# Time and Order

slide credits: H. Kopetz, P. Puschner

# Why do we need a notion of time?

- Event identification and generation
  - State before vs. after the event
- Event ordering
  - Causal order (e.g., *a* may only have caused *b* if *a* happened before *b*)
  - Temporal order (e.g., flight booking: who was first, *A* in VIE or *B* in LA?)
- Coordination – coordinated action at specified time
- Duration – measurement / control

  (e.g., X-ray: exposure time, video: gap between frames)
- Modeling of physical time
  - Comply to laws/dynamics of physics (*second*, physical time, real time)
  - Read input, produce output “at the right time” (e.g., control loops)

# Causal and Temporal Order

## Causal Order

- Deduced from “causal dependency” between events
- Reichenbach: *“If event e1 is a cause of event e2, then a small variation (a mark) in e1 is associated with a small variation in e2, whereas small variations in e2 are not necessarily associated with small variations in e1.”*
- Bunge: *“If a Cause happens, then (and only then) the Event is always produced by it.”*

## Temporal Order

- Deduced from timestamps of physical time

# Causal and Temporal Order (2)

Example

Two events

*e1* … someone enters a room
*e2* … the telephone starts to ring

Two cases

*e1* occurs after *e2* → causal dependency possible
*e2* occurs after *e1* → causal dependency unlikely

- Causal order implies temporal order
- Temporal order is necessary but not sufficient to establish causal order

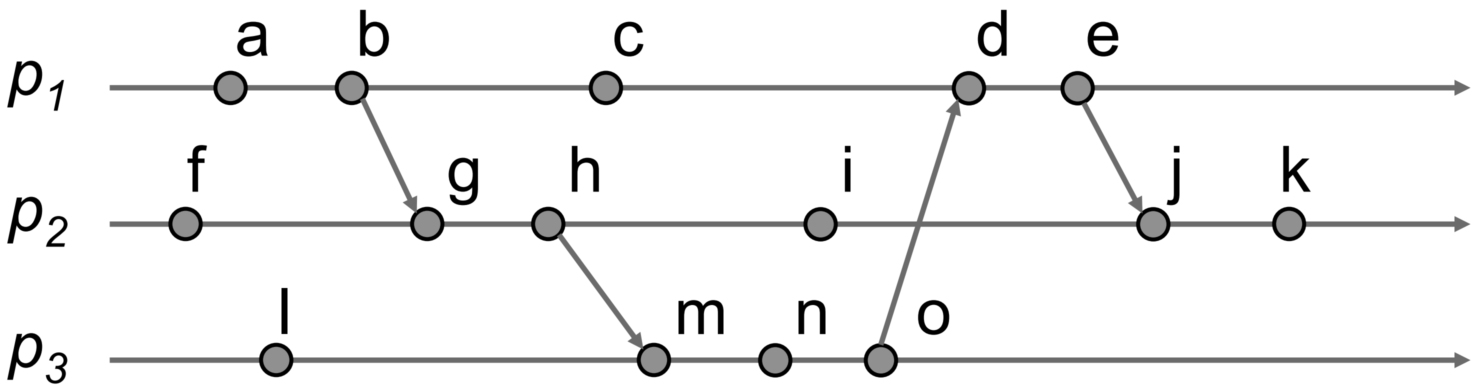
# Causal Order of Computer-generated Events

Partial order for computer-generated events

$a \rightarrow b$ … $a$ causes $b$ (happened before, causal dependence)

1. If $a$, $b$ … events within a sequential process and
   $a$ is executed before $b$
   then: $a \rightarrow b$
2. If $a$ … send event of a message by process $p_i$ and
   $b$ … receive event of the message by process $p_k$
   then: $a \rightarrow b$
3. $\rightarrow$ is transitive

# Causal Order of Computer-generated Events (2)



# Logical Clocks

- Represent information about causal dependency
- Do not use physical time
- Events are “time”-stamped using monotonically increasing counters

| | |
|---|---|
| Events | $a$, $b$ with $a \rightarrow b$ |
| Timestamps | $C(a)$, $C(b)$ |

- Desirable properties
  - $a \rightarrow b \quad \Rightarrow \quad C(a) < C(b)$ … monotonicity, consistency
  - $a \rightarrow b \quad \Leftrightarrow \quad C(a) < C(b)$ … strong consistency

# Lamport‘s Logical Clocks

- Logical clocks of processes $p_i$ represent the local views of global time
- Non-negative integer $C_i$ represents the local clock of $p_i$
- Clock update rules:

  R1: $p_i$ increments $C_i$ for each local event (e.g., event, send):

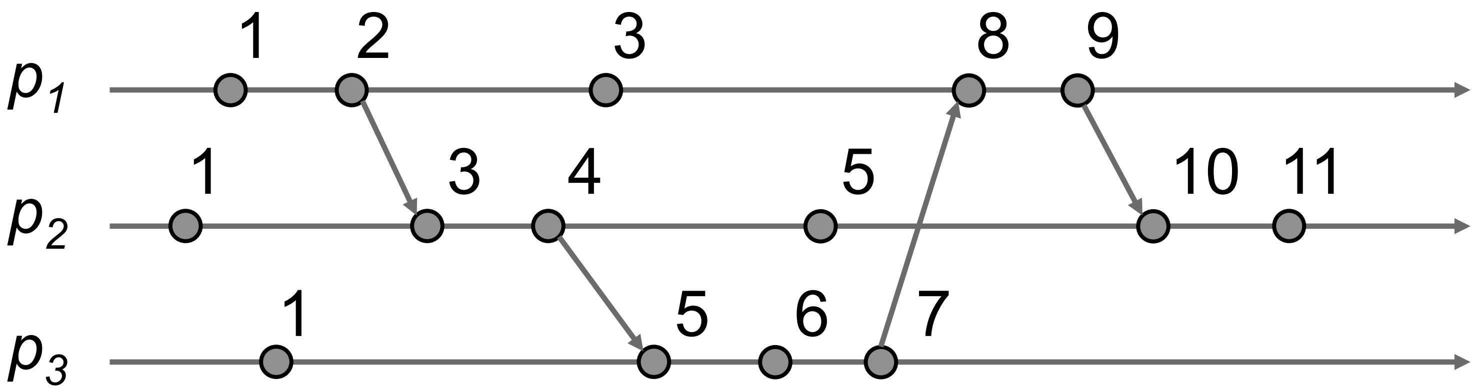
  $C_i = C_i + 1;$

  R2: each message transports the value of the sender’s clock, $C_{msg}$

  R3: when $p_i$ receives a message with timestamp $Cmsg$:

  $C_i = \max(C_i, C_{msg}); C_i = C_i + 1;$

# Lamport‘s Logical Clocks (2)




- Consistency: $a \rightarrow b \;\Rightarrow\; C(a) < C(b)$
- Total ordering: timestamps $(t, i)$: $t$ … time, $i$ … process number
  total order relation $\prec$ on events $a$, $b$ with timestamps $(t, i)$, $(u, j)$

$$a \prec b \;\Leftrightarrow\; (t < u \text{ or } (t = u \text{ and } i < j))$$

- No strong consistency: $C(a) < C(b) \not\Rightarrow a \rightarrow b$

# Vector Time (Fidge, Mattern, Schmuck)

- $n$-dimensional vector $V_i[1..n]$ at $p_i$ with

  $V_i[i]$ … value of local logical clock of $p_i$

  $V_i[k]$ … $p_i$'s knowledge about local time at $p_k$
- Clock update rules:

  R1: $p_i$ updates $V_i[i]$ for each local event (e.g., event, send):

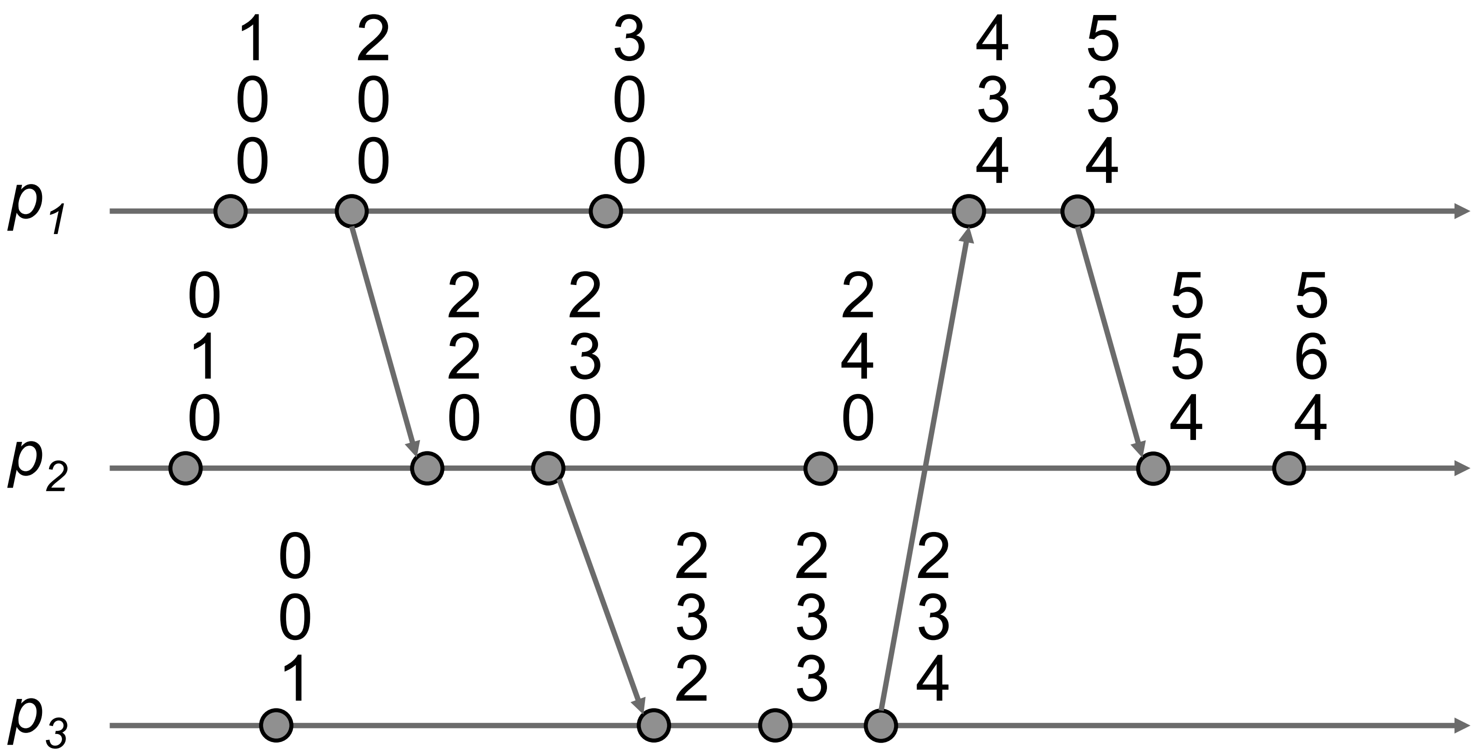
  $V_i[i] = V_i[i] + 1;$

  R2: each message transports sender's clock values

  R3: when $p_i$ receives a message with timestamp $V$:

  $1 \leq k \leq n$: $V_i[k] = \max(V_i[k], V[k]);$
  $V_i[i] = V_i[i] + 1;$

# Vector Time (2)



# Vector Time (3)

Event relations

event $a$ on $p_i$ with timestamp $Va$
event $b$ on $p_k$ with timestamp $Vb$

- $a \rightarrow b \;\Leftrightarrow\; \forall i\!: Va[\,i\,] \leq Vb[\,i\,]$ and $\exists i\!: Va[\,i\,] < Vb[\,i\,]$
- $a \parallel b \;\Leftrightarrow\; \exists i,k\!: \mathrm{Va}[\,i\,] > Vb[\,i\,]$ and $Va[\,k\,] < Vb[\,k\,]$

- Vector clocks are strongly consistent:
  By examining the timestamps of two events *a* and *b* one can determine if *a* and *b* are causally related

# Temporal Order

Continuum of real time modeled by

- a directed timeline, consisting of
- an infinite set $\{T\}$ of instants with

  i. $\{T\}$ is an ordered set,
  i.e., for any two instants $p$ and $q$ either: $p$ and $q$ are simultaneous, $p$ precedes $q$, or $q$ precedes $p$

  ii. $\{T\}$ is a dense set,
  for any instants $p \neq r$ there is at least one $q$ between $p$ and $r$


Temporal order: total order of instants on the timeline

# Events and Durations

Event … is happening at an instant of time

Duration … section of the timeline

Note

- An event does not have a duration
- If two events occur at the identical instant they are called simultaneous
- Events are partially ordered
  In a distributed system, a total order can be established by using process numbers (see Lamport's order)

# Physical Clocks

## Clock

- Counter plus oscillator
- Microticks are generated by periodical increments of the counter, following some law of physics

## Reference clock (*z*)

- Perfect clock of an external observer
- Duration between two ticks is much smaller than duration of any interval to be observed with our clocks (e.g., $10^{-15}$ sec)

Granularity of a clock *c*: nominal number of microticks of *z* between any consecutive microticks of *c*

$$g^c = z(microtick^c_{i+1}) - z(microtick^c_i)$$

# Physical Clocks (2)

## Timestamp

- The timestamp of an event is the state of the clock immediately after the occurrence of the event
- Notation: *clock*(*event*), e.g., *z*(*event*)
- Digitalization error of timestamps due to clock granularity

# Clock Drift

Real clocks deviate from the reference clock

Clock drift

$$drift^k_i = \frac{z(microtick^k_{i+1}) - z(microtick^k_i)}{g^k}$$

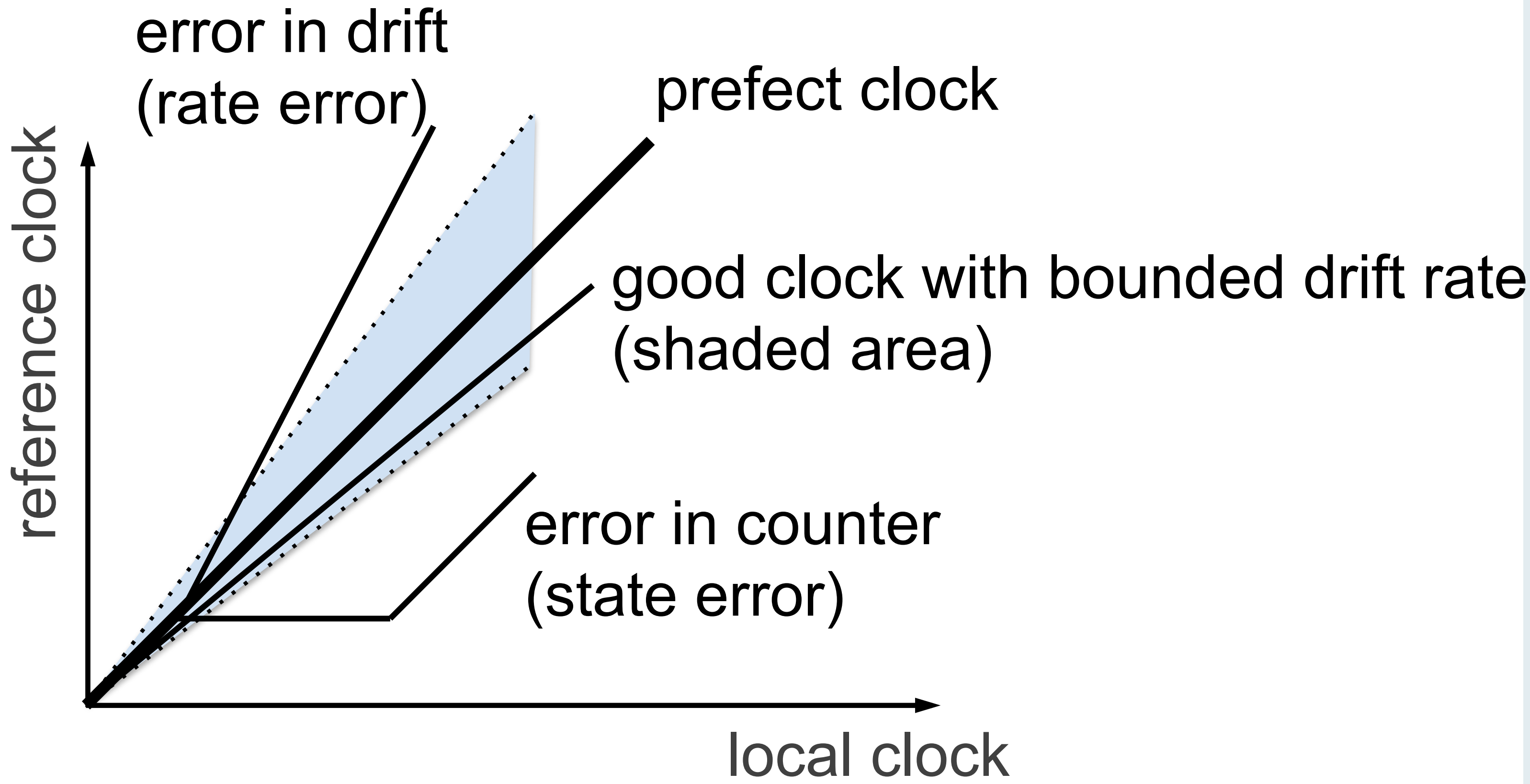
Drift rate

$$\rho^k_i = \left| \frac{z(microtick^k_{i+1}) - z(microtick^k_i)}{g^k} - 1 \right|$$

Drift rate of perfect clock: 0

Drift rate of real clocks: $10^{-8} \ldots 10^{-2}$

# Failure Modes of Clocks



# Precision

Offset between two clocks *j* and *k* at tick *i*

$$offset^{jk}_i = \left| z(microtick^j_i) - z(microtick^k_i) \right|$$

Precision of an ensemble of clocks {1,…,*n*} at macrotick *i*

$$\Pi_i = \max_{j,\,k} \{ offset^{jk}_i \}$$

Internal clock synchronization: mutual resynchronization of an ensemble of clocks in order to maintain a bounded precision

# Accuracy

Offset between clock *k* and the reference clock *z* at tick *i*

$$offset^{k,z(k)}_i = \left| z(microtick^k_i) - z(microtick^{z(k)}_i) \right|$$

Accuracy denotes the maximum offset of a given clock from the reference clock during a time interval of interest

External clock synchronization: resynchronization of a clock with the reference clock

If all clocks of an ensemble are externally synchronized with accuracy $A$, then the ensemble is internally synchronized with a precision $\Pi \le 2A$.

# Time Standards

## International Atomic Time (TAI)

- physical time standard
- defines the second as the duration of  9 192 631 770 periods of the radiation of a specified transition of the Cesium 133 atom.
- chronoscopic timescale, i.e., a timescale without discontinuities.
- defines the epoch, the origin of time measurement, as Jan. 1, 1958 at 00:00:00 hours

# Time Standards (2)

## Universal Time Coordinated (UTC)

- astronomical time standard, basis for the time on the "wall clock".
- duration of the second conforms to the TAI standard
- number of seconds in an hour occasionally modified by inserting a leap second into UTC to maintain synchrony between the wall-clock time and the astronomical phenomena, like day and night.

# Adjusting Time can be Tricky ...

Insertion of a leap second at midnight, New Year's Eve 1995, caused a glitch that affected the time signal for the AP radio broadcast network for hours.

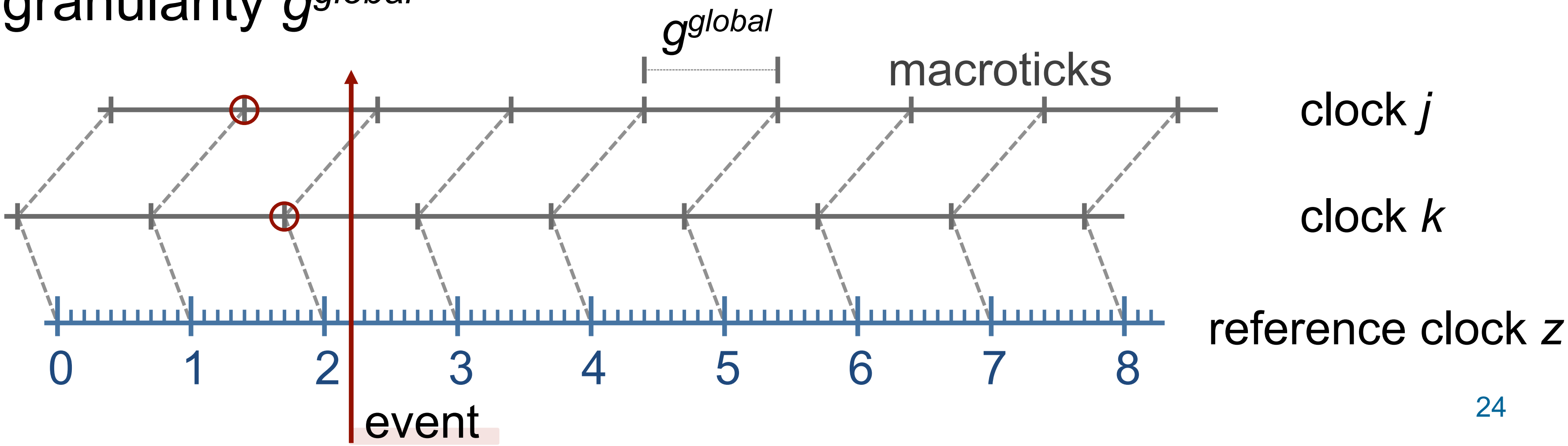
Sequence of events:

1. The day increments to January 1, 1996, 00:00:00.
2. The clock is set back one second, to 23:59:59.
3. The clock continues running.
4. The day changes again. Suddenly it is January 2, 00:00:00.

# Global Time

In a distributed system we need a global notion of time to generate event timestamps ➯ “Global Time”

- Global time is an abstract notion, real clocks are not prefect
- Local clocks of nodes approximate global time
- Macroticks form the local representation of global time with granularity $g^{global}$



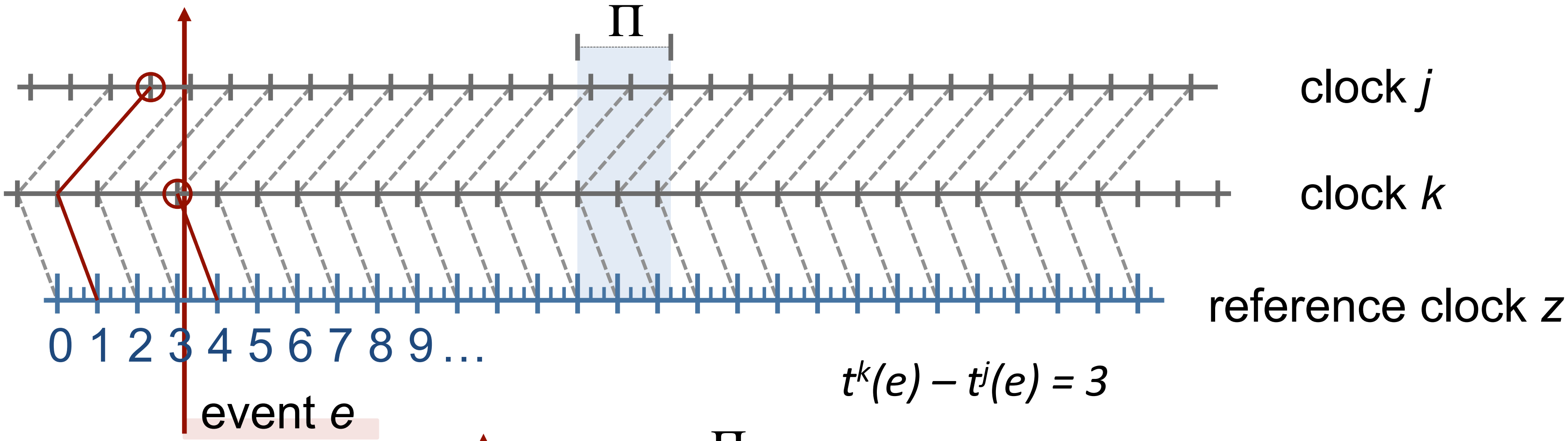
# Absence of a Global Timebase

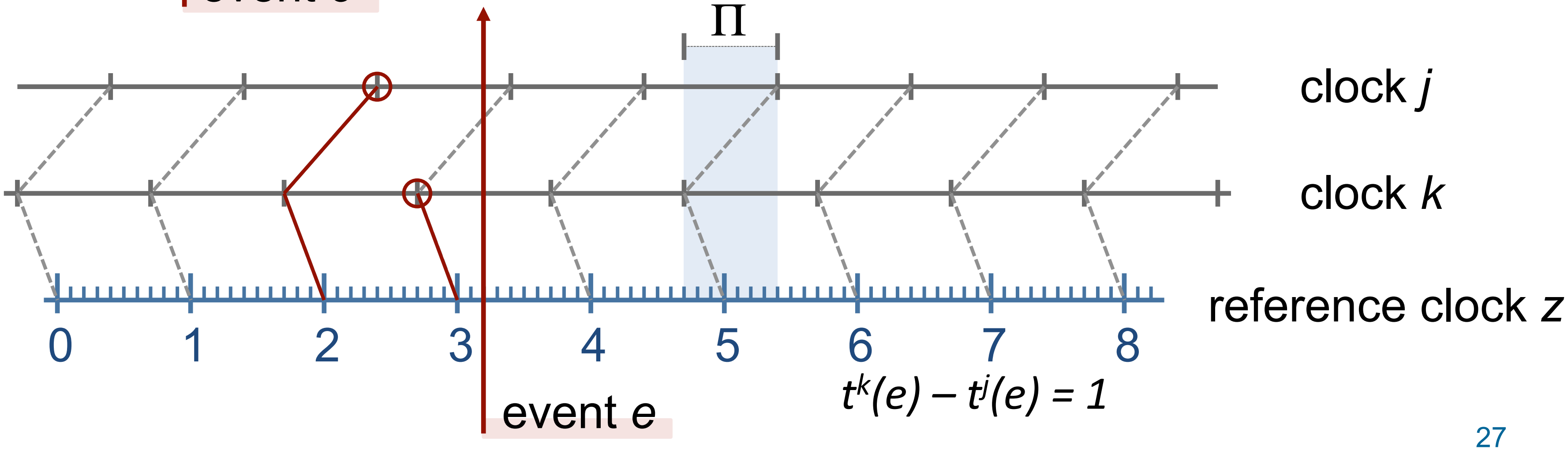
- *n* independent local time references
  ➪ only timestamps from the same clock can be related.
- Interval measurements between events observed at different nodes are limited by the end-to-end communication jitter.
- Delay jitter of communication system determines the jitter in non-local control loops
  ➪ unacceptable for many real-time control applications.
- No knowledge of precise point in time of measurement of process variables
  ➪ state estimation is very difficult

# Requirements for a Global Timebase

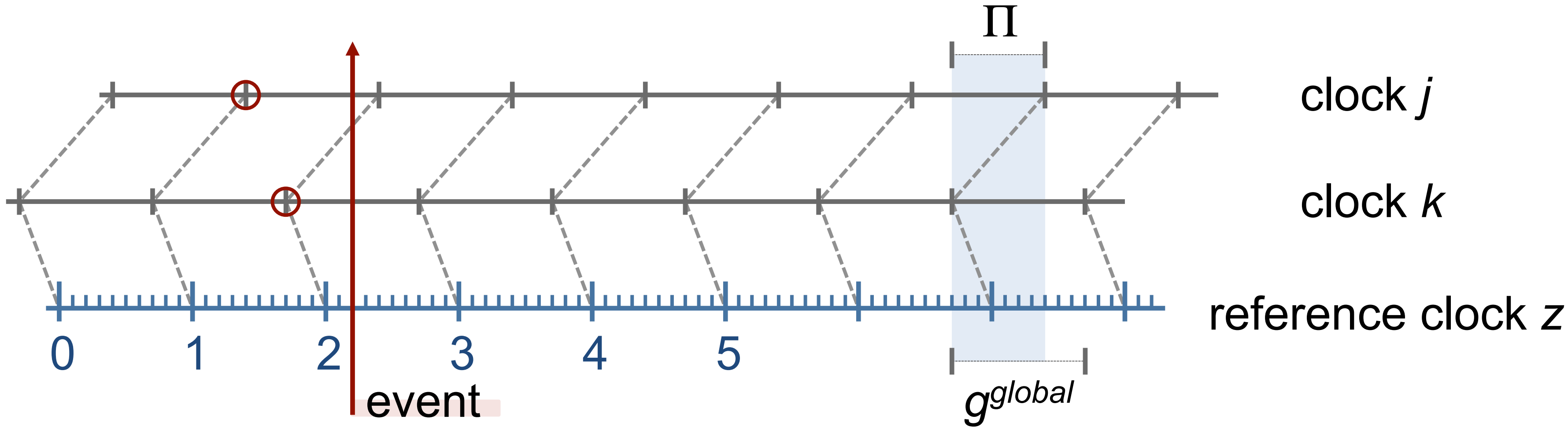
- Chronoscopic behaviour
  (i.e., no discontinuities, even at points of resynchronization)
- Known precision $\Pi$
- High dependability
- Metric of physical second

# Choosing the Right Granularity



# Reasonableness Condition



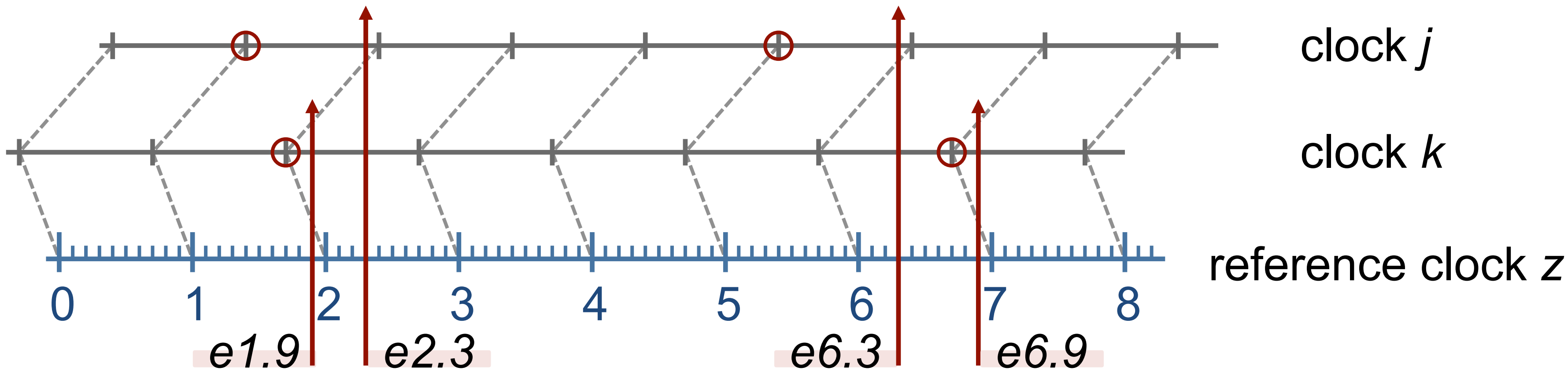

Global time *t* is reasonable if for all local implementations:

$$g^{global} > \Pi$$

The reasonableness condition ensures that:

- the synchronization error is less than one macrogranule
- for any event *e*: $| t^j(e) - t^k(e) | \leq 1$

# Timestamps and Temporal Order




$z(e1.9) < z(e2.3)$

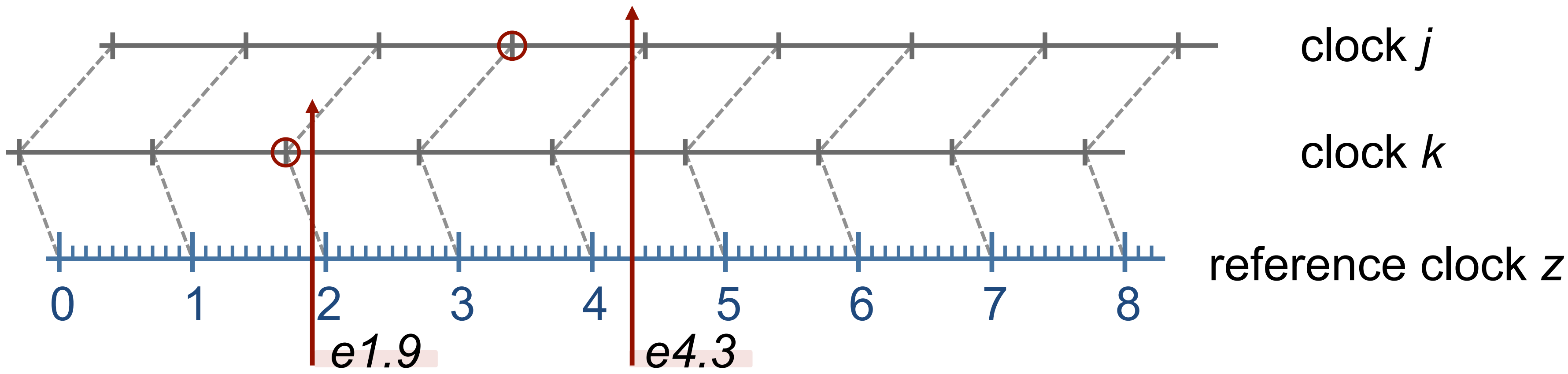
$t^k(e1.9) > t^j(e2.3)$

$z(e6.9) - z(e6.3) = 0.6$

$t^k(e6.9) - t^j(e6.3) = 2$

To reconstruct the temporal order of two events, the (global) timestamps of the events have to differ by at least two ticks.

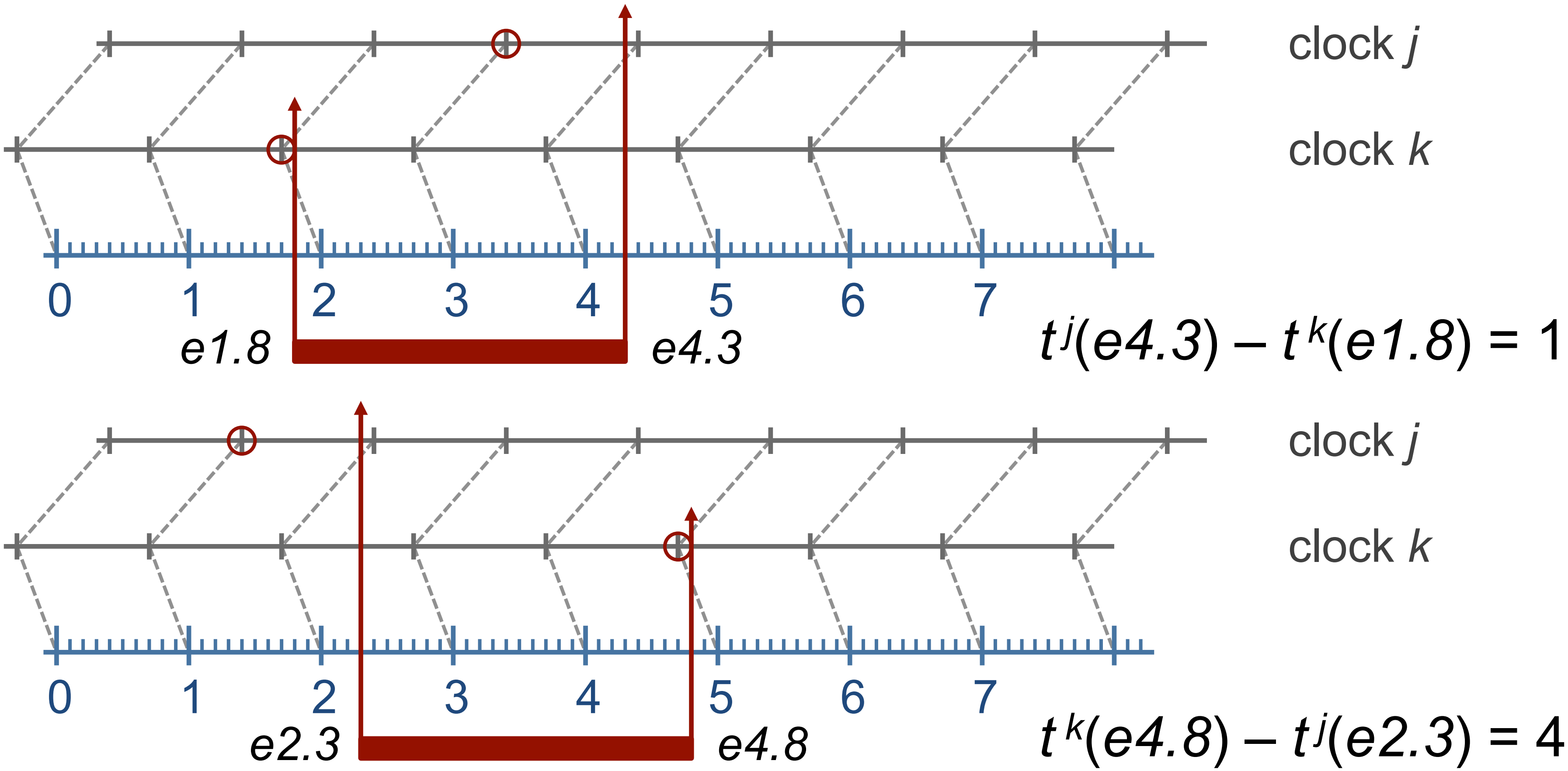
# Timestamps and Temporal Order (2)




$$z(e4.3) - z(e1.9) = 2.4$$
$$t^j(e4.3) - t^k(e1.9) = 1$$

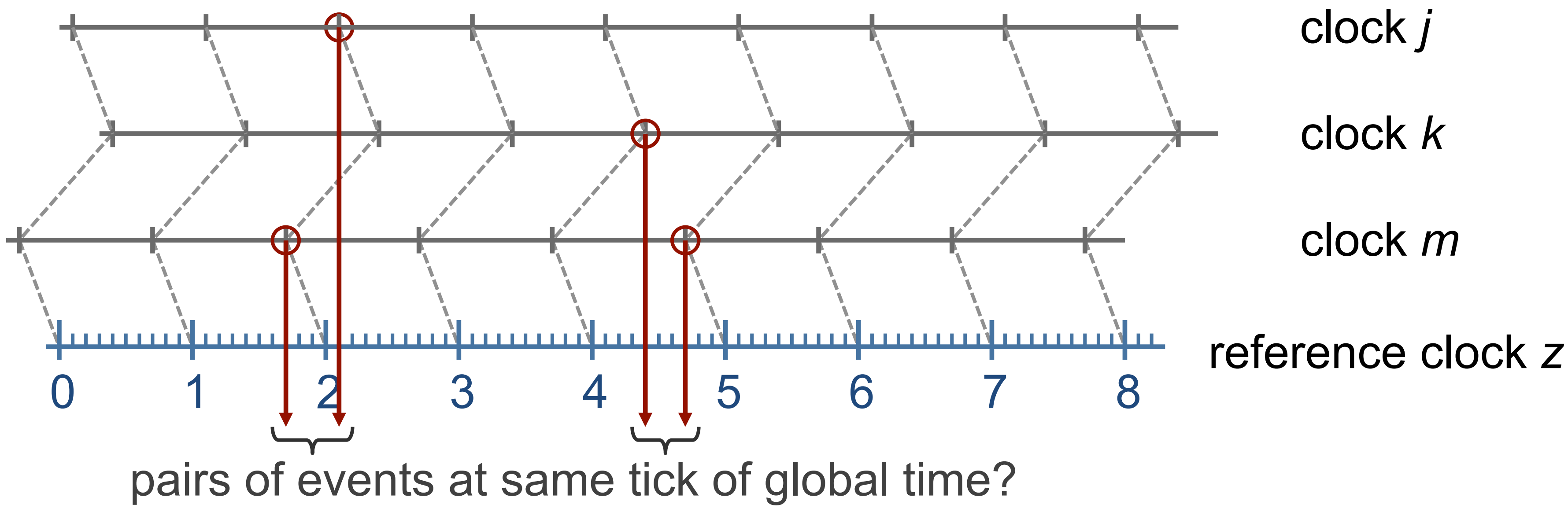
A time distance of $2g^{global}$ between two events is not sufficient to determine their temporal order (if $t^j(a) - t^k(b) = 1$) .

# Measurement of Durations




Real duration: $d_{obs} - 2g^{global} < d^z_{true} < d_{obs} + 2g^{global}$

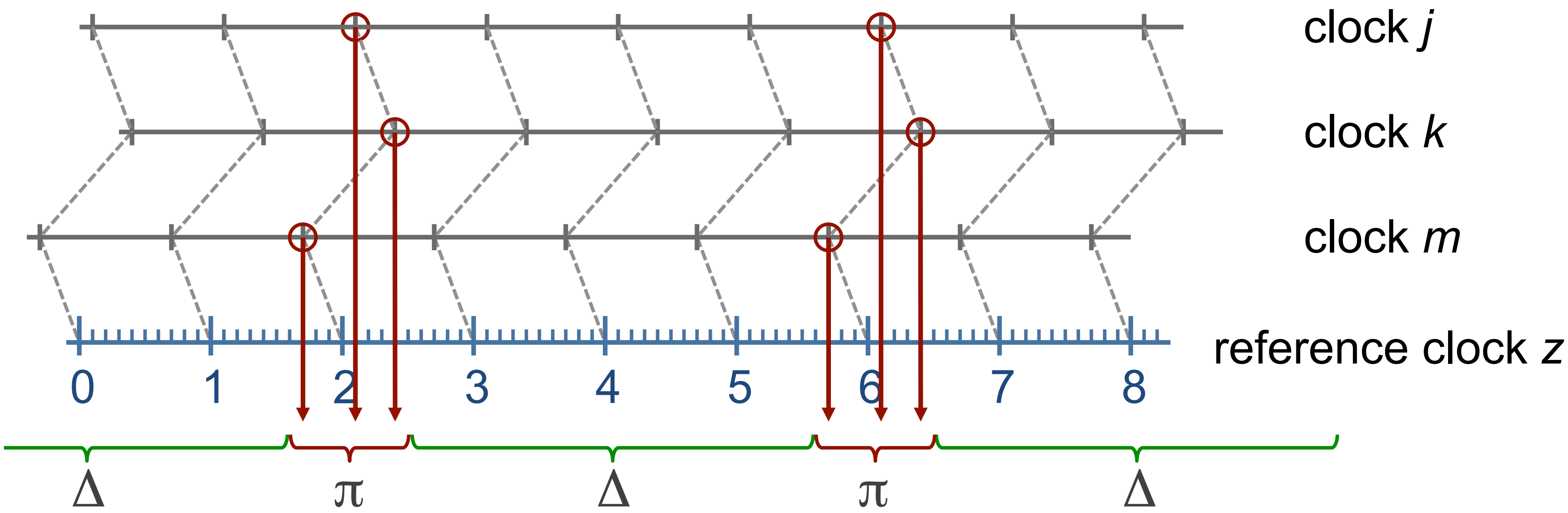
# Temporal Relationship between Generated Events




Assumption: nodes generate events at clock ticks

An external observer cannot reconstruct whether local timestamps of generated events are equal or not

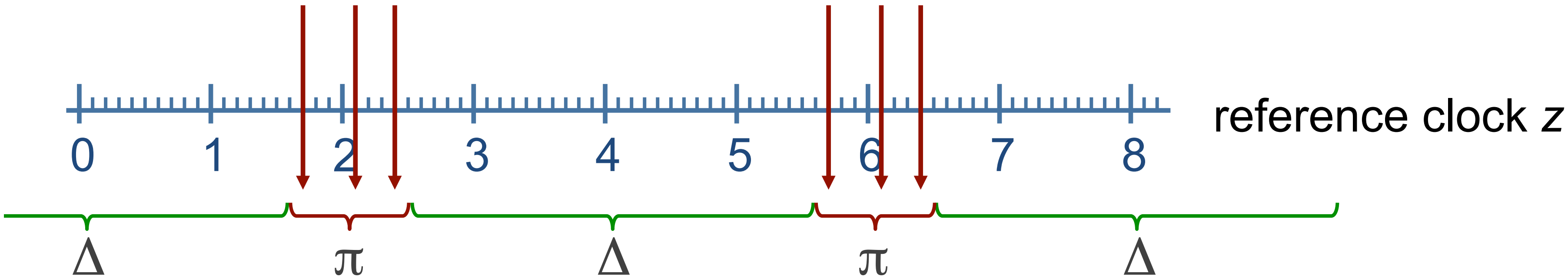
# π/Δ-Precedence of Sets of Events




Given durations $\pi$ and $\Delta$ ($\pi << \Delta$), a set of events $E=\{e_i\}$ is $\pi/\Delta$-precedent, if the following condition holds for all $e_j, e_k \in E$:

$$\big(|\,z(e_j) - z(e_k)\,| \leq \pi\big) \text{ or } \big(|\,z(e_j) - z(e_k)\,| > \Delta\big)$$

# Dense Time and Sparse Time

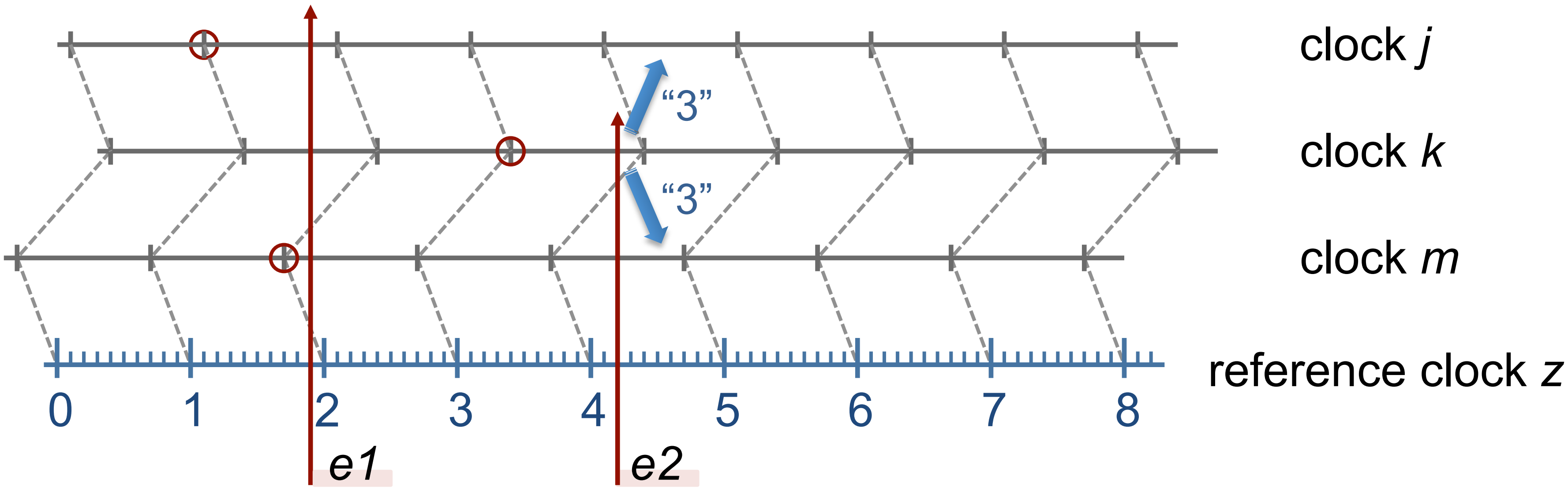



Dense timebase: events are allowed to occur at any time.

Sparse timebase (π/Δ-sparse timebase):

events are only allowed to occur within the time intervals of activity π, followed by an interval of silence Δ.

# Agreement on Observed Events – Dense Time




Nodes *j* and *m* observe *e1*, node *k* observes *e2.*

Node *k* reports observation about *e2* to nodes *j* and *m*.

➪ Nodes *j* and *m* draw different conclusions about event order.

# Agreement on Observed Events – Dense Time

Conclusions from observations made so far:

- If a single event is observed by two nodes, the local timestamps for the event may differ by one tick.

  ⇨ an explicit agreement protocol (communication between the nodes) is needed to establish a consistent view about the global time of the event occurrence.

- If two events occur on a dense timeline, then it is impossible to deduce the temporal order in all cases if the events occur within an interval of duration < $3g^{global}$.

  ⇨ explicit agreement is needed for arbitrary event sets.

# Agreement on Event Order – Sparse Time

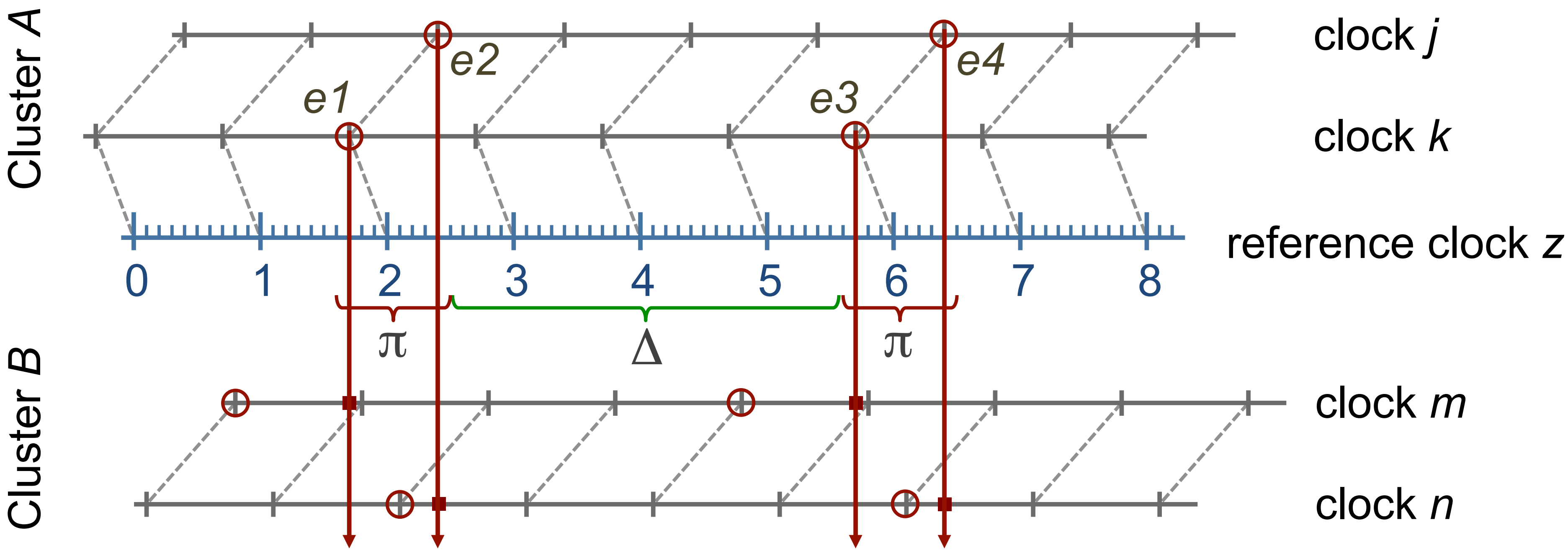
Assume: 2 computation clusters *A*, *B*

- within each cluster clocks are synchronized ($g = g^{global}$)
- no synchronization between *A* and *B*
- Cluster *A* generates events that have to be ordered by *B*:

  *B* must be able to determine order resp. simultaneity of all observed events

⇨ Timebase of *A* has to be 1*g*/4*g*-sparse;
a 1*g*/3*g*-sparse timebase is not sufficient (see next slide)

# Agreement on Event Order – Sparse Time (2)




*e1*, *e2* … generated in same activity interval: $t^n(e2) - t^m(e1) = 2$

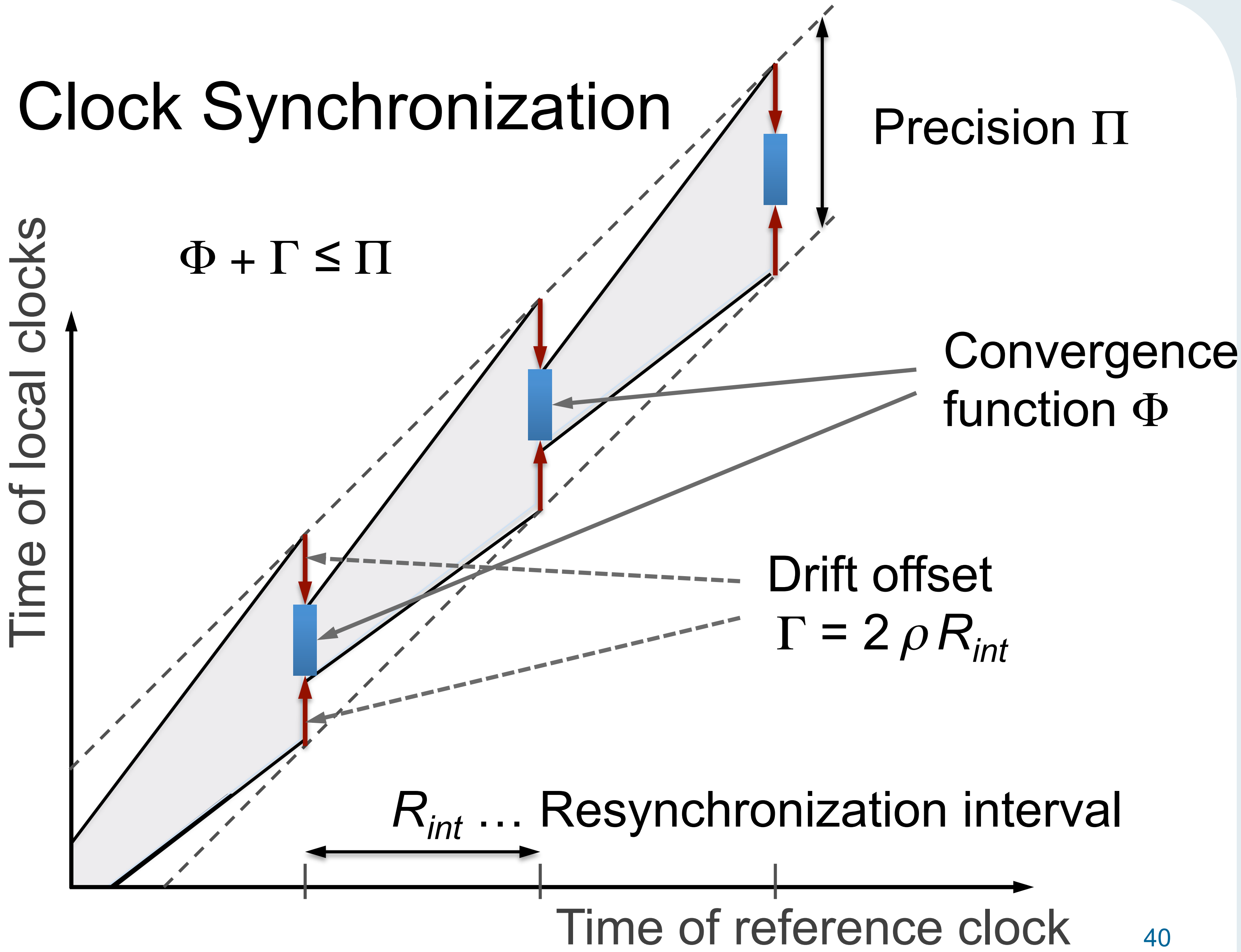
*e2*, *e3* … gen. in different activity interval: $t^m(e3) - t^n(e2) = 2$

# Fundamentals in Time Measurement

Given a distributed system with a reasonable global timebase, with granularity $g^{global}$ :

- If a single event is observed by two nodes, the local timestamps for the event may differ by one tick.
- Duration measurement: $d_{obs} - 2g^{global} < d^{z}_{true} < d_{obs} + 2g^{global}$
- The temporal order of two events $e_1$, $e_2$ can be deduced from their timestamps if $| t^j(e_1) - t^k(e_2) | \geq 2$ .
- The temporal order of events can always be deduced if the event set is $0/3g^{global}$-precedent.

# Internal Clock Synchronization



# Synchronization Condition

To keep the clocks internally synchronized with precision $\Pi$, the synchronization condition must hold:

$$\Phi + \Gamma \leq \Pi$$

$\Phi$ … convergence function: max. offset after synchronization; depends on synchronization algorithm and message latency jitter $\varepsilon$, the transmission-time difference between fastest and slowest message; $\varepsilon = d_{max} - d_{min}$

$\Gamma$ … drift offset: divergence of free-running clocks; $\Gamma = 2\,\rho\,R_{int}$

$R_{int}$ … resynchronization interval

# Central Master Algorithm

- Master node sends periodic synchronization messages, containing its local time
- Slaves adjust local clocks
  - Record local arrival time of sync. message
  - Compute difference master clock – local clock
  - Correct difference by latency (local parameter)
  - Correct local clock
- Precision of Central Master Algorithm

$$\Pi_{central} = \varepsilon + \Gamma$$

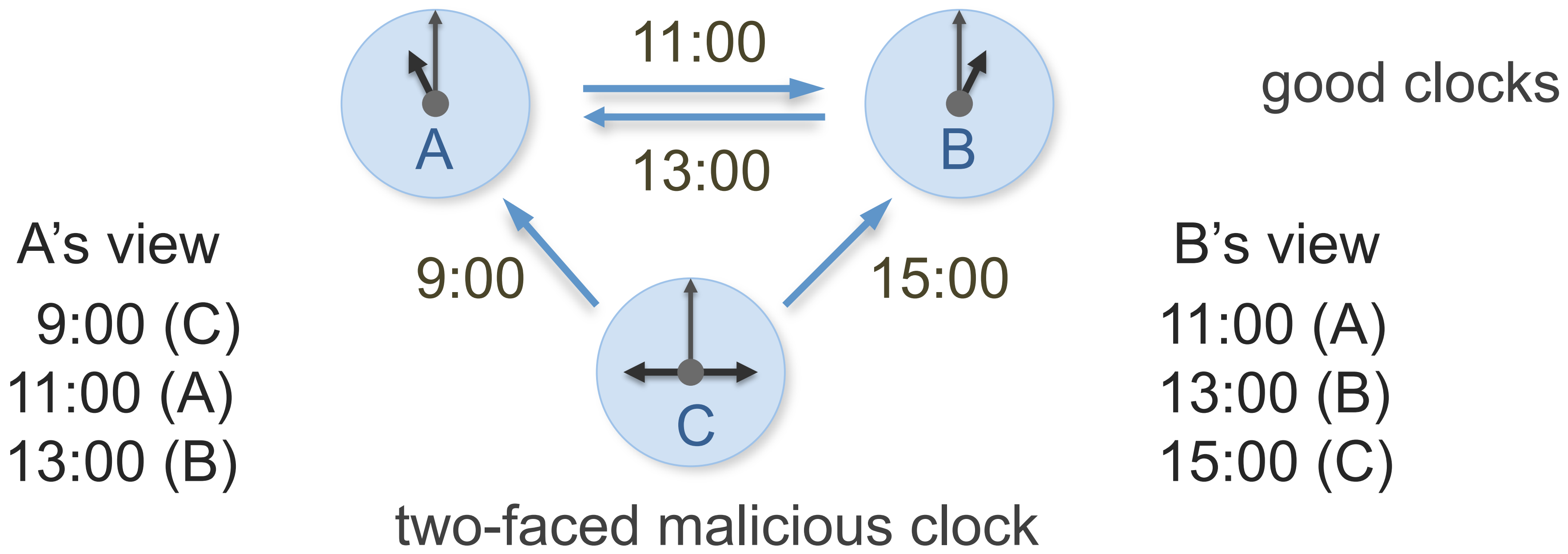
# Distributed Clock Synchronization

Use of distributed algorithms to provide fault tolerance;

Typically three phases:

- Nodes exchange messages and acquire information about global-time counters at other nodes.
- Every node analyzes collected information (error detection) and executes the convergence function to compute a correction term for its local global-time counter
- Every node adjusts its local time counter by its correction term

# Malicious (Byzantine) Clocks

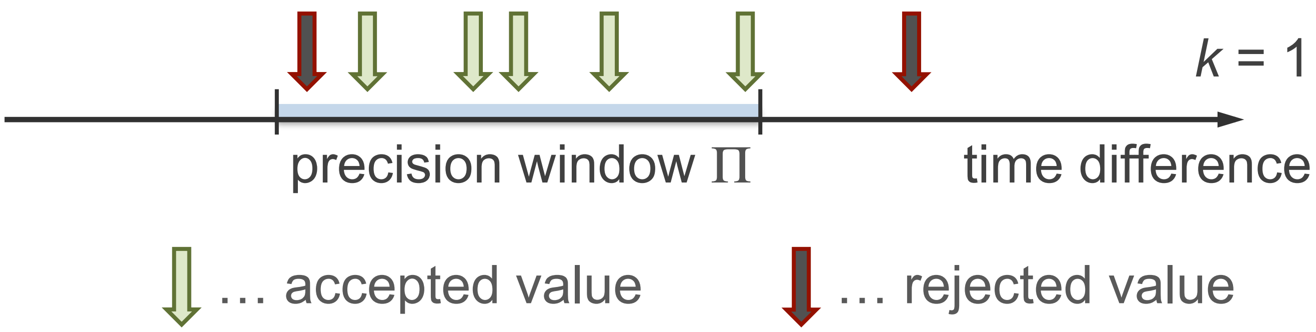



For clock synchronization in the presence of *k* Byzantine clocks the number of clocks *N* must be: $N \geq 3k + 1$

# Fault-Tolerant Average (FTA) Algorithm

Computation of correction term:

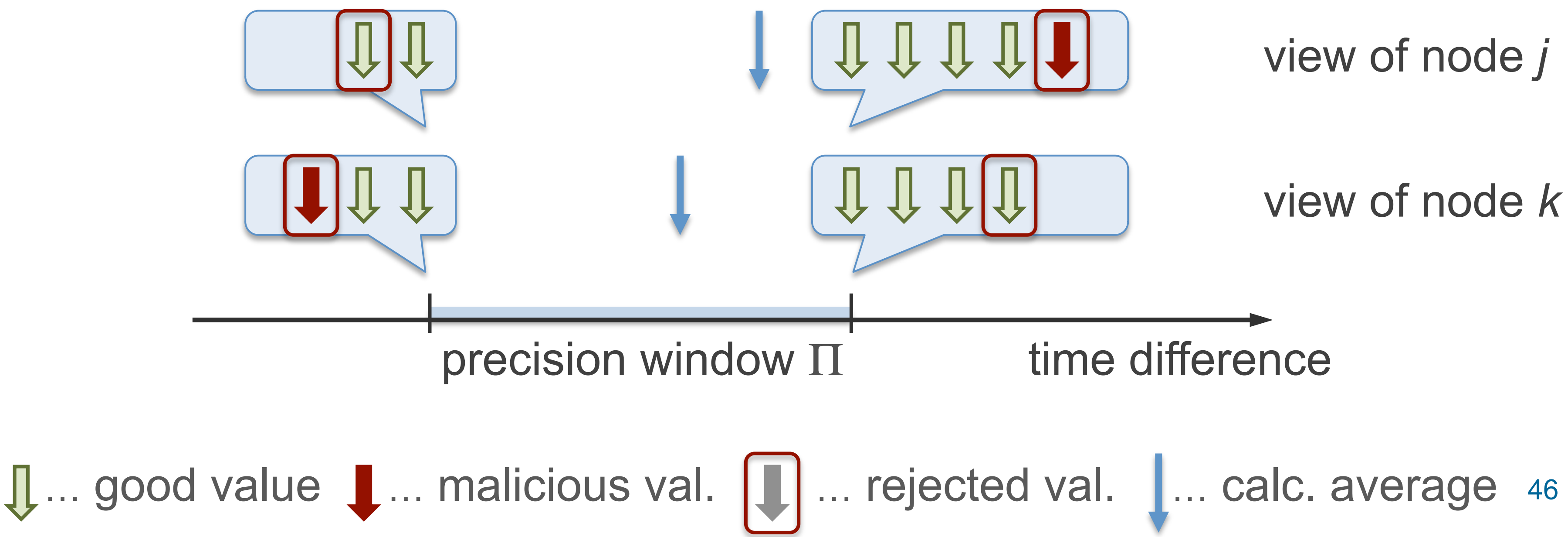
- Calculation of differences between local clock and all other clocks
- Sorting of clock-difference values
- Elimination of *k* smallest and *k* largest values
  (*k* ... max number of erroneous clocks)
- Correction term: average of remaining $N – 2k$ time differences



# FTA Algorithm – Effect of Byzantine Clock

Worst-case effect of a Byzantine node:

- Byzantine time values at different ends of precision window
- Error term of a Byzantine error: $E_{byz} = \Pi / (N - 2k)$



# Precision of the FTA Algorithm

## Convergence Function

$$\Phi(N, k, \varepsilon) = k\,\Pi / (N - 2k) + \varepsilon$$

## Precision

$$\Pi(N, k, \varepsilon, \Gamma) = (\varepsilon + \Gamma)\,\frac{N - 2k}{N - 3k} = (\varepsilon + \Gamma)\,\mu(N, k)$$

$\mu(N, k)$ is called the Byzantine error term

number of nodes $N$

| | $\mu(N, k)$ | 4 | 5 | 6 | 7 | 10 | 15 | 20 |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 1.5 | 1.33 | 1.25 | 1.14 | 1.08 | 1.06 |
| $k$ | 2 | | | | 3 | 1.5 | 1.22 | 1.14 |
| | 3 | | | | | 4 | 1.5 | 1.27 |

# Interactive Consistency Algorithm

Eliminates Byzantine error term

- After collecting the time values of all other clocks, every node sends its view of the clock ensemble to all other clocks
  ➪ extra communication round!
- Nodes have global view; can identify Byzantine nodes
- Correction based on matrix of time vectors of all views
- $\mu(N, k) = 1$

# Limit to Internal Clock Synchronization

Lundelius and Lynch show limits of clock synchronization:

The best achievable precision even with perfect clocks is

$$\Pi_{opt} = \varepsilon \, (1 - 1/N)$$

# Clock-Synchronization Quality Parameters

- Drift offset $\Gamma = 2\,\rho\,R_{int}$
- Delay jitter $\varepsilon = d_{max} - d_{min}$
- Byzantine failures: rare events
- Clock synchronization algorithms: effect on sync. quality is small compared to delay jitter

# Keeping the Drift Offset Small

Minimize relative drift rates of clocks

- Use rate master with precise clock in each cluster
- Adjust rates of local clocks to rate of the master
- Use state correction in FTA
  ➪ mask errors in rate correction of local clocks

# Jitter of Synchronization Messages

Message jitter $\varepsilon$ depends on where message timestamps are inserted and interpreted

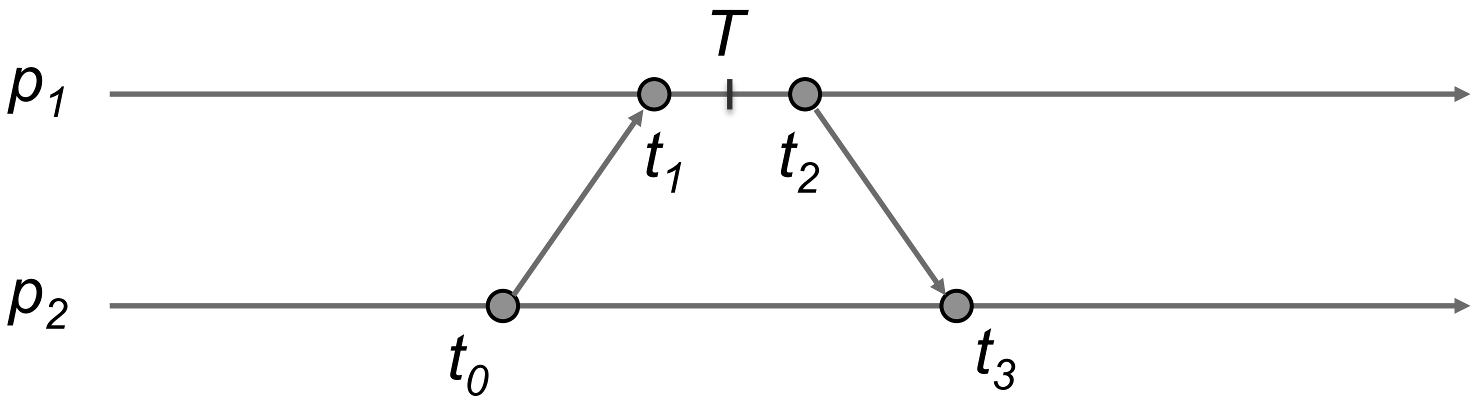
| Message assembly/interpretation | appr. range of jitter |
|---|---|
| Application software level | 500 $\mu s$ … 5 $ms$ |
| Operating system kernel | 10 $\mu s$ … 100 $\mu s$ |
| Hardware: communication controller | < 10 $\mu s$ |

# Quality Attributes of a Global Time Base

- Precision
- Accuracy
- Fault tolerance: number and types of faults the system of clocks can tolerate
- Blackout survivability: blackout duration that can be tolerated without losing synchronism

# Cristian's Algorithm

Request time and evaluate reply




Time-request from $p_2$ to $p_1$ at $t_0$

Reply from $p_1$ arrives at $t_3$: contains $T$, round-trip time $d = t_3 - t_0$

Clock sync: $p_2$ sets local time to $T + d / 2$

Clock sync. error $\leq d / 2$

# Network Time Protocol (NTP)

- Built on idea of Christian's algorithm
- Hierarchy of time servers
  - Class 1: connected to atomic clocks, GPS clocks
  - Class 2: receive time from Class 1 servers, synchronize with other Class 2 devices
  - Class 3: receive time from Class 2 servers, …
- Clock correction based on statistical analysis of $t_0$ … $t_3$ of multiple clock readings

Precision Time Protocol (PTP) builds on NTP; PTP uses hardware support for clock synchronization

# External Clock Synchronization

Synchronize clock ensemble to an external time reference

Example: GPS, achievable accuracy below 1μs

Complementary properties of internal/external synchronization:

- Internal clock synchronization:
  high availability, good short-time stability
- External clock synchronization:
  long-term stability, possibly lower availability

Promising combination:
gateway to external time reference = rate master for internal synchronization

# Lessons Learned

- Why we need time …
- Temporal and causal order
- Logical time (Lamport time, vector time)
- Event, duration
- Global time and clocks
- Internal clock synchronization
- External clock synchronization