```
TBopt
% for the two firms (Firm A and Firm B) of the Simplified
Oligopolistic
% Optimal Influence model which also involves a single consumer C. The
% of the model parameters (Sopt, Xopt, Popt, Qopt, Fopt) are also
computed.
% Specifically, the internal model parameters whose optimal values are
% be determined during the grid searching optimization process are the
% follwing:
% (i):
        [TAopt, TBopt]
% (ii): [SAopt,SCopt,SBopt]
% (iii): [XAopt,XBopt]
% (iv): [PAopt, PBopt] (These are the optimal prices!!!)
% (v): [QAopt,QBopt]
% (vi): [FAopt FBopt FA_rev_opt FB_rev_opt FA_cost_opt FB_rev_opt]
% The grid searching process will be conducted within the 8-
dimensional
% space defined by the Cartesian product of the following external
model
% parameters:
         LA (direct influnce exerted by consumer C on Firm A).
% (i):
        LB (direct influnce exerted by consumer C on Firm B).
% (ii):
% (iii): PA (initial belief consumer C holds for product A).
         PB (initial belief consumer C holds for product B).
% (iv):
% (v):
         M (sensitivity coefficient).
% (vi): K (sensitivity coefficient).
% (vii): C (marginal cost).
% (viii): G (marginal influence cost or Gamma).
% IMPORTANT NOTE!!!
% Mind that the underlying continuous game may not have an equlibrium
point
% for any given configuration of the external parameters. Since all
% internal parameters to be determined accept positive values, the
value of
% (-1) will be utilized in order to indicate the absence of an
 equilibrium
% point for a particular configuration of the external parameters.
% Define ranges and corresponding increment step for each external
parameter
% of the Simplified Oligopolistic Optimal Influence model.
clc
clear
```

% This script file computes the optimal investment levels TAopt and

```
% Construct a cell array storing the names of model parameters.
ParamsNames = {'LA','LB','PA','PB','M','K','C','G'};
% Set the varying parameter index.
ParamIndex = 1;
% Parameter #1
LA MIN = 0.01;
LA MAX = 1.00;
LA\_STEP = 0.01;
% Parameter #2
LB_MIN = 0.50;
LB MAX = 0.50;
LB\_STEP = 0.01;
% Parameter #3
PA_MIN = 0.80;
PA\_MAX = 0.80;
PA\_STEP = 0.01;
% Parameter #4
PB MIN = 0.80;
PB_MAX = 0.80;
PB\_STEP = 0.01;
% Parameter #5
M MIN = 0.40;
M_MAX = 0.40;
M STEP = 0.01;
% Parameter #6
K_MIN = 0.10;
K_MAX = 0.10;
K STEP = 0.01;
% Parameter #7
C_MIN = 0.0;
C_MAX = 0.0;
C\_STEP = 0.01;
% Parameter #8 (G or Gamma!!!)
% G = 0.2; % Mind that G is the Gamma parameter defined elsewhere.
% Determine the minimum value for G.
alpha0 = (K_MAX*M_MAX - 2) / (M_MIN^2 - 4);
beta0 = (2*K_MAX - M_MIN) / (M_MIN^2 - 4);
Fmax = \max(alpha0^2, (3*beta0^2+2*alpha0*beta0));
Lmin = min(LA MIN,LB MIN);
G0 = Fmax / (Lmin^2);
Gfraction = 0.0001; % This value should be set to 1.
G0 = Gfraction * G0;
G_MIN = G0;
G MAX = G0;
G\_STEP = 1.00;
% G STEP = (G MAX - G MIN)/1000;
% Set the corresponding ranges for each parameter of the model.
LA_RANGE = LA_MIN:LA_STEP:LA_MAX;
LB RANGE = LB MIN:LB STEP:LB MAX;
PA_RANGE = PA_MIN:PA_STEP:PA_MAX;
PB_RANGE = PB_MIN:PB_STEP:PB_MAX;
```

```
M_RANGE = M_MIN:M_STEP:M_MAX;
K RANGE = K MIN:K STEP:K MAX;
C_RANGE = C_MIN:C_STEP:C_MAX;
G RANGE = G MIN:G STEP:G MAX;
% Initialize matrices that store fundamental intermediate quantities
 of the
% underlying optimization problem within the simplified oligopolistic
% enviroment as well as the values of the external optimization
parameters
% for each step of the multi-dimensional grid searching process.
Topts = []; % Optimal investment levels.
Sopts = []; % Optimal limiting influences.
Xopts = []; % Optimal limiting beliefs.
Popts = []; % Optimal prices.
Qopts = []; % Optimal quantities.
Fopts = []; % Optimal profits.
FilterFlags = []; % Solution filtering flag which may be indicative of
 a non-existing solution.
DigitAccuracies = []; % Length of maximal sequence of identical digits
 within the obtained optimal solutions.
Params = []; % 8-tuple of the varying optimization parameters.
% Initialize internal solver parameters.
% Set the number of initial solution points.
N = 2000;
% Set the tolerance value for the minimizer. (Preferable value =
 1e-10)
Tolerance = 1e-15;
% Set the Fvals tolerance value for filtering the obtained solutions.
 (Preferable value = 1e-15)
FvalsTolerance = 1e-15;
% Set the derivative tolerance value for filering the obtained
 solutions. (Preferable value = 1e-08).
DerivativeTolerance = 1e-08;
% Set the maximum number of iterations to be conducted by the
 optimizer.
MaxIterations = 2000;
% Set the maximum number of function evaluations to be conducted by
the
% optimizer.
MaxFunctionEvaluations = 20000;
% Set the display flag parameter to 'off' or to 'iter'.
DisplayFlag = 'off';
% Set the minimum digits accuracy.
MinimumDigitsAccuracy = 7;
fprintf('Grid Evaluation Process in Progess...\n');
% Perform the actual grid searching.
for LA = LA RANGE
    for LB = LB_RANGE
        for PA = PA RANGE
            for PB = PB RANGE
                for M = M_RANGE
```

```
for K = K_RANGE
                        for C = C RANGE
                            for G = G RANGE
                                 % Additional parameters definition.
                                 alpha = (K*M - 2) / (M^2 - 4);
                                 beta = (2*K - M) / (M^2 - 4);
                                 gamma = C / (M - 2);
                                 gamma prime = gamma * (M - 1); %
 gamma_prime is the gamma' parameter.
                                 % Set the current params vector.
                                 params = [LA LB PA PB M K C G];
                                 Params = [Params; params];
 [TAopt,TBopt,FilterFlag,DigitsAccuracy] =
 SimplifiedOligopolisticOptimalInfluences(N,DisplayFlag,Tolerance,FvalsTolerance,D
                                 DigitsAccuracy = min(DigitsAccuracy);
                                 T = [TAopt TBopt];
                                 if(FilterFlag==0)
                                     [S,X,P,Q,F] =
RetrieveOptimalModelParameters(T,C,G,LA,LB,PA,PB,alpha,beta,gamma,gamma_prime);
                                 else
                                     S = -1*ones(1,3);
                                     X = -1*ones(1,2);
                                     P = -1*ones(1,2);
                                     Q = -1*ones(1,2);
                                     F = -1*ones(1,6);
                                 end
                                     % Print current solution.
                                param_value_string
= strcat([ParamsNames{ParamIndex} ' = '
num2str(params(ParamIndex),10)]);
                                 fprintf('%s TAopt =
%f TBopt = %f FilterFlag = %d DigitsAccuracy = %d
\n',param_value_string,TAopt,TBopt,FilterFlag,DigitsAccuracy);
                                 Topts = [Topts;T];
                                 FilterFlags =
 [FilterFlags; FilterFlag];
                                 DigitsAccuracies =
 [DigitAccuracies; DigitsAccuracy];
                                 Sopts = [Sopts;S];
                                 Xopts = [Xopts;X];
                                 Popts = [Popts;P];
                                 Qopts = [Qopts;Q];
                                 Fopts = [Fopts;F];
                            end
                        end
                    end
                end
            end
        end
    end
end
```

```
% Set parameters' indices and corresponding values for the parameters
 that
% remain constant.
ConstIndices = setdiff(1:length(params), ParamIndex);
ConstValues = params(ConstIndices);
% Perform plotting operations.
PlotParametersTuples(Topts, Sopts, Xopts, Popts, Qopts, Fopts, Params, ParamIndex, ConstIn
Grid Evaluation Process in Progess...
LA = 0.01 TAopt = 0.109714 TBopt = 0.023527 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.02 TAopt = 0.140984 TBopt = 0.040360 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.03 TAopt = 0.163204 TBopt = 0.055078 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.04 TAopt = 0.180631 TBopt = 0.068361 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.05 TAopt = 0.194869 TBopt = 0.080526 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.06 TAopt = 0.206745 TBopt = 0.091756 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.07 TAopt = 0.216760 TBopt = 0.102170 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.08 \ TAopt = 0.225249 \ TBopt = 0.111853 \ FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.09 TAopt = 0.232454 TBopt = 0.120867 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.1 TAopt = 0.238559 TBopt = 0.129262 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.11 TAopt = 0.243709 TBopt = 0.137078 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.12 TAopt = 0.248021 TBopt = 0.144352 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.13 TAopt = 0.251595 TBopt = 0.151114 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.14 TAopt = 0.254514 TBopt = 0.157392 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.15 TAopt = 0.256849 TBopt = 0.163211 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.16 TAopt = 0.258665 TBopt = 0.168597 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.17 TAopt = 0.260017 TBopt = 0.173572 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.18 TAopt = 0.260955 TBopt = 0.178159 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.19 TAopt = 0.261522 TBopt = 0.182377 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.2 TAopt = 0.261757 TBopt = 0.186248 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.21 TAopt = 0.261698 TBopt = 0.189791 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.22 TAopt = 0.261375 TBopt = 0.193025 FilterFlag = 0
 DigitsAccuracy = 8
```

```
LA = 0.23 TAopt = 0.260817 TBopt = 0.195967 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.24 TAopt = 0.260051 TBopt = 0.198636 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.25 TAopt = 0.259101 TBopt = 0.201047 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.26 TAopt = 0.257987 TBopt = 0.203217 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.27 TAopt = 0.256729 TBopt = 0.205161 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.28 TAopt = 0.255344 TBopt = 0.206894 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.29 TAopt = 0.253848 TBopt = 0.208429 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.3 TAopt = 0.252255 TBopt = 0.209780 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.31 TAopt = 0.250579 TBopt = 0.210959 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.32 TAopt = 0.248830 TBopt = 0.211978 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.33 TAopt = 0.247020 TBopt = 0.212848 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.34 TAopt = 0.245157 TBopt = 0.213581 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.35 TAopt = 0.243250 TBopt = 0.214185 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.36 TAopt = 0.241307 TBopt = 0.214670 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.37 TAopt = 0.239334 TBopt = 0.215045 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.38 TAopt = 0.237337 TBopt = 0.215319 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.39 TAopt = 0.235323 TBopt = 0.215499 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.4 TAopt = 0.233296 TBopt = 0.215593 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.41 TAopt = 0.231260 TBopt = 0.215606 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.42 TAopt = 0.229219 TBopt = 0.215546 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.43 TAopt = 0.227176 TBopt = 0.215419 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.44 TAopt = 0.225136 TBopt = 0.215230 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.45 TAopt = 0.223100 TBopt = 0.214985 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.46 TAopt = 0.221071 TBopt = 0.214687 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.47 TAopt = 0.219051 TBopt = 0.214342 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.48 TAopt = 0.217042 TBopt = 0.213954 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.49 TAopt = 0.215046 TBopt = 0.213527 FilterFlag = 0
 DigitsAccuracy = 8
```

```
LA = 0.5 TAopt = 0.213064 TBopt = 0.213064 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.51 TAopt = 0.211097 TBopt = 0.212568 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.52 TAopt = 0.209147 TBopt = 0.212044 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.53 TAopt = 0.207214 TBopt = 0.211493 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.54 TAopt = 0.205299 TBopt = 0.210919 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.55 TAopt = 0.203404 TBopt = 0.210324 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.56 TAopt = 0.201527 TBopt = 0.209710 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.57 TAopt = 0.199671 TBopt = 0.209079 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.58 TAopt = 0.197836 TBopt = 0.208434 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.59 TAopt = 0.196021 TBopt = 0.207776 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.6 TAopt = 0.194227 TBopt = 0.207107 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.61 TAopt = 0.192454 TBopt = 0.206429 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.62 TAopt = 0.190703 TBopt = 0.205742 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.63 TAopt = 0.188973 TBopt = 0.205049 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.64 TAopt = 0.187264 TBopt = 0.204350 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.65 \text{ TAopt} = 0.185577 \text{ TBopt} = 0.203647 \text{ FilterFlag} = 0
 DigitsAccuracy = 8
LA = 0.66 TAopt = 0.183912 TBopt = 0.202941 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.67 TAopt = 0.182267 TBopt = 0.202232 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.68 TAopt = 0.180644 TBopt = 0.201522 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.69 TAopt = 0.179042 TBopt = 0.200811 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.7 TAopt = 0.177461 TBopt = 0.200101 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.71 TAopt = 0.175901 TBopt = 0.199391 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.72 TAopt = 0.174362 TBopt = 0.198684 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.73 TAopt = 0.172843 TBopt = 0.197978 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.74 TAopt = 0.171345 TBopt = 0.197274 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.75 TAopt = 0.169866 TBopt = 0.196574 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.76 TAopt = 0.168407 TBopt = 0.195878 FilterFlag = 0
 DigitsAccuracy = 8
```

```
LA = 0.77 TAopt = 0.166968 TBopt = 0.195185 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.78 TAopt = 0.165548 TBopt = 0.194497 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.79 TAopt = 0.164148 TBopt = 0.193813 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.8 TAopt = 0.162766 TBopt = 0.193135 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.81 TAopt = 0.161403 TBopt = 0.192461 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.82 TAopt = 0.160058 TBopt = 0.191793 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.83 TAopt = 0.158732 TBopt = 0.191131 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.84 TAopt = 0.157423 TBopt = 0.190475 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.85 TAopt = 0.156132 TBopt = 0.189825 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.86 TAopt = 0.154858 TBopt = 0.189181 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.87 TAopt = 0.153602 TBopt = 0.188543 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.88 TAopt = 0.152362 TBopt = 0.187912 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.89 TAopt = 0.151139 TBopt = 0.187287 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.9 TAopt = 0.149932 TBopt = 0.186669 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.91 TAopt = 0.148742 TBopt = 0.186058 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.92 TAopt = 0.147567 TBopt = 0.185454 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.93 TAopt = 0.146408 TBopt = 0.184856 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.94 TAopt = 0.145264 TBopt = 0.184265 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.95 TAopt = 0.144135 TBopt = 0.183682 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.96 TAopt = 0.143021 TBopt = 0.183105 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.97 TAopt = 0.141922 TBopt = 0.182535 FilterFlag = 0
 DigitsAccuracy = 8
LA = 0.98 TAopt = 0.140838 TBopt = 0.181971 FilterFlag = 0
DigitsAccuracy = 8
LA = 0.99 TAopt = 0.139767 TBopt = 0.181415 FilterFlag = 0
 DigitsAccuracy = 8
LA = 1 TAopt = 0.138711 TBopt = 0.180866 FilterFlag = 0 DigitsAccuracy
 = 8
```

















