plotting the histograms by collected statistics.

One of the important charts is so called the Deviation Chart that plots the trend and probabilistic bounds by rule *3-sigma*. There are also Time Series and XY Chart that draw the simulation results for each run, while the deviation chart is cumulative and it is displayed for the whole Monte-Carlo simulation experiment.

When running the simulation experiment, Aivika creates a Web page containing file `index.html` and corresponding auxiliary files in a separate directory. Then you can open the Web page in your favorite Internet browser to observe the simulation results.

This approach actually allows running thousands of simulation runs within one experiment, when only necessary data are kept in memory. At the same time, the Internet browser becomes a tool for final displaying the results.

We define an `Experiment` object specifying the simulation specs and a number of runs.

```

let experiment = Experiment ()

experiment.Specs <- Model.specs
experiment.RunCount <- 1
  
```

Then we define the `IExperimentProvider` providers that already know how to render the results.

```

let provider1 = ExperimentSpecsProvider ()
let provider2 = TimeSeriesProvider ()
let provider3 = LastValueProvider ()
let provider4 = TableProvider ()
  
```

The most important property of the provider is `Series` or something different with similar name.

So, we could ask the Time Series provider to render three variables A, B and C that we return from the model.

```
provider2.Series <-
  [ ResultSet.findByName "A";
    ResultSet.findByName "B";
    ResultSet.findByName "C"]
  |> ResultTransform.concat
```

By default many providers render all variables that are returned by the model. Therefore, we could omit this assignment statement.

Then we ask the experiment to render HTML by the specified model using our providers.

```
experiment.RenderHtml (Model.model, providers)
  |> Async.RunSynchronously
```

Below is stated a complete simulation experiment script<sup>2</sup> written in F#.

```
// File ChemicalReaction/RunExperiment.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"
#r "../bin/Simulation.Aivika.Experiments.dll"
#r "../bin/Simulation.Aivika.Charting.dll"

#load "Model.fsx"

open System
open System.Web.UI

open Simulation.Aivika
open Simulation.Aivika.Results
open Simulation.Aivika.Experiments
open Simulation.Aivika.Experiments.Web
open Simulation.Aivika.Charting.Web

let experiment = Experiment ()

experiment.Specs <- Model.specs
experiment.RunCount <- 1

let provider1 = ExperimentSpecsProvider ()
let provider2 = TimeSeriesProvider ()
let provider3 = LastValueProvider ()
let provider4 = TableProvider ()

let providers =
  [ provider1 :> IExperimentProvider<HtmlTextWriter>;
    provider2 :> IExperimentProvider<HtmlTextWriter>;
    provider3 :> IExperimentProvider<HtmlTextWriter>;
    provider4 :> IExperimentProvider<HtmlTextWriter> ]

experiment.RenderHtml (Model.model, providers)
  |> Async.RunSynchronously
```

When running this script, we receive in the terminal of Microsoft Windows something like this

---

<sup>2</sup>On Linux and OS X we have to replace assembly `Simulation.Aivika.Charting.dll` with its GTK version as well as have to use the `Simulation.Aivika.Charting.Web.Gtk` namespace.

```
C:\Docs\Projects\Aivika\examples\ChemicalReaction>fsi RunExperiment.fsx
Updating directory experiment
Generated file experiment\TimeSeries(1).png
Generated file experiment\Table(1).csv
Generated file index.html
```

It means that the script created directory `experiment` containing the Web page that we can open in the Internet browser.

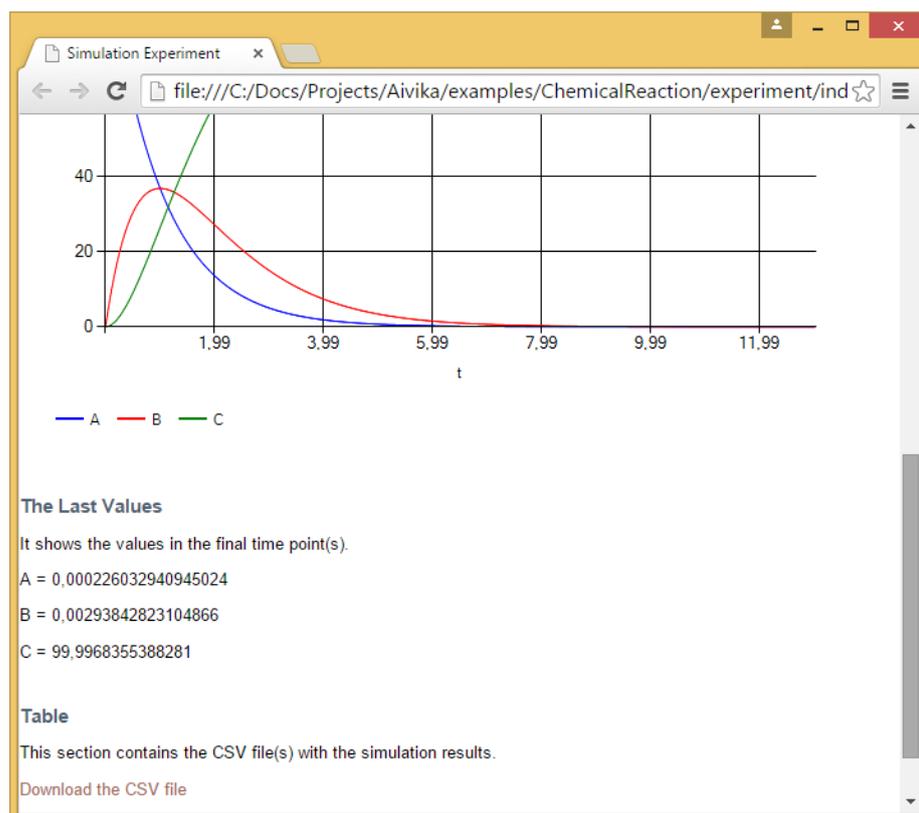


Figure 1.1: The rendered simulation experiment in the Internet browser.

The Web page shows the simulation experiments specs, the time series chart, the last values and provides with a hyper-link to load the CSV file with the results.

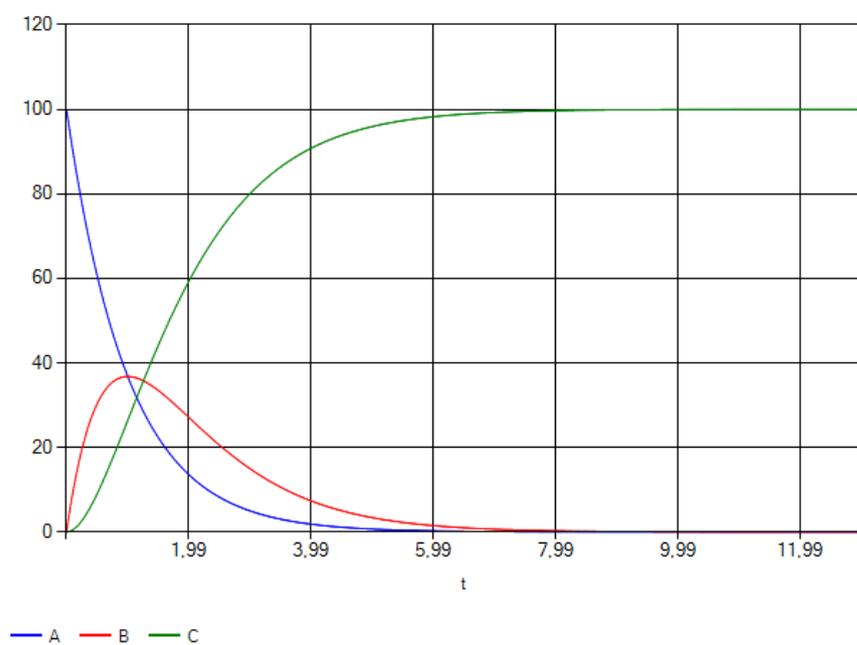


Figure 1.2: The time series chart for chemical reaction.

## Chapter 2

# Discrete Event Simulation

The main focus of the Aivika library is the discrete event simulation (DES). Below are described basic ideas used in Aivika for formalizing the simulation model and reasoning in terms of this paradigm.

### 2.1 Event-oriented Simulation

Under the *event-oriented* paradigm[10, 7] of DES, we put all pending events in the priority queue, where the first event has the minimal activation time. Then we sequentially activate the events removing them from the queue. During such an activation we can add new events. This scheme is also called *event-driven*.

We can use almost the same time-varying function for the event-oriented simulation, which we used for approximating the integrals with help of the `Dynamics` workflow.

```
namespace Simulation.Aivika

type Eventive<'a> = Eventive of (Point -> 'a)
```

The difference is that we can strongly guarantee<sup>1</sup> on level of the type system of F# that the `Eventive` computation is always synchronized with the event queue. Here we imply that every simulation run has an internal event queue, which is contained in the `Run` object.

A key feature of the `Eventive` computation is an ability to specify the event handler that should be actuated at the desired modeling time, when the corresponding event occurs.

```
module Eventive =
    val enqueue : Time -> Eventive<unit> -> Eventive<unit>
```

To pass in a message or some other data to the event handler, we just use a closure when specifying the event handler in the second argument.

The event cancellation can be implemented trivially. We create a wrapper for the source event handler and pass in namely this wrapper to the `enqueue` function. Then the wrapper already decides whether it should call the underlying source event handler. Then we have to provide some means for notifying the

---

<sup>1</sup>Actually, there is a room in Aivika for some hacking that may break this strong guarantee.

wrapper that the source event handler must be cancelled. The Aivika library has the corresponded support.

The same technique of canceling the event can be adapted to implementing the timer and time-out handlers used in the agent-based modeling as it is described later.

To involve in the simulation, the `Eventive` computation must be run explicitly or implicitly within the `Dynamics` computation. The most simple run function is stated below. It actuates all pending event handlers from the event queue relative to the current modeling time and then runs the specified computation.

```
module Eventive =
  val run : Eventive<'a> -> Dynamics<'a>
```

The corresponding computation builder has name `eventive`:

```
let x : Eventive<'a> = eventive { .. }
```

There is a subtle thing related to the `Dynamics` computation. In general, the modeling time flows unpredictably within `Dynamics`, while there is a guarantee that the time is synchronized with the event queue within the `Eventive` computation.

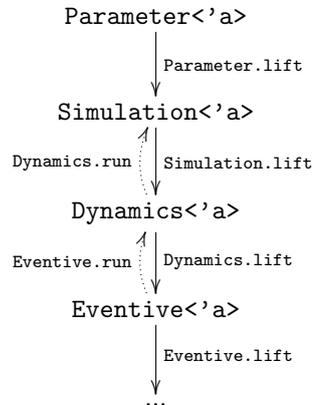
Some other run functions are destined for the most important use cases, when we can run the input computation directly within `Simulation` in the initial and final modeling time points, respectively.

```
module Eventive =
  val runInStartTime : Eventive<'a> -> Simulation<'a>
  val runInStopTime : Eventive<'a> -> Simulation<'a>
```

Following the rule, an arbitrary `Dynamics` computation can be transformed to the `Eventive` computation. The latter in its turn can be transformed to another with help of the corresponding `lift` function.

```
module Eventive =
  val inline lift : Eventive<'a> -> ^m
```

It literally means that the integrals, external parameters and computations on level of the simulation run can be directly used in the event-oriented simulation:



## 2.2 Mutable Reference

Many DES models need a mutable reference. F# already provides with the `ref` reference type. We can use it in the simulation model whenever it can be replaced with the following `Ref` type under simple obligations: the reference must be created within `Simulation` computation and then be used within `Eventive` or another computation to which the latter can be transformed with help of the `lift` function.

```
type Ref<'a>

module Ref =

    val create : 'a -> Simulation<Ref<'a>>

    val read : Ref<'a> -> Eventive<'a>
    val write : 'a -> Ref<'a> -> Eventive<unit>
    val modify : ('a -> 'a) -> Ref<'a> -> Eventive<unit>

    val inc : Ref<int> -> Eventive<unit>
    val dec : Ref<int> -> Eventive<unit>
```

If the standard `ref` reference of F# can be safely replaced with the `Ref` type then we will prefer the former as more easy-to-use.

## 2.3 Example: Event-oriented Simulation

The Aivika distribution contains examples of using the mutable references in the DES models, one of which is provided below. The task itself is described in the documentation of `SimPy`[7].

There are two machines, which sometimes break down. Up time is exponentially distributed with mean 1.0, and repair time is exponentially distributed with mean 0.5. There are two repairpersons, so the two machines can be repaired simultaneously if they are down at the same time. Output is long-run proportion of up time. Should get value of about 0.66.

The corresponding model is as follows.

```
// File MachReplEventDriven/Model.fsx

#nowarn "40"

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"

open Simulation.Aivika
open Simulation.Aivika.Queues
open Simulation.Aivika.Results

let specs = {

    StartTime=0.0; StopTime=10000.0; DT=0.05;
    Method=RungeKutta4; GeneratorType=StrongGenerator
```

```

}

let meanUpTime = 1.0
let meanRepairTime = 0.5

let model = simulation {

  // total up time for all machines
  let totalUpTime = ref 0.0

  let rec machineBroken startUpTime =
    eventive {

      // the machine is broken

      let! finishUpTime =
        Dynamics.time |> Dynamics.lift

      totalUpTime := !totalUpTime
        + (finishUpTime - startUpTime)

      let! repairTime =
        Parameter.randomExponential meanRepairTime
          |> Parameter.lift

      // register a new event

      do! machineRepaired
        |> Eventive.enqueue
          (finishUpTime + repairTime)
    }
  and machineRepaired =
    eventive {

      // the machine is repaired

      let! startUpTime =
        Dynamics.time |> Dynamics.lift

      let! upTime =
        Parameter.randomExponential meanUpTime
          |> Parameter.lift

      // register a new event

      do! machineBroken startUpTime
        |> Eventive.enqueue
          (startUpTime + upTime)
    }

  do! machineRepaired |> Eventive.runInStartTime
  do! machineRepaired |> Eventive.runInStartTime

  let upTimeProp =
    eventive {
      let! t = Dynamics.time |> Dynamics.lift
      return (!totalUpTime / (2.0 * t))
    }
  }

  return
  [ResultSource.From ("upTimeProp", upTimeProp,
    "Long-run proportion of up time \

```

```

        (must be about 0.66)"]
        |> ResultSet.create
}

```

In the simplest case we can use the next simulation experiment that shows the results in the final simulation time.

```

#load "Model.fsx"

open Simulation.Aivika
open Simulation.Aivika.Results

ResultSet.printInStopTime Model.specs Model.model

```

When running it, we receive in the terminal of OS X something like this

```

bash-3.2$ fsharp Run.fsx
-----

// time
t = 10000

// Long-run proportion of up time (must be about 0.66)
upTimeProp = 0,661181942118896

```

Frankly speaking, the use of the event-oriented paradigm may seem to be quite tedious. Aivika supports more high-level paradigms. Later it will be shown how the same task can be solved in a more simple way.

## 2.4 Variable with Memory

Sometimes we need an analog of the mutable reference that would save the history of its values. Aivika defines the corresponding `Var` type. It has almost the same functions with similar type signatures that the `Ref` reference has.

```

type Var<'a>

module Var =

    val create : 'a -> Simulation<Var<'a>>

    val read : Var<'a> -> Eventive<'a>
    val write : 'a -> Var<'a> -> Eventive<unit>
    val modify : ('a -> 'a) -> Var<'a> -> Eventive<unit>

    val inc : Var<int> -> Eventive<unit>
    val dec : Var<int> -> Eventive<unit>

```

However, we can also use the variable in the differential and difference equations requesting for the *first* actual value for *each* time point with help of the following function, actuating the pending events if required.

```

module Var =
    val memo : Var<'a> -> Dynamics<'a>

```

The magic is as follows. The `Var` variable stores the history of changes. When updating the mutable variable, or requesting it for a value at new time

point, the `Var` data object stores internally the value, which was first for the requested time point. Then it becomes constant within the simulation. Therefore, the computation returned by the `memo` function can be used in the differential and difference equations of System Dynamics.

On the contrary, the `read` function returns a computation of the *recent* actual value for the *current* simulation time point. This value is already destined to be used in the discrete event simulation as it is synchronized with the event queue by the very design of the library. Such is the `Eventive` computation that it must be synchronized with the event queue.

In case of need we can freeze temporarily the variable and receive its internal state: triples of time, the first and last values for each time.

```
module Var =
  val freeze : Var<'a> -> Eventive<Time [] * 'a [] * 'a []>
```

The time values returned by this function are distinct and sorted in ascending order.

As a caution, try to avoid using the `Var` variable as it is rather slow in comparison to the standard F# reference and even the `Ref` reference. The usual mistake of novices is to use the variable for accumulating statistics, but Aivika contains optimized data structures designed especially for this task as described in chapter 5.

The `Var` variable is destined for combining the discrete event simulation with ordinary differential equations and difference equations and should be used mainly in such a way.

## 2.5 Process-oriented Simulation

Under the *process-oriented* paradigm[10, 7], we model simulation activities with help of a special kind of processes. We can explicitly suspend and resume such processes. Also we can request for and release of the resources implicitly suspending and resuming the processes in case of need.

Aivika actually supports the process-oriented simulation on two different levels. A higher level which uses streams of data and processors that operate on these streams is considered further. Below is described a lower level, which is a foundation for the higher level, nevertheless.

To model a process, Aivika uses the following type as a basis.

```
type Cont<'a> = Cont of (('a -> Eventive<unit>) -> Eventive<unit>)
```

The corresponding computation builder has name `cont`:

```
let x : Cont<'a> = cont { .. }
```

It is known from the theory of functional programming that we can suspend the `Cont` computation and then resume later. This is one of the main features that distinguishes this computation, being based on so called *continuations*.

A key idea is that the value of type `Cont<unit>` can be reduced to a function of type `Eventive<unit> -> Eventive<unit>`, which is actually an end part of the type signature for the `Eventive.enqueue` function mentioned above. That function enqueues a new event with the desired time of actuating the event handler.

It means that we can take an arbitrary computation of type `Cont<unit>`, suspend it and then resume it at another modeling time with help of the event queue.

This technique allows us to hold the process for the specified time interval. But sometimes we need to passivate the process for indefinite time so that another simulation activity could reactivate it later.

Therefore, we need some data structure to store the continuation that we would receive within the `Cont` computation. The process identifier `ProcId` can play a role of such data structure.

```
data ProcId

module Proc =
  val createId : Simulation<ProcId>
```

Then a *discontinuous process* can be represented with help of the following computation.

```
type Proc<'a> = Proc of (ProcId -> Cont<'a>)
```

The corresponding computation builder has name `proc`:

```
let x : Proc<'a> = proc { .. }
```

We can run the process within the simulation with help of one of the next functions.

```
module Proc =

  val run : Proc<unit> -> Eventive<unit>
  val runUsingId : ProcId -> Proc<unit> -> Eventive<unit>

  val runInStartTime : Proc<unit> -> Simulation<unit>
  val runInStartTimeUsingId : ProcId -> Proc<unit> -> Simulation<unit>

  val runInStopTime : Proc<unit> -> Simulation<unit>
  val runInStopTimeUsingId : ProcId -> Proc<unit> -> Simulation<unit>
```

If the process identifier is not specified then a new generated identifier is assigned when running the process. Every process has always its own unique identifier.

```
module Proc =
  val id : Proc<ProcId>
```

In case of need we can run a sub-process using another identifier.

```
module Proc =
  val usingId : ProcId -> Proc<'a> -> Proc<'a>
```

A characteristic feature of the `Proc` computation is that the process can be hold for the specified time interval through the event queue, following the approach described above in this section.

```
module Proc =
  val hold : Time -> Proc<unit>
```

Nevertheless, the held process can be immediately interrupted and we can request for whether it indeed was interrupted. The information about this is stored until the next call of the `hold` function.

```
module Proc =
  val interrupt : ProcId -> Eventive<unit>
  val isInterrupted : ProcId -> Eventive<bool>
```

It is worth noting to say more about the types of computations returned by these functions. The `Eventive` type of the result means that the computation executes immediately and it cannot be interrupted. On the contrary, the `Proc` type of the result means that the corresponding computation may suspend, even forever. This is very important for understanding.

To passivate the process for indefinite time to reactive it later, we can use the following functions.

```
module Proc =
  val passivate : Proc<unit>
  val isPassivated : ProcId -> Eventive<bool>
  val reactivate : ProcId -> Eventive<unit>
```

Every process can be immediately cancelled, which is important for modeling some activities.

```
module Proc =
  val cancelUsingId : ProcId -> Eventive<unit>
  val cancel<'a> : Proc<'a>
  val isCancelled : ProcId -> Eventive<bool>
```

Sometimes we need to run an arbitrary sub-process with the specified timeout.

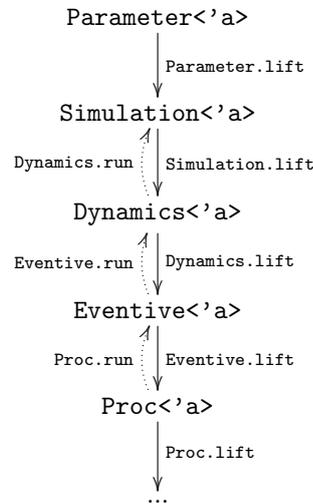
```
module Proc =
  val timeout : Time -> Proc<'a> -> Proc<'a option>
```

If the sub-process executes too long and exceeds the time limit, then it is immediately canceled and `None` is returned within the `Proc` computation. Otherwise; the computed result is returned right after it is received by the sub-process.

Every simulation computation we considered before can be transformed to the `Proc` computation, which in its turn can be transformed to another with help of the corresponding `lift` function, at least, it can be transformed to itself.

```
module Proc =
  val inline lift : Proc<'a> -> ~m
```

It allows using the integrals and external parameters as well as updating the mutable references and variables within the process-oriented simulation. It allows combining the event-oriented and process-oriented simulation.



Another process can be forked and spawn on-the-fly. If that process is not related to the current parent process in any way, then we can run the second process within the `Eventive` computation and then transform the result to the `Proc` computation. There is no need to add a special function. It is enough to have `Eventive.lift` and one of the `Proc` run functions.

```
((p : Proc<unit>) |> Proc.run |> Eventive.lift) : Proc<unit>
```

But if the life cycle of the child process must be bound up with the life cycle of the parent process so that they would be canceled in some order if required, then we should use one of the next functions.

```
module Proc =
  val spawn : Proc<unit> -> Proc<unit>
  val spawnWith : ContCancellation -> Proc<unit> -> Proc<unit>
```

Here the first argument of the second function specifies how two processes are bound.

```
type ContCancellation =
  | CancelTogether
  | CancelChildAfterParent
  | CancelParentAfterChild
  | CancelInIsolation
```

The stated above `timeout` function uses `spawnWith` to run the specified sub-process within time-out.

Also an arbitrary number of the `Proc` computations can be launched in parallel and we can await the completion of all the started sub-processes to return the final result.

```
module Proc =
  val par : Proc<'a> list -> Proc<'a list>
  val par_ : Proc<'a> list -> Proc<unit>
```

The `Proc` computation can be memoized so that the resulting process would always return the same value within the simulation run regardless of that how often the process was requested repeatedly.

```
module Proc =
  val memo : Proc<'a> -> Proc<'a>
```

Using the random number generator and the `hold` function, we can model an activity that is performed for some random time, for example, a processing of item by the machine tool.

Aivika contains a set of built-in random activities that all are defined in the following way.

```
module Proc =

  let randomUniform minimum maximum = proc {

    let! t = Parameter.randomUniform minimum maximum |> Parameter.lift
    do! hold t
    return t
  }

  let randomUniform_ minimum maximum = proc {

    let! t = Parameter.randomUniform minimum maximum |> Parameter.lift
    do! hold t
  }
```

The first function holds the current discontinuous process for a random time interval distributed uniformly and then returns that interval within the computation. The second function performs a side effect only without returning the interval.

Below is provided a list of predefined random activities that hold the current process for a random time interval according to their distributions.

```
module Proc =

  val randomUniform: minimum:float -> maximum:float -> Proc<float>
  val randomUniform_: minimum:float -> maximum:float -> Proc<unit>

  val randomUniformInt: minimum:int -> maximum:int -> Proc<int>
  val randomUniformInt_: minimum:int -> maximum:int -> Proc<unit>

  val randomNormal: mean:float -> deviation:float -> Proc<float>
  val randomNormal_: mean:float -> deviation:float -> Proc<unit>

  val randomExponential: mean:float -> Proc<float>
  val randomExponential_: mean:float -> Proc<unit>

  val randomErlang: beta:float -> m:int -> Proc<float>
  val randomErlang_: beta:float -> m:int -> Proc<unit>

  val randomPoisson: mean:float -> Proc<int>
  val randomPoisson_: mean:float -> Proc<unit>

  val randomBinomial: prob:float -> trials:int -> Proc<int>
  val randomBinomial_: prob:float -> trials:int -> Proc<unit>
```

Thus, the functions described in this section allow efficiently modeling quite complex activities. Nevertheless, the `Proc` computation is still low-level. Aivika supports more high-level computations described further.

## 2.6 Example: Process-oriented Simulation

Let us return to the task that was solved in section 2.3 using the event-oriented paradigm. The problem statement is repeated here. It corresponds to the documentation of SimPy.

There are two machines, which sometimes break down. Up time is exponentially distributed with mean 1.0, and repair time is exponentially distributed with mean 0.5. There are two repairpersons, so the two machines can be repaired simultaneously if they are down at the same time. Output is long-run proportion of up time. Should get value of about 0.66.

Using the processes, we can solve the task in a more elegant way.

```
// File MachRep1/Model.fsx

#I "../../bin"
#r "../../bin/Simulation.Aivika.dll"
#r "../../bin/Simulation.Aivika.Results.dll"

open System

open Simulation.Aivika
open Simulation.Aivika.Results

let specs = {
    StartTime=0.0; StopTime=1000.0; DT=1.0;
    Method=RungeKutta4; GeneratorType=StrongGenerator
}

let meanUpTime = 1.0
let meanRepairTime = 0.5

let model = simulation {
    // total up time for all machines
    let totalUpTime = ref 0.0

    let machine = proc {
        while true do
            let! upTime = Proc.randomExponential meanUpTime
            totalUpTime := !totalUpTime + upTime

            do! Proc.randomExponential_ meanRepairTime
        }

        do! Proc.runInStartTime machine
        do! Proc.runInStartTime machine
    }

    let upTimeProp =
        eventive {
            let! t = Dynamics.time |> Dynamics.lift
            return (!totalUpTime / (2.0 * t))
        }

    return
```

```

[ResultSource.From ("upTimeProp", upTimeProp,
  "Long-run proportion of up time \
(must be about 0.66)")]
  |> ResultSet.create
}

```

The reader can compare this model with the previous one. Conceptually, they do the same thing, use the same event queue and have the same behavior.

Now we will launch a Monte-Carlo simulation with 1000 simultaneous runs. After running the experiment, we will receive a deviation chart, statistics summary and histogram for our single variable that our model returns within Simulation.

```

// File MachRep1/RunExperiment.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"
#r "../bin/Simulation.Aivika.Experiments.dll"
#r "../bin/Simulation.Aivika.Charting.dll"

#load "Model.fsx"

open System
open System.Web.UI

open Simulation.Aivika
open Simulation.Aivika.Results
open Simulation.Aivika.Experiments
open Simulation.Aivika.Experiments.Web
open Simulation.Aivika.Charting.Web

let experiment = Experiment ()

experiment.Specs <- Model.specs
experiment.RunCount <- 1000

let provider1 = ExperimentSpecsProvider ()
let provider2 = DeviationChartProvider ()
let provider3 = LastValueStatsProvider ()
let provider4 = LastValueHistogramProvider ()

let providers =
  [ provider1 :> IExperimentProvider<HtmlTextWriter>;
    provider2 :> IExperimentProvider<HtmlTextWriter>;
    provider3 :> IExperimentProvider<HtmlTextWriter>;
    provider4 :> IExperimentProvider<HtmlTextWriter> ]

experiment.RenderHtml (Model.model, providers)
  |> Async.RunSynchronously

```

Figure 2.1 shows the deviation chart, but figure 2.2 displays the histogram by data collected, when running 1000 simulation runs.

The statistics summary is contained in the generated `index.html` file. In our case it is stated in table 2.1. The actual numbers may differ from experiment to experiment, but the tendency will be always the same.

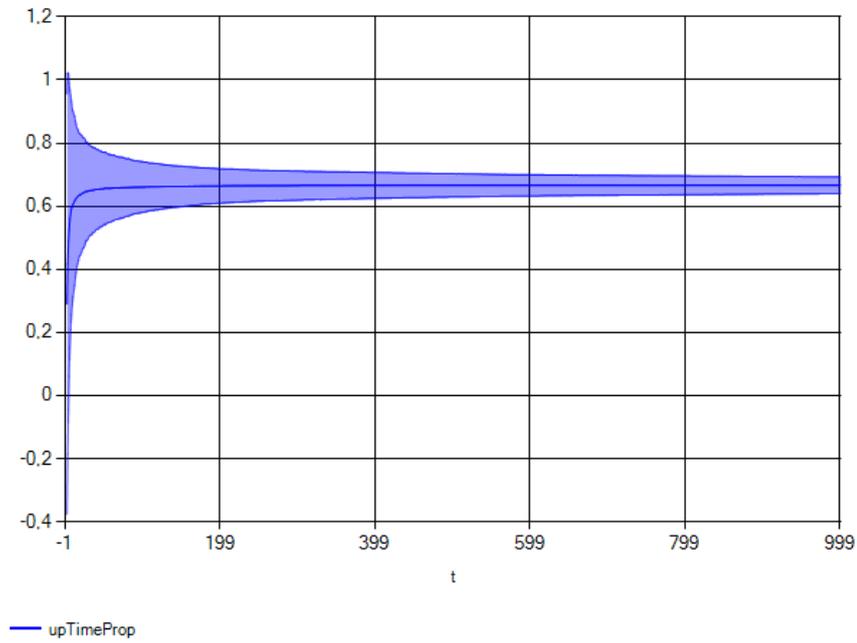


Figure 2.1: The deviation chart for the long-run proportion of up time.

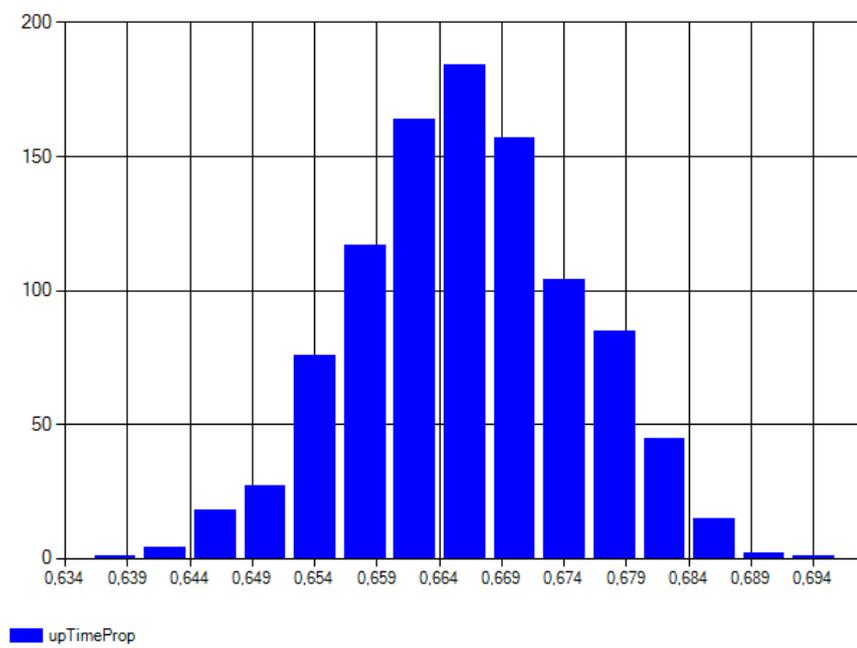


Figure 2.2: The long-run proportion histogram of up time.

Table 2.1: The summary for the long-run proportion of up time.

upTimeProp	
mean	0.666144947122581
deviation	0.00884072527833155
minimum	0.63948370536354
maximum	0.692046918776428
count	1000

There is also another popular paradigm applied to the discrete event simulation. It usually gives more rough simulation results as we have to scale the modeling time. The next two sections show how Aivika supports that paradigm and how we can apply it to solve the same task.

## 2.7 Activity-oriented Simulation

Under the *activity-oriented* paradigm[10, 7] of DES, we break time into tiny increments. At each time point, we look around at all the activities and check for the possible occurrence of events. Sometimes this scheme is called *time-driven*.

An idea is that we can naturally represent the activity as an `Eventive` computation, which we will call periodically through the event queue.

```
module Eventive =
  val enqueueWithTimes: #seq<Time> -> Eventive<unit> -> Eventive<unit>
```

We can also use another predefined function that does almost the same thing, but only it calls the specified computation directly in the integration time points specified by the simulation specs.

```
module Eventive =
  val enqueueWithIntegTimes: Eventive<unit> -> Eventive<unit>
```

Being defined in such a way, the activity-oriented simulation can be combined with the event-oriented and process-oriented ones.

## 2.8 Example: Activity-oriented Simulation

To illustrate the activity-oriented paradigm, let us take our old task that was solved in section 2.3 using the event-oriented paradigm and in section 2.6 using the process-oriented paradigm of DES. The problem statement is repeated here again. It corresponds to the documentation of SimPy.

There are two machines, which sometimes break down. Up time is exponentially distributed with mean 1.0, and repair time is exponentially distributed with mean 0.5. There are two repairpersons, so the two machines can be repaired simultaneously if they are down at the same time. Output is long-run proportion of up time. Should get value of about 0.66.

Now the model looks quite cumbersome. Moreover, we have to scale the modeling time. The time points at which the events occur are not precise any more.

```
// File MachRep1ActivityOriented/Model.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"

open System

open Simulation.Aivika
open Simulation.Aivika.Results

let specs = {

    StartTime=0.0; StopTime=1000.0; DT=0.05;
    Method=RungeKutta4; GeneratorType=StrongGenerator
}

let meanUpTime = 1.0
let meanRepairTime = 0.5

let model = simulation {

    // total up time for all machines
    let totalUpTime = ref 0.0

    let machine () =

        // a number of iterations when working
        let upNum = ref -1

        // a number of iterations when preparing
        let repairNum = ref -1

        // the start up time
        let startUpTime = ref 0.0

        // wait for the break
        let untilBroken = eventive {
            decr upNum
        }

        // wait for the repair
        let untilRepaired = eventive {
            decr repairNum
        }

        // when the tool is broken
        let broken = eventive {

            decr upNum

            // the machine is broken

            let! t = Dynamics.time |> Dynamics.lift
            let! dt = Parameter.dt |> Parameter.lift
            let! repairTime =
                Parameter.randomExponential meanRepairTime
                |> Parameter.lift
```

```

    totalUpTime := !totalUpTime
        + (t - !startUpTime)
    repairNum := int (repairTime / dt)
}

// when the tool is repaired
let repaired = eventive {

    decr repairNum

    // the machine is repaired

    let! t = Dynamics.time |> Dynamics.lift
    let! dt = Parameter.dt |> Parameter.lift
    let! upTime =
        Parameter.randomExponential meanUpTime
        |> Parameter.lift

    startUpTime := t
    upNum := int (upTime / dt)
}

// return a simulation model of the machine
eventive {

    if !upNum > 0 then
        return! untilBroken
    elif !upNum = 0 then
        return! broken
    elif !repairNum > 0 then
        return! untilRepaired
    elif !repairNum = 0 then
        return! repaired
    else
        return! repaired
}

// create two machines

let m1 = machine ()
let m2 = machine ()

// start the machines

do! m1 |> Eventive.enqueueWithIntegTimes
    |> Eventive.runInStartTime

do! m2 |> Eventive.enqueueWithIntegTimes
    |> Eventive.runInStartTime

// return the result

let upTimeProp =
    eventive {
        let! t = Dynamics.time |> Dynamics.lift
        return (!totalUpTime / (2.0 * t))
    }

return
[ResultSource.From ("upTimeProp", upTimeProp,
    "Long-run proportion of up time \

```

```
        (must be about 0.66)"]
        |> ResultSet.create
    }
}
```

It was the model. Now we write the starting script in a separate file.

```
// File MachRep1ActivityOriented/Run.fsx

#load "Model.fsx"

open Simulation.Aivika
open Simulation.Aivika.Results

ResultSet.printInStopTime Model.specs Model.model
```

When running this script on OS X, we receive something similar to the following result.

```
bash-3.2$ fsharp Run.fsx
-----

// time
t = 1000

// Long-run proportion of up time (must be about 0.66)
upTimeProp = 0,66335
```

We saw that the model written in this style is much longer. Nevertheless, the activity-oriented paradigm can be exceptionally useful for modeling some parts that are difficult to represent based on other simulation paradigms.



# Chapter 3

## Resources

This document illustrates how more and more high level concepts can be applied to modeling and simulation, when using Aivika. The resources considered in this chapter are not exception. They are somewhere of intermediate level, but they do allow simplifying many models, nevertheless.

### 3.1 Queue Strategies

Before we proceed to more high level modeling constructs, we need to define the *queue strategies*[17] that prescribe how the competitive requests must be prioritized.

In Aivika the queue strategies are expressed in terms of the `IQueueStrategy` interface, where each queue strategy has its own implementation.

```
type Priority = float

[<Interface>]
type IQueueStorage<'a> =

    abstract IsEmpty : unit -> Eventive<bool>
    abstract Dequeue : unit -> Eventive<'a>
    abstract Enqueue : item:'a -> Eventive<unit>
    abstract Enqueue : priority:Priority * item:'a -> Eventive<unit>

[<Interface>]
type IQueueStrategy =
    abstract CreateStorage<'a> : unit -> Simulation<IQueueStorage<'a>>
```

The queue strategy must implement the `Dequeue` method and one of the `Enqueue` methods. Another method must raise an exception when it is called. The second `Enqueue` method is destined for strategies based on priorities, while the first one is implemented by more simple queue strategies.

There are four predefined queue strategies in Aivika at present:

- FCFS (First Come - First Served), or FIFO (First In - First Out);
- LCFS (Last Come - First Served), or LIFO (Last In - First Out);
- SIRO (Service in Random Order);

- `StaticPriorities` (Using Static Priorities), where the less value means a higher priority.

These strategies are implemented by the corresponded class types.

```
[<Sealed>]
type FCFS =
  interface IQueueStrategy

[<Sealed>]
type LCFS =
  interface IQueueStrategy

[<Sealed>]
type SIRO =
  interface IQueueStrategy

[<Sealed>]
type StaticPriorities =
  interface IQueueStrategy
```

There is also a module that contains predefined values of these types for convenience.

```
module QueueStrategy =

  val FCFS : FCFS
  val LCFS : LCFS
  val SIRO : SIRO
  val staticPriorities : StaticPriorities
```

## 3.2 Resource

A *resource*<sup>[7]</sup> simulates something to be queued for, for example, the machine.

```
[<Sealed>]
type Resource
```

The simplest constructor allows us to create a new resource by the specified queue strategy and initial amount.

```
module Resource =
  val create: strat:#IQueueStrategy -> count:int -> Simulation<Resource>
```

To acquire the resource, we can use the predefined functions like these ones:

```
module Resource =

  val request : Resource -> Proc<unit>
  val requestWithPriority : Priority -> Resource -> Proc<unit>
```

Each of the both suspends the process in case of the resource deficiency until some other simulation activity releases the resource.

```
module Resource =
  val releaseWithinEventive : Resource -> Eventive<unit>
```

There is also a more convenient version of the last function that works within the `Proc` computation, but the provided function emphasizes the fact that releasing the resource cannot block the simulation process and this action is performed immediately.

```
module Resource =
  val release : Resource -> Proc<unit>
```

We can request for the current available amount of the specified resource as well as request for its maximum possible amount and the strategy applied.

```
module Resource =

  val count : Resource -> Eventive<int>
  val maxCount : Resource -> int option
  val strategy : Resource -> IQueueStrategy
```

The second function returns an optional value indicating that the maximum amount could be unspecified when creating the resource.

```
module Resource =
  val createWithMaxCount: strat:#IQueueStrategy
    -> count:int
    -> maxCount:int option
    -> Simulation<Resource>
```

By default, the maximum possible amount is set equaled to the initial amount specified when calling the first constructor `create`.

There are constructors that use the predefined queue strategies. Some of these constructors are provided below.

```
module Resource =

  val createUsingFCFS : count:int -> Simulation<Resource>
  val createUsingLCFS : count:int -> Simulation<Resource>
  val createUsingSIR0 : count:int -> Simulation<Resource>
  val createUsingPriorities : count:int -> Simulation<Resource>
```

Also there are two helper functions each of the both acquires the resource and returns `IDisposable` that in its turn allows releasing the resource regardless of whether the specified process was cancelled or an exception was raised. It can be used together with the `use!` construct of `F#`.

```
module Resource =

  val take : Resource -> Proc<IDisposable>
  val takeWithPriority : Priority -> Resource -> Proc<IDisposable>
```

Finally, we can increase the available amount of the resource to a new value, but not greater than the maximum amount defined when constructing the resource. Then some awaiting processes in the specified number can be awoken and they will acquire the resource.

```
module Resource =
  val incCount : n:int -> Resource -> Eventive<unit>
```

### 3.3 Example: Using Resources

To illustrate how the resources can be used for modeling, let us again take a task from the documentation of SimPy[15].

Two machines, but sometimes break down. Up time is exponentially distributed with mean 1.0, and repair time is exponentially distributed with mean 0.5. In this example, there is only one repairperson, so the two machines cannot be repaired simultaneously if they are down at the same time.

In addition to finding the long-run proportion of up time, let us also find the long-run proportion of the time that a given machine does not have immediate access to the repairperson when the machine breaks down. Output values should be about 0.6 and 0.67.

In Aivika we can solve this task in the following way.

```
// File MachRep2/Model.fsx

#I "../bin/"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"

open System

open Simulation.Aivika
open Simulation.Aivika.Results

let specs = {
    StartTime=0.0; StopTime=1000.0; DT=1.0;
    Method=RungeKutta4; GeneratorType=StrongGenerator
}

let meanUpTime = 1.0
let meanRepairTime = 0.5

let model = simulation {

    // number of times the machines have broken down
    let nRep = ref 0

    // number of breakdowns in which the machine
    // started repair service right away
    let nImmedRep = ref 0

    // total up time for all machines
    let totalUpTime = ref 0.0

    let! repairPerson = Resource.createUsingFCFS 1

    let machine = proc {

        while true do

            let! upTime = Proc.randomExponential meanUpTime
            totalUpTime := !totalUpTime + upTime

            incr nRep
```

```

    let! n =
        Resource.count repairPerson
        |> Eventive.lift

    if n = 1 then
        incr nImmedRep

    do! Resource.request repairPerson
    let! repairTime = Proc.randomExponential meanRepairTime
    do! Resource.release repairPerson
}

do! Proc.runInStartTime machine
do! Proc.runInStartTime machine

let upTimeProp = eventive {

    let! t = Dynamics.time |> Dynamics.lift
    return !totalUpTime / (2.0 * t)
}

let immedTimeProp = eventive {
    return (float !nImmedRep) / (float !nRep)
}

return [ResultSource.From ("upTimeProp", upTimeProp,
    "Long-run proportion of up time \
    (must be about 0.6)");
    ResultSource.From ("immedTimeProp", immedTimeProp,
    "Long-run proportion of the time when \
    immediate access to the repairperson \
    (must be about 0.67)")]
    |> ResultSet.create
}

```

Let's take the following simulation experiment: specs, the deviation chart, statistics summary by last values, the histogram for last values. The number of simultaneous runs is 1000.

```

// File MachRep2/RunExperiment.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"
#r "../bin/Simulation.Aivika.Experiments.dll"
#r "../bin/Simulation.Aivika.Charting.dll"

#load "Model.fsx"

open System
open System.Web.UI

open Simulation.Aivika
open Simulation.Aivika.Results
open Simulation.Aivika.Experiments
open Simulation.Aivika.Experiments.Web
open Simulation.Aivika.Charting.Web

let experiment = Experiment ()

experiment.Specs <- Model.specs

```

```

experiment.RunCount <- 1000

let provider1 = ExperimentSpecsProvider ()
let provider2 = DeviationChartProvider ()
let provider3 = LastValueStatsProvider ()
let provider4 = LastValueHistogramProvider ()

let providers =
  [ provider1 :> IExperimentProvider<HtmlTextWriter>;
    provider2 :> IExperimentProvider<HtmlTextWriter>;
    provider3 :> IExperimentProvider<HtmlTextWriter>;
    provider4 :> IExperimentProvider<HtmlTextWriter> ]

experiment.RenderHtml (Model.model, providers)
  |> Async.RunSynchronously

```

The corresponding histogram is shown on figure 3.1.

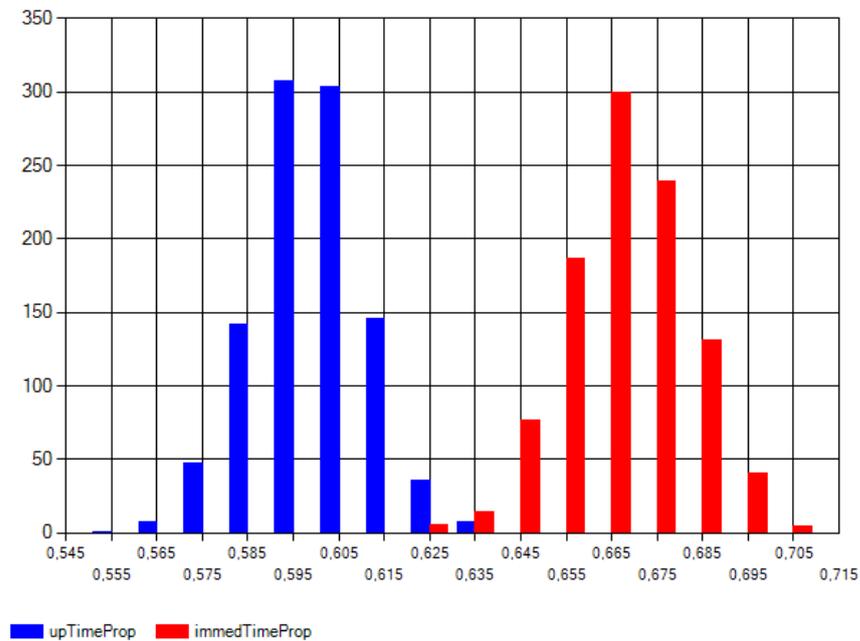


Figure 3.1: The histogram for time proportions, when using the resource.

### 3.4 Example: Passivating and Reactivating Processes

This example illustrates how we can passivate and reactivate processes depending on that whether the resource is free. The task corresponds to the documentation of SimPy[15].

Variation of the previous models described in sections 2.6 and 3.3. Two machines, but sometimes break down. Up time is exponen-

tially distributed with mean 1.0, and repair time is exponentially distributed with mean 0.5. In this example, there is only one repair-person, and she is not summoned until both machines are down. We find the proportion of up time. It should come out to about 0.45.

In Aivika the corresponded model can be defined in the following way.

```
// File MachRep3/Model.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"

open Simulation.Aivika
open Simulation.Aivika.Results

let specs = {
    StartTime=0.0; StopTime=1000.0; DT=1.0;
    Method=RungeKutta4; GeneratorType=StrongGenerator
}

let meanUpTime = 1.0
let meanRepairTime = 0.5

let model = simulation {
    // number of machines currently up
    let nUp = ref 0

    // total up time for all machines
    let totalUpTime = ref 0.0

    let! repairPerson = Resource.createUsingFCFS 1

    let machine pid' = proc {
        incr nUp

        while true do
            let! upTime = Proc.randomExponential meanUpTime
            totalUpTime := !totalUpTime + upTime

            decr nUp

            if !nUp = 1 then
                do! Proc.passivate
            else
                let! n =
                    Resource.count repairPerson
                    |> Eventive.lift

                if n = 1 then
                    do! Proc.reactivate pid'
                    |> Eventive.lift

            do! Resource.request repairPerson
            let! repairTime = Proc.randomExponential meanRepairTime

            incr nUp
    }
}
```

```

        do! Resource.release repairPerson
    }

    let! pid1 = Proc.createId
    let! pid2 = Proc.createId

    do! Proc.runInStartTimeUsingId pid1 (machine pid2)
    do! Proc.runInStartTimeUsingId pid2 (machine pid1)

    let upTimeProp = eventive {
        let! t = Dynamics.time |> Dynamics.lift
        return (!totalUpTime / (2.0 * t))
    }

    return [ResultSource.From ("upTimeProp", upTimeProp,
        "The proportion of up time \
        (must be about 0.45)")]
        |> ResultSet.create
}

```

Let's take the same simulation experiment that we used in section 3.3: show the specs, the deviation chart, statistics summary by last values, the histogram for last values. The number of simultaneous runs is 1000.

```

// File MachRep3/RunExperiment.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"
#r "../bin/Simulation.Aivika.Experiments.dll"
#r "../bin/Simulation.Aivika.Charting.dll"

#load "Model.fsx"

open System
open System.Web.UI

open Simulation.Aivika
open Simulation.Aivika.Results
open Simulation.Aivika.Experiments
open Simulation.Aivika.Experiments.Web
open Simulation.Aivika.Charting.Web

let experiment = Experiment ()

experiment.Specs <- Model.specs
experiment.RunCount <- 1000

let provider1 = ExperimentSpecsProvider ()
let provider2 = DeviationChartProvider ()
let provider3 = LastValueStatsProvider ()
let provider4 = LastValueHistogramProvider ()

let providers =
    [ provider1 :> IExperimentProvider<HtmlTextWriter>;
      provider2 :> IExperimentProvider<HtmlTextWriter>;
      provider3 :> IExperimentProvider<HtmlTextWriter>;
      provider4 :> IExperimentProvider<HtmlTextWriter> ]

experiment.RenderHtml (Model.model, providers)
    |> Async.RunSynchronously

```

The corresponding histogram is shown on figure 3.2. We expect to see a value distributed near 0.45.

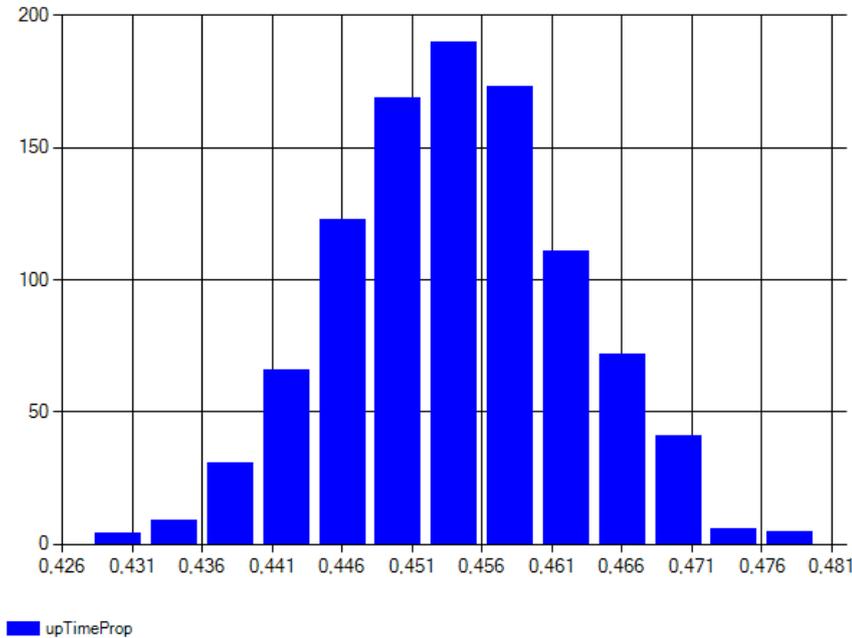


Figure 3.2: The histogram for time proportions, when passivating and reactivating the processes.

## 3.5 Resource Preemption

There are simulation tasks that are modeled better with help of resource preemption. A preemptible resource is like the ordinary resource parameterized by the queue strategy based on static priorities. The numeric priorities are necessary to request for the both resources.

However, there is an important difference. When trying to acquire the ordinary resource, the process suspends in case of resource deficiency. In case of the resource preemption, the resource still can be acquired by the process even if the resource amount is zero, but then another process with less priority must be preempted and its ownership of the resource will be transferred to the current process with higher priority. If the current process that requests for the resource has a less priority then it waits for releasing of the resource as usual.

So, the priority is used not only to range the process requests, but it allows also preempting another process transferring its ownership of the resource to another process.

```
[<Sealed>
type PreemptibleResource

module PreemptibleResource =
```

```

val count : PreemptibleResource -> Eventive<int>
val maxCount : PreemptibleResource -> int option

val create : count:int -> Simulation<PreemptibleResource>
val createWithMaxCount : count:int
                        -> maxCount:int option
                        -> Simulation<PreemptibleResource>

val requestWithPriority : Priority -> PreemptibleResource -> Proc<unit>
val release : PreemptibleResource -> Proc<unit>

val takeWithPriority : Priority -> PreemptibleResource -> Proc<IDisposable>

```

Note that here the resource must be released only within the corresponded `Proc` computation. It is important to know what namely processes took an ownership of the resource. Having this information, we can preempt one of these processes if another process with higher priority requests for the resource.

As it was true with the static priority queue strategy, the less value means a higher priority.

Since the resources can be preempted at any time, we can either increase or decrease the available amount of the resource.

```

module PreemptibleResource =

    val incCount: n:int -> PreemptibleResource -> Eventive<unit>
    val decCount: n:int -> PreemptibleResource -> Eventive<unit>
    val alterCount: n:int -> PreemptibleResource -> Eventive<unit>

```

Section 6.9 illustrates an example of using the resource preemption.

## Chapter 4

# Signals and Tasks

The constructs considered in this chapter are closer to programming than to simulation. Nevertheless, they can be very useful for modeling.

### 4.1 Signals

A *signal* is a variation of the standard `IObservable` interface but specialized for modeling.

```
[<AbstractClass; NoEquality; NoComparison>]
type Signal<'a> =

    new: unit -> Signal<'a>
    abstract Subscribe: handler:( 'a -> Eventive<unit> ) -> Eventive<IDisposable>
```

The `Subscribe` method of the signal takes a handler, subscribes the handler for receiving the signal values and then returns a computation of the `IDisposable` object that being invoked unsubscribes the specified handler from receiving the signal.

If we are not going to unsubscribe at all, then we can ignore the nested value of computation.

```
[<AutoOpen>]
module SignalExtensions =

    type Signal<'a> with
        member Add: handler:( 'a -> Eventive<unit> ) -> Eventive<unit>
```

We can treat the signals in a functional way, transforming or merging or filtering them with help of combinators like these ones.

```
module Signal =

    val add : ( 'a -> Eventive<unit> ) -> Signal<'a> -> Eventive<unit>
    val subscribe: ( 'a -> Eventive<unit> ) -> Signal<'a> -> Eventive<IDisposable>

    val map : ( 'a -> 'b ) -> Signal<'a> -> Signal<'b>
    val filter : ( 'a -> bool ) -> Signal<'a> -> Signal<'a>

    val empty<'a> : Signal<'a>
    val merge : Signal<'a> -> Signal<'a> -> Signal<'a>
    val concat : #Signal<'a> list -> Signal<'a>
```

The `Ref` reference and `Var` variable provide signals that notify about changing their state.

```
module Ref =

  val changed : Ref<'a> -> Signal<'a>
  val changed_ : Ref<'a> -> Signal<unit>

module Var =

  val changed : Var<'a> -> Signal<'a>
  val changed_ : Var<'a> -> Signal<unit>
```

We can create an origin of the signal manually. Distinguishing the origin from the signal allows us to publish the signal with help of a pure function. But we must trigger the signal within a computation synchronized with the event queue, though.

```
[<Sealed>]
type SignalSource<'a>

module SignalSource =

  val create<'a> : Simulation<SignalSource<'a>>

  val publish : source:SignalSource<'a> -> Signal<'a>
  val trigger : value:'a -> source:SignalSource<'a> -> Eventive<unit>
```

Moreover, we can register a history of signal values, which can be useful for accumulating the simulation results.

```
[<Sealed>]
type SignalHistory<'a>

module SignalHistory =

  val create : signal:Signal<'a> -> Eventive<SignalHistory<'a>>
  val read : history:SignalHistory<'a> -> Eventive<Time array * 'a array>
```

The `SignalHistory` type is widely used in the implementation of simulation experiments, when the results are accumulated by signals, for which there are predefined signals.

```
module Signal =

  val inTimes : times:#seq<Time> -> Eventive<Signal<Time>>
  val inIntegTimes : Eventive<Signal<Time>>
  val inStartTime : Eventive<Signal<Time>>
  val inStopTime : Eventive<Signal<Time>>
```

To plot the histogram, we accumulate the simulation results by the signal triggered in the final time, while we use the signal triggered in the integration time points to save the CSV table or draw the time series chart. Only we have to transform the source signal to receive the values we need.

Finally, you might notice that the `Cont` and `Signal` computations have a similar definition. This relation is expressed by the following function, where the current process suspends until the next signal value is triggered. Then the signal value is passed in to the process computation, which is resumed.

```
module Proc =
  val await : signal:Signal<'a> -> Proc<'a>
```

In Aivika there is an opposite transformation from the `Proc` computation to a `Signal` value, but it is a little bit complicated as the process can be actually canceled or an exception can be raised within the simulation. The corresponded transformation is defined with help of the `Task` type.

## 4.2 Tasks

A *task* encompasses the process computation started in background.

```
[<Sealed>]
type Task<'a>

module Task =
  val run : comp:Proc<'a> -> Eventive<Task<'a>>
```

Here we run the specified process in background and immediately return the corresponded task within the `Eventive` computation. Later we can request for the result of the underlying `Proc` computation, whether it was finished successfully, or an exception had occurred or the computation was cancelled.

```
type TaskResult<'a> =
  | TaskCompleted of 'a
  | TaskError of exn
  | TaskCancelled

module Task =

  val tryGetResult : task:Task<'a> -> Eventive<TaskResult<'a> option>
  val result : task:Task<'a> -> Proc<TaskResult<'a>>
  val resultReceived : task:Task<'a> -> Signal<TaskResult<'a>>
```

The background task can be cancelled at any time.

```
module Task =

  val cancel : task:Task<'a> -> Eventive<unit>
  val isCancelled : task:Task<'a> -> Eventive<bool>
```

Also we can include the task computation into an arbitrary `Proc` computation, making the former a compound part of the latter.

```
module Task =
  val toProc : task:Task<'a> -> Proc<'a>
```

In some sense the `Signal` and `Task` types complement other simulation computations, which illustrates how deeply different computations can be interconnected to each other, allowing us to define more comprehensive models.



# Chapter 5

## Statistics

An accumulation of statistics is an important part of simulation. Aivika uses an approach, where the statistics summary is treated as an immutable data structure, which simplifies programming and makes the simulation more safe and robust.

There are two different types of statistics that we can collect. The first one is based upon observations, while the latter is based on time-dependent samples.

### 5.1 Statistics based upon Observations

The generic `SamplingStats` data type is used for accumulating statistics based upon observations.

```
type SamplingStats<'a>

module SamplingStats =

    val emptyInts : SamplingStats<int>
    val emptyFloats : SamplingStats<float>

    val fromInts : samples:int array -> SamplingStats<int>
    val fromFloats : samples:float array -> SamplingStats<float>

    val add : sample:'a -> stats:SamplingStats<'a> -> SamplingStats<'a>

    val append : stats1:SamplingStats<'a>
                -> stats2:SamplingStats<'a>
                -> SamplingStats<'a>

    val appendChoice : stats1:Choice<'a, SamplingStats<'a>>
                      -> stats2:SamplingStats<'a>
                      -> SamplingStats<'a>

    val appendSeq : samples:seq<'a>
                  -> stats:SamplingStats<'a>
                  -> SamplingStats<'a>

    val count : stats:SamplingStats<'a> -> int
    val minimum : stats:SamplingStats<'a> -> 'a
    val maximum : stats:SamplingStats<'a> -> 'a

    val mean : stats:SamplingStats<'a> -> float
```

```

val mean2 : stats:SamplingStats<'a> -> float
val variance : stats:SamplingStats<'a> -> float
val deviation : stats:SamplingStats<'a> -> float

val fromIntsToFloats : stats:SamplingStats<int> -> SamplingStats<float>

```

The main idea is that this is an immutable data type. Each time we collect a new sample, we actually create a new instance of the `SamplingStats` type.

An usual mistake of novices is when they try to use rather a heavy-weight `Var` type for collecting statistics. Nevertheless, it is recommended to use the standard `ref` reference or predefined `Ref` type for updating the light-weight `SamplingStats` value, which is a more efficient and more simple approach.

```

let r = ref SamplingStats.emptyFloats
...

r := !r |> SamplingStats.add 1.0
r := !r |> SamplingStats.add 2.0
...

```

The `SamplingStats` values within simulation computation can be returned as a `ResultSource`.

## 5.2 Statistics for Time Persistent Variables

The generic `TimingStats` data type is used for collecting time-dependent statistics.

```

type TimingStats<'a>

module TimingStats =

    val emptyInts : TimingStats<int>
    val emptyFloats : TimingStats<float>

    val add : time:Time -> sample:'a -> stats:TimingStats<'a> -> TimingStats<'a>

    val count : stats:TimingStats<'a> -> int
    val minimum : stats:TimingStats<'a> -> 'a
    val maximum : stats:TimingStats<'a> -> 'a
    val last : stats:TimingStats<'a> -> 'a

    val minimumTime : stats:TimingStats<'a> -> Time
    val maximumTime : stats:TimingStats<'a> -> Time
    val startTime : stats:TimingStats<'a> -> Time
    val lastTime : stats:TimingStats<'a> -> Time

    val sum : stats:TimingStats<'a> -> float
    val sum2 : stats:TimingStats<'a> -> float
    val mean : stats:TimingStats<'a> -> float
    val mean2 : stats:TimingStats<'a> -> float
    val variance : stats:TimingStats<'a> -> float
    val deviation : stats:TimingStats<'a> -> float

    val fromIntsToFloats : stats:TimingStats<int> -> TimingStats<float>

```

The `TimingStats` value is immutable too. As before, it can be used within references.

There is also one function that allows converting the `TimingStats` statistics to its normalized representation based upon observations. We interpolate the former so that it would be statistically similar to the latter by the specified number of pseudo-observations.

```
module TimingStats =  
  val normalise: count:int -> stats:TimingStats<'a> -> SamplingStats<'a>
```

For example, this function is used when plotting the deviation chart for queue sizes. These sizes are time persistent variables, while the deviation chart plots the trend and probabilistic bounds for the statistics based upon observations. Normalizing the queue size statistics by the iteration number, we receive another representation of the queue size by which we can plot the deviation chart.

As before, the `TimingStats` values within simulation computation can be returned as a `ResultSource`.



# Chapter 6

## Queue Network

It is difficult to imagine any complex discrete event simulation without using queues. This chapter introduces the queues and shows how we can model networks based on them.

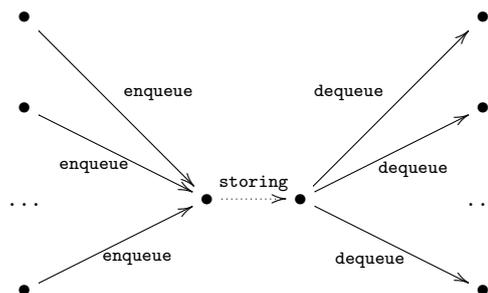
### 6.1 Finite Queues

Sometimes we need a location in the network where entities wait for service[10]. They are modeled in Aivika by finite and infinite queues.

The finite queue is a container of elements.

```
[<Sealed>  
type Queue<'a>
```

To create a new queue, we should specify the queue strategies that will be used for ranging the enqueueing operations, internal storing items in the queue and ranging the dequeueing operations respectively. Also we should specify the queue capacity as the queue is finite.



The enqueueing strategy is used for ranging the enqueueing operations when the queue is full. The storing strategy is used for ranging the items in the queue itself. The dequeueing strategy is used for ranging the dequeueing requests when the queue is empty.

The first and third strategies used for ranging the enqueueing and dequeueing operations usually should be defined as the FCFS strategy, i.e. first come - first serviced, which is the most intuitive and suits the most of needs, while the storing strategy distinguishes the queue itself.

In general case, the queue constructor is as follows.

```
module Queue =
  val create<'si, 'sm, 'so, 'a
    when 'si :> IQueueStrategy and
         'sm :> IQueueStrategy and
         'so :> IQueueStrategy> :
    inputStrat:'si
    -> storingStrat:'sm
    -> outputStrat:'so
    -> maxCount:int
    -> Eventive<Queue<'a>>
```

Fortunately, there are specializations that allow creating new queues using the predefined strategies and these specializations look much shorter.

```
module Queue =
  val createUsingFCFS<'a> : maxCount:int -> Eventive<Queue<'a>>
  val createUsingLCFS<'a> : maxCount:int -> Eventive<Queue<'a>>
  val createUsingSIR0<'a> : maxCount:int -> Eventive<Queue<'a>>
  val createUsingPriorities<'a> : maxCount:int -> Eventive<Queue<'a>>
```

Each of them uses the FCFS strategy for ranging the enqueueing and dequeueing operations, but uses the corresponded queue strategy for internal storing.

For example, the `createUsingFCFS` function uses FCFS for the storing operation too, while the `createUsingPriorities` function creates already a queue that uses the static priorities, when storing a new element.

Unlike other data structures, a queue is created within the `Eventive` computation as we have to know the current simulation time to start gathering the timing statistics for the queue size. The statistics is initiated at time of invoking the computation.

There are different enqueueing functions. The most simple one is provided below.

```
module Queue =
  val enqueue : item:'a -> queue:Queue<'a> -> Proc<unit>
```

It suspends the process if the finite queue is full. Therefore, this action is returned as the `Proc` computation.

Also we can try to enqueue a new item and if the queue is full then the item is counted as lost.

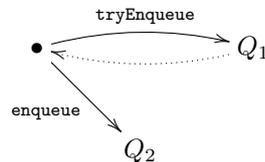
```
module Queue =
  val enqueueOrLost : item:'a -> queue:Queue<'a> -> Eventive<bool>
```

This action cannot already suspend the simulation activity and hence it returns the `Eventive` computation of a flag indicating whether the item was successfully stored in the queue.

There is also a similar function that tries to enqueue a new item, but in case of the full queue the item is not counted as lost and the queue statistics does not change.

```
module Queue =
  val tryEnqueue : item:'a -> queue:Queue<'a> -> Eventive<bool>
```

This function can be useful if we are going to enqueue the item in another queue in case of failure. We try the first queue  $Q_1$ , fail and then enqueue the item in the second queue  $Q_2$ .



If the queue was created by applying the `createUsingPriorities` function then we must enqueue a new element specifying also the storing priority.

```

module Queue =

  val enqueueWithStoringPriority : pm:Priority
    -> item:'a
    -> queue:Queue<'a>
    -> Proc<unit>

  val enqueueWithStoringPriorityOrLost : pm:Priority
    -> item:'a
    -> queue:Queue<'a>
    -> Eventive<bool>

  val tryEnqueueWithStoringPriority : pm:Priority
    -> item:'a
    -> queue:Queue<'a>
    -> Eventive<bool>

```

There are also other enqueueing functions that allow specifying the priority used when ranging the enqueueing operations in case of full queue, but these functions are needed only if we specify the corresponded priority-based enqueueing strategy, when constructing the queue.

As it was mentioned before, the predefined queue constructors use the FCFS strategy for the enqueueing operation that needs no auxiliary priority, when ranging the operations if the queue is full. The first operation will have a priority.

The simplest dequeueing operation suspends the process while the queue is empty. The result is the Proc computation.

```

module Queue =
  val dequeue : queue:Queue<'a> -> Proc<'a>

```

Here the very type signatures specify whether the corresponded function may suspend the simulation activity, or the action is performed immediately.

There are similar dequeueing functions that allow specifying the priority used when ranging the dequeueing operations if the queue is empty. As before, these similar functions are needed only if the queue was constructed by specifying the priority-based dequeueing strategy.

Regarding the predefined queue constructors, they also use the FCFS strategy for ranging the dequeueing operations, which needs no auxiliary priority. The first operation will have a priority if the queue is empty.

The queue has a lot of counters that are updated during the simulation. Actually, these counters are what we are mostly interested in.

For example, we can request the queue for its size and wait time.

```

module Queue =
    val countStats : queue:Queue<'a> -> Eventive<TimingStats<int>>
    val waitTime : queue:Queue<'a> -> Eventive<SamplingStats<Time>>

```

Finally, we can return an arbitrary queue or a list of queues from the model as a `ResultSource`. Then the queue or the list of queues can be used within the simulation experiment.

## 6.2 Infinite Queues

An infinite queue is a container of elements.

```

[<Sealed>]
type InfiniteQueue<'a>

```

Since the queue is infinite, there is no need to range the enqueueing operations as the queue cannot be full. Therefore, a new queue is created by specifying the storing and dequeueing strategies respectively.

```

module InfiniteQueue =
    val create<'sm, 'so, 'a
        when 'sm :> IQueueStrategy and
            'so :> IQueueStrategy> :
        storingStrat:'sm
        -> outputStrat:'so
        -> Eventive<InfiniteQueue<'a>>

```

The storing strategy is used for ranging the elements in the queue itself. The dequeueing strategy is used for ranging the dequeueing operations if the queue is empty.

There are specializations that allow creating new queues based on the pre-defined queue strategies.

```

module InfiniteQueue =
    val createUsingFCFS<'a> : Eventive<InfiniteQueue<'a>>
    val createUsingLCFS<'a> : Eventive<InfiniteQueue<'a>>
    val createUsingSIRO<'a> : Eventive<InfiniteQueue<'a>>
    val createUsingPriorities<'a> : Eventive<InfiniteQueue<'a>>

```

Each of them uses the FCFS strategy, i.e. first come - first serviced, for ranging the dequeueing operations in case of empty queue. The first operation will have a priority.

There are only two enqueueing functions and their actions are performed immediately without suspension. The simulation time remains the same after the operation.

```

module InfiniteQueue =
    val enqueue : item:'a -> queue:InfiniteQueue<'a> -> Eventive<unit>

    val enqueueWithStoringPriority : pm:Priority
        -> item:'a
        -> queue:InfiniteQueue<'a>
        -> Eventive<unit>

```

The second function is used when the queue was created by specifying the priority-based storing strategy. In other cases the first function is used.

The simplest dequeuing operation suspends the process while the queue is empty. The result is the `Proc` computation.

```
module InfiniteQueue =
  val dequeue : queue:InfiniteQueue<'a> -> Proc<'a>
```

It is worth noting again that the type signatures specify whether the corresponding function may suspend the simulation activity, or the action is performed immediately.

There is a similar dequeuing function that allows specifying the priority used when ranging the dequeuing operations if the queue is empty. As before, these similar function is needed only if the queue was constructed by specifying the priority-based dequeuing strategy.

Regarding the predefined queue constructors, they use the `FCFS` strategy for ranging the dequeuing operations, which needs no auxiliary priority. The first operation will have a priority if the infinite queue is empty.

Like the finite queue, the infinite queue has a lot of counter that are updated during the simulation.

For example, we can request the infinite queue for its size and wait time.

```
module InfiniteQueue =

  val countStats : queue:InfiniteQueue<'a> -> Eventive<TimingStats<int>>
  val waitTime : queue:InfiniteQueue<'a> -> Eventive<SamplingStats<Time>>
```

Also we can return an arbitrary infinite queue or a list of such queues from the model as a `ResultSource`. Then the infinite queue or the list of queues can be used within the simulation experiment.

## 6.3 Stream

Many things become significantly more simple for reasoning and understanding after we introduce a concept of *stream* of data distributed sequentially in the modeling time.

```
type StreamItem<'a> =
  | StreamNil
  | StreamCons of 'a * Stream<'a>

and Stream<'a> = Stream of Proc<StreamItem<'a>>
```

The corresponding computation builder has name `stream`:

```
let x : Stream<'a> = stream { .. }
```

It supports `F#` keywords `yield`, `yield!` in obvious way. Also it supports the `let!` and `do!` constructs allowing us to embed arbitrary `Proc` computation in the `Stream` computation.

```
let x : Stream<'a> = stream {
  let a : 'a = ...
  yield a
```

```

...
let p : Proc<'b> = ...
let! b : 'b = p
...
}

```

The `Stream` type is a kind of the *cons-cell*, where the cell is returned within the `Proc` computation. It means that the stream data can be distributed in the modeling time and there can be time gaps between sequential data.

The streams themselves are well-known in the functional programming for a long time[1]. It is obvious that we can map, filter, transform the streams.

```

module Stream =

  val map : ('a -> 'b) -> Stream<'a> -> Stream<'b>
  val mapc : ('a -> Proc<'b>) -> Stream<'a> -> Stream<'b>

  val filter : ('a -> bool) -> Stream<'a> -> Stream<'a>
  val filterc : ('a -> Proc<bool>) -> Stream<'a> -> Stream<'a>

```

Passivating the underlying process forever<sup>1</sup>, we receive a stream that never returns data.

```

module Stream =
  val empty<'a> : Stream<'a>

```

Moreover, we can merge two streams applying the FCFS strategy when enqueueing input data.

```

module Stream =
  val append : Stream<'a> -> Stream<'a> -> Stream<'a>

```

Actually, the latter is a partial case of more general functions that allow concatenating the streams like a *multiplexor*.

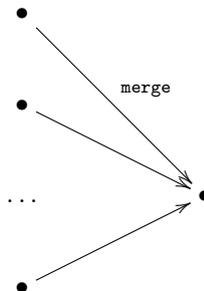
```

module Stream =

  val merge : Stream<'a> list -> Stream<'a>
  val mergeQueueing : #IQueueStrategy -> Stream<'a> list -> Stream<'a>
  val mergePrioritising : #IQueueStrategy
    -> Stream<Priority * 'a> list
    -> Stream<'a>

```

The functions use the resources to concatenate different streams of data.



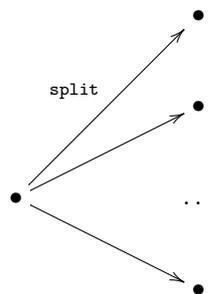
<sup>1</sup>The underlying process can still be canceled, though.

There is an opposite ability to split the input stream into the specified number of output streams like a *demultiplexor*. We have to do it to model a parallel work of services.

```
module Stream =

  val split : int -> Stream<'a> -> Stream<'a> list
  val splitQueueing : #IQueueStrategy -> int -> Stream<'a> -> Stream<'a> list
  val splitPrioritising : #IQueueStrategy
    -> Stream<Priority> list
    -> Stream<'a>
    -> Stream<'a> list
```

These functions use also the resources to split the stream.



An implementation uses an auxiliary function that creates a new stream as a result of the repetitive execution of some process.

```
module Stream =
  val repeat : Proc<'a> -> Stream<'a>
```

A key idea is that many simulation models can be defined as a network of the `Stream` computations.

Such a network must have external input streams, usually random streams like these ones.

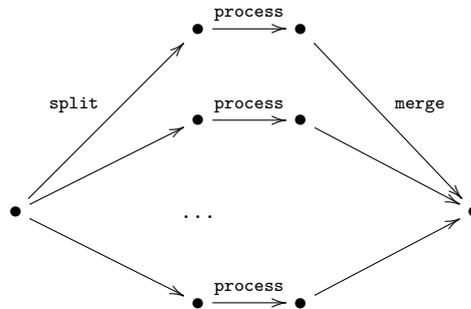
```
module Stream =

  val randomUniform : minimum:float -> maximum:float -> Stream<Arrival<float>>
  val randomUniformInt : minimum:int -> maximum:int -> Stream<Arrival<int>>
  val randomNormal : mean:float -> deviation:float -> Stream<Arrival<float>>
  val randomExponential : mean:float -> Stream<Arrival<float>>
  val randomErlang : beta:float -> m:int -> Stream<Arrival<float>>
  val randomPoisson : mean:float -> Stream<Arrival<int>>
  val randomBinomial : prob:float -> trials:int -> Stream<Arrival<int>>
```

Here a value of type `Arrival<'a>` contains the modeling time at which the external event has arrived, the event itself of type `'a` and the delay time which has passed from the time of arriving the previous event.

```
type Arrival<'a> =
  { Value : 'a;
    Time : float;
    Delay : float option }
```

To process the input stream in parallel, we split the input with help of the `split` function, process new streams in parallel and then concatenate the intermediate results into one output stream using the `merge` function. Later will be provided the `Processor.par` function that does namely this.



To process the specified stream sequentially by some servers, we need a helper function that would read one more data item in advance, playing a role of the intermediate buffer between the servers.

```
module Stream =
  val prefetch : Stream<'a> -> Stream<'a>
```

Now we need the moving force that would run the whole network of streams.

```
module Stream =
  val sink : Stream<'a> -> Proc<unit>
```

It infinitely reads data from the specified stream.

When building queue networks, the following function can be useful too.

```
module Stream =
  val memo : Stream<'a> -> Stream<'a>
```

It memoizes the stream so that the resulting stream would always return the same data within the simulation run.

## 6.4 Processor

Having a stream of data, it would be natural to operate on its transformation which we will call a *processor*:

```
type Processor<'a, 'b> = Stream<'a> -> Stream<'b>
```

We can construct the processors directly from the streams. Omitting the obvious cases, we consider only the most important ones.

A new processor can be created by the specified handling function producing output that can be either pure or the `Proc` computation.

```
module Processor =
  val arr : ('a -> 'b) -> Processor<'a, 'b>
  val arrc : ('a -> Proc<'b>) -> Processor<'a, 'b>
```

Also we can use an accumulator to save the intermediate state of the processor. When processing the input stream and generating an output one, we can update the state.

```
module Processor =
  val accum : ('st -> 'a -> Proc<'st * 'b>) -> 'st -> Processor<'a, 'b>
```

We can involve the `Proc` computation with side effect, when processing every element of the input stream of data.

```
module Processor =
  val within : Proc<unit> -> Processor<'a, 'a>
```

An arbitrary number of processors can be united to work in parallel using the default FCFS queue strategy:

```
module Processor =
  val par : Processor<'a, 'b> list -> Processor<'a, 'b>
```

Its implementation is based on using the multiplexing and demultiplexing functions considered before. We split the input stream, process the intermediated streams in parallel and then concatenate the resulting streams into one output stream.

There are other versions of the `par` function, where we can specify the queue strategies and priorities if required.

To create a sequence of autonomously working processors, we can use the `next` function based on the prefetching function for streams considered above too:

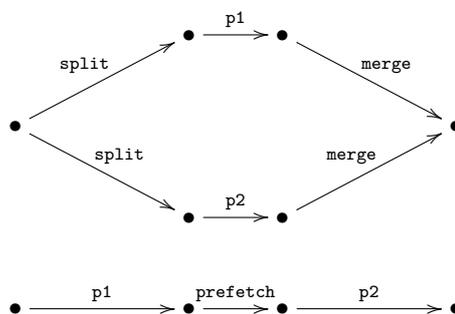
```
module Processor =
  val seq : Processor<'a, 'a> list -> Processor<'a, 'a>
```

For example, having two complementing processors `p1` and `p2`, we can create two new processors, where the first one implies a parallel work, while another implies a sequential processing:

```
let pPar = Processor.par [p1; p2]
let pSeq = Processor.seq [p1; p2]
```

The latter could be written explicitly as

```
let pSeq = p1 >> Stream.prefetch >> p2
```



We could connect two processors `p1` and `p2` directly, but it would be a monolithic processor, where input element is requested only after output element is requested outside.

```
let pWhole = p1 >> p2
```

When creating a sequence of processors, we have to isolate the processors with help of intermediate buffer and the `Stream.prefetch` processor plays a role of such an active buffer that requests one more element in advance.

Table 6.1: Composing Processors.

Function	Description
<code>Processor.par</code>	Parallel processors
<code>Processor.seq</code>	Sequential processors
<code>(&gt;&gt;)</code>	Processor composition

By the same reason we can connect directly to the queue processor as the queue is also an example of active buffer.

Basing on the described approach, we can model quite complex *queue networks* in an easy-to-use high-level declarative manner, which makes the Aivika library similar to some specialized simulation software tools by the capabilities of expression.

Regarding the queues themselves, we can model them using rather general-purpose helper combinators like this one:

```
module Processor =
  val queue : enqueue:('a -> Proc<unit>)
             -> dequeue:Proc<'b>
             -> Processor<'a, 'b>
```

An idea is that there is a plenty of cases how the queues could be united in the network. When enqueueing, we can either wait while the queue is full, or we can count such an item as lost. We can use the priorities for the `Proc` computations that enqueue or dequeue. Moreover, different processes can enqueue and dequeue simultaneously.

Therefore, it was decided to introduce such general-purpose helper functions for modeling the queues, where the details of how the queues are simulated can be shortly described with help of combinators like `enqueue`, `enqueueOrLost` and `dequeue` stated above.

However, there are three predefined combinators that cover the most of cases. Each of them returns a processor corresponding to the queue and adding the desired behavior.

```
module Queue =
  val processor : Queue<'a> -> Processor<'a, 'a>
  val processorWithLost : Queue<'a> -> Processor<'a, 'a>

module InfiniteQueue =
  val processor : InfiniteQueue<'a> -> Processor<'a, 'a>
```

The definition of these three queue processors is quite simple. We pass in two computations to the `queue` combinator. The first computation defines how the input elements are enqueued. The second computation defines how the output elements are dequeued.

```
module Queue =

  let processor (queue: Queue<'a>) =
    Processor.queue
      (fun a -> queue |> Queue.enqueue a)
      (Queue.dequeue queue)

  let processorWithLost (queue:Queue<'a>) =
    Processor.queue
      (fun a -> queue |> Queue.enqueueOrLost_ a |> Eventive.lift)
      (Queue.dequeue queue)

module InfiniteQueue =

  let processor (queue: InfiniteQueue<'a>) =
    Processor.queue
      (fun a -> queue |> InfiniteQueue.enqueue a |> Eventive.lift)
      (InfiniteQueue.dequeue queue)
```

Also we can model the queue networks with loopbacks using the intermediate queues to delay the stream. One of the possible combinators is provided below.

```
module Processor =

  val queueLoopSeq : enqueue:( 'a -> Proc<unit>)
                    -> dequeue:Proc<'c>
                    -> cond:Processor<'c, Choice<'e, 'b>>
                    -> body:Processor<'e, 'a>
                    -> Processor<'a, 'b>
```

Moreover, there are helper functions that hold the active process for a random time interval according to the desired distribution.

```
module Processor =

  val randomUniform : minimum:float -> maximum:float -> Processor<'a, 'a>
  val randomUniformInt : minimum:int -> maximum:int -> Processor<'a, 'a>
  val randomNormal : mean:float -> deviation:float -> Processor<'a, 'a>
  val randomExponential : mean:float -> Processor<'a, 'a>
  val randomErlang : beta:float -> m:int -> Processor<'a, 'a>
  val randomPoisson : mean:float -> Processor<'a, 'a>
  val randomBinomial : prob:float -> trials:int -> Processor<'a, 'a>
```

There is no magic in these random processors. Their definition is quite simple too:

```
module Processor =

  let randomUniform minimum maximum =
    Proc.randomUniform_ minimum maximum |> within
```

An example model that would use the streams and queue processors is provided further in section 6.8.

Using the processors, we can model a complicated enough behavior, for example, we can model the Round-Robbin strategy[17] of the processing.



```

-> Simulation<Server<unit, 'a, 'b>>

val createAccumPreemptible : preemptible:bool
    -> f:( 'state -> 'a -> Proc<'state * 'b>)
    -> state:'state
    -> Simulation<Server<'state, 'a, 'b>>

```

Here the first argument defines whether the underlying process can be preempted, when acquiring the `PreemptibleResource` resource.

To involve the server in simulation, we can use its processor that performs a service and updates the internal counters.

```

module Server =
    val processor : Server<'state, 'a, 'b> -> Processor<'a, 'b>

```

For example, we can request for the statistics of time spent by the server while processing the tasks.

```

module Server =
    val processingTime: Server<'state, 'a, 'b> -> Eventive<SamplingStats<float>>

```

There is one subtle thing. Each time we use the `processor` function, we actually create a new processor that refers to the same server and hence updates the same statistics counters. It can be useful if we are going to gather the statistics for a group of servers working in parallel, although the best practice would still be to use the `processor` function only once per each server.

There are predefined servers that model an activity that holds the underlying process for a random time interval, when processing every input element.

```

module Server =

    val createRandomUniformPreemptible :
        preemptible:bool
        -> minimum:float
        -> maximum:float
        -> Simulation<Server<unit, 'a, 'a>>

    val createRandomUniform :
        minimum:float
        -> maximum:float
        -> Simulation<Server<unit, 'a, 'a>>

    val createRandomUniformIntPreemptible :
        preemptible:bool
        -> minimum:int
        -> maximum:int
        -> Simulation<Server<unit, 'a, 'a>>

    val createRandomUniformInt :
        minimum:int
        -> maximum:int
        -> Simulation<Server<unit, 'a, 'a>>

    val createRandomNormalPreemptible :
        preemptible:bool
        -> mean:float
        -> deviation:float
        -> Simulation<Server<unit, 'a, 'a>>

```

```

val createRandomNormal :
  mean:float
  -> deviation:float
  -> Simulation<Server<unit, 'a, 'a>>

val createRandomExponentialPreemptible :
  preemptible:bool
  -> mean:float
  -> Simulation<Server<unit, 'a, 'a>>

val createRandomExponential :
  mean:float
  -> Simulation<Server<unit, 'a, 'a>>

val createRandomErlangPreemptible :
  preemptible:bool
  -> beta:float
  -> m:int
  -> Simulation<Server<unit, 'a, 'a>>

val createRandomErlang :
  beta:float
  -> m:int
  -> Simulation<Server<unit, 'a, 'a>>

val createRandomPoissonPreemptible :
  preemptible:bool
  -> mean:float
  -> Simulation<Server<unit, 'a, 'a>>

val createRandomPoisson :
  mean:float
  -> Simulation<Server<unit, 'a, 'a>>

val createRandomBinomialPreemptible :
  preemptible:bool
  -> prob:float
  -> trials:int
  -> Simulation<Server<unit, 'a, 'a>>

val createRandomBinomial :
  prob:float
  -> trials:int
  -> Simulation<Server<unit, 'a, 'a>>

```

The definition of these predefined servers with random activity is quite simple, which demonstrates how you can define your own activity.

```

module Server =

  let createRandomUniformPreemptible preemptible minimum maximum =
    createPreemptible preemptible (fun a -> proc {
      do! Proc.randomUniform_ minimum maximum
      return a
    })

  let createRandomUniform minimum maximum =
    createRandomUniformPreemptible false minimum maximum

```

There is no difference between the predefined servers and custom-made ones. Moreover, the resource preemption will work even in case if the activity is defined as a complicated Proc computation with branches, calculations and so on.

## 6.6 Timing Arrivals

Usually, an input of the queue network is expressed as a random stream of `Arrival` values. While processing, we can modify data that come with the arrival. We can add new attributes, remove them and change. It is quite simple as the processors usually work with generic data. In the end we have to measure the time which the specified arrival spent being in a system.

We can do this, using the following data type.

```
[<Sealed>]
type ArrivalTimer

module ArrivalTimer =
  val create : Simulation<ArrivalTimer>
```

An idea is that we pass the input data through a special processor that counts the time spent by arrivals.

```
module ArrivalTimer =
  val processor : ArrivalTimer -> Processor<Arrival<'a>, Arrival<'a>>
```

Then we request the timer for the processing time statistics collected.

```
module ArrivalTimer =
  val processingTime : ArrivalTimer -> Eventive<SamplingStats<float>>
```

Finally, the arrival timer can be returned as a `ResultSource` from the model.

## 6.7 Experiment Providers

The introduced above compound simulation entities such as queues and servers have a lot of counters. Either the compound entity or its specific counter can be returned from the model as a `ResultSource`.

For example, to show the deviation chart and summary for the size statistics of the queue with name `queue1`, we could write:

```
let series : ResultTransform =
  ResultSet.findByName "queue1" >>
  ResultSet.findById QueueCountStatsId

let provider1 = DeviationChartProvider ()
let provider2 = LastValueStatsProvider ()

provider1.Series <- series
provider2.Series <- series
```

The pros of this approach is that the queue has many counters and we can specify precisely those ones we need as well as we can specify how we want to display the simulation results. But such an approach seems to be quite tedious in practice as we usually need a small set of counters for quick analysis, for example, when validating the model.

Therefore, there is another way. An idea is that we can display only the most important information about the simulation entities writing rather small code, where we just define what entity we want to display the information about.

At first, the basic experiment providers are redefined as easy-to-use combinators:

```

namespace Simulation.Aivika.Experiments.Web

module ExperimentProvider =

    /// Shows the experiment specs.
    val experimentSpecs: IExperimentProvider<HtmlTextWriter>

    /// Shows the information about the specified series.
    val description: series:ResultTransform
        -> IExperimentProvider<HtmlTextWriter>

    /// Renders the last values for the specified series.
    val lastValue: series:ResultTransform
        -> IExperimentProvider<HtmlTextWriter>

    /// Renders the CSV file with results for the specified series.
    val table: series:ResultTransform
        -> IExperimentProvider<HtmlTextWriter>

    /// Renders the CSV file with last results for the specified series.
    val lastValueTable: series:ResultTransform
        -> IExperimentProvider<HtmlTextWriter>

    /// Renders the last value statistics for the specified series.
    val lastValueStats: series:ResultTransform
        -> IExperimentProvider<HtmlTextWriter>

```

Similar combinators are defined at level of the charting component for fast creation of the predefined simulation providers.

```

namespace Simulation.Aivika.Charting.Web

module ExperimentProvider =

    /// Renders the time series for the specified series.
    val timeSeries: series:ResultTransform
        -> IExperimentProvider<HtmlTextWriter>

    /// Renders the XY Chart for the specified series.
    val xyChart: seriesX:ResultTransform
        -> seriesY:ResultTransform
        -> IExperimentProvider<HtmlTextWriter>

    /// Renders the deviation chart for the specified series.
    val deviationChart: series:ResultTransform
        -> IExperimentProvider<HtmlTextWriter>

    /// Renders the last value histogram for the specified series.
    val lastValueHistogram: series:ResultTransform
        -> IExperimentProvider<HtmlTextWriter>

```

Now we use the property of providers to be combined, when we can take a list of the providers and create a new one that would behave as a whole.

```

namespace Simulation.Aivika.Experiments

module ExperimentProvider =

    /// Appends two providers.
    val append: p1:IExperimentProvider<'a>
        -> p2:IExperimentProvider<'a>
        -> IExperimentProvider<'a>

```

```

/// Concatenates the specified providers.
val concat: ps:IExperimentProvider<'a> list -> IExperimentProvider<'a>

/// A provider that renders nothing.
val empty<'a> : IExperimentProvider<'a>

```

Using this property, we can create easy-to-use simulation providers for some compound simulation entities displaying only the most important information.

```

namespace Simulation.Aivika.Charting.Web

module ExperimentProvider =

    /// Renders the basic queue properties for the specified series.
    val queue: series:ResultTransform
        -> IExperimentProvider<HtmlTextWriter>

    /// Renders the basic queue properties for the specified series.
    val infiniteQueue: series:ResultTransform
        -> IExperimentProvider<HtmlTextWriter>

    /// Renders the basic server properties for the specified series.
    val server: series:ResultTransform
        -> IExperimentProvider<HtmlTextWriter>

    /// Renders the basic arrival timer properties for the specified series.
    val arrivalTimer: series:ResultTransform
        -> IExperimentProvider<HtmlTextWriter>

```

For example, the queue provider can be defined in the following way.

```

module ExperimentProvider =

    let queue (series: ResultTransform) =
        let series1 = series >> ResultSet.findById QueueCountStatsId
        let series2 = series >> ResultSet.findById QueueWaitTimeId
        let series3 = series >> ResultSet.findById QueueLostCountId
        let series' = ResultTransform.concat [series; series1; series2; series3]
        [ExperimentProvider.description series';
         deviationChart series1;
         ExperimentProvider.lastValueStats series1;
         deviationChart series2;
         ExperimentProvider.lastValueStats series2;
         deviationChart series3;
         ExperimentProvider.lastValueStats series3;
         lastValueHistogram series3]
        |> ExperimentProvider.concat

```

We see that this provider displays the information about the size statistics, wait time and the count of lost items for the queues specified.

## 6.8 Example: Work Stations in Series

To illustrate how the streams and processors can be used for modeling, let us consider a model [10, 17] of two work stations connected in a series and separated by finite queues.

The maintenance facility of a large manufacturer performs two operations. These operations must be performed in series; operation 2 always follows operation 1. The units that are maintained are bulky, and space is available for only eight units including the units being worked on. A proposed design leaves space for two units between the work stations, and space for four units before work station 1. [...] Current company policy is to subcontract the maintenance of a unit if it cannot gain access to the in-house facility.

Historical data indicates that the time interval between requests for maintenance is exponentially distributed with a mean of 0.4 time units. Service times are also exponentially distributed with the first station requiring on the average 0.25 time units and the second station, 0.5 time units. Units are transported automatically from work station 1 to work station 2 in a negligible amount of time. If the queue of work station 2 is full, that is, if there are two units awaiting for work station 2, the first station is blocked and a unit cannot leave the station. A blocked work station cannot server other units.

Below is provided the corresponding simulation model.

```
// File WorkStationsInSeries/Model.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"

open Simulation.Aivika
open Simulation.Aivika.Queues
open Simulation.Aivika.Results

/// the simulation specs
let specs = {

    StartTime=0.0; StopTime=300.0; DT=0.1;
    Method=RungeKutta4; GeneratorType=StrongGenerator
}

/// the mean delay of the input arrivals distributed exponentially
let meanOrderDelay = 0.4

/// the capacity of the queue before the first work places
let queueMaxCount1 = 4

/// the capacity of the queue before the second work places
let queueMaxCount2 = 2

/// the mean processing time distributed exponentially in
/// the first work stations
let meanProcessingTime1 = 0.25

/// the mean processing time distributed exponentially in
/// the second work stations
let meanProcessingTime2 = 0.5

/// the simulation model
let model = simulation {

    // it will gather the statistics about the processing time
```

```

let! arrivalTimer = ArrivalTimer.create

// define a stream of input events
let inputStream = Stream.randomExponential meanOrderDelay

// create a queue in front of the first work stations
let! queue1 =
  Queue.createUsingFCFS queueMaxCount1
  |> Eventive.runInStartTime

// create a queue between the first and second work stations
let! queue2 =
  Queue.createUsingFCFS queueMaxCount2
  |> Eventive.runInStartTime

// create the first work station (server)
let! workStation1 =
  Server.createRandomExponential meanProcessingTime1

// create the second work station (server)
let! workStation2 =
  Server.createRandomExponential meanProcessingTime2

// the entire processor from input to output
let entireProcessor =
  Queue.processorWithLost queue1 >>
  Server.processor workStation1 >>
  Queue.processor queue2 >>
  Server.processor workStation2 >>
  ArrivalTimer.processor arrivalTimer

// start simulating the model
do! inputStream
  |> entireProcessor
  |> Stream.sink
  |> Proc.runInStartTime

// return the simulation results
return [ResultSource.From ("queue1", queue1,
  "Queue no. 1");
  ResultSource.From ("workStation1", workStation1,
  "Work Station no. 1");
  ResultSource.From ("queue2", queue2,
  "Queue no. 2");
  ResultSource.From ("workStation2", workStation2,
  "Work Station no. 2");
  ResultSource.From ("arrivalTimer", arrivalTimer,
  "The arrival timer")]
  |> ResultSet.create
}

/// the model summary
let modelSummary =
  model |> Simulation.map ResultSet.summary

```

The end part shows how we should run the queue network. We have to read permanently data from the output stream generated by the second work station. It initiates the process of receiving data from the queue located in a space between the both work stations. The corresponding queue processor begins requesting the first work station, which in its turn initiates the process of receiving data from the first queue, which processor begins reading data from

the input random stream of data distributed exponentially.

The simulation experiment file mostly mimics the last return function, where the result sources are defined.

```
// File WorkStationsInSeries/RunExperiment.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"
#r "../bin/Simulation.Aivika.Experiments.dll"
#r "../bin/Simulation.Aivika.Charting.dll"

#load "Model.fsx"

open System
open System.Web.UI

open Simulation.Aivika
open Simulation.Aivika.Results
open Simulation.Aivika.Experiments
open Simulation.Aivika.Experiments.Web
open Simulation.Aivika.Charting.Web

let experiment = Experiment ()

experiment.Specs <- Model.specs
experiment.RunCount <- 1000

let queueSeries1 = ResultSet.findByName "queue1"
let queueSeries2 = ResultSet.findByName "queue2"

let serverSeries1 = ResultSet.findByName "workStation1"
let serverSeries2 = ResultSet.findByName "workStation2"

let timerSeries = ResultSet.findByName "arrivalTimer"

let providers =
  [ExperimentProvider.experimentSpecs;
   ExperimentProvider.queue queueSeries1;
   ExperimentProvider.server serverSeries1;
   ExperimentProvider.queue queueSeries2;
   ExperimentProvider.server serverSeries2;
   ExperimentProvider.arrivalTimer timerSeries]

experiment.RenderHtml (Model.model, providers)
  |> Async.RunSynchronously
```

The experiment file creates a lot of information. For brevity, only two charts are provided on figures 6.1 and 6.2. The first chart shows the trend and probabilistic bounds for the first queue size, while the second chart shows the lost item count for the first queue.

## 6.9 Example: A Machine Tool with Breakdowns

The next example<sup>[10]</sup> illustrates the use of resource preemption.

Jobs arrive to a machine tool on the average of one per hour. The distribution of these interarrival times is exponential. During normal operation, the jobs are processed on a first-in, first-out basis. The

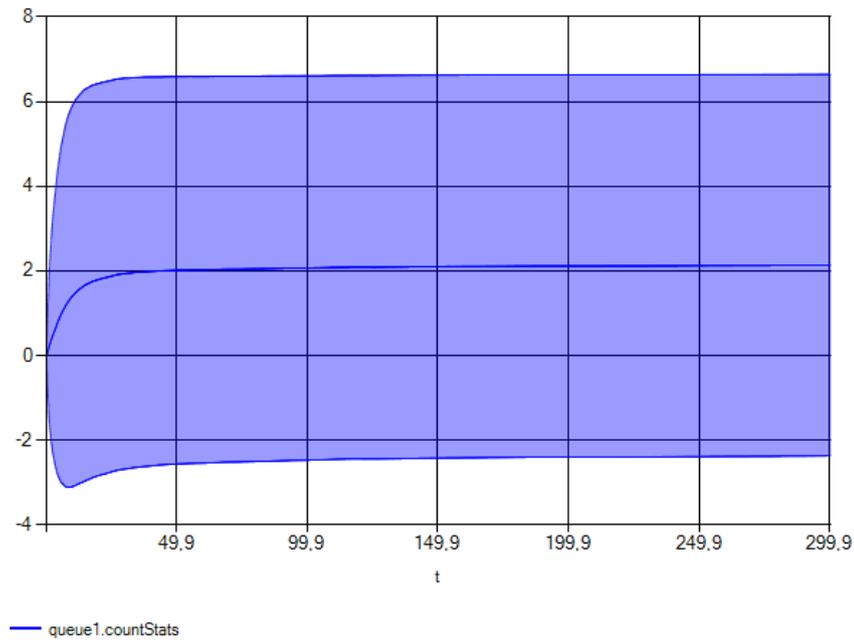


Figure 6.1: The first queue size trend and probabilistic bounds.

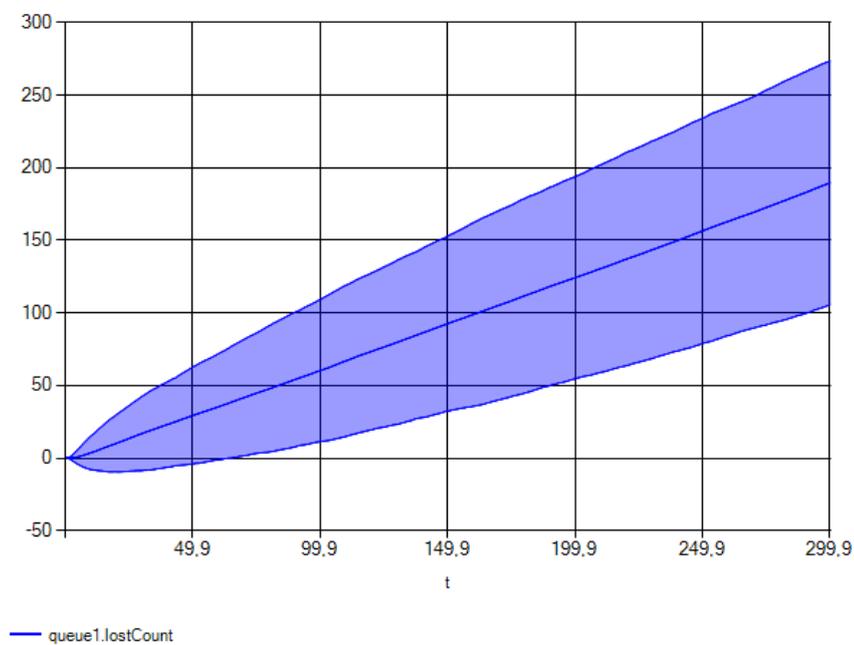


Figure 6.2: The lost item count for the first queue.

time to process a job in hours is normally distributed with a mean of 0.5 and a standard deviation of 0.1. In addition to the processing time, there is a set up time that is uniformly distributed between 0.2 and 0.5 of an hour. Jobs that have been processed by the machine tool are routed to a different section of the shop and are considered to have left the machine tool area.

The machine tool experiences breakdowns during which time it can no longer process jobs. The time between breakdowns is normally distributed with a mean of 20 hours and a standard deviation of 2 hours. When a breakdown occurs, the job being processed is removed from the machine tool and is placed at the head of the queue of jobs waiting to be processed. Jobs preempted restart from the point at which they were interrupted.

When the machine tool breaks down, a repair process is initiated which is accomplished in three phases. Each phase is exponentially distributed with a mean of  $3/4$  of an hour. Since the repair time is the sum of independent and identically distributed exponential random variables, the repair time is Erlang distributed. The machine tool is to be analyzed for 500 hours to obtain information on the utilization of the machine tool and the time required to process a job. Statistics are to be collected for thousand simulation runs.

We create two difference processes. The first process models the processing of jobs, while another models breakdowns. The both processes try to acquire a shared resource but with different preemption priorities, where the breakdown and the further repairing of the machine tool have a higher priority than the processing of jobs.

```
// File MachineBreakdowns/Model.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"

open Simulation.Aivika
open Simulation.Aivika.Queues
open Simulation.Aivika.Results

/// the simulation specs
let specs = {
    StartTime=0.0; StopTime=500.0; DT=0.1;
    Method=RungeKutta4; GeneratorType=StrongGenerator
}

/// How often do jobs arrive to a machine tool (exponential)?
let jobArrivingMu = 1.0

/// A mean of time to process a job (normal).
let jobProcessingMu = 0.5

/// The standard deviation of time to process a job (normal).
let jobProcessingSigma = 0.1

/// The minimum set-up time (uniform).
```

```

let minSetUpTime = 0.2

/// The maximum set-up time (uniform).
let maxSetUpTime = 0.5

/// A mean of time between breakdowns (normal).
let breakdownMu = 20.0

/// The standard deviation of time between breakdowns (normal).
let breakdownSigma = 2.0

/// A mean of each of the three repair phases (Erlang).
let repairMu = 3.0 / 4.0

/// A priority of the job (less is higher)
let jobPriority = 1.0

/// A priority of the breakdown (less is higher)
let breakdownPriority = 0.0

/// The simulation model.
let model: Simulation<ResultSet> = simulation {
  // create an input queue
  let! inputQueue =
    InfiniteQueue.createUsingFCFS
      |> Eventive.runInStartTime
  // a counter of jobs completed
  let! jobsCompleted = ArrivalTimer.create
  // a counter of interrupted jobs
  let jobsInterrupted = ref 0
  // create an input stream
  let inputStream = Stream.randomExponential jobArrivingMu
  // create a preemptible resource
  let! tool = PreemptibleResource.create 1
  // the machine setting up
  let! machineSettingUp =
    Server.createRandomUniformPreemptible
      true minSetUpTime maxSetUpTime
  // the machine processing
  let! machineProcessing =
    Server.createRandomNormalPreemptible
      true jobProcessingMu jobProcessingSigma
  // the machine breakdown
  let machineBreakdown = proc {
    while true do
      do! Proc.randomNormal_ breakdownMu breakdownSigma
      use! h =
        PreemptibleResource.takeWithPriority
          breakdownPriority tool
      do! Proc.randomErlang_ repairMu 3
    }
  }
  // start the process of breakdowns
  do! machineBreakdown
    |> Proc.runInStartTime
  // update a counter of job interruptions
  do! machineProcessing
    |> Server.taskPreempting
    |> Signal.add (fun a -> eventive { incr jobsInterrupted })
    |> Eventive.runInStartTime
  // define the queue network
  let network =
    InfiniteQueue.processor inputQueue >>

```

```

Processor.within
  (PreemptibleResource.requestWithPriority jobPriority tool) >>
Server.processor machineSettingUp >>
Server.processor machineProcessing >>
Processor.within
  (PreemptibleResource.release tool) >>
  ArrivallTimer.processor jobsCompleted
// start the machine tool
do! network inputStream
  |> Stream.sink
  |> Proc.runInStartTime
// return the simulation results in start time
return
  [ResultSource.From ("inputQueue",
    inputQueue, "the queue of jobs");
   ResultSource.From ("machineSettingUp",
    machineSettingUp, "the machine setting up");
   ResultSource.From ("machineProcessing",
    machineProcessing, "the machine processing");
   ResultSource.From ("jobsInterrupted",
    jobsInterrupted, "a counter of the interrupted jobs");
   ResultSource.From ("jobsCompleted",
    jobsCompleted, "a counter of the completed jobs")]
  |> ResultSet.create
}

let modelSummary =
  model |> Simulation.map ResultSet.summary

```

The simulation experiment file mainly repeats the names of sources returned from the model. The Monte-Carlo simulation contains 1000 runs.

```

// File MachineBreakdowns/RunExperiment.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"
#r "../bin/Simulation.Aivika.Experiments.dll"
#r "../bin/Simulation.Aivika.Charting.dll"

#load "Model.fsx"

open System
open System.Web.UI

open Simulation.Aivika
open Simulation.Aivika.Results
open Simulation.Aivika.Experiments
open Simulation.Aivika.Experiments.Web
open Simulation.Aivika.Charting.Web

let experiment = Experiment ()

experiment.Specs <- Model.specs
experiment.RunCount <- 1000

let inputQueue = ResultSet.findByName "inputQueue"

let machineSettingUp = ResultSet.findByName "machineSettingUp"
let machineProcessing = ResultSet.findByName "machineProcessing"

let jobsInterrupted = ResultSet.findByName "jobsInterrupted"

```

```

let jobsCompleted = ResultSet.findByName "jobsCompleted"

let providers =
  [ExperimentProvider.experimentSpecs;
   ExperimentProvider.infiniteQueue inputQueue;
   ExperimentProvider.server machineSettingUp;
   ExperimentProvider.server machineProcessing;
   ExperimentProvider.description jobsInterrupted;
   ExperimentProvider.lastValueStats jobsInterrupted;
   ExperimentProvider.arrivalTimer jobsCompleted]

experiment.RenderHtml (Model.model, providers)
  |> Async.RunSynchronously

```

We receive a lot of results. The chart displaying the utilization of the machine tool is shown on figure 6.3, but the chart of the time required to process a job is illustrated on figure 6.4.

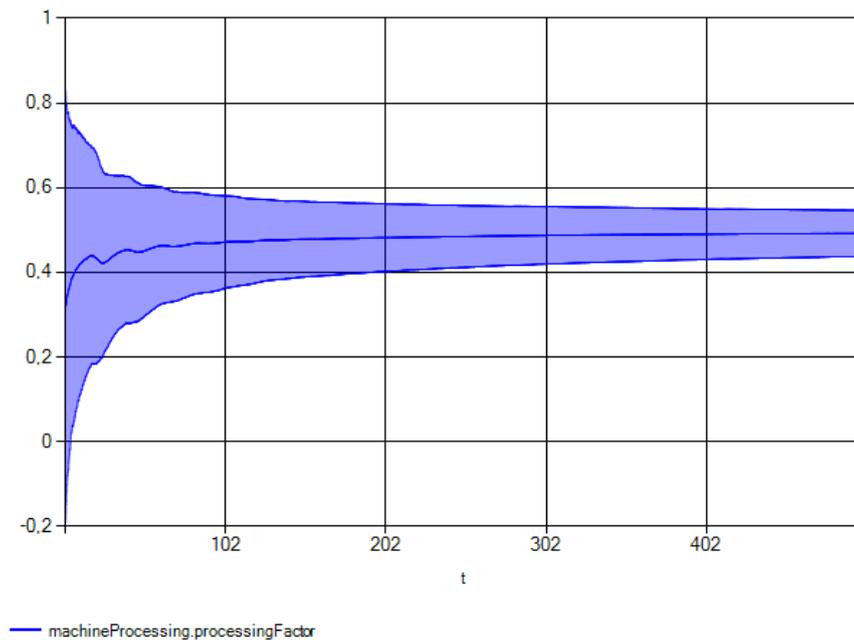


Figure 6.3: The utilization of the machine tool.

The processing time cannot be negative, but it has so huge deviation that the chart shows negative values according to rule 3-sigma.

Actually, we could save the results in CSV files with help of `TableProvider` and then analyze them in R, for example, but now we use the Aivika embedded capabilities that automate the process of quick analysis significantly.

## 6.10 Example: Inspection and Adjustment Stations

This example[10] illustrates how we can model a parallel work of servers. Also it shows how we can create queue networks with loopbacks.

Assembled television sets move through a series of testing stations in the final stage of their production. At the last of these stations, the vertical control setting on the TV sets is tested. If the setting is found to be functioning improperly, the offending set is routed to an adjustment station where the setting is adjusted. After adjustment, the television set is sent back to the last inspection station where the setting is again inspected. Television sets passing the final inspection phase, whether for the first time or after one or more routings through the adjustment station, are routed to a packing area.

The time between arrivals of television sets to the final inspection station is uniformly distributed between 3.5 and 7.5 minutes. Two inspectors work side-by-side at the final inspection station. The time required to inspect a set is uniformly distributed between 6 and 12 minutes. On the average, 85 percent of the sets are routed to the adjustment station which is manned by a single worker. Adjustment of the vertical control setting requires between 20 and 40 minutes, uniformly distributed.

The inspection station and adjustor are to be simulated for 480 minutes to estimate the time to process television sets through the final production stage and to determine the utilization of the inspectors and the adjustors.

The simulation model is as follows.

```
// File InspectionAdjustmentStations/Model.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"

open Simulation.Aivika
open Simulation.Aivika.Queues
open Simulation.Aivika.Results

/// the simulation specs
let specs = {

    StartTime=0.0; StopTime=480.0; DT=0.1;
    Method=RungeKutta4; GeneratorType=StrongGenerator
}

/// the minimum delay of arriving the next TV set
let minArrivalDelay = 3.5

/// the maximum delay of arriving the next TV set
let maxArrivalDelay = 7.5

/// the minimum time to inspect the TV set
let minInspectionTime = 6.0
```

```

/// the maximum time to inspect the TV set
let maxInspectionTime = 12.0

/// the probability of passing the inspection phase
let inspectionPassingProb = 0.85

/// how many are inspection stations?
let inspectionStationCount = 2

/// the minimum time to adjust an improper TV set
let minAdjustmentTime = 20.0

/// the maximum time to adjust an improper TV set
let maxAdjustmentTime = 40.0

/// how many are adjustment stations?
let adjustmentStationCount = 1

let model: Simulation<ResultSet> = simulation {
  // to count the arrived TV sets for inspecting and adjusting
  let! inputArrivalTimer = ArrivalTimer.create
  // it will gather the statistics of the processing time
  let! outputArrivalTimer = ArrivalTimer.create
  // define a stream of input events
  let inputStream =
    Stream.randomUniform minArrivalDelay maxArrivalDelay
  // create a queue before the inspection stations
  let! inspectionQueue =
    InfiniteQueue.createUsingFCFS
    |> Eventive.runInStartTime
  // create a queue before the adjustment stations
  let! adjustmentQueue =
    InfiniteQueue.createUsingFCFS
    |> Eventive.runInStartTime
  // create the inspection stations (servers)
  let! inspectionStations =
    [ for i = 1 to inspectionStationCount do
      yield Server.createRandomUniform
        minInspectionTime maxInspectionTime ]
    |> Simulation.ofList
  // create the adjustment stations (servers)
  let! adjustmentStations =
    [ for i = 1 to adjustmentStationCount do
      yield Server.createRandomUniform
        minAdjustmentTime maxAdjustmentTime ]
    |> Simulation.ofList
  // the line of parallel inspection stations
  let inspectionProcessor =
    inspectionStations
    |> List.map Server.processor
    |> Processor.par
  // the line of adjustment stations
  let adjustmentProcessor =
    adjustmentStations
    |> List.map Server.processor
    |> Processor.par
  // an output stream that comes after the inspection stations
  let rec outputStream = stream {
    let xs: Stream<_> =
      inspectionQueue
      |> InfiniteQueue.dequeue

```

```

        |> Stream.repeat
        |> inspectionProcessor
    for a in xs do
        let! passed =
            Parameter.randomTrue inspectionPassingProb
            |> Parameter.lift
        if passed then
            yield a
        else
            do! adjustmentQueue
                |> InfiniteQueue.enqueue a
                |> Eventive.lift
    }
    // the terminal processor
    and terminalProcessor =
        outputStream
        |> ArrivalTimer.processor outputArrivalTimer
    // the process of adjusting TV sets
    and adjustmentProcess = proc {
        let xs: Stream<_> =
            adjustmentQueue
            |> InfiniteQueue.dequeue
            |> Stream.repeat
            |> adjustmentProcessor
        for a in xs do
            do! inspectionQueue
                |> InfiniteQueue.enqueue a
                |> Eventive.lift
    }
    // the input process
    and inputProcess = proc {
        let xs: Stream<_> =
            inputStream
            |> ArrivalTimer.processor inputArrivalTimer
        for a in xs do
            do! inspectionQueue
                |> InfiniteQueue.enqueue a
                |> Eventive.lift
    }
    // run the process of adjustment
    do! adjustmentProcess
        |> Proc.runInStartTime
    // run the input process
    do! inputProcess
        |> Proc.runInStartTime
    // run the terminal processor
    do! terminalProcessor
        |> Stream.sink
        |> Proc.runInStartTime
    // return the simulation results
    return
    [ResultSource.From ("inspectionQueue", inspectionQueue,
        "the inspection queue");
    ResultSource.From ("adjustmentQueue", adjustmentQueue,
        "the adjustment queue");
    ResultSource.From ("inputArrivalTimer", inputArrivalTimer,
        "the input arrival timer");
    ResultSource.From ("outputArrivalTimer", outputArrivalTimer,
        "the output arrival timer");
    ResultSource.From ("inspectionStations", inspectionStations,
        "the inspection stations");
    ResultSource.From ("adjustmentStations", adjustmentStations,

```

```

        "the adjustment stations"
    ]
    |> ResultSet.create
}

let modelSummary: Simulation<ResultSet> =
    model |> Simulation.map ResultSet.summary

```

We will use the following simulation experiment.

```

// File InspectionAdjustmentStations/RunExperiment.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"
#r "../bin/Simulation.Aivika.Experiments.dll"
#r "../bin/Simulation.Aivika.Charting.dll"

#load "Model.fsx"

open System
open System.Web.UI

open Simulation.Aivika
open Simulation.Aivika.Results
open Simulation.Aivika.Experiments
open Simulation.Aivika.Experiments.Web
open Simulation.Aivika.Charting.Web

let experiment = Experiment ()

experiment.Specs <- Model.specs
experiment.RunCount <- 1000

let inspectionQueue = ResultSet.findByName "inspectionQueue"
let adjustmentQueue = ResultSet.findByName "adjustmentQueue"

let inspectionStations = ResultSet.findByName "inspectionStations"
let adjustmentStations = ResultSet.findByName "adjustmentStations"

let outputTimer = ResultSet.findByName "outputArrivalTimer"

let providers =
    [ExperimentProvider.experimentSpecs;
      ExperimentProvider.infiniteQueue inspectionQueue;
      ExperimentProvider.infiniteQueue adjustmentQueue;
      ExperimentProvider.server inspectionStations;
      ExperimentProvider.server adjustmentStations;
      ExperimentProvider.arrivalTimer outputTimer]

experiment.RenderHtml (Model.model, providers)
    |> Async.RunSynchronously

```

Some of the results are shown on figures 6.5, 6.6 and 6.7.

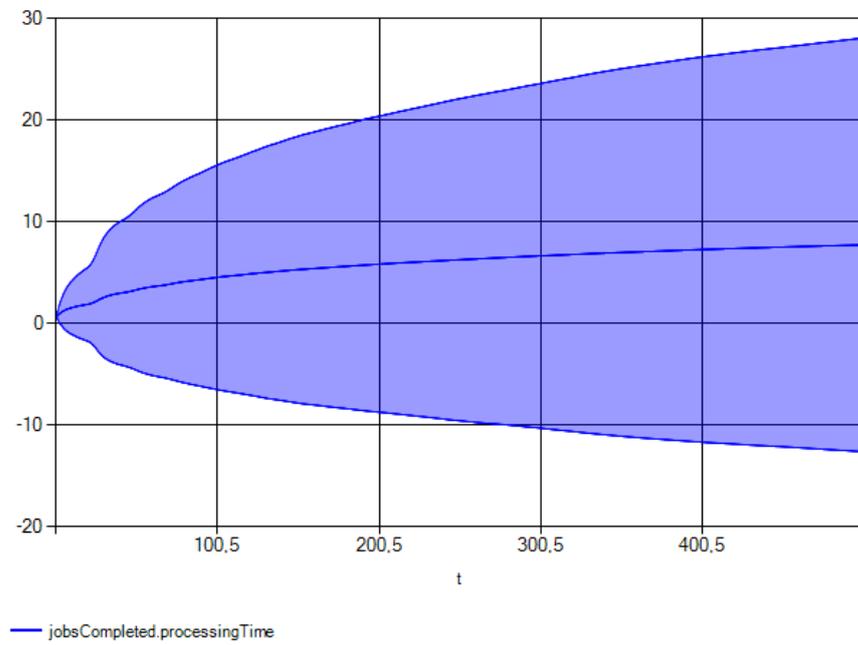


Figure 6.4: The time required to process a job.

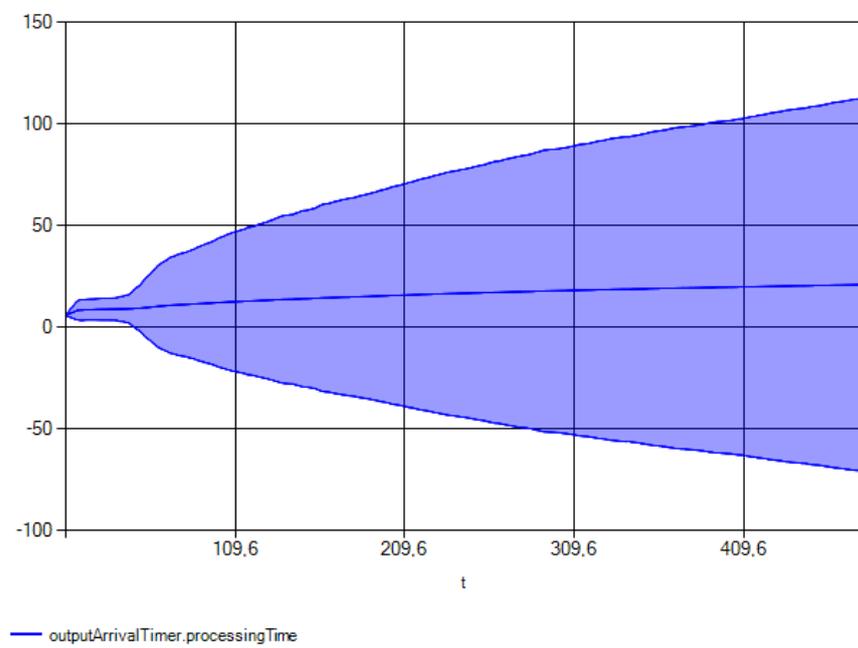


Figure 6.5: The processing time of television sets.

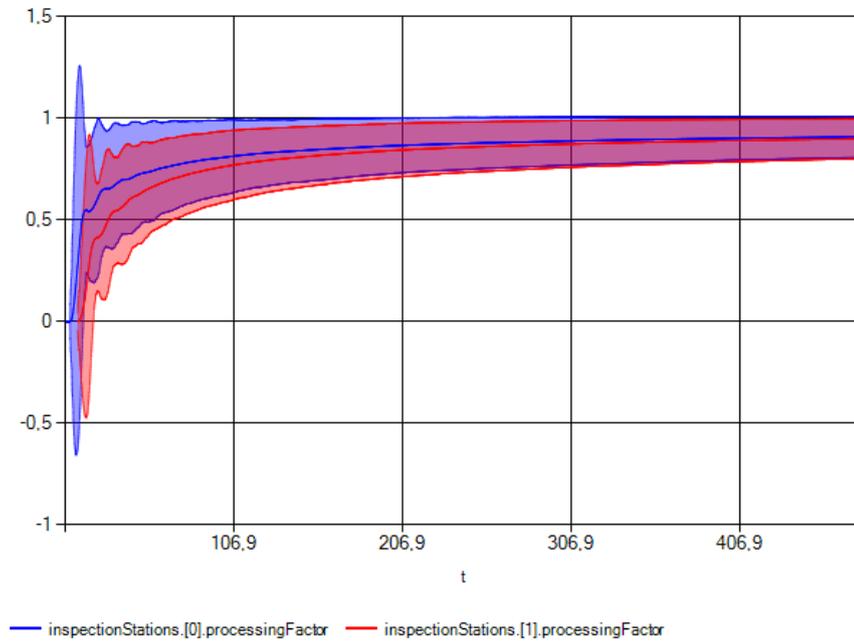


Figure 6.6: The utilization of the inspectors.

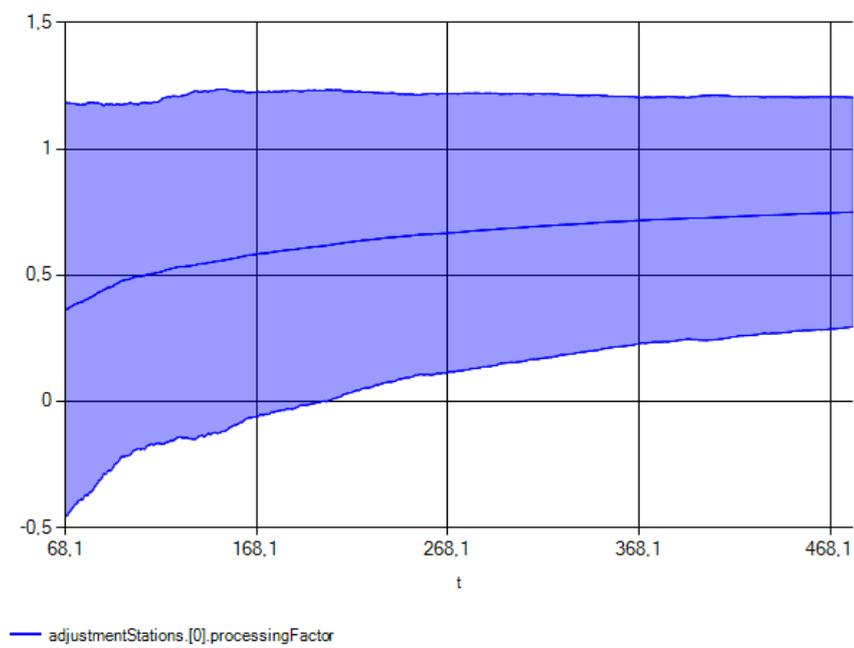


Figure 6.7: The utilization of the adjustor.



## Chapter 7

# Parameters

### 7.1 Latin Square

### 7.2 Reading Data from Excel



## Chapter 8

# System Dynamics

The Aivika library was mainly designed and created for the field of discrete event simulation. However, the library can be used for solving tasks of System Dynamics too. Moreover, the both fields can be naturally combined. Actually, the discrete event simulation computations are implemented on top of the `Dynamics` computation, which is used for System Dynamics.

### 8.1 Memoizing Sequential Computations

A key feature that distinguishes the `Dynamics` computation from the `Eventive` one is that the modeling time flows in an unpredictable order within the former computation.

For example, the past value can be requested from the future: the initial value of the integral can be requested at any time and so on.

Aivika solves this task by ordering and memoizing the computations in integration time points.

```
module Dynamics =  
  
    val memo : comp:Dynamics<'a> -> Dynamics<'a>  
    val memo0 : comp:Dynamics<'a> -> Dynamics<'a>
```

The both functions return a new computation that sequentially calls the input computation in the integration time points by demand and saves the values in array so that the next call in the same integration time point will return the already calculated value without calling the input computation twice.

In other non-integration time points the values are interpolated so that the closest past integration time point is selected, which allows using the resulting computation in the discrete event simulation. Also we cannot request for the future value from the past. Otherwise, we might receive a deadlock. This is a step-wise linear interpolation.

The functions differ in that how they behave when specifying the Runge-Kutta integration method.

The `memo` function is destined for integrating by the Runge-Kutta method. It consumes more memory to allocate an array that stores the values in the intermediate integration time points that are used by this integration technique.

The `memo0` function is more fast. It uses only the basic integration time points as it would always be Euler's integration method.

Usually, the `memo` function is needed only for integrals, while the `memo0` is suitable for other cases.

To emphasize the difference between the integration time points, there are two functions that make the idea more clear.

```
module Dynamics =
  val interpolate : comp:Dynamics<'a> -> Dynamics<'a>
  val discrete : comp:Dynamics<'a> -> Dynamics<'a>
```

The `interpolate` function is used by the `memo` function. This is the very interpolation, when the values in the integration time points including the intermediate integration time points are returned as they are, while in other time points we return a value for the closest past integration time point.

The `discrete` function is used by the `memo0` function. This is the interpolation, where only the basic integration time points are used, i.e. those time points that are defined by Euler's method.

Some simulation software tools for System Dynamics such as Vensim define functions that have an effect, when the value returned by the function changes only in the integration time point regardless on the integration method used. The `discrete` function gives namely this effect.

To complete the picture, there is a function that returns the initial value of the computation that was defined in the start time, but the initial value is returned in the current simulation time.

```
module Dynamics =
  val runInStartTime : comp:Dynamics<'a> -> Simulation<'a>
```

For example, this function can be used for receiving the initial value of the integral.

Returning to the memoization functions, they can be used not only for integrals. They can also be used for creating random processes that can be defined in the differential and difference equations of System Dynamics.

```
module Dynamics =
  val memoRandomUniform : minimum:Dynamics<float>
    -> maximum:Dynamics<float>
    -> Dynamics<float>
  val memoRandomUniformInt : minimum:Dynamics<int>
    -> maximum:Dynamics<int>
    -> Dynamics<int>
  val memoRandomNormal : mean:Dynamics<float>
    -> deviation:Dynamics<float>
    -> Dynamics<float>
  val memoRandomExponential : mean:Dynamics<float> -> Dynamics<float>
  val memoRandomErlang : beta:Dynamics<float>
    -> m:Dynamics<int>
    -> Dynamics<float>
```

```

val memoRandomPoisson : mean:Dynamics<float> -> Dynamics<int>

val memoRandomBinomial : prob:Dynamics<float>
                        -> trials:Dynamics<int>
                        -> Dynamics<int>

```

They are based on the `memo0` function, i.e. the result changes only in the integration time point.

## 8.2 Table Function

Many simulation models of System Dynamics use graphical functions based on tables of pairs  $(x,y)$ .

```

[<Sealed>]
type Table =

  new : xys:(float * float) [] -> Table

  member Lookup : x:float -> float
  member LookupStepwise : x:float -> float

```

The first lookup method uses the linear interpolation, while the second method uses a step-wise linear interpolation.

To make the table functions more easy-to-use, the library defines convenient helper functions to be used in the differential and difference equations.

```

module Dynamics =

  val lookup : x:Dynamics<float> -> tbl:Table -> Dynamics<float>
  val lookupStepwise : x:Dynamics<float> -> tbl:Table -> Dynamics<float>

```

For example, the first function could be trivially defined as

```

module Dynamics =

  let lookup (x: Dynamics<float>) (t: Table) =
    dynamics {
      let! a = x
      return t.Lookup (a)
    }

```

Note the use of the computation expression syntax. It literally means that you can include your own functions in the differential and difference equations as they are actually a system of `Dynamics` computations.

## 8.3 Differential Equations

The integral function signature was stated before and it is repeated here again for convenience.

```

module SD =

  val integ : derivative:Lazy<Dynamics<float>>
            -> init:Dynamics<float>
            -> Dynamics<float>

```

It creates an integral by the specified derivative and initial value.

Comparing to specialized simulation software tools, this function is rather slow but it works. Moreover, it can be used in the combined discrete continuous simulation models.

We can create the ordinary differential equations of almost any complexity based on the `integ` function.

Below is provided an implementation of the  $n$ 'th order exponential smooth function.

```
module SD =

  let smoothN (x: Dynamics<float>) (t: Lazy<Dynamics<float>>) n =

    let rec s = []
      for k = 0 to n-1 do
        if k = 0 then
          yield integ (lazy ((x - s.[k]) /
                               (t.Value / (float n)))) x
        else
          yield integ (lazy ((s.[k-1] - s.[k]) /
                               (t.Value / (float n)))) x []
    in s.[n-1]
```

A key point is that the integrals are just `Dynamics` computations that can be combined in very sophisticated manner.

Earlier we saw a few examples of the discrete event simulation models that extensively used the computation expression syntax of `F#`. The same syntax can be used for extending the differential equations, which allows embed your own functions in the equations.

For example, see a possible implementation of the table lookup function in section 8.2.

Regarding the combination with discrete event simulation, there are two points, at least. An arbitrary `Dynamics` computation can be transformed to the `Eventive` computation. Then, we can use the `Var` data type to create entities that could be used in the differential equations but would be updated from the discrete event simulation.

## 8.4 Difference Equations

The difference equations are much like the differential ones, only another function is used for creating a sum by the specified difference and initial value.

```
module SD =

  val diffsum: difference:Lazy<Dynamics<float>>
              -> init:Dynamics<float>
              -> Dynamics<float>
```

## 8.5 Example: Parametric Financial Model

The next example illustrates a parametric model in combination with the Monte-Carlo simulation. The results received can be useful for the sensitivity analysis. The approach described works for the discrete event simulation too.

We will take the financial model[18] described in Vensim 5 Modeling Guide, Chapter Financial Modeling and Risk. Probably, the best way to describe the model is just to show its equations.

The equations use the `npv` function from System Dynamics. It returns the Net Present Value (NPV) of the stream computed using the specified discount rate, the initial value and some factor (usually 1).

```
module SD =

  let npv stream rate init factor =
    let rec dt' = Parameter.dt |> Parameter.lift
        and df = integ (lazy (- df * rate)) (num 1.0)
        and accum = integ (lazy (stream * df)) init
        in (accum + dt' * stream * df) * factor
```

Also we need a helper conditional combinator that allows simplifying the equations in some cases.

```
module SD =

  val ifThenElse: cond:Dynamics<bool>
                -> thenPart:Dynamics<'a>
                -> elsePart:Dynamics<'a>
                -> Dynamics<'a>
```

After we finished the necessary preliminaries, now we can show how the parametric model can be prepared for the Monte-Carlo simulation.

We represent each external parameter as a `Parameter` computation. To be reproducible within every simulation run, the random parameter must be memoized with help of the `Parameter.memo` function.

```
// File Financial/Model.fsx

#nowarn "40"

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"

open Simulation.Aivika
open Simulation.Aivika.SD
open Simulation.Aivika.Results

/// The simulation specs
let specs =
  { StartTime = 0.0;
    StopTime = 5.0;
    DT = 0.015625;
    Method = RungeKutta4;
    GeneratorType = StrongGenerator }

/// The model parameters.
type Parameters =
  { TaxDepreciationTime : Parameter<float>;
    TaxRate : Parameter<float>;
    AveragePayableDelay : Parameter<float>;
    BillingProcessingTime : Parameter<float>;
    BuildingTime : Parameter<float>;
    DebtFinancingFraction : Parameter<float>;
    DebtRetirementTime : Parameter<float>;
```

```

DiscountRate      : Parameter<float>;
FractionalLossRate : Parameter<float>;
InterestRate      : Parameter<float>;
Price             : Parameter<float>;
ProductionCapacity : Parameter<float>;
RequiredInvestment : Parameter<float>;
VariableProductionCost : Parameter<float> }

/// The default model parameters.
let defaultParams =
{ TaxDepreciationTime = parameter.Return 10.0;
  TaxRate              = parameter.Return 0.4;
  AveragePayableDelay  = parameter.Return 0.09;
  BillingProcessingTime = parameter.Return 0.04;
  BuildingTime         = parameter.Return 1.0;
  DebtFinancingFraction = parameter.Return 0.6;
  DebtRetirementTime  = parameter.Return 3.0;
  DiscountRate         = parameter.Return 0.12;
  FractionalLossRate   = parameter.Return 0.06;
  InterestRate         = parameter.Return 0.12;
  Price                = parameter.Return 1.0;
  ProductionCapacity   = parameter.Return 2400.0;
  RequiredInvestment   = parameter.Return 2000.0;
  VariableProductionCost = parameter.Return 0.6 }

/// Random parameters for the Monte-Carlo simulation.
let randomParams =
let averagePayableDelay = Parameter.randomUniform 0.07 0.11
let billingProcessingTime = Parameter.randomUniform 0.03 0.05
let buildingTime         = Parameter.randomUniform 0.8 1.2
let fractionalLossRate   = Parameter.randomUniform 0.05 0.08
let interestRate         = Parameter.randomUniform 0.09 0.15
let price                = Parameter.randomUniform 0.9 1.2
let productionCapacity   = Parameter.randomUniform 2200.0 2600.0
let requiredInvestment   = Parameter.randomUniform 1800.0 2200.0
let variableProductionCost = Parameter.randomUniform 0.5 0.7
{ defaultParams with
  AveragePayableDelay = Parameter.memo averagePayableDelay;
  BillingProcessingTime = Parameter.memo billingProcessingTime;
  BuildingTime         = Parameter.memo buildingTime;
  FractionalLossRate   = Parameter.memo fractionalLossRate;
  InterestRate         = Parameter.memo interestRate;
  Price                = Parameter.memo price;
  ProductionCapacity   = Parameter.memo productionCapacity;
  RequiredInvestment   = Parameter.memo requiredInvestment;
  VariableProductionCost = Parameter.memo variableProductionCost }

/// This is the model itself that returns experimental data.
let model (ps: Parameters) : Simulation<ResultSet> = simulation {

  let get (x: Parameter<_>) : Dynamics<_> = Parameter.lift x

  let taxDepreciationTime = get ps.TaxDepreciationTime
  let taxRate              = get ps.TaxRate
  let averagePayableDelay  = get ps.AveragePayableDelay
  let billingProcessingTime = get ps.BillingProcessingTime
  let buildingTime         = get ps.BuildingTime;
  let debtFinancingFraction = get ps.DebtFinancingFraction
  let debtRetirementTime  = get ps.DebtRetirementTime
  let discountRate         = get ps.DiscountRate
  let fractionalLossRate   = get ps.FractionalLossRate
  let interestRate         = get ps.InterestRate

```

```

let price                = get ps.Price
let productionCapacity  = get ps.ProductionCapacity
let requiredInvestment  = get ps.RequiredInvestment
let variableProductionCost = get ps.VariableProductionCost

// the equations below are given in an arbitrary order!

let rec bookValue =
  integ (lazy (newInvestment - taxDepreciation)) (num 0.0)
and taxDepreciation = bookValue / taxDepreciationTime
and taxableIncome =
  grossIncome - directCosts - losses
  - interestPayments - taxDepreciation
and production = availableCapacity
and availableCapacity =
  ifThenElse (Dynamics.time .>=. buildingTime)
    productionCapacity (num 0.0)
and accountsReceivable =
  integ (lazy (billings - cashReceipts - losses))
    (billings / (num 1.0 / averagePayableDelay
      + fractionalLossRate))
and awaitingBilling =
  integ (lazy (price * production - billings))
    (price * production * billingProcessingTime)
and billings = awaitingBilling / billingProcessingTime
and borrowing = newInvestment * debtFinancingFraction
and cashReceipts = accountsReceivable / averagePayableDelay
and debt =
  integ (lazy (borrowing - principalRepayment)) (num 0.0)
and directCosts = production * variableProductionCost
and grossIncome = billings
and interestPayments = debt * interestRate
and losses = accountsReceivable * fractionalLossRate
and netCashFlow =
  cashReceipts + borrowing - newInvestment
  - directCosts - interestPayments
  - principalRepayment - taxes
and netIncome = taxableIncome - taxes
and newInvestment =
  ifThenElse (Dynamics.time .>=. buildingTime)
    (num 0.0) (requiredInvestment / buildingTime)
and npvCashFlow =
  npv netCashFlow discountRate (num 0.0) (num 1.0)
and npvIncome =
  npv netIncome discountRate (num 0.0) (num 1.0)
and principalRepayment = debt / debtRetirementTime
and taxes = taxableIncome * taxRate

return
  [ResultSource.From ("netIncome",
    netIncome, "Net income");
   ResultSource.From ("netCashFlow",
    netCashFlow, "Net cash flow");
   ResultSource.From ("npvIncome",
    npvIncome, "NPV income");
   ResultSource.From ("npvCashFlow",
    npvCashFlow, "NPV cash flow")]
  |> ResultSet.create
}

```

Now we can apply the Monte-Carlo simulation to this parametric model, for example, to define how sensitive are some variables to the random external

parameters.

The point is that not only ODEs can be parametric. There is not any difference, whether we integrate numerically, or run the discrete event simulation, or simulate the agents. The external parameters are just `Parameter` computations that can be used within other simulation computations.

Returning to this example, we will use the following simulation experiment.

```
// File Financial/RunExperiment.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"
#r "../bin/Simulation.Aivika.Experiments.dll"
#r "../bin/Simulation.Aivika.Charting.dll"

#load "Model.fsx"

open System
open System.Web.UI

open Simulation.Aivika
open Simulation.Aivika.Results
open Simulation.Aivika.Experiments
open Simulation.Aivika.Experiments.Web
open Simulation.Aivika.Charting.Web

let specs = Model.specs
let model = Model.model Model.randomParams

let experiment = Experiment ()

experiment.Specs <- specs
experiment.RunCount <- 1000

let income =
  [ResultSet.findByName "netIncome";
   ResultSet.findByName "netCashFlow"]
  |> ResultTransform.concat

let cashFlow =
  [ResultSet.findByName "npvIncome";
   ResultSet.findByName "npvCashFlow"]
  |> ResultTransform.concat

let providers =
  [ExperimentProvider.experimentSpecs;
   ExperimentProvider.description income;
   ExperimentProvider.deviationChart income;
   ExperimentProvider.lastValueStats income;
   ExperimentProvider.lastValueHistogram income;
   ExperimentProvider.description cashFlow;
   ExperimentProvider.deviationChart cashFlow;
   ExperimentProvider.lastValueStats cashFlow;
   ExperimentProvider.lastValueHistogram cashFlow]

experiment.RenderHtml (model, providers)
|> Async.RunSynchronously
```

The resulting deviation charts are shown on figures 8.1 and 8.2.

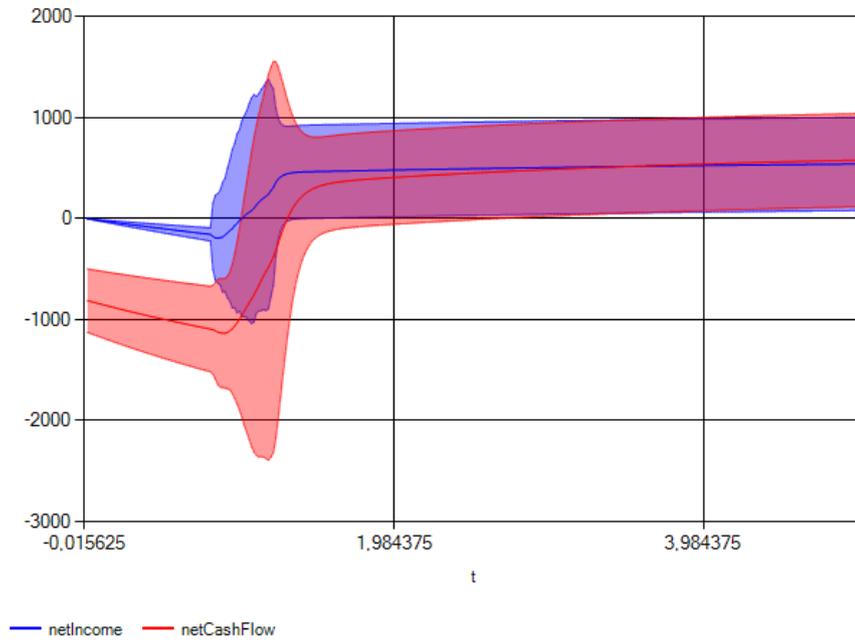


Figure 8.1: The net income and net cash flow.

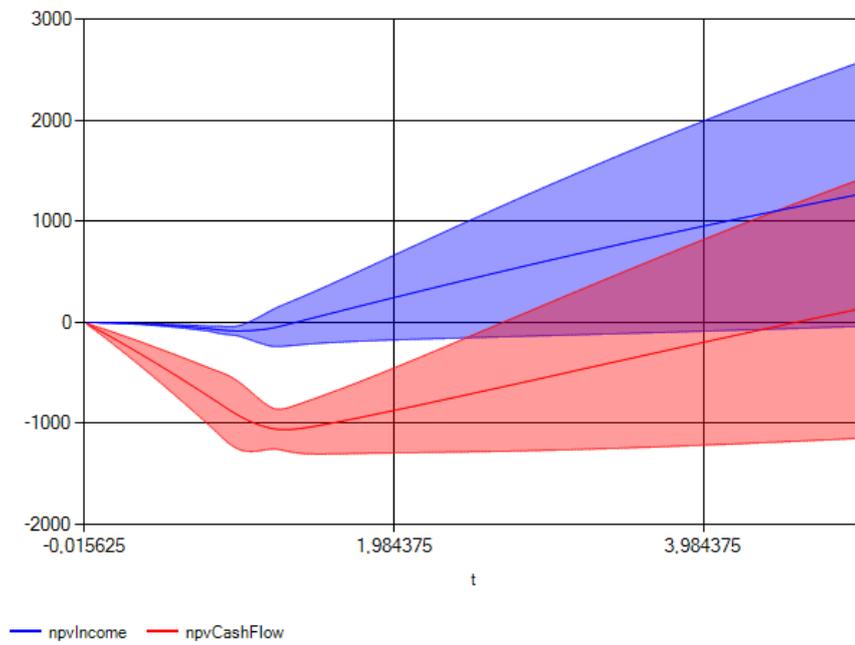


Figure 8.2: The NPV income and cash flow.

## 8.6 Example: Linear Array

This example illustrates the use of arrays. There is no need in special support for them. They can be naturally used with the simulation computations.

Let us take model Linear Array from Berkeley Madonna[6] to demonstrate the main idea.

```
// File LinearArray/Model.fsx

#nowarn "40"

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"

open Simulation.Aivika
open Simulation.Aivika.SD
open Simulation.Aivika.Results

let specs =
  { StartTime = 0.0;
    StopTime = 500.0;
    DT = 0.1;
    Method = RungeKutta4;
    GeneratorType = StrongGenerator }

let model (n: int) : Simulation<ResultSet> = simulation {

  let rec m : Dynamics<float> array =
    [| for i = 1 to n do
      yield integ
        (lazy (q
          + k * (c.[i - 1] - c.[i])
          + k * (c.[i + 1] - c.[i])))
        (num 0.0) |]

  and c : Dynamics<float> array =
    [| for i = 0 to n + 1 do
      if i = 0 || i = n + 1 then
        yield (num 0.0)
      else
        yield (m.[i - 1] / v) |]

  and q = 1.0
  and k = 2.0
  and v = 0.75

  return
    [ResultSource.From("M", m, "M");
     ResultSource.From("C", c, "C")]
    |> ResultSet.create
}
```

Here we create two linear arrays M and C, where the first array consists of integrals. Similarly, we could use arrays in the discrete event simulation or agent-based model.

The simulation experiment shows the both arrays.

```
// File LinearArray/RunExperiment.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"
```

```
#r "../bin/Simulation.Aivika.Experiments.dll"
#r "../bin/Simulation.Aivika.Charting.dll"

#load "Model.fsx"

open System
open System.Web.UI

open Simulation.Aivika
open Simulation.Aivika.Results
open Simulation.Aivika.Experiments
open Simulation.Aivika.Experiments.Web
open Simulation.Aivika.Charting.Web

let experiment = Experiment ()

experiment.Specs <- Model.specs
experiment.RunCount <- 1

let m = ResultSet.findByName "M"
let c = ResultSet.findByName "C"

let provider1 = TimeSeriesProvider ()
let provider2 = TimeSeriesProvider ()

provider1.Series <- m
provider2.Series <- c

let providers =
    [ExperimentProvider.experimentSpecs;
      provider1 :> IExperimentProvider<HtmlTextWriter>;
      provider2 :> IExperimentProvider<HtmlTextWriter>]

experiment.RenderHtml (Model.model 51, providers)
    |> Async.RunSynchronously
```

The charts are provided in figures 8.3 and 8.4.

## 8.7 Example: Bouncing Ball

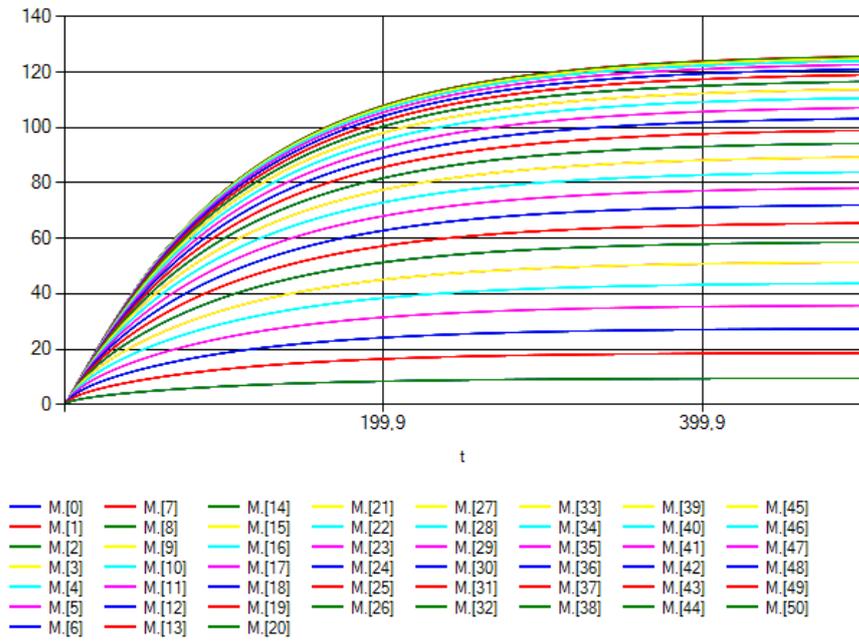


Figure 8.3: The array of integrals.

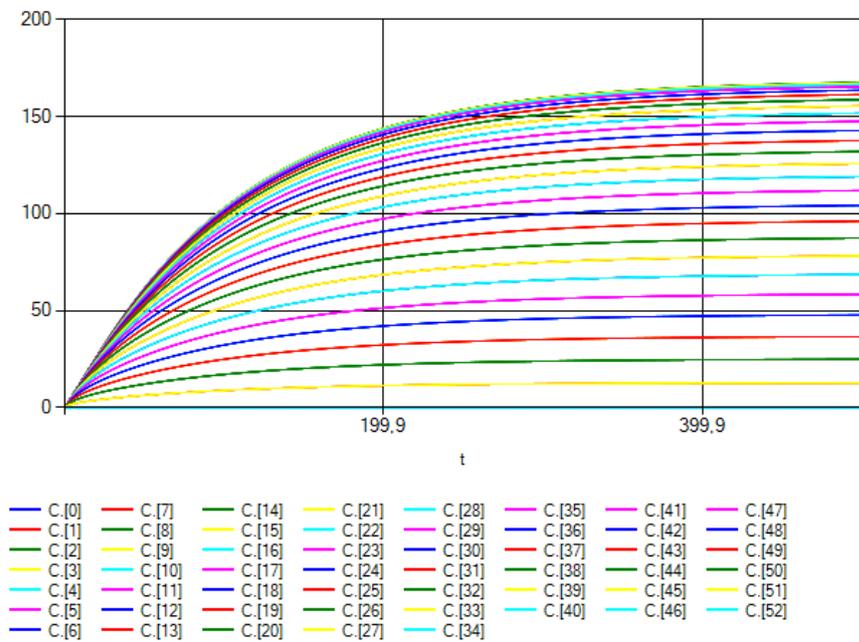


Figure 8.4: The second linear array.

## Chapter 9

# Agent-based Modeling

Aivika supports the agent-based modeling[16] on basic level and this support is well integrated with other simulation computations of the library.

### 9.1 Agents and States

An idea is to try to describe a model as a cooperative behavior of a relatively large number of small agents. The agents can have states and these states can be either active or inactive. We can assign to the state a handler that is actuated under the condition that the state remains active.

We create new agents within the `Simulation` computation, but define the states as dependent objects.

```
type [<Sealed>] Agent =
  ...

and [<AbstractClass>] AgentState =

  new : agent:Agent -> AgentState
  new : parent:AgentState -> AgentState
  ...

module Agent =
  val create : Simulation<Agent>
```

Only one of the states can be selected for each agent at the modeling time. All ancestor states remain active if they were active before, or they become active if they were deactivated. Other states are deactivated if they were active on the contrary.

```
module Agent =

  val selectedState : agent:Agent -> Eventive<AgentState option>
  val selectedStateChanged : agent:Agent -> Signal<AgentState option>
  val selectedStateChanged_ : agent:Agent -> Signal<unit>

module AgentState =
  val select : state:AgentState -> Eventive<unit>
```

The `selectedState` function returns the currently selected state or `None` if the agent was not yet initiated, but the `select` function allows selecting a new

state. The both functions return actions within the `Eventive` computation, which means that the state selection is always synchronized with the event queue.

We can assign the `Eventive` handlers to be performed when activating or deactivating the specified third state during such a selection. They are defined as the object methods.

```
type [<AbstractClass>] AgentState =

  abstract Activate : unit -> Eventive<unit>
  abstract Deactivate : unit -> Eventive<unit>

  default Activate: unit -> Eventive<unit>
  default Deactivate: unit -> Eventive<unit>

  ...
```

When the target state is selected, we can define the next target set if needed.

```
type [<AbstractClass>] AgentState =

  abstract Transit : unit -> Eventive<AgentState option>
  default Transit: unit -> Eventive<AgentState option>

  ...
```

What differs the agents from other simulation concepts is an ability to assign so called *timeout* and *timer handlers*. The timeout handler is an `Eventive` computation which is actuated in the specified time interval if the state remains active. The timer handler is similar, but only the handler is repeated while the state still remains active. Therefore, the timeout handler accepts the time as a pure value, while the timer handler recalculates the time interval within the `Eventive` computation after each successful actualization.

```
module AgentState =

  val addTimeout : Time -> Eventive<unit> -> AgentState
    -> Eventive<unit>

  val addTimer : Eventive<Time> -> Eventive<unit> -> AgentState
    -> Eventive<unit>
```

The implementation is quite simple. By the specified state handler, we create a wrapper handler which we pass in to the `Eventive.enqueue` function with the desired time of actuating. If the state becomes deactivated before the planned time comes then we invalidate the wrapper. After the wrapper is actuated by the event queue at the planned time, we do not call the corresponded state handler if the wrapper was invalidated earlier.

We use the `Eventive` computation to synchronize the agents with the event queue. It literally means that the agent-based modeling can be integrated with other simulation methods within one combined model.

## 9.2 Example: Agent-based Modeling

To illustrate the use of agents, let us take the Bass Diffusion model from the AnyLogic documentation [16].

The model describes a product diffusion process. Potential adopters of a product are influenced into buying the product by advertising and by word of mouth from adopters, those who have already purchased the new product. Adoption of a new product driven by word of mouth is likewise an epidemic. Potential adopters come into contact with adopters through social interactions. A fraction of these contacts results in the purchase of the new product. The advertising causes a constant fraction of the potential adopter population to adopt each time period.

The simulation model is as follows.

```
// File BassDiffusion/Model.fsx

#nowarn "40"

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"

open System
open System.Collections.Generic

open Simulation.Aivika
open Simulation.Aivika.Results

let n = 100 // the number of agents

let advertisingEffectiveness = 0.011
let contactRate = 100.0
let adoptionFraction = 0.015

let specs =
  { StartTime = 0.0; StopTime = 8.0; DT = 0.1;
    Method = RungeKutta4;
    GeneratorType = StrongGenerator }

type PersonContext =
  { PotentialAdopters: int ref;
    Adopters: int ref;
    Persons: List<Person> }

and Person (ctx: PersonContext, agent: Agent) =

  let rec potentialAdopter =
    { new AgentState (agent) with
      member x.Activate () = eventive {
        incr ctx.PotentialAdopters

        // create a timeout that will hold while the state is active

        let! t = Parameter.randomExponential
          (1.0 / advertisingEffectiveness)
          |> Parameter.lift

        do! potentialAdopter
          |> AgentState.addTimeout t
          (AgentState.select adopter)
      }
    }
  }
}
```

```

        member x.Deactivate () = eventive {
            decr ctx.PotentialAdopters
        }
    }
and adopter =
    { new AgentState (agent) with
        member x.Activate () = eventive {
            incr ctx.Adopters

            // create a timer that will hold while the state is active

            let t = Parameter.randomExponential
                (1.0 / contactRate)
                |> Parameter.lift

            let m =
                eventive {
                    let! i = Parameter.randomUniformInt
                        0 (ctx.Persons.Count - 1)
                        |> Parameter.lift

                    do! ctx.Persons.[i].Buy ()
                }

            do! adopter |> AgentState.addTimer t m
        }

        member x.Deactivate () = eventive {
            decr ctx.Adopters
        }
    }

member x.Agent = agent
member x.PotentialAdopter = potentialAdopter
member x.Adopter = adopter

member private x.Buy () = eventive {

    let! st = Agent.selectedState agent

    if st = Some potentialAdopter then

        let! x = Parameter.randomTrue adoptionFraction
            |> Parameter.lift

        if x then
            do! AgentState.select adopter
    }

member x.Init () = eventive {
    ctx.Persons.Add (x)
    do! AgentState.select potentialAdopter
}

let model: Simulation<ResultSet> = simulation {

    let ctx =
        { PotentialAdopters = ref 0;
          Adopters = ref 0;
          Persons = List<_> () }

    for i = 1 to n do

```

```

    let! agent = Agent.create
    let person = Person (ctx, agent)
    do! person.Init () |> Eventive.runInStartTime

    return
      [ResultSource.From ("potentialAdopters",
        ctx.PotentialAdopters, "Potential Adopters");
       ResultSource.From ("adopters",
        ctx.Adopters, "Adopters")]
      |> ResultSet.create
  }

```

The reader can notice that the model uses the same computations that we used for the ordinary differential equations and discrete event simulation.

Now we will define an experiment trying to plot the deviation chart for potential adopters and adopters. Unlike other cases, we will launch a hundred of simulation runs as this model requires more computations because of multiple agents.

```

// File BassDiffusion/RunExperiment.fsx

#I "../bin"
#r "../bin/Simulation.Aivika.dll"
#r "../bin/Simulation.Aivika.Results.dll"
#r "../bin/Simulation.Aivika.Experiments.dll"
#r "../bin/Simulation.Aivika.Charting.dll"

#load "Model.fsx"

open System
open System.Web.UI

open Simulation.Aivika
open Simulation.Aivika.Results
open Simulation.Aivika.Experiments
open Simulation.Aivika.Experiments.Web
open Simulation.Aivika.Charting.Web

let experiment = Experiment ()

experiment.Specs <- Model.specs
experiment.RunCount <- 100

let series =
  [ResultSet.findByName "potentialAdopters";
   ResultSet.findByName "adopters"]
  |> ResultTransform.concat

let providers =
  [ExperimentProvider.experimentSpecs;
   ExperimentProvider.description series;
   ExperimentProvider.deviationChart series]

experiment.RenderHtml (Model.model, providers)
  |> Async.RunSynchronously

```

The resulting chart is shown on figure 9.1

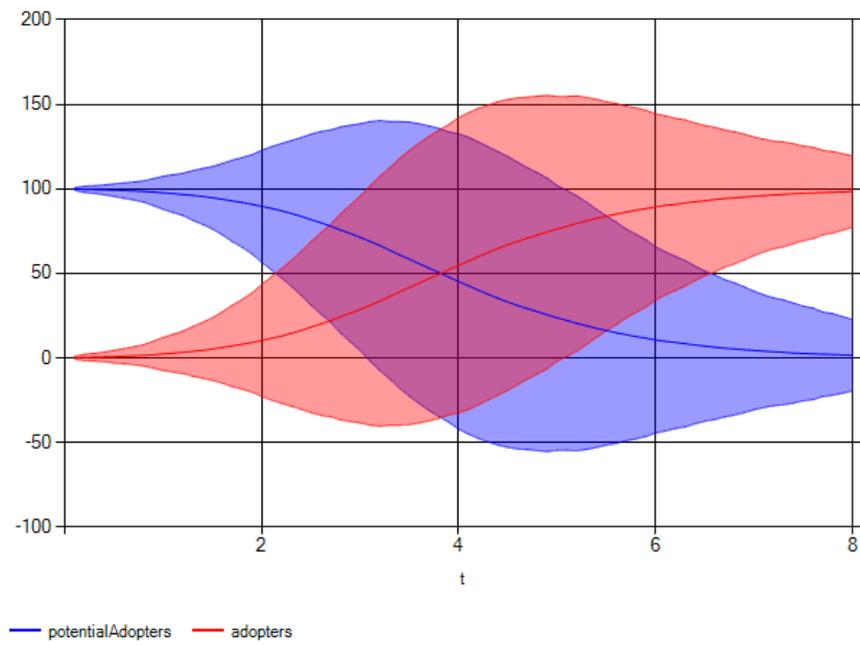


Figure 9.1: The potential adopters and adopters.

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