```
TBopt
% for the two firms (Firm A and Firm B) of the Simplified
Oligopolistic
% Optimal Influence model which also involves a single consumer C. The
% of the model parameters (Sopt, Xopt, Popt, Qopt, Fopt) are also
computed.
% Specifically, the internal model parameters whose optimal values are
% be determined during the grid searching optimization process are the
% follwing:
% (i):
        [TAopt, TBopt]
% (ii): [SAopt,SCopt,SBopt]
% (iii): [XAopt,XBopt]
% (iv): [PAopt, PBopt] (These are the optimal prices!!!)
% (v): [QAopt,QBopt]
% (vi): [FAopt FBopt FA_rev_opt FB_rev_opt FA_cost_opt FB_rev_opt]
% The grid searching process will be conducted within the 8-
dimensional
% space defined by the Cartesian product of the following external
model
% parameters:
         LA (direct influnce exerted by consumer C on Firm A).
% (i):
        LB (direct influnce exerted by consumer C on Firm B).
% (ii):
% (iii): PA (initial belief consumer C holds for product A).
         PB (initial belief consumer C holds for product B).
% (iv):
% (v):
         M (sensitivity coefficient).
% (vi): K (sensitivity coefficient).
% (vii): C (marginal cost).
% (viii): G (marginal influence cost or Gamma).
% IMPORTANT NOTE!!!
% Mind that the underlying continuous game may not have an equlibrium
point
% for any given configuration of the external parameters. Since all
% internal parameters to be determined accept positive values, the
value of
% (-1) will be utilized in order to indicate the absence of an
 equilibrium
% point for a particular configuration of the external parameters.
% Define ranges and corresponding increment step for each external
parameter
% of the Simplified Oligopolistic Optimal Influence model.
clc
clear
```

% This script file computes the optimal investment levels TAopt and

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% Construct a cell array storing the names of model parameters.
ParamsNames = {'LA','LB','PA','PB','M','K','C','G'};
% Set the varying parameter index.
ParamIndex = 1;
% Parameter #1
LA MIN = 0.01;
LA MAX = 1.00;
LA\_STEP = 0.01;
% Parameter #2
LB_MIN = 0.50;
LB MAX = 0.50;
LB\_STEP = 0.01;
% Parameter #3
PA_MIN = 0.80;
PA\_MAX = 0.80;
PA\_STEP = 0.01;
% Parameter #4
PB MIN = 0.80;
PB_MAX = 0.80;
PB\_STEP = 0.01;
% Parameter #5
M MIN = 0.50;
M_MAX = 0.50;
M STEP = 0.01;
% Parameter #6
K_MIN = 0.30;
K_MAX = 0.30;
K STEP = 0.01;
% Parameter #7
C_MIN = 0.0;
C_MAX = 0.0;
C\_STEP = 0.01;
% Parameter #8 (G or Gamma!!!)
% G = 0.2; % Mind that G is the Gamma parameter defined elsewhere.
% Determine the minimum value for G.
alpha0 = (K_MAX*M_MAX - 2) / (M_MIN^2 - 4);
beta0 = (2*K_MAX - M_MIN) / (M_MIN^2 - 4);
Fmax = \max(alpha0^2, (3*beta0^2+2*alpha0*beta0));
Lmin = min(LA MIN,LB MIN);
G0 = Fmax / (Lmin^2);
Gfraction = 0.0001; % This value should be set to 1.
G0 = Gfraction * G0;
G_MIN = G0;
G MAX = G0;
G\_STEP = 1.00;
% G STEP = (G MAX - G MIN)/1000;
% Set the corresponding ranges for each parameter of the model.
LA_RANGE = LA_MIN:LA_STEP:LA_MAX;
LB RANGE = LB MIN:LB STEP:LB MAX;
PA_RANGE = PA_MIN:PA_STEP:PA_MAX;
PB_RANGE = PB_MIN:PB_STEP:PB_MAX;
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M_RANGE = M_MIN:M_STEP:M_MAX;
K RANGE = K MIN:K STEP:K MAX;
C_RANGE = C_MIN:C_STEP:C_MAX;
G RANGE = G MIN:G STEP:G MAX;
% Initialize matrices that store fundamental intermediate quantities
 of the
% underlying optimization problem within the simplified oligopolistic
% enviroment as well as the values of the external optimization
parameters
% for each step of the multi-dimensional grid searching process.
Topts = []; % Optimal investment levels.
Sopts = []; % Optimal limiting influences.
Xopts = []; % Optimal limiting beliefs.
Popts = []; % Optimal prices.
Qopts = []; % Optimal quantities.
Fopts = []; % Optimal profits.
FilterFlags = []; % Solution filtering flag which may be indicative of
 a non-existing solution.
DigitAccuracies = []; % Length of maximal sequence of identical digits
 within the obtained optimal solutions.
Params = []; % 8-tuple of the varying optimization parameters.
% Initialize internal solver parameters.
% Set the number of initial solution points.
N = 2000;
% Set the tolerance value for the minimizer. (Preferable value =
 1e-10)
Tolerance = 1e-15;
% Set the Fvals tolerance value for filtering the obtained solutions.
 (Preferable value = 1e-15)
FvalsTolerance = 1e-15;
% Set the derivative tolerance value for filering the obtained
 solutions. (Preferable value = 1e-08).
DerivativeTolerance = 1e-08;
% Set the maximum number of iterations to be conducted by the
 optimizer.
MaxIterations = 2000;
% Set the maximum number of function evaluations to be conducted by
the
% optimizer.
MaxFunctionEvaluations = 20000;
% Set the display flag parameter to 'off' or to 'iter'.
DisplayFlag = 'off';
% Set the minimum digits accuracy.
MinimumDigitsAccuracy = 7;
fprintf('Grid Evaluation Process in Progess...\n');
% Perform the actual grid searching.
for LA = LA RANGE
    for LB = LB_RANGE
        for PA = PA RANGE
            for PB = PB RANGE
                for M = M_RANGE
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for K = K_RANGE
                        for C = C RANGE
                            for G = G RANGE
                                 % Additional parameters definition.
                                 alpha = (K*M - 2) / (M^2 - 4);
                                 beta = (2*K - M) / (M^2 - 4);
                                 gamma = C / (M - 2);
                                 gamma prime = gamma * (M - 1); %
 gamma_prime is the gamma' parameter.
                                 % Set the current params vector.
                                 params = [LA LB PA PB M K C G];
                                 Params = [Params; params];
 [TAopt,TBopt,FilterFlag,DigitsAccuracy] =
 SimplifiedOligopolisticOptimalInfluences(N,DisplayFlag,Tolerance,FvalsTolerance,D
                                 DigitsAccuracy = min(DigitsAccuracy);
                                 T = [TAopt TBopt];
                                 if(FilterFlag==0)
                                     [S,X,P,Q,F] =
RetrieveOptimalModelParameters(T,C,G,LA,LB,PA,PB,alpha,beta,gamma,gamma_prime);
                                 else
                                     S = -1*ones(1,3);
                                     X = -1*ones(1,2);
                                     P = -1*ones(1,2);
                                     Q = -1*ones(1,2);
                                     F = -1*ones(1,6);
                                 end
                                     % Print current solution.
                                param_value_string
= strcat([ParamsNames{ParamIndex} ' = '
num2str(params(ParamIndex),10)]);
                                 fprintf('%s TAopt =
%f TBopt = %f FilterFlag = %d DigitsAccuracy = %d
\n',param_value_string,TAopt,TBopt,FilterFlag,DigitsAccuracy);
                                 Topts = [Topts;T];
                                 FilterFlags =
 [FilterFlags; FilterFlag];
                                 DigitsAccuracies =
 [DigitAccuracies; DigitsAccuracy];
                                 Sopts = [Sopts;S];
                                 Xopts = [Xopts;X];
                                 Popts = [Popts;P];
                                 Qopts = [Qopts;Q];
                                 Fopts = [Fopts;F];
                            end
                        end
                    end
                end
            end
        end
    end
end
```

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% Set parameters' indices and corresponding values for the parameters
 that
% remain constant.
ConstIndices = setdiff(1:length(params), ParamIndex);
ConstValues = params(ConstIndices);
% Perform plotting operations.
PlotParametersTuples(Topts, Sopts, Xopts, Popts, Qopts, Fopts, Params, ParamIndex, ConstIn
Grid Evaluation Process in Progess...
LA = 0.01 TAopt = 0.127375 TBopt = 0.000735 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.02 TAopt = 0.158432 TBopt = 0.008307 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.03 TAopt = 0.180451 TBopt = 0.017536 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.04 TAopt = 0.198097 TBopt = 0.027414 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.05 TAopt = 0.212930 TBopt = 0.037525 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.06 TAopt = 0.225681 TBopt = 0.047661 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.07 TAopt = 0.236767 TBopt = 0.057705 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.08 \text{ TAopt} = 0.246454 \text{ TBopt} = 0.067583 \text{ FilterFlag} = 0
 DigitsAccuracy = 8
LA = 0.09 TAopt = 0.254930 TBopt = 0.077247 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.1 TAopt = 0.262336 TBopt = 0.086663 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.11 TAopt = 0.268783 TBopt = 0.095806 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.12 TAopt = 0.274363 TBopt = 0.104657 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.13 TAopt = 0.279153 TBopt = 0.113202 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.14 TAopt = 0.283222 TBopt = 0.121430 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.15 TAopt = 0.286631 TBopt = 0.129333 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.16 TAopt = 0.289433 TBopt = 0.136907 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.17 TAopt = 0.291679 TBopt = 0.144148 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.18 TAopt = 0.293415 TBopt = 0.151055 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.19 TAopt = 0.294685 TBopt = 0.157630 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.2 TAopt = 0.295527 TBopt = 0.163876 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.21 TAopt = 0.295979 TBopt = 0.169796 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.22 TAopt = 0.296076 TBopt = 0.175395 FilterFlag = 0
 DigitsAccuracy = 8
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LA = 0.23 TAopt = 0.295850 TBopt = 0.180682 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.24 TAopt = 0.295333 TBopt = 0.185663 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.25 TAopt = 0.294552 TBopt = 0.190346 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.26 TAopt = 0.293534 TBopt = 0.194743 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.27 TAopt = 0.292303 TBopt = 0.198861 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.28 TAopt = 0.290883 TBopt = 0.202712 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.29 TAopt = 0.289294 TBopt = 0.206306 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.3 TAopt = 0.287556 TBopt = 0.209654 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.31 TAopt = 0.285688 TBopt = 0.212767 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.32 TAopt = 0.283705 TBopt = 0.215657 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.33 TAopt = 0.281623 TBopt = 0.218333 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.34 TAopt = 0.279456 TBopt = 0.220808 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.35 TAopt = 0.277216 TBopt = 0.223091 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.36 TAopt = 0.274916 TBopt = 0.225193 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.37 TAopt = 0.272565 TBopt = 0.227125 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.38 TAopt = 0.270174 TBopt = 0.228895 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.39 TAopt = 0.267749 TBopt = 0.230514 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.4 TAopt = 0.265301 TBopt = 0.231991 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.41 TAopt = 0.262834 TBopt = 0.2333335 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.42 \ TAopt = 0.260356 \ TBopt = 0.234553 \ FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.43 TAopt = 0.257872 TBopt = 0.235654 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.44 TAopt = 0.255387 TBopt = 0.236645 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.45 TAopt = 0.252906 TBopt = 0.237534 FilterFlag = 0
 DigitsAccuracy = 9
LA = 0.46 TAopt = 0.250432 TBopt = 0.238328 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.47 TAopt = 0.247968 TBopt = 0.239032 FilterFlag = 0
 DigitsAccuracy = 9
LA = 0.48 TAopt = 0.245518 TBopt = 0.239653 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.49 TAopt = 0.243085 TBopt = 0.240198 FilterFlag = 0
 DigitsAccuracy = 9
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LA = 0.5 TAopt = 0.240670 TBopt = 0.240670 FilterFlag = 0
 DigitsAccuracy = 9
LA = 0.51 TAopt = 0.238276 TBopt = 0.241076 FilterFlag = 0
DigitsAccuracy = 9
LA = 0.52 TAopt = 0.235904 TBopt = 0.241419 FilterFlag = 0
 DigitsAccuracy = 9
LA = 0.53 TAopt = 0.233556 TBopt = 0.241706 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.54 TAopt = 0.231233 TBopt = 0.241939 FilterFlag = 0
 DigitsAccuracy = 9
LA = 0.55 TAopt = 0.228936 TBopt = 0.242123 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.56 TAopt = 0.226667 TBopt = 0.242261 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.57 TAopt = 0.224425 TBopt = 0.242358 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.58 TAopt = 0.222211 TBopt = 0.242415 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.59 TAopt = 0.220025 TBopt = 0.242436 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.6 TAopt = 0.217869 TBopt = 0.242425 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.61 TAopt = 0.215742 TBopt = 0.242383 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.62 TAopt = 0.213645 TBopt = 0.242313 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.63 TAopt = 0.211576 TBopt = 0.242217 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.64 TAopt = 0.209538 TBopt = 0.242098 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.65 TAopt = 0.207528 TBopt = 0.241958 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.66 TAopt = 0.205548 TBopt = 0.241797 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.67 TAopt = 0.203596 TBopt = 0.241619 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.68 TAopt = 0.201674 TBopt = 0.241424 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.69 TAopt = 0.199779 TBopt = 0.241214 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.7 TAopt = 0.197914 TBopt = 0.240991 FilterFlag = 0
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LA = 0.71 TAopt = 0.196076 TBopt = 0.240755 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.72 TAopt = 0.194265 TBopt = 0.240508 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.73 TAopt = 0.192482 TBopt = 0.240252 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.74 TAopt = 0.190726 TBopt = 0.239986 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.75 TAopt = 0.188996 TBopt = 0.239713 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.76 TAopt = 0.187293 TBopt = 0.239432 FilterFlag = 0
 DigitsAccuracy = 8
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LA = 0.77 TAopt = 0.185615 TBopt = 0.239145 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.78 TAopt = 0.183962 TBopt = 0.238853 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.79 TAopt = 0.182334 TBopt = 0.238556 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.8 TAopt = 0.180731 TBopt = 0.238254 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.81 TAopt = 0.179152 TBopt = 0.237949 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.82 TAopt = 0.177597 TBopt = 0.237642 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.83 \text{ TAopt} = 0.176065 \text{ TBopt} = 0.237331 \text{ FilterFlag} = 0
DigitsAccuracy = 8
LA = 0.84 TAopt = 0.174556 TBopt = 0.237019 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.85 TAopt = 0.173069 TBopt = 0.236705 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.86 TAopt = 0.171605 TBopt = 0.236390 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.87 TAopt = 0.170162 TBopt = 0.236075 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.88 TAopt = 0.168740 TBopt = 0.235759 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.89 TAopt = 0.167340 TBopt = 0.235443 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.9 TAopt = 0.165960 TBopt = 0.235127 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.91 TAopt = 0.164600 TBopt = 0.234812 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.92 TAopt = 0.163259 TBopt = 0.234497 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.93 TAopt = 0.161939 TBopt = 0.234183 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.94 \text{ TAopt} = 0.160637 \text{ TBopt} = 0.233871 \text{ FilterFlag} = 0
DigitsAccuracy = 8
LA = 0.95 TAopt = 0.159354 TBopt = 0.233560 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.96 TAopt = 0.158089 TBopt = 0.233250 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.97 TAopt = 0.156843 TBopt = 0.232943 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.98 TAopt = 0.155614 TBopt = 0.232637 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.99 TAopt = 0.154402 TBopt = 0.232333 FilterFlag = 0
 DigitsAccuracy = 8
LA = 1 TAopt = 0.153207 TBopt = 0.232031 FilterFlag = 0 DigitsAccuracy
 = 8
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