1. Consider random variables X and Y with joint pdf $f(x,y) = C(x+2y), 0 < x < 2, 0 < y < 1$.

   (a) Find C so that f is a valid pdf.

   (b) Find the marginal pdf of X, $f_X(x)$.

   (c) Find the marginal pdf of Y, $f_Y(y)$.

   (d) Find the conditional pdf $f_{Y|X}(y|x)$.

   (e) Find the probability that $P(0 < X < 1, 0 < Y < 2)$.

   (f) Find $\mathbb{E}X$.

   (g) Find $\mathbb{E}Y$.

   (h) Find $\mathbb{E}XY$.

   (i) Find $Cov(X,Y)$.

2. Suppose $X_1, X_2, \cdots, X_{12} \overset{iid}{\sim} Pois(3)$. Use the Central Limit Theorem to approximate $P(\bar{X} > 4)$.

3. Let X_1, X_2 be iid from a continuous distribution. Prove that $P(X_1 < X_2) = \frac{1}{2}$.

4. Consider the sequence of random variables $\{\bar{X}_n = \frac{1}{n}X_n\}, n \in \mathbb{N}$, where $X_n \sim Binom(n,p)$.

   (a) Why can the CLT be applied to this sequence? What is the limiting distribution of the sequence?

   (b) What is the approximate distribution of the sample odds: $\frac{\bar{X}_n}{1-\bar{X}_n}$?

   (c) What is the approximate distribution of the log odds: $\log\left(\frac{\bar{X}_n}{1-\bar{X}_n}\right)$?

   Note: The population *log odds*, equal to $\log(\frac{p}{1-p}) = \text{logit}(p)$, plays a central role in logistic regression and is an important quantity in many epidemiological studies.

5. Suppose $x_1, x_2, \cdots, x_n$ be a random sample from $Exp(\lambda)$.

   (a) Using the CLT, describe the asymptotic distribution of $\bar{x}_n$?

   (b) Use the Delta Method to find the asymptotic distribution of $\frac{1}{\bar{x}}$.

6. One way to write Chebychev's Inequality is

$$P(|\bar{X}_n - \mu| \geq k\sigma) \leq \frac{1}{nk^2}.$$

   Consider a sample $X_1, X_2, \cdots, X_n \overset{iid}{\sim} N(0,1)$.

   (a) What is the distribution of $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$?

   (b) If $n = 9$, what is the Chebychev bound for the probability that the sample is more than 2 standard deviations from the true population mean?

   (c) How do you think the Chebychev bound compares to the true probability $P(|\bar{X}_n| > 2)$ (using the distribution you identified in (a) above)?

7. Suppose $\bar{X}$ is the mean of 100 observations from a population with mean μ and variance $\sigma^2 = 9$. Find limits between which $\bar{X} - \mu$ will lie with probability at least .90. Use both Chebychev's Inequality and the Central Limit Theorem and comment on each.