Adaptives:

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Abstract

1 Introduction

2 Definition and first examples

- Collectives
- Multi-collectives
- Dynamical systems
- Multi-categories
- Initial and terminal

2.1 PROPS

Definition 2.1. A *PROP* P consists of a set of objects $\mathrm{Ob}(P)$ and a symmetric strict monoidal category whose monoid of objects is freely generated by $\mathrm{Ob}(P)$. A morphism of PROPs is a strict monoidal functor that preserves generating objects. ◊

While it is technically convenient to regard a PROP as a monoidal category, it can also be thought of as a structure similar to a polycategory, with a set of objects $\mathrm{Ob}(P)$ and sets of many-to-many morphisms $\mathrm{Hom}(x_1, ..., x_m; x'_1, ..., x'_n)$ equipped with composition, units, and permutations of the domain and codomain objects. These many-to-many morphisms correspond to $\mathrm{Hom}(x_1 \otimes \cdots \otimes x_m, x'_1 \otimes \cdots \otimes x'_n)$ in the corresponding monoidal category.