STAT 9120: Homework 1

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Assigned: 2024-08-22

Due September 1st by 11:59 PM *Submit completed assignment on D2L.*

1. *Definitions* Write the definition for each of the listed terms.

a. Convergence in Distribution
b. Convergence in Probability
c. Eigenvalue
d. Idempotent matrix
e. Projection matrix
f. Positive Defnite matrix

2. State Slutsky's theorem.

3. State the Weak Law of Large Numbers.

4. State the Central Limit Theorem.

5. State the Mann-Wald (AKA Continuous Mapping) Theorem

6. State Taylor's Remainder Theorem.

7. Prove $\frac{\mathcal{U}^2(\theta)}{\mathcal{I}\theta} \xrightarrow{D} \chi^2_1$ for $\theta \in \mathbb{R}$. (Hint: Recall, for $\mathcal{U}_i \overset{iid}{\sim} \mathcal{N}(0, \mathcal{I}(\theta)$, we have $\frac{1}{\sqrt{n}} \sum_{i=1}^n \mathcal{U}_i(\theta) \xrightarrow{D} \mathcal{N}(0, \mathcal{I}(\theta))$ by the CLT. Apply Mann-Wald).

8. Prove $2[\ell(\hat{\theta}_n) - \ell(\theta)] \xrightarrow{D} \chi^2_1$ for $\theta \in \mathbb{R}$. (Hint: Second order Taylor expansion of $\ell(\theta)$ about $\ell(\hat{\theta}_n)$).)

9. a. For

$$A = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}, x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

compute $Q(x) = x^T A x$.

b. Convert your answer from part (a) to "standard form": $Q(x) = g(x_1 - x_2) + h(x_1 x_2)$ for some polynomial functions g and h.

10. For X and $n \times p$ full rank matrix, $n > p$, show that $H = X(X^T X)^{-1} X^T$ is a projection matrix.