```
TBopt
% for the two firms (Firm A and Firm B) of the Simplified
Oligopolistic
% Optimal Influence model which also involves a single consumer C. The
% of the model parameters (Sopt, Xopt, Popt, Qopt, Fopt) are also
computed.
% Specifically, the internal model parameters whose optimal values are
% be determined during the grid searching optimization process are the
% follwing:
% (i):
        [TAopt, TBopt]
% (ii): [SAopt,SCopt,SBopt]
% (iii): [XAopt,XBopt]
% (iv): [PAopt, PBopt] (These are the optimal prices!!!)
% (v): [QAopt,QBopt]
% (vi): [FAopt FBopt FA_rev_opt FB_rev_opt FA_cost_opt FB_rev_opt]
% The grid searching process will be conducted within the 8-
dimensional
% space defined by the Cartesian product of the following external
model
% parameters:
         LA (direct influnce exerted by consumer C on Firm A).
% (i):
        LB (direct influnce exerted by consumer C on Firm B).
% (ii):
% (iii): PA (initial belief consumer C holds for product A).
         PB (initial belief consumer C holds for product B).
% (iv):
% (v):
         M (sensitivity coefficient).
% (vi): K (sensitivity coefficient).
% (vii): C (marginal cost).
% (viii): G (marginal influence cost or Gamma).
% IMPORTANT NOTE!!!
% Mind that the underlying continuous game may not have an equlibrium
point
% for any given configuration of the external parameters. Since all
% internal parameters to be determined accept positive values, the
value of
% (-1) will be utilized in order to indicate the absence of an
 equilibrium
% point for a particular configuration of the external parameters.
% Define ranges and corresponding increment step for each external
parameter
% of the Simplified Oligopolistic Optimal Influence model.
clc
clear
```

% This script file computes the optimal investment levels TAopt and

```
% Construct a cell array storing the names of model parameters.
ParamsNames = {'LA','LB','PA','PB','M','K','C','G'};
% Set the varying parameter index.
ParamIndex = 3;
% Parameter #1
LA MIN = 0.50;
LA MAX = 0.50;
LA\_STEP = 0.01;
% Parameter #2
LB_MIN = 0.50;
LB MAX = 0.50;
LB\_STEP = 0.01;
% Parameter #3
PA_MIN = 0.00;
PA\_MAX = 1.00;
PA\_STEP = 0.01;
% Parameter #4
PB MIN = 0.50;
PB_MAX = 0.50;
PB\_STEP = 0.01;
% Parameter #5
M MIN = 0.90;
M_MAX = 0.90;
M STEP = 0.01;
% Parameter #6
K_MIN = 0.10;
K_MAX = 0.10;
K STEP = 0.01;
% Parameter #7
C_MIN = 0.0;
C_MAX = 0.0;
C\_STEP = 0.01;
% Parameter #8 (G or Gamma!!!)
% G = 0.2; % Mind that G is the Gamma parameter defined elsewhere.
% Determine the minimum value for G.
alpha0 = (K_MAX*M_MAX - 2) / (M_MIN^2 - 4);
beta0 = (2*K_MAX - M_MIN) / (M_MIN^2 - 4);
Fmax = \max(alpha0^2, (3*beta0^2+2*alpha0*beta0));
Lmin = min(LA MIN,LB MIN);
G0 = Fmax / (Lmin^2);
Gfraction = 0.20; % This value should be set to 1.
G0 = Gfraction * G0;
G_MIN = G0;
G MAX = G0;
G\_STEP = 1.00;
% G STEP = (G MAX - G MIN)/1000;
% Set the corresponding ranges for each parameter of the model.
LA_RANGE = LA_MIN:LA_STEP:LA_MAX;
LB RANGE = LB MIN:LB STEP:LB MAX;
PA_RANGE = PA_MIN:PA_STEP:PA_MAX;
PB_RANGE = PB_MIN:PB_STEP:PB_MAX;
```

```
M_RANGE = M_MIN:M_STEP:M_MAX;
K RANGE = K MIN:K STEP:K MAX;
C_RANGE = C_MIN:C_STEP:C_MAX;
G RANGE = G MIN:G STEP:G MAX;
% Initialize matrices that store fundamental intermediate quantities
 of the
% underlying optimization problem within the simplified oligopolistic
% enviroment as well as the values of the external optimization
parameters
% for each step of the multi-dimensional grid searching process.
Topts = []; % Optimal investment levels.
Sopts = []; % Optimal limiting influences.
Xopts = []; % Optimal limiting beliefs.
Popts = []; % Optimal prices.
Qopts = []; % Optimal quantities.
Fopts = []; % Optimal profits.
FilterFlags = []; % Solution filtering flag which may be indicative of
 a non-existing solution.
DigitAccuracies = []; % Length of maximal sequence of identical digits
 within the obtained optimal solutions.
Params = []; % 8-tuple of the varying optimization parameters.
% Initialize internal solver parameters.
% Set the number of initial solution points.
N = 2000;
% Set the tolerance value for the minimizer. (Preferable value =
 1e-10)
Tolerance = 1e-15;
% Set the Fvals tolerance value for filtering the obtained solutions.
 (Preferable value = 1e-15)
FvalsTolerance = 1e-15;
% Set the derivative tolerance value for filering the obtained
 solutions. (Preferable value = 1e-08).
DerivativeTolerance = 1e-08;
% Set the maximum number of iterations to be conducted by the
 optimizer.
MaxIterations = 2000;
% Set the maximum number of function evaluations to be conducted by
the
% optimizer.
MaxFunctionEvaluations = 20000;
% Set the display flag parameter to 'off' or to 'iter'.
DisplayFlag = 'off';
% Set the minimum digits accuracy.
MinimumDigitsAccuracy = 7;
fprintf('Grid Evaluation Process in Progess...\n');
% Perform the actual grid searching.
for LA = LA RANGE
    for LB = LB_RANGE
        for PA = PA RANGE
            for PB = PB RANGE
                for M = M_RANGE
```

```
for K = K_RANGE
                        for C = C RANGE
                            for G = G RANGE
                                 % Additional parameters definition.
                                 alpha = (K*M - 2) / (M^2 - 4);
                                 beta = (2*K - M) / (M^2 - 4);
                                 gamma = C / (M - 2);
                                 gamma prime = gamma * (M - 1); %
 gamma_prime is the gamma' parameter.
                                 % Set the current params vector.
                                 params = [LA LB PA PB M K C G];
                                 Params = [Params; params];
 [TAopt,TBopt,FilterFlag,DigitsAccuracy] =
 SimplifiedOligopolisticOptimalInfluences(N,DisplayFlag,Tolerance,FvalsTolerance,D
                                 DigitsAccuracy = min(DigitsAccuracy);
                                 T = [TAopt TBopt];
                                 if(FilterFlag==0)
                                     [S,X,P,Q,F] =
RetrieveOptimalModelParameters(T,C,G,LA,LB,PA,PB,alpha,beta,gamma,gamma_prime);
                                 else
                                     S = -1*ones(1,3);
                                     X = -1*ones(1,2);
                                     P = -1*ones(1,2);
                                     Q = -1*ones(1,2);
                                     F = -1*ones(1,6);
                                 end
                                     % Print current solution.
                                param_value_string
= strcat([ParamsNames{ParamIndex} ' = '
num2str(params(ParamIndex),10)]);
                                 fprintf('%s TAopt =
%f TBopt = %f FilterFlag = %d DigitsAccuracy = %d
\n',param_value_string,TAopt,TBopt,FilterFlag,DigitsAccuracy);
                                 Topts = [Topts;T];
                                 FilterFlags =
 [FilterFlags; FilterFlag];
                                 DigitsAccuracies =
 [DigitAccuracies; DigitsAccuracy];
                                 Sopts = [Sopts;S];
                                 Xopts = [Xopts;X];
                                 Popts = [Popts;P];
                                 Qopts = [Qopts;Q];
                                 Fopts = [Fopts;F];
                            end
                        end
                    end
                end
            end
        end
    end
end
```

```
% Set parameters' indices and corresponding values for the parameters
 that
% remain constant.
ConstIndices = setdiff(1:length(params), ParamIndex);
ConstValues = params(ConstIndices);
% Perform plotting operations.
PlotParametersTuples(Topts, Sopts, Xopts, Popts, Qopts, Fopts, Params, ParamIndex, ConstIn
Grid Evaluation Process in Progess...
PA = 0 TAopt = 0.264821 TBopt = 0.259462 FilterFlag = 0 DigitsAccuracy
 = 8
PA = 0.01 TAopt = 0.265399 TBopt = 0.259103 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.02 TAopt = 0.265926 TBopt = 0.258749 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.03 TAopt = 0.266405 TBopt = 0.258399 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.04 TAopt = 0.266836 TBopt = 0.258052 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.05 TAopt = 0.267220 TBopt = 0.257708 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.06 TAopt = 0.267557 TBopt = 0.257368 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.07 \text{ TAopt} = 0.267848 \text{ TBopt} = 0.257031 \text{ FilterFlag} = 0
 DigitsAccuracy = 8
PA = 0.08 TAopt = 0.268095 TBopt = 0.256696 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.09 TAopt = 0.268298 TBopt = 0.256363 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.1 TAopt = 0.268457 TBopt = 0.256032 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.11 TAopt = 0.268573 TBopt = 0.255704 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.12 TAopt = 0.268647 TBopt = 0.255376 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.13 TAopt = 0.268678 TBopt = 0.255050 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.14 TAopt = 0.268669 TBopt = 0.254726 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.15 TAopt = 0.268618 TBopt = 0.254402 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.16 TAopt = 0.268526 TBopt = 0.254079 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.17 TAopt = 0.268394 TBopt = 0.253756 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.18 TAopt = 0.268223 TBopt = 0.253434 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.19 TAopt = 0.268011 TBopt = 0.253111 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.2 TAopt = 0.267760 TBopt = 0.252789 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.21 TAopt = 0.267470 TBopt = 0.252466 FilterFlag = 0
 DigitsAccuracy = 8
```

5

```
PA = 0.22 TAopt = 0.267140 TBopt = 0.252143 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.23 TAopt = 0.266772 TBopt = 0.251819 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.24 TAopt = 0.266365 TBopt = 0.251494 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.25 TAopt = 0.265919 TBopt = 0.251168 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.26 TAopt = 0.265435 TBopt = 0.250841 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.27 TAopt = 0.264913 TBopt = 0.250512 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.28 TAopt = 0.264352 TBopt = 0.250181 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.29 TAopt = 0.263752 TBopt = 0.249848 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.3 TAopt = 0.263114 TBopt = 0.249514 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.31 TAopt = 0.262438 TBopt = 0.249177 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.32 TAopt = 0.261723 TBopt = 0.248837 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.33 TAopt = 0.260970 TBopt = 0.248495 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.34 TAopt = 0.260177 TBopt = 0.248149 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.35 TAopt = 0.259346 TBopt = 0.247801 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.36 TAopt = 0.258476 TBopt = 0.247449 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.37 TAopt = 0.257566 TBopt = 0.247093 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.38 TAopt = 0.256617 TBopt = 0.246734 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.39 TAopt = 0.255628 TBopt = 0.246370 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.4 TAopt = 0.254599 TBopt = 0.246002 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.41 TAopt = 0.253529 TBopt = 0.245629 FilterFlag = 0
 DigitsAccuracy = 9
PA = 0.42 TAopt = 0.252419 TBopt = 0.245251 FilterFlag = 0
 DigitsAccuracy = 9
PA = 0.43 TAopt = 0.251268 TBopt = 0.244868 FilterFlag = 0
 DigitsAccuracy = 9
PA = 0.44 TAopt = 0.250075 TBopt = 0.244480 FilterFlag = 0
 DigitsAccuracy = 9
PA = 0.45 TAopt = 0.248840 TBopt = 0.244086 FilterFlag = 0
 DigitsAccuracy = 9
PA = 0.46 TAopt = 0.247563 TBopt = 0.243686 FilterFlag = 0
 DigitsAccuracy = 9
PA = 0.47 TAopt = 0.246242 TBopt = 0.243279 FilterFlag = 0
 DigitsAccuracy = 9
PA = 0.48 TAopt = 0.244878 TBopt = 0.242865 FilterFlag = 0
 DigitsAccuracy = 9
```

```
PA = 0.49 TAopt = 0.243470 TBopt = 0.242445 FilterFlag = 0
 DigitsAccuracy = 9
PA = 0.5 TAopt = 0.242017 TBopt = 0.242017 FilterFlag = 0
DigitsAccuracy = 9
PA = 0.51 TAopt = 0.240519 TBopt = 0.241581 FilterFlag = 0
 DigitsAccuracy = 9
PA = 0.52 TAopt = 0.238974 TBopt = 0.241137 FilterFlag = 0
 DigitsAccuracy = 9
PA = 0.53 TAopt = 0.237382 TBopt = 0.240685 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.54 TAopt = 0.235742 TBopt = 0.240223 FilterFlag = 0
DigitsAccuracy = 9
PA = 0.55 \text{ TAopt} = 0.234054 \text{ TBopt} = 0.239752 \text{ FilterFlag} = 0
DigitsAccuracy = 9
PA = 0.56 TAopt = 0.232315 TBopt = 0.239270 FilterFlag = 0
 DigitsAccuracy = 9
PA = 0.57 TAopt = 0.230526 TBopt = 0.238779 FilterFlag = 0
 DigitsAccuracy = 9
PA = 0.58 TAopt = 0.228685 TBopt = 0.238276 FilterFlag = 0
 DigitsAccuracy = 9
PA = 0.59 TAopt = 0.226791 TBopt = 0.237761 FilterFlag = 0
 DigitsAccuracy = 9
PA = 0.6 TAopt = 0.224842 TBopt = 0.237234 FilterFlag = 0
 DigitsAccuracy = 9
PA = 0.61 TAopt = 0.222838 TBopt = 0.236695 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.62 TAopt = 0.220776 TBopt = 0.236141 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.63 TAopt = 0.218656 TBopt = 0.235573 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.64 TAopt = 0.216475 TBopt = 0.234991 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.65 TAopt = 0.214232 TBopt = 0.234392 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.66 TAopt = 0.211925 TBopt = 0.233776 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.67 TAopt = 0.209552 TBopt = 0.233142 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.68 TAopt = 0.207110 TBopt = 0.232490 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.69 TAopt = 0.204598 TBopt = 0.231817 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.7 TAopt = 0.202012 TBopt = 0.231123 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.71 TAopt = 0.199350 TBopt = 0.230407 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.72 TAopt = 0.196609 TBopt = 0.229666 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.73 TAopt = 0.193785 TBopt = 0.228899 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.74 TAopt = 0.190875 TBopt = 0.228106 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.75 TAopt = 0.187874 TBopt = 0.227282 FilterFlag = 0
 DigitsAccuracy = 8
```

```
PA = 0.76 TAopt = 0.184779 TBopt = 0.226427 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.77 TAopt = 0.181585 TBopt = 0.225539 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.78 TAopt = 0.178285 TBopt = 0.224614 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.79 TAopt = 0.174875 TBopt = 0.223649 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.8 TAopt = 0.171348 TBopt = 0.222642 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.81 TAopt = 0.167696 TBopt = 0.221588 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.82 TAopt = 0.163911 TBopt = 0.220484 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.83 TAopt = 0.159984 TBopt = 0.219325 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.84 TAopt = 0.155903 TBopt = 0.218105 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.85 TAopt = 0.151657 TBopt = 0.216817 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.86 TAopt = 0.147231 TBopt = 0.215455 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.87 TAopt = 0.142608 TBopt = 0.214010 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.88 TAopt = 0.137769 TBopt = 0.212471 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.89 TAopt = 0.132691 TBopt = 0.210825 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.9 TAopt = 0.127343 TBopt = 0.209058 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.91 TAopt = 0.121692 TBopt = 0.207150 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.92 TAopt = 0.115693 TBopt = 0.205077 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.93 TAopt = 0.109289 TBopt = 0.202807 FilterFlag = 0
DigitsAccuracy = 8
PA = 0.94 TAopt = 0.102404 TBopt = 0.200298 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.95 TAopt = 0.094934 TBopt = 0.197493 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.96 TAopt = 0.086729 TBopt = 0.194304 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.97 TAopt = 0.077559 TBopt = 0.190600 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.98 TAopt = 0.067031 TBopt = 0.186153 FilterFlag = 0
 DigitsAccuracy = 8
PA = 0.99 TAopt = 0.054369 TBopt = 0.180505 FilterFlag = 0
DigitsAccuracy = 8
PA = 1 TAopt = 0.037491 TBopt = 0.172421 FilterFlag = 0 DigitsAccuracy
```

















