```
TBopt
% for the two firms (Firm A and Firm B) of the Simplified
Oligopolistic
% Optimal Influence model which also involves a single consumer C. The
% of the model parameters (Sopt, Xopt, Popt, Qopt, Fopt) are also
computed.
% Specifically, the internal model parameters whose optimal values are
% be determined during the grid searching optimization process are the
% follwing:
% (i):
        [TAopt, TBopt]
% (ii): [SAopt,SCopt,SBopt]
% (iii): [XAopt,XBopt]
% (iv): [PAopt, PBopt] (These are the optimal prices!!!)
% (v): [QAopt,QBopt]
% (vi): [FAopt FBopt FA_rev_opt FB_rev_opt FA_cost_opt FB_rev_opt]
% The grid searching process will be conducted within the 8-
dimensional
% space defined by the Cartesian product of the following external
model
% parameters:
         LA (direct influnce exerted by consumer C on Firm A).
% (i):
        LB (direct influnce exerted by consumer C on Firm B).
% (ii):
% (iii): PA (initial belief consumer C holds for product A).
         PB (initial belief consumer C holds for product B).
% (iv):
% (v):
         M (sensitivity coefficient).
% (vi): K (sensitivity coefficient).
% (vii): C (marginal cost).
% (viii): G (marginal influence cost or Gamma).
% IMPORTANT NOTE!!!
% Mind that the underlying continuous game may not have an equlibrium
point
% for any given configuration of the external parameters. Since all
% internal parameters to be determined accept positive values, the
value of
% (-1) will be utilized in order to indicate the absence of an
 equilibrium
% point for a particular configuration of the external parameters.
% Define ranges and corresponding increment step for each external
parameter
% of the Simplified Oligopolistic Optimal Influence model.
clc
clear
```

% This script file computes the optimal investment levels TAopt and

```
% Construct a cell array storing the names of model parameters.
ParamsNames = {'LA','LB','PA','PB','M','K','C','G'};
% Set the varying parameter index.
ParamIndex = 5;
% Parameter #1
LA MIN = 0.50;
LA MAX = 0.50;
LA\_STEP = 0.01;
% Parameter #2
LB_MIN = 0.50;
LB MAX = 0.50;
LB\_STEP = 0.01;
% Parameter #3
PA_MIN = 0.40;
PA\_MAX = 0.40;
PA\_STEP = 0.01;
% Parameter #4
PB MIN = 0.40;
PB_MAX = 0.40;
PB\_STEP = 0.01;
% Parameter #5
M MIN = 0.00;
M_MAX = 1.00;
M STEP = 0.01;
% Parameter #6
K_MIN = 0.50;
K_MAX = 0.50;
K STEP = 0.01;
% Parameter #7
C_MIN = 0.0;
C_MAX = 0.0;
C\_STEP = 0.01;
% Parameter #8 (G or Gamma!!!)
% G = 0.2; % Mind that G is the Gamma parameter defined elsewhere.
% Determine the minimum value for G.
alpha0 = (K_MAX*M_MAX - 2) / (M_MIN^2 - 4);
beta0 = (2*K_MAX - M_MIN) / (M_MIN^2 - 4);
Fmax = \max(alpha0^2, (3*beta0^2+2*alpha0*beta0));
Lmin = min(LA_MIN,LB_MIN);
G0 = Fmax / (Lmin^2);
Gfraction = 1.50; % This value should be set to 1.
G0 = Gfraction * G0;
G_MIN = G0;
G MAX = G0;
G\_STEP = 1.00;
% G STEP = (G MAX - G MIN)/1000;
% Set the corresponding ranges for each parameter of the model.
LA_RANGE = LA_MIN:LA_STEP:LA_MAX;
LB RANGE = LB MIN:LB STEP:LB MAX;
PA_RANGE = PA_MIN:PA_STEP:PA_MAX;
PB_RANGE = PB_MIN:PB_STEP:PB_MAX;
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M_RANGE = M_MIN:M_STEP:M_MAX;
K RANGE = K MIN:K STEP:K MAX;
C_RANGE = C_MIN:C_STEP:C_MAX;
G RANGE = G MIN:G STEP:G MAX;
% Initialize matrices that store fundamental intermediate quantities
 of the
% underlying optimization problem within the simplified oligopolistic
% enviroment as well as the values of the external optimization
parameters
% for each step of the multi-dimensional grid searching process.
Topts = []; % Optimal investment levels.
Sopts = []; % Optimal limiting influences.
Xopts = []; % Optimal limiting beliefs.
Popts = []; % Optimal prices.
Qopts = []; % Optimal quantities.
Fopts = []; % Optimal profits.
FilterFlags = []; % Solution filtering flag which may be indicative of
 a non-existing solution.
DigitAccuracies = []; % Length of maximal sequence of identical digits
 within the obtained optimal solutions.
Params = []; % 8-tuple of the varying optimization parameters.
% Initialize internal solver parameters.
% Set the number of initial solution points.
N = 2000;
% Set the tolerance value for the minimizer. (Preferable value =
 1e-10)
Tolerance = 1e-15;
% Set the Fvals tolerance value for filtering the obtained solutions.
 (Preferable value = 1e-15)
FvalsTolerance = 1e-15;
% Set the derivative tolerance value for filering the obtained
 solutions. (Preferable value = 1e-08).
DerivativeTolerance = 1e-08;
% Set the maximum number of iterations to be conducted by the
 optimizer.
MaxIterations = 2000;
% Set the maximum number of function evaluations to be conducted by
the
% optimizer.
MaxFunctionEvaluations = 20000;
% Set the display flag parameter to 'off' or to 'iter'.
DisplayFlag = 'off';
% Set the minimum digits accuracy.
MinimumDigitsAccuracy = 7;
fprintf('Grid Evaluation Process in Progess...\n');
% Perform the actual grid searching.
for LA = LA RANGE
    for LB = LB_RANGE
        for PA = PA RANGE
            for PB = PB RANGE
                for M = M_RANGE
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for K = K_RANGE
                        for C = C RANGE
                            for G = G RANGE
                                 % Additional parameters definition.
                                 alpha = (K*M - 2) / (M^2 - 4);
                                 beta = (2*K - M) / (M^2 - 4);
                                 gamma = C / (M - 2);
                                 gamma prime = gamma * (M - 1); %
 gamma_prime is the gamma' parameter.
                                 % Set the current params vector.
                                 params = [LA LB PA PB M K C G];
                                 Params = [Params; params];
 [TAopt,TBopt,FilterFlag,DigitsAccuracy] =
 SimplifiedOligopolisticOptimalInfluences(N,DisplayFlag,Tolerance,FvalsTolerance,D
                                 DigitsAccuracy = min(DigitsAccuracy);
                                 T = [TAopt TBopt];
                                 if(FilterFlag==0)
                                     [S,X,P,Q,F] =
RetrieveOptimalModelParameters(T,C,G,LA,LB,PA,PB,alpha,beta,gamma,gamma_prime);
                                 else
                                     S = -1*ones(1,3);
                                     X = -1*ones(1,2);
                                     P = -1*ones(1,2);
                                     Q = -1*ones(1,2);
                                     F = -1*ones(1,6);
                                 end
                                     % Print current solution.
                                param_value_string
= strcat([ParamsNames{ParamIndex} ' = '
num2str(params(ParamIndex),10)]);
                                 fprintf('%s TAopt =
%f TBopt = %f FilterFlag = %d DigitsAccuracy = %d
\n',param_value_string,TAopt,TBopt,FilterFlag,DigitsAccuracy);
                                 Topts = [Topts;T];
                                 FilterFlags =
 [FilterFlags; FilterFlag];
                                 DigitsAccuracies =
 [DigitAccuracies; DigitsAccuracy];
                                 Sopts = [Sopts;S];
                                 Xopts = [Xopts;X];
                                 Popts = [Popts;P];
                                 Qopts = [Qopts;Q];
                                 Fopts = [Fopts;F];
                            end
                        end
                    end
                end
            end
        end
    end
end
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% Set parameters' indices and corresponding values for the parameters
 that
% remain constant.
ConstIndices = setdiff(1:length(params), ParamIndex);
ConstValues = params(ConstIndices);
% Perform plotting operations.
PlotParametersTuples(Topts, Sopts, Xopts, Popts, Qopts, Fopts, Params, ParamIndex, ConstIn
Grid Evaluation Process in Progess...
M = 0 TAopt = 0.075856 TBopt = 0.075856 FilterFlag = 0 DigitsAccuracy
 = 8
M = 0.01 TAopt = 0.075889 TBopt = 0.075889 FilterFlag = 0
 DigitsAccuracy = 8
M = 0.02 \text{ TAopt} = 0.075925 \text{ TBopt} = 0.075925 \text{ FilterFlag} = 0
 DigitsAccuracy = 8
M = 0.03 TAopt = 0.075964 TBopt = 0.075964 FilterFlag = 0
 DigitsAccuracy = 8
M = 0.04 TAopt = 0.076007 TBopt = 0.076007 FilterFlag = 0
 DigitsAccuracy = 8
M = 0.05 \text{ TAopt} = 0.076053 \text{ TBopt} = 0.076053 \text{ FilterFlag} = 0
 DigitsAccuracy = 8
M = 0.06 TAopt = 0.076103 TBopt = 0.076103 FilterFlag = 0
 DigitsAccuracy = 8
M = 0.07 \text{ TAopt} = 0.076156 \text{ TBopt} = 0.076156 \text{ FilterFlag} = 0
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M = 0.08 \text{ TAopt} = 0.076213 \text{ TBopt} = 0.076213 \text{ FilterFlag} = 0
 DigitsAccuracy = 8
M = 0.09 TAopt = 0.076274 TBopt = 0.076274 FilterFlag = 0
 DigitsAccuracy = 8
M = 0.1 \text{ TAopt} = 0.076338 \text{ TBopt} = 0.076338 \text{ FilterFlag} = 0
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M = 0.11 TAopt = 0.076405 TBopt = 0.076405 FilterFlag = 0
 DigitsAccuracy = 8
M = 0.12 \text{ TAopt} = 0.076476 \text{ TBopt} = 0.076476 \text{ FilterFlag} = 0
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M = 0.13 \text{ TAopt} = 0.076551 \text{ TBopt} = 0.076551 \text{ FilterFlag} = 0
 DigitsAccuracy = 8
M = 0.14 \text{ TAopt} = 0.076629 \text{ TBopt} = 0.076629 \text{ FilterFlag} = 0
 DigitsAccuracy = 8
M = 0.15 \text{ TAopt} = 0.076711 \text{ TBopt} = 0.076711 \text{ FilterFlag} = 0
 DigitsAccuracy = 8
M = 0.16 \text{ TAopt} = 0.076797 \text{ TBopt} = 0.076797 \text{ FilterFlag} = 0
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M = 0.17 \text{ TAopt} = 0.076886 \text{ TBopt} = 0.076886 \text{ FilterFlag} = 0
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M = 0.18 \text{ TAopt} = 0.076979 \text{ TBopt} = 0.076979 \text{ FilterFlag} = 0
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M = 0.19 \text{ TAopt} = 0.077076 \text{ TBopt} = 0.077076 \text{ FilterFlag} = 0
 DigitsAccuracy = 8
M = 0.2 TAopt = 0.077177 TBopt = 0.077177 FilterFlag = 0
 DigitsAccuracy = 8
M = 0.21 TAopt = 0.077281 TBopt = 0.077281 FilterFlag = 0
 DigitsAccuracy = 8
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M = 0.22 TAopt = 0.077389 TBopt = 0.077389 FilterFlag = 0
 DigitsAccuracy = 8
M = 0.23 \text{ TAopt} = 0.077501 \text{ TBopt} = 0.077501 \text{ FilterFlag} = 0
 DigitsAccuracy = 9
M = 0.24 TAopt = 0.077617 TBopt = 0.077617 FilterFlag = 0
 DigitsAccuracy = 9
M = 0.25 \text{ TAopt} = 0.077737 \text{ TBopt} = 0.077737 \text{ FilterFlag} = 0
 DigitsAccuracy = 8
M = 0.26 \text{ TAopt} = 0.077860 \text{ TBopt} = 0.077860 \text{ FilterFlag} = 0
 DigitsAccuracy = 9
M = 0.27 TAopt = 0.077988 TBopt = 0.077988 FilterFlag = 0
 DigitsAccuracy = 8
M = 0.28 \text{ TAopt} = 0.078120 \text{ TBopt} = 0.078120 \text{ FilterFlag} = 0
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M = 0.29 \text{ TAopt} = 0.078255 \text{ TBopt} = 0.078255 \text{ FilterFlag} = 0
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M = 0.3 \text{ TAopt} = 0.078395 \text{ TBopt} = 0.078395 \text{ FilterFlag} = 0
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M = 0.31 \text{ TAopt} = 0.078539 \text{ TBopt} = 0.078539 \text{ FilterFlag} = 0
 DigitsAccuracy = 9
M = 0.32 \text{ TAopt} = 0.078687 \text{ TBopt} = 0.078687 \text{ FilterFlag} = 0
 DigitsAccuracy = 9
M = 0.33 \text{ TAopt} = 0.078840 \text{ TBopt} = 0.078840 \text{ FilterFlag} = 0
 DigitsAccuracy = 9
M = 0.34 \text{ TAopt} = 0.078996 \text{ TBopt} = 0.078996 \text{ FilterFlag} = 0
 DigitsAccuracy = 9
M = 0.35 \text{ TAopt} = 0.079157 \text{ TBopt} = 0.079157 \text{ FilterFlag} = 0
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M = 0.36 \text{ TAopt} = 0.079322 \text{ TBopt} = 0.079322 \text{ FilterFlag} = 0
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M = 0.42 \text{ TAopt} = 0.080407 \text{ TBopt} = 0.080407 \text{ FilterFlag} = 0
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M = 0.46 TAopt = 0.081224 TBopt = 0.081224 FilterFlag = 0
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M = 0.47 TAopt = 0.081441 TBopt = 0.081441 FilterFlag = 0
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M = 0.48 TAopt = 0.081662 TBopt = 0.081662 FilterFlag = 0
 DigitsAccuracy = 9
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M = 0.51 TAopt = 0.082358 TBopt = 0.082358 FilterFlag = 0
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M = 0.52 TAopt = 0.082600 TBopt = 0.082600 FilterFlag = 0
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M = 0.53 \text{ TAopt} = 0.082848 \text{ TBopt} = 0.082848 \text{ FilterFlag} = 0
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M = 0.54 \text{ TAopt} = 0.083101 \text{ TBopt} = 0.083101 \text{ FilterFlag} = 0
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M = 0.55 \text{ TAopt} = 0.083360 \text{ TBopt} = 0.083360 \text{ FilterFlag} = 0
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M = 0.56 \text{ TAopt} = 0.083625 \text{ TBopt} = 0.083625 \text{ FilterFlag} = 0
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M = 0.57 \text{ TAopt} = 0.083895 \text{ TBopt} = 0.083895 \text{ FilterFlag} = 0
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M = 0.59 \text{ TAopt} = 0.084453 \text{ TBopt} = 0.084453 \text{ FilterFlag} = 0
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M = 0.61 \text{ TAopt} = 0.085036 \text{ TBopt} = 0.085036 \text{ FilterFlag} = 0
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M = 0.62 TAopt = 0.085336 TBopt = 0.085336 FilterFlag = 0
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M = 0.63 \text{ TAopt} = 0.085643 \text{ TBopt} = 0.085643 \text{ FilterFlag} = 0
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M = 0.64 \text{ TAopt} = 0.085956 \text{ TBopt} = 0.085956 \text{ FilterFlag} = 0
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M = 0.65 \text{ TAopt} = 0.086276 \text{ TBopt} = 0.086276 \text{ FilterFlag} = 0
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M = 0.66 \text{ TAopt} = 0.086603 \text{ TBopt} = 0.086603 \text{ FilterFlag} = 0
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M = 0.67 \text{ TAopt} = 0.086936 \text{ TBopt} = 0.086936 \text{ FilterFlag} = 0
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M = 0.68 TAopt = 0.087277 TBopt = 0.087277 FilterFlag = 0
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M = 0.69 \text{ TAopt} = 0.087624 \text{ TBopt} = 0.087624 \text{ FilterFlag} = 0
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M = 0.7 \text{ TAopt} = 0.087979 \text{ TBopt} = 0.087979 \text{ FilterFlag} = 0
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M = 0.71 \text{ TAopt} = 0.088341 \text{ TBopt} = 0.088341 \text{ FilterFlag} = 0
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M = 0.72 \text{ TAopt} = 0.088710 \text{ TBopt} = 0.088710 \text{ FilterFlag} = 0
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M = 0.73 TAopt = 0.089088 TBopt = 0.089088 FilterFlag = 0
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M = 0.74 \text{ TAopt} = 0.089472 \text{ TBopt} = 0.089472 \text{ FilterFlag} = 0
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M = 0.75 \text{ TAopt} = 0.089865 \text{ TBopt} = 0.089865 \text{ FilterFlag} = 0
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M = 0.78 TAopt = 0.091093 TBopt = 0.091093 FilterFlag = 0
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M = 0.79 \text{ TAopt} = 0.091520 \text{ TBopt} = 0.091520 \text{ FilterFlag} = 0
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M = 0.8 TAopt = 0.091955 TBopt = 0.091955 FilterFlag = 0
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M = 0.81 \text{ TAopt} = 0.092399 \text{ TBopt} = 0.092399 \text{ FilterFlag} = 0
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M = 0.82 \text{ TAopt} = 0.092852 \text{ TBopt} = 0.092852 \text{ FilterFlag} = 0
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M = 0.83 \text{ TAopt} = 0.093315 \text{ TBopt} = 0.093315 \text{ FilterFlag} = 0
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M = 0.84 TAopt = 0.093787 TBopt = 0.093787 FilterFlag = 0
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M = 0.85 \text{ TAopt} = 0.094270 \text{ TBopt} = 0.094270 \text{ FilterFlag} = 0
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M = 0.86 \text{ TAopt} = 0.094762 \text{ TBopt} = 0.094762 \text{ FilterFlag} = 0
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M = 0.87 \text{ TAopt} = 0.095264 \text{ TBopt} = 0.095264 \text{ FilterFlag} = 0
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M = 0.88 \text{ TAopt} = 0.095777 \text{ TBopt} = 0.095777 \text{ FilterFlag} = 0
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M = 0.89 TAopt = 0.096301 TBopt = 0.096301 FilterFlag = 0
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M = 0.9 TAopt = 0.096836 TBopt = 0.096836 FilterFlag = 0
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M = 0.93 \text{ TAopt} = 0.098510 \text{ TBopt} = 0.098510 \text{ FilterFlag} = 0
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M = 0.94 TAopt = 0.099092 TBopt = 0.099092 FilterFlag = 0
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M = 0.96 TAopt = 0.100293 TBopt = 0.100293 FilterFlag = 0
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M = 0.98 \text{ TAopt} = 0.101547 \text{ TBopt} = 0.101547 \text{ FilterFlag} = 0
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M = 0.99 \text{ TAopt} = 0.102194 \text{ TBopt} = 0.102194 \text{ FilterFlag} = 0
 DigitsAccuracy = 9
M = 1 TAopt = 0.102856 TBopt = 0.102856 FilterFlag = 0 DigitsAccuracy
```

















