1. (a) Load the `PlantGrowth` data set in `R` with the command

   ```
   data("PlantGrowth")
   ```

   You can read about the data set with the command

   ```
   ?PlantGrowth
   ```

   (b) Find the mean of the three treatment groups using the `aggregate` function.

   (c) Similarly, find the sample standard deviation of the three treatment groups.

   (d) Write a function in `R` to perform a two-sample t-test to test whether the means from group 1 and group 2 are significantly different under the assumption of equal variance. Use the following function definition:

   ```
   my.ttest <- function(grp1, grp2, alpha = 0.05) {
   # Code to perform calculations goes here...
   ...
   # Return these values
   return(data.frame(t.diff, P, df)
   }
   ```

   (e) Use your t-test function to test whether the plants in treatment group 1 had significantly different growth on average compared to the plants in the control group.

   (f) Compare the results from the previous part to the results given by the `t.test` function with the argument $var.equal = TRUE$.

   (g) Plot the reference distribution and indicate the value of the test statistic on the plot.

   (h) State the conclusion of the hypothesis test in context.

2. Consider a random sample $x_1, x_2, \cdots, x_n \overset{iid}{\sim} Unif(0, \theta)$.

   (a) Find the method of moments estimator of θ, call it $\hat{\theta}_{MoM}$.

   (b) Show that $\hat{\theta}_{MoM}$ is unbiased.

   (c) What is the asymptotic distribution of the $\hat{\theta}_{MoM}$?

   (d) Find the likelihood function $\mathcal{L}(\theta|\mathbf{x})$.

   (e) Find the maximum likelihood estimator of θ, call it $\hat{\theta}_{MLE}$.

   (f) Use `R` to generate a random sample $x_1, x_2, \cdots, x_{20}$ from $Unif(0, 10)$. Use `set.seed(1234)` before you generate your random sample. Plot the likelihood function. Calculate the $\hat{\theta}_{MLE}$ for this sample. Indicate this value on the plot along with the true value $\theta = 10$.

   (g) Generate $B = 1000$ samples each of size $n = 20$ from a $Unif(0, 1)$ distribution. Plot the empirical sampling distribution of $\hat{\theta}_{MoM}$.

   (h) The MLE $\hat{\theta}_{MLE}$ is biased. Write an `R` simulation to approximate the bias of $\hat{\theta}_{MLE}$ for a sample of size $n = 20$. Use $B = 1000$ samples.

   (i) Use the simulation to estimate the variance of $\hat{\theta}_{MoM}$ and the variance of $\hat{\theta}_{MLE}$.

   (j) The *Mean-squared Error* (MSE) of an estimator $\tilde{\theta}$ of parameter θ is defined as

   $$MSE_{\theta}(\tilde{\theta}) = Bias^2_{\theta}(\tilde{\theta}) + Var_{\theta}(\tilde{\theta}).$$

   Approximate $MSE_{\theta=1}(\hat{\theta}_{MoM})$ and $MSE_{\theta=1}(\hat{\theta}_{MLE})$ using your `R` simulation.